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Wing Mass Formula for Twin Fuselage Aircraft

Sergei V. Udin* and William J. Anderson†
University of Michigan, Ann Arbor, Michigan 48109

A formula is derived to calculate structural wing mass. This formula can be applied to twin fuselage aircraft, conventional single-body aircraft and some other unconventional aircraft (such as the Voyager). The approach is particularly useful in the first stages of preliminary aircraft design and in optimization programs where the wing-mass calculation time is an important characteristic. The concept model assumes a nontapered inboard wing section, a tapered outboard wing section and fuel stored only in the outboard wing. The theory for the wing-mass estimation is described. Unlike the other mass formulae where mass spanwise distribution is considered by an "unloading coefficient," the present method integrates the mass spanwise distribution with the air load spanwise distribution. This allows more precise consideration of the wing geometry and mass unloading. There are no simplifications applied and the formula completely reflects the initial concept model. Good comparison with statistical data for single body aircraft is obtained.

Nomenclature

A = aspect ratio
a = gravitational acceleration
b = wing span
c = wing chord
d = skin thickness
E_T = effective airfoil thickness coefficient
g_d = design g (overload factor)
H = spanwise distribution of specific wing thickness ratio, $T(z)/T_r$
h = outboard wing thickness taper ratio, T_e/T_r
K = relative coefficient of structural mass
k = factor
M = spanwise distribution of reduced bending moment
m = relative mass (mass divided by aircraft mass)
m_c^i = doubled relative mass of i-numbered concentrated mass located on wing
m_s^* = previously iterated or expert-estimated wing structure mass
n = number of concentrated loads on wing
p = wing loading (total mass divided by wing area)
Q = spanwise distribution of reduced shear resultant (internal to wing)
q = spanwise distribution of reduced running load
S = wing area
T = absolute airfoil thickness
t = wing thickness ratio (absolute airfoil thickness at z divided by wing chord at z)
Z = spanwise absolute coordinate
z = spanwise relative coordinate (spanwise absolute coordinate divided by half-span)
σ_u = ultimate direct stress
σ_{u_s} = ultimate shear stress
Λ = half-chord sweep
λ = outboard wing taper ratio (end chord divided by root chord)
μ = absolute aircraft mass
ρ = density of structural material
ψ = outboard wing airfoil area taper ratio, $\approx c_e T_e / c_r T_r = h\lambda = \lambda^2(t_e/t_r)$

Subscripts

a = aerodynamic
abs = absolute (not reduced) value
ail = ailerons
c = concentrated load
e = end of wing
f = fuselage
flap = flaps
fu = fuel
i = inboard wing section
M = reduced bending moment
man = manufacturing
mid = middle
o = outboard wing section
Q = reduced shear force
r = root of wing
rib = ribs
s = wing structure
sk = load-free wing skin
sl = service life
tw = twist moment

Introduction

THERE are many advantages of twin fuselage aircraft (TFA) compared with conventional aircraft as range and payload increase. The theory of similarity predicts reduction of the payload capability when the dimensions of an aircraft are increased.[1] Application of TFA can improve the situation. This scheme can be used for subsonic[2,3] and supersonic aircraft. For the latter, the interference between fuselages improves drag characteristics.[4] A multibody scheme has been considered for high-speed transport aircraft.[5]

The preliminary design of a TFA is not a simple task, because there is not much experience or a database for this aircraft type. Therefore, the mass formula for TFA must be derived with statistical coefficients that consider only common structural design characteristics such as the mass penalty for providing service life or the joint-mass penalty.[1] The formula should consider all particulars of TFA in the computational model because there is no chance to improve the formula accuracy by comparing predicted results with experience. Another difficulty is the uncertainty of the flexible dynamic twist moment between fuselages. This moment depends on aircraft stiffness and capabilities of the asymmetric load aerodynamic compensation. The formula presented does not take into consideration flexible dynamic twist moment between fuselages.

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*Visiting Research Scholar, Aerospace Engineering Department; permanent affiliation: Graduate Student, Department 101, Moscow Aviation Institute, Volocolamsky shosse 4, Moscow 125080, Russia.

†Professor of Aerospace Engineering, Aerospace Engineering Department. Senior Member AIAA.

Usually designers do not trust a formula if they do not know which assumptions and simplifications were made during the formula derivation, therefore, a complete derivation is presented. There are no simplifications during the derivation, and so the resulting formula completely corresponds to the concept model. As a consequence, any differences between calculated mass and an actual wing mass may be clearly revealed. This will simplify formula improvement in the future. Additional details about the derivation of the wing mass formula are given in report form.[6,7]

Theoretical Method

The relative mass of wing structure is a sum of components

$$m_s = k_{sl} k_{tw} k_{man}(m_M + m_Q) + m_{rib} + m_{ail} + m_{sk} + m_{flap} \tag{1}$$

The manufacturing factor is defined[1] as

$$k_{man} = 1 + k_1 + k_2 + k_3 + k_4 + k_5 + k_6 + k_7 \tag{2}$$

where k_i have expert values within bounds presented in Table 1. This coefficient k_{man} may be decreased as low as 1.3 for large, advanced-technology aircraft.[1] The service life factor k_{sl} is ultimate stress divided by panel fatigue stress.

