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Techniques for Estimating Uncertainty Propagation in Probabilistic Design of Multilevel Systems

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In probabilistic design of multilevel systems, the challenge is to estimate uncertainty propagation since outputs of subsystems at lower levels constitute inputs of subsystems at higher levels. Three uncertainty propagation estimation techniques are compared in this paper in terms of numerical efficiency and accuracy: root sum square (linearization), distribution-based moment approximation, and Taguchi-based integration. When applied to simulation-based, multilevel system design optimization under uncertainty, it is investigated which type of applications each method is best suitable for. The probabilistic formulation of the analytical target cascading methodology is used to solve the multilevel problem. A hierarchical bi-level engine design problem is employed to investigate unique features of the presented techniques for uncertainty propagation. This study aims at helping potential users to identify appropriate techniques for their applications.

1. Introduction

Multilevel system design refers to the optimization process of large, complex engineering systems that are decomposed into a hierarchy of subsystems. Since the subsystems are coupled, their interactions need to be taken into consideration to achieve consistent designs. Analytical target cascading (ATC) is a methodology that takes these interactions into account during the early stages of the design optimization process [1].

In recent years, design guidelines and standards are being adjusted to incorporate the concept of uncertainty into the early design and product development stage [2]. In response to these new requirements, the ATC formulation has been extended to solve probabilistic design optimization problems [3].

Probabilistic design of multilevel systems does not only entail the difficulty of formulating and solving non-deterministic optimization problems; it is also quite challenging to model the mechanism of uncertainty propagation throughout the multilevel hierarchy. Outputs of subsystems at lower levels constitute inputs of subsystems at higher levels. It is thus necessary to estimate the statistical information of these outputs (that are inputs of subsystems at higher levels) with adequate accuracy without requiring a huge amount of raw data. In previous work [4], we have taken advantage of the ATC formulation that enables the use of first-order Taylor series [5] for approximating nonlinear responses. The ATC consistency constraints do not allow large deviations from the incumbent expansion point (which are the mean values of the design variables) during the optimization process. In this manner, not only can we linearize nonlinear responses, but we can also consider them as normally distributed if all the random variables they depend on were also normally distributed. Although large approximation errors of expected values for the nonlinear responses are avoided, the convergence rate of the ATC process can be low since many iterations involving small "steps" may be necessary. In addition, this estimation technique may exhibit relatively large errors when approximating higher-order statistical moments [3,4,6].

This paper considers two alternative methods for estimating statistical moments of nonlinear responses of random variables. The first method generates approximate probability density functions to be numerically integrated [3]. The second method uses numerical quadrature rules motivated by Taguchi-type experimental designs [7]. The scope of this paper is to investigate the stability, accuracy, and efficiency of these two methods when applied on simulation-based, multilevel system design optimization problems, and to determine which type of applications each method is best suitable for.

2. Probabilistic Design of Multilevel Systems

The major characteristic of hierarchically decomposed multilevel systems is that outputs (responses) of lower-level subsystems are inputs of higher-level subsystems. We take advantage of this functional dependency among levels and use analytical target cascading to solve probabilistic design problems of multilevel systems. The ATC process formulates and solves a design optimization problem for each of the elements in a multilevel hierarchy, e.g., the one shown in Figure 1.

