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Minimum-Fuel Aeroassisted Coplanar Orbit Transfer Using Lift Modulation
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Abstract

Minimum-fuel trajectories and lift controls are computed for aeroassisted coplanar transfer from high orbit to low orbit. The optimal aeroassisted transfer requires less fuel than the all-propulsive Hohmann transfer for a wide range of high orbit to low orbit transfers. The optimal control program for the atmospheric portion of the transfer is to fly at maximum positive L/D initially to recover from the downward plunge, and then, to fly at negative L/D to level off the flight, such that the vehicle skips out of the atmosphere with a flight path angle near zero degrees. To avoid excessive heating rates, the vehicle flies initially at high angle of attack in order to slow down higher in the atmosphere, allowing recovery from the downward plunge, which occurs subsequently using the maximum positive L/D, to take place at a lower atmospheric density, or equivalently, at a higher altitude.

Introduction

When orbital transfer is required and there is an atmosphere-bearing celestial body in the vicinity, it may be advantageous to utilize aerodynamic force in effecting the transfer. In this paper, we present an investigation of aeroassisted coplanar transfer from a circular orbit of radius \( r_1 \) to a concentric circular orbit of radius \( r_2 \), where \( r_1 \) is greater than \( r_2 \) (Fig. 1). We will consider the orbits to be about the Earth, however much of the analysis is more generally applicable. Our assumptions are as follows. The vehicle has a lifting configuration; and the lift can be modulated by varying the angle of attack. Lift modulation is the sole means of controlling the flight path in the atmosphere, propulsion being used only outside the atmosphere. The vehicle has a high-thrust propulsion system so that applications of the thrust can be considered to produce impulsive velocity changes (\( \Delta V \)) and the fuel consumption for an orbital transfer is thus indicated by the characteristic velocity, the sum of the \( \Delta V \)s needed to effect the transfer.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta V_{i} )</td>
<td>impulsive change in ( V )</td>
</tr>
<tr>
<td>( \Delta V )</td>
<td>to deorbit from ( HE0 )</td>
</tr>
<tr>
<td>( \Delta V_{2} )</td>
<td>to circularize at ( LEO )</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>( C_{L}/C_{L}^* )</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>( E_{p}/(v_{p}v) )</td>
</tr>
<tr>
<td>( \mu )</td>
<td>gravitational constant multiplied by mass of Earth</td>
</tr>
<tr>
<td>( \rho )</td>
<td>atmospheric density</td>
</tr>
<tr>
<td>( \rho_{0} )</td>
<td>value of ( \rho ) at ( H = 40 \text{ km} )</td>
</tr>
<tr>
<td>( \tau )</td>
<td>( (t_{f}/t_{h})/\mu R )</td>
</tr>
</tbody>
</table>

Subscripts

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e )</td>
<td>value at atmospheric entry</td>
</tr>
<tr>
<td>( f )</td>
<td>value at atmospheric exit</td>
</tr>
<tr>
<td>( p )</td>
<td>value at hypothetical perigee of transfer orbit from ( HE0 ) to atmosphere</td>
</tr>
</tbody>
</table>

Fig. 1 Aeroassisted coplanar orbit transfer.
transfer must involve only a single atmospheric pass. And finally, the atmospheric properties, the vehicle's aerodynamic properties, the equations of motion, and the initial position and velocity of the vehicle are all known precisely.

Under these assumptions, we determine the minimum-fuel aeroassisted transfer, the fuel requirements of which are then compared to those of the minimum-fuel all-propulsive transfer. The characteristics of the minimum-fuel trajectory during the atmospheric portion of the aeroassisted transfer are examined in detail. In addition, the effect of a vehicle heating constraint on the atmospheric trajectory is determined.

The motivation for this study stems from the current interest in orbital transfer vehicles (OTVs). These vehicles would transfer spacecraft from a space shuttle to higher and/or different inclination orbits. In the case of a space-based OTV, the vehicle is then required to return, after delivering its cargo, to rendezvous with either a shuttle or a space operating center. The OTV maneuvers which could potentially benefit from aeroassist are the orbital plane change and the transfer from high Earth orbit (HEO) to low Earth orbit (LEO). The present study concerns only the latter.

