A ROCKET MEASUREMENT OF UPPER ATMOSPHERE DENSITY

BY PASCHEN'S LAW

HALDON L. SMITH
HAROLD C. EARLY
NELSON W. SPENCER

The research reported in this document is published for technical information only, and does not necessarily represent recommendations or conclusions of the sponsoring agency.

Project 2096

AIR FORCE CAMBRIDGE RESEARCH CENTER, U.S. AIR FORCE
CONTRACT AF 19(604)-545

May 1955
ACKNOWLEDGEMENT

The authors wish to acknowledge the assistance of Mr. H. S. Sicinski for carrying through the calculation of minimum surface density from Alphatron pressure measurements, and for his helpful suggestions in the interpretation of the Paschen's law density data.
OBJECTIVE

The original goal of this research was to develop a new method of rocket instrumentation for obtaining a direct measurement of the ambient air density of the upper atmosphere. This objective was modified, however, in favor of a more readily realizable system for determining the surface density existing on the nose cone of a supersonic missile. The measurement was based upon the principle of Paschen's law and depended upon a measurement of the breakdown potential of a special 6-inch spark gap which formed part of the rocket nose. The experimental investigation and the construction of the rocket-borne equipment were carried on in the electrical engineering laboratories of the University of Michigan. An Aerobee rocket carrying this equipment was flown 13 September 1951 at the Holloman Air Force Base, Alamogordo, New Mexico.
ABSTRACT

The air density in the region adjacent to the surface of a supersonic cone in the upper atmosphere has been measured by means of a spark-breakdown technique based on Paschen's law. A spark gap was incorporated into the nose of an Aerobee rocket and consisted of alternate metal and dielectric laminations. The breakdown path was essentially parallel to the surface of the cone and provided for a measurement without changing the air flow past the missile.

The spark-breakdown technique was evaluated by measurements taken in a large vacuum chamber, and investigations were made into the statistical deviation from the mean calibration curve, and of the consequences of initial ionization. The effects of the high air velocity upon the breakdown were also considered. The Paschen's law density measurements have been compared with data derived from Alphatron pressure gages included in the rocket instrumentation and were found to have a consistently lower value. This discrepancy is believed to be due largely to the influence of the boundary layer upon the location of the discharge path.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENT</td>
<td>ii</td>
</tr>
<tr>
<td>OBJECTIVE</td>
<td>iii</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF ILLUSTRATIONS</td>
<td>vii</td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. PASCHEN'S LAW CONSIDERATIONS</td>
<td>1</td>
</tr>
<tr>
<td>III. EXPERIMENTAL CONSIDERATIONS</td>
<td>4</td>
</tr>
<tr>
<td>PLACEMENT OF ELECTRODES</td>
<td>4</td>
</tr>
<tr>
<td>RANGE OF MEASUREMENT</td>
<td>4</td>
</tr>
<tr>
<td>INDEPENDENCE OF AIR VELOCITY</td>
<td>5</td>
</tr>
<tr>
<td>REPETITION RATE OF MEASUREMENT</td>
<td>6</td>
</tr>
<tr>
<td>IV. ALTERNATIVE SPARK GAP DESIGNS</td>
<td>6</td>
</tr>
<tr>
<td>DOUBLE ROD GAP</td>
<td>6</td>
</tr>
<tr>
<td>SINGLE ROD GAP</td>
<td>7</td>
</tr>
<tr>
<td>LAMINATED CONE GAP</td>
<td>9</td>
</tr>
<tr>
<td>V. EXPERIMENTAL PROGRAM</td>
<td>9</td>
</tr>
<tr>
<td>VACUUM APPARATUS</td>
<td>10</td>
</tr>
<tr>
<td>ELECTROLYTIC FLUID MAPPER</td>
<td>10</td>
</tr>
<tr>
<td>OBSERVATION OF DISCHARGE PATH</td>
<td>11</td>
</tr>
<tr>
<td>EFFECTS OF WIND</td>
<td>15</td>
</tr>
<tr>
<td>EFFECTS OF STRAY IONIZATION</td>
<td>16</td>
</tr>
<tr>
<td>VI. FINAL DESIGN</td>
<td>16</td>
</tr>
<tr>
<td>SPARK GAP STRUCTURE</td>
<td>17</td>
</tr>
<tr>
<td>ELECTRICAL CIRCUITRY</td>
<td>19</td>
</tr>
<tr>
<td>1. Control</td>
<td>19</td>
</tr>
<tr>
<td>2. Sawtooth Sweep</td>
<td>19</td>
</tr>
<tr>
<td>3. Telemetering Procedure</td>
<td>24</td>
</tr>
</tbody>
</table>
# TABLE OF CONTENTS (cont.)

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>VII. Calibration</td>
<td>28</td>
</tr>
<tr>
<td>VIII. Rocket Firing and Data Reduction</td>
<td>33</td>
</tr>
<tr>
<td>IX. Results of 13 September 1951 Firing</td>
<td>34</td>
</tr>
<tr>
<td>X. Utilization of Surface Density Information</td>
<td>37</td>
</tr>
<tr>
<td>XI. Discussion</td>
<td>38</td>
</tr>
<tr>
<td>XII. Results</td>
<td>41</td>
</tr>
</tbody>
</table>
LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Paschen's law curve for plane parallel electrodes in dry air.</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>Single rod gap.</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>Functional sketch of nose cone.</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>Plot of pre-breakdown electric field in gap section.</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>Photograph of breakdown around cone gap inside vacuum chamber.</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>Partially assembled nose cone.</td>
<td>18</td>
</tr>
<tr>
<td>7</td>
<td>Sawtooth voltage applied to spark gap.</td>
<td>20</td>
</tr>
<tr>
<td>8</td>
<td>Block diagram of breakdown experiment circuitry.</td>
<td>21</td>
</tr>
<tr>
<td>9</td>
<td>Breakdown circuit schematic (unpressurized).</td>
<td>22</td>
</tr>
<tr>
<td>10</td>
<td>Breakdown circuit schematic (pressurized).</td>
<td>23</td>
</tr>
<tr>
<td>11</td>
<td>Pressurized circuitry of voltage breakdown experiment.</td>
<td>25</td>
</tr>
<tr>
<td>12</td>
<td>Aft portion of instrumentation.</td>
<td>26</td>
</tr>
<tr>
<td>13</td>
<td>Functional diagram of breakdown experiment calibration.</td>
<td>30</td>
</tr>
<tr>
<td>14</td>
<td>Mean calibration curve and data from a typical calibration run.</td>
<td>32</td>
</tr>
<tr>
<td>15</td>
<td>Minimum local cone wall density vs. altitude, 0436, 13 September 1951.</td>
<td>35</td>
</tr>
</tbody>
</table>
I. INTRODUCTION

A density measurement based upon Paschen's law has several very attractive aspects as a method for obtaining an independent measurement of one of the flow parameters about a supersonic cone. For a given spark gap, the variation in sparking voltage depends essentially on the air density and is independent of temperature as such over a wide range. Also, if the gas discharge breakdown takes place with a sufficiently short formative time lag, the effects of the missile's motion through the surrounding air are unimportant, permitting calibration in stationary air. This type of measurement also provides a valuable means of cross-checking the data now being derived from other methods of instrumentation.

The design of the equipment for this experiment involved a great many choices between alternative methods of operation. A systematic evaluation of all possibilities would have necessitated a lengthy research program, hence many decisions were somewhat arbitrary. It is likely that further development would result in very substantial improvements in the reliability and accuracy of the measurement.