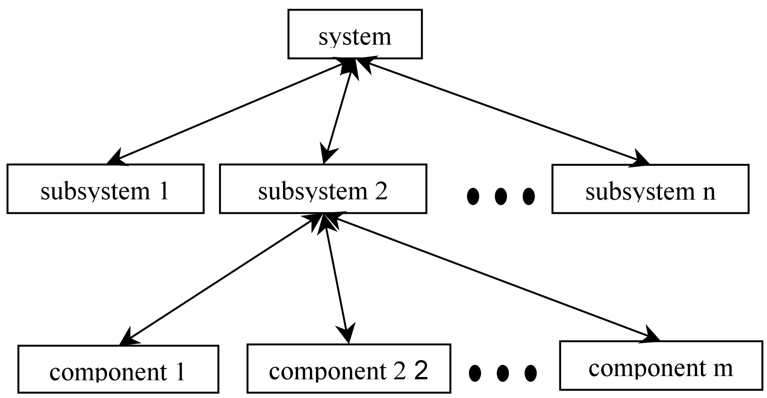


Figure 1: Example of multilevel system hierarchy

In the following general mathematical formulation, subscripts i and j are used to denote level and element, respectively. For each element j at level i, the set C_{ij} includes the elements that are "children" of this element. Responses r_{ij} are functions of children responses $r_{(i+1)k_1},\ldots,r_{(i+1)k_{c_{ij}}}$, local design variables x_{ij}, and shared design variables y_{ij}, i.e.,

$$\mathrm{r}_{ij} = f_{ij}\left(\mathrm{r}_{(i+1)k_1},\ldots,\mathrm{r}_{(i+1)k_{c_{ij}}},\mathrm{x}_{ij},\mathrm{y}_{ij}\right) = f_{ij}\left(\mathrm{z}_{ij}\right), \tag{1}$$

where $\mathrm{C}_{ij} = \left\{k_1, k_2, \cdots, k_{c_{ij}}\right\}$. Shared design variables are restricted to exist only among elements at the same level having the same parent.

In the presence of uncertainty, random design variables and parameters are symbolized by the use of upper case letters and represented by their mean values, responses are represented by the expectations of nonlinear responses, and constraints are formulated probabilistically. Mathematically, the probabilistic design optimization problem associated with each problem in the multilevel hierarchy is formulated as

$$\begin{aligned}
\underset{\mu_{\tilde{\mathrm{X}}_{ij}},\varepsilon_{ij}^r,\varepsilon_{ij}^y}{\text{minimize}} \quad & \left\| E\left[R_{ij}\right] - \mu_{R_{ij}}^u \right\|_2^2 + \left\| \mu_{Y_{ij}} - \mu_{Y_{ij}}^u \right\|_2^2 + \varepsilon_{ij}^r + \varepsilon_{ij}^y \\
\text{subject to} \quad & \sum_{k \in C_{ij}} \left\| \mu_{R_{(i+1)k}} - E\left[R_{(i+1)k}\right]^l \right\|_2^2 \le \varepsilon_{ij}^r \\
& \sum_{k \in C_{ij}} \left\| \mu_{Y_{(i+1)k}} - \mu_{Y_{(i+1)k}}^l \right\|_2^2 \le \varepsilon_{ij}^y \\
& P\left[g_{ij}\left(Z_{ij}\right) > 0\right] \le P_f
\end{aligned} \tag{2}$$

with $R_{ij} = \mathrm{f}_{ij}\left(R_{(i+1)k_1},\ldots,R_{(i+1)k_{c_{ij}}},X_{ij},Y_{ij}\right) = \mathrm{f}_{ij}\left(Z_{ij}\right)$, where the vector of design variables $\mu_{\tilde{X}_{ij}}$ consists of vectors $\mu_{Z_{ij}}$, $\mu_{Y_{(i+1)k_1}},\ldots,\mu_{Y_{(i+1)k_{c_{ij}}}}$ and $g_{ij}\left(Z_{ij}\right)$ denote local design inequality constraints. $E[\bullet]$ denotes expected value of a random variable, $P[\bullet]$ represents the probability of an event, and P_f is a vector of assigned probabilities of failure, i.e., probabilities of violating constraints. Tolerance optimization variables ε^r and ε^y are introduced for coordinating responses and shared variables, respectively. Superscripts (subscripts) $u(l)$ are used to denote response and shared variable values that have been obtained at the parent (children) problem(s), and have been cascaded down (passed up) as design targets (consistency parameters).

Figure 2 illustrates the information flow of the ATC process at element j in level i. Assuming that all the parameters have been updated using the solutions obtained at the parent- and children- problems, Problem (2) is solved to update the parameters of the parent- and children- problems. This process is repeated until the tolerance optimization variables in all problems cannot be reduced any further. The top-level element of the hierarchy is a special case; the responses cascaded from above are the given system design targets, and since this is the only element of the level, there exist no shared variables.