The basic sequence of events for the aeroassisted HEO to LEO coplanar orbit transfer is as follows. Referring to Fig. 1, the transfer begins with a tangential retroburn (AV1) at HEO which injects the vehicle into an elliptical transfer orbit with a hypothetical target perigee inside the atmosphere. At point E, the vehicle enters the atmosphere. As the vehicle flies through the atmosphere, some of its kinetic energy is converted to heat, and consequently, upon skipping out of the atmosphere (at point F), the apogee of the orbit is decreased to the distance r2. Finally, at the new apogee, a second tangential burn (AV2) is executed to circularize and thereby achieve the desired LEO. The minimum-fuel aeroassisted transfer is that which has the minimum characteristic velocity, AV1 + AV2. The flight path for the minimum-fuel transfer is effected by the AV1 magnitude, which controls the atmospheric entry, and the lift coefficient as a function of time during the atmospheric flight, which controls the exit and hence determines the required AV2.

The above version of an aeroassisted transfer is somewhat restrictive. Firstly, a tangential AV1 at HEO is not always the most fuel efficient means of effecting a transfer to specified atmospheric entry conditions. The justification for this restriction is that, for the range of HEO to LEO transfers considered in this paper, the additional fuel savings, if any, offered by non-tangential or multiple-impulse transfers, are small. Furthermore, multi-impulse transfers are often times impractical. For a more general treatment of the minimum-fuel deorbit, see Ref. 3. Secondly, the apogee of the transfer orbit, that which injects the vehicle into the atmosphere, does not necessarily need to be at the distance r2. However, for the one-impulse transfer from exit to LEO, this tangency turns out to be a property that is consistent with the minimum characteristic velocity transfer.

Analytic Solution for an Idealized Optimal Transfer

Consider an aeroassisted HEO to LEO transfer which proceeds as follows. Referring to Fig. 1, a tangential retroburn, AV1, at HEO injects the vehicle into an elliptical transfer orbit with perigee at the distance R. When the vehicle is at perigee, its lifting capability (in this case, negative lift) is employed to effect flight along the boundary of the atmosphere (i.e., along a circular orbit of radius R). Flight along the boundary is continued until sufficient velocity has been depleted (by atmospheric drag) such that, upon reducing the lift to zero, the vehicle ascends on an elliptical orbit to an apogee at r2. Finally, at r2, a tangential or multiple-impulse burn, AV2, is executed to achieve the desired LEO. The idealizations here are 1) that the atmospheric density at R is sufficient to generate enough drag to slow the vehicle in a reasonable amount of time and 2) that the vehicle has sufficient lift to maintain flight along the atmospheric boundary.

Now compare the characteristic velocity of this idealized transfer with that of any realistic aeroassisted transfer. A realistic transfer would require a larger AV1 to ensure sufficient penetration into the atmosphere such that the required velocity is depleted before skipping back out, given the limited lifting capability of the vehicle. Thus the characteristic velocity is a lower bound for aeroassisted transfers. Secondly, for the one-impulse transfer from atmospheric exit to LEO, exit with a flight path angle of zero degrees (γf = 0°) into an elliptical transfer orbit, tangent to LEO at apogee, leads to the minimum circularizing AV2. The corresponding exit speed Vf = \(\sqrt{\frac{\mu}{R(\mu + R)}}\) + AV2. Any other exit pair (Vf, γf) will lead to a higher AV2. Consequently, the characteristic velocity, AV1 + AV2, for this idealized aeroassisted transfer is a lower bound for the characteristic velocity of any realistic aeroassisted transfer.

An analytic expression for this lower bound can be derived. Let

\[ a_1 = \frac{r_1}{R}, \quad a_2 = \frac{r_2}{R}, \quad \text{and} \quad \Delta v_1 = \frac{AV_1}{\sqrt{\mu/R}}. \]

The elliptical grazing trajectory requires an impulse

\[ \Delta v_1 = \sqrt{1/a_1} - \sqrt{2/[a_1(a_1 + 1)]} \quad (1) \]

The second impulse used to circularize the orbit at r2 is

\[ \Delta v_2 = \sqrt{1/a_2} - \sqrt{2/[a_2(a_2 + 1)]} \quad (2) \]