The structural analysis is based on slender beam (Euler-Bernoulli) theory. The mass of elements required for twist moment is proportional to the mass required for bending moment.[1] The twist moment mass has been considered by the twist moment factor k_{tw}. It depends on the cosine of sweep, taper, and aspect ratio. For conventional aircraft,[1] the formula for k_{tw} may be suggested

$$k_{tw} = 1 + \frac{0.015\sqrt{A}(1 + 2\lambda)}{(1 + \lambda)\cos\Lambda} \tag{3}$$

Usually the spanwise distribution of structure, fuel, and engine mass are considered through an "unloading coefficient."[1] Derivation of this coefficient will be more difficult if a complex form of wing or multibody fuselage scheme is used. The proposed approach implies consideration of the spanwise distribution of mass simultaneously with the lift distribution. The reduced quantities[1] at any point z are

$$q = \frac{q_{abs}(b/2)}{a\mu G_d}, \qquad Q = \frac{Q_{abs}}{a\mu G_d}, \qquad M = \frac{M_{abs}}{a\mu G_d(b/2)} \tag{4}$$

Usually the approximate spanwise lift distribution q_a has a standard form, exists in the preliminary design stage and may be considered at any spanwise point. Linear or quadratic approximation of the q_a curve can be recommended.[1] The reduced shear force $Q_a(z)$ and the reduced bending moment $M_a(z)$ due to aerodynamic load are then

$$Q_a = \int_z^1 q_a \, dz, \qquad M_a = \int_z^1 Q_a \, dz \tag{5}$$

Table 1 Bounds on manufacturing coefficients

k_1	Stepped thickness (rather than tapered)	0.10–0.13
k_2	Dead joint mass penalty	0.15–0.30
k_3	Standard thickness of webs, ribs, and other elements	0.10–0.13
k_4	Joint fittings and joint defects	0.10–0.15
k_5	Plus tolerances	0.04–0.09
k_6	Manufacturing thicknesses	0.03–0.05
k_7	Breakdown joint mass penalty	0.10–0.20
k_{man}	Manufacturing factor	1.62–2.05

The fuel mass spanwise distribution q_f depends on the airfoil area as taken by Badiagin.[8] The wing structure mass spanwise distribution q_s can be approximately obtained as a function of the chord spanwise distribution. The spanwise distributions of reduced shear and bending moment caused by a concentrated load (e.g., an engine) are

$$\text{for } 0 < z < z_c: \qquad Q_c = 1, \qquad M_c = z_c^i - z \tag{6}$$

$$\text{for } z_c < z < 1: \qquad Q_c = 0, \qquad M_c = 0 \tag{7}$$

We define the relative coefficient of structural mass caused by shear and the relative coefficient of structural mass caused by bending moment

$$K_Q = \int_0^1 \left|\frac{Q_{sum}}{\cos\Lambda}\right| dz = \int_0^1 \left|\frac{Q_a - Q_{fu}m_{fu} - Q_s m_s^* - \sum_{i=1}^{n} Q_c^i m_c^i}{\cos\Lambda}\right| dz \tag{8}$$

$$K_M = \int_0^1 \left|\frac{M_{sum}}{H\cos\Lambda}\right| dz = \int_0^1 \left|\frac{M_a - M_{fu}m_{fu} - M_s m_s^* - \sum_{i=1}^{n} M_c^i m_c^i}{H\cos\Lambda}\right| dz \tag{9}$$

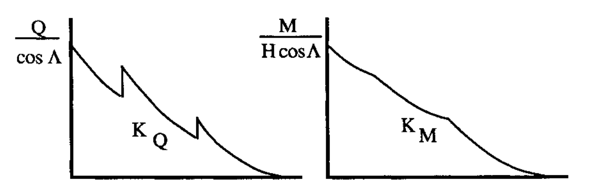
The K_Q and K_M are the areas under the curves of relative shear and bending moment (Fig. 1). According to Eq. (4) the estimated relative mass of structure counteracting the shear and the estimated relative structural mass counteracting the bending moment (without consideration of manufacturing and service life) are

$$m_Q = \frac{\rho}{\mu}\int_0^{b/2} \frac{Q_{abs}(Z)}{\sigma_{u_s}\cos\Lambda} dZ = \frac{\rho g_d a}{\sigma_{u_s}}\frac{b}{2}\int_0^1 \frac{Q_{sum}}{\cos\Lambda} dz = \frac{\rho g_d a}{2\sigma_{u_s}}\sqrt{\frac{\mu A}{p}} K_Q \tag{10}$$

$$m_M = 2\frac{\rho}{\mu}\int_0^{b/2} \frac{E_T M_{abs}(Z)}{T(Z)\sigma_u \cos\Lambda} dZ = 2\frac{\rho g_d a}{\sigma_u T_r}\frac{b^2}{4} E_T \int_0^1 \frac{M_{sum}}{H\cos\Lambda} dz = \frac{\rho g_d a}{2\sigma_u}\frac{\mu}{p} A \frac{E_T}{T_r} K_M \tag{11}$$

The coefficient E_T considers the effective airfoil thickness and difference between loading on lower and upper wing panels. The approximate value of E_T is[1]

$$E_T \approx 1.1\left(\frac{4T_2}{T_1 + 2T_2 + T_3}\right)^2 \approx 1.2\text{–}1.4 \tag{12}$$



a) Relative spanwise coordinate, z b) Relative spanwise coordinate, z

Fig. 1 Relative coefficients K_Q and K_M.

where T_1 is the first wing spar thickness, T_2 is the maximum airfoil thickness, and T_3 is the most rearward wing spar thickness. Actual operating stresses and twist moment are considered by factors in Eq. (1). If the upper and lower torsion box panels have been made from different materials then average values can be used

$$\rho = [(\rho_{\text{upper panel}} + \rho_{\text{lower panel}})/2]$$
$$\sigma = [(\sigma_{\text{upper panel}} + \sigma_{\text{lower panel}})/2] \tag{13}$$

Masses of other elements (e.g., flaps and ailerons) may be taken from existing methods.

This theoretical derivation improves the accuracy of the wing mass formula by a more detailed calculation of mass spanwise distribution as compared to the method of Sheinin and Kozlowsky.[1] It includes all physical relations that are important for optimization programs.

Formula Derivation

The geometry of a TFA wing is shown in Fig. 2. It is assumed that the inboard wing section is not tapered, and the inboard/outboard wing joint is located at the fuselage centerline.

