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# OPTIMAL WING CONFIGURATION OF A TETHERED SATELLITE IN FREE MOLECULAR FLOW

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Abstract

In this paper we present an analysis of the Tethered Satellite/Wing System (TS/WS) in free molecular flow (at altitude of 142 km) in planar motion subject to impulse moments. The focus of this mission is to demonstrate and validate deployment and retrieval operations of a Tethered System, conduct hypersonic aerothermodynamic research., and validate Tethered Satellite System, or TSS, operations in the Earth's upper atmosphere. The analysis indicates that a wing system could provide stable flight over a wide range of conditions.

Nomenclature

| | |
|---|---|
| $\alpha$ | Angle of attack |
| $\alpha_o$ | Wing inclination |
| $\alpha(t)$ | Angular displacement of the TS/WS |
| $\alpha(t_m)$ | Maximum angular overshoot |
| $\alpha_{tm}(S,\alpha_o,d_6)$ | $\alpha(t_m)$ as a function of wing area, wing inclination, and boom length |
| $\beta$ | Impulse magnitude factor |
| C | The aerodynamic damping derivative |
| $C_d$ | Drag coefficient |
| $C_l$ | Lift coefficient |
| $C_m$ | Moment coefficient |
| $C_{mq}$ | Pitching damping coefficient |
| CM | Center of mass |
| D | Drag |
| $\delta$ | Applied impulse moment |
| $d_6$ | Boom length |
| $I^o_{cm}$,I | Moment of inertia of the TS/WS |
| L | Lift |
| $l_{ref}$ | Wing length |
| M | Moment |
| $M_{oz}$ | The moment resultant acting on the system |
| S | Wing area |
| $S_r$ | Speed ratio |
| $t_m$ | Time of maximum angular overshoot |
| v | Satellite speed |
| $\omega_n$ | Natural frequency of the TS/WS |
| $\zeta$ | Damping ratio of the TS/WS |

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System Parameters

| | |
|---|---|
| A | Shuttle orbiter altitude = 230 km |
| $D_t$ | Drag on satellite = 0.3 N (from reference 6) |
| l | Position along the tether measured from the orbiter = 88 km |
| i | Inclination of the orbiter plane relative to the equator = $50^o$ |
| $\rho$ | Density of the atmosphere = $3.358 \times 10^{-9}$ kg/m$^3$ (from reference 11) |
| $R_o$ | Radius of the Earth = 6378.14 km |
| $T_w$ | Wall temperature ≈ $310^o$ k (from reference 12) |
| wt | Wing thickness = 0.048 m |

## I. Introduction

The Tethered Satellite System, in conjunction with the Space Shuttle, will provide a new means for remote exploration of the Earth's upper atmosphere and ionosphere. To investigate the Earth's upper atmosphere, payloads of 200 to 500 kg will be lowered to distances of 100 km from the orbiter ( The Tethered Satellite System [TSS] will investigate altitudes roughly between 130 and 220 km above the Earth )[1,5,14]. Stable flight is imperative for the success of the mission. One possible control strategy is the use of a flat plate passive wing system attached to a boom mounted on the TSS. Fig. 1 shows a top view of the TS/WS as modeled in this study. The goal of this project was to find feasible and optimal values for wing area, boom length, and wing inclination to assure stable flight and acceptable peak "overshoot".

The model derived here is based on square flat plate wings. The TS/WS center of mass, CM, is located on the centerline of the boom, and the tether connection passes through the CM. The wing and boom dimensions are constrained by the practical fact that the system must "fit" within the space shuttle cargo bay.[9]

It is further assumed that the wings will remain in a fixed position, that the TS/WS is a symmetric body, and that the aluminum wing structure incorporates a protective coating to protect it from the hostile upper atmosphere and hypersonic velocities. This initial analysis is limited to *planar motion.*

## II. Background

Initial conceptual modeling of the TS/WS has been completed and was presented at the 29th AIAA Aerospace Sciences Meeting[15]; others have focused research in the areas of system modeling, aerodynamics and dynamics.