Thus the total characteristic velocity for the idealized aeroassisted transfer is

\[ \Delta v_A = \Delta v_1 + \Delta v_2 \quad (3) \]
Compare this to the characteristic velocity for the all-propulsive Hohmann transfer which is
\[ \Delta v_H = \sqrt{\frac{1}{a_1} - \sqrt{\frac{2a_2}{a_1(a_1 + a_2)} + \sqrt{\frac{2a_1}{(a_2 + a_1)}}} - \sqrt{\frac{1}{a_2}} } \]

The curve plotted in Fig. 2 represents pairs \((a_1, a_2)\) for which \(\Delta v_A = \Delta v_H\). For pairs below the curve, \(\Delta v_A < \Delta v_H\), i.e., the idealized aerodynamic transfer requires less fuel. For example, idealized aerodynamic transfer from geostationary orbit to LEO requires less fuel than the Hohmann transfer, if the LEO radius, \(r_2\) is less than about 12,000 km.

Furthermore, it is assumed that the drag polar is parabolic, that is
\[ C_D = C_D + K \lambda^2 \]

With this relation, using \(C_L\) as a control corresponds physically to using pitch modulation to shape the trajectory. It is convenient to use a normalized lift control
\[ \lambda = C_L / C_L^* \]

where \(C_L^*\) is the lift coefficient corresponding to the maximum lift-to-drag ratio \(E^*\). In terms of \(C_D\) and \(K\), we have
\[ C_L = \sqrt{C_D / K} ; \quad C_D = 2C_D^* ; \quad E^* = \frac{1}{2\sqrt{KC_D}} \]

Using the following dimensionless variables and parameters
\[ h = H/H_e ; \quad \nu = \sqrt{\mu} / \sqrt{R} ; \quad \tau = \frac{\nu}{H_e} \sqrt{\nu / R} \]
\[ \frac{\rho}{\rho_e} = b = R / R_e ; \quad B = \frac{\rho}{\rho_e} C_L^* \]

the equations of motion can be rewritten as
\[ \frac{dh}{\tau} = \nu \sin \gamma \]
\[ \frac{d\nu}{\tau} = -\frac{B^2}{2E^*} \nu^2 - \frac{b}{(b-1+h)} \sin \gamma \]
\[ \frac{d\gamma}{\tau} = 3b \lambda \nu + \tan \gamma \left[ \frac{\nu}{(b-1+h)} - \frac{b}{(b-1+h)\nu} \right] \]

Besides being preferable for numerical computation, the dimensionless equations of motion (10) focus attention on the critical aerodynamic parameters which affect flight, namely, the lift loading coefficient \(B\) and the maximum lift-to-drag ratio \(E^*\). Again \(\lambda\) is the modulated lift control, scaled such that \(\lambda = 1\) corresponds to flight at the maximum lift-to-drag ratio.

**The Optimization Problem**

The optimization problem is to find the magnitude of the tangential \(\Delta v_1\) and the lift control \(\lambda\), as a function of time, which minimize the total characteristic velocity
\[ \Delta v_1 + \Delta v_2 = \sqrt{\frac{1}{a_1} - \nu_e \cos \gamma_e / a_1} + \sqrt{\frac{1}{a_2} - \nu_f \cos \gamma_f / a_2} \]
where \( A = E^* p_y/(v p_v) \). In determining this rule, we have used the fact that the Hamiltonian is a quadratic function of \( \lambda \) whose second derivative with respect to \( \lambda \) is negative.

The adjoint variables satisfy the necessary conditions

\[
\frac{d\lambda}{dt} = -E^* F \cos \gamma + \frac{v \cos \gamma}{E^* (b-1+h)^2} \left( \frac{2b\lambda^2}{E^*} + bE^* \right)
\]

and

\[
\frac{dF}{dt} = -\frac{2b\sin \gamma}{(b-1+h)^3} + \frac{v A \cos \gamma}{E^* (b-1+h)^2} \left( v - \frac{2b}{(b-1+h)v} \right)
\]

where

\[
\delta' = \frac{d\delta}{dh} = \frac{1}{\delta_0} \frac{d\delta}{dh}
\]

Writing the Hamiltonian in terms of \( \lambda \) and \( F \), we have

\[
\mathcal{H} = F \sin \gamma - \frac{B}{2E^*} \delta v (1 + \lambda^2) - \frac{b \sin \gamma}{(b-1+h)^2 v}
\]

