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HUMAN POWER PRODUCTION IN A CAGED SITUATION

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HUMAN POWER PRODUCTION IN A CAGED SITUATION

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Abstract

Mechanical efficiencies are calculated for a human doing work in a standing and stooping cycle while enclosed in a cage. An unsteady force is generated which does useful work in oscillating the cage on its suspension system. Such a vertical pumping motion has been proposed for a man-powered ornithopter. The theorem of virtual work provides the efficiency expression. Analog simulation reveals that square wave force exitation is more efficient than sinusoidal or triangular. Design curves show some unexpected requirements for matching man and machine, and very poor efficiency if care is not taken. Losses are due to gravity and human inability to store energy in unloading portions of the cycle. A spring-dashpot suspension allows efficiencies of up to 88% in cases involving sinusoidal exitation. A freely-floating suspension (the flight situation) allows only 64% efficiency for harmonic exitation. Some improvement can be made by adding toe straps to the human and/or by forcing the cage in a square wave. The novel feature, making this work differ from ordinary vibration work, is the "switching" logic needed to distinguish loading and unloading portions of the cycle.

Symbols

B	half amplitude of nondimensional leg extension
c	equivalent viscous damping coefficient of suspension
F	half amplitude of leg force
$\mathcal{F}$	external force on cage
f(t)	dimensionless periodic function
k	spring constant of suspension
M	cage mass
m	human mass
P	power
S	half amplitude of stroke
T	period of oscillation
W	work
x(t)	cage position
y(t)	human c.g. position
z(t)	arbitrary function
α	phase lag, Equation 19
δW	virtual work
δW_{human}	internal work by human during virtual displacement
ϵ	biological penalty factor
ζ	damping ratio, $2\sqrt{kM}$
η	efficiency
ϕ	phase lag, Equation 12
ω	circular frequency
ω_n	natural frequency of cage alone $\sqrt{k/M}$

Introduction

Humans use rotary and reciprocating cycles of motion in many work and play situations including rowing boats, riding bicycles and bouncing on trampolines. Power production and efficiency of some kinds of motion have been carefully studied, especially the rotary motion used in pedalling a bicycle or flying a man-powered plane and particularly in the British literature.(1)(2) In 1972, Grant Smith proposed a rigid wing ornithopter propelled by a pilot moving in a standing and stooping cycle within the fuselage.(3) It was believed that this is a highly efficient method of transmitting energy to the fuselage and then to the airstream. The question of how efficiently energy can in fact be transmitted from pilot to fuselage in such a caged situation is by no means trivial, however, and may be the critical part of the power cycle.

The purpose of this paper is to study the efficiency and power generation of a "caged" human being moving in a vertical, closed cycle of motion. The direct application is to man-powered flight, but a general approach will be taken in terms of the suspension of the cage so that other man/cage situations can be covered.

The human is enclosed in a rigid cage and isolated from the outside world (Figure 1). The cage position x(t) and the human's c.g. y(t) are measured from an inertial frame. The external force on the cage would be found from the aircraft's stability derivatives or from a given suspension system.

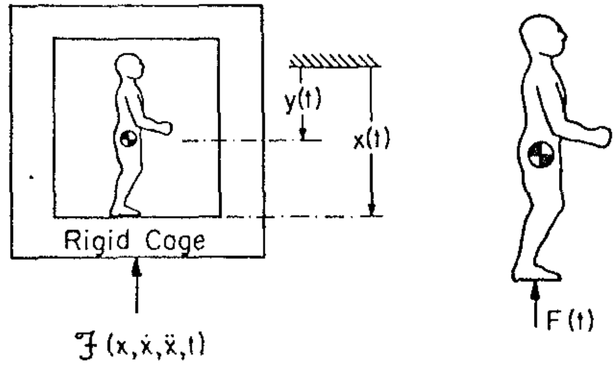

Figure 1. Man in Caged Situation

The novel thing making this problem different from a standard vibration problem is that the human does work in a irreversible way. He cannot absorb work on the portions of the cycle where the cage and gravity do work on him (the so called "unloading" portion). The cage and gravity can accelerate the human and create kinetic energy, but any portion of a leg stroke where cage forces and gravity tend to further this stroke must release energy in heat. One must be careful in analyzing the cycle, then, to not consider this component of work as useful mechanical energy.

The role of gravity is important in this study. Gravity can cause a loss of efficiency because it creates a dead weight which must be cycled through all motion. Gravity never does any net work over a closed cycle, but provides a bias force which does affect the division of the cycle into loading and unloading portions. Physiologists and engineers have realized that gravity represents a loss term in human motion, such as running a race, and can account for the portion of energy lost when a vertical cyclic motion is performed on a hard surface.(1) Gravity's role in the man/cage combination is more subtle, but still must be regarded as the chief culprit in the energy loss.

