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# FURTHER ADVANCES IN THE INTEGRATED STRUCTURAL OPTIMIZATION SYSTEM (ISOS)

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## ABSTRACT

This paper summarizes our recent efforts in implementing an Integrated Structural Optimization System (ISOS), introduced in our earlier publications. The premise of this system is to reduce the need for the so-called *engineering intuition* at the conceptual level of structural design. We have emphasized automation from the early stages of this project. The system has been successfully implemented for two-dimensional structures. Three-dimensional extensions to the system have been more challenging. We have taken initial steps to meet this challenge, and briefly discuss these steps in this article. Our earlier activities focused on static responses of structures. Recently, dynamic capabilities have been added to the system and are discussed here together with several examples.

## 1. INTRODUCTION

The field of structural optimization was initiated by Michell in his classic work [1] at the turn of the century. Due to computational complexities, no further progress was made in that field until the 1960's, when computer-assisted structural analysis and numerical optimization techniques became accessible. The last three decades witnessed intensive research and development work in this field.

Early work in structural optimization focused on proportioning the dimensions of the structures to obtain an optimum design. (The work by Schmitt [2] is one of the earliest articles on truss optimization and is frequently viewed as the start of modern structural optimization.) In the literature, proportion optimization is commonly referred to as *sizing optimization*.

Varying geometry of structures was a natural step to advance the capabilities of structural optimization. In the literature, this type of optimization is referred to as *shape optimization*. A recent survey on shape optimization is given by Haftka and Grandhi [3]. For both sizing and shape problems, the design topology remains unaltered during the optimization procedure. Design topology is characterized, for example, by the number of holes in a solid structure or the number and connectivity of nodes in a skeletal structure.

Only in the late 1980's topology optimization has started drawing attention by researchers. A survey of the rigorous research on this topic is provided by Suzuki and Kikuchi [4]. Further background work on both rigorous and heuristic techniques for topology optimization can be found in Ref. [5]. Kikuchi and Bendsoe [6] solved the so-called generalized layout problem (GLP) using a homogenization method. GLP is posed as a material distribution problem to obtain the stiffest possible topology given a specified amount of material. References [4, 6] should be consulted for details on the homogenization method.

Our goal for the past four years has been to incorporate a rigorous topology-generation tool, i.e., homogenization, into an overall design system, the Integrated Structural Optimization System (ISOS). Our previous publications [5, 7, 8] bear witness to our success, but our efforts are ongoing. This article includes only a short review of ISOS and its different phases; for further details on different capabilities and the theory underlying the various phases of ISOS the reader should consult References [5, 7, 8]. The recent capabilities added to ISOS are provided together with a two-dimensional practical example solved using the system. Additionally, this article discusses optimization of structures with respect to their dynamic response, i.e., eigenfrequency response.

The remainder of this article is organized as follows. Section 2 describes briefly the overall three-phase design process of ISOS and provides brief descriptions for each phase. In Section 3, a two-dimensional design example is presented. Section 4 provides some insight to the three-dimensional aspects of ISOS together with an example. Section 5 discusses the theory for topology and shape optimization of structures with respect to their dynamic (eigenfrequency) response, and Section 6 provides some examples for this type of activity of ISOS. Finally, some concluding remarks are given in Section 7.

## 2. THREE-PHASE DESIGN PROCESS

A three-phase design process is outlined in this section. The overview of the system and brief discussions on each phase are given here. As emphasized earlier, readers interested in more details about ISOS or each of its phases should consult References [5,7, 8].

A schematic diagram of ISOS is given in Fig. 1. ISOS was initially viewed as a three-phase procedure [5, 7], which was subsequently augmented by a fourth phase as the system evolved [8]. The core of this system is Phase I or *topology generation* using homogenization. The inputs to this phase are an initial design domain (i.e., packaging requirements), boundary conditions (load and displacement), global specifications (for functionality of the design), and a specified volume constraint. The output of Phase I is a density array. From a design point of view, the results of this procedure are not entirely satisfactory - examples in this article will illustrate this fact. Kikuchi and Bendsoe mention in Ref. [6] (the first article on topology optimization using homogenization) the need for further processing the outputs of this topology-generation tool. Addressing this issue was the rationale for implementing ISOS.

Phase II serves as a conversion module fulfilling two basic purposes. First, Phase II transfers the design representation from a density array (output of Phase I) into a form suitable for detailed design optimization or Phase III. Second, if any other non-structural constraints must be imposed on the design, Phase II is capable of handling those.