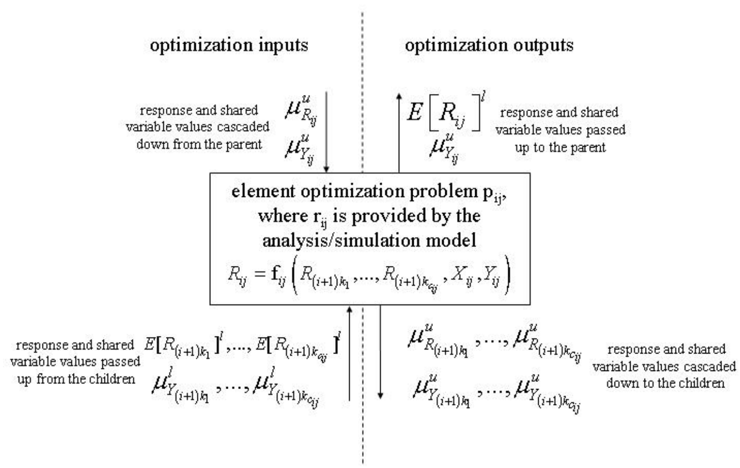


Figure 2: ATC information flow at element j, level i

We optimize with respect to the means of the random variables, and utilize an analytical first-order reliability method (FORM) to evaluate the reliability of satisfying the probabilistic constraints. Specifically, we adopt the hybrid mean value (HMV) algorithm to compute the most probable point (MPP) for each constraint at each iteration of the optimization process [2]. It is emphasized that first-order methods yield exact results only if the limit-state (i.e., constraint) responses are linear, and random variables are normally distributed and uncorrelated. If these assumptions are violated, the obtained results are only approximate. Nevertheless, these methods are used widely in literature due to their simplicity and efficiency despite their relative inaccuracy.

3. Uncertainty Propagation Techniques

The solution of a probabilistic design problem requires information on the distribution and moments of the random design variables and parameters. Typically, this information is given or postulated at the bottom level of a probabilistic multilevel system design problem. However, since the outputs of lower-level problems constitute inputs to higher-level problems, we must propagate the uncertainty information as accurate as possible to solve the higher-level problems and the overall multilevel design problem. In this section we present two alternative techniques for estimating uncertainty propagation.

3.1 Advanced Mean Value Based Distribution Generation and Moment Estimation

The main idea of this technique is to perform a reliability analysis on the output response (i.e., the nonlinear response of the random variables) using FORM for a sufficiently large range of reliability targets, e.g., from $\beta = 4$ (with corresponding probability of failure $P_f = \Phi(-\beta) = 0.00003$) to $\beta = -4$ (with $P_f = \Phi(-\beta) = 0.99997$). Once the most probable point is found (we use the HMV method [2]), the output response is evaluated at this point to provide the "corrected" function value for the corresponding probability of failure [8]. With the cumulative density function (CDF) available, one can then differentiate numerically to obtain the probability density function (PDF) [9]. We use central differences to obtain second-order accurate approximations. Finally, we integrate numerically, using spline interpolation to estimate response values that lie between the available "discrete" points of the PDF, to compute moments. As will be shown later by means of preliminary numerical results, this method is quite accurate. However, it can be inefficient depending on how the "β-range" is "discretized".

3.2 Taguchi-based Integration and Moment Estimation

3.2.1 Output Statistical Moment Modeling: Numerical Integration on Input Domain

One purpose of statistical moment estimation stems from the robust design optimization, which attempts to minimize the quality loss [10,11], which is a function of the statistical mean and standard deviation. Several methods are proposed to estimate the first two statistical moments of the output response. Analytically, the statistical moments are expressed in an integration form as

$$\begin{aligned} E\left[R\right] &= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} R(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \\ E\left[(R(\mathbf{x}) - \mu_R)\right]^k &= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} (R(\mathbf{x}) - \mu_R)^k f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \end{aligned} \tag{3}$$

