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Technical Notes

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Virtual Origin of Incompressible and Supersonic Turbulent Bluff-Body Wakes

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Introduction

BLUFF-BODY turbulent wakes arise in many practical applications; they are widely used, for example, as flameholders to stabilize combustion processes under both subsonic and supersonic flow conditions. These include strut-type fuel injectors of various types found in scramjet engines, where the additional complication of fuel injection into the near-field recirculation zone immediately behind the bluff body can also be important. In practice, scaling laws are often used to analyze the wakes produced by such bluff bodies.

For incompressible and supersonic planar, turbulent, bluff-body wakes,[1,2] the corresponding far-field scaling laws are of the form

$$\delta/\vartheta = c_\delta (x/\vartheta)^{\frac{1}{2}} \tag{1a}$$

$$u_0/U_\infty = c_u (x/\vartheta)^{-\frac{1}{2}} \tag{1b}$$

where x is the downstream location in the wake, $u_0(x) \equiv U_0 - U_\infty$ is the wake centerline velocity deficit, with $U_0(x)$ the centerline mean streamwise velocity, and U_∞ the freestream velocity, $\delta(x)$ is the local wake width, and $\vartheta \equiv D/\rho_\infty U_\infty^2$ is the momentum thickness of the wake, with D the net drag per unit span produced by the wake generator.

The scalings in Eqs. (1a) and (1b) for such turbulent wakes are based on the fact that the drag D is an invariant of the flow. As a consequence, it remains relevant to the momentum transport process within the wake at all downstream distances x, while the relative influence of all other aspects of the wake generator become smaller with increasing downstream distance. The far field of the wake refers to x locations sufficiently large that the drag is the only remaining dynamically relevant aspect of the wake source. The scalings for $u_0(x)$ and $\delta(x)$ are then determined by the downstream distance, the net drag, the fluid density, and the freestream velocity, which leads to the forms in Eqs. (1a) and (1b). In effect, the wake source has been replaced by a singularity that removes momentum from the freestream at the same rate as the actual source. However, as shown schematically in Fig. 1, if the downstream coordinate x is to denote the distance measured from the actual wake source, then the shift x_0 in the position of this equivalent point singularity relative to the actual source must be accounted for. Here we present a simple procedure that allows the value of this "virtual origin" to be determined for incompressible and supersonic turbulent bluff-body wakes.

Subsonic Wake

Figure 1a shows a typical bluff-body wake generated by a body with frontal dimension d. The location of the equivalent point singularity in Fig. 1b relative to the coordinate origin $x = 0$ is the virtual origin, denoted x_0. The wake generated by the actual source can be related to the corresponding ideal wake by taking the shift caused by x_0 into account in the power-law scalings in Eqs. (1a) and (1b) as

$$\delta/\vartheta = c_\delta[(x - x_0)/\vartheta]^{\frac{1}{2}} \tag{2a}$$

$$u_0/U_\infty = c_u[(x - x_0)/\vartheta]^{-\frac{1}{2}} \tag{2b}$$

The virtual origin can thus be obtained from experimental data by extrapolating in a plot of $(\delta/\vartheta)^2$ or $(u_0/U_\infty)^{-2}$ against (x/ϑ), as done in Ref. 1 for incompressible wakes produced by various wake generators and in Ref. 2 for supersonic wakes at $M_\infty = 2$ and 3.

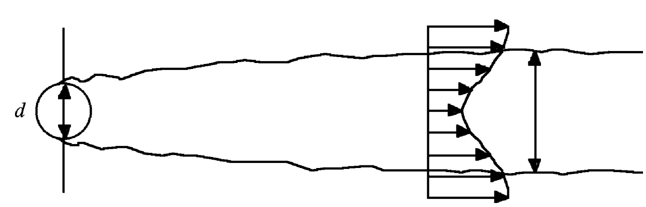
However it is possible to obtain an expression for x_0 by a simple procedure that also reveals various parametric effects on the virtual origin. Define the initial width δ_0 of the wake as indicated in Fig. 1b, namely,