In the preliminary conceptual model presented in reference 15, planar motion of the TS/WS was examined for flight in an ideal atmospheric environment. The preliminary analysis indicated that the wing system could provide stable flight. The model however ignored the transition flight regime and the aerodynamic damping coefficient derived was for a TSS/cone frustrum in a contimuum flow.

The conceptual design and mission requirements of the Tethered Satellite spherical model have also been posed. The TSS will be lowered to an altitude of 130 km where it will conduct it's studies. Upon completion it will be further lowered until the tether breaks due to heating, or is severed from the shuttle.[1,2] The satellite characteristics (the spherical portion) include a mass of 500 kg and a diameter of 1.25 m[6,13].

The TSS satellite will fly in a very low pressure atmosphere where the Knudsen number is roughly 1 to 30 depending on the altitude. Since the TS/WS will fly in tandem with the Space Shuttle orbiter at a speed of roughly 7,400 m/sec, the vehicle will travel at "hypersonic" velocities in a rarified gas. In this range of Knudsen number there are two regimes of flight that must be considered: transition flow (~100 km - 140 km) and free molecular flow (>140 km). This paper will examine flight in free molecular flow. NASA has studied the aerodynamic effects on the spherical part of the satellite and the connecting tether, including drag and aerodynamic heating[6]; also studied were conceptual models of a TSS wing system with a 45° half-angle cone frustrum attached to a 1 m diameter spherical satellite[12].