A brief description is given of some of the spark gap designs that were considered, with their merits and limitations in addition to a description of the apparatus which was finally constructed and flown. A discussion of the data reduction and results of the 13 September 1951 flight is also given.

II. PASCHEN'S LAW CONSIDERATIONS

The relation known as Paschen's law expresses the voltage at which gas discharge breakdown will occur between plane parallel electrodes. As usually stated, the sparking potential for a given gas is a function of the gas pressure, $p$, and the gap distance, $d$, and depends only upon the product $pd$, i.e.,

$$V_s = f(pd).$$
Figs. 1(a) and 1(b) are curves of Paschen's law for dry air. The abscissa of these curves is expressed in terms of the product \( p d \) because this is a convenient means of stating experimental data. However, the fundamental quantity that determines the breakdown voltage is the number of mean free paths between the electrodes. This quantity is proportional to the product \( p d \), where \( p \) is the gas density. Since most curves are taken at constant temperature, the form of the curve is the same for either variable. Actually, when the sparking voltage is expressed as a function of \( p d \), the breakdown voltage is essentially independent of temperature over a wide range. This independence of temperature, in connection with the present application, is very fortunate since density can be measured without a significant ambiguity arising from an unknown temperature.

As shown in Fig. 1(a), the minimum sparking voltage for air is approximately 300 volts, which occurs at a point where the (pressure x gap distance) product is about 6 mm x mm of Hg. The part of the curve to the right of this minimum can be verified experimentally in a fairly straightforward manner. The region to the left of the minimum presents a much more evasive experimental situation because the sparking potential is less for a long gap spacing than for a shorter one. Hence, the spark will tend to seek some devious roundabout path for which the breakdown voltage is less than for the shorter direct path. For this reason, a density-measuring experiment must be designed to operate only on that portion of the Paschen's law curve which lies to the right of the minimum sparking potential (high density branch of the curve).

The curves shown in Fig. 1 are for a uniform field (parallel plate electrodes). If the electrode geometries are such that the field is not uniform, then partial breakdown or corona may occur near one or both electrodes at a lower voltage than is required for complete breakdown. Under such conditions a modified form of Paschen's law is still applicable where the breakdown voltage is found by an integration process across the gap.

The partial breakdown condition is undesirable from the standpoint of the rocket measurement, because a corona current of negative or positive ions is subject to the motion of the air relative to the electrodes. The magnitude of such a streaming dependence has not yet been measured and depends, of course, upon the amount of the discrepancy between corona onset voltage and breakdown voltage.

---


2. Loeb, *loc. cit.*

FIG. 1a

PASCHEN'S LAW CURVE

FOR

PLANE PARALLEL ELECTRODES IN DRY AIR

VOLTS

PRESSURE X GAP LENGTH (mm x mm Hg)

FIG. 1b

PASCHEN'S LAW CURVE

FOR

PLANE PARALLEL ELECTRODES IN DRY AIR

VOLTS

PRESSURE X GAP LENGTH (mm x mm Hg)
III. EXPERIMENTAL CONSIDERATIONS

The design of this experiment involved a succession of compromise choices between a variety of conflicting requirements. Some of the more important experimental considerations are outlined below.

PLACEMENT OF ELECTRODES

In order to obtain meaningful data, the density measurement must be made in a region where the nature of the air flow is known. This means that the breakdown must occur either 1) in the undisturbed air ahead of the rocket where ambient conditions obtain or 2) behind a shock wave of known characteristics for which the flow conditions can be calculated. The first case might be approximated by means of spark-gap electrodes consisting of rods or probes projecting ahead of the rocket and of such small diameter that the intensity of the shock wave produced by them would be negligibly small. In the second case, where the breakdown occurs at a given position behind a known shock wave, the data must be treated through use of aerodynamic principles to obtain the ambient conditions. The reduction of such data involves many factors such as cone geometry and missile velocity, and must be corrected for the effects of yaw and boundary layer.

"Sphere gap" or Ragowski electrodes are generally used in a Paschen's law type of measurement, but they were not considered because of the obvious aerodynamic difficulties. The choice of the best type of spark gap was of major importance for this experiment. A discussion of the various factors affecting this choice is given in the next section.

RANGE OF MEASUREMENT

The anticipated maximum altitude of the missile determined the lower limit of density to be measured. For this missile it was anticipated that zenith would be about 70 kilometers, corresponding to a density of about $10^{-7}$ g/cc or a static pressure at room temperature of 0.045 mm of Hg. In order to determine this density and still keep to the right of the Paschen's law curve minimum, a gap length of approximately 20 cm (8 in.) is indicated [See Fig. 1(a)].

On the other hand, the maximum air density, which can be measured with a given spark gap spacing, is restricted by the voltage obtainable. A maximum of 4,000 to 5,000 volts represented a practical limit for a power
supply which could be conveniently packaged into the available space in an Aerobee rocket. This value would provide for a measurement of densities over a range of about 100 to 1 and thus represented a reasonable goal for a first experiment.

INDEPENDENCE OF AIR VELOCITY

An air density measurement should be independent of the velocity of the air stream past the missile. To fulfill this requirement the breakdown process must take place rapidly enough so that the distance moved by the missile during the breakdown is negligible. There are at least two ways to achieve such an independence of air velocity.

(1) If the voltage gradient throughout the spark gap is sufficiently uniform, the breakdown process will take place in a few microseconds when a critical voltage gradient is reached. During such a short interval the movement of the missile through the air is negligibly small. However, if the electric field in the gap is not uniform, partial breakdown will occur in the regions of high field strength and produce a space charge of ionized air molecules which could be subject to distortion by the air motion. Hence, partial breakdown from a non-uniform gradient would tend to be sensitive to missile velocity.

(2) An alternative method for avoiding the effects of relative air velocity is to apply the voltage to the spark gap in the form of a short pulse of the order of a few microseconds in duration. During such an interval, the missile will move only a fraction of an inch, and a calibration of the breakdown characteristics made in stationary air would thus be valid for large air velocities. The voltage pulses would have to be spaced far enough apart so that any ionization created by one pulse would not affect the succeeding one. The amplitude of successive pulses would be continuously increased until a breakdown was recorded. A suitable pulse technique would avoid the problems of a non-uniform field and the resulting wind effects; however, a great deal of time and expense would be required to develop the precise electronic equipment for generating and measuring the variable pulses of several thousand volts amplitude.
REPETITION RATE OF MEASUREMENT

During the flight of the missile, it is desirable to obtain as many separate density measurements as practicable. A straightforward method of measurement (if the electrode geometry provides a uniform electric field) is gradually to increase the voltage across the spark gap until breakdown occurs. If the rate of rise of voltage is not too rapid, this procedure results in an essentially d-c breakdown situation. This system suggests the use of a periodically repeating "saw-tooth" wave of voltage across the electrodes. In order to cover the required range from 0 to 5,000 volts with a rate of rise slow enough to avoid over-volting the gap, a period of several seconds is required for the saw-tooth sweep. This sets a limit upon the number of determinations which can be made during the time of flight.

Further, it is important that any electrical interference which might be created by the gas discharge breakdown should be kept to a minimum so as not to disturb other instrumentation on the same missile. This requires that the sparks must not occur too often. The use of the pulse technique previously mentioned would, in general, greatly complicate the problem of electrical interference.

IV. ALTERNATIVE SPARK GAP DESIGNS

A preliminary investigation was made of several different arrangements for producing spark breakdown in the vicinity of the nose of an Aerobee rocket. The salient features of these alternative designs are outlined below.