$$\delta_0 \equiv \delta(x = 0) \tag{3}$$

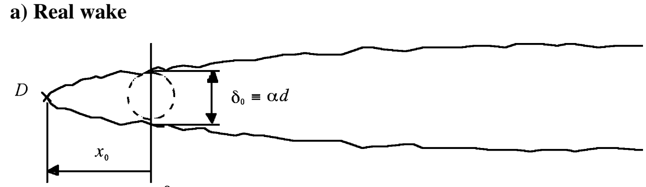
As Fig. 1 suggests, δ_0 should be proportional to the frontal dimension d as

$$\delta_0 \approx \alpha d \tag{4}$$

where the proportionality factor α will depend (weakly) on the body shape via the details of the boundary-layer separation process. Most two-dimensional bluff-body shapes give $\alpha \approx 1.5$, and for essentially



a) Real wake



b) Theoretical (ideal) wake

Fig. 1 Schematic representation of incompressible planar turbulent bluff-body wakes, showing a) actual wake produced by bluff body with frontal scale d and b) ideal far-field wake produced by appropriately located equivalent point singularity to account for initial wake width in scaling laws via the virtual origin x_0.

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all such body shapes $1 \le \alpha < 2$. Substituting Eqs. (3) and (4) in Eq. (2a) then gives

$$x_0 = -\alpha^2 d^2 \Big/ c_\delta^2 \vartheta \tag{5}$$

Introducing the drag coefficient C_D of the wake generator, namely,

$$C_D = D \Big/ \tfrac{1}{2}\rho_\infty U_\infty^2 d \tag{6}$$

and noting that

$$\vartheta = \tfrac{1}{2} C_D d \tag{7}$$

then gives the virtual origin for incompressible, planar, turbulent, bluff-body wakes as

$$x_0/\vartheta = -4\alpha^2 \Big/ c_\delta^2 C_D^2 \tag{8}$$

Note that C_D and α in Eq. (8) are closely related via free streamline theory, and depend on the bluff-body shape. For instance, for a circular cylinder $C_D \approx 1$ at large Reynolds number, and taking $\alpha \approx 1.5$ and using $c_\delta = 0.28$ from Ref. 1, the value of x_0/ϑ from Eq. (8) becomes

$$x_0/\vartheta = -123 \tag{9}$$

which is in the range of experimental values reported for unforced incompressible planar turbulent wakes from circular cylinders in Ref. 1. In Eq. (8) the effect of the bluff-body shape on the virtual origin appears via the drag coefficient C_D, and to a lesser extent through α, and the effect of any changes in the far-field wake appears via the scaling constant c_δ. These effects might account for the wide variation in x_0 values obtained experimentally for various wake generators in Ref. 1.

A corresponding result can be obtained for axisymmetric wakes. In that case the far-field power-law scalings analogous to Eqs. (1a) and (1b) are

$$\delta/\vartheta = c_\delta[(x - x_0)/\vartheta]^{\frac{1}{3}} \tag{10a}$$

$$u_0/U_\infty = c_u[(x - x_0)/\vartheta]^{-\frac{2}{3}} \tag{10b}$$

and the momentum radius ϑ is

$$\vartheta^2 = \tfrac{1}{8} C_D d^2 \tag{11}$$

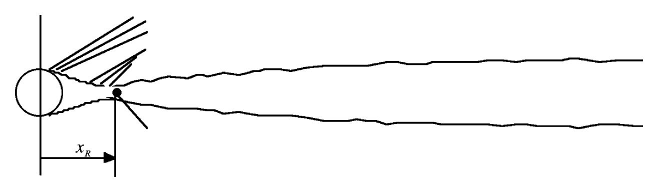
giving the result analogous to that in Eq. (8) for the virtual origin of incompressible axisymmetric wakes as

$$x_0/\vartheta = -8^{\frac{3}{2}}\alpha^3 \Big/ c_\delta^3 C_D^{\frac{3}{2}} \tag{12}$$

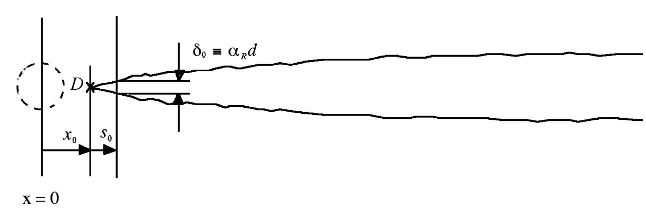
For wakes generated by spheres at large Reynolds number, $C_D \approx 0.5$, and $\alpha \approx 0.5$, and using $c_\delta = 1.14$ from experimental data in Ref. 3 gives $x_0/\vartheta \approx -5.4$, which is in good agreement with the measured value $x_0/\vartheta \approx -2.4$ reported for the sphere wake in Ref. 3.