Equations of Motion

Assume that the mass of the human is concentrated at his c.g., and that the loading passes entirely through his legs. (The analysis is also valid if hand supports are available, but is conceptually easier without them.) The motion studied here is periodic because best measures of energy production and efficiency of operation are derived from a closed cycle of motion.

Forces due to cage suspension, whether mechanical or an aerodynamic equivalent for unsteady motion, will be $\mathcal{F}(x, \dot{x}, \ddot{x}) = c\dot{x} + kx$. Any apparent mass effects due to fluid forces on the cage would be included in the cage inertial term $M\ddot{x}$. The force $F(t)$ in the human leg is defined positive in compression and has a gravitational bias $F(t) = F\,f(t) + mg$, where $f(t)$ is periodic with half amplitude of unity and F is the measure of force amplitude. The coordinates x and y are defined so that

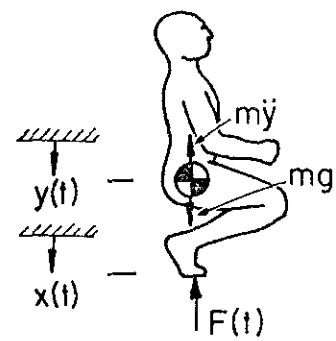

Figure 2. Equilibrium of Human

$x - y = 0$ when the human is half crouched. The equations of motion are simply

$$m\ddot{y} = -F\,f(t) \qquad (1)$$

$$M\ddot{x} + c\dot{x} + kx = F\,f(t) + (M+m)\,g \qquad (2)$$

The steady state, oscillatory solutions to these equations are easily found.

Efficiency

The system efficiency will be at first defined in purely mechanical terms. Efficiency is the ratio of useful mechanical work done by the human on the cage to the total mechanical work done by the human. Work is defined in reference to the inertial frame.

The useful work done by the man on the cage per cycle is

$$W_{useful} = \int_0^T F(t)\,\dot{x}(t)\,dt \qquad (3)$$

One might be tempted to include as useful work that which the man does against gravity, on the grounds that it will be returned to the cage at some later time in the cycle. This is fallacious because all work done by the man on the cage must be accomplished through his legs (his only point of contact) and is hence already included in Equation 3.

The total mechanical work done by the man can best be seen by applying the principle of virtual work and D'Alembert's principle. Consider the human, acting upon by gravity and the cage, during a virtual displacement.

$$\delta W = \delta W_{human} + mg\,\delta y - m\ddot{y}\,\delta y - F(t)\,\delta x = 0 \quad (4)$$

The internal work δW_{human} must be supplied to maintain the energy balance at any instant. It does not include any wasted energy (e.g., blood circulation, flapping arms) that does not contribute to vertical equilibrium. Adding force equilibrium, $F(t) = mg - m\ddot{y}$, one obtains

$$\delta W_{human} = (mg - m\ddot{y})\ (\delta x - \delta y) \qquad (5)$$

In retrospect, this energy argument gives the correct results, since the work increment (as seen by the human) is logically the force in his legs times the extension of his legs. Note that this work is done against gravity as well as against the cage.

There is a problem of interpretation if the increment of work in Equation 5 is negative. The loading and unloading portions of the cycle depend on the signs of the factors:

$(mg - m\ddot{y})$	$(\delta x - \delta y)$	case
+	+	loading
+	-	unloading
-	+	unloading
-	-	loading

The latter two cases correspond to tension in the legs and are obtainable only if straps hold the feet to the floor. In purely mechanical terms, there is no penalty to the system during the unloading cycle, since the human does no mechanical work in rejecting the heat. When comparing this reciprocating cycle with a rotary pedalling cycle (which usually has no unloading stroke), one might want to include a biological penalty factor for making the human pass through an unloading portion of a cycle. This could be done by defining the total human work on a cycle as

$$W_{human} = \int_0^T \left\{ (mg - m\ddot{y})\ (\dot{x} - \dot{y}) \right\} dt \qquad (6)$$

where the curly brackets are defined for an arbitrary function:

$$\left\{ z(t) \right\} \equiv \left\{ \begin{array}{lll} z(t) & \text{when} & z(t) \geq 0 \\ \epsilon\ |z(t)| & \text{when} & z(t) < 0 \end{array} \right\} \qquad (7)$$

A choice of biological factor $\epsilon = 0.33$, say, would then allow a fairer comparison between the current cycle and a rotary cycle. In the long run, however, human experiments should be run on the vertical reciprocating cycle to find the true physiological penalty for unloading. This would reduce, for instance, the useful long time horsepower output of a human, which is approximately 0.45 for a rotary cycle.[1]

The efficiency is now obtained by the ratio of useful to total work

$$\eta = \frac{\int_0^T F(t)\ \dot{x}\ dt}{\int_0^T \left\{ (mg - m\ddot{y})\ (\dot{x} - \dot{y}) \right\} dt} \qquad (8)$$

This expression is evaluated analytically for sine wave forcing and by analog simulation for sine, square and triangle forcing.

