THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

THE INTERACTION OF A DETONATION WAVE
WITH AN INERT BOUNDARY

William P. Sommers

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy in The
University of Michigan
1961

March, 1961

IP-501
ACKNOWLEDGMENTS

It has been the author's privilege to have had the following gentlemen serve on his doctoral committee:

Professor Richard B. Morrison, Co-Chairman
Professor William Mirsky, Co-Chairman
Doctor Theodore P. Cotter, Los Alamos Scientific Lab.
Professor Arthur G. Hansen
Professor R. S. B. Ong, presently on leave
Professor J. Louis York

The author is particularly indebted to Professor Richard B. Morrison, who invariably provided the encouragement, understanding and assistance when most needed throughout the period of this research. Professor Mirsky's efforts as Co-Chairman of the committee are gratefully acknowledged, as is the help given by Professors Hansen, Ong and York. Doctor Cotter, now returned to Los Alamos Scientific Laboratory, contributed valuable advice pertaining to the liquid explosive aspects of the study. The author also wishes to thank Professor Thomas C. Adamson, Jr. for his considerate help and interest in this study.

Particular thanks are due the entire staff of the Aircraft Propulsion Laboratory, who generously contributed their assistance on many occasions. Much of the success of this investigation is the result of their accumulated experience in the fields of gas dynamics and instrumentation.

Finally, the author wishes to thank the Institute of Science and Technology for the financial support provided during the last year of this work. Prior to this period, the author benefited from fellowships given by The Texas Company and The University of Michigan.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ACKNOWLEDGMENTS</strong></td>
<td>11</td>
</tr>
<tr>
<td><strong>LIST OF TABLES</strong></td>
<td>V</td>
</tr>
<tr>
<td><strong>LIST OF FIGURES</strong></td>
<td>vi</td>
</tr>
<tr>
<td><strong>NOMENCLATURE</strong></td>
<td>xi</td>
</tr>
<tr>
<td>I INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Summary of Gaseous Detonation Research</td>
<td>2</td>
</tr>
<tr>
<td>1.2 Summary of Condensed Explosives Research</td>
<td>4</td>
</tr>
<tr>
<td>1.3 Purpose of This Investigation</td>
<td>9</td>
</tr>
<tr>
<td>II THEORY OF DETONATION</td>
<td>11</td>
</tr>
<tr>
<td>2.1 One-Dimensional Analysis</td>
<td>11</td>
</tr>
<tr>
<td>2.2 Thermodynamic Equation of State</td>
<td>14</td>
</tr>
<tr>
<td>2.3 Chapman-Jouguet Detonation Relationships for Perfect Gases</td>
<td>21</td>
</tr>
<tr>
<td>III EXPERIMENTAL ARRANGEMENT AND PROCEDURE</td>
<td>24</td>
</tr>
<tr>
<td>3.1 Required Experimental Conditions</td>
<td>24</td>
</tr>
<tr>
<td>3.2 Experimental Arrangement</td>
<td>24</td>
</tr>
<tr>
<td>3.3 Detonation Tubes</td>
<td>26</td>
</tr>
<tr>
<td>3.4 Equipment for Velocity Measurement</td>
<td>31</td>
</tr>
<tr>
<td>3.5 Photographic Equipment</td>
<td>33</td>
</tr>
<tr>
<td>3.6 Gas Metering System</td>
<td>37</td>
</tr>
<tr>
<td>3.7 Test Sections</td>
<td>39</td>
</tr>
<tr>
<td>3.8 Operating Procedure</td>
<td>41</td>
</tr>
<tr>
<td>IV EXPERIMENTAL RESULTS</td>
<td>47</td>
</tr>
<tr>
<td>4.1 Axially Symmetric Detonation Experiments</td>
<td>47</td>
</tr>
<tr>
<td>4.2 Two-Dimensional Detonation Experiments, Test Section II</td>
<td>57</td>
</tr>
<tr>
<td>4.3 Two-Dimensional Detonation Experiments, Test Section III</td>
<td>58</td>
</tr>
<tr>
<td>4.3.1 Air Boundary Results</td>
<td>61</td>
</tr>
<tr>
<td>4.3.2 Helium Boundary Results</td>
<td>80</td>
</tr>
<tr>
<td>V THEORETICAL ANALYSIS AND COMPARISON WITH EXPERIMENTIAL RESULTS</td>
<td>95</td>
</tr>
<tr>
<td>5.1 Selection of Idealized Flow Model</td>
<td>95</td>
</tr>
<tr>
<td>5.2 Prediction of Shock and Interface Angles</td>
<td>97</td>
</tr>
<tr>
<td>5.3 Comparison of Theoretical and Experimental Results</td>
<td>107</td>
</tr>
</tbody>
</table>
### TABLE OF CONTENTS (CONT'D)

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.3.1 Idealized Model Results</td>
<td>107</td>
</tr>
<tr>
<td>5.3.2 Time-Unsteady Influences</td>
<td>111</td>
</tr>
<tr>
<td>5.3.3 Effect of Wave Curvature</td>
<td>120</td>
</tr>
<tr>
<td>5.3.4 Influence of Diffusion and Mixing Along the Jet Boundary</td>
<td>123</td>
</tr>
<tr>
<td>VI DISCUSSION OF RESULTS</td>
<td>127</td>
</tr>
<tr>
<td>6.1 Phenomenological Description of Boundary Interaction Process</td>
<td>129</td>
</tr>
<tr>
<td>6.2 Generalization of Analysis Through Use of Acoustic Impedance</td>
<td>132</td>
</tr>
<tr>
<td>6.3 Application of Results to Liquid Explosives</td>
<td>139</td>
</tr>
<tr>
<td>VII CONCLUSIONS</td>
<td>144</td>
</tr>
<tr>
<td>APPENDIX A. SAMPLE CALCULATION OF SHOCK AND INTERFACE ANGLE</td>
<td>147</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>151</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Typical Gaseous Detonation Properties</td>
<td>4</td>
</tr>
<tr>
<td>II</td>
<td>Typical Condensed Explosive Detonation Properties</td>
<td>6</td>
</tr>
<tr>
<td>III</td>
<td>Results of Calculations for the Solid Explosive PETN, Using Three Different Equations of State</td>
<td>19</td>
</tr>
<tr>
<td>IV</td>
<td>Detonation Tube Specifications</td>
<td>29</td>
</tr>
<tr>
<td>V</td>
<td>Explosive Gas Supply System Characteristics</td>
<td>37</td>
</tr>
<tr>
<td>VI</td>
<td>Flow Mach Numbers of Various Explosive Mixtures, and Gas Boundary Combinations</td>
<td>99</td>
</tr>
<tr>
<td>VII</td>
<td>Computed Values of Shock and Interface Angles</td>
<td>106</td>
</tr>
<tr>
<td>VIII</td>
<td>Comparison of Shock Angles Computed Using Steady Flow and Pseudo Shock Tube Analyses</td>
<td>118</td>
</tr>
<tr>
<td>IX</td>
<td>Comparison of Shock and Interface Angles Using Idealized Model and Approximate Curved Wave Analysis</td>
<td>123</td>
</tr>
<tr>
<td>X</td>
<td>Ratio of Acoustic Impedance of Boundary Material to that of Nitromethane</td>
<td>142</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>1</td>
<td>One-Dimensional Planar Wave</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>Hugoniot Curve</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>Schematic Sketch of Experimental System</td>
<td>25</td>
</tr>
<tr>
<td>4</td>
<td>Photograph of Entire Experimental System</td>
<td>27</td>
</tr>
<tr>
<td>5</td>
<td>Photograph of the Left End of Equipment Showing Detonation Tube B, Spark Source Box, Time Delay Unit (front of lower shelf) and 30 kV Power Supply (rear of lower shelf)</td>
<td>28</td>
</tr>
<tr>
<td>6</td>
<td>Photograph of Detonation Tube A</td>
<td>30</td>
</tr>
<tr>
<td>7</td>
<td>Ionization Probe Assembly</td>
<td>36</td>
</tr>
<tr>
<td>8</td>
<td>Circuit Diagram of Phototube Detector</td>
<td>36</td>
</tr>
<tr>
<td>9</td>
<td>Test Section II After Particularly Energetic Experiment</td>
<td>40</td>
</tr>
<tr>
<td>10</td>
<td>Test Section III and Typical Ionization Probe</td>
<td>42</td>
</tr>
<tr>
<td>11</td>
<td>Cross-Sectional Sketch of Test Section III</td>
<td>43</td>
</tr>
<tr>
<td>12</td>
<td>Experimental Detonation Velocity for Hydrogen-Oxygen Mixtures vs. Hydrogen Content, Corrected to Infinite Diameter Tube</td>
<td>45</td>
</tr>
<tr>
<td>13</td>
<td>Schlieren Photograph of Stable Explosive Jet Issuing from Detonation Tube A</td>
<td>48</td>
</tr>
<tr>
<td>14</td>
<td>Schlieren Photographs of Detonation Failing to Propagate Up Unconfined Column of Explosive; 79% Hydrogen-21% Oxygen with Air Boundary: Tube A</td>
<td>49</td>
</tr>
<tr>
<td>15</td>
<td>Explanation of Flow Details Visible in Figure 14</td>
<td>51</td>
</tr>
<tr>
<td>16</td>
<td>Dimensionless Distance of Wave from Tube Exit vs. Elapsed Time for 79% Hydrogen-21% Oxygen Mixture with Air Boundary: Tube A</td>
<td>53</td>
</tr>
<tr>
<td>17</td>
<td>Velocity of Leading Shock Wave vs. Dimensionless Distance from Tube Exit for 79% Hydrogen-21% Oxygen Mixture with Air Boundary: Tube A</td>
<td>54</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>18</td>
<td>Schlieren Photographs of the Detonation of 79% Hydrogen-21% Oxygen Within a Saran Wrap Detona-</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>tion Tube</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 78% Hydrogen-22% Oxygen Mixture</td>
<td>62</td>
</tr>
<tr>
<td></td>
<td>with Air Boundary: Test Section III, Tube B.</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 67% Hydrogen-33% Oxygen Mixture</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td>with Air Boundary: Test Section III, Tube B.</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 50% Hydrogen-50% Oxygen Mixture</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td>with Air Boundary: Test Section III, Tube B.</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 35% Hydrogen-65% Oxygen Mixture</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>with Air Boundary: Test Section III, Tube B.</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>Schlieren Photographs of 78% Hydrogen-22% Oxygen Detonation with Air Boundary: Test Section III.</td>
<td>67</td>
</tr>
<tr>
<td>24</td>
<td>Interpretive Sketch of Flow Field Associated with Detonation next to Gaseous Boundary in Test Section III</td>
<td>67</td>
</tr>
<tr>
<td>25</td>
<td>Schlieren Photographs of 67% Hydrogen-33% Oxygen Detonation with Air Boundary: Test Section III.</td>
<td>69</td>
</tr>
<tr>
<td>26</td>
<td>Schlieren Photographs of 35% Hydrogen-65% Oxygen Detonation with Air Boundary: Test Section III.</td>
<td>71</td>
</tr>
<tr>
<td>27</td>
<td>Measured Shock and Interface Angles vs. Dimensionless Distance of Detonation Wave from Tube Exit for 78% Hydrogen-22% Oxygen Mixture with Air Boundary</td>
<td>73</td>
</tr>
<tr>
<td>28</td>
<td>Measured Shock and Interface Angles vs. Dimensionless Distance of Detonation Wave from Tube Exit for 67% Hydrogen-33% Oxygen Mixture with Air Boundary</td>
<td>73</td>
</tr>
<tr>
<td>29</td>
<td>Measured Shock and Interface Angles vs. Dimensionless Distance of Detonation Wave from Tube Exit for 50% Hydrogen-50% Oxygen Mixture with Air Boundary</td>
<td>74</td>
</tr>
<tr>
<td>30</td>
<td>Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 40% Methane-60% Oxygen Mixture with Air Boundary: Test Section III, Tube B.</td>
<td>76</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 30% Methane-70% Oxygen Mixture with Air Boundary: Test Section III, Tube B.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Page 77</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 19% Methane-81% Oxygen Mixture with Air Boundary: Test Section III, Tube B.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Page 78</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>Schlieren Photographs of 19% Methane-81% Oxygen Detonation with Air Boundary: Test Section III.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Page 79</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>Schlieren Photographs of 30% Methane-70% Oxygen Detonation with Air Boundary: Test Section III.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Page 79</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>Measured Shock and Interface Angles vs. Dimensionless Distance of Detonation Wave from Tube Exit for 40% Methane-60% Oxygen Mixture with Air Boundary.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Page 81</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>Measured Shock and Interface Angles vs. Dimensionless Distance of Detonation Wave from Tube Exit for 30% Methane-70% Oxygen Mixture with Air Boundary.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Page 81</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 17% Propane-83% Oxygen Mixture with Air Boundary: Test Section III, Tube B.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Page 82</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 78% Hydrogen-22% Oxygen Mixture with Helium Boundary: Test Section III, Tube B.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Page 84</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 67% Hydrogen-33% Oxygen Mixture with Helium Boundary: Test Section III, Tube B.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Page 85</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 50% Hydrogen-50% Oxygen Mixture with Helium Boundary: Test Section III, Tube B.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Page 86</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 35% Hydrogen-65% Oxygen Mixture with Helium Boundary: Test Section III, Tube B.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Page 87</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>Schlieren Photographs of 67% Hydrogen-33% Oxygen Detonation with Helium Boundary: Test Section III.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Page 89</td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>Schlieren Photographs of 35% Hydrogen-65% Oxygen Detonation with Helium Boundary: Test Section III.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Page 91</td>
<td></td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>44</td>
<td>Schlieren Photograph Enlargement of 35% Hydrogen-65% Oxygen Detonation - Helium Boundary Interaction: Test Section III</td>
<td>92</td>
</tr>
<tr>
<td>45</td>
<td>Measured Shock Angle vs. Dimensionless Distance of Detonation Wave from Tube Exit for 76% Hydrogen-22% Oxygen Mixture with Helium Boundary</td>
<td>94</td>
</tr>
<tr>
<td>46</td>
<td>Measured Shock Angle vs. Dimensionless Distance of Detonation Wave from Tube Exit for 67% Hydrogen-33% Oxygen Mixture with Helium Boundary</td>
<td>94</td>
</tr>
<tr>
<td>47</td>
<td>Idealized Flow Model of Detonation Wave-Boundary Interaction</td>
<td>96</td>
</tr>
<tr>
<td>48</td>
<td>Shock Detachment Angle vs. Mach Number as a Function of the Specific Heat Ratio</td>
<td>101</td>
</tr>
<tr>
<td>49</td>
<td>Specific Volume Ratio Across Chapman-Jouguet Detonation Wave vs. Mol Fraction of Hydrogen for Hydrogen-Oxygen Mixtures</td>
<td>103</td>
</tr>
<tr>
<td>50</td>
<td>Comparison of Theoretical and Experimental Shock and Interface Angles for Oxygen-Hydrogen Mixtures with Air Boundary</td>
<td>108</td>
</tr>
<tr>
<td>51</td>
<td>Comparison of Theoretical and Experimental Shock and Interface Angles for Oxygen-Hydrogen Mixtures with Helium Boundary</td>
<td>109</td>
</tr>
<tr>
<td>52</td>
<td>Comparison of Theoretical and Experimental Shock and Interface Angles for Oxygen-Methane Mixtures with Air Boundary</td>
<td>110</td>
</tr>
<tr>
<td>53</td>
<td>Schlieren Enlargement of Stoichiometric Hydrogen-Oxygen Mixture next to Air and Comparison with Theoretical Results</td>
<td>112</td>
</tr>
<tr>
<td>54</td>
<td>Schlieren Enlargement of Stoichiometric Hydrogen-Oxygen Mixture next to Helium and Comparison with Theoretical Results</td>
<td>113</td>
</tr>
<tr>
<td>55</td>
<td>Pseudo Shock Tube Analysis for Lateral Expansion of Combustion Products</td>
<td>115</td>
</tr>
<tr>
<td>56</td>
<td>Distance vs. Time Plot of Conditions at Tube Exit for 67% Hydrogen-33% Oxygen Mixture with Air Boundary</td>
<td>119</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>57</td>
<td>Distance vs. Time Plot of Conditions at Tube Exit for 67% Hydrogen-33% Oxygen Mixture with Helium Boundary</td>
<td></td>
</tr>
<tr>
<td>58</td>
<td>Schematic of Triple Point with Curved Waves Represented by Small, Straight Line Segments</td>
<td></td>
</tr>
<tr>
<td>59</td>
<td>Hydrogen Concentration vs. Dimensionless Width at ( x' = 2.0 ) for 78% Hydrogen-22% Oxygen Next to Air</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>Hydrogen Concentration vs. Dimensionless Width at ( x' = 2.0 ) for 78% Hydrogen-22% Oxygen Next to Helium</td>
<td></td>
</tr>
<tr>
<td>61</td>
<td>Schematic of Actual Detonation Wave-Boundary Interaction</td>
<td></td>
</tr>
<tr>
<td>62</td>
<td>Pressure Ratio Across Oblique Shock vs. Velocity of Sound Ratio for Stoichiometric Hydrogen-Oxygen and Various Gas Boundaries</td>
<td></td>
</tr>
<tr>
<td>63</td>
<td>Oblique Shock Angle vs. Velocity of Sound Ratio for Stoichiometric Hydrogen-Oxygen and Various Gas Boundaries</td>
<td></td>
</tr>
<tr>
<td>64</td>
<td>Pressure Ratio Across Oblique Shock vs. Acoustic Impedance Ratio for Stoichiometric Hydrogen-Oxygen and Various Gas Boundaries</td>
<td></td>
</tr>
</tbody>
</table>
NOMENCLATURE

A  Constant, equal to $V_D^2/p_{E_1}a_{E_1}$, Equation (6.24)
a  Speed of sound
B  Constant, equal to $V_D/p_{I_1}$, Equation (6.25)
B_2  Second virial coefficient
B_3  Third virial coefficient
B_4  Fourth virial coefficient
b  Temperature independent second virial coefficient
C  Concentration, molecules/cm$^3$
C-J  Chapman Jouguet condition
c_p  Specific heat at constant pressure
D  Inside diameter of axially symmetric tube
D_{12}  Binary diffusion coefficient, cm$^2$/sec.
E  Point corresponding to tube exit
e  Internal energy per unit mass
f  Function in Equation (2.18)
g  Adjustable constant in Equation (2.18)
h  Enthalpy per unit mass
j  Adjustable constant in Equation (2.18)
K  Covolume constant in Equation (2.18)
k  Mol fraction
l  Adjustable constant in Equation (2.18)
M  Mach number
m  Molecular weight
n  Number of mols
p  Pressure
NOMENCLATURE (CONT’D)

$Q$  Enthalpy of reaction per unit mass

$R$  Universal gas constant

$r$  Adjustable constant in Equation (2.18)

$Re$  Reynolds number

$T$  Absolute temperature

$t$  Time

$u$  Velocity

$u_0$  Velocity of explosive jet and boundary gas, assuming both flow at same velocity

$V$  Wave velocity

$v$  Specific volume

$V_E$  Flow velocity of explosive mixture

$V_i$  Interface or contact surface velocity

$V_S$  Shock velocity

$V_T$  Velocity parallel to wave front

$V_\infty$  Detonation velocity in infinite diameter tube

$W$  Width of two-dimensional test section, 0.50 in.

$w$  Particle velocity

$\dot{w}$  Weight flow per second

$x$  Axis in detonation velocity direction

$x'$  Dimensionless coordinate, $x/ZW$ or $x/D$

$y$  Axis perpendicular to explosive jet axis, $x$

$y'$  Dimensionless coordinate, $y/ZW$ or $y/D$

$Z$  Acoustic impedance, $\rho a$

$\alpha$  Thermodynamic function of $T$ and $v$ in Equation (2.12), the "covolume"; Angle which detonation makes with vertical
NOMENCLATURE (CONT'D)

\( \beta \)  
Angle between normal to wave front and flow velocity behind detonation wave

\( \gamma \)  
Ratio of specific heats

\( \delta \)  
Interface angle

\( \Theta \)  
Shock angle

\( \Theta_{\text{max}} \)  
Shock detachment angle

\( \lambda \)  
Dummy variable in Equation (5.20)

\( \mu \)  
Mach angle

\( \nu \)  
Prandtl-Meyer angle

\( \rho \)  
Mass density

\( \sigma_{12} \)  
Molecular potential parameter, Equation (5.18)

\( \Omega_{12} \)  
Lennard-Jones potential integral, Equation (5.18)

Subscripts

1  Conditions ahead of detonation or shock wave

2  Conditions at Chapman-Jouguet plane or rear of shock wave

3  Conditions in explosive following Prandtl-Meyer expansion; Conditions behind contact surface

4  High pressure conditions in pseudo shock tube analysis

D  Detonation

E  Explosive

I  Inert Boundary

i  Denotes type of species

t  Stagnation conditions
Superscripts

' Refers to conditions calculated using pseudo shock tube analysis, Section V.2

" Refers to conditions calculated from approximate curved wave analysis, Section V.3

Explosive Mixtures

All mixture compositions given are on a volumetric basis; e.g., 78% hydrogen-22% oxygen.
I. INTRODUCTION

Combustion is generally divided into two types, deflagration and detonation. Deflagration is a slow, subsonic process governed by diffusion, thermal action, and molecular transport phenomena, while detonation is a supersonic process in which the combustion is initiated by a shock wave. Deflagration and detonation may occur in either gaseous, liquid or solid media. Regardless of the medium the same fundamental mechanisms are involved. Detonation in any explosive consists of a shock wave which initiates combustion, and a reaction zone in which chemical energy is released, thereby furnishing the energy for the continued propagation of the shock.