Spanwise Distribution of Reduced Aerodynamic Quantities

The aerodynamic load spanwise distribution q_a is related to the chord,[1] and

$$q_a(1) = \lambda q_a(z_f) \tag{14}$$

According to Eq. (4) and using the equivalence of lift force to aircraft weight on the basis of TFA geometry, we obtain

$$q_a(z_f)z_f + \frac{q_a(z_f) + q_a(1)}{2}(1 - z_f) = 1 \tag{15}$$

Using a linear equation, one takes the spanwise distribution of reduced running load

for $0 < z < z_f$ (inboard wing):

$$q_a = \frac{2}{z_f(1 - \lambda) + \lambda + 1} \tag{16}$$

for $z_f < z < 1$ (outboard wing):

$$q_a = 2\frac{[(1 - z)/(1 - z_f)](1 - \lambda) + \lambda}{z_f(1 - \lambda) + \lambda + 1} \tag{17}$$

The relative shear resultant $Q_a(z)$ due to aerodynamic force is

for $z_f < z < 1$: $\quad Q_a = \int_z^1 q_a \, dz$

$$= \frac{(1 - \lambda)(1 - z)^2/(1 - z_f) + 2\lambda(1 - z)}{z_f(1 - \lambda) + \lambda + 1} \tag{18}$$

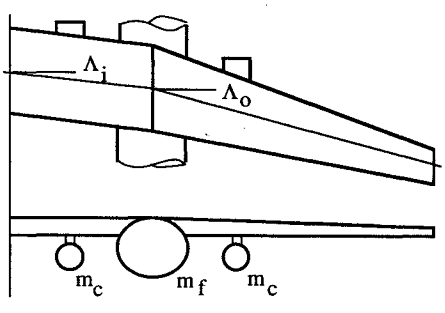


Fig. 2 TFA wing geometry.

for $0 < z < z_f$: $\quad Q_a = \int_z^{z_f} q_a \, dz + Q_a(z_f)$

$$= 1 - \frac{2z}{z_f(1 - \lambda) + \lambda + 1} \tag{19}$$

The spanwise distribution of reduced bending moment due to aerodynamic force is

for $z_f < z < 1$: $\quad M_a = \int_z^1 Q_a \, dz$

$$= \frac{[\frac{1}{3}(1 - z)^3/(1 - z_f)](1 - \lambda) + \lambda(1 - z)^2}{z_f(1 - \lambda) + \lambda + 1} \tag{20}$$

for $0 < z < z_f$: $\quad M_a = \int_z^{z_f} Q_a \, dz + M_a(z_f)$

$$= z_f - z - \frac{z_f^2 - z^2 - \frac{1}{3}(1 - z_f)^2(1 + 2\lambda)}{z_f(1 - \lambda) + \lambda + 1} \tag{21}$$

The spanwise distributions of q_a, Q_a, and M_a are shown in Fig. 3.

Spanwise Distribution of Reduced Fuel Mass Quantities

First, the formula for maximum fuel mass that may be located in the outboard wing, must be derived. The wing fuel volume is[8]

$$V = 0.7\frac{t_{\text{mid}}S_o^{1.5}}{\sqrt{A_o}}; \qquad t_{\text{mid}} = \frac{t_r + \lambda t_e}{1 + \lambda} \tag{22}$$

From geometric relations one may write

$$S_o = S\frac{S_o}{S_i + S_o} = \frac{\mu}{p}\left[\frac{(1 - z_f)(1 + \lambda)}{2z_f + (1 - z_f)(1 + \lambda)}\right] \tag{23}$$

If the fuel is kerosene then the relative fuel mass in the outboard wing section is[8]

$$m_{\text{fu}} = 800\frac{V}{\mu} = 560\frac{t_{\text{mid}}}{p^{1.5}}\sqrt{\frac{\mu}{A}}\left[\frac{(1 - z_f)(1 + \lambda)}{2z_f + (1 - z_f)(1 + \lambda)}\right]^{1.5} \tag{24}$$

This formula shows that for a heavy TFA it is possible to locate all fuel in the outboard wing, e.g., if $z_f < 0.35$; $\lambda > 0.35$; $p < 700$ kg/m^2; $m > 400{,}000$ kg then $m_{\text{fu}} > 0.3$ for all other useful parameters. Locating fuel in the fuselage is better than in the inboard wing because the fuel weight unloads the inboard wing section. We will assume that the inboard wing section does not contain fuel.

The spanwise distribution of reduced values caused by fuel weight must be derived. It is assumed that the fuel tank area $S_{\text{fu}}(z)$ is proportional to the airfoil (wing cross section) area $S_{\text{airf}}(z)$, and the airfoil area is proportional to the chord $c(z)$ multiplied by airfoil thickness $T(z)$. An approximation is made by assuming a linear relation (Fig. 4) for fuel distribution

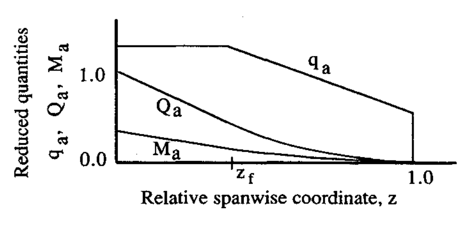


Fig. 3 Spanwise distribution of reduced quantities caused by aerodynamic force.