Analysis of Spring-Dashpot System

Consider the system of Figure 1 where both spring and dashpot external forces act. A harmonic forcing function $f(t) = \sin \omega t$ is chosen. The oscillatory, steady state solutions of Equations 1 and 2 are desired. Transients and static deflections are neglected. The solution is simply

$$x(t) = X \sin (\omega t - \phi) \qquad (9)$$

and

$$y(t) = Y \sin \omega t \qquad (10)$$

where

$$\frac{Xk}{F} = \frac{1}{\sqrt{[1 - (\omega/\omega_n)^2]^2 + (2\zeta\omega/\omega_n)^2}} \qquad (11)$$

$$\phi = \tan^{-1} \frac{2\zeta\omega/\omega_n}{1 - (\omega/\omega_n)^2} \qquad 0 \leq \phi \leq 180^\circ \quad (12)$$

$$\frac{Y m \omega^2}{F} = 1 \qquad (13)$$

$$\frac{\omega}{\omega_n} \equiv \sqrt{\frac{k}{M}} \qquad (14)$$

$$\zeta \equiv \frac{c}{2\sqrt{kM}} \qquad (15)$$

Note that the cage motion $x(t)$ lags the force $f(t)$ by angle ϕ and the human c.g. $y(t)$ is in phase with the force (by virtue of the opposing sign conventions chosen for $F(t)$ and $y(t)$).

The velocity difference $\dot{x} - \dot{y}$ appears in the expression for work done by the human. A non-dimensional velocity difference is defined

$$\frac{\dot{x} - \dot{y}}{X\omega} \equiv B \cos (\omega t - \alpha) \qquad (16)$$

which implies

$$\frac{x - y}{X} = B \sin (\omega t - \alpha) \qquad (17)$$

and α is seen as the phase angle by which the leg expansion $x - y$ lags the force. This angle plays a very important role in the efficiency results. From Equations 8-12, one finds

$$B = \sqrt{\sin^2\phi + (\cos\phi - Y/X)^2} \qquad (18)$$

$$\alpha = \tan^{-1} \frac{\sin\phi}{\cos\phi - Y/X} \qquad 0 \leq \alpha \leq 180^\circ \qquad (19)$$

$$Y/X = (\omega_n/\omega)^2 (M/m)\sqrt{[1 - (\omega/\omega_n)^2]^2 + (2\zeta\omega/\omega_n)^2} \quad (20)$$

Using the dynamic results above, one calculates the useful work per cycle

$$(W_{useful}/FX) = \pi \sin\phi \qquad (21)$$

The total human work per cycle requires some algebraic work. By limiting the forcing to $F/mg \leq 1.0$, however, the cycle consists of only one loading and one unloading portion and the integration is tractable. The case $F/mg \geq 1.0$ will not be attempted analytically.

$$(W_{human}/mg\,X) = 2(1+\epsilon)B + \frac{\pi}{2}(1-\epsilon)B(F/mg)\sin\alpha \quad (22)$$

The sinusoidal cycle efficiency for $F/mg \leq 1.0$ is hence

$$\eta = \frac{W_{useful}}{W_{human}} \quad (23)$$

$$= \frac{\pi\,(F/mg)\,\sin\phi}{B\left[2(1+\epsilon) + \frac{\pi}{2}(1-\epsilon)(F/mg)\sin\alpha\right]} \quad (24)$$

which can be viewed as

$$\eta = \text{function}\,(\omega/\omega_n,\ \zeta\ ,\ F/mg\ ,\ M/m\ ,\ \epsilon\ ,\ f(t)\,) \quad (25)$$

The problem has been solved to this point in terms of dimensionless ratios and needs to be discussed in those terms. On the other hand, we must scale the problem to human dimensions to see which ranges of efficiency can be reached. Many operating conditions are inaccessible because of human power or leg stroke limitations. The useful power developed by the human is

$$P_{useful} = \frac{1}{2}\,F\,X\,\omega\,\sin\phi \quad (26)$$

and the half stroke S is defined in Equation 17.

$$S = X\,B \quad (27)$$

Analog Simulation

The analog computations were performed on the Applied Dynamics/Four Analog-Hybrid computer located in The University of Michigan Simulation Research Center. Generally, conventional programming techniques were employed. The equations were written and programmed in terms of dimensionless variables by normalizing with respect to either mg or Mg. In addition, time scaling was utilized to run the problem in either 100 or 1000 times real time. In principle, the differential equation for the human requires two open-ended integrations to compute the c.g. position. In practice, this kind of programming is very sensitive to any amplifier drift or D.C. offset voltages. Hence very light damping and spring forces were incorporated to stabilize the position of the human. The values of the damping and spring constants employed were approximately .1 lb/ft/sec and .1 lb/ft, respectively.

The wavy bracketed function in the denominator of the efficiency expression was programmed using diode networks which were both biased to compensate for diode break-over voltage and calibrated to determine and correct for diode conduction resistances. The integrations in the efficiency quantity were computed by integrating over a time much longer than a cycle, rather than integrating over a single closed cycle. Generally, the period of integration was greater than two hundred times the characteristic period of oscillation of the system. Also, all transients were allowed to decay out before any efficiency measurements were performed.