The objective of the present study is to determine the influence of an inert, compressible boundary on a detonation wave. The process of interaction between a detonation and such a boundary is to be studied analytically and experimentally through the use of gaseous explosives and various inert, gaseous boundaries. Under these circumstances detonation ceases to be a one-dimensional phenomenon, thus embodying in this investigation a boundary condition which, to the author's knowledge, has not previously been considered in gaseous detonation work. This study arose from consideration of the interaction between a condensed explosive (i.e., liquid or solid) detonation wave and the explosive container which is generally a solid. Understanding of this latter process will be acquired through use of the gaseous model as an analogue.

Because the investigation reported herein is based upon previous detonation research in condensed and gaseous explosives, a brief summary of the pertinent work in both fields will be given.
1.1 Summary of Gaseous Detonation Research

The study of detonation in gaseous explosives has progressed rapidly following the initial identification of the phenomenon by Berthelot and Vieille\(^1\) and independently by Mallard and LeChatelier\(^2\) in 1881. Shortly after the discovery of the supersonic form of combustion, Chapman\(^3\) and Jouguet\(^4\) formulated the hydrodynamic theory which explained in large part the characteristics observed experimentally. It was shown that an analogy exists between detonation waves and the non-reactive shock waves which had been discussed previously by Riemann\(^5\) and others. This led to the correct description of a detonation as a shock wave followed closely by an extremely rapid combustion process. After the initial formulation of the theory by Chapman and Jouguet, many investigators undertook the detailed study of detonation, both analytically and experimentally. Most of this work up through the mid-1940's is summarized at length in books by Jost and Croft,\(^6\) and Lewis and VonElbe.\(^7\)

In the last ten years the study of detonation in gases has been actively pursued from both the chemical and hydrodynamic aspects. Excellent summaries and bibliographies of recent gaseous detonation work may be found in References 15 and 16. Morrison\(^8\) emphasized the hydrodynamic features of detonation, and was eminently successful in clarifying the dynamic properties of the process. Studies such as those performed by Moyle\(^9\) and Gealer\(^10\) showed that the theoretical analysis could give agreement with experimental values of velocity within several percent, and correctly predict the influence of initial temperature and pressure over a broad range. A unique and extremely useful method for the investigation of detonation was acquired with the successful stabilization of a
hydrogen-air detonation in the open jet of an underexpanded supersonic nozzle. This is reported by Nicholls\(^{(11)}\).

Much understanding of the detonation reaction zone in gaseous explosives has been achieved through the efforts of such investigators as Hirschfelder and Curtiss,\(^{(12)}\) Linder, Curtiss and Hirschfelder,\(^{(13)}\) Kistiakowsky and Kydd,\(^{(30)}\) and Adamson.\(^{(14)}\) This work has aided in the determination of reaction zone lengths, the chemical kinetics involved, and the influence of the transport terms.

One feature common to virtually all of the work mentioned above is the treatment of a gaseous detonation wave as a "one-dimensional" process; i.e., the wave front is assumed to be straight, normal to the detonation velocity direction, and the gases are assumed to enter and leave perpendicular to the wave front. The justification for a one-dimensional theoretical treatment lies in the degree of "confinement" normally present in any gaseous detonation experiment. Because the pressures produced behind the wave are generally in the order of 15 to 50 atmospheres, almost any solid material has sufficient physical strength to contain the explosion products without significantly yielding or allowing lateral expansion. Thus shock tubes (or "detonation tubes") made of steel, brass, glass and other materials have all been used in gaseous detonation work. Table I gives theoretically determined properties of detonation for some typical gaseous explosives.
TABLE I
TYPICAL GASEOUS DETONATION PROPERTIES

<table>
<thead>
<tr>
<th>Explosive (by Vol.)</th>
<th>( V_D ) (ft/sec.)</th>
<th>( P_2 ) (atm.)</th>
<th>( T_2 ) (°R)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{H}_2 + .667 \text{ O}_2 )</td>
<td>8620.</td>
<td>17.64</td>
<td>6518.</td>
<td>(9)</td>
</tr>
<tr>
<td>( \text{H}_2 + .25 \text{ O}_2 )</td>
<td>11262.</td>
<td>17.43</td>
<td>6185.</td>
<td>(9)</td>
</tr>
<tr>
<td>( \text{H}_2 + 0.5 \text{ O}_2 )</td>
<td>9206.</td>
<td>18.05</td>
<td>6450.</td>
<td>(34)</td>
</tr>
<tr>
<td>( \text{H}_2 + 0.5 \text{ O}_2 + 0.75\text{He} )</td>
<td>10499.</td>
<td>17.6</td>
<td>6142.</td>
<td>(34)</td>
</tr>
<tr>
<td>( \text{H}_2 + 0.5 \text{ O}_2 + 1.5\text{He} )</td>
<td>11260.</td>
<td>17.11</td>
<td>5877.</td>
<td>(34)</td>
</tr>
<tr>
<td>( \text{H}_2 + 0.5 \text{ O}_2 + 2.5\text{He} )</td>
<td>11854.</td>
<td>16.32</td>
<td>5575.</td>
<td>(34)</td>
</tr>
<tr>
<td>( \text{CH}_4 + 2 \text{ O}_2 + 7.45\text{N}_2 + 0.089\text{A} )</td>
<td>5911.</td>
<td>17.22</td>
<td>5012.</td>
<td>(29)</td>
</tr>
<tr>
<td>( \text{C}_2\text{H}_6 + 3.5 \text{ O}_2 + 13.05\text{N}_2 + 0.155\text{A} )</td>
<td>5909.</td>
<td>17.98</td>
<td>5071.</td>
<td>(29)</td>
</tr>
<tr>
<td>( \text{C}_3\text{H}_8 + 5 \text{ O}_2 + 18.64\text{N}_2 + 0.222\text{A} )</td>
<td>5900.</td>
<td>18.36</td>
<td>5088.</td>
<td>(29)</td>
</tr>
<tr>
<td>( \text{C}_2\text{H}_2 + 3.0 \text{ O}_2 )</td>
<td>7650.</td>
<td>29.5</td>
<td>7335.</td>
<td>(37)</td>
</tr>
<tr>
<td>( \text{C}_2\text{H}_2 + \text{ O}_2 )</td>
<td>9660.</td>
<td>43.5</td>
<td>8060.</td>
<td>(37)</td>
</tr>
</tbody>
</table>

1.2 Summary of Condensed Explosives Research

The investigation of detonation in condensed explosives, although not really a field apart from that of detonation in gases, followed a somewhat different line of development. Detonation of condensed explosives had its beginning in the 1860's, when Nobel discovered and made use of nitroglycerine's properties. From this time until the 1940's research in the field of condensed explosives was almost entirely of an experimental nature. The hydrodynamic theory of detonation derived for
gaseous explosives was not successfully extended to liquids and solids until work by Kistiakowsky and Wilson,\textsuperscript{17} Cook,\textsuperscript{18} Jones\textsuperscript{19} and others appeared in the 1940's.

The primary deterrent to theoretical analysis was the lack of a satisfactory thermodynamic equation of state for the combustion products. Because of the high pressures generated behind the wave, the reaction products cannot be considered ideal gases. The detonation pressures are a factor of $10^3$ to $10^4$ higher than for gaseous explosives which, in fact, lead to the practical uses for condensed explosives. Analytical procedures using a virial equation of state were developed by Paterson,\textsuperscript{20} while Cowan and Fickett\textsuperscript{21} utilized an extremely complicated equation of state. Still other theoretical equations of state have been developed by Cook\textsuperscript{24} and Jones\textsuperscript{19}. Recent reviews of condensed explosive research can be found in a paper by Jacobs\textsuperscript{22} and in books by Taylor\textsuperscript{23} and Cook.\textsuperscript{24}

Table II gives some of the theoretical results obtained by various investigators for a sampling of liquid and solid explosives. As a matter of interest the author sought out some experimentally measured velocity of sound values for condensed explosives, and calculated the Mach numbers which appear in Table II. Only a few sound velocities could be found for this purpose (\textsuperscript{40} and \textsuperscript{41}).

Several interesting observations can be made concerning the data in Table II. First, the Mach numbers of detonation, although limited in number, appear to group around 3.0 for solid explosives and around 5.0 for liquid explosives. This contrasts with the case in gaseous explosives, where the detonation Mach numbers spread all the way from 3. to 10.
<table>
<thead>
<tr>
<th>Solids</th>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
<th>$\rho_2/\rho_1$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pentetranitrate (PETN)</td>
<td>1.50</td>
<td>1.22</td>
<td>1.22</td>
<td>(25)</td>
</tr>
<tr>
<td>Nitroglycerine</td>
<td>1.60</td>
<td>5.26</td>
<td>3.29</td>
<td>(25)</td>
</tr>
<tr>
<td>Ethyl Nitrate</td>
<td>1.10</td>
<td>3.79</td>
<td>3.79</td>
<td>(24)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Liquefied Explosives</th>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
<th>$\rho_2/\rho_1$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nitroglycerate</td>
<td>1.60</td>
<td>5.26</td>
<td>3.29</td>
<td>(25)</td>
</tr>
<tr>
<td>Methyl Nitrate</td>
<td>1.13</td>
<td>4.77</td>
<td>4.31</td>
<td>(24)</td>
</tr>
<tr>
<td>Nitrocellulose</td>
<td>1.15</td>
<td>4.81</td>
<td>3.73</td>
<td>(24)</td>
</tr>
<tr>
<td>Ethyl Nitrate</td>
<td>1.10</td>
<td>3.79</td>
<td>3.79</td>
<td>(24)</td>
</tr>
</tbody>
</table>
Second, the density ratio across the detonation wave varies within a fairly narrow range from about 1.2 to 1.4. The liquid explosive density ratios are near 1.25 and those for the solids nearer 1.3. This fact leads to a simplified method for estimating the pressure behind a detonation wave in condensed explosives, which will be given in Section II.2.

Ever since the development of high speed photographic equipment it has been known that detonation waves in condensed explosives are literally never one-dimensional, but rather are always curved. This is due to the virtual impossibility of confining a condensed explosive and preventing lateral expansion of the reaction products. The high values of pressure at the rear of the wave cause any presently known container to yield and become plastic. As a result of this effect an experimentally measured detonation velocity must always be qualified by stating what type of confinement the explosive was subjected to during the experiment. The detonation velocity is observed to be a function of the charge radius, the wall material, and in some cases the wall thickness. Only in extremely large diameter charges does the detonation velocity approach that calculated by the one-dimensional, hydrodynamic theory.

Several approximate theories have been advanced as possible explanations for the relationship between the curvature of a condensed explosive detonation wave and the type of confinement employed. The two main theories are the "curved-front theory" proposed by Eyring, et al.\(^{(26)}\) and the "expanding-jet theory" by Jones.\(^{(27)}\) Neither theory purports to explain the actual process taking place, but rather is aimed at providing
a means by which existing experimental data may be correlated. Even in this regard the theories are regarded as inadequate\(^{(23, 24)}\).  

Another phenomenon associated with the effect of confinement on condensed explosives is the existence of a "failure diameter" or minimum diameter below which stable Chapman-Jouguet detonation cannot be established.* A qualitative explanation offered for this is that the energy expended in moving the container walls laterally reduces that left to drive the shock wave. It is reasoned that if the explosive charge is small enough in diameter this effect can become large in relation to the total amount of energy being liberated by the combustion process. For reasons not presently evident, the failure diameter is observed to vary significantly with the container material used with a given condensed explosive.  

Campbell, Malin and Holland\(^{(28)}\) found that the liquid nitromethane would detonate (at about 75°F) in brass tubes of 3 mm i.d. and 1.6 mm wall thickness, and in dural tubes of 4.8 mm i.d. and 1.6 mm wall thickness, but would fail in glass tubes below 17 mm i.d. Even more significantly they found that lining a glass tube with only a two-mil thick layer of aluminum foil reduced the failure diameter of the glass toward that corresponding to a tube made entirely of dural. Additional experiments eliminated chemical seeding, catalytic action or surface smoothness as possible explanations for the results. Yet it is apparent that the

\* A critical diameter is observed also in gaseous explosives. This is due to the combined effects of viscosity, heat conduction and radiation to the walls. Gaseous failure diameters are generally smaller than those in condensed explosives.
interaction of the boundary with the detonation plays an important role in the propagation of the wave. Médard\textsuperscript{(31)} has also reported results similar to those cited above.

The lack of an explanation of the boundary interaction process is undoubtedly due to the extreme difficulty involved in obtaining either quantitative or qualitative information. Shadowgraph or schlieren photographic techniques are not successful when used with condensed explosives, even though employed with a transparent explosive and container. The high density of the reaction products, the presence of unburned carbon, the highly turbulent and expanding flow field behind the detonation wave, and the large changes in the index of refraction throughout the flow field all serve to obscure the results.

To compound the problem, one must contend with the awesome destructive power of condensed explosives, a feature which guarantees annihilation of any instrumentation in the near vicinity of the experiment. Added to these purely experimental difficulties are the fundamental problems associated with condensed explosives, relating to their thermodynamic properties and equations of state. In summary, the difficulties involved in research of this nature are formidable.

1.3 Purpose of This Investigation

The influence of the container and its material properties on the detonation characteristics of a liquid or solid explosive has neither been explained nor described analytically. It is not possible, on the basis of previous research, to specify even the proper mechanism which causes anomalous results such as those reported by Campbell.\textsuperscript{(28)} The
interaction of a detonation wave with a compressible boundary is, however, an interesting problem which it would seem is primarily of a hydrodynamic nature.

The difficulties concomitant with condensed explosive research were powerful persuasion to seek solution to the question in a more amenable environment. This suggested the use of a gaseous explosive next to an inert gaseous boundary as an analogue. This arrangement provides the same features as present in the aforementioned liquid experiments; viz., the explosive is confined in a container which becomes compressible (or plastic) under the influence of the detonation pressure.

The purpose of this investigation is therefore to study the boundary interaction between a gaseous detonation wave and an inert gaseous boundary with the emphasis being placed on hydrodynamic effects. Both analytical and experimental procedures are to be employed. Following this study it is hoped some aspects of the boundary interaction phenomena observed in condensed explosives will be more understandable through use of the analogy.
II THEORY OF DETONATION

The theoretical relations describing detonation have been developed in great detail in many previous studies (e.g. references 8 and 32). Therefore only a brief review of the salient features will be given here.

2.1 One-Dimensional Analysis

Consider, as in Figure 1, a stationary, exothermic wave of unspecified nature existing in a constant area duct. Combustible material flows through the wave from left to right, upstream conditions being denoted by (1) and downstream conditions by (2).

![Diagram of one-dimensional planar wave](image)

Figure 1. One-Dimensional Planar Wave.

Assuming steady flow, negligible friction and viscosity effects along the boundaries, and the wave to be a discontinuity, the following relations may be written:

**Conservation of Mass:**

\[ \rho_1 u_1 = \rho_2 u_2 \]  \hspace{1cm} (2.1)
Conservation of Momentum:
\[ p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \] (2.2)

Conservation of Energy:
\[ h_1 + \frac{u_1^2}{2} + Q = h_2 + \frac{u_2^2}{2} \] (2.3)

Where \( Q \) = enthalpy of reaction per unit mass.

Combining Equations (2.1) and (2.2) gives
\[ (\rho_1 u_1)^2 = (\rho_2 u_2)^2 = \frac{p_2 - p_1}{v_1 - v_2} \] (2.4)

As has been noted elsewhere (8), since the left side of Equation (2.4) is always positive, the signs of the numerator and denominator on the right side must be identical. If \( p_2 \) is greater than \( p_1 \), \( v_1 \) must be greater than \( v_2 \) and the exothermic wave is a detonation. If \( p_1 \) is greater than \( p_2 \), \( v_2 \) must be greater than \( v_1 \) and the wave is a deflagration process.

If Equation (2.4) is used to eliminate the velocity terms in Equation (2.3), the Hugoniot equation is obtained.
\[ h_1 - h_2 + Q = \frac{1}{2} (p_1 - p_2) (v_1 + v_2) \] (2.5)

Since \( h = pv \), Equation (2.5) can also be written as:
\[ e_1 - e_2 + Q = \frac{1}{2} (p_1 + p_2) (v_2 - v_1) \] (2.6)

Whether the material in the duct is solid, liquid or gaseous, use of the proper equation of state will allow the internal energy to be written as a function of any two state variables, e.g., pressure and volume. Employment of this fact allows Equation (2.6) to be expressed entirely in terms of \( p \) and \( v \). Figure 2 is a schematic representation of the result, and is the well known Hugoniot curve.
Figure 2. Hugoniot Curve.

The tangent through the point marked C-J in Figure 2 has the slope (as has the tangent through point D):

$$\left(\frac{dp_2}{dv_2}\right)_{C-J} = \frac{p_2 - p_1}{v_2 - v_1}$$  \hspace{1cm} (2.7)

A result of the first law of thermodynamics is that

$$Tds = dd + pdv$$  \hspace{1cm} (2.8)

Using Equations (2.6) and (2.7) in (2.8) shows that

$$ds = 0$$  \hspace{1cm} (2.9)

at point C-J. This is equivalent to

$$\left|\frac{dp_2}{dv_2}\right|_{C-J} = \rho_2^2 a_2^2$$  \hspace{1cm} (2.10)

Finally, using Equation (2.10) in (2.4) shows that

$$u_2 = a_2$$  \hspace{1cm} (2.11)

Thus, the point marked C-J is the Chapman-Jouguet point. At this point the velocity of the reaction products leaving the wave is exactly equal to the local speed of sound, or stated hydrodynamically, the flow is at Mach one relative to the wave front. This establishes the most unique aspect of a detonation wave, and in fact is the condition which allows a complete theoretical description of the phenomenon.
For detonation in a closed end tube, as used in the experiments of this study, the Chapman-Jouguet condition is the only stable solution. The velocity corresponding to the Chapman-Jouguet point is that generally observed in a detonation tube if sufficient length of tube is provided. The inherent stability of the Chapman-Jouguet solution has been firmly established by previous investigators (7 and 31).

In order to predict theoretically the detonation properties of a given explosive, the values of five unknowns must be determined: \( p_2, T_2, v_2, V_D \) and \( w_2 \). In the present notation, \( V_D = u_1 \) and \( w_2 = V_D - a_2 \). Thus far, four equations have been formulated for this purpose: Equations (2.1, 2.2, 2.3 and 2.11). The fifth equation required, and the one which relates specifically to whether the explosive is liquid, solid or gaseous, is the equation of state.

An additional complication, which will not be discussed in detail here, is the determination of \( Q \) for use in Equation (2.3). Since the chemical energy released is in general dependent upon the final pressure and temperature, it must be determined by a chemical equilibrium solution. One would not expect ideal gas equilibrium constants to apply to the reaction products of liquid or solid explosives, and thus in this instance the solution also depends on whether the explosive is gaseous or not. This complication has been discussed in detail by Paterson (20) and Cook (24).

2.2 Thermodynamic Equation of State

In Section 1.2 it was stated that thermodynamic equations of state for condensed explosives are exceedingly complex and difficult to
use in hydrodynamic developments. A summary of these equations follows, which not only demonstrates this but gives some insight into the physics of the detonation of condensed and gaseous media as well. An additional effect of the summary is to strengthen the desire to solve the interaction problem for condensed explosives using the gaseous analogy.

The general equation of state can be written as:

$$ pv = nRT + \alpha(T, v)p $$  \hspace{1cm} (2.12)

where $\alpha$ is an unspecified function of the two state variables, temperature and specific volume, and is sometimes referred to as the "covolume."

For an ideal gas the function $\alpha$ is zero and the perfect gas law is obtained

$$ pv = nRT $$  \hspace{1cm} (2.13)

The assumption of ideal gases is usually made in the calculation of gaseous detonation properties and is a very satisfactory description of both the explosive mixture and the explosive products. The effect of the relatively high pressure at the Chapman-Jouguet plane, compared to atmospheric, is amply compensated for by the high temperatures present.

For liquid and solid explosives the problem is far more complex. The very high densities involved (e.g., 2 gm/cm$^3$) make it quite difficult to specify the equation of state. Since the state variables ahead of the detonation are always known, the equation of state generally sought is that which pertains only to the reaction products. Thus, one of the convenient aspects of gaseous detonation theory, that of being able to apply the same equation of state on both sides of the wave, is lost.

The simplest assumption that can be made concerning $\alpha$ in Equation (2.12) is that it is a constant. Some calculations have been performed for
condensed explosives using this form of the equation of state, but according to Taylor (23) the results are quite unsatisfactory.

The next level of sophistication is to treat $\alpha$ as a function of either $v$ or $p$ alone. That is:

\[
\alpha = \alpha(v) \quad (2.14) \\
\alpha = \alpha(p) \quad (2.15)
\]

It can be shown that Equation (2.14) implies the internal energy is a function of temperature only (as is true for an ideal gas). Hence this form of an equation of state discounts intermolecular potential energy, a questionable assumption at the extreme pressures encountered with condensed explosives. In spite of this apparent flaw, Cook (18) used this form of the equation and experimental values of detonation velocity to derive an empirical equation of state for condensed explosives. Using his results, Cook was able to predict detonation velocities for other solid explosives with reasonably good agreement. It will be shown subsequently that this is not sufficient proof of an equation of state's suitability.

Jones (19) used Equation (2.15) along with experimental values of detonation velocity and, like Cook, deduced an equation of state he hoped would apply to most condensed explosives. This equation also proved fairly successful in predicting detonation velocities.