DOUBLE ROD GAP

In this arrangement two parallel rod or probe electrodes would project ahead of the rocket, and the breakdown would occur between them. If the spacing between the electrodes is small compared to their diameter, a sufficiently uniform field exists in the gap. However, if the diameter of the electrodes is small compared to their spacing, a highly non-uniform field exists, and the breakdown characteristics would be affected by the wind distortion of the pre-breakdown space charge.

Since the required gap length is about 8 in., very-large-diameter electrodes would be necessary to provide a uniform gradient in the gap.
These would be quite unstable for the nose of an Aerobee rocket. An additional complication would arise due to shock waves produced by such electrodes.

If a suitable pulse technique were used, a double probe gap arrangement with very small diameter rods might prove feasible. Partial breakdown near the electrodes would occur at a voltage considerably lower than would be required to produce complete breakdown of the gap. However, this highly non-uniform field and partial breakdown could be tolerated since any ion space charge formed by one pulse would be blown away before the succeeding pulse was applied, and each pulse would be applied to a fresh sample of air.

SINGLE ROD GAP

In another scheme, a single small-diameter tapered rod would project axially ahead of the missile as shown in Fig. 2. An 8-in. section of the base of this rod just ahead of the missile would be made of an insulating material. Ahead of this insulating section, the rod would be composed of metal, which would serve as one electrode of the spark gap. The nose cone of the rocket would act as the other electrode.

For this arrangement, the breakdown would occur along the length of the probe across the insulating section. Presumably, the tip of the rod would be made of dielectric material to decrease corona formation. This design is very "clean" from an aerodynamic standpoint; if the rod were constructed as a cone of sufficiently small angle, the shock wave from its tip would have negligible intensity, permitting the measured density to be essentially ambient.

This design presents the problem of breakdown over a dielectric surface. The sparking potential under this condition is usually lower and more erratic than across an air gap of the same spacing, due to the accumulation of stray electric charge on the surface, which tends to distort the electric gradient. If the surface charge could be controlled by maintaining the potential of every point along the insulating section at some predetermined value, it would be possible to eliminate erratic breakdown behavior. Further, it seems reasonable that the formation of pre-breakdown corona in the gap could be minimized if the potential distribution along the probe surface were properly maintained.

Several methods for controlling the potential distribution along the surface of an insulating section were tried. One system, which showed considerable promise at first, was to use a resistive coating over a supporting dielectric base. Several types of coatings were tried. One surface employed lampblack rubbed into a sheet of rubber; another used
Fig. 2. Single rod gap.
lampblack suspended in glyptol enamel and painted on glass. However, the resistance of these surfaces was difficult to control and was quite non-uniform. Another resistance coating was a layer of platinum sputtered onto glass. This was likewise abandoned because of poor adhesion between the glass and the platinum, and also because of the difficulties of controlling the amount of platinum deposited and, consequently, the resistance of the coating.

A system which finally did prove suitable for controlling the surface potential distribution used a gap section consisting of alternate metal and dielectric discs which formed a laminated "stack". The potential distribution along this "stack" was controlled by connecting each of the metal discs to an appropriate point on a resistance voltage divider. If sufficient laminations are used, this scheme approximates a continuous resistance coating. It also has the advantage that the potential distribution can easily be adjusted by changing the value of the resistors in the voltage divider.

LAMINATED CONE GAP

The difficulty with the single-rod gap described above is the non-uniform electric field which exists about the electrodes. For a small-diameter rod, the radial gradient at the electrode surface is considerably greater than the longitudinal gradient along the spark path parallel to the rod. In order to avoid partial breakdown and the formation of space charge, the electric field must not, at any point, greatly exceed the average gradient throughout the gap.

This problem can be solved by using a rod large enough in diameter so that the radial gradient is not appreciably higher than the average longitudinal gradient. Inasmuch as a rod of larger diameter was indicated, it seemed logical to incorporate the gap as an integral portion of the missile nose cone. This would require, of course, that the measurements be made behind a strong shock wave and that the resulting data be treated by an aerodynamic analysis to obtain the ambient density.

V. EXPERIMENTAL PROGRAM

The preliminary investigation of alternative spark-gap designs outlined above was carried out in an 18-in. bell jar with appropriate vacuum pumps and pressure gages. Each type of gap was evaluated by comparing its breakdown characteristics with the Paschen's law curve in Fig. 1 and by
observing visually the path of the spark discharge and any pre-breakdown corona.

Since the results of the preliminary tests of the laminated gap were quite favorable, a thorough investigation was made of its properties and the problems associated with it. Cylindrical geometry was used for most of the tests in place of the conical shape eventually required, because of the greater ease of fabrication and analysis. These test gaps were about 6 in. long and were composed of stacks of 1/4-in.-thick brass or magnesium disks 4 in. in diameter, separated by thin rubber disks. An axial hole through the center of the stack provided space for the voltage divider resistances.

VACUUM APPARATUS

A steel vacuum box, 32 by 28 by 30 in., having a 1-in.-thick plate glass window on one end was used for subsequent tests of the laminated gap. A chamber of this size was necessary in order to evaluate the large gap lengths required, while minimizing extraneous effects due to the walls of the vacuum box. A National Research Alphatron pressure gage provided pressure information, and a mercury thermometer was used for determining the gas temperature, thus permitting a density determination.

In order to maintain the purity of the air in the box, a continuous fast-pumping vacuum system was used, and dry air was fed into the box at a rate sufficient to obtain the desired equilibrium pressure. For the initial experiments, a 15 cu ft/min mechanical vacuum pump was used, but more accurate results were later obtained through the use of a 3-in. D.P.I. oil diffusion pump having a capacity of 100 liters per second. The air fed into the chamber was thoroughly dried by passing it over "Dehydrite" drying agent. The drying was quite essential in order to obtain reproducible results.

ELECTROLYTIC FLUID MAPPER

An electrolytic fluid mapper was constructed in order to study the electric field which existed in the vicinity of the spark gap prior to breakdown. Since the laminated type of gap design had cylindrical symmetry, a relatively simple fluid mapper was quite suitable for obtaining two-dimensional field plots. The effects of changing the potential distribution along the laminated gap surface (by changing the resistors between gap segments) could readily be observed, and thus the mapper proved to be of considerable value for determining the conditions which produced the most uniform field in the gap.
Several modifications of the laminated type of gap were tried, including double gaps and guard-ring arrangements. A wide variety of potential distributions along the gap surface were also investigated. These designs were evaluated; first, by obtaining an electric field plot from the fluid mapper and, secondly, by observing the discharge path and the breakdown characteristics in the vacuum chamber. Figure 3 is a sketch of the design of the nose cone and gap which was finally developed. The details of gap construction illustrated in this figure will be discussed in the next section. Figure 4 is a plot of the electric field obtained by the fluid mapper technique for the gap design shown in Fig. 3.

The fluid mapping analysis was also very useful in conjunction with several auxiliary experiments. In one of these tests, a floating probe in the form of a ring around one of the electrodes was employed to investigate the field distortion just prior to breakdown. In another experiment, the potential of the vacuum box with respect to the spark-gap electrodes was varied in order to determine if the presence of the walls of the box had any effect upon the breakdown characteristics. It was shown that the walls of the vacuum box had a negligible effect upon the breakdown if the box was maintained at a potential roughly midway between the electrode potentials.