Supersonic Wakes

To obtain corresponding results for supersonic bluff-body wakes, additional account must be taken of the more complicated geometry of the near-field recompression region in Fig. 2 on the virtual origin x_0 as indicated in Fig. 3. In this case, the initial width δ_0 of the wake is determined from the width of the "neck" formed by the recompression in Fig. 4, which is shifted downstream of the actual wake source by the length x_R of the recompression region. If the coordinate x again measures the distance from the actual wake source,



a) Real wake



b) Ideal wake

Fig. 3 Schematic representation of supersonic planar turbulent bluff-body wakes, showing a) actual wake produced by bluff body and b) ideal far-field wake produced by appropriately located equivalent point singularity to account for near-field base flow recompression region on virtual origin shift x_0 in scaling laws. Compare with corresponding incompressible wake in Fig. 1.

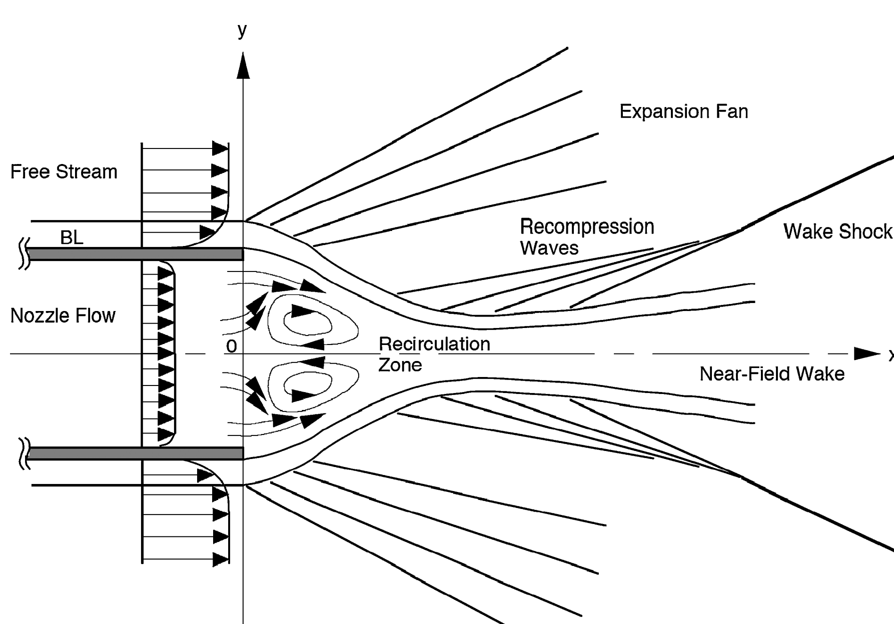


Fig. 2 Schematic indicating major features of the near-field supersonic wake just beyond slot jet exit, showing wake expansion and recompression waves above wake centerline, as well as coupling between slot jet flow rate and base flow structure and size.

then the location x_0 of the equivalent point singularity is given by

$$x_0 = x_R + s_0 \tag{13}$$

where s_0 is the location of the point singularity relative to the neck. In direct analogy with Eq. (5), s_0 for planar supersonic bluff-body wakes is obtained as

$$s_0 = -\alpha_R^2 d^2 \big/ c_\delta^2 \vartheta \tag{14}$$

where α_R is now the proportionality constant between the neck width δ_0 and the size d of the wake generator. To determine the length x_R of the recompression region in Fig. 5, the Prandtl–Meyer turning angle β associated with the expansion fan is obtained by the isentropic pressure ratio (p_0/p_∞), where p_0 is the base pressure and p_∞ the freestream pressure. When the mass injection rate at the wake source is sufficiently small for its effect on δ_0 to be neglected, as is the case in Fig. 4, then