The most recent attempts at formulating a thermodynamic equation of state for condensed explosive products involve the use of a virial equation of state by Paterson (25), and an empirical equation of state by Cowan and Pickett (21), who refined an earlier attempt of Kistiakowsky and Wilson (17). The virial equation of state takes the form of an expansion about the perfect gas law and is written as:
pv = nRT \left( 1 + \frac{B_2}{v} + \frac{B_3}{v^2} + \frac{B_4}{v^3} + \ldots \right) \quad (2.16)

Where $B_2$, $B_3$ and $B_4$ are the second, third and fourth virial coefficients respectively and are functions of temperature but not pressure. For non-attracting rigid spherical molecules, however, the virial coefficients are also independent of temperature. Paterson made use of the virial equation in this latter sense where the coefficients have been computed from kinetic theory (33).

\[ pv = nRT \left(1 + \frac{b}{v} + 0.625\frac{b^2}{v^2} + 0.287\frac{b^3}{v^3} + 0.193\frac{b^4}{v^4} \right) \quad (2.17) \]

The term $b$ is the second virial coefficient for the gas, i.e., four times the volume of the rigid spherical molecules as in van der Waal's equation. Because the reaction products are made up of several gases, the second virial coefficient of the mixture is taken as the linear summation of the concentration of each species times its molar coefficient.

This equation also has the disadvantage mentioned earlier of ignoring the intermolecular potential energy.

The empirical equation used by Cowan and Fickett (21) consists of substituting in Equation (2.12) the relation

\[ \alpha(T,v) = nRT \frac{fG}{p} \quad (2.18) \]

where \[ f = K(T + \beta) v^{-1} \]

The covolume constant $K$ was taken as an additive constant by summing the values for each type of molecule present in the reaction products. Each of the separate constants making up $K$ were adjusted empirically to obtain the best general agreement between observed and computed velocities.

Thus \[ K = \sum k_i r_i \quad (2.19) \]
where $k_i$ is the mol fraction of component $i$ and $r_i$ is a special covolume constant for each of the species. All of the constants ($j$, $k$, $r$ and $g$) were treated as adjustable in an effort to obtain a set which would give agreement with experimental data.

In spite of such a wide latitude in the choice of empirical constants, the results obtained from this equation of state are not entirely satisfactory. Cowan and Fickett state its use should be restricted to explosives similar to those used to determine the constants, namely RDX and TNT. Although fairly good agreement is found between measured and computed velocities, the authors of the equation express doubt in the accuracy of the pressures calculated.

As mentioned earlier, agreement between computed and measured values of velocity does not constitute a complete test of an equation of state. It has been demonstrated that the predicted values of velocity, pressure and density behind the detonation wave are relatively insensitive to the equation of state. The temperature, on the other hand, is quite strongly a function of the state equation. This is clearly illustrated in Table III, which contains calculations for the solid explosive PETN (Pentaerythritol tetranitrate) performed using three different equations of state. Notice the relatively good agreement among the three values of pressure and velocity, but the wide variations in predicted temperatures. Experimental measurement of the temperature at the Chapman-Jouguet plane (an extremely difficult problem) would therefore constitute an excellent check on the equation of state.

Because of the difficulties involved in applying equations of state of the type mentioned above, some investigators have turned to the
TABLE III
RESULTS OF CALCULATIONS FOR THE SOLID EXPLOSIVE, PETN, USING THREE DIFFERENT EQUATIONS OF STATE

A. Virial equation of state, Equation 2.17, (20) and (23)

B. Covolume a function of pressure only, Equation 2.15, (25)

C. Theoretical equation with one adjustable constant, (35)

(Terms in brackets are interpolated values)

<table>
<thead>
<tr>
<th>$\rho_1$</th>
<th>$V_D(\text{obs.})$</th>
<th>$V_D(\text{calc.})$</th>
<th>$V_D(\text{calc.})$</th>
<th>$P_2$</th>
<th>$P_2$</th>
<th>$P_2$</th>
<th>$T_2$</th>
<th>$T_2$</th>
<th>$T_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>g/cm³</td>
<td>m/sec.</td>
<td>m/sec.</td>
<td>m/sec.</td>
<td>X10⁻³ atm.</td>
<td>X10⁻³ atm.</td>
<td>X10⁻³ atm.</td>
<td>K</td>
<td>K</td>
</tr>
<tr>
<td>1.50</td>
<td>7430–7600</td>
<td>8150.</td>
<td>-</td>
<td>188.</td>
<td>206.</td>
<td>-</td>
<td>5340.</td>
<td>3490.</td>
<td>-</td>
</tr>
<tr>
<td>1.40</td>
<td>7000.</td>
<td>-</td>
<td>7010.</td>
<td>-</td>
<td>(175.)</td>
<td>186.</td>
<td>-</td>
<td>(3730.)</td>
<td>2630.</td>
</tr>
<tr>
<td>1.00</td>
<td>5520.</td>
<td>5550.</td>
<td>5530.</td>
<td>73.8</td>
<td>(84.)</td>
<td>88.</td>
<td>5150.</td>
<td>(4350.)</td>
<td>3650.</td>
</tr>
<tr>
<td>0.75</td>
<td>4700.</td>
<td>4520.</td>
<td>4760.</td>
<td>41.9</td>
<td>(50.)</td>
<td>53.</td>
<td>5060.</td>
<td>(4650.)</td>
<td>4060.</td>
</tr>
<tr>
<td>0.40</td>
<td>3710.</td>
<td>-</td>
<td>3660.</td>
<td>-</td>
<td>(20.)</td>
<td>19.</td>
<td>-</td>
<td>(5000.)</td>
<td>4525.</td>
</tr>
</tbody>
</table>
use of approximate equations designed primarily to give qualitative rather than quantitative results. Erkman (43) and Drummond (44), for example, used the following equation in the solution to the expansion process undergone by the detonation products of a solid explosive:

\[ p = \alpha \rho^c - b \quad (2.20) \]

where \( p \) and \( \rho \) are the pressure and density, respectively, and \( \alpha \), \( c \) and \( b \) are arbitrary constants. The equation was selected merely on the basis of resembling the isentropic relationship for a gas. One of the sets of constants employed was \( c = 2.8 \) and \( b = 0 \).

Concerning simplifications, it was stated in Section I.2 that the apparent constancy of the specific volume ratio across a condensed explosive detonation wave could be used to estimate the detonation pressure. This results from substituting Equation (2.1) in (2.2) and noting \( p_1 \ll p_2 \).

Thus \( p_2 \approx (\rho_1 u_1)^2 (v_1 - v_2) \quad (2.21) \)

For condensed explosives it was observed from Table II, \( v_1 \approx 1.25 v_2 \).

Therefore \( p_2 \approx \frac{\rho_1 u_1^2}{5} \quad (2.22) \)

Knowing the detonation velocity allows the computation of the detonation pressure within about 10% using Equation (2.22). Equation (2.22) also expresses the interesting fact that approximately 20% of the momentum entering the detonation wave ends up as static pressure behind the wave. The rest must be taken up in the dynamic pressure. (The dynamic pressure is the term in the Bernoulli equation which corresponds to the kinetic energy),
2.3 Chapman-Jouguet Detonation Relationships for Perfect Gases.

Many useful relationships can be found when the explosive and explosive products may be treated as perfect gases. One of the identities which makes this possible is the simple expression for the speed of sound in a perfect gas:

\[ a^2 = \frac{\gamma p}{\rho} \]  

(2.23)

Dividing the conservation of momentum (Equation 2.2) equation through by \( p_1 \) and rearranging, one has:

\[ \frac{p_2}{p_1} = 1 + \frac{\rho_1 u_1^2}{p_1} - \frac{\rho_2 u_2^2}{p_1} = 1 + \frac{\rho_1 u_1^2}{p_1} (1 - \frac{v_2}{v_1}) \]  

(2.24)

Employing 2.23 in 2.24 yields:

\[ \frac{p_2}{p_1} = 1 + \gamma_1 \frac{M_D^2}{\gamma_2} (1 - \frac{v_2}{v_1}) \]  

(2.25)

If Equation (2.2) is again used with (2.23) and, in addition, the Chapman-Jouguet condition \( (M_2 = 1, \text{i.e., } u_2 = a_2) \) is accounted for, one finds the following relation:

\[ \frac{p_2}{p_1} = \frac{1 + \gamma_1 \frac{M_D^2}{\gamma_2}}{1 + \gamma_2} \]  

(2.26)

It was shown Equation (2.11) that the gases behind a Chapman-Jouguet detonation wave move at the local sonic velocity relative to the front. Equation (2.4) may therefore be written as

\[ \gamma_1 \frac{M_D^2}{\gamma_2} = \frac{1 - p_2/p_1}{v_2/v_1 - 1} \]  

(2.27)
Using Equation (2.26) for the pressure ratio and recognizing that for ideal gases

\[
\frac{T_2}{T_1} = \frac{m_2}{m_1} \frac{P_2 V_2}{P_1 V_1} \tag{2.28}
\]

where \( m_1 \) and \( m_2 \) are respectively the average molecular weight upstream and downstream of the detonation wave, one finds:

\[
\frac{T_2}{T_1} = \frac{m_2}{m_1} \frac{\gamma_2}{\gamma_1} \frac{(1 + \gamma_1 M_D^2)^2}{M_D^2(1 + \gamma_2)^2} \tag{2.29}
\]

Rewriting Equation (2.29), again with the use of (2.28) and (2.26), the following expression may be found:

\[
\frac{V_2}{V_1} = \frac{\gamma_2}{\gamma_1} \frac{(1 + \gamma_1 M_D^2)}{M_D^2(1 + \gamma_2)} \tag{2.30}
\]

If the specific heats, \( c_{p1} \) and \( c_{p2} \), are assumed constant and the enthalpy base to be zero when the temperature equals zero, the energy equation (2.3) becomes

\[
c_{p1} T_1 + \frac{u_1^2}{2} + Q = c_{p2} T_2 + \frac{u_2^2}{2} \tag{2.31}
\]

Recalling that

\[
c_p = \frac{\gamma R}{\gamma - 1} \quad a^2 = \gamma RT
\]

\[
M = \frac{u}{a} \quad M_2 = 1.0
\]

it follows that

\[
\frac{T_2}{T_1} = \frac{1}{\gamma_1 - 1} + \frac{M_D^2}{2} + \frac{Q}{\gamma_1 RT_1} \tag{2.32}
\]

Equations (2.26, 2.29, 2.30 and 2.32) relate the significant ratios (pressure, specific volume and temperature) across the detonation wave.
in terms of the detonation Mach number, the ratio of specific heats, the ratio of molecular weights and the heat added. The expressions are exact for Chapman-Jouguet detonation insofar as the specific heats are assumed constant and the gases ideal.
III. EXPERIMENTAL ARRANGEMENT AND PROCEDURE

3.1 Required Experimental Conditions.

In order to study the interaction of a gaseous detonation wave with an inert gaseous boundary, a means of placing the explosive inside, or at least next to, the boundary is required. This must be done in such a manner that mixing and diffusion effects between the gases are minimized. A system of flowing gases is thus suggested in which the flow is laminar.

It was decided to employ hydrogen and oxygen mixtures as the explosive, as they are easy to handle and their detonation characteristics are well documented. It was thought other explosives might be used in addition, if the significance of the results could be extended by doing so.

A wide variety of inert gases exists giving many possibilities for the boundary gas. Helium, argon, and the other monatomic gases were considered for use, as well as air, since it is readily available and also inert, as long as it is unmixed with a combustible.

The low average molecular weights of hydrogen-oxygen mixtures suggested using a vertical flow in order to obtain an inherently stable system due to buoyant forces. The experimental arrangement using a vertical detonation tube and a steady flow of explosive was selected with all of these requirements in mind.

3.2 Experimental Arrangement.

Figure 3 is a schematic sketch of the experimental system. The oxygen and hydrogen were introduced into the base of a vertical detonation tube through separate stainless steel lines, each equipped with a
Figure 3. Schematic Sketch of Experimental System.
filter and sonic metering orifice. A third line run into the base of the tube carried 100 psi air. This was used following each experiment to purge and cool the detonation tube.

An overall view of the equipment is shown in Figure 4. The bottled hydrogen and oxygen are visible at the right behind the Hewlett-Packard timer. The vertical detonation tube is to the left of center of the picture. Also visible is the focusing lens, knife edge, and camera of the schlieren system. The spark source and collimating lens are in the box on top of the optical bench and to the left of the detonation tube. The large pressure gauge at the extreme right side of the picture is not pertinent to the present study. A closer view of the left end of the equipment is shown in Figure 5.

The experiment was devised in such a manner that only one photograph of the boundary interaction process could be taken per detonation. This was done primarily for convenience. Multiple spark source systems appeared to be unnecessarily complex while a high-speed framing camera of sufficient speed was not available. The emphasis was therefore placed on obtaining highly repeatable experiments in order that single photographs taken at various intervals during the interaction process might be pieced together to show the entire sequence of events. Many experiments with the arrangement described showed the data to be highly repeatable, a variation of 1% or less in velocity being noted generally for each mixture ratio. To insure further against variations in the data, tests on any one combination of explosive mixture and boundary were always performed during one continuous period of running.

3.3 Detonation Tubes.

Two detonation tubes were employed in the experiments, tube A made of round, soft steel pipe, and tube B a rectangular, welded construction
Figure 4. Photograph of Entire Experimental System.
Figure 5. Photograph of the left end of equipment showing Detonation Tube B, Spark Source Box, Time Delay Unit (front of lower shelf) and 30 KV Power Supply (rear of lower shelf).
steel tube. The physical characteristics of the tubes are given in Table IV. Photographs of tubes A and B are shown in Figures 6 and 5, respectively.

**TABLE IV. DETONATION TUBE SPECIFICATIONS**

<table>
<thead>
<tr>
<th>Tube</th>
<th>Material</th>
<th>Internal Dimensions</th>
<th>Distance to First Probe</th>
<th>Distance Between Velocity Measuring Probes</th>
<th>Total Length of Tube</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Steel</td>
<td>0.30 in. I. D.</td>
<td>24 in.</td>
<td>18 in.</td>
<td>48 in.</td>
</tr>
<tr>
<td>B</td>
<td>Steel</td>
<td>0.375 x 0.500 in.</td>
<td>32 in.</td>
<td>17.6 in.</td>
<td>53.5 in.</td>
</tr>
</tbody>
</table>

A critical dimension for each tube is the distance from the explosive igniter to the location of the first ionization probe. This is the distance in which the flame must undergo transition to a stable Chapman-Jouguet detonation wave. Bollinger and Edse (36) found it took from 25 in. (60% hydrogen - 40% oxygen) to 70 in. (78% hydrogen - 22% oxygen) for transition to occur. However, they also noted the induction distance was inversely proportional to the level of turbulence. Bollinger and Edse's experiments were performed on a quiescent mixture of explosive gases, while in the present instance experiments were to be performed in a steadily flowing mixture. Since a flowing mixture should have higher turbulence levels than one which is stationary, the induction distance provided was thought to be acceptable.

To verify this a series of preliminary experiments were performed with tube A, in which only the wave velocity was measured. For the entire range of mixture ratios, from 35% hydrogen to 78% hydrogen by volume, a sequence of 24 runs at each of five mixture ratios showed less than 1%
variation in velocity about the average for that mixture ratio. Because it is highly unlikely that the same velocities would be recorded if the detonation were still undergoing a phase of transition when it reached the first probe, the results were interpreted as justifying the use of only 2\(^{1/2}\) in. of induction distance.

Although tube B was provided with 32 in. of induction distance, it was checked in the same manner as tube A. In addition, since tube B was eventually used for virtually all the experimental data taken in this investigation, the sufficiency of the induction distance provided was further checked by installing an additional pair of ionization probes, the lowest being located approximately 6 inches further from the igniter than the closest probe of the first pair. Additional experiments were run using two timers, one for each pair of probes. These runs showed the wave velocity was the same over each interval utilized. The timers were then interchanged and the runs repeated. Again the velocities were found to be the same. Thus ample induction distance was found to have been provided in each detonation tube.

The igniter in both detonation tubes consisted of a 1\(^{1/2}\) volt model airplane engine glow plug. This was at one point replaced by a spark plug, but since identical results were obtained in either case, a glow plug was used in almost all of the experiments performed.

3.4 Equipment for Velocity Measurement.

Two electronic timers were utilized in the study, a Model 524B Hewlett-Packard 10 megacycle counter, and a Berkley 1 megacycle counter, model No. 5120. The Hewlett-Packard timer measures time intervals down
to 0.1 μ sec, to ± 0.1 μ sec, accuracy. The Berkley timer is accurate to ± 1.0 μ sec. Since the longest time intervals measured during the experiments were those associated with the velocity measurements, the Berkley timer was generally utilized for this purpose.

Ionization probes were used to detect the passage of the detonation wave in the tube. The probes were constructed by twisting two 0.020 in. enameled magnet wires together and inserting them through a small teflon tube so that the ends protrude. The teflon tube was in turn placed inside a 1/8 in. pipe-to-1/4 in. tube stainless steel "Ermeto" coupling. Using a pair of diagonal clippers, the ends of the two twisted wires were clipped off close to the teflon, leaving an ideal ionization probe; viz., two copper wire ends with sharp points electrically insulated from each other, and separated by only two layers of varnish. Finally, the entire assembly was threaded into the detonation tube. A sketch of an assembled probe is shown in Figure 7, and a photograph in Figure 10.

Very little difficulty was experienced with the probes "shorting out." When they did it was usually caused by the presence of condensed water collecting on the tips following a test run. The short was generally corrected by the drying action provided by the purge air. If a probe could not be restored in this fashion, it was merely removed, the wire twisted through another fraction of an inch and the extended ends again clipped off.

The ionization probes act as a shorting switch in the grid of an 884 thyatron tube. Two such thyatrons were part of a triggering unit built previously by Aircraft Propulsion Laboratory personnel and
used in this investigation. Units similar to the one used have been described previously in the literature (8, 9 and 10). The grid of the thyratron was connected to a minus forty-five volt battery through a 1/2 megohm potentiometer. One side of the probe was connected directly to ground, the other to the grid of the thyratron. The potentiometer was adjusted such that the thyratron was on the verge of firing. The passage of the ionized gases associated with the detonation wave shorted the probe, thus decreasing the grid bias on the thyratron and causing it to fire. When the thyratron fired, the 180 volt positive plate voltage provided the signal to either start or stop the timer.

The velocity of the wave was obtained by dividing the distance between the probes by the time interval measured on the timer. Time intervals ranging from 130 μ sec to 240 μ sec were recorded in the course of the experiments run. The maximum overall error in measuring the velocity was believed to be 1-1/2%, a major portion of this being due to the Berkley's tolerance of ±1 μ sec. The next biggest contribution to the error was the measured distance between the probes. All other factors were negligible compared to these two.

3.5 Photographic Equipment.

The photographic equipment employed was a simple schlieren system (39) using a pair of 3.0 in. achromatic lenses of 15 in. focal length. The collimating lens was located as close as possible to the test section, while the focusing lens was located the necessary two focal lengths from the test section. To prevent fogging of the negative due to the luminosity of the detonation process, a small pin hole was
inserted just upstream of the knife edge. The pin hole was large enough that it would not interfere with the cutoff provided by the knife edge.

A spark gap built by Aircraft Propulsion Laboratory personnel provided the necessary light for the schlieren system. The spark gap utilized ten 500 mm farad capacitors, Sprague type 20 DK-25 rated at 20 KV, arranged in a circle between two metal plates. The capacitors were charged, on the average, to 28 KV and discharged at the proper instant by the 15 KV pulse output of the time delay unit. The duration of light provided was found to be less than 0.1 μ sec.

The spark source and collimating lens were located in a large sealed plastic box. This was done primarily for safety purposes, but had the added advantage of providing a controlled atmosphere in which the spark gap could operate. Several dozen bags of dessicant were placed in the box, which was then sealed shut. This provided a dry atmosphere, thereby allowing the gap to be set as small as possible for a given voltage charge. This was found to give the shortest possible spark discharge times and the most repeatable results.

A variable time delay unit based on the standard resistance-capacitance type circuit was used to vary the instant at which photographs were taken. The time delay could be varied from ten to several thousand μ sec. The signal used to start the time delay unit was provided by the firing of a 2050 thyatron. Probe #3 (see Figure 3) was connected to the grid of the thyatron in a manner similar to that used in the velocity measurements. The passage of the detonation shorted the probe to ground, thereby firing the thyatron. At some time later, the interval having been determined previously by the setting of a
potentiometer in the time delay unit, a second 2050 thyratron fired thus causing a sharply peaked signal to be induced in a coil. The high voltage output of the coil (approximately 15 KV) was used to trigger the spark gap.

The elapsed time interval between the wave passing probe #3 and the spark flash was recorded by the Hewlett-Packard timer. The timer was started by the signal generated at the firing of the first 2050 thyratron and stopped by the output signal of a phototube detector placed in the line of sight of the spark source. The circuit diagram for the phototube is shown in Figure 8.

Royal Ortho cut film, 4 in. x 5 in., was used for most of the photographs taken. Polaroid 200 speed film was also used in the early experiments in order that the results might be viewed immediately. Both films proved to be compatible with the light source.

The camera back used was either a Speed Graphic 4 x 5 or a standard Land Polaroid. No shutter was necessary, as extraneous light was prevented from reaching the film by the bellows, the small pin hole, and by operating in a darkened room.

The knife edge was generally set at 45° to the horizon. This was selected over either a vertical or horizontal knife edge in order to record the position of the interface, shock, and detonation all in one photograph.

During the time tube A was employed in this investigation, a shadowgraph was used in addition to the schlieren system described. The shadowgraph was erected at 90° to the schlieren optical axis and used a 100 watt zirconium light source. The shadowgraph was operated continuously during an experiment and used as a visual check on the stability
Figure 7. Ionization Probe Assembly

Figure 8. Circuit Diagram of Phototube Detector
of the jet. It was found that the jet had a tendency to undulate due to local disturbances. By observing the jet in the shadowgraph and only igniting the mixture during a stable period, any influence of this effect on the data was avoided.