(24)

Since the equations of motion (10) do not depend explicitly on time and the final time is not prescribed, we have the Hamiltonian integral

\[
\mathcal{H} = 0
\]

(25)

Now, rather than the original six differential equations, we have five, namely, Eqs. (10) for the three states, and Eq. (22) and Eq. (23) for \( \lambda \) and \( F \). Integration of these equations will yield extremal trajectories for a number of problems which differ only in the entry and exit conditions which must be satisfied. Besides having reduced the dimension from six to five, this formulation has the distinct advantage that

\[
A = \frac{E^* p_y}{v p_v}
\]
four of the five dependent variables are physical variables. (Actually, only has physical meaning if \( |\lambda| < \lambda_{\text{max}} \).) This situation eases the difficulty in guessing unknown initial conditions during the course of solving the boundary value problem. The nonphysical variable \( F \) can almost always (\( \sin y \neq 0 \)) be computed from the other four using the Hamiltonian integral (24). Indeed one might use the Hamiltonian integral to eliminate the need for solving the differential equation for \( F \). However, to avoid the difficulty in evaluating \( F \) at the singularity, \( \sin y = 0 \), we shall integrate the equation for \( F \) instead and use the Hamiltonian integral as a check on the accuracy of the numerical integration.

The heating rate, \( H_R \), along the atmospheric trajectory is computed according to the equation

\[
H_R = (3.08 \times 10^{-4}) \rho^{1/2} V^3 \rho_0 \tag{26}
\]

where \( \rho \) is the atmospheric density in kg/km\(^2\) and \( V \) is the speed in km/s. Eq. 26 gives the convective heating rate for a sphere with a radius of one meter, under conditions of laminar flow. Since only relative changes are of concern, this model will suffice.

**Method of Numerical Solution**

We shall only concern ourselves with minimizing \( \Delta V_2 \). Although \( \Delta V_1 \) is the larger of the two burns, the difference between the value of \( \Delta V_1 \), required to target to a perigee at the atmospheric boundary, \( R = 6498 \) km, and that required to target to a perigee at the surface of the Earth, is a mere 13 m/s. In contrast, \( \Delta V_2 \) is very sensitive to the values of the exit parameters \( v_f \) and \( y_f \). For example, the \( \Delta V_2 \) required for a given HEO to LEO transfer can increase by 100 m/s or more for each degree above zero in the exit flight path angle, \( y_f \).

Knowing that a skip trajectory with \( y_f = 0^\circ \) leads to the minimum \( \Delta V_2 \) at the LEO to which the ascending orbit is tangent, we employed the following approach to compute minimum-fuel trajectories and controls. A target perigee, \( r_p \), is chosen and, from this, the entry parameters \( v_e \) and \( y_e \) are determined according to the equations

\[
v_e^2 = 2\left[1 - 1/(a_1 + a_p)\right] \tag{27}
\]

\[
\cos^2 y_e = \left[2a_1 - (2 - v_e^2) a_1^2\right]/v_e^2 \tag{28}
\]

Then, using the computed values of \( v_e \) and \( y_e \), \( h_e = 1 \), and a pair (\( v_e, F_p \)) as initial conditions, Eqs. 10, 22, and 23 are integrated from \( t = 0 \) to \( t = 1 \), using Eq. 20 to determine the lift control. The pair (\( \dot{v}_e, F_p \)) is determined by choosing \( \dot{v}_e \) and using Eq. 24 to solve for the corresponding value of \( F_p \), with \( v, \gamma \), and \( h \) as specified above. The integration is performed by a variable order, linear, multistep predictor-corrector routine of the Adams-Moulton type,\(^7\) with the local absolute error controlled to less than 1.0\( \times 10^{-8} \) for each of the five dependent variables. In all cases studied, it has been possible to find, by iterative search, a value of \( \dot{v}_e \) such that \( y_f = 0^\circ \) at exit. The corresponding value of \( v_f \) determines the apogee of the transfer orbit, following exit, and hence, the LEO to which the vehicle is optimally transferred. As the value of \( r_p \) is lowered from \( R, y_f = 0^\circ \) continues to be reachable, but the exit speed decreases, resulting in lower LEO transfers. There is a certain critical value of \( r_p \), below which, the lifting capability of the vehicle is insufficient to effect a skip trajectory.