$$\delta_0 \approx d/4 \tag{15}$$

giving $\alpha_R \approx 0.25$. For large injection rates, the neck width δ_0 will increase to accommodate the mass flow, and this will lead to an increase in α_R and thereby to a change in the virtual origin x_0; evidence of this can be seen in Fig. 6, where at large injection rates δ_0 becomes comparable to d (i.e., $\alpha_R \to 1$). In general, the recirculation zone length $x_R \approx (d/2)\cot\beta$. For low rates of mass injection into the base region, the turning angle β in Fig. 4 is roughly 20 deg, giving

$$x_R \approx 1.4d \tag{16}$$

Combining Eqs. (13) with (14) and (16), together with Eq. (7), then gives the virtual origin for the supersonic wake as

$$x_0/\vartheta = (1/C_D)\left(\cot\beta - 4\alpha_R^2 \big/ c_\delta^2 C_D\right) \tag{17}$$

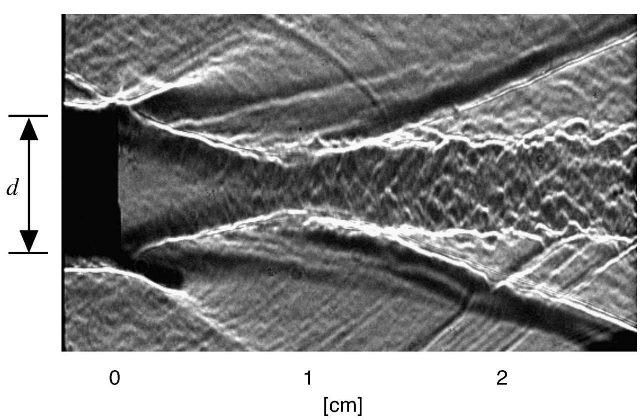


Fig. 4 Shadowgraph images showing base flow region and early self-similar wake region for the Mach 2 wake for zero mass injection rate into the base flow region.

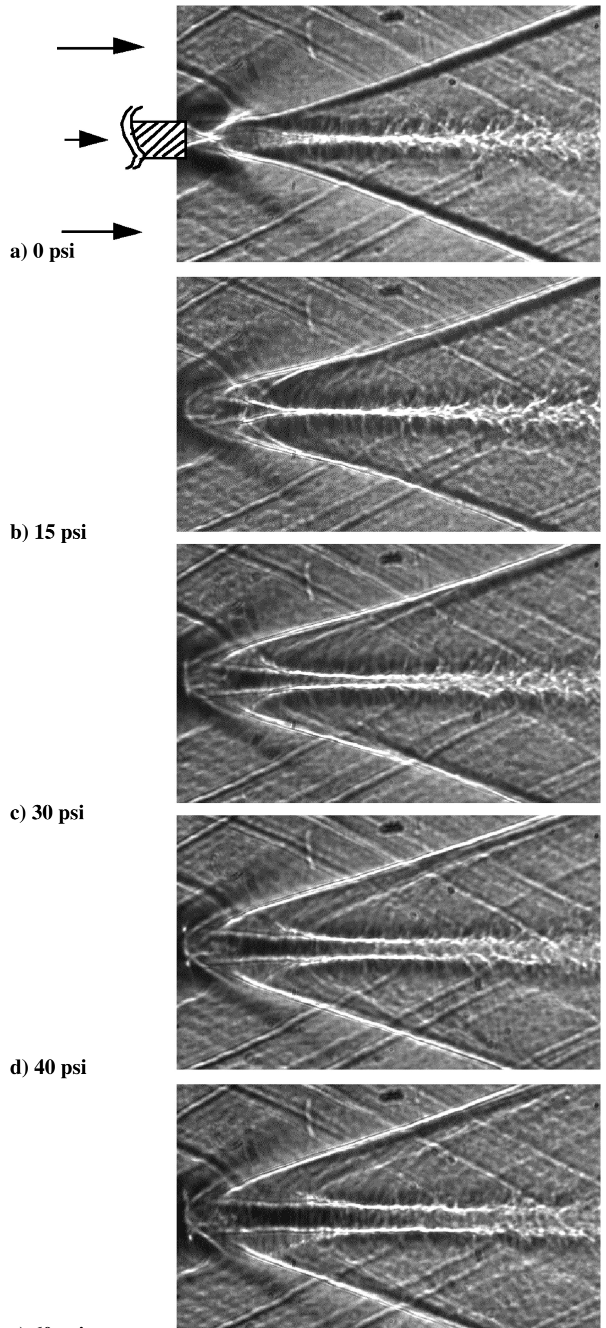


Fig. 6 Shadowgraph images for the Mach 2 wake showing effect of base flow mass injection rate m_0, as determined by supply pressure, on wave pattern and near-field length.