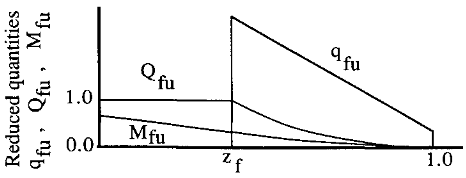


Fig. 4 Spanwise distribution of reduced quantities caused by fuel weight.

between the fuselage and wing tip. The extreme values are

$$\frac{q_{fu}(z_f)}{q_{fu}(1)} = \frac{S_{fu}(z_f)}{S_{fu}(1)} = \frac{T_r c_r}{T_e c_e} = \frac{1}{\psi} \tag{25}$$

where the area under the q_{fu} curve equals 1

$$\frac{q_{fu}(z_f) + q_{fu}(1)}{2}(1 - z_f) = 1 \tag{26}$$

Then

$$q_{fu}(z_f) = \frac{2}{(1+\psi)(1-z_f)}; \qquad q_{fu}(1) = \frac{2\psi}{(1+\psi)(1-z_f)} \tag{27}$$

$$\text{for} \quad 0 < z < z_f: \qquad q_{fu} = 0 \tag{28}$$

$$\text{for} \quad z_f < z < 1: q_{fu} = 2\frac{(1-z)(1-\psi) + (1-z_f)\psi}{(\psi+1)(1-z_f)^2} \tag{29}$$

Spanwise distribution of reduced shear force caused by fuel weight is

$$\text{for} \quad z_f < z < 1: \qquad Q_{fu}(z) = \int_z^1 q_{fu}\,dz$$

$$= \frac{(1-z)^2(1-\psi) + 2\psi(1-z_f)(1-z)}{(\psi+1)(1-z_f)^2} \tag{30}$$

$$\text{for} \quad 0 < z < z_f: \qquad Q_{fu}(z) = \int_z^{zf} q_{fu}\,dz + Q_{fu}(z_f) = 1 \tag{31}$$

Spanwise distribution of reduced bending moment caused by fuel weight is

$$\text{for} \quad z_f < z < 1: \qquad M_{fu}(z) = \int_z^1 Q_{fu}\,dz$$

$$= \frac{\frac{1}{3}(1-z)^3(1-\psi) + \psi(1-z_f)(1-z)^2}{(\psi+1)(1-z_f)^2} \tag{32}$$

$$\text{for} \quad 0 < z < z_f: \qquad M_{fu}(z) = \int_z^{zf} Q_{fu}\,dz + M_{fu}(z_f)$$

$$= z_f - z + \frac{(1-z_f)(1+2\psi)}{3\psi+3} \tag{33}$$

Figure 4 illustrates q_{fu}, Q_{fu}, and M_{fu}.

Spanwise Distribution of Reduced Wing Structure Quantities

A linear approximation of wing mass spanwise distribution is recommended.[1] For a conventional tapered wing, the reduced running load caused by structural weight q_s^* is

$$q_s^*(0) = (2 + 1.2\lambda)/(\lambda + 1); \qquad q_s^*(1) = 0.8\lambda/(\lambda + 1) \tag{34}$$

A TFA has two different portions of wing with different tapers so that there are two subranges of wing mass spanwise distribution: inboard $q_{si}(z)$ and outboard $q_{so}(z)$. The proportions of TFA are taken as (parameters noted by asterisk relates to the conventional aircraft)

$$(b^*/m_s^*)q_{s1}^*(0) = (b_i/m_{si})q_{si}(z_f); \qquad (b^*/m_s^*)q_{s1}^*(1) = (b_i/m_{si})q_{si}(0) \tag{35}$$

$$(b^*/m_s^*)q_{s2}^*(1) = (b_o/m_{so})q_{so}(1); \qquad (b^*/m_s^*)q_{s2}^*(0) = (b_o/m_{so})q_{so}(z_f) \tag{36}$$

Proceeding from the principle of smooth mass distribution,[1] we write

$$q_{si}(z_f) = q_{so}(z_f) \tag{37}$$

The inboard wing taper ratio equals 1, then

$$\frac{m_{so}}{m_{si}} = \frac{b_o q_{s1}^*(0)}{b_i q_{s2}^*(0)} = \frac{(1-z_f)(1+\lambda)}{z_f(1.25 + 0.75\lambda)} \tag{38}$$

After some simplifications

$$q_{si}(z_f) = q_{so}(z_f) = \frac{8 + 4.8\lambda}{z_f(1-\lambda) + 4(1+\lambda)} \tag{39}$$

$$q_{so}(1) = \frac{3.2\lambda}{z_f(1-\lambda) + 4(1+\lambda)}; \qquad q_{si}(0) = \frac{2 + 1.2\lambda}{z_f(1-\lambda) + 4(1+\lambda)} \tag{40}$$

A piecewise linear curve may be used to represent q_s

for $z_f < z < 1$:

$$q_s = \frac{(8 + 1.6\lambda)(1-z)/(1-z_f) + 3.2\lambda}{z_f(1-\lambda) + 4(1+\lambda)} \tag{41}$$

for $0 < z < z_f$:

$$q_s = \frac{(z/z_f)(6 + 3.6\lambda) + 2 + 1.2\lambda}{z_f(1-\lambda) + 4(1+\lambda)} \tag{42}$$

The reduced shear force caused by wing mass spanwise distribution is

$$\text{for} \quad z_f < z < 1: \qquad Q_s(z) = \int_z^1 q_s\,dz$$

$$= \frac{[(4 + 0.8\lambda)(1-z)^2/(1-z_f)] + 3.2\lambda(1-z)}{z_f(1-\lambda) + 4(1+\lambda)} \tag{43}$$

$$\text{for} \quad 0 < z < z_f: \qquad Q_s(z) = \int_z^{zf} q_s\,dz + Q_s(z_f)$$

$$= 1 - \frac{[3(z^2/z_f) + 2z](1 + 0.6\lambda)}{z_f(1-\lambda) + 4(1+\lambda)} \tag{44}$$

The reduced bending moment caused by wing mass spanwise distribution is

for $z_f < z < 1$:

$$M_s = \frac{\frac{1}{3}(4 + 0.8\lambda)(1-z)^3/(1-z_f)] + 1.6\lambda(1-z)^2}{z_f(1-\lambda) + 4(1+\lambda)} \tag{45}$$

for $0 < z < z_f$:

$$M_s = z_f - z + \frac{[(z^2 - 2z_f^2 + (z^3/z_f)](1 + 0.6\lambda) + (\frac{4}{3} + 1.867\lambda)(1 - z_f)^2}{z_f(1 - \lambda) + 4(1 + \lambda)} \tag{46}$$

Figure 5 shows q_s, Q_s, and M_s.

