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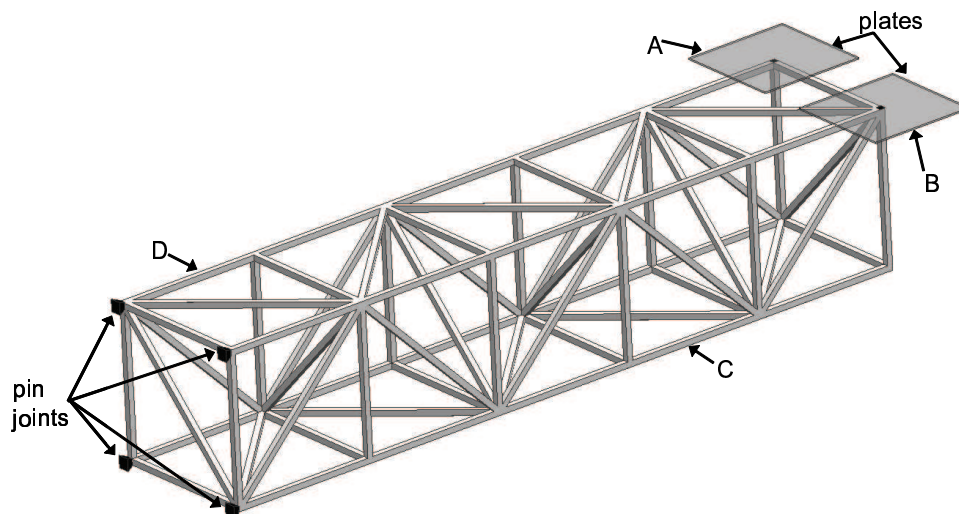# AIAA -2007-2057

Sensor Placement for Damage Detection in Nonlinear Systems using System Augmentations

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Currently, most sensor placement methodologies are focused on maximizing the controllability and observability of the monitored structure. Recently there have been several sensor placement techniques proposed for damage detection. The work herein provides an integrated sensor placement and reduced order health assessment approach that can be applied to both linear and nonlinear structures. The method uses the idea that often the damageable regions (hot spots) of the system are known in advance and therefore the modes that are sensitive to changes in these hot spots should be the ones exploited for damage detection. Generally,the sensors are placed near the hot spots. However if that is not possible, or if additional sensors are being used, then a generalized effective independence distribution vector method is applied for the remaining sensors. The partial eigenvector information is expanded to the full space using the knowledge that damage is limited to the hot spots of the system. Modal based damage detection methods such as minimum rank perturbation theory (MRPT) can then be used to solve for the damage in a linear system. Also, an alternative damage identification by hot spot projection (DIHSP) method is a novel additional option for the damage detection presented herein. Nonlinear systems are handled by forming (higher dimensional) augmented linear systems that follow the same trajectory of the nonlinear system when projected onto the physical (lower dimensional) space. The sensor placement methodology for nonlinear systems is similar, but it requires that sensors be placed at the location of all nonlinearities as well as the hot spots. The damage can be detected using the multiple augmentations generalized MRPT approach previously developed by the authors or by DIHSP. Numerical simulations of the methodology are presented for linear and nonlinear 5-bay frame structures.

Nomenclature

$\mathbf{A}_i$	partial eigenvalue problem
$\mathbf{B}$	damage location matrix
$\mathbf{E}_i$	effective independence distribution vector
$\mathbf{F}$	external excitation
$\mathbf{F}_f$	filtered external excitation
$\mathbf{M}, \mathbf{K}$	original FEM mass and stiffness matrices
$\Delta\mathbf{K}$	perturbation to the stiffness
$\Delta\bar{\mathbf{K}}$	projected perturbation to stiffness
$\Delta\bar{\mathbf{K}}'_i$	projected perturbation to stiffness scenarios
N	size of full FEM model
$\mathbf{P}_d$	sensitive damaged modes
$\mathbf{P}_{dm}$	measured portion of sensitive damaged modes
$\mathbf{c}$	time varying vector
$\mathbf{d}_i$	damage location vector of the ith mode
n	number of measured modes
$\mathbf{r}$	eigenvector sensitivity rating
s	number of damage scenarios
$\mathbf{v}_{di}$	ith damaged eigenvector
$\mathbf{v}_{hi}$	ith healthy eigenvector
$\mathbf{x}$	coordinate vector
$\mathbf{x}_f$	filtered coordinate vector
$\mathbf{x}_m$	measured portion of the coordinate vector
$\mathbf{x}_{mf}$	filtered measured portion of the coordinate vector
$\mathbf{x}_u$	unmeasured portion of the coordinate vector
Λ_d	damaged eigenvalue matrix
Φ, Ψ	vector expansion matrices
α_i	level of damage in ith projected damage scenario
ϵ	tolerance level
λ_{di}	ith damaged eigenvalue

Introduction

Large and complex air and space structures are being placed in new and extreme conditions for extended periods of time. As a result, the need for robust and accurate health monitoring techniques continues to grow. These health monitoring techniques would ideally have sensor information at all the degrees of freedom of a finite element model used for monitoring the integrity of the structure. Practically, however, due to cost, weight and accessibility issues a limited number of locations can be instrumented.