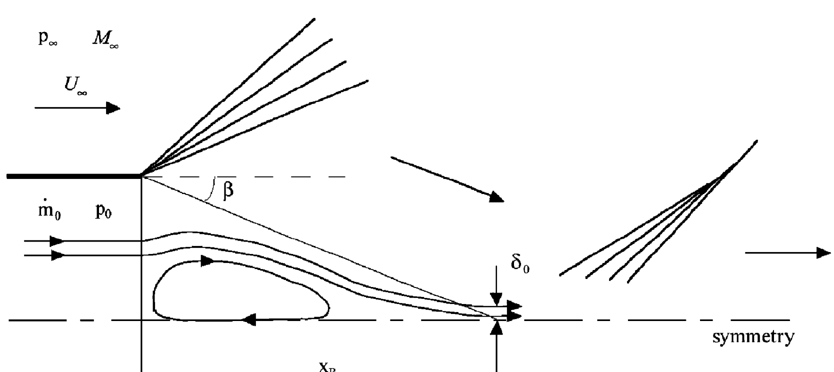


Fig. 5 Effect of base flow mass injection rate m_0 on near-field recompression length x_R and thus on virtual origin x_0. Prandtl–Meyer turning angle β set by pressure-matching requirement between freestream pressure p_∞ and injection pressure p_0 (see Fig. 6) gives $x_R \approx (d/2)\cot\beta$.

This gives the parametric effects on the virtual origin of the bluff-body shape via C_D, the freestream Mach number via β, and the mass injection rate via α_R and β.

For negligible mass injection, Eq. (17) indicates a large downstream shift in the virtual origin relative to the incompressible value $x_0/\vartheta \approx -123$ from Eq. (8). Again taking $C_D \approx 1$ and $c_\delta \approx 0.29$, Eq. (17) gives

$$x_0/\vartheta \approx 0 \tag{18}$$

which is in good agreement with the large downstream shift of x_0/ϑ seen in supersonic wakes in Ref. 2. Specifically, at $M_\infty \approx 3$ Ref. 2 found $x_0/\vartheta \approx 5$ from the scaling in Eq. (1a) for the widths $\delta(x)$ of the measured mean velocity deficit profiles, and $x_0/\vartheta \approx 12$ from the scaling in Eq. (1b) for the measured centerline mean velocity deficit $u_c(x)$. Both values are qualitatively consistent with the large downstream shift relative to the measured incompressible two-dimensional value $x_0/\vartheta \approx -128$ and are furthermore in reasonable quantitative agreement with the result in Eq. (17) for the measured scaling constant $c_\delta \approx 0.26$ from Ref. 2.

Moreover, Eq. (17) indicates that the effects of local self-induced forcing seen in Ref. 2 for confined supersonic bluff-body wakes, as large-scale vortical structures in the wake pass through the reflected near-field expansion wave, can alter the virtual origin caused by the difference in the unforced and forced values of the scaling constant c_δ. The crossover from the forced value $c_\delta \approx 0.35$ in Ref. 2 at $M_\infty \approx 2$ to the unforced value $c_\delta \approx 0.26$ at $M_\infty \approx 3$ should, based on Eq. (17), be accompanied by a small upstream shift in the virtual origin. Such a shift agrees with the experimentally measured values of the virtual origin in Ref. 2, where $x_0/\vartheta \approx 22$ at $M_\infty \approx 2$ shifted upstream to $x_0/\vartheta \approx 5$ at $M_\infty \approx 3$.

Conclusions

The results in Eqs. (8), (12), and (17), respectively, provide estimates of the virtual origin in the far-field scaling laws in Eqs. (2a) and (2b) and (10a) and (10b) for incompressible and supersonic turbulent bluff-body wakes and allow various parametric influences on the virtual origin to be understood.

Acknowledgments

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