Using numerical integration, the statistical moments of output response are approximated through numerical integration on the input domain as

$$\begin{aligned} E\left[R\right] &\cong \bar{\mu}_R \\ &= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} R(\mathbf{x}) \prod_{i=1}^{n} f_{X_i}(x_i) dx_1 \cdots dx_n \\ &= \sum_{j_1=1}^{m} w_{j_1} \cdots \sum_{j_n=1}^{m} w_{j_n} R(\mu_1 + \alpha_{j_1}, \cdots, \mu_n + \alpha_{j_n}) \\ E\left[R(\mathbf{x}) - \mu_R\right]^k &\cong \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} (R(\mathbf{x}) - \mu_R)^k \prod_{i=1}^{n} f_{X_i}(x_i) dx_1 \cdots dx_n \\ &= \sum_{j_1=1}^{m} w_{j_1} \cdots \sum_{j_n=1}^{m} w_{j_n} \left[R(\mu_1 + \alpha_{j_1}, \cdots, \mu_n + \alpha_{j_n}) - \bar{\mu}_R \right]^k \end{aligned} \tag{4}$$

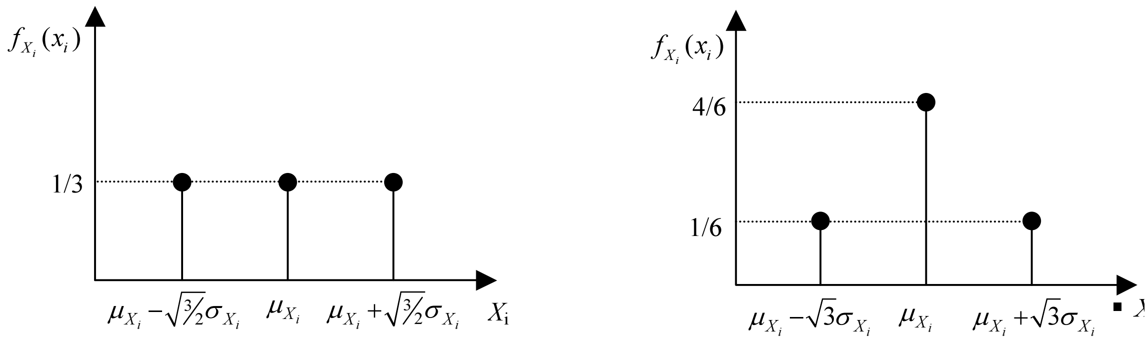
For application, Taguchi [11,12] proposed an experimental design approach for statistical tolerance design with a three-level (m=3) factorial experiment, which are composed of low, center, and high levels as

$$\left\{ w_1, w_2, w_3, \alpha_1, \alpha_2, \alpha_3 \right\} = \left\{ \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}, -\sqrt{3/2}, 0, \sqrt{3/2} \right\}$$

Three-level factorial experiment is modified by D'Errico and Zaino [13] by employing distinctive weights at different levels as

$$\left\{ w_1, w_2, w_3, \alpha_1, \alpha_2, \alpha_3 \right\} = \left\{ \tfrac{1}{6}, \tfrac{4}{6}, \tfrac{1}{6}, -\sqrt{3}, 0, \sqrt{3} \right\}$$

Thus, the modified three-level factorial experiment improved numerical accuracy in estimating the statistical moments of output response. In numerical integration, three weights for X_i are used to approximate the probability density of X_i at three different probability levels. From the statistical point of view, the modified three-level factorial experiment is meaningful, since many random input variables follows the rule of high density near the mean and low density at the tail of statistical distribution, as shown in Fig. 3.



(a) Taguchi Method [11] (b) D'Errico and Zaino Method [13]

Figure 3: Three-level numerical integration on the input domain

In the experimental method, the computation of the moment could be very expensive for a large number of design and/or random parameters, since the number of function evaluations or experiments required is $N = 3^n$ where *n* is a number of design and random parameters. Thus, this method is not used in this paper.