A trajectory and control computed in this manner is optimal in the following sense. Firstly, the necessary conditions (10), (22), and (23), the entry and exit conditions (15) and (16), and the relations (13) and (14) are satisfied. Secondly, the lift control satisfies the constraint (19). Thirdly, the near-zero degree flight path angle at exit ensures that the circularizing \( \Delta V_2 \), to achieve the LEO to which the post-exit orbit is tangent, is the absolute minimum, when compared to those for all other aeroassisted transfers from the same HEO to the same LEO. (The exit flight path angles achieved, as indicated in Table 1, are a few tenths of a degree. The iteration on \( r_p \) was stopped at this point because the associated value of \( \Delta V_2 \) was within 8 m/s of the lower bound set by the idealized transfer.) Fourthly, although only \( \Delta V_2 \) has been minimized, the characteristic velocity, \( \Delta V_1 + \Delta V_2 \), is very close to the absolute minimum. The value of \( \Delta V_1 \) for the cases shown in Table 1 is within 10.4 m/s of the lower bound on \( \Delta V_1 \) given by the idealized transfer. Thus, the characteristic velocity can not get much smaller. More important, however, is whether the trajectory and/or control would change significantly, as the characteristic velocity is reduced the last few meters per second. Numerical experience indicates that they do not. As the exit flight path angle is reduced, the associated trajectory is changing very little. Indeed, the value of \( v_e \) is being changed only slightly (parts in \( 10^6 \) or less) to get the exit angle below a few tenths of a degree. This level of change in \( v_e \) affects most of the trajectory almost negligibly, but extends the trajectory, in order to achieve the lower exit angles. Furthermore, as mentioned above, zero exit flight path angle atmospheric trajectories, there is a one-to-one mapping, based on numerical experience, from values of \( r_p \) to values of \( v_f \), and hence, to the LEOs for which the \( \Delta V_2 \) is a minimum. Therefore, if \( r_p \) were increased in order to decrease \( \Delta V_1 \), the corresponding \( \Delta V_2 \) would be greater. Given the low sensitivity of \( \Delta V_1 \) to changes in the value of \( r_p \), it is unlikely that the characteristic velocity could be reduced much, if any, by adjusting \( r_p \). In conclusion, a trajectory and control, computed in the manner described above, is a good approximation to that with the absolute minimum characteristic velocity and, henceforth, we shall refer to such a solution as a minimum-fuel solution.

When a heating rate constraint is imposed, the solution procedure is somewhat different. We follow an approach used in Ref. 6. The heating rate for a skip trajectory reaches its maximum value shortly after entry, in a monotonic fashion (see Fig. 4). It then decreases, as expected, during the remainder of the flight, although some oscillation may occur. In order to satisfy a heating rate constraint, \( H_R \leq (H_R)_{\text{max}} \), we shall assume that it
is sufficient to control the first peak of the heating rate function, such that the peak value is equal to \( (H_R)_{\text{max}} \). Furthermore, we shall assume that, once this peak value is reached, flight does not continue on the constraint boundary. These assumptions allow us to solve the constrained problem in two stages, each requiring an iteration on only one parameter.

In the first stage, we begin at \( \tau = 0 \) as in the unconstrained case, except that now the goal is to choose \( \lambda_e \) such that \( H_R = (H_R)_{\text{max}} \) at the time when the derivative of the heating rate with respect to time is equal to zero. Once this value of \( \lambda_e \) is found, an extremal trajectory up to the peak heating rate is determined. The second stage is to find a value for \( \lambda_r = \lambda_p \) such that when Eqs. 10, 22, and 23 are integrated from the time of the peak heating rate to atmospheric exit, the exit flight path angle is zero degrees. Indeed, in the cases studied, it has been possible to find such values of \( \lambda_e \) and \( \lambda_p \). Thus, the functions \( \lambda \) and \( F \) are, in general, discontinuous at the time of the peak heating rate; the states \( h, v, \) and \( \gamma \) are always continuous.