Most current sensor placement methodologies are focused on maximizing the controllability and observability of the healthy structure. For example, Cherng[1]

identified the optimal placement of sensors and actuators for controllability and observability. That method examines the whole structure and selects sensor locations to maximize the signal to noise ratio in the system. Other approaches examine ways to minimize the information entropy norm, which is a measure of the uncertainty in the model parameter estimates. For example, Yuen et al.[2] proposed a sensor placement method designed for system identification and based on reducing entropy. That method requires choosing a number of damageable areas (each with an associated parameter) and placing an equal number of sensors to minimize the uncertainty in parameter estimates. Another technique, called the effective independence distribution vector (EIDV) method, selects sensor locations that make the mode shapes of interest as linearly independent as possible while capturing as much information as possible in the desired mode shapes in the measured data.[3]

Recently, there have been proposed several techniques which are focused on sensor placement for damage detection. Cobb and Liebst[4] discussed one of the first such approaches. That sensor placement technique makes no assumption about damage location and, instead, focuses on a sensitivity analysis to find the degrees of freedom which maximize the changes due to damage in the observable partial eigenstructure. The method does not control which sections of the system will be detectable. Finally, other techniques are based on maximizing the Fischer information matrix to find the optimum sensor placement for damage detection.[5,6]

The method herein uses reduced order modeling combined with an eigenvector sensitivity analysis to find which eigenvectors are most sensitive to the damageable regions of interest. These damageable regions of interest are based on certain regions of the system being known as the most likely points of damage (hot spots). This differs significantly from classical reduced order modeling (ROM) techniques[7–13] which model the dynamics of the system and, therefore, are interested in the first few modes of the system. Instead, herein reduced order health assessment (ROHA) methodology is developed to capture the *change* in dynamics, which leads to different modes being of interest.

The work herein develops a sensor placement methodology specifically designed for damage detection. It places sensors at the hot spots of the system. If additional sensors need to be placed, or if certain characteristics of the hot spots of the system make placing sensors difficult or impossible, then a generalized EIDV can be formulated to place the remaining sensors.

The physical measured displacements and forcing of the structure are filtered in the frequency domain to keep only frequencies near the eigenvectors used in the projection matrix. Modal information corresponding to the measurement locations can then be extracted. This (partial) modal information can be expanded by an approach that enforces that damage can only occur in the

hot spots of the system. Any number of modal based damage detection methods, such as minimum rank perturbation theory[14–17] (MRPT) or optimal matrix update approaches,[18,19] can be used to calculate the damage. However, to provide additional filtering, in this paper a method called damage identification by hot spot projection (DIHSP) is presented.

One of the advantages of the integrated sensor placement and damage detection methodology demonstrated herein is that it can be applied to both linear and nonlinear systems when the nonlinear system can be modeled by appropriate augmented linear systems previously proposed by the authors.[20,21] These augmented linear systems are of higher dimension than their corresponding nonlinear systems. If the augmented systems are projected into the lower dimension space of the nonlinear system, they will follow a single trajectory of the nonlinear system. Linear modal extraction methods can be used on augmented linear systems if the identification method uses a forcing that is known but not prescribed such as direct system parameter identification[22] (DSPI) or vector backward auto-regressive with exogenous (VBARX) modeling.[23] A linear damage detection methodology called generalized MRPT (GMRPT) has been previously developed by the authors for these augmented systems.[20,21]

In this work, the ROHA methodology is detailed for detecting damage in the hot spots of linear and nonlinear (augmented) systems with few measurements. Next, the methodology for optimal sensor placement is laid out for linear and nonlinear (augmented) systems. Then the MRPT and DIHSP damage detection methodologies are detailed. Finally, linear and nonlinear 5-bay frames are used for various tests illustrating the effectiveness of the proposed techniques.

Methodology

In this section, a reduced order health assessment (ROHA) methodology for determining the full mode shape of linear and nonlinear (augmented) systems from partial measurement data is explained. Additionally, an improved sensor placement algorithm is introduced for linear and nonlinear systems when ROHA is employed. The modal based damage detection technique MRPT is overviewed and then DIHSP is presented. Finally, filtering algorithms to reduce the effects of noise are discussed.

Reduced Order Health Assessment

In this section, the procedure for extracting the full mode shapes that are most sensitive to damages in the hot spots of the system is outlined.

Modal based damage detection techniques are only effective when the modes that are used are sensitive to the damage that occurs. ROHA is therefore formulated to determine the full mode shapes (that are most sensitive to changes in the hot spots of the system) from limited sensor information. These damages are chosen on

the basis that, in many structural systems, the hot spot locations are known. A sensitivity rating $\mathbf{r}$ of the eigenvectors to the s different damage scenarios is evaluated as

$$r_i = \sum_{j=1}^{s} \mathbf{v}_{hi}^T \mathbf{M} \mathbf{v}_{dij}, \tag{1}$$

where i denotes the eigenvector number, j denotes the damage scenario, and $\mathbf{v}_h$ and $\mathbf{v}_d$ are the (augmented) eigenvectors for the healthy and damaged systems, respectively. A projection matrix that consists of the n eigenvectors with the highest sensitivity (lowest r_i) is called $\mathbf{P}$. The dimensions of $\mathbf{P}$ are $N \times n$, where N is the size of the full system and $n << N$.

To accurately extract the partial modes of the system corresponding to the sensitive eigenvectors, the measured forcing and positions of the system must be filtered appropriately. Consider the case of having n sensors that measure the degrees of freedom $\mathbf{x}_m$ of the full model. The remaining unmeasured degrees of freedom are denoted by $\mathbf{x}_u$. The forcing $\mathbf{F}$ is measured. In the nonlinear case, $\mathbf{x}_m$ would contain all the augmented variables and $\mathbf{F}$ would contain the augmented forcing. The indices of the degrees of freedom of the system are re-ordered such that the measured degrees of freedom of the system are the first degrees of freedom of the system. Then, the full coordinates of the system, $\mathbf{x}$ can be written as

$$\mathbf{x} = \left[\begin{array}{c} \mathbf{x}_m \\ \mathbf{x}_u \end{array} \right]. \tag{2}$$

The modal content of $\mathbf{x}_m$ and $\mathbf{F}$ can be filtered by taking a Fast Fourier Transform (FFT) of $\mathbf{x}_m$ and $\mathbf{F}$ and filtering out all frequencies except the ones near the healthy natural frequencies of the desired reduced modes. The filtered frequency domain data can then be returned to the time domain via an inverse FFT, yielding $\mathbf{x}_{mf}$ and $\mathbf{F}_f$ corresponding to a filtered $\mathbf{x}_m$ and $\mathbf{F}$, respectively.