3.2.2 Output Statistical Moment Modeling: Numerical Integration on Output Domain

In Section 3.2.1, statistical moments of output response are estimated through numerical integration on the input domain, making it very expensive for reliability-based robust design optimization. In this paper, the proposed method directly identifies uncertainty propagation using numerical integration on the output domain. Unlike Eq. (3), the statistical moment calculation is carried out by

$$E[R]^1 = \int_{-\infty}^{\infty} r f_R(r) dr = \mu_R$$
$$E[(R-\mu_R)]^k = \int_{-\infty}^{\infty} (r-\mu_R)^k f_R(r) dr \tag{5}$$

where $f_R(r)$ is a probability density function of *R*. To approximate the statistical moments of *R* accurately, *N*-point numerical quadrature technique can be used as

$$E[R]^1 = \mu_R \cong \sum_{i=1}^{N} w_i r_i \quad \text{and}$$
$$E[R-\mu_R]^k \cong \sum_{i=1}^{N} w_i (r_i - \mu_R)^k \ \text{ for } 2 \le k \le 5 \tag{6}$$

At minimum, the three-point integration (*N*=3) is required to maintain a good accuracy in estimating first two statistical moments. By solving Eq. (6), three levels and weights on the output domain are obtained as $\{r_1, r_2, r_3\} = \{r_{\beta=-\sqrt{3}}, r(\boldsymbol{\mu}_\mathbf{X}), r_{\beta=+\sqrt{3}}\}$ and $\{w_1, w_2, w_3\} = \{\frac{1}{6}, \frac{4}{6}, \frac{1}{6}\}$, as shown in Fig. 4. In general, upper and lower levels are not symmetrically located, as shown in Fig. 4.

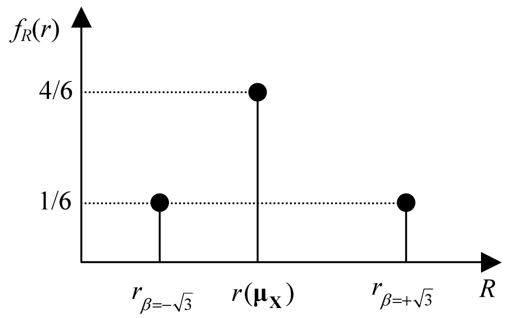


Figure 4: Three-level numerical integration on the output domain

Using the three-level numerical integration on the output domain, the first two statistical moments in Eq. (5), the mean and standard variation of the output response are approximated to be

$$
\begin{aligned}
E\left[R\right]^1 &= \mu_R \cong \tfrac{1}{6} r_{\beta=-\sqrt{3}} + \tfrac{4}{6} r(\boldsymbol{\mu}_{\mathbf{X}}) + \tfrac{1}{6} r_{\beta=+\sqrt{3}} \\
E\left[R-\mu_R\right]^2 &= \sigma_R^2 = \int_{-\infty}^{\infty} \left(r-\mu_R\right)^2 f_R(r)dr \\
&\cong \tfrac{1}{6}(r_{\beta=-\sqrt{3}}-\mu_R)^2 + \tfrac{1}{6}(r_{\beta=+\sqrt{3}}-\mu_R)^2
\end{aligned}
\quad (7)
$$

Since the statistical moments of output response are estimated through a numerical integration on the output (or performance) domain, this method is called performance moment integration (PMI) method. In the PMI method, $r_{\beta=-\sqrt{3}}$ and $r_{\beta=+\sqrt{3}}$ are obtained through reliability analyses [2,8,9] at $\beta = \pm\sqrt{3}$ confidence levels. In this paper, the hybrid mean value (HMV) method is used for reliability analysis [2].