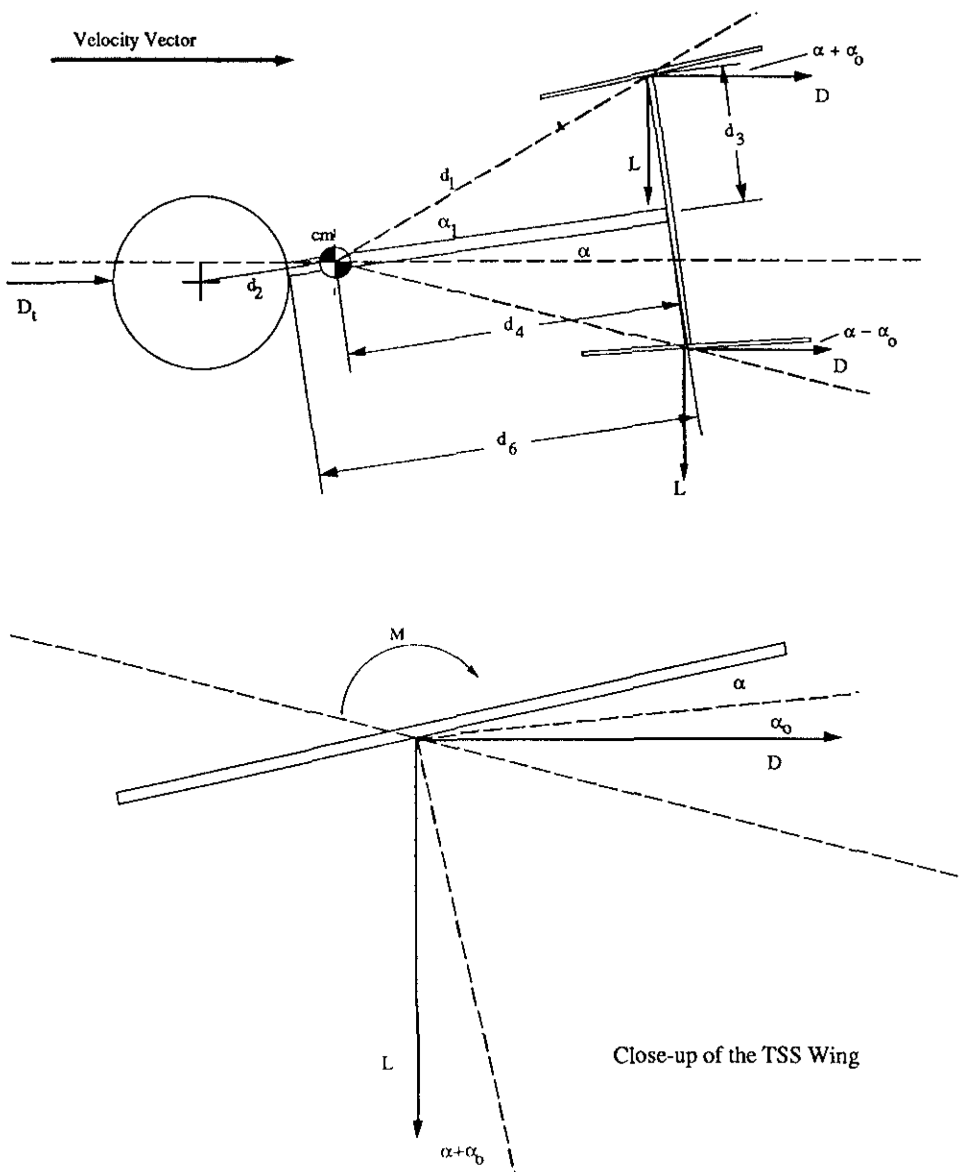


**Fig. 1 Top view of the Tethered Satellite/Wing System (TS/WS) with parameters and values used throughout the report.**

In this article we examine a passive flat plate wing system. Aerodynamic models of the lift, drag, moment and other coefficients for a flat plate in free molecular flow have been developed by Hayes and Probstein[7], Kogan[10], and Blick[3]. Blick's model for flat plate aerodynamics was incorporated into the TS/WS simulation.

Optimization was completed through a parametric study with the computer application *Mathematica*[18].

## III. Model Development

The lift, drag, and moment acting on any given wing design are given by

$$\text{LIFT} = L = C_l\ 0.5\rho\, v^2\, S \quad (1)$$

$$\text{DRAG} = D = C_d\ 0.5\rho\, v^2\, S \quad (2)$$

$$\text{MOMENT} = M = C_m\ 0.5\rho\, v^2\, S\ l_{ref} \quad (3)$$

Blick[3] analyzed and derived the aerodynamic coefficients as a function of angle of attack for a flat plate disk. Applying his result to the TSS flat plat wings leads to:

$$C_d = 2\sin\alpha + \left(\sqrt{\pi}/S_r\right)\sin^2\alpha \quad (4)$$

$$C_l = -\left(\sqrt{\pi}/S_r\right)\sin\alpha\cos\alpha \quad (5)$$

$$C_m = \left(-d_4\sqrt{\pi}\sin\alpha - 2\,S_r d_4 \sin^2\alpha + 2\,S_r d_3(\cos\alpha)\,(\sin\alpha)\right)/\left(S_r\, l_{ref}\right) \quad (6)$$

$$C_{mq} = \left(-S\sqrt{\pi} - 12 d_4^2\sqrt{\pi} - (4\,S)\,S_r\sin\alpha - 24\,S_r d_3^2\sin\alpha - 48\,S_r d_4^2\sin\alpha + 24\,S_r d_3 d_4\cos\alpha\right)/\left((12\,S)\,S_r\right) \quad (7)$$

where $d_3$ and $d_4$ are defined in Fig. 1, and

$$S_r = \frac{v}{\sqrt{2RT_w}} \quad (8)$$

$$v = (R_o + A - 1)\left[\sqrt{\frac{GM}{(R_o + A)^3}} - \omega_o\cos i\right] \quad (9)$$

For the specified environment, $S_r$ = 17.74 and v = 7376 m/sec. Diffuse reflection with full surface accommodation was employed in the analysis, consistent with typical spacecraft surfaces.

Results for $C_l$, $C_d$, and $C_{mq}$ for one of the wings and for $C_m$ of the TS/WS as a function of $\alpha$ are shown in Fig. 2.