The coordinates of the system can be formulated in modal coordinates as

$$\mathbf{x} = \left[\begin{array}{c} \mathbf{x}_m \\ \mathbf{x}_u \end{array} \right] = \sum_{i=1}^{n} c_i(t) \mathbf{v}_{ki} + \sum_{j=n+1}^{N} c_j(t) \mathbf{v}_{rj}, \tag{3}$$

where $\mathbf{v}_k$ corresponds to the kept damaged modes, $\mathbf{v}_r$ corresponds to the removed damaged modes and $c_i(t)$ are time varying coefficients. The filtering process that produces $\mathbf{x}_{mf}$ and $\mathbf{F}_f$ essentially forces $c_j(t)$ to be zero for $j = n+1 \ldots N$, which gives the following filtered response in modal coordinates

$$\mathbf{x}_f = \left[\begin{array}{c} \mathbf{x}_{mf} \\ \mathbf{x}_{uf} \end{array} \right] = \sum_{i=1}^{n} c_i(t) \mathbf{v}_{ki} = \mathbf{P}_d \mathbf{c}, \tag{4}$$

where

$$\begin{aligned} \mathbf{P}_d &= [\mathbf{v}_{k1}, \mathbf{v}_{k2}, \ldots, \mathbf{v}_{kn}], \\ \mathbf{c} &= [c_1(t), c_2(t), \ldots, c_n(t)]^T. \end{aligned}$$

The input data, into a modal analysis approach, such as DSPI, is the filtered measurements $\mathbf{x}_{mf}$ and $\mathbf{F}_f$ and the output is $\mathbf{P}_{dm}$, where the m corresponds to the degrees of freedom that relate to the measured ones in $\mathbf{x}$.

The next step is to expand each mode from $\mathbf{P}_{dm}$ to the full space using the fact that damage is limited to the hot spots of the system. A damage location vector $\mathbf{d}$ used in MRPT,[17] can be defined as follows (when, for example, damage only occurs in the stiffness matrix of an undamped system)

$$\mathbf{d}_i \equiv (\lambda_{di}^2 \mathbf{M} + \mathbf{K}) \mathbf{v}_{di} = \Delta \mathbf{K} \mathbf{v}_{di}, \tag{5}$$

where $\mathbf{d}_i$ is the perturbation equation for the ith eigenvector $\mathbf{v}_{di}$ and eigenvalue λ_{di}. Matrices $\mathbf{M}$ and $\mathbf{K}$ are the healthy mass and stiffness matrices, and $\Delta \mathbf{K}$ is the change in the healthy stiffness matrix.

The entries in $\mathbf{d}_i$ that correspond to degrees of freedom in the system that are undamageable (not in the hot spots) are known to be exactly zero. Therefore, if the number of measured degrees of freedom is equal to or greater than the number of degrees of freedom that are damageable, then there are at least as many equations (from Eq. 5) as unknowns, and the inverse problem can be solved to obtain a unique solution. Two matrices $\mathbf{\Phi}_i$ and $\mathbf{\Psi}_i$ can be defined such that

$$\left[\begin{array}{cc} \mathbf{\Phi}_i & \mathbf{\Psi}_i \end{array} \right] = \lambda_{di}^2 \tilde{\mathbf{M}} + \tilde{\mathbf{K}}, \tag{6}$$

where $\tilde{\mathbf{M}}$ and $\tilde{\mathbf{K}}$ are composed only of the rows of $\mathbf{M}$ and $\mathbf{K}$ that correspond to the undamageable degrees of freedom. If the system is nonlinear, then the augmented degrees of freedom are not contained in $\tilde{\mathbf{M}}$ and $\tilde{\mathbf{K}}$ (even though they are undamageable). Combining Eq. 5 and Eq. 6 one can write

$$\begin{aligned} \mathbf{0} &= \left[\begin{array}{cc} \mathbf{\Phi}_i & \mathbf{\Psi}_i \end{array} \right] \left[\begin{array}{c} \mathbf{v}_{mi} \\ \mathbf{v}_{ui} \end{array} \right] \\ \mathbf{0} &= \mathbf{\Phi}_i \mathbf{v}_{mi} + \mathbf{\Psi}_i \mathbf{v}_{ui} \\ \mathbf{v}_{ui} &= -\mathbf{\Psi}_i^+ \mathbf{\Phi}_i \mathbf{v}_{mi}, \end{aligned} \tag{7}$$

where the $\mathbf{v}_{mi}$ corresponds to the measured portion of the ith eigenvector given by the modal analysis technique and $\mathbf{v}_{ui}$ to the corresponding unmeasured portion of that eigenvector and $\mathbf{\Psi}_i^+$ is the pseudo-inverse of $\mathbf{\Psi}_i$. The system is well conditioned if the number of measured degrees of freedom is equal to or greater than the number of degrees of freedom that are damageable, and $\mathbf{\Psi}_i$ is full rank. A way to ensure that $\mathbf{\Psi}_i$ is full rank is in the proper choice of sensor locations, which will be discussed in the next subsection.

Sensor Placement

In this section the sensor placement methodology is explained for linear and nonlinear systems.