4. Numerical Examples

4.1 Single-level Propagation Examples

Two nonlinear analytical examples and one vehicle crashworthiness for side-impact simulation example are used to demonstrate the aforementioned techniques. For abbreviation purposes, the method presented in Section 3.1 is called distribution-based method (DBM) in this paper. Monte Carlo simulation (MCS) with one million samples and root sum square (RSS) method are used for numerical comparison. Experimental methods [11,13] are not used for comparison because it would be too expensive even though it would be as accurate as MCS. Statistical non-normality of the response functions is represented by skewness and kurtosis. Skewness is a measure of symmetry of probability density function (a normal distribution has a skewness value of 0). Kurtosis is a measure of relative peakness/flatness of probability density function to normal distribution, which has a kurtosis value of 3.

The first analytical example, the response is

$$R_1(\mathbf{X}) = 1 - X_1^2 X_2 / 20 \quad (8)$$

For this example, the input random parameters are modeled as $X_i \sim N(5.0, 0.3)$ for i=1,2. As shown in Table 1 and Fig. 5, the probabilistic distribution of the first response is close to a normal distribution with a moderate rate of skewness and kurtosis. Thus, RSS, DBM, and PMI show overall a good accuracy in estimating the first two statistical moments of responses.

The second analytical example response is

$$R_2(\mathbf{X}) = -e^{X_1-7} - X_2 + 10\,. \quad (9)$$

The input random parameters are modeled as $X_i \sim N(6.0, 0.8)$ for i=1,2. As shown in Table 1, the RSS method yields a large approximation error of 107% for the second moment, whereas the DBM and PMI methods are much more accurate for both the mean and standard deviation.

The last single-level example R_3 is the pubic force from a side impact simulation [14], which is modeled with input uncertainties of Gumbel distribution and 10% coefficient of variation. Even though the stochastic response is highly skewed with large kurtosis, the PMI method seems to predict the first two statistical moments accurately, whereas the RSS could yield larger errors. DBM results are not available for this example.

Table 1: Single-level examples

	Mean				Standard Deviation				Skew.	Kurt.
	RSS	DBM	PMI	MCS	RSS	DBM	PMI	MCS		
R_1	-5.2500	-5.286	-5.286	-5.2719	0.8385	0.842	0.8411	0.8405	-0.26	3.11
Error, %	0.415	-0.259	-0.259	--	-0.238	0.17	0.071	--	--	--
R_2	3.6321	3.6029	3.6082	3.4937	1.9386	0.9013	0.8800	0.9349	-0.57	7.13
Error, %	3.961	3.125	3.277	--	107.4	-3.593	-5.872	--		
R_3	-1.4100	N/A	-1.4135	-1.4291	0.0632	N/A	0.0685	0.0708	-0.99	4.93
Error, %	1.337	N/A	1.092	--	-10.73	N/A	-3.248	--	--	--

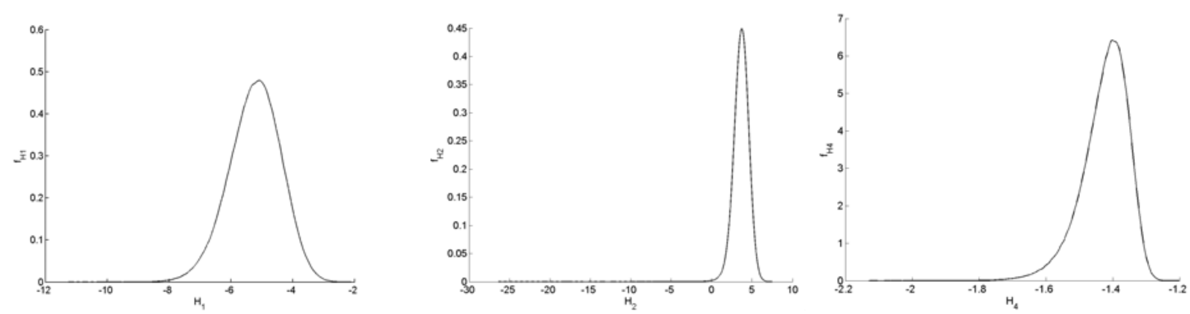


Figure 5: PDF of R_1 (left), R_2 (middle), and R_3 (right)

4.2 Bi-level Propagation Example

In this section we demonstrate how the aforementioned techniques are used to estimate uncertainty propagation in multilevel systems. The probabilistic formulation of the ATC process is used to solve a simple yet illustrative simulation example. An engine is considered at the system (top) level, which is decomposed into subsystems (bottom level) representing the piston-ring/cyliner-liner subassembly of the cylinders. Although an engine has multiple cylinders, they are all designed to be identical. For this reason, only one subsystem is considered.

