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MEASUREMENTS AND PREDICTIONS OF TURBULENCE GENERATION IN HOMOGENEOUS PARTICLE-LADEN FLOWS

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MEASUREMENTS AND PREDICTIONS OF TURBULENCE GENERATION IN HOMOGENEOUS PARTICLE-LADEN FLOWS

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Abstract

The overall properties of turbulence generated by uniform fluxes of monodisperse spherical particles moving through a uniform flowing gas were studied both theoretically and experimentally. Mean and fluctuating values, probability density functions and energy spectra of streamwise and cross-stream velocities were measured within a counter-flowing particle/air wind tunnel using laser velocimetry. Test conditions included nearly monodisperse glass spheres having diameters of 0.5, 1.1 and 2.2 mm, particle Reynolds numbers of 106, 373 and 990, mean particle spacings of 13-208 mm, particle volume fractions less than 0.003%, and direct rates of dissipation of turbulence by particles less than 4%. Velocity fluctuations and PDFs were predicted using volume fraction weighted conditional averages of the properties of the wake disturbances and the turbulent inter-wake region. The relative turbulence intensities were correlated with a dimensionless dissipation factor developed during a previous study of inter-wake turbulence. The PDFs of streamwise velocities were not Gaussian, with negative skewness and higher kurtosis than a Gaussian distribution due to the presence of wake disturbances. The measurements and predictions of the above properties agree with each other very well. Finally, the streamwise energy spectra demonstrate both -1 and -5/3 decay regions, with the former resulting from contributions due to the presence of the laminar-like turbulent wakes of particles.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>C_d</td>
<td>particle drag coefficient</td>
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<tr>
<td>D</td>
<td>dissipation factor, Eq. (18)</td>
</tr>
<tr>
<td>d_p</td>
<td>particle diameter</td>
</tr>
<tr>
<td>E_s(k)</td>
<td>streamwise energy spectrum</td>
</tr>
<tr>
<td>f</td>
<td>frequency</td>
</tr>
<tr>
<td>f_p</td>
<td>weighted average of wake passing frequencies</td>
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<tr>
<td>f_w</td>
<td>wake volume fraction</td>
</tr>
<tr>
<td>k</td>
<td>wave number, 2π/λ</td>
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<tr>
<td>L_a</td>
<td>streamwise integral length scale</td>
</tr>
<tr>
<td>L_w</td>
<td>wake length</td>
</tr>
<tr>
<td>ε</td>
<td>characteristic width of laminar-like wakes</td>
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<tr>
<td>ε_k</td>
<td>Kolmogorov length scale, (ν^{3/4})^{1/4}</td>
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<tr>
<td>ε_p</td>
<td>mean particle spacing, Eq. (1)</td>
</tr>
<tr>
<td>ν^*</td>
<td>particle number flux</td>
</tr>
<tr>
<td>PDF</td>
<td>probability density function</td>
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<tr>
<td>Re_p</td>
<td>particle Reynolds number, d_pU_p/ν</td>
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<tr>
<td>Re_t</td>
<td>turbulence Reynolds number, Re_t = λ U/ν</td>
</tr>
<tr>
<td>r</td>
<td>radial distance in laminar-like wakes</td>
</tr>
<tr>
<td>S</td>
<td>skewness of PDF</td>
</tr>
<tr>
<td>s</td>
<td>distance in streamwise direction</td>
</tr>
<tr>
<td>t_k</td>
<td>Kolmogorov time scale, (ν/ε)^{1/4}</td>
</tr>
<tr>
<td>U_p</td>
<td>mean streamwise relative velocity of a particle</td>
</tr>
<tr>
<td>u</td>
<td>streamwise gas velocity</td>
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<tr>
<td>U_d</td>
<td>mean streamwise velocity defect in wakes</td>
</tr>
<tr>
<td>U_w</td>
<td>streamwise inter-wake air velocity</td>
</tr>
<tr>
<td>U_{iw}</td>
<td>mean streamwise inter-wake air velocity</td>
</tr>
</tbody>
</table>