In the cases where there is exactly the same number of degrees of freedom as sensors, and the sensors can be placed anywhere, the sensors are placed at the hot spots
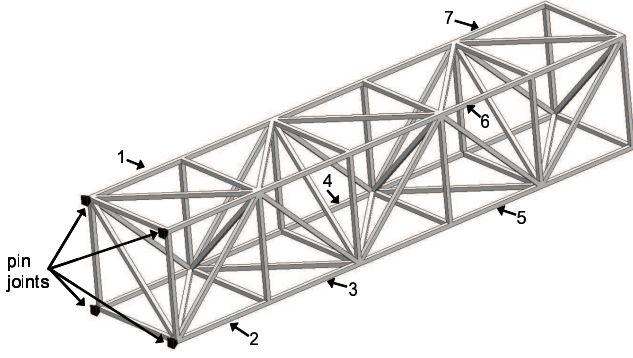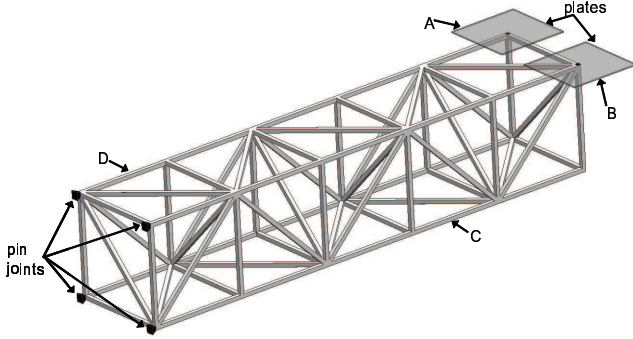
Fig. 1   A linear 5-bay frame structure

Fig. 2   A linear 5-bay structure with 2 plates which introduce cubic stiffness nonlinearities

of the system. For the nonlinear case, sensors must also be placed at the degrees of freedom that contain the nonlinearities.

In some systems additional sensors may be used to reduce the effects of measurement noise. Also, some hot spots of the system may not allow the placement of sensors nearby. In either of these cases, a generalized EIDV[3] method can be used to place the remaining sensors, as follows.

The goal of the generalized EIDV sensor placement methodology is to find the locations of sensors that lead to the largest minimum singular values of $\boldsymbol{\Psi}_i$ for all i, where $\boldsymbol{\Psi}_i$ contains $N - n$ columns of $\mathbf{A}_i$ where

$$\mathbf{A}_i = \lambda_{di}^2 \tilde{\mathbf{M}} + \tilde{\mathbf{K}}. \tag{8}$$

EIDV[3] can be used on $\mathbf{A}_i$ to determine which columns of $\mathbf{A}_i$ contribute the least to the rank of $\mathbf{A}_i$, and then remove them. The procedure for EIDV is to form the matrix $\mathbf{E}_i$ given by

$$\mathbf{E}_i = \mathbf{A}_i^T (\mathbf{A}_i \mathbf{A}_i^T)^{-1} \mathbf{A}_i. \tag{9}$$

Matrix $\mathbf{E}_i$ is an idempotent matrix with the property that its trace equals its rank. The entry along the diagonal with the lowest value corresponds to the smallest contribution to the rank, and therefore the corresponding column can be removed. The matrix $\mathbf{A}_i$ is then recalculated without the lowest column, and the process is repeated.

Since there are n matrices $\mathbf{A}_i$ to be optimized over at each step, the generalized EIDV requires n matrices $\mathbf{E}_i$ to be formed simultaneously. Then, the entries of the diagonals of each $\mathbf{E}_i$ are squared. Finally, the diagonals are summed, the column corresponding to the minimum value is removed, and the process is repeated.

In practice the damaged natural frequencies are not known until after damage occurs, therefore healthy natural frequencies must be used in Eq. 8 in order to calculate $\mathbf{A}_i$ for sensor placement.

For nonlinear systems modeled through augmentation the procedure is very similar. The only difference is that columns associated with the augmented degrees of freedom and the linear degrees of freedom that contain the nonlinearity are removed from $\mathbf{A}_i$ at the beginning of the procedure along with the sensors that can be placed at the hot spots. This is done because those degrees of freedom are required to be measured to form the augmented system.[20]

Damage Detection Methodology: MRPT

A variety of modal based damage detection methodologies can use the modes given by Eq. 7 to predict damage. In this section a modal based approach called MRPT[17] is discussed.

MRPT was developed on the basis that damage often initially occurs in localized regions of the system. Therefore, a minimum rank solution to the perturbation equations can be used. For a system with damage in stiffness only (and no damping), the perturbation equations are defined in Eq. 5.

The minimum rank solution to the perturbation equations for $\Delta\mathbf{K}$ (using MRPT and a subspace selection algorithm) is given by

$$\Delta\mathbf{K} = \mathbf{B}\mathbf{Z}(\mathbf{Z}^T\mathbf{B}^T\mathbf{P}_d\mathbf{Z})^{-1}\mathbf{Z}^T\mathbf{B}^T, \tag{10}$$

where

$$\begin{aligned} \mathbf{B} &= [\mathbf{d}_1, \mathbf{d}_2, \dots, \mathbf{d}_n], \\ \mathbf{Z} &= \mathbf{V}\boldsymbol{\Sigma}_\epsilon, \\ \mathbf{B} &= \mathbf{U}\boldsymbol{\Sigma}_\epsilon\mathbf{V}^T, \end{aligned}$$

with $\mathbf{U}$, $\boldsymbol{\Sigma}$, and $\mathbf{V}$ forming the standard singular value decomposition of matrix $\mathbf{B}$, while $\boldsymbol{\Sigma}_\epsilon$ contains the singular values that are greater than a tolerance level ϵ.