3.6 Gas Metering System.

The sonic orifices metering the gas supply were sized such that very low speed flow would be provided through the detonation tube. It was extremely important that this flow be laminar throughout the test section. Table V gives the orifice sizes used and the calculated gas flow rates for the hydrogen and oxygen mixtures in tube B. Flow rates were not calculated for tube A, as observations showed the flow to be laminar and quite stable for the only mixture ratio tested in this tube.

An assumption made throughout this investigation is that the velocity of the jet is entirely negligible compared to the detonation wave velocity. The slowest detonation velocity encountered was 6,360 ft/sec. for the 35% hydrogen-65% oxygen mixture. For this, the worst case, the jet velocity was less than 0.3% of the detonation velocity. The assumption was therefore justified.

<table>
<thead>
<tr>
<th>Explosive (by volume)</th>
<th>( O_2 ) Orifice in. dia.</th>
<th>( H_2 ) Orifice in. dia.</th>
<th>( \dot{m}_{O_2} \times 10^{-4} ) lbm/sec.</th>
<th>( \dot{m}_{H_2} ) lbm/sec.</th>
<th>( V_E ) ft/sec.</th>
<th>( R_E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>78% H_2-22% O_2</td>
<td>0.019</td>
<td>0.01</td>
<td>5.0</td>
<td>1.1</td>
<td>22.8</td>
<td>1400</td>
</tr>
<tr>
<td>67% H_2-33% O_2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>7.5</td>
<td>0.95</td>
<td>22.6</td>
<td>1800</td>
</tr>
<tr>
<td>50% H_2-50% O_2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>9.5</td>
<td>0.56</td>
<td>18.5</td>
<td>2100</td>
</tr>
<tr>
<td>35% H_2-65% O_2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>11.3</td>
<td>0.54</td>
<td>17.6</td>
<td>2400</td>
</tr>
</tbody>
</table>
The orifices used were not calibrated. Instead, the flow through the oxygen orifice was calculated, using a flow coefficient of 1.0 (38), and the hydrogen flow found as that needed to satisfy the known explosive mixture ratio. This approach could be employed since the mixture ratio was found directly from the velocity measurement and not from a flow measurement (see Section 3.8). The Reynolds numbers were calculated based on the hydraulic radius of the detonation tube.

Liquid Carbonic Commercial Grade hydrogen, rated 99.95% minimum purity, and Liquid Carbonic oxygen, 99.5% minimum purity, were used in the experiments. For the experiments using methane, Phillips Pure Grade, 99% minimum purity, was employed.

The gas flows were regulated by pressure regulators attached to the bottles. In order to insure repeatable results, regulators having very little drift were necessary. The regulators found acceptable for this purpose were an Air Reduction 2-stage Style 8496 regulator for the oxygen, and a Meco type-J regulator for the fuel.

Complete mixing of the explosive gas components was insured by directing the two flows at each other from opposite sides of the tube. Turbulence downstream of the sonic orifices thus aided in the mixing process. A small mixing chamber was also provided at the base of both detonation tubes. These features, coupled with the high molecular diffusion rate of hydrogen, apparently provided sufficient mixing stimulus, as the repeatability of the velocity data could not have been achieved otherwise.

Helium, used as the boundary gas in some experiments, was also metered through an uncalibrated sonic orifice. The regulator pressure was merely adjusted to give laminar flow in the test section.
3.7 Test Sections.

The test section used in conjunction with detonation tube A (test section I) was literally the room in which the experiments were run. A stable jet of hydrogen and oxygen (79% H₂ - 21% O₂ was the only mixture used) was established out the end of the tube. By eliminating all drafts and motions near the jet, an extremely well defined laminar flow could be maintained. Figure 13 shows the result of this procedure.

Two test sections were used in conjunction with detonation tube B; one, a large twelve-inch wide "fish bowl" arrangement (test section II), and the second, a smaller unit having the form of a three-sided shock tube (test section III). Figure 9 shows test section II following one of the experiments performed using a mixture of 78% hydrogen and 22% oxygen by volume, in which the glass plates were destroyed. This was the last experiment performed using this test section, not because of the accident encountered, but rather because of the disappointing results obtained. (see Section 4.2).

The glass plates in section II were approximately 12 in. wide, 12 in. high and 1/2 in. thick. They were mounted 3/8 in. apart and the entire assembly was securely mounted on top of tube B. The explosive mixture flowed through the tube, up through a rectangular hole 3/8 in. x 1/2 in. in the base of the test section (visible in Figure 9), and finally up between the plates of glass. The purpose of this arrangement was to provide a two-dimensional jet of explosive, constrained on two sides by the glass walls, and having two sides exposed to a gas boundary. The top of the test section directly above the jet was left open so that the jet could flow through. In initial experiments performed with this apparatus,
Figure 9. Test Section II After Particularly Energetic Experiment.
air was employed as the boundary gas although it was planned to fill the "fish bowl" with other inert gases for later experiments. These experiments were not run, however, due to difficulties described in Chapter IV.

Test section III was the test chamber actually employed in the experiments which finally made it possible to detonate a gaseous explosive next to a gas boundary. The majority of the data reported herein was obtained using this test section. A photograph of it appears in Figure 10.

Figure 11 is a cross-sectional sketch of the test assembly, showing the major points of interest. For experiments utilizing air as the boundary, the part marked "Insert" was removed. This then meant the explosive jet was exposed to a gas boundary from point "E" upward to point "F." For the cases in which helium was used as the inert boundary gas, the insert was installed and the helium introduced through the 3/16 in. diameter hole drilled through the base of the test section. The extension of the insert above the level of "E" helped maintain the helium boundary separate from the surrounding air. As noted in the sketch, the helium boundary provided was 3/8 in. wide.

3.8 Operating Procedure.

All of the electronic equipment was allowed a full two hours' warm-up period prior to taking any data. Both timers were kept on "stand-by" at all times other than when experiments were being run, which meant the crystal ovens were maintained at constant temperature. This was found desirable for the life of the equipment.
Figure 10. Test Section III and Typical Ionization Probe.
Figure 11. Cross-Sectional Sketch of Test Section III.
As noted in Section 3.6, the explosive composition was not
determined from a measurement of the gas flow rates. Such small ori-
fices had to be used to meter the flow that the potential error in this
method appeared prohibitive. It was also deemed impractical to use a
pre-mixed explosive composition, since in order to maintain a steady
flow of explosive the detonation tube would have had to connect into
the pre-mix chamber through an open valve—a potentially hazardous
condition. Therefore, it was decided to make use of the rather exten-
sive data already available for hydrogen and oxygen mixtures.

Figure 12 was taken from Reference 9, p. 78, and is the result
of considerable experimentation by Moyle and several previous investi-
gators. The values of velocity were converted to an infinite diameter
tube by Moyle, using the equation

$$V_\infty = V_D + \frac{22.8}{\text{Dia.}}$$

(3.1)

where "Dia." refers to the tube diameter, measured in inches, and the
velocities are in ft/sec. For the tubes used in the present study, the
correction to $V_D$ was 76 ft/sec. for tube A and 52 ft/sec. for tube B.
Thus in all cases the correction was less than 1% of the measured veloc-
ity.

The information presented in Figure 12 was used to determine
the hydrogen-oxygen mixture ratios. Preliminary experiments were run
each day, prior to taking any photographs. Tentative pressure settings
were picked for the regulators and the explosive mixture established.
The mixture was ignited and the detonation velocity measured. Correct-
ing this velocity to an infinite diameter tube, the mol fraction of
hydrogen was read from Figure 12. This therefore determined the composition
Figure 12. Experimental Detonation Velocity for Hydrogen-Oxygen Mixtures vs. Hydrogen Content, Corrected to Infinite Diameter Tube.
which must have existed in the tube prior to detonating the mixture. Experiments were run until the pressure settings were determined which yielded the desired explosive composition for repeated tests. This system provided a sufficiently accurate determination of explosive composition with a minimum of complexity. For the methane-oxygen experiments, velocity versus composition data was obtained from Morrison’s work (8).

Having determined the necessary gas regulator settings the electronic equipment was readied by adjusting the thyatrons in the triggering unit and the time delay unit to the verge of firing. A time delay consistent with the wave velocity was set. The explosive mixture was turned on and allowed to run for approximately a full minute prior to igniting it. This period was allowed so that all the air would be exhausted from the detonation tube by the ejector action of the flowing explosive. The slide was pulled from the camera and the mixture ignited. The passage of the wave up the tube initiated both the velocity measurement and the spark. Immediately following the spark, the switch to the rheostat supplying the 30,000 volt power supply with current was turned off. This prevented the spark gap from recharging and firing a second time, thus spoiling the photograph. Sufficient time was provided for turning the switch off, prior to the second spark, by the presence of the 200 megohm resistor shown in Figure 3. The explosive gases were then turned off, the purge air turned on, and the detonation tube allowed to cool back to room temperature.

In order to obtain a complete sequence of photographs of any one explosive and boundary combination, the time delay was altered slightly and the experiment repeated. This was done until sufficient data were obtained.
IV. EXPERIMENTAL RESULTS

Initially, the experimental portion of this study amounted to a trial and error search for conditions under which a gaseous explosive could be detonated while exposed to a gaseous interface. Three different methods were attempted, the third being successful. The methods used are described in the order in which they were undertaken.

The schlieren photographic results presented show the detonation wave boundary interaction process within approximately a three inch zone, measured from the tube exit. This was picked as a practical limit on the length of the explosive jet which could be maintained reasonably straight and well defined. It was thought that extending the field of view past this point would not be particularly beneficial and possibly misleading, as the gaseous interface becomes less "ideal" with increasing distance.

4.1 Axially-Symmetric Detonation Experiments

From the viewpoint of proving that a gaseous explosive could be detonated inside an inert gas "container," the results of these experiments were disappointing, or more correctly, a failure. However, they furnished information which was used in formulating subsequent experiments. These experiments also produced some extremely interesting hydrodynamic effects, and by far the most beautiful schlieren photographs taken during this investigation.

It was first necessary to establish a stable, laminar jet extending upward from detonation tube A. Ample proof that this was accomplished is shown in Figure 13. This schlieren photograph was taken using a vertical knife edge.
Some of the photographic results obtained using this detonation tube are shown in Figure 14. The jet is not visible in Figures (a), (b) or (c), as a horizontal knife edge was used for these photographs. Notice that the wave front is rounded almost immediately after leaving the tube. This in itself is a good indication that "failure" or quenching of the detonation has already occurred. In Figure (b) a Mach disc is visible within the expansion products issuing from the tube.

Figures 14c and 14d were taken of two different experiments at the same time delay. Exposure (c) was obtained using a horizontal knife edge; exposure (d) with a vertical knife edge. Besides illustrating the repeatability of the experiments, some interesting features of the flow are apparent in these two pictures.
Figure 14. Schlieren Photographs of Detonation Failing to Propagate Up Unconfined Column of Explosive; 79% Hydrogen - 21% Oxygen with Air Boundary: Tube A.
Figure 15 is an interpretive sketch of the flow shown in photographs l4c and l4d. The explosive jet can be seen ahead of the shock wave in l4d. The shock is tapered or "pointed" because it is propagating faster through the hydrogen-oxygen jet than through the air. For the conditions shown, the speed of sound of the jet is 2110 ft/sec while that for the air is 1130 ft/sec. Thus one would expect the shock to move roughly twice as fast through the jet as through the air.

The sharply defined, blunt nose following the shock is a gas interface. The gas ahead and behind the shock wave, up to the interface, is air. Behind the interface is the mixture of hydrogen and oxygen. This fact is verified by the density shifts apparent in Figure l4c. A change in color tone from dark to light demarks the shock wave when crossing from in front to behind the shock; i.e., less dense to more dense gas, respectively. The opposite shift in tone is noted when crossing the line denoted as the "gas interface." Since at the same pressure and temperature the mixture of hydrogen and oxygen is less dense than air, this confirms the interpretation of this line as the gas interface which must be present. The shape of the interface and the shock configuration ahead of it is in many respects suggestive of the flow conditions resulting from supersonic flow over a blunt body.

The combustion "wavelets" pointed out in Figure 15 are characteristic of gaseous combustion. These indicate a combustion process is taking place outside of the detonation tube. However, the great distance between this combustion zone and the shock front makes it evident that the energy released by the combustion process is not driving the shock wave. The process is therefore not a detonation.
Figure 15. Explanation of Flow Details Visible in Figure 14.
The distance of the shock front from the tube exit is shown as a function of time in Figure 16. It is apparent that the wave front is decelerating rapidly. Figure 17 is a plot of the wave velocity versus distance from the tube exit, constructed from Figure 16. Within one tube diameter the wave velocity drops more than 20% and eventually, as shown in Figure 17, approaches asymptotically the speed of sound of the explosive mixture. Thus the detonation wave originating in the detonation tube did not propagate up the column of explosive, but rather was quite effectively quenched upon leaving the confinement of the tube. A few additional experiments were run for leaner mixtures of hydrogen and oxygen, but the results were similar to the ones given above. At this point of the investigation, it was not apparent just what conditions would be necessary to detonate a gaseous explosive inside a gaseous container. There undoubtedly is a size effect associated with the experiments; i.e., assuming it possible to obtain detonation under gaseous confinement conditions, one would expect a minimum or failure diameter exists. It was therefore possible that the mixture had failed to detonate due merely to using too small a tube diameter.

Prior to making any major alterations in the experimental arrangement, an endeavor was made to place an order of magnitude on the confinement necessary to obtain satisfactory results. The last three inches of the end of the detonation tube were machined down to an outside diameter of 0.4 in. (see Figure 6). "Extensions" of the detonation tube were then built from commercially available "Saran Wrap" (0.0005 in. thick). The Saran Wrap was molded into a cylinder by wrapping just one

* "Saran Wrap" is produced by the Dow Chemical Company, Midland, Michigan, and is a clear plastic wrapping material.
Figure 16. Dimensionless Distance of Wave from Tube Exit vs. Elapsed Time for 79% Hydrogen - 21% Oxygen Mixture with Air Boundary: Tube A
Figure 17. Velocity of Leading Shock Wave vs. Dimensionless Distance from Tube Exit for 79% Hydrogen-21% Oxygen Mixture with Air Boundary: Tube A.
layer of it around a mechanical pencil and overlapping the edge about 1/4 in. The edge was sealed by the inherent clinging tendency of the plastic itself. The Saran Wrap cylinder was then slipped over the end of the detonation tube and allowed to extend upward approximately two inches, being held in place by tape. A flow of explosive, 79% hydrogen-21% oxygen, was established through the tube and the mixture ignited as before. Examples of the results obtained are shown in Figure 18.

Figure 18. Schlieren Photographs of the Detonation of 79% Hydrogen-21% Oxygen Within a Saran Wrap Detonation Tube.
Figure 18a shows the detonation wave 1.9 inches, or over six diameters, from the end of the metal tube. The wave front is clearly visible through the transparent Saran Wrap, and is one-dimensional -- just as it is inside a metal detonation tube. Again, just as in a metal tube, the wave has moved at constant velocity to the point at which it exists in Figure 18a. This was shown by calculating the distance the wave would move from probe #3 during the time interval between the shorting of that probe and the occurrence of the spark (this interval having been measured on the Hewlett-Packard timer, see Figure 3). This calculated distance was found to be 1.9 in., the same as that measured on the photograph.

Notice that a shock has moved through the Saran Wrap and has the appearance of a truncated cone. The plastic tube, which was originally cylindrical, is bulging out at its base where it is taped to tube A. The two spherical disturbances visible at the left and right edges of the plastic detonation tube are leaks caused by the gradual breakdown of the Saran Wrap due to thermal effects.

Figure 18b is a picture of an identical experiment, taken at a larger time delay. The detonation wave has already left the end of the extended detonation tube. Leaks noted in Figure 18a are also apparent in this photograph.

Saran Wrap is quite sensitive to heat. It curls and wrinkles rather quickly when a match is held six or even eight inches under it. For that reason the leaks are believed to be due to heat effects. As a result of this effect, the plastic cylinders were always destroyed by the experiment.
The fact that a thin plastic sheet, half one-thousandth of an inch thick, would prove to be "infinite" confinement for an explosive mixture of hydrogen and oxygen was surprising and encouraging. The experiment was interpreted as providing qualitative evidence that confinement of a detonation wave did not necessarily depend upon the physical strength of the container, but rather in some way upon its elastic properties. It was therefore thought that under some situations a gaseous container would be sufficient confinement for a gaseous detonation.

4.2 Two-Dimensional Detonation Experiments, Test Section II.

Because some difficulties were encountered in interpreting the photographs of three-dimensional results, a two-dimensional test configuration was evolved. Two-dimensional flow has the added advantage that shocks of the same Mach number as in three-dimensional flow are stronger; that is, have higher pressure ratios. Since both these features would aid in the establishment and interpretation of the desired results, test section II was incorporated.

Very few experiments were performed using this test section, as extreme difficulty was encountered in maintaining a stable jet. As described in Section 3.7 (see Figure 9), the explosive jet was to pass between the two 12, in. wide glass plates and flow out the top of the test section. However, the jet showed a propensity to lean toward one or the other ends of the section. This was probably caused by small eddies induced by the jet in the lower corners of the test section. To try to prevent the jet from leaning, the opening at the top of the test section was covered with masking tape, save for a 1 in. wide strip directly above the opening into the detonation tube. Although this eliminated some of the
leaning tendency, the boundary of the jet did not remain well defined. The eddies present in the corners of the test chamber encouraged mixing along the jet boundaries.

No photographs of the experiments performed with this test arrangement are shown, as they provided no conclusive results. Because of the warped nature of the jet, a distance-time plot of the wave front could not be made. In spite of this, some of the schlieren photographs showed the reaction zone to be narrow and flat in appearance near the tube exit. It was therefore thought that detonation might have continued for at least a short distance into the "unconfined" region. Unfortunately, little confidence could be placed in the results.

Figure 9 illustrates another reason, in addition to the mixing problems encountered, why test section II was discarded after very few experiments. Taping the top of the test section may have led to a reflected shock. Whether due to a reflected shock or whether the explosive mixture had in fact detonated, it is apparent that the disturbance was sufficiently strong to pulverize the 1/2 in. thick glass plates. This occurrence, coupled with the unsatisfactory results already obtained, suggested another approach should be tried.

4.3 Two-Dimensional Detonation Experiments, Test Section III.

The experiences described in Section 4.1 and 4.2 prompted the design of test section III which, like test section II, allowed a two-dimensional study of the detonation-boundary interaction process. This test section, although providing an explosive jet with the same width as before (0.5 in.), was effectively twice as wide since it provided solid confinement on three rather than two sides of the detonation wave. One
wall thus acts as the centerline of a detonation wave that is twice as wide. This is a satisfactory assumption as long as boundary layer effects are negligible along the wall. No evidence was found which contradicted this assumption and, as expected, the detonation wave was normal to the wall in all cases.

Several variations of this test section were made, and all proved to be satisfactory in that they demonstrated that a gaseous explosive would detonate in a region having a gaseous boundary. The last version of section III is shown in Figures 10 and 11. The only difference between this and earlier versions is in the external shape of the nozzle fairing (see Figure 11) and in the width of glass used.

The changes in the nozzle fairing were made in an attempt to keep the mixing region as narrow as possible. The three types of fairing used appear in Figure 25, exposures (f), (g) and (h). Little difference was found in the width of the mixing zone due to changing from the square shoulder (Figure 25h) to the triangular fairing (Figure 25g) to the smooth nozzle fairing (all other exposures in Figure 25), although it was thought the last mentioned did give slightly better results than the other two. This was the version used for virtually all of the experimental data reported in this study.

The test section was originally constructed with glass only as wide as the jet (for example, Figure 26a). This arrangement proved unsatisfactory, as the oblique shock wave, which extends out into the boundary gas, becomes three-dimensional as soon as it extends beyond the glass edge. Therefore the glass width was increased so that the entire process being photographed would remain two-dimensional.
As noted above, test section III accomplished what I and II had not: the detonation wave produced in the detonation tube would continue to propagate at Chapman-Jouguet velocity through a region in which the confinement was provided by an inert gas. Not all combinations of explosive mixtures and gaseous boundaries were equally effective in this regard.

The proof of whether or not a particular combination of explosive and boundary provided the proper confinement conditions was obtained in two forms: by studying the change in wave velocity as a function of time, and by observing the characteristics of the reaction zone visible in the photographs. The latter method, although qualitative in nature, provides confirming information for the velocity trends.

The change in wave velocity with time is presented in the form of distance versus time plots of the wave front. The plots were constructed from a sequence of at least 24 photographs for each mixture ratio. The position of the wave (or "distance of wave from tube exit") was measured from the tube exit (point "E" in Figure 1t) to the wave front at the wall. These measurements were made on 3x enlargements of the original 4 x 5 negatives, and are estimated to be accurate to ± 0.02 in, in most cases. Figure 19 is an example of these results.

On each distance versus time plot a dotted line is given, whose slope corresponds to the average wave velocity measured electronically for all of the test runs making up the graph. A comparison of the electronically measured velocity with that obtained from the sequence of photographs illustrates not only the accuracy and repeatability of the photographic system, but also aids in the determination of whether or not
a wave is slowing down. Note that the photographic data extends down to negative values of \( x' \) in most cases (Figure 20, for example). Data points in this region were obtained by taking photographs of the wave prior to its reaching the tube exit. This helped substantiate the accuracy of the photographically determined velocity plots, and also allows comparison of the detonation wave shape under infinite confinement conditions with that under compressible confinement.