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\[ \bar{u}_{iw} = \text{streamwise r.m.s. fluctuating air velocity of inter-wake turbulence} \]
\[ u_{iw}' = \text{contribution of inter-wake turbulence to streamwise r.m.s. velocity fluctuations} \]
\[ u_k = \text{Kolmogorov velocity scale, } (v_e)^{1/4} \]
\[ u_w = \text{streamwise velocity in laminar-like wakes} \]
\[ \bar{u}_w = \text{contribution of mean streamwise velocities in laminar-like wakes to overall mean streamwise velocities} \]
\[ u_{wm}' = \text{contribution of mean streamwise velocities in laminar-like wakes to overall streamwise r.m.s. velocity fluctuations} \]
\[ u_{wf}' = \text{contribution of streamwise velocity fluctuations in laminar-like wakes to overall streamwise r.m.s. velocity fluctuations} \]
\[ V_w = \text{wake volume} \]
\[ v = \text{cross stream gas velocity} \]
\[ x = \text{streamwise distance} \]
\[ \varepsilon = \text{rate of dissipation of turbulence kinetic energy} \]
\[ \lambda = \text{Taylor microscale of turbulence} \]
\[ \theta = \text{wake momentum diameter, } d_p(C_d/8)^{1/2} \]
\[ \nu = \text{molecular kinematic viscosity of air} \]
\[ \nu_t = \text{effective turbulent viscosity of air} \]

**Superscripts**

\( \overline{()}_t = \text{mean value} \)
\( ()' = \text{r.m.s. fluctuating value} \)
\( ()^2 = \text{mean square fluctuating value} \)

**Introduction**

Turbulence generation is the direct disturbance of the continuous-phase velocity field by the wakes of dispersed-phase objects. It supplements the conventional production of turbulence due to mean velocity gradients in the continuous-phase. Turbulence generation is most important when the dispersed-phase objects have large relative velocities (large Reynolds numbers) and relatively large relaxation times compared to characteristic turbulence times. Such conditions are typical of many practical dispersed multiphase flows having significant separated-flow effects, e.g., sprays, particle-laden jets, bubbly jets, rainstorms, etc. In spite of its importance, however, turbulence generation has not received much attention so that current understanding and capabilities for predicting its properties are very limited.

Earlier observations of flows resulting from turbulence generation in this laboratory have involved uniform fluxes of nearly monodisperse spherical particles moving at near terminal velocities in still water\(^1\) and in still air.\(^2\) The resulting flows are homogeneous and stationary with turbulence production entirely due to turbulence generation. The local rate of dissipation of turbulence kinetic energy mainly controls continuous-phase turbulence properties in these flows and it can be found as the local rate of loss of particle mechanical energy per unit volume. All other continuous-phase properties of the flow (moments, probability density functions, spectra, etc.), however, are not known and must be related to particle properties and dissipation rates. Due to the lack of adequate knowledge about particle wakes in turbulent environments at intermediate Reynolds numbers, the stochastic theory of these studies failed to predict turbulence properties effectively except the relative turbulence intensities. Later studies\(^3\)\(^-\)\(^5\) in this laboratory showed that particle wakes at such conditions differed from wakes in still environments at the same Reynolds numbers. Instead, these wakes scaled in the same way as laminar wakes but mixed much faster due to the presence of turbulence; thus, they were called laminar-like turbulent wakes.\(^3\)\(^4\)\(^5\)

A recent study of turbulence generation in upflowing air\(^6\) in this laboratory showed that flows caused by turbulence generation actually contain laminar-like turbulent wakes and a relatively large inter-wake turbulent region. It was also found that relative turbulence intensities were proportional to the square root of dissipation rate in accord with a simplified stochastic theory of turbulence generation flow properties described in Refs. 1 and 2. Other properties, however, were not explained by the stochastic theory and exhibited features not seen in conventional homogeneous turbulence such as non-Gaussian streamwise velocity PDF's and temporal spectra with 1 power decay regions in addition to the usual -5/3 power inertial decay region.

The turbulent inter-wake region of flows caused by turbulence generation was subsequently studied using a conditional sampling technique in this laboratory\(^7\)\(^8\) in order to understand the unexplained overall properties of flows resulting
from turbulence generation. The turbulent inter-wake region was found to be homogeneous and isotropic with both streamwise and cross-stream velocity PDF's having near Gaussian distributions and spectra similar to conventional homogeneous isotropic turbulence. The relative turbulence intensities in the turbulent inter-wake regions could be correlated effectively using a dimensionless dissipation factor developed by analogy to grid-generated turbulence. The old dissipation factor used in Refs. 1 and 2 was also found to be similar to this dissipation factor.

Motivated by these observations, the objective of the present investigation was to develop a prediction method of the overall properties of turbulence generation and to evaluate this method using measurements of overall flow properties resulting from turbulence generation.

**Experimental Methods**

**Apparatus**

The apparatus consisted of a vertical counterflow wind tunnel with upflowing air moving toward the suction side of a blower and freely-falling particles introduced at the top of the wind tunnel using a particle feeder, see Refs. 6 and 9 for a complete description. The air flow path consisted of a rounded inlet, a flow straightener and a 16:1 contraction ratio to the 305 x 305 mm cross section windowed test section. The particle flow path consisted of a variable-speed particle feeder, a screen array particle dispersion section, a honeycomb particle flow straightener and a particle acceleration section to yield near terminal velocity particles at the test section. The particles were collected in a flexible plastic funnel below the air inlet.