For those analog results which can be directly compared with digital calculations ($f(t) = \sin\omega t$, $F/mg \leq 1.0$), agreement within 2% was typically found. Thus the analog results can be considered as a check on the digital calculations as well as a means of extending f(t) to nonharmonic cases.

Numerical Results for the Spring-Dashpot System

In spite of the simplicity of the dynamical system, the efficiency expression, Equation 25, shows complex dependence on some of its parameters, notable ω/ω_n and ζ. Apparently the "switching" behavior in the denominator of the efficiency expression causes the interesting results.

A. Reference Case

Results will be presented in detail for a reference, or baseline, case. This case will bring out the complexity of the ω/ω_n and ζ dependence. The more moderate effects of F/mg, M/m, ϵ and f(t) will be discussed later. The reference case is

$$\eta = \text{function}\,(\omega/\omega_n,\ \zeta\ ,\ 1.0,\ 0.3333,\ 0.\ ,\ \sin\omega t) \quad (26)$$

The force ratio F/mg is unity, which is the highest value possible without causing negative g forces on the pilot and requiring toe straps. The mass ratio M/m = 0.3333 is chosen as a typical for a 150 lb man flying a 50 lb ultralight aircraft. The biological penalty factor is set at zero, concentrating attention on the mechanical aspects of the problem.

Figure 3 is a contour chart showing constant elevation lines of efficiency. The maximum efficiency reached for this baseline case is 88%, occurring on a horseshoe shaped curve. For a given cage/man configuration, at low damping, there are two frequencies for which efficiency is maximum. The lower intercept of this curve on the ordinate corresponds to resonance of the cage and man locked together and moving in phase on the spring support at $\omega = \sqrt{k/M+m}$. The upper intercept is the resonance of the cage alone on the support at $\omega = \sqrt{k/M}$. (Later calculations will show that the higher branch of the horseshoe is not physically obtainable for a human in a ultralight aircraft.) The phase angle α is 90° at all points on this optimum efficiency horseshoe, and this appears to be the general criterion for maximum efficiency. Elevation contours for less than .50 effiency are not given in this figure.

To study the reference case further, cuts of the efficiency surface will be made parallel to the frequency and damping axes. Figures 4 and 5 are

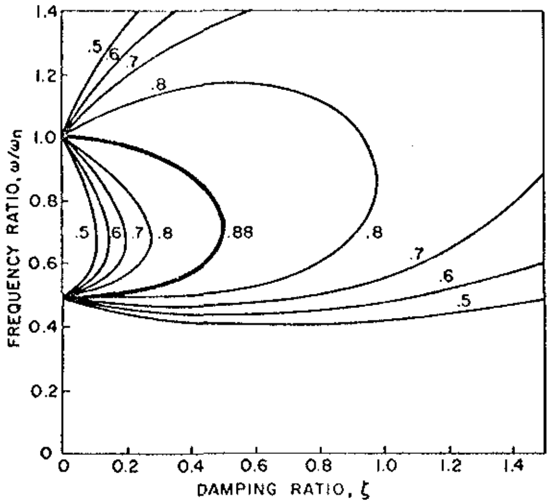

Figure 3. Efficiency Contours for Sinusoidal Loading. ($\epsilon = 0$, $F/mg = 1.0$, $M/m = 0.3333$). Contours omitted for $\eta < .50$.

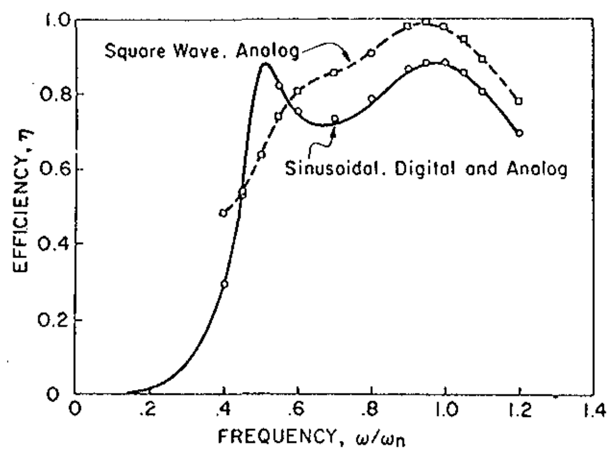

Figure 4. Frequency Sweep. $\zeta = 0.2$, $F/mg = 1.0$, $\epsilon = 0$, $M/m = 0.3333$