OBSERVATION OF DISCHARGE PATH

The path followed by the discharge was of considerable interest. It was desirable to have the breakdown region as far removed from the cone surface as possible, in order to avoid the complexities of the air boundary layer which are present for supersonic flow conditions. It was found experimentally that for a given gap and potential distribution along the laminated stack the location of the visible spark path depended upon the air density. It tended to be farther away from the surface at lower densities and became progressively closer as the density increased. Changing the potential distribution along the surface of the gap also had a moderate influence upon the discharge path.

Figure 5 is a photograph of a gap section under test, taken through the plate glass window which covered one end of the vacuum box. This photograph is a composite exposure showing the portion of the nose being tested and the actual breakdown across the gap. The pressure inside the vacuum chamber when the picture was taken was about 0.6 mm Hg. Two or three breakdowns were superimposed to produce this photograph. For these conditions, the discharge path midway between the electrodes is approximately 1 in. away from the surface of the cone.

There is some evidence that the initial stages of the breakdown take place along a path which is farther from the cone wall than the path observed
Fig. 3. Functional sketch of nose cone.
FIG. 4
PLOT OF PRE-BREAKDOWN ELECTRIC FIELD IN GAP SECTION.
Fig. 5. Photograph of breakdown around cone gap inside vacuum chamber.
in Fig. 5. Flux maps obtained from the fluid mapper show that this would be the case if the breakdown followed the undistorted pre-breakdown electric field lines. The actual discharge path is influenced by several factors depending ultimately upon "least energy" considerations. There is always a tendency to shorten the length of a discharge column so that as the spark channel grows it will tend to enlarge more rapidly on the short side next to the cone. However, the loss of charged particles from the spark plasma will be greater next to the cone wall because of diffusion to this surface and recombination on it. The luminous path which can be observed is probably the end result of this growth process and represents a more or less equilibrium situation. The initial breakdown streamers probably occur along a path which is certainly no closer to the cone than the observable luminous plasma and in all probability is farther out, following more nearly along the electric flux lines.

EFFECTS OF WIND

The possible effect of supersonic air flow upon the accuracy of the measurement was a factor which could not be evaluated in the vacuum chamber. The only experiment made in this direction consisted of mounting a small propeller-type fan near the spark gap. A 3-in.-diameter fan turning at 8000 rpm did not produce enough air flow to have any observable effect on the breakdown voltage.

The sparking potential would presumably be independent of stream velocity if the breakdown occurred by avalanche and streamer propagation, and no pre-breakdown space charge was formed. This condition was not completely realized in the final laminated gap design. The field map in Fig. 4 shows a greater field strength near the electrodes than in the center of the gap, although this potential distribution resulted in conditions for which no pre-breakdown glow or dark current could be detected.

It seems unlikely that any significant error in density measurement was caused by wind effect, since calculations indicated that the pre-breakdown field strength in the weakest field part of the gap was sufficient to cause an ion drift velocity of approximately 38 times the missile velocity.

As an additional consideration the cathode was selected as the downstream electrode so that the positive ions were moving in the same direction as the air stream. The electron drift velocity is so high in comparison that the effect of stream velocity on them should be insignificant.

One undesirable effect of air motion is that due to yaw of the missile. This causes the air to flow past the cone in a skewed manner and results in an unsymmetrical density distribution around the circumference of
the spark gap. Under these conditions the spark would take place on the side of the gap where the density is at a minimum. The error in the measurement caused by this effect can presumably be corrected, since the attitude of the missile is known from gyroscopic data, and the relations for air flow around yawed cones are available in tabular form for small angles of yaw.

EFFECTS OF STRAY IONIZATION

There was some uncertainty regarding the possible effect of stray ionization and ozone upon the breakdown characteristics of the gap. To assist in evaluating this factor, a test was made with an auxiliary discharge running in one corner of the same vacuum chamber in which the gap was being tested. This auxiliary glow discharge took place between two electrodes about 3 in. apart and operated at a current of about 0.5 amperes. For most of the density range concerned, the effect upon the breakdown characteristics was not detectable. For the lowest densities investigated, however, there was a slight lowering of the breakdown potential which amounted to about 10% at 0.1 mm Hg.

A small amount of initial ionization was desirable in order to minimize the statistical deviation from the Paschen's law curve. In the vacuum box, this ionization was obtained from a source of gamma radiation in the vicinity of the gap. In the case of the missile instrumentation, this ionization was produced by radiation which came from alphatron pressure gages located just aft of the gap section as seen in Fig. 3. The intensity of this radiation was about 15 to 25 milliroentgens per hour at the cathode surface.

A further precaution was taken to minimize extraneous effects from the ozone in the upper atmosphere, in that the missile was fired at night, at which time the ozone layer is least active. Also, a night firing avoided the problem of having to account for the photoelectric emission from the cathode. This problem would result from the intense solar ultra-violet radiation at high altitudes.

VI. FINAL DESIGN

The final design for the rocket-borne spark gap and the associated electrical circuitry was rather arbitrary in certain respects, as a thorough investigation of the various alternatives was not warranted for this first experiment. The guiding considerations were simplicity and reliability,
consistent with the objectives of this first experiment. A substantial increase in the accuracy of this type of density measurement would presumably result from a program of further development.

THE SPARK GAP STRUCTURE

The general features of the rocket nose-cone design are shown in Fig. 3, while Fig. 6 is a photograph of the partially assembled nose cone. The gap section was a laminated, truncated cone incorporated as an integral part of the missile nose, consisting of 24 magnesium disks separated by 1/16-in.-thick Lamicoid sheets (Lamicoid is a phenolic laminate). The 6 top and 6 bottom magnesium disks were relatively thin (1/8 in.), since it was desirable to have greater control of the voltage gradient in these regions. In the center of the gap where control of the gradient was not as critical, the disks were 1/4-in. thick. The whole stacked assembly was held together by four 1/2-in. fiber bolts passing parallel to the cone axis through close-fitting holes in the magnesium and Lamicoid laminations. The base of the gap section (cathode) was fastened to the rest of the rocket by means of a breech-block type of screw consisting of three sections of 1/8- by 1/8-in.-square thread, while the tip of the cone (forward portion of the anode) ahead of the gap, was secured by an ordinary screw thread.

It was necessary to provide for a ram-pressure measurement in connection with the Alphatron pressure measurements. The ram-pressure Alphatron was located aft of the gap section, and a glass connecting tube extended through the gap section to the gage, as shown in Fig. 3. This ram pressure turbulence could not exceed 20 in., due to gas-conductance considerations, and thus fixed the maximum aft position of the spark gap section.

In order to obtain the most desirable electric field throughout the gap, as discussed in section IV, it was, in general, desirable to maximize the diameter of the gap section. However, with the aft electrode (cathode) limited to 20 in. from the cone tip and with a gap length of about 8 in., the resulting anode diameter was only about 3 in. and resulted in a somewhat undesirably strong radial field at its surface. Thus, as a compromise to minimize the radial field at the anode, the gap section length was reduced to 6 in., resulting in an anode diameter of 3.67 in. This shortened gap length did not seriously restrict the low density limit of the gap because of the tendency for the discharge to curve out away from the surface as seen in Fig. 5.

A 1-in.-diameter clearance hole was provided along the geometric axis of the gap section, in order to accommodate the glass tubing that connected the ram pressure port at the tip of the nose cone with the proper Alphatron gage. This hole also served to pass the high voltage cable to the
Fig. 6. Partially assembled nose cone.
top of the gap, and to accommodate the voltage-gradient-determining resistors which were connected between each magnesium lamination.