4.3.1 Air Boundary Results.

Four fuels were used in these experiments: hydrogen, methane, ethane and propane. Only the results for hydrogen-oxygen and methane-oxygen mixtures will be reported in detail, as the mixing boundary between air and explosive mixtures of ethane and propane were much too wide and poorly defined for meaningful interpretation. This was caused by the heaviness of the gases (i.e., the large molecular weights of propane and ethane), which meant no buoyant forces were present, as with the hydrogen and methane mixtures, to help maintain a stable vertical jet.

Four mixtures of hydrogen and oxygen were tested next to an air boundary: 78% hydrogen-22% oxygen, 67% hydrogen-33% oxygen, 50% hydrogen-50% oxygen, and 35% hydrogen-65% oxygen (all mixture ratios are given on a per volume basis). Distance versus time graphs for these combinations appear in Figures 19 through 22.

It is apparent from Figure 19 that the mixture of 78% hydrogen-22% oxygen detonated at constant velocity for about one tube width (1.0 in.) past the tube exit, but began to quench shortly after, as evidenced by the decay in velocity. The scatter in the data at the
Figure 19. Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 76% Hydrogen-22% Oxygen Mixture with Air Boundary: Test Section III, Tube B.
Figure 20. Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 67% Hydrogen-33% Oxygen Mixture with Air Boundary: Test Section III, Tube B.
Figure 21. Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 50% Hydrogen-50% Oxygen Mixture with Air Boundary: Test Section III, Tube B.
Figure 22. Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 35% Hydrogen-65% Oxygen Mixture with Air Boundary: Test Section III, Tube B.
highest values of \( x' \) are due to the thickening of the reaction front and the attendant difficulty in making accurate distance measurements. This thickening of the reaction zone is apparent in Figure 23. Figures 20 and 21 are for 67% hydrogen-33% oxygen and 50% hydrogen-50% oxygen mixtures, respectively. In both cases the explosive continued to detonate at Chapman-Jouguet velocity throughout the region observed, with no signs of slowing down as \( x' \) increases. The stoichiometric mixture (67% hydrogen-33% oxygen) appears to sustain the most stable detonation, as shown in the photographs of Figure 25.

Figure 22 is the distance versus time plot for 35% hydrogen-65% oxygen mixture. This was the leanest mixture for which repeatable ignition and full transition to Chapman-Jouguet detonation velocity could be obtained. It is evident from Figure 22 that before this wave had traveled one tube width (1.0 in.) from the tube exit, it had begun to slow down. Somewhat more data scatter appears in this plot than in those for richer mixtures, as the reaction zone is thicker and more difficult to locate accurately (see Figure 27).

Figure 23 is a sequence of three schlieren photographs showing the change in shape of the wave front of the 78% hydrogen-22% oxygen mixture as the distance from the tube exit increases. In photograph (a) the wave front is still quite straight.* Longitudinal striae, so characteristic of hydrogen-oxygen detonation waves, can be seen trailing the

* Two glass edges appear in some pictures due to one of the glass plates working loose and becoming slightly skewed with respect to the test section. At the same time this sometimes caused one of the external gaskets to extend into the field of view, giving the test section an uneven appearance. Neither of these occurrences affected the quality of the information obtained.
Figure 23. Schlieren Photographs of 78% Hydrogen-22% Oxygen Detonation with Air Boundary: Test Section III.

Figure 24. Interpretive Sketch of Flow Field Associated with Detonation next to Gaseous Boundary in Test Section III.
reaction zone. The edge of the explosive mixture ahead of the wave is also visible to some extent. In all three pictures the oblique shock in the air boundary shows up clearly, as does another shock front that appears to be moving parallel to the face of the nozzle fairing. This shock front is the result of leakage of the combustion products between the nozzle fairing and the glass plates. An interpretive sketch of these details is shown in Figure 24.

The width of the reaction zone is clearly increasing with the distance of the wave from the tube exit. In Figure 23c the characteristic striae have disappeared and the reaction zone appears quite thick and turbulent. Both from this picture and Figure 19 it is concluded that the detonation wave is in the process of being quenched.

Figure 25 is a sequence of eight schlieren photographs showing in detail the change in detonation wave shape with time of a 67% hydrogen-33% oxygen (stoichiometric) mixture with an air boundary. Photograph (a) shows the wave front when it was at the tube exit. Note the narrowness of the reaction zone and the straightness of the wave front. In each successive picture the curvature of the wave front increases slightly until in the last three photographs the wave front appears to have reached a stable configuration. Note that striae still trail the wave front in the picture (h), concurring with the results in Figure 20 that the mixture is still detonating. This was the first sequence of photographs from which it was concluded that the confinement provided by a gaseous boundary is sufficient for the continued propagation of a Chapman-Jouguet detonation. An enlargement of one of these photographs appears in Figure 53, in Section V.
Figure 25. Schlieren Photographs of 67% Hydrogen-33% Oxygen Detonation with Air Boundary: Test Section III.
Photographs of the 50% hydrogen-50% oxygen mixture are not shown because of their similarity to those in Figures 23 and 25. This mixture also continued to detonate throughout the field of view, as shown by the $x'$ vs. $t$ plot in Figure 21, although at the furthest point which could be seen the reaction zone was thickening. It therefore appears that this mixture may have quenched shortly after leaving the field of view.

Figure 26 shows the results obtained using a 35% hydrogen-65% oxygen mixture next to air. For these conditions it was quite difficult to maintain a narrow mixing zone between the explosive jet and the air. Better conditions could have been maintained, probably, if the air flow had been blown adjacent to the explosive, rather than just induced. A higher velocity explosive jet might also have cured the difficulty, as this explosive is relatively heavy compared to the air. Both of these solutions were ignored, however, as it was apparent the mixture would not detonate anyway. The failure of the wave extends right to the wall only a short distance from the tube exit, demonstrating that the turbulent mixing zone had no particular influence on the quenching observed.

In exposure (a), the detonation wave is still inside the detonation tube. The reaction zone, even under these ideal conditions, is quite thick, as this mixture ratio is close to the lower limit of detonable hydrogen-oxygen mixtures. As the wave propagates away from the tube exit, the shock front is seen to separate from the combustion zone at the jet boundary. This effect continues to move inward toward the wall until in the last exposure the shock front is entirely separated from the combustion zone, and the detonation has been completely quenched.
Figure 26. Schlieren Photographs of 35% Hydrogen-65% Oxygen Detonation with Air Boundary: Test Section III.
The shock angle Θ, associated with a particular explosive and boundary material, is an important characteristic of the interaction process. The same is true of the interface angle δ. These two angles are labeled in Figure 24. By viewing the schlieren negatives in an enlarger at an enlargement of three times, and laying a straight edge along the shock front, shock angles could be measured reasonably well. By repeating measurements a number of times on a given set of photographs, the probable error was found to be ± 2°.

The reason for this large possible error is apparent in the enlargements shown in Figures 53 and 54 in Chapter V. Because of the curvature of the detonation wave front, the shock wave has considerable curvature near the intersection point of the two waves. It was necessary to judge just what constituted the representative portion of the shock front. It will be shown in Chapter V that even were the detonation wave front perfectly straight, one would expect the shock front to have some curvature.

In general, the same difficulties found in measuring shock angles were also associated with the measurement of the interface angle δ. An additional problem was present in cases where the detonation wave was close to the tube exit. In these cases it was sometimes impossible to distinguish clearly between the shock front and the interface, even in the enlarger. In these instances only the shock angle was measured.

Figures 27, 28, and 29 are the results of these measurements for explosive mixtures containing 78%, 67%, and 50% hydrogen, respectively. Measurements were not made for the mixture of 35% hydrogen-65% oxygen, as the detonation quenched rapidly.
Figure 27. Measured Shock and Interface Angles vs. Dimensionless Distance of Detonation Wave from Tube Exit for 75% Hydrogen-22% Oxygen Mixture with Air Boundary.

Figure 28. Measured Shock and Interface Angles vs. Dimensionless Distance of Detonation Wave from Tube Exit for 67% Hydrogen-33% Oxygen Mixture with Air Boundary.
Figure 29. Measured Shock and Interface Angles vs. Dimensionless Distance of Detonation Wave from Tube Exit for 50% Hydrogen-50% Oxygen Mixture with Air Boundary.
There is a trend apparent in the aforementioned plots; as the hydrogen content is decreased from 78% to 67% to 50%, the shock angle increases from 23° to 25° to 30°, respectively. A similar trend in the values of the interface angles is shown.

Distance versus time graphs for three methane-oxygen mixtures are given in Figures 30, 31, and 32. The detonation waves in the richest (40% methane by vol.) and leanest (19% methane by vol.) both show signs of quenching at about one-and-one-half tube widths (1.50 in.) from the tube exit. The velocity trace of the 30% methane-70% oxygen mixture is the most stable of the three. Morrison(8) notes that the upper detonability limit on methane-oxygen mixtures occurs at about 42% methane by volume, and lower limit at about 10% methane. Thus, as in the case of hydrogen and oxygen mixtures, a mixture ratio near stoichiometric proves to be the most stable under the influence of the detonation-boundary interaction.

Photographs of the 19% methane and 30% methane mixtures are given in Figures 33 and 34, respectively. It is apparent from Figure 33 why so much data scatter occurred in Figure 32; it was caused by the highly irregular nature of the wave front for this explosive. The reaction zone for a mixture of 19% methane and 81% oxygen is extended and quite complicated, even under perfect confinement conditions, as Figure 33a shows.

Figure 34 shows the results for 30% methane-70% oxygen. In this case the reaction zone is thinner than for the leaner mixture, but still relatively thick compared to that for hydrogen-oxygen mixtures. Even for this mixture ratio, Figure 34c shows that the detonation is
Figure 30. Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 40% Methane-60% Oxygen Mixture with Air Boundary: Test Section III, Tube B.
Figure 31. Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 30% Methane-70% Oxygen Mixture with Air Boundary: Test Section III, Tube B.
Figure 32. Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 19% Methane-
61% Oxygen Mixture with Air Boundary: Test Section III, Tube B.
Figure 33. Schlieren Photographs of 19% Methane - 81% Oxygen Detonation with Air Boundary: Test Section III.

Figure 34. Schlieren Photographs of 30% Methane - 70% Oxygen Detonation with Air Boundary: Test Section III.
probably being extinguished by the influence of the air boundary, as the characteristic coarse striae normally present in methane detonations have disappeared.

Figures 35 and 36 give the measured shock and interface angles for the 40\% methane and 30\% methane mixtures, respectively. It was not possible to make these measurements for the mixture of 19\% methane-81\% oxygen because of the rapid quenching experienced by this wave.

Figure 37 is a distance versus time trace for a mixture of 17\% propane-83\% oxygen, by volume. Although photographs are not given for this case because of the poor mixing zone mentioned earlier, it is evident the mixture detonated very readily through the region in which the gaseous boundary existed. The extremely small amount of scatter in this data is due to the very thin reaction zone of propane-oxygen mixtures, and thus the ease with which the wave front could be accurately located.

4.3.2 Helium Boundary Results

The helium boundary results differ from those presented previously in that they were obtained following the completion of most of the theoretical analysis to be given in the following chapter (V). These experiments were performed specifically to test the accuracy of the theoretical analysis, with which it had been predicted that helium would give a far different boundary interaction than air (see Figure 51).

The same four compositions of hydrogen and oxygen were employed in these experiments as in those run with an air boundary. As mentioned earlier (Section 3.6), the helium flow rate was determined by "observation." That is, the helium flow rate was adjusted to that which provided a laminar
Figure 35. Measured Shock and Interface Angles vs. Dimensionless Distance of Detonation Wave from Tube Exit for 40% Methane-60% Oxygen Mixture with Air Boundary.

Figure 36. Measured Shock and Interface Angles vs. Dimensionless Distance of Detonation Wave from Tube Exit for 30% Methane-70% Oxygen Mixture with Air Boundary.
Figure 37. Dimensionless Distance of Wave from Tube Exit vs. Time Delay for
17% Propane-83% Oxygen Mixture with Air Boundary: Test Section III,
Tube B.
mixing region. Following this determination it was calculated the helium velocity was approximately 10 ft/sec. during all the tests performed.

Prior to performing the helium boundary experiments, the optical system was raised about 0.50 in. in order to observe the boundary interaction over a longer distance. This shows up in the following figures in that the values of $x^1$ extend to higher values than they did previously. This change was consistent with the fact that the explosive jet could generally be maintained laminar and well defined for a longer distance using helium than had been possible with air. The use of a blown jet of helium, rather than an induced flow, was undoubtedly the primary factor in bringing this about.

The piece marked "Insert" in Figure 11 was installed during most of the helium runs. It proved helpful in maintaining the jet of helium laminar, and in preventing mixing between the helium and air. It also assured a pure helium boundary was provided. In a few runs, the extension of the insert from point "G" to "H" (Figure 11) was removed in order to check its effect on the shock position recorded. It was found that the insert had no noticeable effect on the boundary interaction process, and thus could be left installed in order to take advantage of its stabilizing influence on the jet.

Figures 38 through 41 are the distance vs. time graphs for these experiments. In Figure 38 it is apparent the mixture of 78% hydrogen and 22% oxygen is again quenched by the influence of the gaseous boundary condition. Figures 39 and 40 show that both the stoichiometric mixture of hydrogen and oxygen and the 50% mixture detonate throughout the field of view. Thus, helium appears to be as adequate a container for these mixtures
Figure 38. Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 78% Hydrogen-22% Oxygen Mixture with Helium Boundary; Test Section III, Tube B.
Figure 40. Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 50% Hydrogen- 
50% Oxygen Mixture with Helium Boundary: Test Section III, Tube B.
Figure 41. Dimensionless Distance of Wave from Tube Exit vs. Time Delay for 35% Hydrogen-65% Oxygen Mixture with Helium Boundary: Test Section III, Tube B.
as was air. The leanest mixture tested, 35% hydrogen-65% oxygen, quenches due to the detonation-boundary interaction after approximately one inch of travel; i.e., \( x' = 1.0 \). This is illustrated in Figure 41.

Slightly more data scatter is present in these plots than in the air boundary data, but it was the fault of the equipment rather than anything inherent in the experiment. Just prior to making these runs a malfunction of the Hewlett-Packard timer occurred, which prevented it from stopping properly on the signal provided by the phototube circuit. The timer would, however, function properly when operated off the thyatron triggering unit. Therefore the two timers were switched in jobs: the more accurate timer, the Hewlett-Packard, was used for the velocity measurement; the less accurate, the Berkley, had to be used to record the time delay (see Figure 3). This resulted in a loss in accuracy of the time delay measurements.

Figure 42 is a sequence of schlieren photographs of the stoichiometric hydrogen-oxygen mixture next to the helium boundary. The most obvious difference between these pictures and those obtained for an air boundary (Figure 25) is the much larger shock angle, \( \theta \). It is also apparent in exposures (b) through (f) that the intersection point between the detonation wave and the shock wave forms a discontinuity in the wave front, i.e., rather than the waves intersecting in a smooth curve as before, they have dissimilar rates of curvature where they meet. This effect is less evident in the exposures where the wave is furthest from the tube exit. An enlargement of one of these pictures appears in Figure 54.

The characteristic striae of oxygen-hydrogen detonation waves appear in all of the exposures in this sequence, verifying the continued detonation of the explosive. The gas interface is not as evident in these
Figure 42. Schlieren Photographs of 67% Hydrogen-33% Oxygen Detonation with Helium Boundary: Test Section III.
photographs as in previous examples shown. One reason for this is that
it is distorted and spread out by the shock that reflects from the "Insert."
This reflected shock appears as a slightly curved white streak running
roughly parallel to the extension on the insert. Since the flow at the
Chapman-Jouguet plane is at Mach 1.0, relative to the wave, any disturb-
ances behind the wave, such as the reflected shock, cannot influence its
properties.

Picture (d) in this sequence shows the extension of the insert
removed. Pictures such as this verify that the reflected shock from the
extension does not noticeably affect the interaction phenomenon.

Figure 43 is a sequence of schlieren photographs of the 35%
hydrogen-65% oxygen detonation-boundary interaction process. As pre-
dicted theoretically in Section 5.3.1, under these circumstances the shock
is detached; that is, the weak oblique shock solution does not hold.
(This will be discussed further in Chapter V.) The shock is no longer
oblique and $\Theta$ is therefore indeterminate. The result is the rather
unique flow pattern shown. In exposure (g), for instance, the shock actu-
ally is ahead of the detonation wave. An enlargement of one of these pic-
tures is shown in Figure 44.

Close inspection of the photographs shows the existence of a
"lambda shock" at the point where the shock in the boundary gas intersects
with the insert extension. This is especially evident in exposures (d)
and (e) in Figure 43. The spots that appear in some of these photographs
are water droplets left from the condensed vapor of the previous experi-
ment. In some of the pictures it will be noted that the helium-explosive
mixing region is turbulent. This appeared to be due to the relative heav-
iness of this lean a mixture of oxygen and hydrogen.
Figure 43. Schlieren Photographs of 35% Hydrogen-65% Oxygen Detonation with Helium Boundary: Test Section III.
Figure 44. Schlieren Photograph Enlargement of 35% Hydrogen-
65% Oxygen Detonation - Helium Boundary Interaction:
Test Section III.
Photographs of the 78% hydrogen and 50% hydrogen mixtures are not included, due to their similarity to those already shown. The boundary interaction process of the 78% hydrogen-22% oxygen was virtually identical to that for the stoichiometric mixture, while the 50% hydrogen-50% oxygen pictures showed the shock pattern to be somewhat similar to that in Figures 43 and 44.

Figures 45 and 46 give the measured shock angles for the 78% hydrogen and 67% hydrogen mixtures, respectively. Shock angles were not measured for the other two cases because of the lack of meaning of this measurement under detached shock conditions. Because of the influence of the reflected shock on the interface, the interface angles could not be measured accurately. Therefore they are also omitted.
Figure 45. Measured Shock Angle vs. Dimensionless Distance of Detonation Wave from Tube Exit for 70% Hydrogen-20% Oxygen Mixture with Helium Boundary.

Figure 46. Measured Shock Angle vs. Dimensionless Distance of Detonation Wave from Tube Exit for 67% Hydrogen-33% Oxygen Mixture with Helium Boundary.
V. THEORETICAL ANALYSIS AND COMPARISON WITH EXPERIMENTAL RESULTS

5.1 Selection of Idealized Flow Model

It was shown in the last chapter that a gaseous explosive would continue to detonate while under the confinement of an inert gas. The experiments which demonstrated this were two-dimensional in nature. In order to understand further the interaction of the detonation wave with an inert boundary, it is instructive to study an idealized model of the process. The flow configuration which will be considered theoretically is illustrated in Figure 47.

It is assumed that both the explosive and inert gas are flowing steadily (without mixing or diffusing) at the velocity $V_D$, corresponding to the Chapman-Jouguet velocity of the explosive mixture. A detonation wave is postulated to exist in the explosive mixture. Note that in the selected coordinate system, the x-y axis is fixed to the detonation wave, which in turn is fixed in space. Although the two gases move at the same velocity, they will not in general have the same flow Mach number, because of having different velocities of sound. It will be seen shortly that for virtually any combination of explosive and boundary gas the Mach numbers of both flows will be greater than one. Therefore, a shock wave must exist in the inert gas in order to adjust the pressure in that flow in accordance with the high pressures behind the detonation wave. The gases leaving the detonation wave turn though a Prandtl-Meyer expansion fan in order to be aligned with the flow behind the shock wave.
Figure 47. Idealized Flow Model of Detonation Wave-Boundary Interaction.
Initially, it will be assumed that neither the detonation wave nor the shock wave has any curvature. Although it has already been seen in the schlieren photographs presented that this is not the case, it is a satisfactory first approximation which, in fact, allows virtually all aspects of the interaction process to be explained. Some of the restrictions placed on the problem will be relaxed presently in order to determine their influence on the solution. The assumption of perfect gases is made throughout the analysis as previous gaseous detonation studies have demonstrated this to be correct (e.g. Reference 9 and 10).

5.2 Prediction of Shock and Interface Angles

The solutions for the shock angle $\Theta$ and the interface angle $\delta$ is made possible by the condition that the pressure and the flow direction must be the same on both sides of the gas interface. Thus the problem becomes a trial and error solution, in which conditions in the boundary gas and the detonation products are matched.

The information necessary to perform the calculations consists of $V_D$, the detonation velocity; the molecular weights of the boundary gas and explosive mixture, $m_I$ and $m_E$ respectively; the pertinent specific heat ratios, $\gamma_{I_1}$, $\gamma_{E_1}$ and $\gamma_{E_2}$; and the initial temperature, $T_1$.

Using the specific heat ratios, molecular weights and initial temperature, the sound velocity and initial Mach numbers can be computed for both gases.

$$a_{E_1}^2 = \frac{\gamma_{E_1}RT_1}{m_{E_1}} (5.1)$$

$$a_{I_1}^2 = \frac{\gamma_{I_1}RT_1}{m_{I_1}} (5.2)$$
The flow Mach numbers thus are:

\[ M_{E1} = \frac{V_D}{a_{E1}} \quad \text{and} \quad M_{I1} = \frac{V_D}{a_{I1}} \quad (5.3) \]

where \( M_{E1} = M_D \).

Calculations of this type performed for the explosive-boundary combinations of this study, and some additional cases, are given in Table VI.

For a stoichiometric mixture of hydrogen and oxygen, changing the boundary gas from helium to sulfurhexafluoride (SF₆) changes the Mach number of the boundary gas from 2.78 to 20.72, respectively. It should thus be possible to obtain a wide variation of interaction processes depending upon the boundary gas used. Note that all of the values of \( M_{I1} \) in Table VI are greater than one, the lowest being 1.91 for the combination of 35% hydrogen and 65% oxygen with a helium boundary.