**Instruments**

Measurements included particle number fluxes by sampling and particle and gas velocities by a traversible laser velocimetry (LV) system. The particle measurements are described in Ref. 6; therefore, the following discussion will be limited to the gas velocity measurements. A single-channel, dual-beam, forward-scatter, frequency-shifted LV was used, based on the 514.5 nm line of an argon-ion laser. Streamwise and cross-stream velocities were found by rotating the optics accordingly. A beam expander was used to yield a measuring volume diameter and length of 55 and 425 \( \mu \text{m} \), respectively. The air flow entering the wind tunnel was seeded with oil drops having a 1 \( \mu \text{m} \) nominal diameter for the air velocity measurements. Velocities were found from the low-pass filtered analog output of a burst counter signal processor. The combination of frequency shifting plus a constant sampling rate of the analog output of the signal processor eliminated effects of directional bias and ambiguity and velocity bias. Because of their small impaction efficiencies due to their small size, the LV seeding particles were not collected by the glass spheres. Heavy seeding levels were used so that effects of LV step noise were deferred to scales roughly an order of magnitude larger than the Kolmogorov scales.

**Test Conditions**

Test conditions are summarized in Table 1. Particle properties were measured as described in Ref. 3. Assuming that the particles are falling randomly, the mean particle spacing is given by

\[ \ell_p = \left( \frac{(U_p - \bar{U})}{n_p} \right)^{1/3} \]

which yields values of 13-208 mm with corresponding particle volume fractions less than 0.003%. The direct dissipation of turbulence kinetic energy (dissipation) by particles was less than 4%, found as described in Ref. 1; therefore; dissipation can be found from the rate of turbulence generation by particles, as follows:

\[ \varepsilon = \frac{\pi n_p d_p^2 C_d U_p}{8} \]

Given \( \varepsilon \), the Kolmogorov scales can be computed from their definitions. For present dissipation rates, relative turbulence intensities due to turbulence generation were in the range 0.2-5.0%.

Evaluation of the apparatus is discussed by Chen et al. and Chen. Measurements of particle number fluxes and phase velocities showed that they varied less than experimental uncertainties over the central 205 x 205 mm cross section of the flow, extending 200 mm in the streamwise direction, which surrounded the location where measurements were made. Similar measurements showed that flow properties varied less than experimental uncertainties as a function of time. Thus, present flows were properly homogeneous and stationary with turbulence produced by turbulence generation.

**Theoretical Methods**

Consideration of the overall turbulence properties of turbulence generation processes will
begin with a discussion of conditional averaging. The mean streamwise velocities of the overall flow can be expressed as a combination of the mean velocities within the turbulent inter-wake region, \( \bar{u}_{iw} \), and the average contribution due to mean velocities in the laminar-like wakes, \( \bar{u}_w \), as follows:

\[
\bar{u} = (1-f_w)\bar{u}_w + f_w \bar{u}_{iw}
\]

where \( f_w \) is the volume fraction of the wake regions and \( \bar{u}_{iw} \) is defined as the volume averaged mean velocity of a laminar-like turbulent wake, i.e.,

\[
\bar{u}_{iw} = \frac{1}{V_w} \int_0^{L_w} \int_{-d_w/2}^{d_w/2} u_w(r,x)2\pi r dr dx
\]

where the mean velocity in the wakes is \( u_w = \bar{u}_{iw} - U_p (\bar{u}_d / U_p) \). The characteristic radius of the wakes, \( \ell_w \), and mean velocity defect, \( \bar{u}_d \), are both defined in Refs. 4 and 5. The wake length, \( L_w \), was defined as the streamwise distance between the particle and the point where the mean wake defect velocity at wake centerline is equal to the ambient turbulence intensity level, i.e.,

\[
L_w/d_p = C_d Re U_p/(32 \bar{u}_{iw}).
\]

The wake volume of a single particle, \( V_w \), is defined to have a conical shape with a height of \( L_w \) and radius at each streamwise distance of \( 2\ell \). The resulting \( V_w \) can be expressed as \( V_w = 4\pi d_w \ell_w^2 / Re_c \). The resulting theoretical estimation of \( \bar{u} \) due to contributions from both the wake disturbances and the turbulent inter-wake region can be reduced to the following:

\[
\bar{u} = \bar{u}_{iw} - f_w (1-e^{-2})\bar{u}_{tw}
\]

where \( \bar{u}_{iw} \) is the streamwise velocity fluctuation in inter-wake region. For all present test conditions, the second term in Eq. (5) contributed less than 0.6% to the overall expression, which was even smaller than the experimental uncertainties of \( \bar{u} \) (which were 1.2%). Thus, the present overall mean streamwise velocities were approximated using the mean streamwise inter-wake velocities alone (\( \bar{u} \approx \bar{u}_{iw} \)).