The most difficult problem associated with the laminated gap was to determine the optimum value for the 25 voltage-gradient-determining resistors. This was accomplished with the aid of the electrolytic fluid mapper previously described. Figure 4 is a plot of the electric field for the final design and also shows the percentage of the total resistance between adjacent magnesium segments. As there are many suitable combinations of resistance, no effort was made to extend this investigation beyond the requirements of this experiment.

As a precautionary measure, glass wool was packed around the ram-pressure tube and the high-voltage cable so as to prevent the gap from breaking down internally, as occurred several times in laboratory tests. The glass wool completely eliminated this difficulty.

ELECTRICAL CIRCUITRY

The electrical circuitry associated with the spark-breakdown air-density measurement had to provide: (1) the proper external switching control of the apparatus, (2) a sawtooth waveshape of voltage from 0 to 5,000 volts, and (3) a satisfactory method for supplying breakdown information to the missile telemetering system. Numerous arrangements for achieving these requirements were considered; however, in the interests of reliability and simplicity the following arrangement was adopted. Figure 8 is a functional block diagram of the associated circuitry, while a complete schematic wiring diagram is given in Figs. 9 and 10.

1. Control.—The control function was performed by a Ledex rotary solenoid stepping relay (Fig. 9) which applied power to the various components in the desired manner. It was actuated from the control panel in the block house. The main power was furnished from 12- and 24-volt battery supplies included in the instrumentation.

2. Sawtooth Sweep.—The sawtooth voltage sweep which was applied across the gap is illustrated in Fig. 7. This sweep had a maximum value of about 5,000 volts and a period of about 3.5 seconds. As the voltage increased, a point was reached at which the gap broke down, this point varying with the density of the air surrounding the gap.

The sawtooth sweep originated with a motor-driven Variac (Fig. 9) which furnished 400-cycle power of varying amplitude to operate the high-voltage power supply (Fig. 10). As the Variac was rotated, the power to the high-voltage supply increased from 0 to 115 volts in a saw-tooth manner.
Fig. 7. Sawtooth voltage applied to spark gap.

\[
\frac{dv}{dt} \approx 1.5 \text{ KV/SEC}
\]

\[
5 \text{ KV MAX.}
\]

\[
\sim 3.5 \text{ SEC.}
\]
FIG. 8

BLOCK DIAGRAM OF BREAKDOWN EXPERIMENT CIRCUITRY
Fig. 9. Breakdown circuit schematic (unpressurized).
Fig. 10. Breakdown circuit schematic (pressurized).
The 115-volt, 400-cycle power in turn was supplied by a rotary inverter operating on the main 12-volt missile power. The Variac was a General Radio type 200B, which operated well at 400 cycles. A fiber cam lifted the Variac brush for 65° of rotation over the junction of the windings to provide the return to 0 volts; thus, the voltage rise lasted for 295° of rotation, or about 2.9 seconds.

The high voltage power supply (Fig. 10) was a conventional voltage doubler using selenium diodes for rectifiers rather than thermionic tubes. This eliminated the difficult problem of filament voltage supplies and saved considerable space. A capacitor (C3 - Fig. 10) was placed directly across the lead to the spark gap so that breakdown occurred as a capacitor discharge with an abrupt drop in voltage at that instant. A resistance (R1 - Fig. 10) was used to effectively isolate the power supply from the condenser and gap during breakdown.

Because of the high voltages employed, many of the circuits had to be pressurized. Thus, the high voltage power supply, indicating circuits, and calibration circuits (i.e., all the circuitry shown in Fig. 10) were placed in the airtight brass container partially illustrated in Fig. 11. Electrical connections were made through a pressure-tight Amphenol fitting; however, a section of RG8U coaxial cable was used to connect the high voltage supply to the gap, and this cable was passed through the cover by means of a stuffing gland arrangement. All components were mounted on the cover of the can to permit accessibility.

The pressurized container is shown in Fig. 12 labeled "0- to 5000-volt power supply" while the remaining unpressurized voltage breakdown circuitry (schematic, Fig. 9) may be seen directly below it. The 115-volt, 400-cycle rotary inverter can also be seen in Fig. 12.

3. Telemetering Procedure.—Consideration was given to two ways in which the breakdown information could be telemetered, namely, (a) as a signal voltage proportional to the breakdown voltage, or (b) as a time measurement of the interval between the start of the sweep and the time at which breakdown occurred. For a telemetering signal with a maximum excursion of 5 volts, an amplitude modulated signal can be read with an accuracy of about one part in 50. The telemetering system had a prf of 569 pulses per second and, since each sawtooth sweep lasted about 3.5 seconds, this provided a time base with high resolving power. A comparison of the two systems showed that the time measurement procedure had inherently far greater accuracy and thus was adopted.

The method of timing the sweep to breakdown, of course, required a knowledge of the shape of the voltage sweep applied to the gap. This was obtained from prefight calibration. Additionally, in order to minimize the
Fig. 11. Pressurized circuitry of voltage breakdown experiment.
Fig. 12. Aft portion of instrumentation.
errors caused by variations of battery voltage and circuit components during flight, several points along the rise of the applied voltage sweep were indicated by means of "marker" signals. Deviations from the preflight calibration of the applied sawtooth could thus be determined by means of these reference markers.

The most useful criterion of breakdown was the abrupt drop in voltage across the gap which occurred at the instant of breakdown. The magnitude of this voltage drop varied anywhere from 35 to 5,000 volts, depending upon the density. Thus, it was necessary to provide clipping and amplifying stages \( T_1, T_2, T_3 \) - Fig. 10 in order to provide a uniform signal as an indication of each breakdown, and hence each density determination. The breakdown-indicating signal was applied to the missile telemetering system through a cathode follower mixer stage (output mixer - Fig. 8 and \( T_{11} \) - Fig. 9) along with calibration marker pulses to be mentioned subsequently.

A negative-going signal from the indicating circuits was also used to de-energize the power supply once the gap had fired. This was achieved by means of a thyatron-and-relay arrangement \( (T_4 \) - Fig. 9) which interrupted the power from the Variac to the high voltage supply, the power supply being operative when the relay was energized. The relay was operated from the 115-volt, 400-cycle supply by placing appropriate capacitance and resistance across the relay coil \( (C_7 \) and \( R_{10} \) - Fig. 9). The thyatron was normally conducting on the positive half cycles of the 400-cycle power, but the negative trigger pulse from the indicating circuit at the time of breakdown was applied to its grid, thus preventing conduction and de-energizing the relay coil. The relay remained de-energized for the balance of the sweep, but was reset for the succeeding sweep by means of a cam switch mounted on the Variac (Fig. 9) and actuated during the Variac "dead time."

Calibration circuits provided marker pulses whenever the sweep voltage reached a value of about 350, 800, 1400, and 3000 volts, each pulse being of different, and hence distinguishing, length. These marker pulses were obtained from small neon glow tubes which, under the proper conditions, had a consistent firing potential. It was found that General Electric type NE-2 neon lamps were satisfactory for this use, having a variation in firing voltage of a fraction of one percent if exposed to moderately bright light or \textit{gamma} radiation. To insure this accuracy in flight, a small radium source was placed close to the neon tubes, which were clustered to accommodate the radiation pattern. The calibrating neon lights and radium source can be seen in Fig. 11.