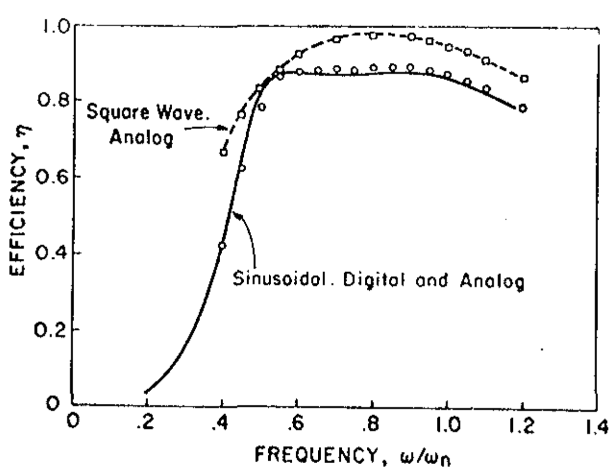

Figure 5. Frequency Sweep. $\zeta = 0.4$, $F/mg = 1.0$, $\epsilon = 0$, $M/m = 0.3333$

frequency "sweeps" and contain some analog data (the points) as well as analytical results. The analog data confirms the nature of the horseshoe curve for the sine wave exitation. One also sees that the square wave excitation is more efficient over most of the operating range. The case of $\zeta = 0.4$ in Figure 5 shows a rather high, broad plateau of efficiency for sinusoidal exitation which would provide desirable operating characteristics. Figure 6 gives various cuts of the efficiency surface parallel to the damping axis. The value of η is discontinuous at $\omega/\omega_n = 1.0$ and $\zeta = 0$. This point is not physically accessible because infinite power is needed to operate there.

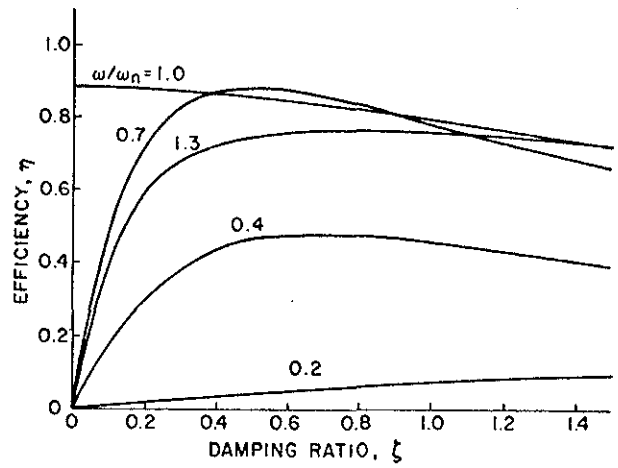

Figure 6. Typical Variation of Efficiency with Damping. ($F/mg = 1.0$, $\epsilon = 0$, $M/m = 0.3333$, Sine Wave Force).

B. General Case

Having developed some intuition about the efficiency surface for the reference case, one can now do parameter studies. Figure 7 indicates that

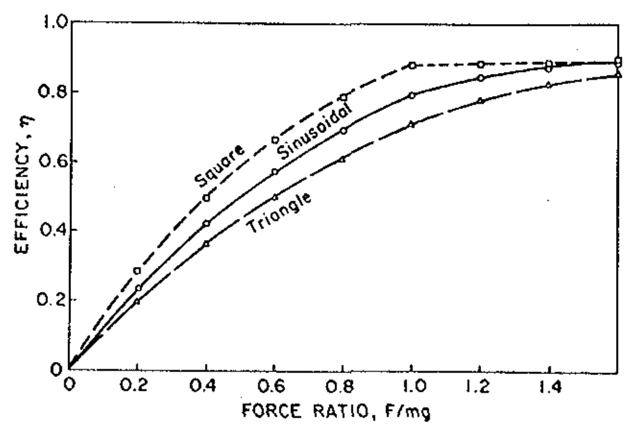

Figure 7. Comparison of Sinusoidal, Square, and Triangular Forcing Functions. ($\epsilon = 0$, $\zeta = 1.0$, $\omega/\omega_n = 1$, $M/m = 0.3333$). Analog Computer Results.

...wave is the most efficient forcing func- ...th sine wave next and triangular wave least ...t. This seems to be the general rule, with ...inor exceptions where the sine and square ...everse positions of supremacy. Figure 7 ...hows that large F/mg causes high efficiency. ...means that as the unsteady force in the ...'s legs tends to dominate the gravitational ... the cycle becomes more efficient. Gravity ... be viewed as the loss factor. If this cyclic ...ss were done in a horizontal plane, one would ... $F/mg \to \infty$ with $\eta \to 1.0$, at least for the sine ...ve. (The square wave may not reach this limit.)

The biological penalty factor ϵ is studied in ...gure 8. It has an adverse affect on sine, square ...d triangular (not shown) cases, with a monotonic ...decrease in efficiency as ϵ is raised.