If the system is an augmented linear one, damage will first be calculated in the nonlinear parameters using the multiple augmentations approach discussed previously by the authors.[21]

Damage Detection Methodology: DIHSP

This section introduces an alternative damage detection methodology called the damage identification by hot spot projection (DIHSP) method. This approach has been developed on the basis that damage is constrained to a linear combination of s possible damage scenarios.
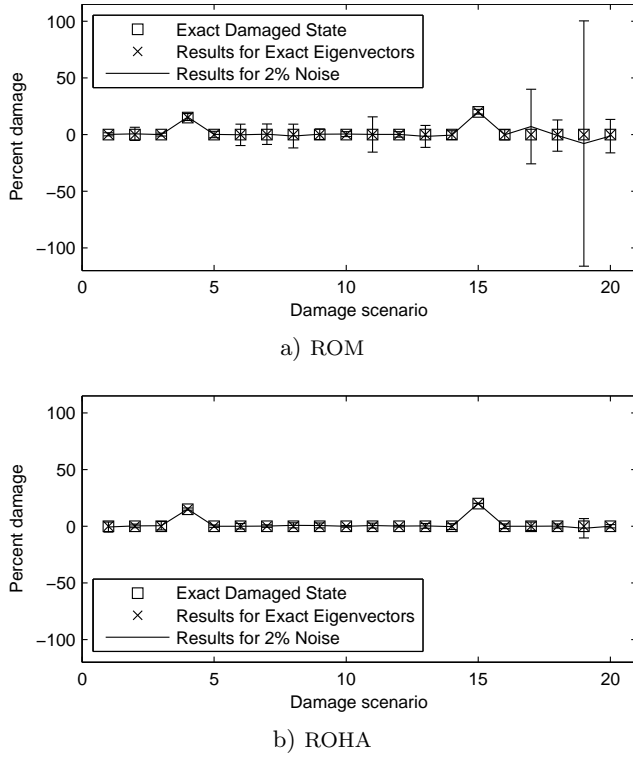
a) ROM

b) ROHA

Fig. 3 Predicted damage in the linear 5-bay frame with damage at two locations and 2% random eigenvector noise using the first 20 modes of the system (a) and using the 20 most sensitive modes to damage in the hot spots (b)

The eigenvalue problem for all n damaged modes can be written as

$$\mathbf{MP}_d\mathbf{\Lambda}_d^2 + \mathbf{KP}_d = \mathbf{\Delta K P}_d, \qquad (11)$$

where

$$\mathbf{\Lambda}_d = diag[\lambda_{d1}, \lambda_{d2}, \dots, \lambda_{dn}],$$

and can be projected onto a reduced space by premultiplying Eq. 11 by $\mathbf{P}_d^T$ to obtain

$$\mathbf{P}_d^T\mathbf{MP}_d\mathbf{\Lambda}_d^2 + \mathbf{P}_d^T\mathbf{KP}_d = \mathbf{P}_d^T\mathbf{\Delta K P}_d = \mathbf{\Delta \bar{K}}. \qquad (12)$$

Using the knowledge that only certain regions of the system are damageable, different reduced damage scenarios, $\mathbf{\Delta \bar{K}'}$ can be generated as a set of basis matrices for damage in the reduced space. A requirement of these basis matrices is that they are linearly independent. These basis matrices are calculated by transforming the physical damage scenarios to the reduced ones using the following relation

$$\mathbf{P}_d^T\mathbf{\Delta K}_i\mathbf{P}_d = \mathbf{\Delta \bar{K}'_i}, \qquad (13)$$

where $\mathbf{\Delta K}_i$ is the damage in the stiffness matrix in the full (augmented) space, and $i = 1 \dots s$ corresponds to the damage scenario. The damage predicted can then be written as

$$\mathbf{\Delta \bar{K}} = \sum_{i=1}^{s} \mathbf{\Delta \bar{K}'_i}\alpha_i, \qquad (14)$$

where α_i corresponds to the level of damage corresponding to the ith damage scenario.

If the matrices from Eq. 14 are transformed into column vectors by stacking each column of the matrix on top of each other $n^2 \times 1$ vectors will result. If $n^2 > s$ and the $\mathbf{\Delta \bar{K}'_i}$ matrices are independent, an over determined set of equations results for calculating the damage, and α_i can be obtained from Eq. 14.

Filtering Algorithms

There are two filtering algorithms that can be used with this methodology to reduce the effects of noise.

The first filtering algorithm uses the fact that the damage scenarios affect different natural frequencies of the system. This filtering can be implemented by first determining which natural frequencies are changed significantly by each damage scenario in the damage range of interest. After damage occurs, the natural frequencies can be inspected to see which ones were affected. Finally, any damage scenario that would cause a change in a natural frequency of the system that remains unaffected is eliminated as a possible damage scenario. This information can be used in Eq. 13. Essentially, basis matrices $\mathbf{\Delta \bar{K}'_i}$ do not have to be computed for the eliminated damage scenarios, which filters out any damage that would be erroneously predicted in that space (due to measurement noise). This filtering algorithm is particularly powerful for cases where damages occur in a only a few damage scenarios.

The second filtering algorithm is based on the fact that the minimum singular values of the different $\mathbf{\Psi}_i$ matrices can be different in scale. A threshold value can be used such that if the minimum singular value for a given i is lower then the threshold, then that $\mathbf{\Psi}_i$ would not be used (in turn the eigenvector it corresponds to would not be calculated). This filtering algorithm is important because the singular values of $\mathbf{\Psi}_i$ are not known until after damage occurs so that the damage natural frequencies can be measured.