Four similar circuits employing the neon tubes were used to obtain the four desired marker pulses \( (\text{Circuits A, B, C, and D - Fig. 10}) \). Considering any one of these circuits separately, a balanced resistor-capacitor
voltage divider was placed across the high voltage bus to the gap. The neon tube was across the lower leg of the divider, and as the gap voltage increased, a point was reached at which the tube would fire, depending upon the divider ratio. A 60-volt biasing battery was employed so as to increase the effective firing voltage of the neon tubes and thus reduce the divider ratio, in this way minimizing the effects of any changes in component values after the calibration checks.

When the neon tube fired, a positive signal with an abrupt rise and an exponential decay appeared across the tube load resistance (R₃ - Fig. 10). This pulse was coupled to the cathode follower mixer stage (T₈ - Fig. 9) and so appeared on the telemeter film with the breakdown pulse. The length and shape of each marker pulse was varied by adjusting the values of the coupling capacitor and resistor (C₃ and R₄ - Fig. 10), thus giving it a distinguishing appearance on the telemeter film.

It was found necessary to add a mono-stable multivibrator circuit (T₆ and T₇ - Fig. 9) in order to eliminate extraneous signals from the calibration circuit at the time of breakdown. This circuit was triggered from the indicating circuit (T₃ - Fig. 10) and biased the marker pulse output below cut-off for about 1/2 second.

As a means of obtaining qualitative information in the event of telemeter failure, the voltage existing across the spark gap was displayed on a meter which was photographed by the missile-borne camera. A cathode follower (T₅ - Fig. 9) was included in the circuit to operate the 1-MA movement of this meter. Provisions were made so that this voltage could also be displayed on a meter in the control box in the blockhouse. This indication furnished a check of the functioning of the equipment immediately prior to flight.

VII. CALIBRATION

A reasonably reliable calibration was required since the interpretation of density data from the spark-breakdown experiment was based upon a comparison of flight values with corresponding results obtained under known conditions. However, it was considered unnecessary to secure a high order of accuracy since this first flight was to be primarily an exploratory experiment for evaluating this method of instrumentation.

The information obtained from the telemetering record was basically a time measurement, i.e., the time between the marker pulses and the breakdown pulse. In order to derive a density from these data, it was necessary
to interpret the time measurement as a voltage and then to convert the voltage to a density. Three different preflight calibration steps were thus required: (1) a determination of the shape of the sawtooth voltage sweep applied to the spark gap, (2) a measurement of the voltages at which the "marker" signals occurred, and (3) an experiment to obtain the relation between the breakdown voltage of the spark gap and the density of the surrounding air.

Figure 13 illustrates the most successful method which was tried for calibrating the sweep, marker signals, and breakdown voltage. The calibrating circuit consists essentially of a potentiometer bridge with a d-c oscilloscope used as an instantaneous balance (zero) indicating device. A balanced resistor capacitor voltage divider \((R_1C_1, R_2C_2)\) was placed across the gap because of the high voltages involved. This divider provided a working signal of convenient magnitude. The gap voltage was thus recorded in terms of an arbitrary reference scale determined by the divider ratio.

In this arrangement the "ground" potential of the oscilloscope could be set at any desired value by means of the potentiometer \(R_3\). When the oscilloscope trace was at the zero line on the scope face, zero voltage existed across the input terminals, and the voltage across the lower leg of the voltage divider was the same as read on the laboratory voltmeter placed across the potentiometer. Thus, the bridge was in balance at the instant that the scope trace crossed the zero line. The oscilloscope sweep was triggered to begin at the same time as the saw-tooth voltage applied to the gap and was adjusted to have approximately the same duration. A Tektronix type 512 oscilloscope was used for these measurements. This arrangement provided a convenient and reasonably accurate method for obtaining simultaneous measurements of voltage and time.

The saw-tooth sweep applied to the gap was not strictly linear, being affected by many factors (such as characteristics of the rectifiers in the power supply). Therefore, it was necessary to obtain a point-by-point plot of its exact shape. The time scale used for calibration and data reduction was normalized so as to allow for slight variations in the rate of rise and total period of the sawtooth. This normalization was achieved, in effect, by matching slopes for corresponding voltage intervals. By plotting the gap voltage in this way it was found that the relative shape of the sawtooth was very nearly constant for moderate variation of battery voltage.

A measurement of the voltages at which the marker signals occurred was also determined by the arrangement in Fig. 13. The telemetering cathode follower stage had a sufficiently low output impedance to produce large, sharp, negative-going signals on the ascending scope trace when tied to the scope ground as illustrated. The voltage (on the reference scale) at which
Fig. 13. Functional diagram of breakdown experiment calibration.
each of the marker signals occurred was measured by adjusting the potentiometer $R_3$ so that the oscilloscope trace just touched the zero line at the leading edge of the marker pulse.

Further, the gain of the vertical amplifier of the oscilloscope was adjusted so as to obtain a known vertical deflection factor. Slight variations of the voltage at which these signals occurred could easily be measured by noting the highest point reached by the trace with respect to the scope zero line. It was possible to determine variations of voltage to an accuracy of better than one part in one thousand with this arrangement.

The third calibration step consisted of determining the relation between the breakdown potential of the spark gap and the density of the air surrounding it. The gap and portions of the nose cone fore and aft of the gap section were placed in the vacuum box as described earlier (See Fig. 5). A three-in. oil-diffusion pump was used to evacuate the chamber in order to obtain a very high pumping speed. Thoroughly dried air was bled into the chamber at a rate sufficient to maintain the desired equilibrium density. This method greatly reduced contamination from unknown vapors. When slow pumping speeds were used, some variation was noticed in the breakdown voltage at the lower density, probably due to contamination. A National Research Alphatron ionization type gage was used to measure the box pressure and a mercury thermometer gave the temperature, making a calculation of density possible.

The voltage at which breakdown occurred was determined in the same manner as described above for the marker signals. The variations in the value of breakdown voltage were also measured in the same way. By recording these variations, for 20 or 30 consecutive sweeps, it was possible to make a statistical evaluation of the breakdown behavior at each equilibrium density. This procedure was followed over the complete range of densities and for various pumping speeds, etc. In this way, a reasonably accurate and reproducible calibration of breakdown voltage vs. vacuum box density was obtained. The variation between different runs was usually within one standard deviation of the mean, and the distribution about the mean for all runs was roughly Gaussian (normal distribution).

Figure 14 shows the mean calibration curve obtained and the experimental points and standard deviation for a typical run. The scatter of points between the densities of $1.5 \times 10^{-6}$ and $3.0 \times 10^{-6}$ g/cc was due primarily to errors in reading the pressure gage and not in a determination of voltage, as appears to be indicated in Fig. 14. Also shown in this figure, for the sake of comparison, is the theoretical Paschen's law curve for a parallel plate electrode gap of 157 mm spacing (6 in.). The probable reason that the measured breakdown voltage was higher than the theoretical Paschen's
FIG. 14
MEAN CALIBRATION CURVE AND DATA FROM A TYPICAL CALIBRATION RUN
law curve is that the actual breakdown path length is greater than the 157 mm assumed for the theoretical curve. This increase in path length is discussed on pages 11 and 15.

VIII. ROCKET FIRING AND DATA REDUCTION

An Aerobee rocket, carrying the voltage breakdown air density measuring equipment, was fired 13 September 1951 at Holloman Air Force Base, New Mexico at 0436. This time was selected because it was a few minutes before sunrise at an altitude of 100 miles. Firing at this time of day minimized the problems arising from the sun's ultra-violet radiation, as mentioned previously. The rocket reached a maximum altitude of about 70 kilometers, and useful data were obtained from approximately 32 to 65 kilometers. The instrumentation appeared to function normally and, upon recovery and return to Ann Arbor, it was sufficiently intact to be operated.