The motion of a rigid body in a plane about a fixed point can be given by the equation:

$$M_{oz} = I_{cm}^{o}\ddot{\alpha} \quad (10)$$

Where,

$$M_{oz} = \left(\frac{\partial M}{\partial \alpha}\right)_{\alpha=0}\alpha + \left(\frac{\partial M}{\partial \dot{\alpha}}\right)_{\dot{\alpha}=\alpha=0}\dot{\alpha} + \beta\,\delta \quad (11)$$

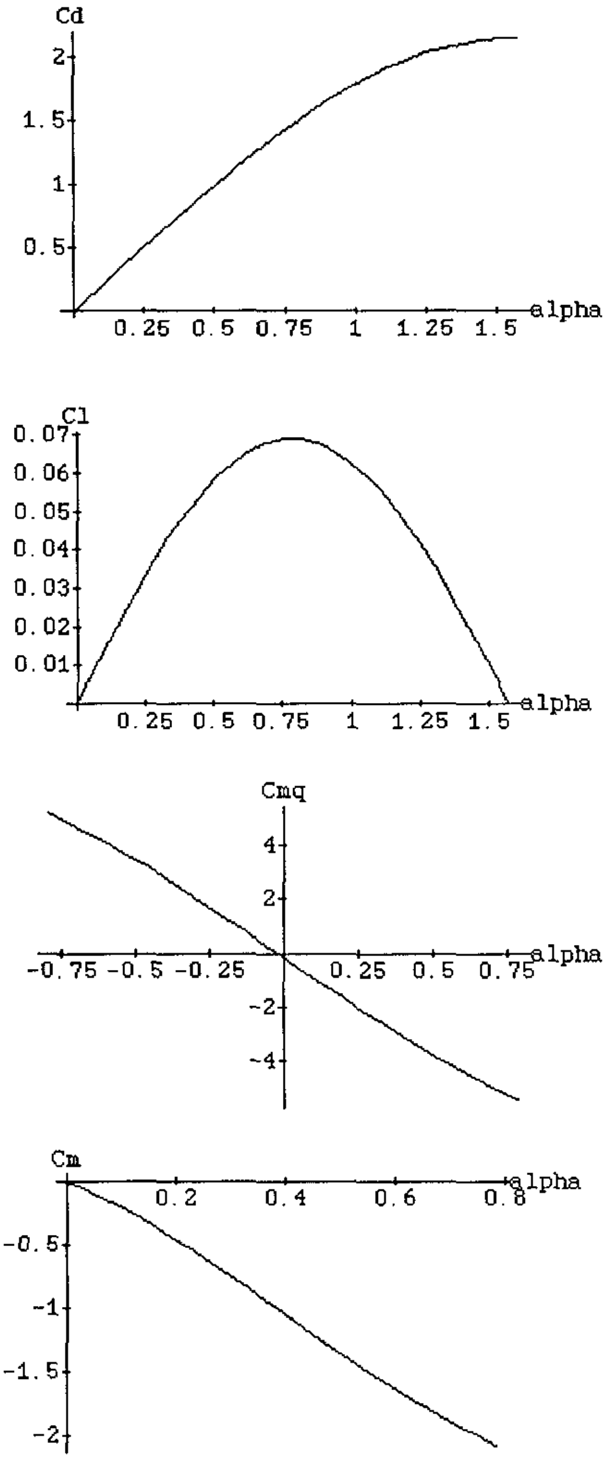


**Fig. 2 Drag coefficient ($C_d$), lift coefficient ($C_l$), the pitching damping coefficient ($C_{mq}$) of the flat plate wing, and the moment coeffiecient ($C_m$) of the TS/WS as a *function of* $\alpha$ *(alpha, in radians)***

$$\left(\frac{\partial M}{\partial \alpha}\right)_{\alpha=0} = \frac{\partial C_m}{\partial \alpha}(0.5\,\rho v^2)\,S\,l_{ref} = -K \quad (12)$$

$$\left(\frac{\partial M}{\partial \dot\alpha}\right)_{\dot\alpha=\alpha=0} = C_{mq}(0.5\,\rho v^2)\,S\,(l_{ref}/v) = -C \quad (13)$$

The $\beta\delta$ term is an applied impulse moment on the TS/WS, representing sudden flight environment changes. Applying the linearized Eqs. 6 and 7, we get a differential equation in the form of

$$I\ddot\alpha + C\dot\alpha + K\alpha = \beta\,\delta \quad (14)$$

or,

$$\ddot\alpha + 2\zeta\omega_n\dot\alpha + \omega_n^2\alpha = \hat{F}\,\delta \quad (14a)$$

where,

$$\zeta = \text{damping ratio} = \frac{C}{2\sqrt{KI}} \quad (15)$$

$$\omega_n = \text{natural frequency} = \sqrt{\frac{K}{I}} \quad (16)$$

$$\hat{F} = \frac{\beta}{I} \quad (17)$$

C, K and I are dependent on the wing area S, the boom length $d_6$, and the wing inclination $\alpha_o$. Also, due to the rarefied environment, the resulting value of C is much smaller than K.