The system simulation predicts engine performance in terms of brake-specific fuel consumption. The ring/liner subassembly simulation takes as inputs the surface roughness of the ring and the liner and the Young's modulus and hardness and computes power loss due to friction. The root mean square (RMS) of asperity height is used to represent asperity roughness, which is assumed to be normally distributed with a coefficient of variation of 15-25%. The engine simulation takes then as input the power loss and computes brake-specific fuel consumption of the engine. Commercial software packages were used to perform the simulations. Detailed descriptions of the problem can be found in [3,4].

Since we have some information about the uncertainty associated with surface roughness of the ring and the liner, we begin at the bottom level. The bottom-level ATC problem is formulated as

$$
\begin{aligned}
&\min_{\mu_{X_{11}},\mu_{X_{12}},x_{13},x_{14}} \quad \left(E\left[R_1\right]-\mu_{R_1}^u\right)^2 \\
&\text{s.t.} \quad P\left(\text{liner wear rate} > 2.4\times10^{-12} m^3/s\right) \le P_f \\
&\qquad P\left(\text{blow-by} > 4.25\times10^{-5} kg/s\right) \le P_f \\
&\qquad P\left(\text{oil consumption} > 15.3\times10^{-3} kg/hr\right) \le P_f \\
&\qquad P\left(X_{11} < 1\mu\text{m}\right) \le P_f \\
&\qquad P\left(X_{11} > 10\mu\text{m}\right) \le P_f \\
&\qquad P\left(X_{12} < 1\mu\text{m}\right) \le P_f \\
&\qquad P\left(X_{12} > 10\mu\text{m}\right) \le P_f \\
&\qquad 340GPa \ge x_{13} \ge 80GPa \\
&\qquad 240BHV \ge x_{14} \ge 150BHV \\
&\text{where} \quad R_1 = f_1(X_{11},X_{12},x_{13},x_{14})
\end{aligned}
\tag{10}
$$

and is solved first assuming that $\mu_{R_1}^u = 0$. When the optimal solution is found, uncertainty propagation associated with the random response R_1 must be estimated in order to solve the top-level problem, which is formulated as

$$
\begin{aligned}
&\min_{\mu_{R_1},\varepsilon^R} \quad \left(E\left[R_0\right]-T\right)^2 + \varepsilon^R \\
&\text{s.t.} \quad \left(\mu_{R_1} - E\left[R_1\right]^l\right)^2 \le \varepsilon^R \\
&\text{where} \quad R_0 = f_0(R_1)
\end{aligned}
\tag{11}
$$

Once this problem is solved, we have an update for $\mu_{R_1}^u$, and we solve the bottom-level problem again. This process is repeated until the optimization variables in both problems don't change significantly anymore.

It was found that the response of the bottom-level simulation (power loss) is highly nonlinear, while the response of the top-level simulation (fuel consumption) was almost linear. Therefore, we focus on the results of the bottom-level simulation, which are the more interesting anyway since it is the power loss that is propagated in the bi-level system (as the output of the bottom level simulation that constitutes the input of the top level simulation). The DBM method enables the generation of the highly nonlinear, multi-modal probability density function (shown in Fig. 6). The histogram obtained using Monte Carlo simulation with one million samples confirms the accuracy of the DBM-predicted PDF. Note that the presented plots and numerical results are based on the obtained optimal ring/liner design.