The telemetering record of the voltage-breakdown experiment was reasonably complete. Therefore, the supplementary information from the missile-borne camera was not used. A reduction of the telemetering film consisted roughly of three steps: (1) a numerical tabulation of values obtained from the telemetering record, (2) a determination of the rate of rise of the sawtooth, i.e., a reduction of the tabulated data to the normalized time scale, and (3) a determination of the density by the use of a calibration curve.

The telemetering record consisted of several hundred feet of 35 mm film with a continuous string of dots running the entire length of the film. The lateral displacement of the dots provided the indication of the marker signals or breakdown. Each of these signals produced displacements in a characteristic manner. The job of tabulating the telemetering film consisted essentially of counting the number of dots between these different signals. The dots occurred with a repetition frequency of 569 per second and, since there were approximately 100 seconds of useful data, this represented a rather formidable counting task.

To facilitate this job, a grid of numbered lines was constructed so that the spacing between lines corresponded approximately to the spacing between the dots as they appeared on the film reader. The marks on the grid could be lined up with the dots on the film by making slight adjustments of the grid as the film was moved through the reader, so it was relatively easy to obtain a one-to-one correspondence between the dots and the lines on the grid without having to perform any counting. When a marker or breakdown signal was observed, the number of the corresponding grid mark was recorded.
With this method it was possible to read through interfering signals which often looked much the same as the desired signals. Also, a good estimate of the number of dots between signals was obtained when "holes" occurred in the record. The telemetering film from two different recording stations was read and, since the "holes" in the two films were often complementary, breakthrough data were obtained which were about 80% complete. A reasonably good estimate of the remaining data was also achieved.

The second step in data reduction consisted of calculating the number of dots corresponding to a unit of the normalized time scale. This step was essentially a calculation of the rate of rise of the sawtooth for the intervals between the marker signals being observed. This calculation was based upon information from the preflight calibration experiments and the assumption that the marker signals occurred at the same voltages in both cases.

The third step was to derive the density corresponding to each breakdown. A calibration curve was constructed which gave density directly in terms of the normalized time scale. By applying the appropriate rate of rise to the tabulated number of dots at breakdown, the density corresponding to each breakdown measurement could be read directly from this signal calibration curve.

IX. RESULTS OF 13 SEPTEMBER 1951 FIRING

The densities obtained from the spark-breakdown experiment are plotted in Fig. 15 as a function of altitude. These densities are to be construed as the minimum density which existed in the region adjacent to the surface of the missile nose cone at the time of breakdown. They are the direct "raw" data from the Paschen's law density measurement and, as such, are not immediately usable for obtaining ambient conditions but require corrections for several effects. Only the data for the ascent (from 32 to 65 km) are plotted in Fig. 15, although values were obtained through zenith and for the descent. The data for the descent are in rough agreement with the values shown in Fig. 15.

The spark-breakdown experiment was evaluated by comparing the density information derived from it with values of surface density calculated from the Alphatron pressure measurements. The Alphatron pressure gages constituted the primary instrumentation carried by this rocket.¹ These calculated densities

FIG. 15
MINIMUM LOCAL CONE WALL
DENSITY VS. ALTITUDE
0436, 13 SEPTEMBER 1951

DENSITIES DERIVED FROM BREAKDOWN EXPERIMENTS
DENSITIES DERIVED FROM ALPHATRON PRESSURE MEASUREMENTS
are also shown in Fig. 15 and are believed to be very close to the true density which existed adjacent to the cone and just on top of the boundary layer region.

In general, a supersonic missile is a yawing body—that is, the longitudinal axis of the missile is not aligned with the velocity vector of the air flowing past it. This slight, sidewise motion through the surrounding air causes an unequal distribution of density around the circumference of the cone. A gas discharge breakdown will prefer a region of lowest density; therefore, the densities measured by breakdown should always correspond to the minimum surface density which exists at that time.

It is possible to calculate a correction factor by which the "minimum yawed density" can be converted to a value for the surface density on a hypothetical unyawed cone and vice versa. This correction factor is based on an expansion of perturbation coefficients involving the angle of yaw. The yaw angle can be calculated for any instant from a knowledge of the missile trajectory and data from the missile-borne gyroscope. In Fig. 15, the calculated densities based upon the Alphatron pressure measurements have been modified to include the effects of yaw, and therefore are the calculated minimum cone wall densities. The yaw correction factors used in Fig. 15 varied from 0.988 to 0.916, so the discrepancy between the surface density on an unyawed cone and the minimum cone-wall density experienced in this experiment is a relatively small factor.

These calculated densities were obtained from a knowledge of Mach number and ambient density derived from the primary instrumentation flown. This instrumentation provided for a measurement of (1) the ratio of the nose-cone tip (impact) pressure to the pressure at a point on the cone wall, (2) the instantaneous angle between the rocket's longitudinal axis and a fixed reference system, and (3) the magnitude of the missile velocity vector in the same reference system.

A comparison of the Paschen's law-derived densities with corresponding densities calculated from an established method of measurement was considered essential for the proper overall evaluation of the Paschen's law breakdown experiment. The choice of the minimum local surface density appeared to provide the most realistic basis for this comparison and facilitated the interpretation of the results. As can be seen in Fig. 15, the densities resulting from the Paschen's law experiment are consistently lower than the calculated density based on pressure measurements. Also, the spark-breakdown densities exhibit a larger variation about a line through their mean value than is present for the calculated (true) densities. The probable reasons for these discrepancies are discussed in Section XI.
X. UTILIZATION OF SURFACE DENSITY INFORMATION

In a general sense, it is necessary to make three independent measurements in order to determine the flow about a supersonic cone. A reliable technique has been developed at the University of Michigan in which the total head pressure and the cone-wall pressure on a right circular cone are measured.\(^1\) This is combined with velocity information obtained from ground-based radar beacons in order to obtain a measurement of ambient atmospheric conditions. The cone-wall density derived from a Paschen's law type of measurement offers the possibility of obtaining a fourth independent measurement, and thus provides a possible means for cross-checking the present measurements, or for eliminating the complex ground-radar installation required to measure missile velocity.

The most effective way to utilize the surface density information probably would be to incorporate into the missile instrumentation an additional experiment for measuring the impact or total head density. The ratio of total head to surface density would lead to the calculation of a Mach number. This could be used to cross-check the present system for obtaining Mach number, but in order to arrive at ambient conditions this would require velocity data of some sort.

The difficulty with this method is that the density ratio exhibits a very slow variation with Mach number. Therefore, it would be necessary to obtain four-place accuracy in density in order to obtain three-place accuracy in Mach number. This is to be contrasted with the existing technique of measuring pressures, in which the pressure ratio changes rapidly with Mach number.

Another way in which the surface-density measurement might be used would be to combine it with the present total head and cone-wall pressure measurements in the following manner: The pressure measurements would be used to derive a Mach number in the manner currently being employed. The cone-wall density and cone-wall pressure could be combined to obtain a cone-wall temperature. With the Mach number already obtained, it would be possible to find the temperature ahead of the shock from the cone-wall temperature. This would be the ambient temperature. By combining this with the pressure and Mach number already determined, all of the ambient conditions could be derived. This system would free the missile from the necessity of the ground-based radar tracking which is now needed for obtaining velocity data.