Conditional Average of Velocity Fluctuations
Similar to the mean velocities, streamwise velocity fluctuations can be predicted using results of conditional sampling theory. The square of mean overall streamwise velocity fluctuations can be estimated as follows,

\[
u'^2 = f_w (\bar{u}_{wm}^2 + \bar{u}_{wf}^2) + (1-f_w)\bar{u}_{iw}^2
\]

where \( \bar{u}_{wm} \) is the contribution of the mean velocities of laminar-like wakes to the overall velocity fluctuations, \( \bar{u}_{wf} \) is the contribution of the velocity fluctuations in the wakes to the overall velocity fluctuations, and \( \bar{u}_{iw} \) is the contribution of inter-wake turbulence to the overall velocity fluctuations. The contribution of inter-wake turbulence to the overall velocity fluctuations (\( \bar{u}_{iw}^2 = (\bar{u}_{iw} - \bar{u})^2 \)) is not exactly equal to the velocity fluctuations of the inter-wake turbulence due to the small difference between the mean velocities; but these velocity fluctuations can be expressed in terms of inter-wake velocity fluctuations, \( \bar{u}_{iw} \), using the present mean velocity approximation (\( \bar{u} \approx \bar{u}_{iw} \)) as follows:

\[
\bar{u}_{iw}^2 = \left[ 1 + f_w^2 (1-e^{-2})^2 \right] \bar{u}_{iw}^2
\]

The contribution of the mean velocities of the wakes was obtained from the volume average of the mean wake velocities of a single wake as follows,

\[
u_{wm}^2 = \frac{1}{V_w} \int_{d_w/2}^{L_w} \int_{-d_w/2}^{d_w/2} u_{wm}^2 (r,x)2\pi r dr dx
\]

where

\[
u_{wm}^2 = u_{wm}^2 = \frac{C_d Re U_p (d_p)}{32 \bar{u}_d x} \exp \left( \frac{-Re_c r^2}{4d_p x} \right)
\]

Integration of Eq. (8) encounters a singularity at \( x=0 \) which corresponds to the center of the particle and is not a part of the real wake disturbance. Thus, the integration of Eq. (8) was found by integrating from \( x=d_w/2 \) to \( L_w \) which properly corresponds to the continuous phase portion of the wake disturbance. The resulting contribution of wake mean velocities to overall streamwise velocity fluctuations can be expressed as follows:
The velocity fluctuations in laminar-like turbulent wakes are always larger than or equal to the ambient velocity fluctuations in the turbulent inter-wake region. Thus, the contribution of velocity fluctuations in the laminar-like wakes can be written as follows,

\[ \frac{u^2_{wf}}{u^2_{iw}} = 1 + \frac{u^2_{wf}}{u^2_{iw}} \] (11)

Substituting Eq. (11) into Eq. (6) yields,

\[ \overline{u^2} = f_w (\overline{u^2_{wm}} + \overline{u^2_{wf}}) + \overline{u^2_{iw}} \] (12)

The second term in Eq. (11) can be found from the volume average of \( \frac{u^2_{wf}}{u^2_{iw}} \), which can be obtained by fitting the data of Wu and Faeth. The present approach involved a -1 power exponential fit with respect to streamwise distance to approximate the data in order to carry out the volume average integration. Thus, \( \frac{u^2_{wf}}{u^2_{iw}} \) can be written as follows:

\[ \frac{u^2_{wf}}{u^2_{iw}} = e^{-1.8} \left( \frac{u_{iw}}{U_p} \right)^{-1.56} \left( \frac{x}{d_p} \right)^{-1}, \quad 1 \leq \frac{x}{d_p} \leq e^{0.5} \left( \frac{u_{iw}}{U_p} \right)^{-1.3} \] (13)

The correlation coefficient of \( \frac{u^2_{wf}}{u^2_{iw}} \) is 0.989 and the standard deviations of the fitted powers -1.8 and -1.56 are 0.26 and 0.084, respectively. The correlation coefficient of the range of \( x/d_p \) is 0.986 and the standard deviations of the powers -0.5 and -1.3, are 0.47 and 0.15, respectively. Then, volume averaged integration yields,

\[ \overline{u^2_{wf}} = \frac{83.1}{C_d^2 R_e^2} \overline{u^2_{iw}} \left[ e^{-0.5} \left( \frac{u_{iw}}{U_p} \right)^{-1.3} - 1 \right] \] (14)

Hence, the overall streamwise r.m.s. velocity fluctuations can be predicted using Eqs. (7), (10), (12) and (14). Cross-stream velocity fluctuations were found to be the same as streamwise velocity fluctuations except that the mean wake velocity contribution is zero for all test conditions and that the cross-stream velocity fluctuations in wakes are equal to ambient velocity fluctuations for the present test conditions with the 0.5 mm diameter particles.