The solution of the differential equation is

$$\alpha(t) = \frac{\hat{F}}{\omega_n\sqrt{1-\zeta^2}}\,e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}\,t \quad (18)$$

The *maximum overshoot* , $\alpha(t_m)$, of the system can be found by taking the derivative of the equation of motion and setting it equal to zero. Solving for $t_m$, yields

$$t_m = \frac{\cos^{-1}\zeta}{\omega_n\sqrt{1-\zeta^2}} \quad (19)$$

Substitution of (19) into (18) gives

$$\alpha(t_m) = \frac{\hat{F}}{\omega_n\sqrt{1-\zeta^2}}\,e^{-\zeta\cos^{-1}\zeta/\sqrt{1-\zeta^2}}\sin\left(\cos^{-1}\zeta\right) \quad (20)$$

which is a function of S, $d_6$, and $\alpha_o$. Equation (20) is the objective function to be minimized. The goal is to find values of S, $d_6$, and $\alpha_o$ which provide the minimum value for $\alpha(t_m)$.

The constraints are:

g1: $\alpha \le \alpha_{max} = 2^o$
g2: $\alpha \ge \alpha_{min} = -2^o$ (21)

g3: $\alpha_o \le \alpha_{o\,max} = 45^o$
g4: $\alpha_o \ge \alpha_{o\,min} = 0^o$
g5: $d_6 \le d_{6\,max} = 6$ m
g6: $d_6 \ge d_{6\,min} = 2$ m
g7: $S \le S_{max} = 5.3\ m^2$ (21)
g8: $S \ge S_{min} = 0.1\ m^2$
g9: $CM \le d_6$
g10: $CM \ge CM_{min} = 0.2$ m
g11: $\partial C_m/\partial\alpha < 0$

A parametric study showed that $\alpha$ stays within the $\pm 2^o$ limits imposed by g1 and g2 for all feasible values of S, $d_6$ and $\alpha_o$. Hence g1 and g2 can be left out of the computation. Similarly, g9 and g10 (illustrated in fig. 1) are never violated, so they can also be removed from the computation. From Fig. 2 one can see that the slope of the moment coefficient is negative, hence g11 is also never violated. Obviously it would be prudent to test any "optimal" design to be sure that g1, g2, g9 g10 and g11 are satisfied.

## IV. Model Solution

A parametric study indicated that the solution occurs at the maximum value for $d_6$. Accordingly the boom length was fixed at 6 m, and values for S and $\alpha_o$ which would provide the minimum "maximum peak overshoot" were determined. A parametric study of the objective function shows the optimal solution to occur at $S = 5.3\ m^2$, $\alpha_o = 0$ rads and $d_6 = 6$ m. Figure 3 shows a plot of the peak overshoot in radians as a functions of S and $\alpha_o$ for an impulse magnitude factor ($\beta$) equal to 1. Numerical values of the optimal solutions for different values of $\beta$ are presented in Table 1. The results show that stable flight should occur for an impulse magnitude factor less than 7.6. A conceptual model of the optimal TS/WS is shown in Fig. 4.