XI. DISCUSSION

The data presented in Fig. 15 shows a systematic discrepancy between the density values derived from the Paschen's law experiment and those obtained from the Alphatron pressure measurements. It is believed that the pressure-calculated values are very close to the true densities which existed just outside of the boundary layer region. Therefore, this discrepancy must be due to either a systematic error in the experimental procedure or a fallacy in the initial assumptions upon which this experiment is based.

An analysis of the probable errors in the calibration, data reduction, etc. indicated that the experimental procedure should be accurate to at least five parts out of a hundred. The discrepancy in Fig. 15 is considerably greater than this; therefore, it is necessary to reconsider the basic assumptions underlying this experiment, which are:

1. That the breakdown behavior of the spark gap is unaffected by the supersonic air stream adjacent to the cone, i.e., that the breakdown occurs in the same manner in a supersonic flow as in static air.

2. That the boundary layer region is negligibly thin, and thus the indicated density is the density that exists on top (outside) of the boundary layer.

The validity of the first assumption would seem to depend primarily upon the duration of the formative time lag required for breakdown once the sparking potential has been reached, and upon the formation of any pre-breakdown space charge. There is considerable experimental evidence that at high densities and for parallel electrodes, the formative time lag is a small fraction of a microsecond. However, fairly recently (since this research was undertaken) there has been some evidence that at low densities (near the Paschen's law minimum) and for a small amount of over voltage, the formative time lags may be as long as several hundred microseconds.¹

However, as discussed in this report, calculations of ion mobility indicate that the breakdown process should not be seriously affected by a


long formative time lag. It is not known to what extent these factors affect the present results. However, it is probable that they would tend to lower the breakdown potential and thus might be a partial explanation for the discrepancy in Fig. 15. A thorough study of the effects of the relative stream velocity past the gap should be undertaken before this type of experiment is carried any further.

The second assumption is admittedly an approximation of the truth and probably represents the weakest point in the experiment. Some form of boundary layer always exists, and the breakdown has to take place through this region of varying density. The assumption made in this experiment is that the boundary layer is negligibly thin, and that the major part of the breakdown process occurs outside of this region.

Present information on boundary layer thickness indicates that it varies from 1/16 to 1/2 in. in thickness for the range of velocities and densities involved here. Observations of the breakdown in the test vacuum chamber showed that the luminous region of the discharge was at a distance of between 1/4 and 2-1/2 in. from the cone surface (see Fig. 5), and thus occupied a position out from the cone wall equivalent to about four boundary layer thicknesses. Furthermore, as pointed out earlier, the actual breakdown processes probably occur along electric flux lines and are therefore even farther away from the surface than indicated by the position of the luminous plasma.

In spite of the thickness considerations, the boundary layer presents a serious difficulty because of the relatively large variation in density across it. The density is always less on the cone surface than on top of the boundary layer and, for the conditions encountered in this experiment, the change may be as large as a factor of two. With this thin film of less-dense air immediately adjacent to the cone there is the possibility that the complete breakdown process could take place wholly within the boundary layer, without curving out away from the cone as observed in still air. Even if the breakdown did pass through the boundary layer, the effective indicated density would be some average value resulting from an integration over the particular path chosen. Thus, if an appreciable part of the breakdown path was contained in this region of varying density, the resulting integrated density would have a value lower than the true surface density.

It is the opinion of the authors that the major factor causing the discrepancies shown in Fig. 15 is a result of this boundary layer effect. The three Paschen's law-derived densities at altitudes of 32, 34, and 36 kilometers differ by a factor of about two from the calculated (true) cone-wall densities. At these altitudes the missile has its highest velocity,
and the change of density through the boundary layer would be greatest. This suggests that for these three points the breakdown took place wholly within the boundary layer. Between the point at 36 kilometers and 39 kilometers there appears to be a jump in the values of the breakdown-determined densities, indicating that at higher altitudes when the missile was moving more slowly the breakdown path was only partially contained within the boundary layer.

At correspondingly higher altitudes the discrepancy between the Paschen's law densities and calculated densities became less and less. This can be correlated with the fact that as the missile slowed down the variation in density across the boundary layer became progressively smaller. Just below zenith the breakdown-determined densities appeared to jump around and then assumed an appreciably higher value, which might indicate that at this point supersonic flow broke up and the boundary layer disappeared.

If this same type of experiment were to be repeated, a major amount of consideration would, of course, have to be given to the effect of the boundary layer upon the path of the spark breakdown. A great deal of wind tunnel investigation would be called for and other possible spark gap arrangements should be reconsidered. Further consideration should also be given to the amount of formative time lag involved in this type of breakdown and to the effects of stray space charge upon the breakdown characteristics. The use of a saw-tooth sweep of applied voltage appeared to be quite satisfactory. However, the procedure for telemetering the breakdown information and obtaining in-flight calibration should be thoroughly reworked. It would be desirable to code the breakdown data in such a way that "holes" or interference in the telemetering record would not result in the loss of timing accuracy, as occurred in the present experiment. Also, minor refinements in the preflight calibration would result in greatly increased accuracy.

There appears to be a method by which it is possible to measure the ambient atmospheric density directly by means of a gas discharge breakdown. It would be free from the complications introduced by the boundary layer and missile yaw and not require an aerodynamic analysis to obtain ambient conditions. This method would utilize a converging beam of microwave energy focused to a point well ahead of the rocket. This energy would be obtained from an oscillator in a rocket and focused by a microwave lens mounted in the missile nose. Radio-frequency breakdown in air is a function of pulse length, field strength, and density. A measurement of the pulse length and/or field strength would thus provide an indication of density.

Work on this system was carried on at the University of Michigan from September 1951 to March 1952. The necessary lens system was designed, and breakdown was obtained in a vacuum chamber for a moderate range of densities. The authors believe that this system has many unexplored potentials, but it has not been carried forward because of lack of financial support.
XII. RESULTS

The original objective of this research was to investigate the possibility of obtaining a direct measurement of ambient atmospheric density from a rocket, through the use of a gas-discharge breakdown. Based upon the principle of Paschen's law, a measurement of the sparking potential of a suitably designed spark gap would give a direct measure of the density of the air in the gap.

Although this original objective could not be fulfilled the Paschen's law spark-breakdown technique was pursued and tested under flight conditions as a possible means for obtaining a reliable measurement of cone-wall density. As discussed in this report, any feasible method for obtaining a "d-c breakdown" type of measurement from a rocket requires that the breakdown take place in the region adjacent to the surface of the missile. Therefore, such a measurement must be made behind the main shock wave from the missile and, in order to obtain ambient values, it is necessary to know the air-flow conditions existing about the rocket at the time of measurement. However, these data are potentially useful as an independent measure of one of these flow parameters and, when appropriately combined with other measured quantities, could lead ultimately to a calculation of ambient conditions.

The results of this experiment are summarized in Fig. 15, which is a plot of the minimum local cone-wall density as a function of altitude for an Aerobee rocket fired on 15 September 1951. The densities derived from the Paschen's law data are lower than the densities calculated from pressure measurements made on the same flight. The primary reason for this discrepancy is believed to be the influence of the boundary layer upon the path of the spark breakdown.

For an exploratory experiment of this type, the authors feel that the work was successful and that satisfactory data were obtained. No serious difficulties with the experimental procedure or malfunctions of the equipment were encountered. The results of this research indicate that a gas-discharge breakdown method of measuring density is feasible, but that adjacent to the surface of a supersonic cone the breakdown appears to be influenced by the presence of a boundary layer.