The inert boundary gas flow is similar to supersonic flow over a wedge where \( \delta \), in Figure 47, is analogous to the wedge angle. It was found most convenient to start the shock and interface angle computation by first determining what the detachment angle, \( \Theta_{\text{max}} \), is for the given conditions. The relationship between the shock and deflection angle at a given Mach number in the gas boundary is:

\[ \tan \delta = 2 \cot \Theta \frac{M_{I1}^2 \sin^2 \Theta - 1}{M_{I1}^2 (\gamma_{I1} \cos 2\Theta + 2)} \quad (5.4) \]

Differentiating Equation (5.4) with respect to \( \Theta \) and setting it equal to zero, a relationship for the maximum shock angle \( \Theta_{\text{max}} \) is


**TABLE VI**

FLOW MACH NUMBERS OF VARIOUS EXPLOSIVE MIXTURES
AND GAS BOUNDARY COMBINATIONS

\[ T_1 = 535^\circ \text{R} \]

<table>
<thead>
<tr>
<th>Explosive</th>
<th>Boundary</th>
<th>( V_D ) ft/sec</th>
<th>( S_{I1} ) ( \times 10^{-3} )</th>
<th>( S_{E1} )</th>
<th>( m_{I1} ) gm/mol</th>
<th>( m_{E1} ) gm/mol</th>
<th>( M_{I1} )</th>
<th>( M_{E1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen and Oxygen</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>78% H₂ - 22% O₂</td>
<td>Air</td>
<td>10.8</td>
<td>1.4</td>
<td>1.4</td>
<td>8.6</td>
<td>29.</td>
<td>9.55</td>
<td>5.19</td>
</tr>
<tr>
<td>67% H₂ - 33% O₂</td>
<td>Air</td>
<td>9.25</td>
<td>1.4</td>
<td>1.4</td>
<td>11.9</td>
<td>29.</td>
<td>8.18</td>
<td>5.20</td>
</tr>
<tr>
<td>50% H₂ - 50% O₂</td>
<td>Air</td>
<td>7.55</td>
<td>1.4</td>
<td>1.4</td>
<td>17.0</td>
<td>29.</td>
<td>6.68</td>
<td>5.14</td>
</tr>
<tr>
<td>35% H₂ - 65% O₂</td>
<td>Air</td>
<td>6.35</td>
<td>1.4</td>
<td>1.4</td>
<td>21.5</td>
<td>29.</td>
<td>5.61</td>
<td>4.83</td>
</tr>
<tr>
<td>78% H₂ - 22% O₂</td>
<td>Helium</td>
<td>10.8</td>
<td>1.66</td>
<td>1.4</td>
<td>8.6</td>
<td>4.</td>
<td>3.25</td>
<td>5.19</td>
</tr>
<tr>
<td>67% H₂ - 33% O₂</td>
<td>Helium</td>
<td>9.25</td>
<td>1.66</td>
<td>1.4</td>
<td>11.9</td>
<td>4.</td>
<td>2.78</td>
<td>5.20</td>
</tr>
<tr>
<td>50% H₂ - 50% O₂</td>
<td>Helium</td>
<td>7.55</td>
<td>1.66</td>
<td>1.4</td>
<td>17.0</td>
<td>4.</td>
<td>2.27</td>
<td>5.14</td>
</tr>
<tr>
<td>35% H₂ - 65% O₂</td>
<td>Helium</td>
<td>6.35</td>
<td>1.66</td>
<td>1.4</td>
<td>21.5</td>
<td>4.</td>
<td>1.91</td>
<td>4.83</td>
</tr>
<tr>
<td>67% H₂ - 33% O₂</td>
<td>Argon</td>
<td>9.25</td>
<td>1.67</td>
<td>1.4</td>
<td>11.9</td>
<td>40.</td>
<td>8.78</td>
<td>5.20</td>
</tr>
<tr>
<td>67% H₂ - 33% O₂</td>
<td>Sulfur-</td>
<td>9.25</td>
<td>1.094</td>
<td>1.4</td>
<td>11.9</td>
<td>146.</td>
<td>20.72</td>
<td>5.20</td>
</tr>
<tr>
<td>Methane and Oxygen</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40% CH₄ - 60% O₂</td>
<td>Air</td>
<td>8.1</td>
<td>1.4</td>
<td>1.38</td>
<td>25.6</td>
<td>29.</td>
<td>7.16</td>
<td>6.76</td>
</tr>
<tr>
<td>30% CH₄ - 70% O₂</td>
<td>Air</td>
<td>7.55</td>
<td>1.4</td>
<td>1.39</td>
<td>27.2</td>
<td>29.</td>
<td>6.68</td>
<td>6.49</td>
</tr>
<tr>
<td>19% CH₄ - 81% O₂</td>
<td>Air</td>
<td>6.6</td>
<td>1.4</td>
<td>1.39</td>
<td>28.96</td>
<td>29.</td>
<td>5.84</td>
<td>5.84</td>
</tr>
<tr>
<td>Propane and Oxygen</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17% C₃H₈ - 83% O₂</td>
<td>Air</td>
<td>7.9</td>
<td>1.4</td>
<td>1.34</td>
<td>34.0</td>
<td>29.</td>
<td>6.98</td>
<td>7.71</td>
</tr>
<tr>
<td>7% C₃H₈ - 93% O₂</td>
<td>Air</td>
<td>6.4</td>
<td>1.4</td>
<td>1.37</td>
<td>32.8</td>
<td>29.</td>
<td>5.66</td>
<td>6.07</td>
</tr>
<tr>
<td>Ethane and Oxygen</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22% C₂H₆ - 78% O₂</td>
<td>Air</td>
<td>7.7</td>
<td>1.4</td>
<td>1.37</td>
<td>31.6</td>
<td>29.</td>
<td>6.81</td>
<td>7.18</td>
</tr>
</tbody>
</table>
\[
\sin^2 \Theta_{\text{max}} = \frac{1}{\gamma_{I_1} M_{I_1}^2} \left[ \frac{\gamma_{I_1}+1}{4} M_{I_1}^2 - 1 + \sqrt{\left(\gamma_{I_1}+1\right) \left(1 + \frac{\gamma_{I_1}-1}{2} M_{I_1}^2 + \frac{\gamma_{I_1}+1}{16} M_{I_1}^4\right)} \right].
\]

(5.5)

For a given Mach number and boundary gas, one is thus able to determine the maximum possible shock angle. If an oblique shock solution exists, the shock angle \( \Theta \) must be equal to, or less than, the value of \( \Theta_{\text{max}} \). For convenience, \( \Theta_{\text{max}} \) was plotted as a function of the Mach number \( M_{I_1} \) for three values of \( \gamma_{I_1} \): 1.10, 1.40 and 1.67. This information is presented in Figure 48.

Since the combustion products leave the Chapman-Jouguet plane at Mach 1.0 and turn through a Prandtl-Meyer expansion until they are parallel to the interface, the Prandtl-Meyer angle \( \nu \) must equal the interface angle \( \delta \) (see Figure 47). In undergoing an expansion, resulting from turning an angle equal to \( \nu \), the gas accelerates to some higher Mach number. The relation between this new Mach number, \( M_{E_3} \), and \( \nu \) is:

\[
\nu = \frac{\gamma_{E_2} + 1}{\sqrt{\gamma_{E_2} - 1}} \arctan \left[ \frac{\sqrt{\gamma_{E_2} + 1}}{\sqrt{\gamma_{E_2} - 1}} (M_{E_3}^2 - 1) \right] - \arctan \sqrt{M_{E_3}^2 - 1}
\]

(5.6)

where \( \nu \) is in radians. This expression is based on the assumption that \( \gamma_{E_2} \) remains constant during the expansion process.

The value of \( \gamma_{E_2} \) must be determined for each explosive and is a function of the temperature, pressure and composition of the products. In practice it is found that \( \gamma_{E_2} \) is usually close to 1.20. For the calculations made in the present study, \( \gamma_{E_2} \) was determined in two ways, depending upon whether the fuel used was hydrogen or a hydrocarbon. The
Figure 46. Shock Detachment Angle vs. Mach Number as a Function of the Specific Heat Ratio.
values of $\gamma_{E_2}$ for hydrocarbons were estimated from information in Reference 29. The values of $\gamma_{E_2}$ for the four hydrogen-oxygen mixtures were obtained from Moyle's theoretical results.\(^{9}\)

The method of obtaining $\gamma_{E_2}$ and the detonation wave pressure ratio $P_{E_2}/P_{E_1}$ from Moyle's work is facilitated by the use of the specific volume ratio across the wave $v_{E_1}/v_{E_2}$. Moyle computed this ratio for various hydrogen-oxygen mixtures. His results appear in Figure 49. Using Equation (2.24), Moyle's results enable one to solve for the detonation pressure ratio.

$$\frac{P_{E_2}}{P_{E_1}} = 1 + \gamma_{E_1} M_{E_1}^2 (1 - \frac{v_{E_2}}{v_{E_1}}) \quad (5.7)$$

Employing Equation (2.26) and the pressure ratio $P_{E_2}/P_{E_1}$, the value of $\gamma_{E_2}$ is found

$$\gamma_{E_2} = \frac{P_{E_1}}{P_{E_2}} \left(1 + \gamma_{E_1} M_{E_1}^2\right) - 1 \quad (5.8)$$

For all four oxygen-hydrogen mixtures used in this study, $\gamma_{E_2}$ was found to be approximately 1.22.

At the end of the Prandlt-Meyer expansion, the ratio of the local stagnation pressure to the static pressure is:

$$\left(\frac{P_{E_1}}{P_{E}}\right)_3 = \left(1 + \frac{\gamma_{E_2} - 1}{2} M_{E_3}^2\right) \frac{\gamma_{E_2}}{\gamma_{E_2} - 1} \quad (5.9)$$

At the Chapman-Jouguet plane the ratio of stagnation to static pressure is:

$$\left(\frac{P_{E_1}}{P_{E}}\right)_2 = \left(1 + \frac{\gamma_{E_2} - 1}{2}\right) \frac{\gamma_{E_2}}{\gamma_{E_2} - 1} \quad (5.10)$$
Figure 49. Specific Volume Ratio Across Chapman-Jouguet Detonation Wave vs. Mol Fraction of Hydrogen for Hydrogen-Oxygen Mixtures.

REF. 9

$P_i = 14.7 \text{ Psi.}$

$T_i = 300 \text{ }^\circ\text{K}$
The Prandtl-Meyer expansion process is ideally isentropic. It will not depart substantially from this as the effects of heat transfer and viscosity are negligible. Therefore the stagnation pressure at the Chapman-Jouguet plane is equal to that at the end of the expansion. This allows the pressure ratio \( \frac{p_{E2}}{p_{E1}} \), which is the static pressure ratio across the entire process undergone by the explosive gas, to be determined.

\[
\frac{p_{E3}}{p_{E1}} = \left( \frac{p_{E2}}{p_{E1}} \right) \left( \frac{p_{E_2}}{p_{E}} \right) \left( \frac{p_{E}}{p_{E_2}} \right)^{\frac{2}{\gamma_1}}
\]

This pressure ratio must equal that across the shock in order for the pressures on either side of the gas interface to be equal. The shock pressure ratio is:

\[
\frac{p_{I2}}{p_{I1}} = 1 + \frac{2\gamma_1}{\gamma_1 + 1} \left( M_{I_1}^2 \sin^2 \theta - 1 \right)
\]

where it is assumed \( \gamma_{I_1} = \gamma_{I_2} \).

The information necessary to compute the shock angle \( \theta \) and the interface angle \( \delta \) is now complete. The computation was performed in the following fashion: following the determination of \( M_{I_1} \) and \( M_{E_1} \), \( \theta_{\text{max}} \) was computed or read from Figure 48. For the first trial, a value of \( \theta \) equal to, or less than \( \theta_{\text{max}} \) was then assumed. The value of \( \delta \) corresponding to the assumed value of \( \theta \) was calculated from Equation (5.4). Utilizing Moyle's information, Figure 49, or that contained in Reference 29, \( \gamma_{E_2} \) was found and used in Equation (5.9) to solve for \( \left( \frac{p_{E_2}}{p_{E}} \right)^{\frac{2}{\gamma_1}} \). The pressure ratio, \( \frac{p_{E3}}{p_{E1}} \) was then obtained through the use of Equations (5.10) and (5.11), and the previously determined value of \( \left( \frac{p_{E_2}}{p_{E}} \right)^{\frac{2}{\gamma_1}} \).
Using the assumed shock angle, $\frac{p_{I_2}}{p_{I_1}}$ was computed from Equation (5.12). Since the static pressures ahead of the detonation wave are equal, i.e., $p_{E_1} = p_{I_1}$, the correct value of $\varphi$ has been determined when $\frac{p_{I_2}}{p_{I_1}}$ equals $\frac{p_{E_3}}{p_{E_1}}$. If this is not the case for the assumed value of $\varphi$, the calculation is repeated. If $\frac{p_{E_3}}{p_{E_1}}$ was found to be greater than $\frac{p_{I_2}}{p_{I_1}}$, the assumed shock angle must be increased. If the opposite was true, the assumed value of shock angle must be decreased.

A possible solution is that for $\varphi$ assumed equal to $\varphi_{\text{max}}$, $\frac{p_{E_3}}{p_{E_1}}$ will be greater than $\frac{p_{I_2}}{p_{I_1}}$. For this case the shock is detached, i.e., an oblique shock solution does not exist. This means the turning angle, $\delta$, demanded is greater than the boundary gas can negotiate through an oblique shock. For this condition the flow behind the shock is subsonic. There is no rigorous method at present of theoretically analyzing this condition.

The results of these calculations are given in Table VII. A sample calculation is shown in Appendix A.

It may be observed in Table VII that the computed angles for the fuels ethane, propane and methane are similar in magnitude to those for hydrogen. It is because of this that these fuels were not pursued further in the experimental portion of this study. The shock angles for hydrogen and oxygen mixtures next to a helium boundary are quite different from those computed for an air boundary. This should therefore furnish an excellent means of checking the theoretical results.

It should be pointed out that although an expansion was assumed to exist in the reaction products behind the detonation wave, it is not
### TABLE VII

**COMPUTED VALUES OF SHOCK AND INTERFACE ANGLES**

\[ T_1 = 535^\circ R \]

<table>
<thead>
<tr>
<th>Explosive</th>
<th>Boundary</th>
<th>( \theta_{\text{max}} )</th>
<th>( \delta )</th>
<th>( \vartheta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen and Oxygen</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>78% H₂ - 22% O₂</td>
<td>Air</td>
<td>67.5</td>
<td>12.6</td>
<td>17.4</td>
</tr>
<tr>
<td>67% H₂ - 33% O₂</td>
<td>Air</td>
<td>67.4</td>
<td>14.3</td>
<td>20.0</td>
</tr>
<tr>
<td>50% H₂ - 50% O₂</td>
<td>Air</td>
<td>67.1</td>
<td>16.3</td>
<td>23.3</td>
</tr>
<tr>
<td>35% H₂ - 65% O₂</td>
<td>Air</td>
<td>66.8</td>
<td>17.3</td>
<td>25.7</td>
</tr>
<tr>
<td>78% H₂ - 22% O₂</td>
<td>Helium</td>
<td>62.7</td>
<td>22.9</td>
<td>43.6</td>
</tr>
<tr>
<td>67% H₂ - 33% O₂</td>
<td>Helium</td>
<td>62.7</td>
<td>24.4</td>
<td>51.5</td>
</tr>
<tr>
<td>50% H₂ - 50% O₂</td>
<td>Helium</td>
<td>62.9</td>
<td>Detached Shock</td>
<td></td>
</tr>
<tr>
<td>35% H₂ - 65% O₂</td>
<td>Helium</td>
<td>63.5</td>
<td>Detached Shock</td>
<td></td>
</tr>
<tr>
<td>67% H₂ - 33% O₂</td>
<td>Argo₂</td>
<td>63.2</td>
<td>12.1</td>
<td>18.9</td>
</tr>
<tr>
<td>67% H₂ - 33% O₂</td>
<td>SF₆</td>
<td>77.8</td>
<td>8.0</td>
<td>9.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Methane and Oxygen</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>40% CH₄ - 60% O₂</td>
<td>Air</td>
<td>67.1</td>
<td>20.0</td>
<td>27.1</td>
</tr>
<tr>
<td>30% CH₄ - 70% O₂</td>
<td>Air</td>
<td>67.0</td>
<td>18.8</td>
<td>25.9</td>
</tr>
<tr>
<td>19% CH₄ - 81% O₂</td>
<td>Air</td>
<td>66.9</td>
<td>19.7</td>
<td>28.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Propane and Oxygen</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>17% C₃H₈ - 83% O₂</td>
<td>Air</td>
<td>67.2</td>
<td>21.5</td>
<td>29.1</td>
</tr>
<tr>
<td>7% C₃H₈ - 93% O₂</td>
<td>Air</td>
<td>66.8</td>
<td>20.6</td>
<td>29.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ethane and Oxygen</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>22% C₂H₆ - 78% O₂</td>
<td>Air</td>
<td>67.1</td>
<td>20.9</td>
<td>28.5</td>
</tr>
</tbody>
</table>
obvious that this must always be the case. It was thought some situations might be found in which a shock would exist. For this case the pressure ratio \( p_{i_2}/p_{i_1} \) would be greater than \( p_{e_2}/p_{e_1} \). No example of this was found, however, as the theoretical calculations performed all indicated that an expansion wave must exist. For the case in which a detonation wave is directed at, rather than parallel to, another gas, situations where a shock is reflected back into the detonation products do exist.\(^{(8)}\)

5.3 Comparison of Theoretical and Experimental Results

The theoretical results given in Section 5.2 are for an idealized model of the detonation boundary interaction. A comparison of these results with those obtained from the experiments will be made prior to delving further into the mechanics of the flow. In spite of the simplifications made, it will be seen that surprisingly good agreement is found between the theoretical and experimental results.

5.3.1 Idealized Model Results

Experiments were performed for four hydrogen-oxygen mixtures next to both air and helium boundaries, and for three methane-oxygen mixtures with an air boundary. Shock and interface angles were measured for those cases in which the explosive continued to detonate some distance past the tube exit. These results are plotted in Figures 50, 51 and 52, along with the corresponding theoretical results taken from Table VII.

The computed and measured shock and interface angles are seen to be in good agreement. In Figure 50 the theoretical values are within 4 to 6 degrees of the measured values and have the same trend with changing hydrogen content. In Figure 51 only two measured values of the shock
Figure 50. Comparison of Theoretical and Experimental Shock and Interface Angles for Oxygen-Hydrogen Mixtures with Air Boundary.
Figure 51. Comparison of Theoretical and Experimental Shock and Interface Angles for Oxygen-Hydrogen Mixtures with Helium Boundary.
Figure 52. Comparison of Theoretical and Experimental Shock and Interface Angles for Oxygen-Methane Mixtures with Air Boundary.
angle could be plotted because of the detachment of the shock noted in the photographic results, Figure 43. The shock angle for the mixture of 78% hydrogen-22% oxygen is in rather poor agreement with the theoretical value. The reason disagreement exists between the theoretical and experimental angles is discussed in Section 5.3.3.

The correct prediction that the shock would just be detached for the 50% hydrogen-50% oxygen, and well into the detachment region for the 35% hydrogen-65% oxygen mixture, constitutes excellent proof of the fundamental validity of the theoretical analysis. As mentioned earlier in Chapter IV, these theoretical results prompted use of helium as a boundary. The results illustrated in Figures 50 and 51 thus verify the essential accuracy of the idealized theoretical model.

The results shown in Figure 52 further substantiate that the boundary interaction process is essentially as described in Section 5.2. Although the computed and measured values of shock and interface angle differ by from 6 to 8 degrees, the theory again predicts the correct trend with varying mixture composition.

Figures 53 and 54 show enlargements of schlieren photographs of stoichiometric hydrogen-oxygen mixtures next to air and helium, respectively. Adjacent to each photograph is a scale drawing of the theoretical results.

5.3.2 Time-Unsteady Influences

One of the assumptions made in the idealized model theory was that the flow associated with the interaction process is steady. This was not the case in the actual experiments performed, however, as several unsteady flow effects were unavoidably present.
Figure 53. Schlieren Enlargement of Stoichiometric Hydrogen-Oxygen Mixture next to Air and Comparison with Theoretical Results.
Figure 54. Schlieren Enlargement of Stoichiometric Hydrogen-Oxygen Mixture next to Helium and Comparison with Theoretical Results.
When a detonation wave is formed in a closed end tube, a so-called "Taylor expansion wave" trails the wave front. This means the pressure behind the Chapman-Jouguet plane drops toward a plateau value of about one-third of the Chapman-Jouguet pressure, according to Morrison's results.\(^8\) For a detonation tube 50 inches long, this decline in pressure takes place over approximately a 30 inch distance behind the wave front. Thus, as the detonation wave leaves the tube exit and propagates into the gas boundary region, the pressure behind the wave decreases both because of expansion of the products out into the boundary gas and because of the trailing expansion. Neither of these effects is accounted for in the flow of Figure 47. Recalling that the flow is Mach one at the Chapman-Jouguet plane, it is apparent that neither of the expansion effects discussed can influence the properties of the detonation wave. However, the shape of the shock in the boundary gas can be affected, and it is this aspect that is treated in the following analysis.

Since the trailing expansion is reasonably gradual in nature, no great error is incurred by assuming that for a short distance behind the wave front the pressure is equal to that at the C-J plane. Using this assumption, the effect of the lateral expansion of the combustion products can easily be computed.

At the instant at which the detonation wave leaves the confines of the shock tube, that is, travels past point "E" in Figure 55, the combustion products are free to expand to atmospheric pressure. When the detonation wave is still close to point E, this lateral expansion will be the predominant factor in establishing the location of the shock wave
Figure 55. Pseudo Shock Tube Analysis for Lateral Expansion of Combustion Products.
in the boundary gas. As the detonation wave continues to propagate away from the tube exit, the lateral expansion will become less important and the shock position should correspond more closely to that calculated from the steady flow model.