### Conditional Averages of PDF and Skewness

More insight about the effect of particle wake disturbances on the overall turbulence properties of homogeneous disperced flows dominated by turbulence generation can be obtained from the PDFs of velocity fluctuations. The prediction of PDFs can be obtained similar to the methods used for predicting mean and fluctuating velocities; i.e., the PDF of overall velocities is a combination of the PDF of inter-wake turbulence and laminar-like wakes weighted by their volume fractions, as the follows,

\[ PDF(u) = f_w PDF(u_w) + (1-f_w) PDF(u_{iw}) \] (15)

where PDF\( (u_{iw}) \) is a Gaussian distribution normalized by corresponding cross-stream overall velocity fluctuations because the streamwise overall velocity fluctuations are enlarged by the contributions from the laminar-like wakes; and PDF\( (u_w) \) is the corresponding streamwise velocity PDF within a laminar-like turbulent wake considering both the mean velocity and velocity fluctuation distributions, which was obtained by sampling and statistical analysis using a computer program due to the difficulty of obtaining an analytical expression for this result. Cross-stream velocity PDFs were obtained similarly without considering the mean velocity (which is zero) contribution of particle wakes.

Theoretically, a prediction of skewness is possible using methods similar to the predictions of velocity fluctuations from the present predictions of PDF\( (u) \) and PDF\( (v) \). Since the skewness of the inter-wake turbulence is theoretically zero, the skewness of the overall flow can be expressed as the follows:

\[ S(u) = f_w \frac{1}{u^3} \int_{u_{iw}/2}^{L_w} \int_{r=0}^{2\pi} \int_{x} \left( \frac{u^3}{u_w} (r, x) \right) 2\pi r dxdx \] (16)

Using approximations similar to those used to find velocity fluctuations then yields,

\[ S(u) \equiv -\frac{f_w}{3} \left( \frac{C_d R_e U_p}{16 u_{iw}} - 1 \right) \left( \frac{u_{iw}}{\overline{u}} \right)^3 \] (17)
The singularity of the integration to find \( S(u) \) was handled by the same manner as the approach used for velocity fluctuations.

**Results and Discussion**

**Velocity Fluctuations**

Measurements of streamwise and cross-stream relative turbulence intensities as well as present predictions of streamwise and cross-stream relative turbulence intensities are plotted in Fig. 1 as a function of the dissipation factor, \( D \), defined in Refs. 7 and 8, as follows,

\[
D = \varepsilon \rho (C_d/8)^{1/2} \left[ \pi U_p^2 \right] \tag{18}
\]

The results of earlier measurements in water and air due to Parthasarathy and Faeth\(^1\) and Mizukami et al.\(^2\) are also shown on the figure. All three experiments used the same 0.5, 1.1, and 2.2 mm spherical glass particles. The best fit correlations found by linear regression of the measurements had powers of 0.56 and 0.48 for the streamwise and cross-stream relative turbulent intensities with standard deviations of these powers of 0.03 and 0.02, respectively. As a result, there is no statistical significance between the powers of these fits and the power of 1/2 found by Parthasarathy and Faeth\(^1\) and Mizukami et al.\(^2\). Thus, for consistency with the earlier work, the present measurements were re-fitted by forcing the power to be 1/2. The new fits are also plotted in Fig. 1. These correlations can be written as follows:

\[
\left( \frac{\overline{u}}{U_p} \right) = 13.4 D^{1/2} \tag{19}
\]

and

\[
\left( \frac{\overline{v}}{U_p} \right) = 8.4 D^{1/2} \tag{20}
\]

where the standard deviations of the coefficients of Eqs. (19) and (20) are both 1.5 and the correlation coefficients of these fits are both 0.96.

The predictions of streamwise and cross-stream relative turbulence intensities vary slightly with particle size as illustrated in Fig. 1. The different predictions for different particle sizes were caused by the different laminar-like turbulent wake properties for the different particle sizes when Eq. (8) was evaluated. Uncertainties about inter-wake turbulence properties of the three sizes also contributed to the differences. Vortex shedding effects also are evident in cross-stream relative turbulence intensities. In particular, the lack of vortex shedding for 0.5 mm particles (Re<300) makes the relative turbulence intensities smaller than results for the larger particles where vortex shedding occurs. The predictions of both streamwise and cross-stream relative turbulence intensities in Fig. 1 are reasonably good and they exhibit very small deviations from the measurements. The qualitative behavior of the predictions is also correct, with relative turbulence intensities at a given value of \( D \) tending to increase as the particle size decreases for both the measurements and the predictions.