Numerical Results

To demonstrate the methodology presented, a numerical analysis of linear and nonlinear frame structures was implemented. The linear frame structure shown in Fig. 1 consists of 70 steel beams connected at 24 nodes, 4 of which are pinned to the ground, leading to 132 degrees of freedom. The damageable portions of the linear system were chosen as the longitudinal stiffness of the 20 longitudinal (horizontal) beams. The nonlinear frame structure shown in Fig. 2 consists of the same linear frame structure as in Fig. 1 with the addition of two plates connected to the frame at their center. These plates are clamped to ground at the perimeter and introduce two cubic stiffness nonlinearities to the structure. The damageable portions of the nonlinear system were chosen as the same 20 longitudinal beams and the two nonlinear plates. Hence, the size of the augmented linear
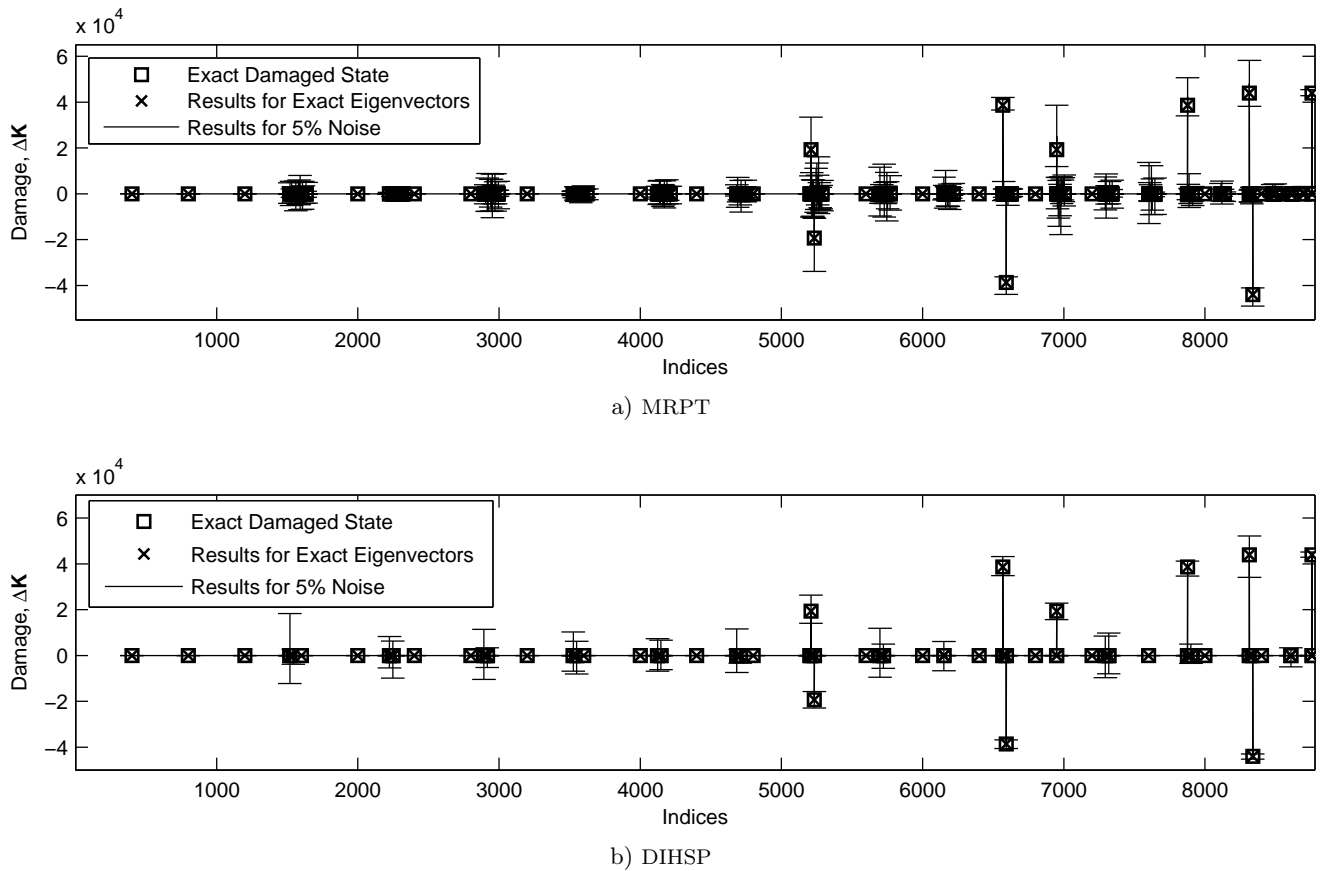
a) MRPT

b) DIHSP

Fig. 4   Predicted damage in the linear 5-bay frame with damage at three locations and 5% random eigenvector noise using 20 sensors to find damage using MRPT (a) and using DIHSP (b)

system is 134 degrees of freedom.

In the following sections several important aspects of the methodology are highlighted. First, the differences between ROM and ROHA are explored. Second, a comparison case is setup for MRPT and DIHSP. Next, the effects of the filtering algorithm and placement of additional sensors are illustrated. Finally, a damage case for the nonlinear frame system is examined.

Sensors were placed at the hot spots of the system for all the results obtained. In the linear system that implies that the sensors measure the longitudinal position of the 20 longitudinal beams. The nonlinear system has the same 20 sensors as the linear system, and one measuring the vertical displacement of the center of each plate.

ROHA vs. ROM

This section highlights the differences between a ROM type method and ROHA. ROM is designed to predict the system dynamics. In contrast, ROHA is designed to predict *changes* in the system dynamics. ROM uses the first (dominant) several modes of the system to capture the dynamics of the system. This differs from ROHA which uses the eigenvectors most sensitive to changes in the hot spots of the system.