Spanwise Distribution of Reduced Concentrated Load Quantities

The computational model has n_i, i-numbered inboard wing concentrated loads, and n_o, j-numbered outboard wing concentrated loads. The relative mass of such a load is $\frac{1}{2}m_c^i$ or $\frac{1}{2}m_c^j$ (i.e., m_c^i or m_c^j is the mass of both symmetric loads located in both halves of the wing). The relative coordinate of a concentrated load in the inboard wing section is z_c^i, and in the outboard wing z_c^j. Reduced shear and bending moment distributions caused by a concentrated load were presented in Eqs. (6) and (7).

The fuselage is also a concentrated load and for our case it is characterized by the relative mass

$$m_f = 1 - m_s^* - m_{\text{fu}} - \sum_{i=1}^{n_i} m_c^i - \sum_{j=1}^{n_Q} m_c^j \tag{47}$$

Spanwise Distribution of Specific Wing Thickness Ratio

The inboard wing specific thickness ratio H does not depend on z and equals 1. Using the linear equation for the outboard wing ($z_f < z < 1$)

$$H = \frac{1}{T_r}\left(\frac{1 - z}{1 - z_f}(T_r - T_e) + T_e\right) = \frac{1 - z}{1 - z_f}(1 - h) + h \tag{48}$$

T_r must be derived. The wing area is

$$S = S_o + S_i = \frac{c_r + c_e}{2}b(1 - z_f) + c_r b z_f = c_r b\left(\frac{1 + \lambda}{2}(1 - z_f) + z_f\right) \tag{49}$$

where

$$S = \mu/p; \qquad b = \sqrt{SA} \tag{50}$$

and then

$$T_r = t_r c_r = \frac{2t_r}{(1 + \lambda)(1 - z_f) + 2z_f}\sqrt{\frac{\mu}{pA}} \tag{51}$$

Structural Mass Required by Shear Force

The integral in Eq. (8) must be represented for the inboard and outboard wing as

$$K_Q = K_{Q_i} + K_{Q_o} \tag{52}$$

where

$$K_{Q_i} = \frac{1}{\cos\Lambda_i}\int_0^{z_f}\left|Q_a - Q_{\text{fu}}m_{\text{fu}} - Q_s m_s^* - Q_f m_f - \sum_{j=1}^{n_Q} Q_c^j m_c^j - \sum_{i=1}^{n_i} Q_c^i m_c^i\right| \mathrm{d}z \tag{53}$$

Usually the integrand in Eq. (53) is negative in the range $(0, z_f)$ for TFA (Fig. 6). For an analytical integration, one desires that the quantity within the absolute brackets does not change sign in the range of integration so that the integral of absolute value is the absolute value of the integral. This assumption will be discussed later. Using Eqs. (8), (19), (31), and (44), we have

$$K_{Q_i} = \frac{1}{\cos\Lambda_i}\left[\frac{z_f^2}{z_f(1 - \lambda) + \lambda + 1} - \frac{z_f^2(2 + 1.2\lambda)m_s^*}{z_f(1 - \lambda) + 4\lambda + 4} + \sum_{i=1}^{n_i}(z_f - z_c^i)m_c^i\right] \tag{54}$$

In the same manner

$$K_{Q_o} = \frac{1}{\cos\Lambda_o}\int_{z_f}^{1}\left|Q_a - Q_{\text{fu}}m_{\text{fu}} - Q_s m_s^* - \sum_{j=1}^{n_Q} Q_c^j m_c^j\right| \mathrm{d}z \tag{55}$$

Assume that the quantities within the absolute brackets are positive (Fig. 6). Further, according to Eqs. (7), (18), (30), (43), and (47), we obtain

$$K_{Q_o} = \frac{1}{\cos\Lambda_o}\left[\frac{(1 - z_f)^2}{3}\left(\frac{1 + 2\lambda}{z_f(1 - \lambda) + \lambda + 1} - \frac{(1 + 2\psi)m_{\text{fu}}}{(\psi + 1)(1 - z_f)} - \frac{(4 - 5.6\lambda)m_s^*}{z_f(1 - \lambda) + 4\lambda + 4}\right) - \sum_{j=1}^{n_i}(z_c^j - z_f)m_c^j\right] \tag{56}$$

According to Eq. (10), estimated structural mass required by shear (without consideration of manufacturing and service life) is

$$m_Q = \frac{\rho g_d a}{2\sigma_{u_s}}\sqrt{\frac{\mu A}{p}}(K_{Q_i} + K_{Q_o}) \tag{57}$$

The actual operating stresses are considered in Eq. (1).

Structural Mass Required by Bending Moment

The integral in Eq. (9) can be represented as

$$K_M = K_{M_i} + K_{M_o} \tag{58}$$

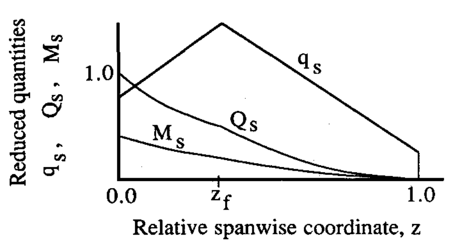


Fig. 5 Spanwise distribution of reduced quantities caused by structural weight.