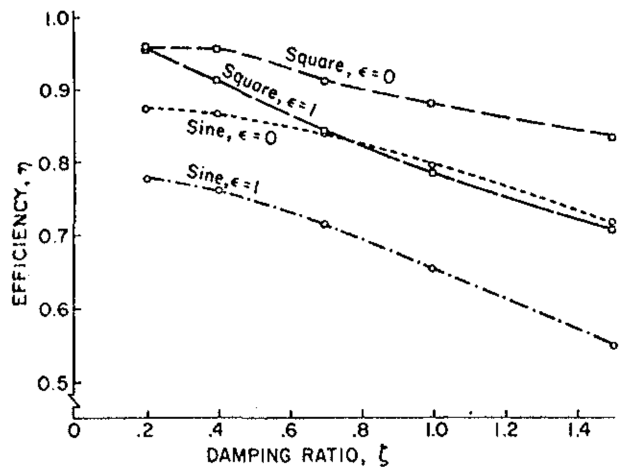

Figure 8. Effect of Penalty Factor ϵ on Square and Sine Wave Cases. $\omega/\omega_n = 1.0$, $M/m = 0.3333$, $F/mg = 1.0$. Analog Data.

The remaining parameter is mass ratio M/m. It will be studied through its effect on the optimum efficiency contour, in the next section.

C. Optimum "Horseshoe"

Digital results have shown optimum efficiency to occur for motion where $\alpha = 90^\circ$. Since

$$\alpha = \tan^{-1} \frac{\sin \phi}{\cos\phi - Y/X}$$

this occurs at

$$\cos \phi = Y/X \tag{27}$$

and the maximum efficiency reached is

$$\eta_{max} = \frac{\pi\ (F/mg)}{2\,(1+\epsilon) + (1-\epsilon)\,\frac{\pi}{2}\,\frac{F}{mg}} \tag{28}$$

This η_{max} is not a function of ω/ω_n, ζ or M/m. For example, the value of η_{max} for the reference case (Figure 3) is

$$\eta_{max} = \frac{\pi}{2 + \pi/2} = 0.7988 \tag{29}$$

It was earlier found that the optimum value of α, and hence η_{max}, was obtained on the horseshoe shaped curve in Figure 3. If the mass ratio is changed, the same $\eta_{max} = 0.7988$ is obtainable, but on a different frequency horseshoe. Figure 9 consists of a family of horseshoe curves as would be found from efficiency elevation charts for different mass ratios. These frequencies were actually found by solving Equation 27 for ω/ω_n as a function of M/m and ζ.

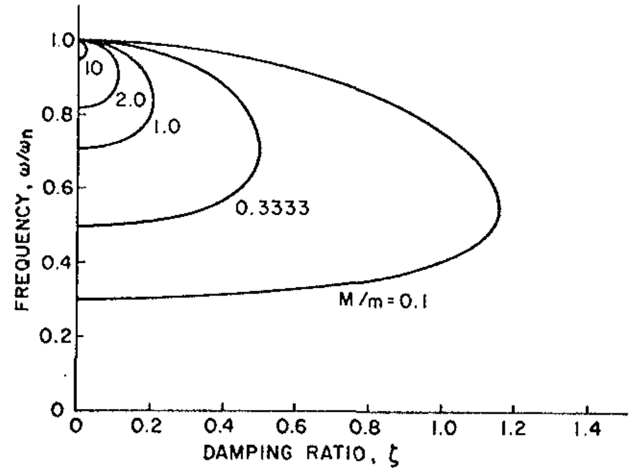

Figure 9. Optimum Frequency, Leading to 88% Efficiency. ($\epsilon = 0$, $F/mg = 1.0$, Sine Wave Force)

Finally, η_{max} from Equation 28 is plotted in Figure 10. This gives the maximum efficiency possible for all sinusoidal forcing without toe straps. These values may or may not be dynamically obtainable for specific values of ω/ω_n, ζ and M/m.

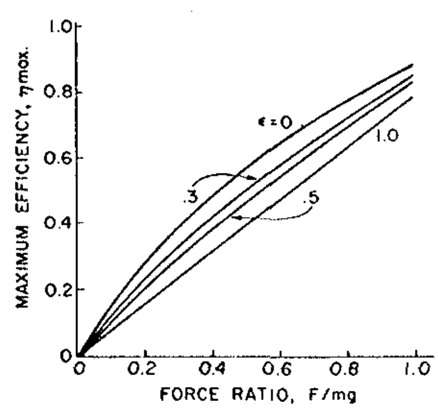

Figure 10. Maximum Efficiency Kinematically Possible

D. Scaling of System to Human Capabilities

For a human, the half stroke S has an upper limit of roughly 1.5 feet and any cycle requiring motion greater than this is unrealizable. Also, the human is limited to power output of less than 1500 ft lb/sec, even for short periods of time.[1] Again, cycles calling for more power than this are unattainable.