The ratio of the coefficients in Eqs. (19) and (20) suggests that the anisotropy of the overall flow, \( \overline{u}/\overline{v} \), is roughly 1.59. This degree of anisotropy for the overall flow is much larger than the anisotropy of the turbulent inter-wake region due to the presence of streamwise mean velocity disturbances from particle wakes. The ranges of the anisotropy values for each test condition can be summarized as follows: 1.5-2.5 for Parthasarathy and Faeth\(^1\), 1.3-4.0 for Mizukami et al.\(^2\) and 0.9-3.2 for the present measurements.

**PDF**

Similar to earlier considerations of the PDF's of velocities within the turbulent inter-wake region in Refs. 7, 8 and 9, plots of PDF's using a logarithmic scale help reveal specific features of the flow. Thus, plots of the PDF's of streamwise and cross-stream velocities for the three particle sizes, considering two particle loadings for each size, are illustrated in Figs. 2-4. Results shown on
the present predictions along with a standard Gaussian distribution for reference purposes. The present PDFs do not agree with earlier observations of Parthasarathy and Faeth\textsuperscript{1} and Mizukami et al.\textsuperscript{2} for dispersed flows in stagnant baths, which yielded Gaussian PDFs for both velocity components, with at most a slight upward bias (roughly 10\% when averaged over all test conditions) of the PDF(u) near its most probable value. This behavior agrees with the present behavior of the PDF(v), which is nicely fitted by Gaussian PDFs for both loadings of all particle sizes. In contrast, the present PDF(u) are more peaked and somewhat skewed toward negative velocities compared to the mean velocity, i.e., the PDF(u) exhibit greater kurtosis and skewness than the nearly Gaussian PDF(v).

Similar to earlier observations, the PDFs are relatively independent of particle loading. The measured and predicted PDFs are also in relatively good agreement, which is encouraging. The predicted PDFs of cross-stream velocities are nearly Gaussian due to the lack of contributions of mean wake velocities. The PDFs of streamwise velocities, however, are never well approximated by a Gaussian distribution and become progressively more peaked (or have progressively increasing kurtosis) as the particle size (or particle Reynolds number) decreases. All these characteristics can be explained from the properties of the streamwise and cross-stream velocity records, and the disturbances due to particle wakes seen in the velocity records in Ref. 6, as discussed next.

First of all, the disturbances due to laminar-like turbulent wakes make a relatively small contribution to the velocity PDFs of the overall flow. The contribution to the PDF(v) is also due entirely to turbulence in the turbulent inter-wake region and in the particle wake disturbances. Both these contributions are represented quite well by Gaussian distributions and it is not surprising that their combined effect is adequately fitted by a Gaussian distribution, as well. Naturally, the turbulence contributions to the PDF(u) behave in the same manner; therefore, effects of mean velocities in wake disturbances are mainly responsible for the departure of the PDF(u) from a Gaussian distribution. First of all, the mean velocities in the laminar-like turbulent wake disturbances only contribute negative velocities compared to the mean velocities of the flow and these velocities generally are rather large negative velocities compared to turbulence levels in the turbulent inter-wake region, see Refs. 6, 7 and 8. Thus, these contributions are responsible for the upward bias, compared to the Gaussian distribution, seen at large negative velocities in Figs. 2-4. This upward bias is compensated by the behavior of the PDF(u) at positive velocities where the maximum PDF(u) shifts slightly to positive velocities while the PDF(u) at large positive velocities is biased downward from the Gaussian distribution. The narrower (or more peaked) PDF(u) near the mean velocity condition as the particle size (or Reynolds number) decreases, then follows from the reduced levels of turbulence in the wake disturbances. Such reduced disturbances concentrate the effects of wake disturbances to the mean velocity distributions which implies reduced velocity variations and thus a more peaked PDF(u). This behavior is particularly promoted by laminar-like wake disturbances where streamwise turbulence intensities approach unity due to the unstable nature of wake flows, see Wu and Faeth.\textsuperscript{4,5}

The main reason for the different PDF(u) of Parthasarathy and Faeth\textsuperscript{1} and Mizukami et al.\textsuperscript{2}, and the present study, follows from the much improved laser velocimetry measuring conditions of the present study. In particular, improved seeding and reduced turbulence intensities allowed the near-wake region of the disturbances due to particle wakes to be resolved for the streamwise velocity records. In contrast, these disturbances were not resolved during the measurements of Parthasarathy and Faeth\textsuperscript{1} and Mizukami et al.\textsuperscript{2} This point was easily demonstrated during the present experiments by reducing seeding levels so that the spikes seen in velocity records in Ref. 6 due to wake disturbances were rarely resolved; the corresponding PDF(u) then became more Gaussian similar to the results of Parthasarathy and Faeth\textsuperscript{1} and Mizukami et al.\textsuperscript{2}