The initial rush of gas in the lateral, or \( y \)-direction, can be approximated by a one-dimensional shock tube analysis. The high pressure of the C-J plane is analogous to the high pressure side of the diaphragm. The atmosphere represents the low pressure side of the diaphragm. The diaphragm of this pseudo shock tube is "ruptured" when the detonation wave passes point \( E \) in Figure 55.

The basic shock tube relation is: \(^{(46)}\)

\[
\frac{p_4'}{p_1'} = \frac{p_2'}{p_1'} \left[ 1 - \frac{(\gamma_4'-1)(a_1'/a_4')(p_2'/p_1'-1)}{\sqrt{2\gamma_4'} \sqrt{2\gamma_1' + (\gamma_1'+1)(p_2'/p_1'-1)}} \right]^{-2\gamma_4'-1} \tag{5.13}
\]

where the primes are used to distinguish between the use of the nomenclature in this instance and that used previously in the detonation calculations. \( p_4' \) equals \( p_{E2} \), the pressure at the C-J plane; \( p_1' \) is the atmospheric pressure; \( p_2' \) is the pressure behind the shock traveling in the \( y \)-direction; \( \gamma_4' \) equals \( \gamma_{E2} \); and \( \gamma_1' \) equals \( \gamma_{I1} \) (see Figure 55). Equation (5.13) allows the calculation of the pressure ratio \( (p_2'/p_1') \) across the shock wave which, at the breaking of the diaphragm, will move into region \( 1' \) of the shock tube. \( p_4'/p_1' \) is the initial pressure ratio across the diaphragm.

Using \( p_2'/p_1' \), the velocity of this shock can be calculated from the following equation:

\[
V_{S'} = a'_1 \left( \frac{\gamma_1'-1}{2\gamma_1'} \frac{p_2'}{\sqrt{2\gamma_1'}} \right)^{1/2} \tag{5.14}
\]
Since, within the assumed conditions, the shock moving in the y-direction will travel at constant velocity, and whereas the detonation wave also travels at constant velocity in the x-direction, these two velocities may be used to calculate $\Theta'$, where this is the angle defined by the expression:

$$\tan \Theta' = \frac{V_S}{V_D} \quad (5.15)$$

$\Theta'$ is thus defined analogously to $\Theta$, the shock angle in Figure 47. One would expect that when the detonation is close to the exit, the measured shock angle will be closer to $\Theta'$ than to $\Theta$. As the distance of the wave from the exit increases, the measured shock angle should approach $\Theta$, the steady state value.

Values of $V_S$ and $\Theta'$ were computed for the four hydrogen-oxygen mixtures of this study and with both the air and helium boundaries. These values are tabulated in Table VIII, where they are compared to the values of $\Theta$ calculated previously using the idealized steady flow analysis.

For the hydrogen and oxygen mixtures next to an air boundary, $\Theta'$ and $\Theta$ are roughly equal. This coincidence means that, neglecting other factors, the measured shock angle should be almost independent of the distance the detonation has traveled in the x-direction. This observation is verified by the measured values of shock angle versus distance given in Figures 27, 28 and 29.

For hydrogen and oxygen next to helium, $\Theta'$ is considerably less than $\Theta$, and thus one would expect the measured shock angle to increase as the distance of the detonation wave from the exit increases.
TABLE VIII

COMPARISON OF SHOCK ANGLES COMPUTED USING STEADY FLOW AND PSEUDO SHOCK TUBE ANALYSES

<table>
<thead>
<tr>
<th>Explosive</th>
<th>Boundary</th>
<th>$V_D$ (ft/sec)</th>
<th>$V_S$ (ft/sec)</th>
<th>$\theta'$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>78% H$_2$ - 22% O$_2$</td>
<td>Air</td>
<td>10800</td>
<td>3340</td>
<td>17.2</td>
<td>17.4</td>
</tr>
<tr>
<td>67% H$_2$ - 33% O$_2$</td>
<td>Air</td>
<td>9250</td>
<td>3285</td>
<td>19.6</td>
<td>20.0</td>
</tr>
<tr>
<td>50% H$_2$ - 50% O$_2$</td>
<td>Air</td>
<td>7550</td>
<td>3085</td>
<td>22.4</td>
<td>23.3</td>
</tr>
<tr>
<td>35% H$_2$ - 65% O$_2$</td>
<td>Air</td>
<td>6350</td>
<td>2880</td>
<td>24.4</td>
<td>25.7</td>
</tr>
<tr>
<td>78% H$_2$ - 22% O$_2$</td>
<td>Helium</td>
<td>10800</td>
<td>7540</td>
<td>34.9</td>
<td>43.6</td>
</tr>
<tr>
<td>67% H$_2$ - 33% O$_2$</td>
<td>Helium</td>
<td>9250</td>
<td>7290</td>
<td>38.3</td>
<td>51.5</td>
</tr>
<tr>
<td>50% H$_2$ - 50% O$_2$</td>
<td>Helium</td>
<td>7550</td>
<td>6890</td>
<td>42.2</td>
<td>-</td>
</tr>
<tr>
<td>35% H$_2$ - 65% O$_2$</td>
<td>Helium</td>
<td>6350</td>
<td>6390</td>
<td>45.2</td>
<td>-</td>
</tr>
</tbody>
</table>

This is seen to be the case as shown in Figures 45 and 46, although the detonation wave curvature also contributes to this effect (see Section 5.3.3). Thus the unsteady aspects of the experiments could only have influenced the measured shock and interface angles when the detonation wave was reasonably close to the tube exit.

The lateral expansion process is not one-dimensional as assumed above; rather, the shock formed at point E and propagated into the boundary will be cylindrical in nature, and thus decay more rapidly than the one-dimensional wave assumed. However, the reasonableness of the shock-tube analysis used is illustrated in Figures 56 and 57. The circled points are measured shock wave positions, while the remainder of the graph is the result of the theoretical calculations.
Figure 56. Distance vs. Time Plot of Conditions at Tube Exit for 67% Hydrogen-33% Oxygen Mixture with Air Boundary.

Figure 57. Distance vs. Time Plot of Conditions at Tube Exit for 67% Hydrogen-33% Oxygen Mixture with Helium Boundary.
5.3.3 Effect of Wave Curvature

In the schlieren photographs shown previously (Figures 53 and 54), it was observed that the detonation wave and the shock wave in the boundary gas are curved. Ignoring the cause for the present, the implication of curvature is that the flow behind the wave must be rotational, rather than irrotational as tacitly assumed in the idealized model analysis given in Section 5.3.1. The entropy gradient normal to the streamlines behind a curved shock is a function of the radius of curvature of the streamlines, the velocity gradient normal to the streamlines, and the stagnation enthalpy gradient normal to the streamlines. Generally, the smaller the shock curvature, the less the flow behind it will depart from irrotationality.

In the present analysis it did not prove necessary to include the rotational features of the flow behind the detonation and shock. Initially it was ignored because of the observation that the waves associated with the boundary interaction are not significantly curved (save for the detached shock, which is an intractable case anyway). Subsequently, however, it was found the ideal model results were entirely satisfactory in providing the proper description of the interaction process and in determining the significant parameters in the solution. This further justified the assumption made.

The effect of the detonation wave curvature on the predicted shock angle can be approximated in the following manner. An enlargement of the area about the triple point "0" for the case of curved waves appears in Figure 58. In this figure it is assumed that close to the triple point, small segments of curved waves can again be represented
Figure 58. Schematic of Triple Point with Curved Waves Represented by Small, Straight Line Segments.
as straight lines. The detonation wave is at an angle $\alpha$ with the vertical. For small values of $\alpha$ as a first approximation, the velocity normal to the detonation wave may be taken as the Chapman-Jouguet velocity, which will greatly facilitate the calculations. This assumption will cause slight errors in predicting the flow parameters behind the detonation wave, but will not alter the general result. In this case, to fix the detonation wave in space, the gases must enter the wave front at the velocity $V_D / \cos \alpha$. The resulting velocity vectors fore and aft of the detonation wave are sketched in Figure 58. The shock and interface angle symbols are $\Theta''$ and $\delta''$, respectively, to distinguish them from the idealized model values. Since $V_{T_2} = V_{T_1}$ and $V_2 < V_D$, the gas leaving the C-J plane must be turned upward relative to the direction at which it entered the wave. This in turn means $\Theta''$ must be greater than the $\Theta$ which was calculated for the case, $\alpha = 0$. For flow situations in which $\Theta$ was calculated to be less than, but close to $\Theta_{\text{max}}$, the consideration of $\alpha > 0$ could conceivably lead to detached shock situations.

For the two examples which follow, $\alpha$ was assumed equal to 10°. Measurements of $\alpha$ made from the photographs taken during this study show it to vary between 10° and 20°, right at the jet edge. The cosine of 10° is 0.9542, and thus the assumption that the flow perpendicular to the detonation front travels at the C-J velocity is not greatly in error. Two conditions were considered: 67% hydrogen-33% oxygen next to air, and the same explosive next to helium. The results of calculations for $\Theta''$ and $\delta''$ are given in Table IX.
TABLE IX

COMPARISON OF SHOCK AND INTERFACE ANGLES USING IDEALIZED MODEL AND APPROXIMATE CURVED WAVE ANALYSIS

<table>
<thead>
<tr>
<th>Explosive</th>
<th>Boundary</th>
<th>$\Theta_{\text{max}}$ Deg.</th>
<th>$\delta$ Deg.</th>
<th>$\delta''$ Deg.</th>
<th>$\phi$ Deg.</th>
<th>$\phi''$ Deg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>67% H$_2$ - 33% O$_2$</td>
<td>Air</td>
<td>67.4</td>
<td>14.3</td>
<td>16.4</td>
<td>20</td>
<td>22.3</td>
</tr>
<tr>
<td>67% H$_2$ - 33% O$_2$</td>
<td>Helium</td>
<td>62.7</td>
<td>24.4</td>
<td>27.1</td>
<td>51.5</td>
<td>61</td>
</tr>
</tbody>
</table>

Several observations can be made from the results in Table IX. First, if a method existed for taking the curvature of the detonation wave into account in the theoretical solution, one would undoubtedly find shock angles closer to those measured (see Figures 50, 51 and 52). This is especially true of the results in Figure 51, as it has been shown that a small increase in $\alpha$ causes a large increase in shock angle, for shock angles near the maximum. This same fact explains the discontinuity in wave shape at the triple point, which was noted in Figure 42. Because the curvature of the detonation wave significantly increases the shock angle for the helium boundary case, it is apparent the shock may be detached in situations where the idealized flow model predicts it is not. This is undoubtedly the cause of the local "bulge" in the shock wave in the exposures of Figure 42. The flow field associated with this form of boundary interaction is thus quite complex.

5.3.4 Influence of Diffusion and Mixing Along the Jet Boundary

The curvature of the detonation wave has been noted in the photographic results and its influence discussed in Section 5.3.3. It
is possible that some, if not all, of the detonation wave curvature is the result of dilution of the explosive mixture along the boundary which, in turn, is due to mixing and diffusion effects. It is necessary, therefore, to place an upper limit on the effects of mixing and diffusion between the explosive and the inert gas.

As long as the boundary between the gases is laminar in nature, the effects of mechanical mixing are negligible. Since this was the case throughout most of the experiments, only mass diffusion due to concentration gradients need be considered. Because the temperature and pressure are the same on both sides of the jet boundary, the diffusion process is equimolar. For equimolar binary diffusion, "Fick's Law," i.e., the diffusion equation, applies\(^{(33)}\)

\[
\frac{\partial C}{\partial t} = D_{12} \frac{\partial^2 C}{\partial y^2} \tag{5.16}
\]

where \(C\) is the concentration in molecules/cm\(^2\), \(D_{12}\) is the binary diffusion coefficient in cm\(^2\)/sec., \(t\) is the time, and \(y\) is in the direction of diffusion.

If, in the present case, it is assumed that the explosive mixture and the inert boundary gas are flowing parallel and at the same velocity, \(u_0\), Equation (5.16) may be transformed using \(t = x/u_0\). It has been shown elsewhere by Adamson\(^{(47)}\) that the velocity profile does not greatly influence the concentration gradient. Equation (5.16) thus becomes:

\[
u_0 \frac{\partial C}{\partial x} = D_{12} \frac{\partial^2 C}{\partial y^2} \tag{5.17}
\]
Two cases of diffusion will be considered: case I, hydrogen-oxygen mixture flowing next to air; case II, hydrogen-oxygen mixture flowing next to helium. Only one explosive mixture, 78% hydrogen-22% oxygen, will be employed in the calculations, as it should be representative of the results for all mixtures. Also, the high hydrogen content should help exaggerate the rapid diffusing qualities of hydrogen, thereby giving conservative results.

For case I it can be noted that the explosive mixture and the air boundary have roughly the same oxygen content, viz., 21% and 22% by volume, respectively. Thus, as a first approximation this case can be reduced to a binary diffusion process between the hydrogen of the explosive and the nitrogen of the air, thereby implying that the oxygen is merely a "solvent." The binary diffusion coefficient for hydrogen and nitrogen was calculated from the following equation, taken from Reference 33, page 539:

$$D_{12} = \frac{0.002629 \sqrt{\frac{T^3(m_1+m_2)}{2m_1m_2}}}{p \sigma_{12}^2 \varrho_{12}}$$

(5.18)

where $m_1$ and $m_2$ are the molecular weights of species 1 and 2, respectively; $T$ is the temperature in degrees Kelvin; $p$ is the pressure in atmospheres; $\sigma_{12}$ is a molecular potential parameter defined in Reference 33; and $\varrho_{12}$ is a tabulated integral in Reference 32, derived from the Lennard-Jones potential. Inserting the proper values in Equation (5.18), the binary diffusion coefficient for hydrogen and nitrogen is found to be 0.764 cm$^2$/sec.

For case II, three binary diffusion coefficients were calculated, using Equation (5.18), for the following gas pairs: hydrogen-helium, hydrogen-oxygen, and helium-oxygen. The binary diffusion
coefficients were found to be 1.56, 0.804, and 0.741 cm /sec., respectively. The diffusion process is thus dominated by the hydrogen-helium concentration gradient. Case II was therefore reduced to a binary diffusion problem by assuming the hydrogen-oxygen and helium-oxygen diffusional effects were relatively unimportant.

The boundary conditions necessary to solve Equation (5.17) can now be specified. It is desired to calculate the concentration profile of the hydrogen in both cases mentioned above. The hydrogen concentration will be taken as unity in the undiluted mixture of 78% hydrogen-22% oxygen. The boundary conditions thus are:

\[ C = 1.0 \text{ and } \frac{\partial C}{\partial y} = 0 \text{ at } y = +\infty \]
\[ C = 0 \text{ and } \frac{\partial C}{\partial y} = 0 \text{ at } y = -\infty \]
\[ C = 1.0 \text{ at } x = 0 \text{ and } 0 < y < +\infty \]
\[ C = 0 \text{ at } x = 0 \text{ and } -\infty < y < 0 \] (5.19)

The solution to Equation (5.17) which satisfies the boundary conditions above is well known,

\[ C = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\sqrt{u_o}}{2 \sqrt{xD_{12}}} \right) \right] \]

where

\[ \text{erf} \left( \frac{\sqrt{u_o}}{2 \sqrt{xD_{12}}} \right) = \frac{2}{\sqrt{\pi}} \int_0^{\frac{\sqrt{u_o}}{2 \sqrt{xD_{12}}}} e^{-\lambda^2} d\lambda \] (5.20)

The velocity, \( u_o \), is taken as the arithmetic mean velocity of the explosive and boundary gas. Using the jet velocities given in Table V and the helium velocity of 10 ft/sec., \( u_o \) was calculated to be 11.4 ft/sec. for case I, and 16.4 ft/sec. for Case II.
The computed concentration profiles at the dimensionless distance $x' = 2.0$ ($x = 2.0 \text{ in.}$), are shown in Figures 59 and 60. Satisfactory agreement was found between the computed concentration profiles and the mixing zone widths observed in the schlieren photographs. In Figure 59 the concentration profile for $v_o = 5.7 \text{ ft/sec.}$, half of the arithmetic mean velocity, is also shown to demonstrate the relatively small effect of velocity.

Dilution of an explosive usually results in a lower detonation velocity. Diluting hydrogen-oxygen along the jet boundary with air, for example, would thus result in curvature of the detonation wave front. Comparison of the results in Figure 59 with the schlieren photographs given previously of the interaction process shows the curvature of the detonation wave to extend farther in toward the wall than does the diffusion zone. Hence diffusion alone does not explain the front curvature.

The case of diluting hydrogen and oxygen with helium, due to diffusion effects along the jet boundary, constitutes an even more convincing argument against the wave curvature being due entirely to diffusion effects. Data given in Table I show that as a stoichiometric hydrogen-oxygen mixture is diluted with increasing amounts of helium, the detonation velocity increases. If diffusion were the only effect influencing the wave curvature, a detonation wave next to a helium boundary should curve forward, rather than backward.
Figure 59. Hydrogen Concentration vs. Dimensionless Width at $x' = 2.0$ for 78% Hydrogen-22% Oxygen next to Air.

Figure 60. Hydrogen Concentration vs. Dimensionless Width at $x' = 2.0$ for 78% Hydrogen-22% Oxygen Next to Helium.
VI. DISCUSSION OF RESULTS

6.1 Phenomenological Description of Boundary Interaction Process

On the basis of the results acquired thus far, it is possible to construct a physical description of the detonation wave - boundary interaction. Treating the detonation as a finite thickness wave, rather than as a discontinuity, the interaction process is represented in Figure 61.

Once the detonation is considered as having some thickness due to the existence of a finite ignition delay, the reason for curvature of the wave front becomes apparent. The gas that enters the detonation wave front undergoes a considerable pressure rise to something in the order of twice the C-J pressure, according to the "von Neumann spike theory" (24). For situations in which the boundary is compressible under these pressures, the gas that flows into the wave front enters a region having lateral pressure relief. The streamlines of this flow will thus bend toward the gas boundary; the net effect will be that both the temperature and pressure of this explosive will decrease from the values it had directly behind the shock front. Compensating the temperature decay due to expansion will be the temperature rise due to chemical energy release. However, the lower pressures mean the reaction will not go as far toward completion as it would have, had "perfect" confinement been provided. There is therefore less heat available to drive the shock, and the detonation wave will bend backward.

The flow behind the shock front is subsonic, but in turning it accelerates and at some point reaches Mach 1.0. This is denoted as
the "sonic line." The departure of the detonation wave from one-dimensionality results in an extremely complex flow field in which subsonic, sonic and supersonic regions lie in juxtaposition. The rigorous solution to the flow system illustrated in Figure 6.1 thus requires the simultaneous solution of flows described by elliptic, parabolic and hyperbolic equations, and the use of a proper chemical description of the reaction zone.

The experimental portion of this study demonstrated that some combinations of gaseous explosive and boundary result in a low enough temperature and pressure at the edge of the detonation wave that the chemical reaction is quenched, or at least limited enough that stable detonation can no longer be maintained. In these cases this effect has been seen to propagate from the boundary toward the wave center until the entire wave is extinguished. The best gaseous boundary or container, therefore, is that which results in the highest value of pressure behind the oblique shock in the boundary, since it is this pressure to which the gas entering the detonation wave front seeks to expand. The parameter which determines this pressure is the acoustic impedance, as will be demonstrated in the next section.

The oblique shock wave in the boundary is drawn slightly curved in Figure 6.1. This must be so, irrespective of the conditions in the detonation reaction zone. The shift in the coordinate system made in order to stabilize the detonation wave in space overlooks the fact that only a finite amount of energy is available in the detonation process. Therefore, in the actual case, the shock must decay to a Mach line at some distance away from the edge of the detonation wave.
Figure 61. Schematic of Actual Detonation Wave-Boundary Interaction.
6.2 Generalization of Analysis Through Use of Acoustic Impedance

It has been reasoned in the previous section that the higher the pressure behind the oblique shock in the boundary, the better the confinement properties of the boundary gas. In an attempt to determine what parameter of the boundary interaction problem best correlates the oblique shock pressure ratio, a large number of calculations were performed for a stoichiometric hydrogen-oxygen mixture next to various boundary gases. These results are plotted in Figure 62.

As the velocity of sound of the boundary decreases, relative to that of the explosive mixture, the pressure ratio across the oblique shock increases. The pressure ratio, \( p_{I_2}/p_{I_1} \), has an upper limit, however, as it cannot exceed the pressure ratio across the detonation wave. The pressure ratio also has a lower limit, corresponding to the detachment of the shock in the boundary. Thus, according to these results, detachment of the shock is an indication that the explosive is poorly confined by the boundary gas.

The oblique shock angle, \( \theta \), is plotted as a function of the velocity of sound ratio, \( a_{E_1}/a_{I_1} \), in Figure 63. As the velocity of sound of the boundary gas decreases relative to that of the explosive, the shock angle decreases. Thus the best gaseous container gives not only the highest shock pressure ratio, \( p_{I_2}/p_{I_1} \), but also the smallest shock angle, \( \theta \). Note that the shock angle versus velocity of sound ratio is virtually a straight line on a log-log plot. The upper limit on this curve is determined by the detachment angle for a given Mach number.

For a higher Mach number of detonation, such as associated with acetylene or ether, the curves in Figure 62 shift to higher values
Figure 62. Pressure Ratio Across Oblique shock vs. Velocity of Sound Ratio for Stoichiometric Hydrogen-Oxygen and Various Gas Boundaries.
Figure 63. Oblique Shock Angle vs. Velocity of Sound Ratio for Stoichiometric Hydrogen-Oxygen and Various Gas Boundaries.
of pressure ratios, but retain their same character. In this respect the results obtained are a function of the explosive.