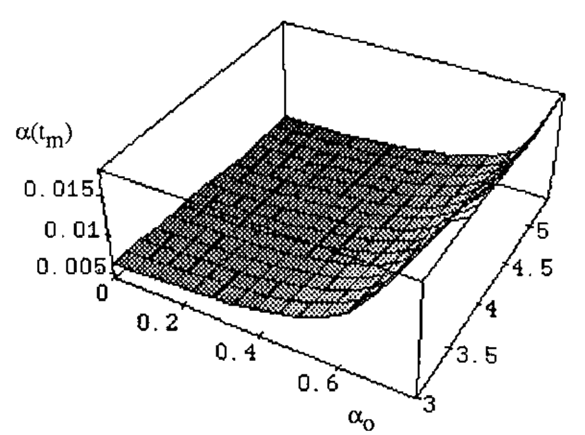


**Fig. 3 Peak Overshoot(in radians) as a function of S and $\alpha_o$,**

**Table 1 Resultant Peak Overshoot due to an Impulse Moment ( β δ )**

| $\beta$ | Peak Overshoot (radians) | Peak Overshoot (degrees) |
|---|---|---|
| 0.5 | 0.002305 | 0.1321 |
| 1.0 | 0.004611 | 0.2642 |
| 1.5 | 0.006917 | 0.3963 |
| 2.0 | 0.009222 | 0.5284 |
| 2.5 | 0.01152 | 0.6605 |
| 3.0 | 0.01383 | 0.7926 |
| 3.5 | 0.01614 | 0.9247 |
| 4. 0 | 0.01844 | 1.056 |
| 4.5 | 0.02075 | 1.188 |
| 5.0 | 0.02305 | 1.321 |
| 5.5 | 0.02536 | 1.453 |
| 6. 0 | 0.02766 | 1.585 |
| 6.5 | 0.02997 | 1.717 |
| 7.0 | 0.03228 | 1.849 |
| 7.5 | 0.03458 | 1.981 |
| 8.0 | 0.03689 | 2.113 |

## V. Future Work

In the future several additional tasks will be addressed. The first of these tasks will be to develop an aerodynamic model of the TS/WS in the transition flight regime (Knudsen numbers between 1 - 10) and to develop models of the TS/WS flight characteristics. Though little detail is still known about the Thermosphere, Bird has developed a technique known as the *Direct Simulation Monte Carlo Method* (DSMC) for accurately modeling on a computer gas flows over surfaces.[4] The software will be used to determine the lift and drag coefficients, aerodynamic heating, and flow characteristics of the TS/WS. Because of the TS/WS's complex shape a *body fitting curvilinear coordinate system* will be employed in the modeling process.[15,16] The target goal of this task will be to develop a mathematical model defining the behaviour of the TS/WS which can be integrated into the dynamic model.

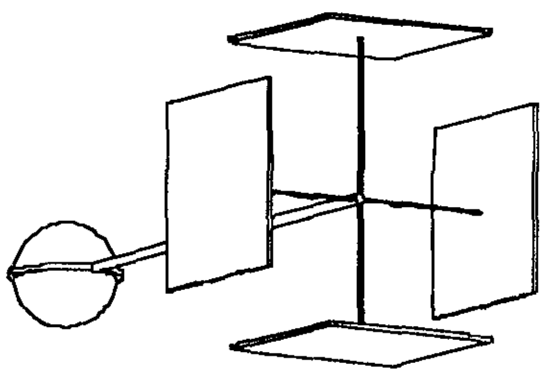

**Fig 4 Conceptual Model of the "optimal" TS/WS**

Other work will include further development of the dynamic model development and constraint identification. The key goals of this task will be to model the dynamic motion of the TS/WS (taking into account orbital eccentricity and motion in three dimensions), to identify stable modes, and to develop an objective function which will characterize the motion and incorporate the aerodynamic model. It is expected that the system stability will be analyzed using a combination of both analytical (including using a Lagrangian approach) and numerical techniques. This task will also identify those constraints on the model which have not been identified, and to consider a variety of loading conditions the TS/WS will encounter in flight. Such analyses will allow us to identify and deal with worst case scenarios. Once the system has been modeled, a range of feasible system configurations, and "best" system configuration will be identified. A weight constraint will be included to allow additional payloads to be taken up with the TS/WS, to allow increased orbiter altitude, and/or to allow for additional scientific experiments on board the TS/WS.

## VI. Conclusion

These results indicate that an appropriately configured passive wing system could provide stable flight for a TS/WS in free molecular flow subject to impulse moments. As a result, sensing probes on the satellite could accurately measure and study the Earth's upper atmosphere. The next step will be to investigate the behaviour of the system under different loading conditions, transitional flow and in 3D motion.

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