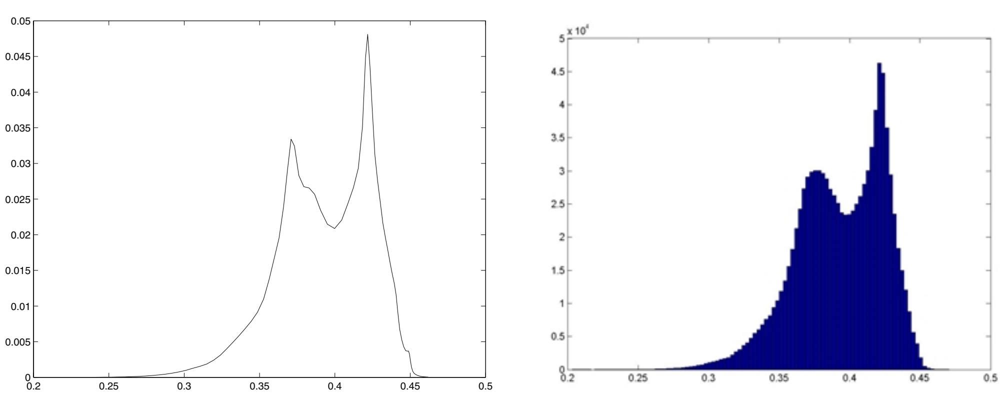


Figure 6: DBM-generated PDF of power loss (left) and MCS histogram (right)

As shown in Table 2, although all methods predict the mean of power loss accurately, RSS is highly inaccurate when estimating the standard deviation. PMI exhibits a smaller error, while DBM yields excellent prediction, due to the fact that it generates distribution information. The PMI error is mostly due to the statistical nonlineariy of bimodal distribution. PMI requires two reliability analyses for each of the three measures, whereas DBM requires about 11 reliability analyses to discretize the "β-range" in order to generate the probability density function.

Table 2: Numerical accuracy of moments for simulation-based example

	Mean				Standard Deviation			
	RSS	PMI	DBM	MCS	RSS	PMI	DBM	MCS
R_4	0.3950	0.3920	0.3923	0.3932	0.0482	0.0297	0.0310	0.0311
Error, %	0.5	-0.3	-0.2	-	55.0	-4.6	-0.45	-

Table 3 compares the optimal design of the ring/liner obtained using the three methods. It is clear that solving the probabilistic multilevel design problem using the error-prone RSS method yields the wrong results, which highlights the importance of accurate estimation of uncertainty propagation. The results obtained using DBM or PMI are very similar for this example.

Table 3: Comparison of ring/liner optimal design

Design variable	Initial	RSS optimum	DBM optimum	PMI optimum
X_{11}: ring surface roughness, [μm]	5.000	4.264	4.000	4.000
X_{12}: liner surface roughness, [μm]	5.000	6.001	6.150	6.113
x_{13}: liner Young's modulus, [GPa]	200	80	80	80
x_{14}: liner hardness, [BHV]	200	220.84	240	239.8

5. Discussion and Conclusions

Two alternative techniques for estimating uncertainty propagation in probabilistic design of multilevel systems were presented in the paper. The first method generates approximate probability density functions, which are then integrated numerically to obtain statistical moments (distribution-based method, DBM). The second method uses numerical quadrature rules to estimate statistical moments of output response (performance moment integration, PMI). The methods were successfully applied to model the uncertainty propagation mechanism by estimating statistical moments efficiently and accurately. The scope of this paper was to investigate the stability, accuracy, and efficiency of these two methods when applied on simulation-based, multilevel system design optimization problems, and to determine which type of applications each method is best suited for.

It was found that both DBM and PMI estimate statistical moments accurately for nonlinear responses with high skewness and kurtosis. Thus, PMI and DBM successfully carried out the probabilistic design optimization of multilevel hierarchical system. The methods were compared to the Root Sum Square (RSS) method and Monte Carlo simulation was performed to compare the estimation of statistical moments. PMI and DBM are more accurate to assess statistical moments than RSS. PMI can be useful for many nonlinear engineering systems, since it is computationally inexpensive yet accurate. On the other hand, DBM can be more accurate but is computationally more expensive. Finally, DBM should be used when the probabilistic design of multilevel systems requires generating distributions of nonlinear responses, i.e., when moments are not adequate to model propagation of uncertainties.

Acknowledgments

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