The other properties of Figs. 2-4 then follow from the well known effects of the properties of the velocity signal on the values of the skewness and kurtosis of the PDF(u), see Tennekes and Lumley.\textsuperscript{10} In particular, the spikes always contribute streamwise negative velocity signals based on the results illustrated in Ref. 6. This implies a corresponding positive bias of the PDF(u) or negative skewness, based on the well known properties of PDFs.\textsuperscript{10} The streamwise skewness of the overall flow is generally larger than the skewness of inter-wake turbulence. On the other hand, the cross-stream skewness exhibits both positive and negative values near zero for both inter-wake turbulence and overall flows. The average values of skewness are -1.32 and -0.021,
with standard deviations of 0.72 and 0.24, for streamwise and cross-stream velocities, respectively. The resulting predicted values of skewness from Eq. (17) and from the predicted PDF(u) are roughly -2 which is within the range of measured data. Considering the relatively large uncertainties of skewness (50%-70%), the agreement between predictions and measurements is satisfactory.

**Spectra**

Streamwise and cross-stream energy spectra were transformed from measured temporal spectra using Taylor's hypothesis and normalized by velocity fluctuations and integral length scales which were obtained from the integral time scale similar to the method used for inter-wake turbulence as described in Refs. 7 and 8. The resulting spectra of the streamwise velocity fluctuations of the overall flow are plotted as functions of $kL_a$ in Fig. 5. Results are shown for various particle sizes and fluxes. The LDV conditions for these measurements were good so that effects of step noise were deferred until conditions near Kolmogorov frequencies were approached. Correlations of energy spectra for approximate isotropic turbulence for streamwise velocity fluctuations are also shown on the plot for comparison with the present measurements. The isotropic turbulence spectra represent a simplification of isotropic turbulence discussed by Hinze, where the actual -5/3 power decay of the inertial range of isotropic turbulence is approximated by a -2 power decay.

An interesting feature of the spectra of Fig. 5 is that they decay over a rather large range of $kL_a$ (roughly four decades) even though present particle Reynolds numbers are not large (less than 1,000). Much of this behavior is typical of other homogeneous turbulence fields where disturbances due to grids (with relatively small grid element Reynolds numbers) yield turbulent flows having extensive inertial ranges. Additional evidence supporting the explanation that the -1 power decay region is caused by the mean velocities within wake disturbances was that the -1 power decay region was associated with values of $kL_a/2\pi$ on the order of unity, which roughly corresponds to the characteristic frequency of a wake passing through the measuring volume. Larger frequencies exhibit -5/3 power decay behavior which is similar to conventional turbulence and involves contributions from turbulence in both the wake disturbances and the turbulent inter-wake region.

The energy spectra of Fig. 5 also provided other evidence of direct contributions of mean velocities in wake disturbances. In particular, the spectra exhibit rather prominent -1 and -5/3 decay regions as $kL_a$ increases. The -1 power decay region is not seen in either conventional turbulent flows or in inter-wake turbulence, but based on the approximate stochastic analysis of Parthasarathy and Faeth, such behavior is typical of the contributions of mean velocities in particle wakes to the spectra. In order to verify the contributions from mean wake velocities, FFT analysis of mean streamwise velocities in single laminar-like turbulent wakes was carried out to analyze their contribution to the spectra. The temporal spectra for velocity profiles at ten different relative radial positions for each particle size and ambient turbulence intensity that was considered were computed. The spectra were then averaged according to the probabilities of the presence of these positions and then normalized by probability-weighted average of the passing frequencies of velocity profiles at these radial positions. The resulting weighted average spectra are illustrated in Fig. 6 for 0.5, 1.1 and 2.2 mm diameter particles in the presence of ambient relative turbulence intensities of 0.52%, 0.20% and 0.15%, respectively. The spectra clearly exhibit a nearly -1 power decay. This finding confirms that the region of -1 power decay seen in overall spectra of flows caused by turbulence generation is caused by the mean velocity distributions of laminar-like turbulent wake disturbances.

Conclusions

This investigation considered the overall properties of homogeneous turbulence generated by uniform fluxes of monodisperse spherical particles moving at near terminal velocities through air at normal temperature and pressure. Present test
conditions can be summarized as follows: 0.5, 1.1 and 2.2 mm nominal diameter glass particles; particle Reynolds numbers of 106, 344, and 990; particle fluxes of 0.5-1000 kpart/m's; mean particle spacings of 13-209 mm; dissipation rates of turbulence kinetic energy of 0.01-1.2 m²/s³; streamwise relative turbulence intensities of 0.15-3.68%; particle volume fractions less than 0.003% and direct dissipation of turbulence by particles less than 4%. The major conclusions of the study are as follows.