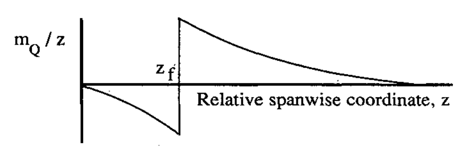


Fig. 6 Mass distribution required by shear force.

where

$$K_{M_i} = \int_0^{z_f} \left| \frac{M_a - M_{\mathrm{fu}} m_{\mathrm{fu}} - M_s m_s^* - M_f m_f - \sum_{j=1}^{n_O} M_c^j m_c^j - \sum_{i=1}^{n_i} M_c^i m_c^i}{H \cos \Lambda_i} \right| \mathrm{d}z \tag{59}$$

The quantity within the absolute brackets is assumed positive. Using Eqs. (9), (21), (33), (46), and (47) one has

$$\begin{aligned} K_{M_i} = \frac{z_f}{\cos \Lambda_i} \Bigg[& \frac{(1 - z_f)^2(1 + 2\lambda) - 2z_f^2}{3(z_f(1 - \lambda) + \lambda + 1)} \\ & - \frac{(1 - z_f)(1 + 2\psi)m_{\mathrm{fu}}}{3\psi + 3} \\ & - \frac{(\frac{4}{3} + 1.87\lambda)(1 - z_f)^2 - \frac{17}{12} z_f^2(1 + 0.6\lambda)}{z_f(1 - \lambda) + 4(\lambda + 1)} m_s^* \\ & - \sum_{j=1}^{n_O} z_c^j m_c^j - \sum_{i=1}^{n_i} \frac{(z_c^i)^2 - z_f^2}{2z_f} m_c^i \Bigg] \end{aligned} \tag{60}$$

In the same manner

$$K_{M_o} = \int_{z_f}^{1} \left| \frac{M_a - M_{\mathrm{fu}} m_{\mathrm{fu}} - M_s m_s^* - \sum_{j=1}^{n_O} M_c^j m_c^j}{H \cos \Lambda_o} \right| \mathrm{d}z \tag{61}$$

Using Eqs. (8), (20), (32), and (45), one may write

$$\begin{aligned} K_{M_o} = \frac{1 - z_f}{(1 - h)\cos \Lambda_o} \Bigg\{ & \frac{(1 - z_f)^2}{3(1 - h)^3} \left(\frac{1}{3} - \frac{3}{2} h \right. \\ & \left. + 3h^2 - \frac{11}{6} h^3 + h^3 \log h \right) \left(\frac{1 - \lambda}{z_f(1 - \lambda) + \lambda + 1} \right. \\ & \left. - \frac{(1 - \psi)m_{\mathrm{fu}}}{(1 + \psi)(1 - z_f)} - \frac{(4 + 0.8\lambda)m_s^*}{z_f(1 - \lambda) + 4 + 4\lambda} \right) \\ & + \left(\frac{1 - z_f}{1 - h} \right)^2 (\tfrac{1}{2} - 2h + \tfrac{3}{2}h^2 - h^2 \log h) \\ & \cdot \left(\frac{\lambda}{z_f(1 - \lambda) + \lambda + 1} - \frac{\psi m_{\mathrm{fu}}}{(\psi + 1)(1 - z_f)} \right. \\ & \left. - \frac{1.6\lambda m_s^*}{z_f(1 - \lambda) + 4\lambda + 4} \right) - \sum_{j=1}^{n_O} m_c^j \Bigg[z_c^j - z_f \\ & - \left(h \frac{1 - z_f}{h - 1} - 1 + z_c^j \right) \log \left(\frac{1 - z_c^j}{1 - z_f} \right. \\ & \left. \left. \cdot (1 - h) + h \right) \Bigg] \Bigg\} \end{aligned} \tag{62}$$

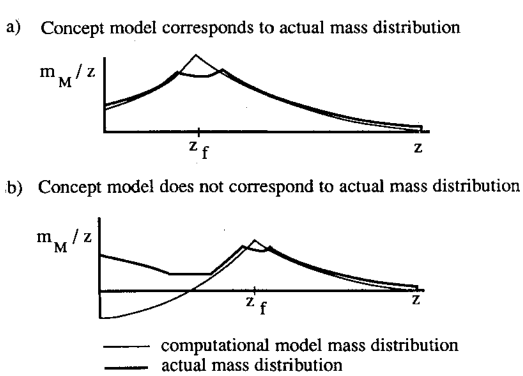


Fig. 7 Fuselage location effect upon mass needed to carry bending moment.

Assume that $h \neq 1$. Otherwise division by zero will occur. If the formula is to be used for a nontapered wing, the integral in Eq. (61) must be rederived with $H = 1$.

Using Eqs. (11) and (51) one can estimate mass of elements caused by bending moment without consideration of manufacturing and service life:

$$m_M = \frac{\rho g_d a}{2\sigma_u} A^{1.5} \sqrt{\frac{\mu}{p}} E_T \frac{(\lambda + 1)(1 - z_f)}{2t_r} (K_{M_i} + K_{M_o}) \tag{63}$$

The actual operating stresses are considered in Eq. (1).

Twist Moment Factor for TFA

The TFA wing has two sections with different sweep and taper. On the basis of Eq. (3) we have

$$k_{tw} = 1 + \frac{0.0225\sqrt{b^2/S_i}}{\cos \Lambda_i} + \frac{0.015\sqrt{b_o^2/S_o}(1 + 2\lambda)}{(\lambda + 1)\cos \Lambda_o} \tag{64}$$

After simplification

$$\begin{aligned} k_{tw} = 1 + & \sqrt{A}[2z_f + (1 - z_f)(1 + \lambda)] \\ & \cdot \left[\frac{0.0225}{\cos \Lambda_i} \sqrt{\frac{z_f}{2}} + \frac{0.015(1 + 2\lambda)\sqrt{1 - z_f}}{(\lambda + 1)^{1.5}\cos \Lambda_o} \right] \end{aligned} \tag{65}$$

Range of Formula Application

It has been assumed that quantities within absolute brackets in Eqs. (53), (55), (59), and (61) are positive. But in some cases, this is not true. The spanwise distribution of mass needed to carry bending moment is shown in Fig. 7 in accordance with both the concept model and the actual physical bending moment mass. There is a difference between actual and formula mass when z_f increases. The bounds of formula applicability in terms of z_f must be obtained.