Assume a human weight of 150 lb, and a frequency of operation of 1 cps. Power generation and stroke required can be calculated for the reference case (Figures 11, 12). Useful power generated is presented, rather than total human power generated. The area near $\omega/\omega_n = 1.0$ and $\zeta = 0$ is a forbidden area for reasons of both power and stroke required. Combining results of Figures 3, 11 and 12, one can see that for continuous power output in the half-horsepower range, the human/cage system with spring-damper suspension should operate in the vicinity of $\zeta = 0.4$ and $\omega/\omega_n = 0.55 - 0.70$. This also allows a range of horsepower and stroke at efficiencies near 88%. One could better this efficiency only by using toe straps (and negative g loading) while trying to excite the cage with a square wave leg force, both of which would be uncomfortable over a period of time. These extraordinary measures would be used only to improve efficiency and would not improve power or stroke characteristics.

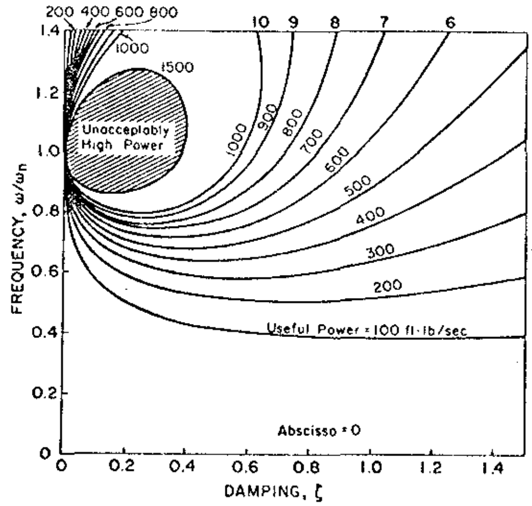

Figure 11. Power Contours for Sinusoidal Loading. ($\epsilon = 0$, $F/mg = 1.0$, $M/m = 0.3333$)

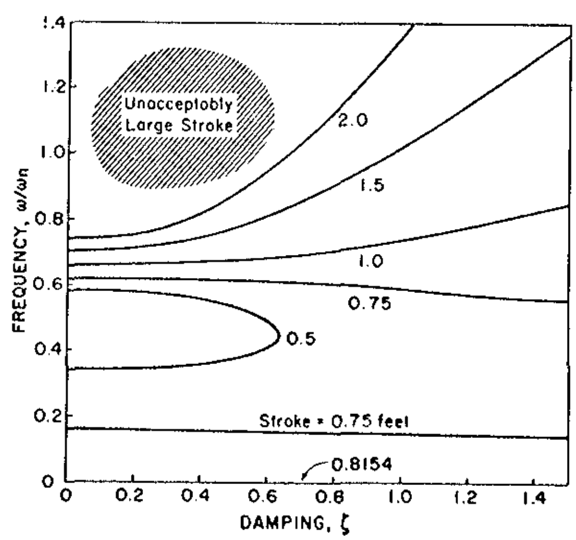

Figure 12. Stroke Contours for Sinusoidal Loading. ($\epsilon = 0$, $F/mg = 1.0$, $M/m = 0.3333$)

Freely Floating Cage

All work to this point has dealt with a cage suspension containing a spring. For a rigid flight vehicle such as the pseudo-ornithopter[3] there is no spring, although there will be apparent mass and equivalent viscous damping forces. If we remove the spring from the cage suspension, and reanalyze the system, one less dimensionless ratio is needed to describe the system. This is because one less characteristic time exists in the problem (the period $\sqrt{M/k}$ has been lost).

The choice of $f(t) = \sin(\omega t)$ leads to

$$x(t) = X \sin(\omega t - \phi)$$

$$y(t) = Y \sin \omega t$$

$$\frac{X\omega c}{F} = \frac{1}{\sqrt{(\frac{\omega M}{c}) + 1}}$$

$$\phi = \tan^{-1} \frac{1}{-(\frac{\omega M}{c})} \qquad 90^\circ \leqslant \phi \leqslant 180^\circ$$

$$\frac{Y m \omega^2}{F} = 1$$

$$\frac{Y}{X} = \frac{M}{m} \sqrt{1 + (\frac{c}{M\omega})^2}$$

and the expressions for B, α, and η are as before in Equations 18, 19, 24. We now can view η as

$$\eta = \text{function } (\omega M/c,\ F/mg,\ M/m,\ \epsilon,\ f(t))$$

A. Reference Case

The solution in terms of dimensionless ratios is given in Figure 13. Again a mass ratio typical of a human and an ultralight aircraft is assumed. The maximum efficiency gained for harmonic oscillation, without toe straps, is 0.64. Square wave exitation, also at $F/mg = 1.0$, has a peak efficiency of 0.74. Equation 28, for maximum