**Minimum Fuel Trajectories**

For all the cases reported below, the transfer is from geostationary Earth orbit (GEO), for which \( r_1 = 42,241 \) km. The radius of the atmosphere is 6498 km. Above this distance, the density is identically zero. Over the altitudes of atmospheric flight, 40-120 km (where the radius of the Earth is taken to be 6378 km), the density is approximated by a fifth-degree Chebyshev polynomial whose coefficients were determined by a least-squares fit to the U.S. Standard Atmosphere, 1976 (Ref. 7). The vehicle mass-to-surface area is 300 kg/m² for all cases.

**Unconstrained**

We begin by presenting some minimum-fuel trajectories, under conditions of unbounded lift (\( \lambda_{\text{max}} = \infty \)) and unconstrained heating rate. Three vehicle configurations were considered, as distinguished by their respective maximum L/D capabilities, namely, 0.845, 1.5, and 2.9. Data from wind tunnel tests is available for vehicles with these maximum L/D capabilities (Refs. 8, 9, and 10, respectively) and the values for the parameters \( C_D_0 \) and \( K \) which appear in the parabolic drag polar were chosen to best fit the data. The values used for the pair \( (C_D_0, K) \) were \((0.21, 1.67), (0.10, 1.11), \) and \((0.017, 1.76)\), respectively.

For each of the three maximum L/D cases, we have fixed \( r_p = 6400 \) km and have searched and found the \( \lambda_e \) such that \( \gamma_f = 0^\circ \). In this manner, a minimum-fuel trajectory for each case was generated. The corresponding LEO orbits, to which the transfers are optimal, are not exactly the same, but are close enough to permit comparisons. The alternative approach of specifying the LEO orbit a priori would require searching on two parameters, \( r_p \) and \( \lambda_e \), in order to determine the minimum-fuel trajectory.

Certain characteristics of the minimum-fuel unconstrained trajectories are given in the first three columns of Table 1. We see that the high L/D vehicle penetrates farthest into the atmosphere and experiences the highest dynamic pressure and heating rate. The low L/D vehicle experiences the highest g-load. For comparison, the Shuttle design limits for dynamic pressure and g-load are 16 kN/m² and 2.5 respectively. Time histories of the state variables for the \( (L/D)_{\text{max}} \times 1.5 \) case are shown in Fig. 3; those for the heating rate,

<table>
<thead>
<tr>
<th>L/D Capability</th>
<th>Heating Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>((L/D)_{\text{max}})</td>
<td>Low</td>
</tr>
<tr>
<td>0.845</td>
<td>1.5</td>
</tr>
<tr>
<td>1.5</td>
<td>6400.0</td>
</tr>
<tr>
<td>2.9</td>
<td>2.303247</td>
</tr>
<tr>
<td>(\lambda_e (\lambda_p))</td>
<td>Exit flight path angle (deg)</td>
</tr>
<tr>
<td>6558.8</td>
<td>6578.7</td>
</tr>
<tr>
<td>(\Delta V_2) (m/s)</td>
<td>26.1</td>
</tr>
<tr>
<td>Ideal (\Delta V_2) for same LEO (m/s)</td>
<td>18.3</td>
</tr>
<tr>
<td>Min altitude (km)</td>
<td>58.8</td>
</tr>
<tr>
<td>Max dynamic pressure (kN/m²)</td>
<td>15.9</td>
</tr>
<tr>
<td>Max convective heating rate for a one meter sphere (W/cm²)</td>
<td>193.1</td>
</tr>
<tr>
<td>Max g's</td>
<td>3.6</td>
</tr>
</tbody>
</table>
dynamic pressure, and g-load are shown in Fig. 4. The behaviors illustrated in these two figures are qualitatively representative of all the cases investigated in this paper.

Fig. 5 shows the lift-to-drag ratio as a function of the time from atmospheric entry for the three cases. A similar pattern is followed in each case. The maximum positive L/D is used initially to recover from the downward plunge. As the flight path angle becomes positive, the maximum negative L/D is used to level off the flight. These first two phases occur within the first four minutes of flight. Of course, although the basic pattern is similar, quantitatively, there are definite differences in the flight characteristics of the three L/D vehicles, as indicated in Table 1. After the first four minutes, a negative L/D is used to maintain flight at a small positive flight path angle in order to achieve the desired shallow exit. The required negative L/D increases, as the flight proceeds, to compensate for the decreasing atmospheric density.