Fig. 3 illustrates the difference in the methods in using the first 20 modes as opposed to the 20 most sensitive modes to damage obtained by using Eq. 1. The case plot-

ted in Fig. 3 is for a 15% loss of longitudinal stiffness in beam 2 and a 20% loss of longitudinal stiffness in beam 6, where these beam numbers are shown in Fig. 1. Standard deviation error bars are plotted for the 100 separate numerical simulations in which 2% random eigenvector noise was added. The x-axis in each plot represents the 20 damage scenarios. The y-axis in each plot represents the percent damage for each scenario. No filtering algorithm was applied for this case and 20 sensors were used. DIHSP was the damage methodology used to obtain the results. Fig. 3(a) corresponds to using the dominant modes of the system while Fig. 3(b) corresponds to using the sensitive modes of the system. In both plots it is shown that for the case of zero noise, damage is predicted exactly. In the case of 2% noise the actual damage is also predicted very accurately. However in Fig. 3(a) there are also large damages predicted by ROM in two other damage scenarios where there is no damage.

DIHSP vs. MRPT

In this section the differences between DIHSP and MRPT are explored. The results for both methodologies in a case with a 15% loss of stiffness in beam 4, 30% loss in beam 5 and 20% loss in beam 7 is plotted in Fig. 4. In the case of 5% random eigenvector noise, 100 separate numerical simulations were performed and standard deviation error bars are plotted. The x-axes
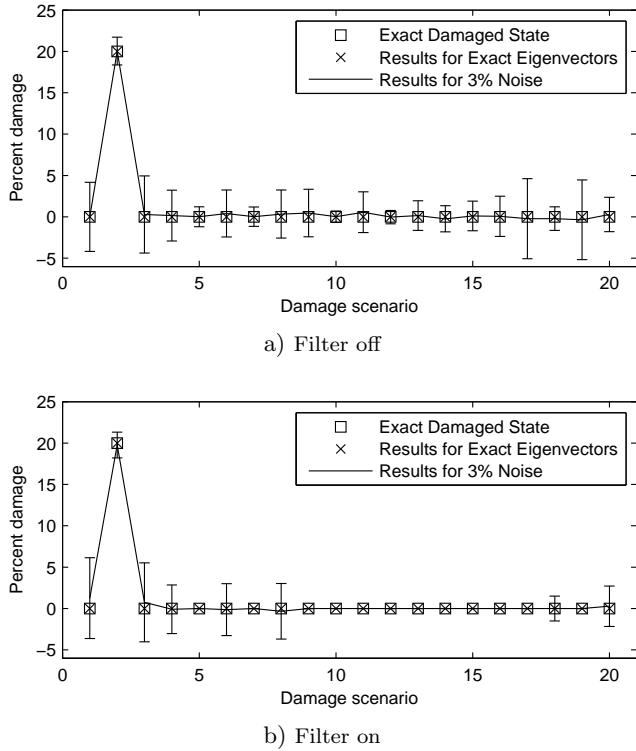
a) Filter off

b) Filter on

Fig. 5 Predicted damage in the linear 5-bay frame with damage at one location and 3% random eigenvector noise using 20 sensors without the frequency shift filter (a) and with the frequency shift filter (b)

in each plot represent the index of a column vector obtained from storing the upper triangular portion of the perturbation stiffness matrix ($\Delta \mathbf{K}$) into a column vector. The y-axes in the plots represent the entries of the difference between the original and updated stiffness matrices $\Delta \mathbf{K}$. In both cases the filtering algorithm that eliminates damage scenarios based on the shift in frequencies was used.

Fig. 4(a) corresponds to the case where MRPT was used to predict damage, while Fig. 4(b) corresponds to the case where DIHSP was used. Damage is predicted exactly by each methodology for the case of zero noise. For the case of 5% noise Fig. 4 shows that DIHSP predicts damage more accurately then MRPT. Also, DIHSP predicts fewer false damages elsewhere in the system.

Filtering

In this section the benefit of the filtering algorithm that eliminates damage scenarios based on the shift in frequencies is demonstrated.

Fig. 5 contains the results for a case with a 20% loss of stiffness in beam 1. The plots are structured in the same way as Fig. 3. Standard deviation error bars are plotted for the 100 separate numerical simulations in which 3% random eigenvector noise was added. Fig. 5(a) corresponds to the case without filtering, and Fig. 5(b) corresponds to the case with filtering. For the case where there is zero noise, both cases predict damage exactly. In the case of 3% noise, both cases predict the damage to a

similar level of accuracy, but Fig. 5(b) has less noise in the other damage scenarios (where there is no damage).

Effect of Additional Sensors

In this section the benefit of using additional sensors is illustrated. Fig. 6 contains the results for a case with a 15% loss of stiffness in beam 2, 25% loss in beam 3, 30% loss in beam 4 and 25% loss in beam 7. The plots are structured in the same way as in Fig. 3. In the case of 10% random eigenvector noise, 100 separate numerical simulations were performed and standard deviation error bars are plotted.