The wing root skin thickness is assumed to be set by strength requirements and is greater than minimum thickness requirements for manufacturing, otherwise, the formula must not be used. We require

$$d_t < \frac{a\mu G_d E_T b M_r}{c_{tb} k_{\mathrm{rib}} T_r \sigma_u k_{\mathrm{man}} k_{\mathrm{sl}} k_{\mathrm{tw}}} \tag{66}$$

where k_{rib} defined by Sheinin and Kozlowsky[1] (approximate value is 2); c_{tb}—torsion box chord (approximate value is $\frac{1}{2}c_r$, c_r in Eq. (51)). According to Eqs. (4), (6), (7), (22), (32), (33), and (45–47), one may write

$$\begin{aligned} d_t < & \frac{a g_d \sqrt{\mu p A^3} E_T[(\lambda + 1)(1 - z_f) + 2z_f]^2}{4 t_r \sigma_u k_{\mathrm{man}} k_{\mathrm{sl}} k_{\mathrm{tw}}} \\ & \cdot \left[\frac{\frac{1}{3}(1 - z_f)^2(1 + 2\lambda) - z_f^2}{z_f(1 - \lambda) + 1 + \lambda} - \frac{(1 - z_f)(1 + 2\psi)}{3\psi + 3} m_{\mathrm{fu}} \right. \\ & - \frac{(\frac{4}{3} + 1.886\lambda)(1 - z_f)^2 - 2z_f^2(1 + 0.6\lambda)}{z_f(1 - \lambda) + 4\lambda + 4} m_s^* \\ & \left. - \sum_{i=1}^{n_i} m_c^i(z_c^i - z_f) - \sum_{j=1}^{n_O} m_c^j(z_c^j - z_f) \right] \end{aligned} \tag{67}$$

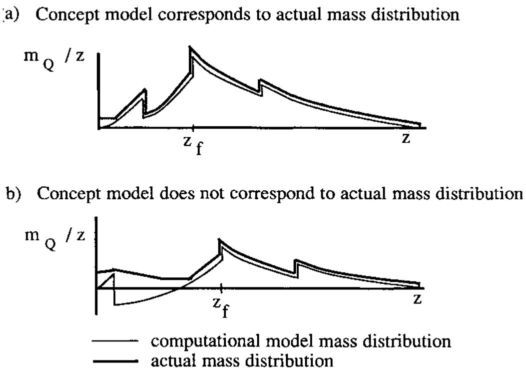


Fig. 8 Fuselage location effect upon mass needed to carry shear resultant.

Equation (67) involves the 4th power of z_f. Results may be obtained by a numerical method. A computer program to solve Eq. (67) indicates that for $z_f \leq 0.3$, $\lambda \geq 0.35$, $m_0 \geq$ 100,000 kg, $d_t \leq 1.5$ mm, and $A \geq 10$ the inequality is true for all other useful parameters. If the actual parameters of a multibody aircraft are outside these limits, then verification of use of the formula must be carried out. The inequality is always true for a single-body aircraft.

The spanwise distributions of relative mass needed to carry shear, in accordance with both the concept model and the actual bending moment mass, are shown on Fig. 8. There is a difference between actual and calculated mass if concentrated forces are located near the longitudinal aircraft axis. The simple relation[1] between the mass caused by bending moment, and the mass caused by shear force for such cases is

$$K_Q = 0.1K_M \tag{68}$$

This formula also can be recommended for moderate-sized aircraft. The mass caused by shear is approximately $\frac{1}{20}$ of the total wing mass and its influence on the formula accuracy is not significant. If the outboard wing is not tapered, then Eq. (61) must be rederived.

Mass of Other Wing Elements

The mass of other elements may be found by existing methods. Some formulae are presented below.

Mass of Ribs

The mass of ribs for TFA is

$$m_{\text{rib}} = m_{r\,\text{rib}} + 2m_{f\,\text{rib}} + m_{\text{row rib}} \approx 0.15m_M \tag{69}$$

where $m_{r\,\text{rib}}$ is root rib mass; $m_{f\,\text{rib}}$ is fuselage rib mass; and $m_{\text{row rib}}$ is row rib mass (that are only aerodynamically loaded). The relative mass of conventional aircraft root ribs $m^*_{r\,\text{rib}}$ is[1]

$$m^*_{r\,\text{rib}} = \left(\frac{1.05 \times 10^{-4}}{\mu}\right)\left(\frac{M_r}{a}\right)\frac{c_r}{2}\sin\Lambda \tag{70}$$

where M_r is the root bending moment (Nm). For twin fuselage aircraft bending moments (Nm) at $z = 0$ and $z = z_f$ are

$$\begin{aligned} M_r(0) &= a\frac{b}{2}\mu g_d M(0) = a\frac{b}{2}\mu g_d \int_0^1 Q\,\mathrm{d}z \\ M_r(z_f) &= a\frac{b}{2}\mu g_d M(z_f) = a\frac{b}{2}\mu g_d \int_{z_f}^1 Q\,\mathrm{d}z \end{aligned} \tag{71}$$

These integrals were seen previously in Eqs. (53) and (54). Considering the sense of the moment, and using Eqs. (51) and (71), we obtain

$$m_{r\,\text{rib}} = \frac{1.05 \times 10^{-4} g_d |K_{Q_o}\cos\Lambda_o - K_{Q_i}\cos\Lambda_i|}{(1+\lambda)(1-z_f) + 2z_f} \cdot \frac{\mu}{p}\sqrt{\frac{A}{a}}\sin|\Lambda_i| \tag{72}$$

$$m_{f\,\text{rib}} = \frac{1.05 \times 10^{-4} g_d K_{Q_o}\cos\Lambda_o}{(1+\lambda)(1-z_f) + 2z_f}\frac{\mu}{p}\sqrt{\frac{A}{a}}\sin|\Lambda_o - \Lambda_i| \tag{73}$$

Note that these formulae do not correspond to the theory of similarity that implies the relative mass varies as mass μ to the $\frac{1}{2}$ power. The initial formula Eq. (70) also does not correspond to the theory of similarity.