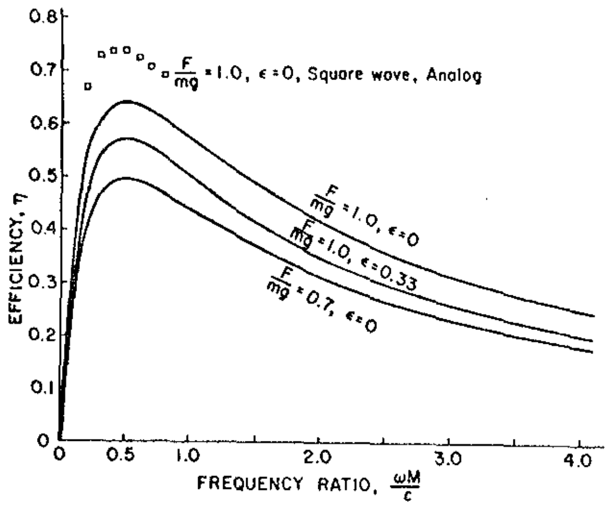

Figure 13. Efficiency of a Man in a Floating Cage (Zero Spring Stiffness) $M/m = 0.3333$

efficiency, does not hold for the floating case, because the optimum phase lag of $\alpha = 90^{\circ}$ is unobtainable dynamically. These peak efficiencies may vary somewhat with mass ratio, then. Efficiency could be raised slightly with toe straps, and the penalty factor ϵ would lower it.

B. Freely Floating Cage Scaled to Human Capabilities

Again choose a human weighing 150 lb. and oscillating at 1 cps (Figure 14). The freely floating

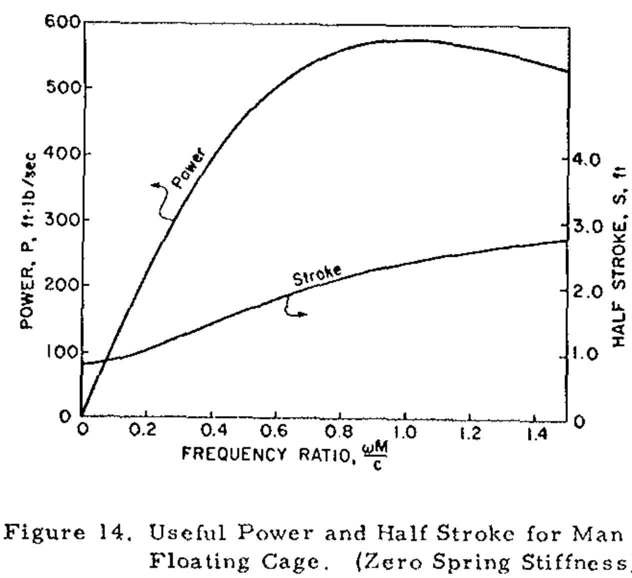

Figure 14. Useful Power and Half Stroke for Man in Floating Cage. (Zero Spring Stiffness) $M/m = 0.3333$, $F/mg = 1.0$

cage has some problems with the impedance match between human and cage. The cage is too "soft" for the human. To operate at peak efficiency at $\omega M/c = 0.5$, the human would have to use a half stroke of 1.62 ft. which is not attainable. This means operating at smaller stroke with slightly lesser efficiency. Nevertheless, the cycle does useful work, and figures such as 13 and 14 can be used to optimize performance for the individual involved. The best operating frequency is $\omega = 0.5\ c/M$ and so the numerical value of c, the equivalent viscous damping coefficient should be found early in the design cycle.

Conclusions

A complete analysis has been done for the dynamics and efficiency of a man performing a vertical work cycle in an enclosed cage. The cage was given an idealized suspension, which in the freely floating case is related to the flight of an airplane. The definition of total work done by the human was the only difficult part of the formulation, with due care taken to handle the unloading portions of the mechanical cycle. The dimensionless variables describing the cyclic process are now well defined. The efficiency of the process can be stated in purely mechanical terms, or the viewpoint can be slightly enlarged to include a biological penalty factor for requiring the human to use a cycle with unloading segments.

The efficiency calculations show that there are definite losses in this work cycle. These are due to gravity and the inability of the human to store energy during unloading portions of the cycle. The efficiency depends in an involved way on the operating frequency and the damping of the cage suspension. Increasing the force ratio F/mg increases efficiency. Increasing the penalty factor ϵ decreases efficiency. Square wave exitation is in almost all cases more efficient than sine wave, and sine wave is always better than triangular exitation.

Optimum efficiency is reached when the leg force $f(t)$ leads the leg extension $x(t) - y(t)$ by 90°. For a given cage suspension with low damping ratio ζ, this condition is met at two frequencies. The lower corresponds to the resonance of the human and cage locked together and oscillating on the spring. The higher corresponds to the cage alone resonating on the spring. Operation at the higher frequency is not attainable for humans because of the large stroke and power requirements. Operation at neither of these resonant frequencies is attainable for the free flight case, making it less efficient.

An earth-bound man/cage system can operate at up to 88% efficiency in a realistic situation using sinusoidal forcing and no toe straps. The comparable case for the flight vehicle yields 64% efficiency. (Neither case is here penalized for the tiring effect of the unloading portions of the cycle.) The lower efficiency of the flight vehicle poses a difficult challenge to the practicality of rigid wing ornithopter flight.

For the caged human, efficiencies vary widely with operating conditions. It is very important to match the characteristics of human and machine using the dynamics considerations discussed here.

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