Fig. 6(a) corresponds to the case where 20 sensors are used and Fig. 6(b) corresponds to the case where 30 sensors are used. The 10 sensors beyond the 20 placed at the hot spots were found using the generalized EIDV method. In the case where there is zero noise, both sensor placements predict damage exactly. In the case of 10% noise, Fig. 6(b) predicts the damage slightly better in the scenarios with damage and predicts significantly fewer false damages than Fig. 6(a).
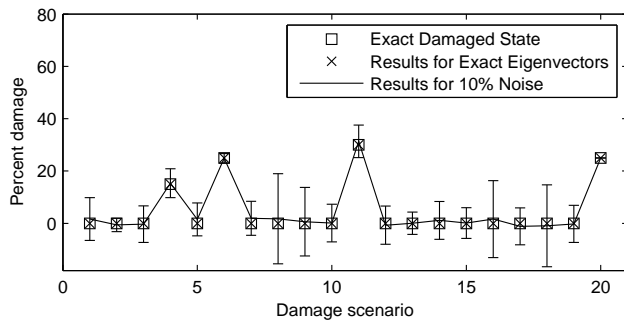
Nonlinear 5-Bay Frame

In this section, the use of ROHA and DIHSP is demonstrated for determining damage in linear and nonlinear elements of the frame shown in Fig. 2.

Fig. 7 contains the results for a case with a 35% loss of stiffness in plate A, 30% loss in plate B, 30% loss in beam C, and 25% loss in beam D where the beams and plates are indicated in Fig. 2. The plot is structured in the same way as Fig. 3. Standard deviation error bars are plotted for the 100 separate numerical simulations in which 3% random eigenvector noise was added. In the case where there is zero noise, exact damage is predicted. In the case where there is 3% noise, the actual damage is predicted very accurately, with some false damages predicted in other scenarios.
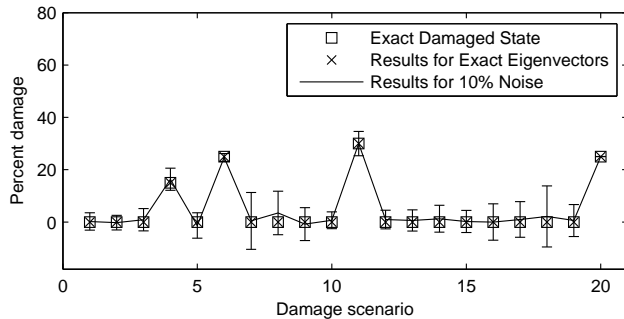
Discussion and Conclusions

Several new ideas have been incorporated into this integrated sensor placement and damage detection methodology.

The first major aspect of the methodology is ROHA. ROHA differs fundamentally from ROM techniques in its goal. ROM techniques are interested in capturing the dynamics of the system. Therefore, they use the dominant modes of the system, which tend to be the modes corresponding to the lowest frequencies of the system. In contrast, ROHA is interested in the change in dynamics, and as a result it uses the modes that are sensitive to changes in the hot spots of the system. ROHA uses a frequency filtering algorithm to remove frequency content away from these selected modes so that the partial eigenstructure obtained corresponds to the desired modes. The partial modes are then expanded to the full space using the information about the possible damage locations. Other algorithms that try to expand the

a) Twenty sensors

b) Thirty sensors

Fig. 6 Predicted damage in the linear 5-bay frame with damage at four locations and 10% random eigenvector noise using 20 sensors (a) and using 30 sensors (b)

modes within the space of a set of the healthy eigenvectors fail because the damage causes the eigenvectors to vary/rotate into the space of a large number of the original healthy eigenvectors.

The next major aspect of the methodology was the improved sensor placement. For the case where the number of sensors equals the number of damageable degrees of freedom, and the hot spots of the system were accessible, the sensors were placed at the hot spots of the system. If additional sensors need to be placed, or if some hot spots cannot have sensors placed there, a generalized EIDV method can be used to place the remaining sensors. EIDV is inherently a quasioptimal method in that it finds the optimal sensor choice at each step, but all the choices together do not necessarily lead to the global optimum sensor placement. Using EIDV to place the remaining sensors is useful since the EIDV procedure effectively searches for the optimal locations for the remaining sensors.

The DIHSP technique is the next major aspect of the methodology. The key advantage DIHSP has over other modal based techniques is that it essentially filters out all damages except the desired damage scenarios. That is also why DIHSP is only truly useful when the damage scenarios are known in advance.

The final novel aspect of the methodology herein is that it was extended to nonlinear (augmented) systems. The methodology as a whole is essentially the same. The only difference is that nonlinear ROHA/DIHSP requires
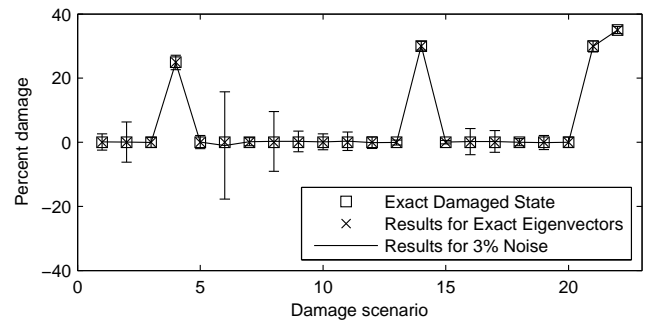

Fig. 7 Predicted damage in the nonlinear 5-bay frame with damage in both cubic stiffnesses and two linear hot spots and 3% random eigenvector noise using 22 sensors

the measurement of the degrees of freedom that contain nonlinearities. This is required in order to form the augmented equations of motion.

This work presents a method to place sensors for damage detection in linear and nonlinear systems. The sensor placement approach is based on determining the eigenvectors most sensitive to changes in damageable hot spots of interest in the system. The full modes are then extracted using ROHA. Damage can then be assessed using any number of modal based approaches or by DIHSP. Nonlinear systems can be handled by this methodology by converting them into augmented linear systems. The algorithms proposed have been demonstrated numerically for linear and nonlinear frame structures. The effectiveness of the proposed method was demonstrated, and the effects of measurement errors were presented.

Acknowledgments

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