The row rib mass is[1]

$$m_{\text{row rib}} = [0.26\sqrt{S_{tb}} + 0.0006(g_d\mu/S)]S_{tb} \tag{74}$$

where S_{tb} is the torsion box area. Using some simple relations we have

$$m_{\text{row rib}} = 0.0919\sqrt{\mu/p^3} + 0.0003g_d \tag{75}$$

Mass of Forward and Rear Parts of Wing Skin

The mass of forward and rear parts of wing skin (not loaded surfaces) in accordance with experience[1] is

$$m_{\text{sk}} = (3/p) \tag{76}$$

Mass of Flaps

The mass of flaps consists of components

$$m_{\text{flap}} = m_{\text{lef}} + m_{\text{tef}} + m_{\text{of}} \tag{77}$$

The relative mass of leading edge flaps is[1]

$$m_{\text{lef}} = \frac{k_{\text{lef}}}{p} \tag{78}$$

where K_{lef} for a Krueger flap is 2.5, and for others is 3.5. The relative mass of trailing-edge flaps, in accordance with the Torenbeek formula[1] is

$$m_{\text{tef}} = \frac{2.706}{\mu} S_f k_f (S_f b_f)^{3/16}\left[\left(\frac{V_f}{100}\right)^2 \frac{\sin a_f \cos\Lambda_f}{t_f}\right]^{3/4} \tag{79}$$

where S_f is trailing edge flaps area, k_f is the statistical coefficient, b_f is the flap length (equals spanwise flap length divided by the cosine of the trailing edge sweep), V_f is flap-down speed, a_f is flap deflection angle, Λ_f is half chord flap sweep, t_f is thickness ratio of upper flap, k_f for double-slotted flap is 1.05, for triple-slotted flap is 1.25, for triple-slotted extending split flap is 1.6, for single-slotted Fowler flap is 1.25, for double-slotted Fowler flap is 1.3, and for triple-slotted Fowler flap is 1.62. For the relative mass of other flap types (brake flaps, spoilers) it is suggested that

$$m_{\text{of}} = (30\tilde{S}_{\text{of}})/p \tag{80}$$

where $\tilde{S}_{\text{of}}$ is the relative area of flap (area of flap divided by area of wing).

Mass of Ailerons

The relative mass of ailerons is[1]

$$m_{\text{ail}} = 0.03\tilde{S}_{\text{ail}}; \qquad \tilde{S}_{\text{ail}} = \frac{S_{\text{ail}}}{S} \tag{81}$$

One can also use Eq. (80) for ailerons.

Table 2 Results of formula usage for conventional aircraft

Aircraft	μ, kg	m_s, actual	k_{man}	m_s, formula	Error, %	Error,[10] %
B-727-100	72,600	0.111	1.9	0.1206	8.7	−0.7
B-747-100	322,000	0.122	1.7	0.1061	−13.1	−13.3
DC-9-30	49,000	0.106	1.9	0.1018	−4.0	−6.5
DC-10-10	195,000	0.114	1.8	0.1069	−6.2	10.4
DC-10-30	252,000	0.106	1.8	0.0989	−6.7	2.4
A-300-B2	137,700	0.145	1.7	0.1423	−1.8	−14.6
C-5A	349,000	0.130	1.7	0.1349	3.8	−1.3
Tu-154	90,000	0.102	1.9	0.1085	6.4	19.5
An-10	54,000	0.0981	1.9	0.0929	−5.3	0.3
An-22	250,000	0.119	1.8	0.1167	−1.9	−9.1
An-24	21,000	0.1142	1.9	0.1189	4.1	16.0
Il-76-T	171,000	0.121	1.8	0.1351	11.7	−14.9

Applications

The wing structural mass for TFA is obtained from Eq. (1) where mass components are defined in Eqs. (57), (63), (69), (76–78), (80), and (81). The twist moment factor for the TFA is Eq. (65). The formula should not be used if initial parameters do not correspond to Eq. (67). If the wing has a concentrated load near the longitudinal aircraft axis, Eq. (54) must be replaced by Eq. (68).

Conventional Single-Body Aircraft

The mass formula [Eq. (1)] can be used for conventional aircraft if z_f equals the fuselage radius. The fuselage (inboard) wing-section geometry and loading are completely considered in this case. The twist moment factor is Eq. (3).

Results for several single body aircraft[9] are presented in Table 2. The accuracy is within (−13.1, +11.7%) and rms error is 7.0%. The manufacturing factors were chosen through engineering judgement without access to manufacturing data. The chosen factors were not altered to drive the error to zero. (Otherwise, all errors could be made zero!)

For comparison, single-body results using the Torenbeek formula for class II estimation[10] are shown as the last column in Table 2. The accuracy is within (−14.9, + 19.5%) and rms error is 11.1%. This equation is not as accurate as the equation developed above, but has the advantage of being simpler.

Details of the examples, comparisons with the Badiagin formula,[1] and spreadsheet calculations, are included in Ref. 7. These will be of help to the practicing engineer.

Other Unconventional Schemes

The formula can also be used for aircraft with three or more bodies, if Eq. (67) is true. The fuselage loads are represented by concentrated loads in this case. Use of Eq. (68) instead of Eq. (53) is recommended. The Voyager[9] corresponds to this case. The formula can also be used to calculate mass required to carry bending moment for the multibody scheme for supersonic aircraft.[5]

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