1. The homogeneous turbulent particle-laden flow resulting from turbulence generation for present test conditions consists of two flow regions; namely, the particle wake disturbance region and the turbulent inter-wake region that surrounds the particle wake disturbances. For present test conditions, the turbulent inter-wake region dominated the flow, comprising 70-98% of the flow volume, while direct wake/wake interactions occurred for less than 25% of the wakes at particular flow cross-sections with most of these interactions being weak (involving peripheral positions of the wakes).

2. Present relative turbulence intensities of the overall flow in both the streamwise and cross-stream directions were in excellent agreement with the earlier measurements of Parthasarathy and Faeth¹ and Mizukami et al.² for particle flows in motionless (in the mean) water and air, respectively. Taken together, these observations cover relative turbulence intensities in the range 0.1-10% and show that relative turbulence intensities are proportional to the square root of the dimensionless dissipation factor, D. Remarkably, this correlation is relatively independent of particle size and Reynolds number over the available range of measurements.

3. Mean streamwise velocities in particle wake disturbances are responsible for many of the unusual features of the velocity properties of the overall flow because the arrival of particle wakes at a given point in the flow is random and these mean velocity contributions cannot be separated from conventional turbulence. These features include large anisotropy values ($\bar{v}/\bar{u} = 1.6$), non-Gaussian PDFs of streamwise velocities, non-Gaussian values of skewness ($S = -1.3$) of streamwise velocities, and prominent -1 regions of the energy spectra of streamwise velocity fluctuations, all due to the large negative mean streamwise velocity spikes in wake disturbances. In contrast, the statistical properties of cross-stream velocity fluctuations behave similar to isotropic turbulence because mean cross-stream velocities in particle wake disturbances are small.

4. Many properties of the overall flow, including relative turbulence intensities, anisotropies, velocity PDFs and skewness were predicted reasonably well based on volume averaged contributions of the conditional averages of these properties within the wake disturbance and inter-wake regions. This includes reasonably good predictions of relative turbulence intensities (with good estimations of effects of dissipation factor and particle size) and anisotropies, as well as the PDF properties of streamwise and cross-stream velocities.

Acknowledgments
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References
⁵Wu, J.-S., and Faeth, G. M., "Effects of Ambient Turbulence Intensity on Sphere Wakes at


<table>
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<tr>
<th>Table 1. Summary of Test Conditions*</th>
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<tbody>
<tr>
<td>Nominal Particle Diameter (mm)</td>
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<tr>
<td>-----------------------------------</td>
</tr>
<tr>
<td>$U_p$ (m/s)</td>
</tr>
<tr>
<td>Re (-)</td>
</tr>
<tr>
<td>$C_d$ (-)</td>
</tr>
<tr>
<td>$\theta$ (mm)</td>
</tr>
<tr>
<td>$n^a (kpart/m^2s)$</td>
</tr>
<tr>
<td>$\ell_p$ (mm)</td>
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<tr>
<td>$\varepsilon$ (m$^2$/s$^3$)</td>
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<tr>
<td>$\ell_K$ (mm)</td>
</tr>
<tr>
<td>$t_K$ (ms)</td>
</tr>
<tr>
<td>$u_K$ (m/s)</td>
</tr>
<tr>
<td>$\overline{U}/U_p$ (%)</td>
</tr>
<tr>
<td>$\overline{V''}/U_p$ (%)</td>
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* Round glass beads (density of 2500 kg/m$^3$) falling in upflowing air at standard temperature and pressure (air density of 1.16 kg/m$^3$ and kinematic viscosity of 15.9 mm$^2$/s) having a mean velocity of 1.1 m/s.
Fig. 1 Measurements and predictions of streamwise and cross-stream relative turbulence intensities for the overall flow as a function of the dissipation factor.

Fig. 2 Streamwise and cross-stream velocity PDF's (logarithmic scale) of the overall flow for 0.5 mm particles.

Fig. 3 Streamwise and cross-stream velocity PDF's (logarithmic scale) of the overall flow for 1.1 mm particles.

Fig. 4 Streamwise and cross-stream velocity PDF's (logarithmic scale) of the overall flow for 2.2 mm particles.
Fig. 5 Measured energy spectra of streamwise velocity fluctuations of the overall flow.

Fig. 6 Simulated temporal power spectra due to the mean velocities of laminar-like turbulent wakes at relative turbulence intensities of 0.52%, 0.20% and 0.15% for 0.5, 1.1 and 2.2 mm diameter particles, respectively.