The trial and error nature of the solution to the boundary interaction process makes it difficult to isolate a particular parameter as the correlating factor. Therefore, the equations necessary for the idealized model solution will be simplified by specializing them all to large values of the Mach numbers, \( M_1 \) and \( M_{E_1} \).*

For oblique shocks, one knows (46):

\[
M_1^2 \sin^2 \theta - 1 = \frac{\gamma + 1}{2} \frac{M_1^2 \sin \theta \sin \delta}{\cos(\theta - \delta)} \tag{6.1}
\]

For small values of \( \theta \) (in which case \( \delta \) is quite small) and large values of \( M_1 \), Equation (6.1) reduces to:

\[
M_1^2 \sin^2 \theta - 1 \approx \frac{\gamma + 1}{2} M_1^2 \tan \theta \delta \tag{6.2}
\]

If \( M_1 \) is very large, then \( \theta \ll 1 \), but \( M_1 \theta \gg 1.0 \). Therefore, Equation (6.2) becomes

\[
\theta \approx \frac{\gamma + 1}{2} \delta \tag{6.3}
\]

Again, for small values of \( \theta \) and \( M_2 > 1.0 \), \( \theta \approx \mu \), where \( \mu \) is the Mach angle, i.e.:

\[
\mu = \arcsin \frac{1}{M_1} \tag{6.4}
\]

Thus

\[
\tan \theta \approx \tan \mu = \frac{1}{\sqrt{M_1^2 - 1}} \tag{6.5}
\]

* This method of analysis was suggested to the author by Professor T. C. Adamson, Jr.
Equation (6.2) can therefore be written as:

\[ M_1^2 \sin^2 \theta - 1 \cong \frac{\gamma + 1}{2} M_1^2 \frac{5}{\sqrt{M_1^2 - 1}} \]  \hspace{1cm} (6.6)

An expression for the pressure ratio across an oblique shock appears in Equation (5.12) as:

\[ \frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma + 1} (M_1^2 \sin^2 \theta - 1) \]  \hspace{1cm} (6.7)

Substituting Equation (6.6) into (6.7), and noting \( M_1^2 \gg 1.0 \), one finds:

\[ \frac{P_2}{P_1} \cong 1 + \gamma M_1^2 \]  \hspace{1cm} (6.8)

For a Prandtl-Meyer expansion the relation between the angle \( \gamma \) and the local Mach number is (46):

\[ \gamma(M) = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1} \left( \sqrt{\frac{\gamma - 1}{\gamma + 1} (M^2 - 1)} \right) - \tan^{-1} \left( \sqrt{M^2 - 1} \right) \]  \hspace{1cm} (6.9)

\( \tan^{-1} x \) can be expanded into the following form for the case in which \( x^2 < 1.0 \):

\[ \tan^{-1} x = x - \frac{1}{3} x^3 + \frac{1}{5} x^5 + \ldots \]  \hspace{1cm} (6.10)

This expansion is applicable in the present case for \( 1.0 < M < 1.414 \) or \( 0 \leq \gamma < 12^\circ \), for \( \gamma_{E_2} = 1.2 \). Letting \( m^2 = M^2 - 1 \) and neglecting terms of higher powers than \( m^3 \), Equation (6.9) becomes:

\[ \gamma(M) \cong 2 \left( \frac{m^3}{\gamma + 1} \right) \]  \hspace{1cm} (6.11)

The static pressure behind the detonation wave is \( P_{E_2} \), that at the end of the Prandtl-Meyer expansion is \( P_{E_3} \). The total to static pressure ratio of each point may be written as:

\[ \frac{P_{Et}}{P_{E2}} = \left( 1 + \frac{\gamma_{E_2} - 1}{2} \right) \frac{\gamma_{E_2}}{1 - \gamma_{E_2}} \]  \hspace{1cm} (6.12)
\[
\left(\frac{PE_2}{PE_3}\right)_3 = (1 + \frac{\gamma_{E_3} - 1}{2} M_{E_3}^2)^{1 - \gamma_{E_3}} \quad (6.13)
\]

Since the expansion process is isentropic, the stagnation pressure is constant. Assuming \( \gamma_{E_2} = \gamma_{E_3} \), the static pressure ratio, \( \frac{PE_2}{PE_3} \), can be found from Equations (6.12) and (6.13).

\[
\frac{PE_2}{PE_3} = \left[ \frac{2 + (\gamma_{E_2} - 1) M_{E_3}^2}{1 + \gamma_{E_2}} \right]^{\frac{\gamma_{E_2}}{\gamma_{E_2} - 1}} \quad (6.14)
\]

Substituting \( m_{E_3}^2 = M_{E_3}^2 - 1 \) into Equation (6.14) leads to:

\[
\frac{PE_2}{PE_3} = \left[ 1 + \left(\frac{\gamma_{E_2} - 1}{\gamma_{E_2} + 1} m_{E_3}^2\right) \right]^{\frac{\gamma_{E_2}}{\gamma_{E_2} - 1}} \quad (6.15)
\]

The binomial series is:

\[
(a + x)^n = a^n + na^{n-1}x + \frac{n(n-1)}{2!} a^{n-2}x^2 + \ldots
\]

for \( x^2 < a^2 \) \quad (6.16)

Applying the binomial series to Equation (6.15) and ignoring terms having powers larger than two, the following is obtained:

\[
\frac{PE_2}{PE_3} \approx 1 + \frac{\gamma_{E_2}}{\gamma_{E_2} + 1} m_{E_3}^2 \quad (6.17)
\]

Since the static pressure ratio across a detonation wave is

\[
\frac{PE_2}{PE_1} = \frac{1 + \gamma_{E_1} M_{E_1}^2}{1 + \gamma_{E_2}} \quad (6.18)
\]

the pressure ratio \( \frac{PE_3}{PE_1} \) is found, with the use of Equation (6.17), to be:

\[
\frac{PE_3}{PE_1} \approx \frac{1 + \gamma_{E_1} M_{E_1}^2}{1 + \gamma_{E_2} M_{E_3}^2} \quad (6.19)
\]
If Equations (6.3), (6.8), (6.11) and (6.19) are combined, along with
the condition
\[
\frac{P_{I2}}{P_{I1}} = \frac{P_{E3}}{P_{E1}},
\]
a relation for \(\frac{P_{I2}}{P_{I1}}\) may be found directly in terms of the initial
conditions and \(\gamma_{E2}\).

\[
\frac{P_{I2}}{P_{I1}} \propto \frac{1 + \gamma_{E1} M_{E1}^2}{1 + \gamma_{E2}} \left\{ 1 + \frac{3}{2} \frac{\gamma_{E2}}{\gamma_{I1} M_{I1}^2} \left( \frac{P_{I2}}{P_{I1}} - 1 \right)^{2/3} \right\}^{2/3} \tag{6.20}
\]

The acoustic impedance of any material is equal to its density
times its speed of sound, \(\rho a\). Although the acoustic impedance generally
only has meaning in acoustic problems*, it occurs in Equation (6.20) and
allows a simple description of the confinement properties of a gas. The
acoustic impedance for a gas can be rewritten with the use of
\(a^2 = \gamma_p / \rho\). The acoustic impedance, defined as \(Z\), thus becomes

\[
Z = \frac{\gamma_p}{a} \tag{6.21}
\]

In Equation (6.20), the terms \(\gamma_{E1} M_{E1}^2\) and \(\gamma_{I1} M_{I1}^2\) occur. Both of
these can be rewritten using the acoustic impedance

\[
\gamma_{E1} M_{E1}^2 = \frac{V_{E1}^2}{P_{E1} a_{E1}} \tag{6.22}
\]

\[
\gamma_{I1} M_{I1}^2 = \frac{V_{I1}}{P_{I1}} \tag{6.23}
\]

* An acoustic wave traveling from one medium into another is transmitted
with maximum efficiency when the acoustic impedances of the two media
are identical. For the case in which the acoustic wave is perpendi-

cular to the interface between the media, no reflected wave will occur
when the media are acoustically "matched;" i.e., when the acoustic
impedances are equal.
For a given explosive and initial pressure, Equations (6.22) and (6.23) become:

\[
\gamma_{E1} \cdot M_{E1}^2 = A \cdot Z_{E1} \tag{6.24}
\]

\[
\gamma_{I1} \cdot M_{I1} = B \cdot Z_{I1} \tag{6.25}
\]

where \( A \equiv \frac{V_D^2}{V_{E1}^2 V_{E1}} \) is a constant, and \( B \equiv \frac{V_D}{P_{I1}} \) is a constant. Equation (6.20) can therefore be written in the following manner:

\[
\frac{P_{I2}}{P_{I1}} \approx \frac{1 + A \cdot Z_{E1}}{1 + \gamma_{E2} \left\{ 1 + \frac{3}{2} \left( \frac{\gamma_{E2} + 1}{\gamma_{I1} \cdot Z_{I1}} \right) \left( \frac{P_{I2}}{P_{I1}} - 1 \right) \right\}^{2/3}} \tag{6.26}
\]

It is apparent from Equation (6.26) that the boundary interaction solution is no longer a function of either \( \gamma_{I1} \) or \( \gamma_{E1} \). For a given explosive at a specified initial pressure, the solution depends upon only the acoustic impedances of the boundary and explosive gases (with the exception of \( \gamma_{E2} \), which is virtually constant at 1.20 for all gaseous explosives). The data plotted in Figure 62 are replotted in Figure 64 as a function of the ratio \( Z_{I1}/Z_{E1} \), along with the approximate solution, Equation (6.26). The idealized model results show that, as predicted by the approximate solution, the influence of \( \gamma_{I1} \) and \( \gamma_{E1} \) is essentially included in the acoustic impedances. Thus, the confinement properties of a gaseous boundary are simply expressed in terms of the acoustic impedance; the higher the value of \( Z_{I1}/Z_{E1} \), the better the gas is as a "container."

6.3 Application of Results to Liquid Explosives

According to the discussion of Section 6.1 and 6.2, the best confinement is provided by the boundary material which causes the
Figure 64. Pressure Ratio Across Oblique Shock vs. Acoustic Impedance Ratio for Stoichiometric Hydrogen-Oxygen and Various Gas Boundaries.
highest static pressure to occur behind the oblique shock in the boundary. The highest possible pressure behind the shock occurs for the case in which the acoustic impedance ratio of the boundary material to the explosive is a maximum. These conclusions, although made on the basis of gaseous explosive research, should be valid for any material if the analogy upon which this study is based is valid. The experiments performed by Campbell, Malin and Holland (23) using glass tubes lined with thin metal foils allow the comparison between condensed and gaseous explosive results to be made quite readily.

It will be recalled (see Introduction) that Campbell, et al. found that the failure diameter associated with a glass tube was considerably larger than that for an aluminum tube, which in turn was larger than that for a brass tube (nitromethane was used as the explosive). Following these results, further experiments led these same authors to conclude:

"Additional experiments suggested that for a given foil thickness, steel was more effective a confining material than aluminum, and tungsten was more effective than steel. The effectiveness was evidenced by the minimum diameter glass tube in which propagation [of the detonation wave] was made possible by the presence of the foil."

The ratio of the acoustic impedance of various wall materials to that of nitromethane is given in Table X. The values necessary to compute the acoustic impedances of the wall materials were taken from Reference 48; those required for nitromethane from Reference 40.

It is evident in Table X that tungsten, the material rated "best" as a container by Campbell and associates, provides the highest ratio of acoustic impedance. In addition, the order of decreasing acoustic
TABLE X
RATIO OF ACOUSTIC IMPEDANCE OF BOUNDARY MATERIAL TO THAT OF NITROMETHANE

\[ a_{E_1} = 1331.5 \text{ m/sec.} \quad c_{E_1} = 1.13 \text{ gm/cm}^3 \]

<table>
<thead>
<tr>
<th>Boundary</th>
<th>( \rho_{I_1} ) gm/cm³</th>
<th>( a_{I_1} ) m/sec.</th>
<th>( Z_{I_1}/Z_{E_1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tungsten</td>
<td>18.6</td>
<td>4300.</td>
<td>53.3</td>
</tr>
<tr>
<td>Platinum</td>
<td>21.37</td>
<td>2690.</td>
<td>38.2</td>
</tr>
<tr>
<td>Steel</td>
<td>7.83</td>
<td>5000.</td>
<td>26.1</td>
</tr>
<tr>
<td>Brass</td>
<td>8.56</td>
<td>3500.</td>
<td>20.0</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2.7</td>
<td>5100.</td>
<td>9.2</td>
</tr>
<tr>
<td>Glass</td>
<td>2.6</td>
<td>5000.</td>
<td>8.6 to 10.4</td>
</tr>
</tbody>
</table>

Impedance ratio is identical to the order of increasing failure diameter. These results, although not entirely conclusive, suggest strongly that the confinement provided a condensed explosive by a solid material is dependent, as in the case of gases, upon the hydrodynamic properties of the materials involved, rather than upon physical strength.

Campbell et al. also noted in their work that if a glass tube were lined for several inches with one-mil thick platinum foil, for the next several inches with one-half-mil foil, and for several more inches with one-fifth-mil foil, a detonation wave in nitromethane would propagate through the first two regions but fail in that lined with only one-fifth-mil of platinum. This result is also in accord with the conclusion that confinement is primarily a hydrodynamic phenomenon. All that a container need do, regardless of how thin, is to result in a sufficiently high pressure behind the oblique shock in the boundary that the detonation wave will continue to propagate. However, if the foil is thin enough that the shock passing through it can reflect off the next interface.
and still return in time to encounter the reaction zone, the foil's characteristics may not be the decisive factor. The shock may be reflected as a rarefaction which, if strong enough, will quench the detonation.

Cotter measured the reaction zone thickness in nitromethane to be approximately .007 in.\(^{(49)}\) Thus, assuming for instance that the shock angle in platinum is 30° and the reflected disturbance returns at an angle of 30°, the foil would only have to be about .002 in. (2 mil) thick to appear infinitely thick to the explosive.

Knowing the shock angle θ, the angle of the reflected disturbance, and the foil thickness would thus provide an indirect means of measuring the reaction zone thickness of a condensed explosive.

It would be interesting to study the results obtained by backing one-fifth-mil thick foil of platinum with a thin foil of tungsten, rather than with glass alone, as was done by Campbell, et al. Changing the impedance match at an interface controls whether the wave reflected is a shock or rarefaction. Forcing the disturbance reflected to be a shock might demonstrate the wave failed due to the type of wave reflected from the platinum-glass interface.
VII. CONCLUSIONS

The experimental portion of this study demonstrated that the confinement provided a gaseous explosive by a compressible boundary is sufficient to maintain a stable Chapman-Jouguet detonation in the explosive. The experiments which established this were two-dimensional in nature. Hydrogen-oxygen mixtures were used extensively as the explosive, and to a lesser extent, methane, ethane and propane-oxygen mixtures. Air and helium were utilized as the compressible, inert boundary gases.

Under compressible boundary confinement the detonation wave was found to depart from the one-dimensional configuration normally observed in detonation or shock tubes. Gaseous detonation waves, imperfectly confined, become curved and in some cases are quenched, the quenching process beginning at the compressible boundary and propagating in toward the wave center. Detonations formed in explosive mixtures near either the lean or rich limits of detonation were the most readily quenched. The bending and quenching of the detonation waves were determined to be a consequence of the hydrodynamic aspects of the detonation-boundary interaction process.

The interaction of a gaseous detonation wave with an inert, gaseous boundary causes a lateral shock wave to exist in the boundary. It was found this shock wave could be either a weak oblique shock or a detached shock, depending on the Mach number of detonation and the thermodynamic properties of the explosive and boundary. An idealized, steady-flow model, in which the shock and detonation waves were treated as vanishingly thin discontinuities, was utilized to analyze the interaction.
process. Theoretical predictions of shock and interface angles were found, which are in satisfactory agreement with values measured from schlieren photographs of the experiments. The theory also correctly predicted the detached shock phenomenon observed experimentally for several mixtures of hydrogen and oxygen next to a helium boundary. This, and the agreement between measured and predicted shock angles, served to verify the essential accuracy of the theoretical model.

Understanding of the detonation-boundary interaction process led to the characterization of "confinement" as a hydrodynamic, rather than a physical strength, property. Confinement of a detonating explosive by a compressible boundary was concluded to be entirely a result of the pressure behind the lateral shock in the boundary. The pressure behind the shock, for a given explosive, was found to be a function of the acoustic impedance ratio of the boundary to the undetonated explosive; the higher this ratio of the acoustic impedance, the higher the pressure behind the lateral shock.

Following the disclosure of the importance of the acoustic impedance ratio in the gaseous case, it was employed as a correlating parameter for condensed explosive experimental results. The outcome suggests strongly that confinement of a condensed explosive detonation wave is also dependent upon the relative hydrodynamic properties of the boundary material and the undetonated explosive. Experimental data available to check this premise, though limited in number, confirm that the "best" container is that which provides the highest ratio of the acoustic impedance of the boundary to the undetonated explosive - the same conclusion as drawn on the basis of gaseous results. This observation strengthens the belief
that a gaseous explosive confined by a gaseous boundary is a satisfactory
and useful analogue for the study of condensed explosive phenomena.

The results of this research suggest the continued use of gase-
ous media for investigation into the failure of detonation waves, con-
densed or otherwise, and the relationship between the container and ex-
plosive.

Surely if a rigorous attempt is to be made at solving simul-
taneously the chemical kinetic and hydrodynamic relationships describing
the interaction of a detonation wave with a compressible boundary, it
will be greatly facilitated by the use of ideal gas assumptions and equa-
tions.

The successful propagation of a stable detonation wave through
a linear region having a gaseous boundary suggests performing the same
experiment in a circular track.* In this case, the stability of the
detonation would be enhanced by the pressure gradient due to centrifugal
forces. Since the release of chemical energy is extremely rapid and
essentially complete in detonation, this concept might prove useful in
the design of extremely small and efficient combustion chambers. Just
prior to the conclusion of this study, a paper was published on this sub-
ject in the Russian literature.(50) This article describes experiments
in which detonation was stabilized in a circular combustion chamber by
continually replenishing the gaseous explosive. Further attention to
this concept of combustion appears justified.

* This observation was made by Professor Richard B. Morrison.
APPENDIX A

SAMPLE CALCULATION OF SHOCK AND INTERFACE ANGLE
A sample computation will be performed for the case of a stoichiometric mixture of hydrogen and oxygen next to a helium boundary. For these conditions the following preliminary information applies:

\[
\begin{align*}
T_1 &= 535^\circ R \\
M_{I_1} &= 4.0 \\
M_{E_1} &= 11.9 \\
M_{I_1} &= 2.78 \\
\gamma_{I_1} &= 1.66 \\
\gamma_{E_1} &= 1.40 \\
V_D &= 9250 \text{ ft/sec.}
\end{align*}
\]

From Figure 49, \(\frac{v_{E_1}}{v_{E_2}}\) is found to be 1.777.

Using Equation (5.7), one finds:

\[
\frac{p_{E_2}}{p_{E_1}} = 1 + \gamma_{E_1} M_{E_1}^2 \left(1 - \frac{v_{E_2}}{v_{E_1}}\right) = 17.65
\]

Equation (5.8) is used to find:

\[
\gamma_{E_2} = \frac{p_{E_1}}{p_{E_2}} \left(1 + \gamma_{E_1} M_{E_1}^2\right) - 1 = 1.218
\]

Equation ((5.5)) is then utilized to find \(\Theta_{\text{max}}\):

\[
\Theta_{\text{max}} = 62.74^\circ
\]

For the first trial, \(\Theta\) will be assumed equal to \(\Theta_{\text{max}}\). Equation (5.4) thus allows the corresponding interface angle, \(\delta\), to be computed. From this one finds

\[
\delta = 26.93^\circ = \nu
\]

Using Equation (5.5) or Reference 47:

\[
M_{E_3} = 1.89
\]
Equations (5.9) and (5.10) are used to solve for \( \left( \frac{PE}{PE_t} \right)_3 \) and \( \left( \frac{PE}{PE_t} \right)_2 \), respectively.

\[
\left( \frac{PE}{PE_t} \right)_3 = 0.1592 \quad \quad \left( \frac{PE}{PE_t} \right)_2 = 0.5606
\]

Equation (5.11) shows

\[
\frac{PE_2}{PE_1} = \left( \frac{PE_2}{PE_1} \right)_3 \left( \frac{PE}{PE_t} \right)_2 = \frac{.1592 \times 17.65}{.5606} = 5.0
\]

The pressure ratio across the oblique shock is found from Equation (5.12):

\[
\frac{P_{l2}}{P_{l1}} = 7.36
\]

Thus,

\[
\frac{PE_3}{PE_1} < \frac{P_{l2}}{P_{l1}}
\]

Therefore, assume a smaller value for the shock angle; assume \( \theta = 51.5^\circ \).

For this value of \( \theta \), \( \beta = 24.37^\circ \). Repeating the steps listed previously, one finds

\[
M_{E3} = 1.814
\]

\[
\left( \frac{PE}{PE_t} \right)_3 = 0.1803
\]

\[
\left( \frac{PE}{PE_t} \right)_2 = 0.5606
\]

\[
\frac{PE_2}{PE_1} = 17.65
\]
Equation (5.11) gives

\[
\frac{P_{E_3}}{P_{E_1}} = \frac{0.1803 \times 17.65}{0.5606} = 5.66
\]

Equation (5.12) gives

\[
\frac{P_{I_2}}{P_{I_1}} = 5.66
\]

Therefore,

\[
\theta = 51.5^\circ
\]

\[
\phi = 24.4^\circ
\]


41. Private Communication from Dr. Andrej Mašek, Physical Chemistry Division, U. S. Naval Ordnance Laboratory, White Oak, Maryland.