**Bounded Lift**

For the vehicle, with \((L/D)_{\text{max}} = 1.5\), wind tunnel data show that the lift coefficient does not exceed 0.9 in absolute value. Thus, we impose the constraint

\[ C_L \leq 0.9 \]

which corresponds to setting \(\lambda_{\text{max}} = 3.0\) in Eq. 20. The resulting lift control is given by the dashed curve in Fig. 6. For comparison, the corresponding curve with \(C_L\) unbounded is given by the solid curve. We see that flight is along the constraint boundary for much of the flight. The important point, however, is that a near zero exit flight path angle is still reachable by proper choice of \(\lambda_e\).

**Constrained Heating Rate**

Using Eq. 26, the heating rate along the minimum-fuel trajectory can be calculated. Referring to this as the unconstrained heating rate, we can ask the question: What is the minimum-fuel trajectory, if the maximum heating...
rate is constrained to be no greater than some fraction of the maximum unconstrained heat rate? In order to see the effect of a heating rate constraint, we again consider the configuration with $(L/D)_{\text{max}} = 1.5$, as described earlier, except that the target perigee is taken to be 6415 km. The minimum-fuel trajectory is computed first with the heating rate unconstrained. The maximum heating rate is found to be 190.8 W/cm² for a reference one meter sphere. Next, the minimum-fuel trajectory is computed with all conditions identical, except that the heating rate is constrained not to exceed 150.0 W/cm². In both cases, the lift coefficient is bounded, as described earlier.

Certain characteristics of the unconstrained and constrained cases are given for comparison in the last two columns of Table 1. In both cases, a near-zero exit flight path angle is reached and the $\Delta V_2$ is within 8 m/s of that for the idealized transfer. With the heating rate constrained, the vehicle does not penetrate the atmosphere as deeply, the maximum dynamic pressure is reduced, but the maximum g-load is increased.

The optimal lift control, for each case, is plotted versus time in Fig.7. We see that the vehicle flies, initially, at $(C_L)_{\text{max}}$, in the constrained case; whereas, in the unconstrained case, $C_L$ is decreasing steadily during the same period. By flying at $(C_L)_{\text{max}}$ initially, and correspondingly at a higher $C_D$, the vehicle slows down higher in the atmosphere, allowing recovery from the downward plunge, which occurs subsequently at the maximum positive L/D, to take place at a lower atmospheric density or equivalently at a higher altitude. In this manner, higher heating rates are avoided.

As a final note, the minimum entry flight path angle from which the vehicle can recover and achieve the prescribed exit state conditions, is raised when a heating rate constraint is imposed (that is, raised with respect to the unconstrained case). The reason is that, if the entry is too steep, even by flying at the maximum positive $C_L$, excessive heating rates cannot be avoided. In the particular case investigated here, an optimal solution was found for entry angles as low as $-6.5^\circ$ $(r_p = 6400 \text{ km})$ in the unconstrained case. In the constrained case, the lowest entry angle, that could be tolerated, was $-6.0^\circ$ $(r_p = 6415 \text{ km})$.

Summary and Conclusions

Under the assumptions and restrictions given in the Introduction, minimum-fuel aeroassisted coplanar transfer from high orbit to low orbit has been considered. An idealized version of the transfer lent itself to analytic treatment and allowed a lower bound on the characteristic velocity for any given HEO to LEO aeroassisted transfer to be determined. In order to examine minimum-fuel transfer under more realistic conditions, an optimization problem was formulated and solved numerically. It was found that for each given HEO to LEO transfer considered, even with bounded lift and/or a heating rate constraint, a characteristic velocity within 10–20 m/s of the lower bound is achievable. Thus, Fig. 2 provides a good indication of the high orbit to low orbit coplanar transfers for which the optimal aeroassisted transfer requires less fuel than the Hohmann transfer.

The characteristic lift program for the atmospheric portion of the minimum-fuel transfer is to fly at the maximum positive L/D initially to recover from the downward plunge, and then, to fly at negative L/D to level off the flight, such that the vehicle skips out of the atmosphere with a flight path angle near zero degrees. This program is modified at the beginning if high heating rates are to be avoided. Flight initially at maximum lift, and correspondingly, high drag, lowers the vehicle's speed higher in the atmosphere, allowing recovery from the downward plunge, which occurs subsequently using the maximum positive L/D, to take place at a lower atmospheric density, or equivalently, at a higher altitude.

References