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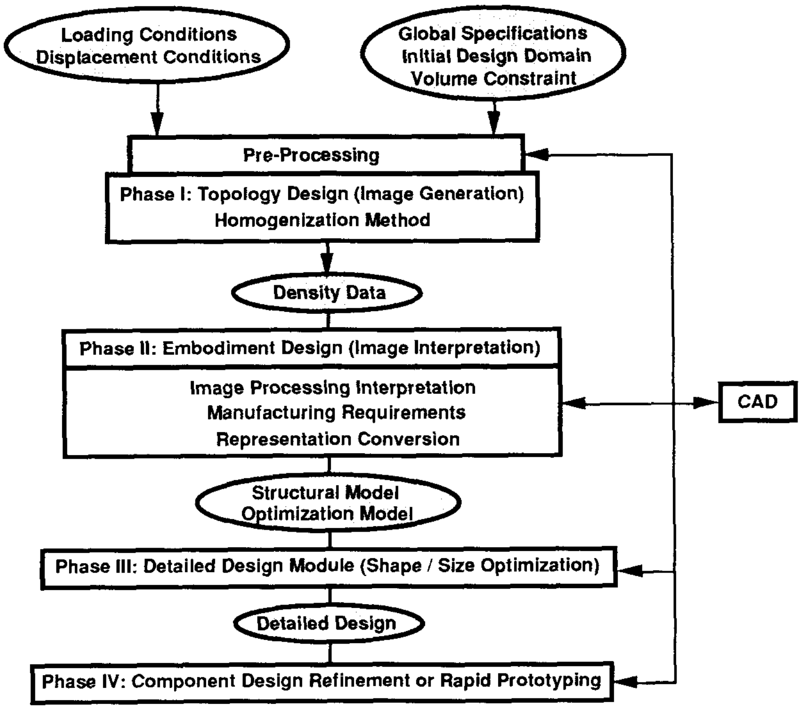


Figure 1. Overview of the Integrated Structural Optimization System (ISOS).

The traditional shape and sizing optimization, explained earlier, are performed in Phase III. Phase I seeks to maximize the total stiffness of a structure by generating the optimum topology and shape given a specified amount of material. This optimization simplifies the design process, and as discussed earlier, cannot be used directly as a design tool. Other performance criteria such as maximum stress, maximum stress gradient, maximum displacement, stability conditions of slender portions of the structure, as well as various geometrical restrictions associated with manufacturing and assembly requirements must be included to design a more realistic and practical structure. To this end a new optimization problem must be formulated including these additional design constraints. However, if the optimal solution to the newly formulated problem yields a significantly different configuration from the result of Phase I, the design process must be restarted at the Phase I level to include additional restrictions suggested by the detailed analysis.

Phase IV includes necessary design refinements or rapid prototyping and testing units. Examples for design refinements after Phase III are given in Ref. [8]. The examples in the present article emphasize the activities in Phases I, II, and III.

## 3. TWO-DIMENSIONAL EXAMPLE: BICYCLE FRAME

The following two-dimensional example provides an introduction to ISOS and its different phases. This example has been suggested and solved previously by Rasmussen and Olhoff [10]. The goal is to optimize the rigidity of a (topologically undefined) bicycle frame. Figure 2 shows a typical bicycle frame along with its loading conditions.

Phase I: An initial design model for this problem is depicted in Fig. 3(a). The functionality of the steering fork is not taken into account at this point. The mesh for Phase I, shown in Fig. 3(a), is 25 x 44, and the size of each square element is 25 mm; the geometric configuration of the problem can be reconstructed using these data. Points (0.0, 100.0) and (1100.0, 100.0) are modeled as a roller and a pinned support, respectively. The coordinate system is as shown in Fig. 3(a). Figure 3(b) shows the output of Phase I for a solid-to-void ratio of 1:9 (density 10%).

Figure 4(a) illustrates a slightly different initial model from the one shown in Fig. 3(a). Here the shaded elements have a prescribed density of 1 and are not subject to design. These elements are necessary to maintain the steering capability of the bicycle through the front fork. It illustrates how functionality of the design is maintained in ISOS. Figure 4(b) shows the output of Phase I for the model shown in Fig. 4(a) for a solid-to-void ratio of 10:67 (density 13%). Comparing Figures 3 and 4 demonstrates the fact that maintaining the steering functionality is a key element in determining the design topology.

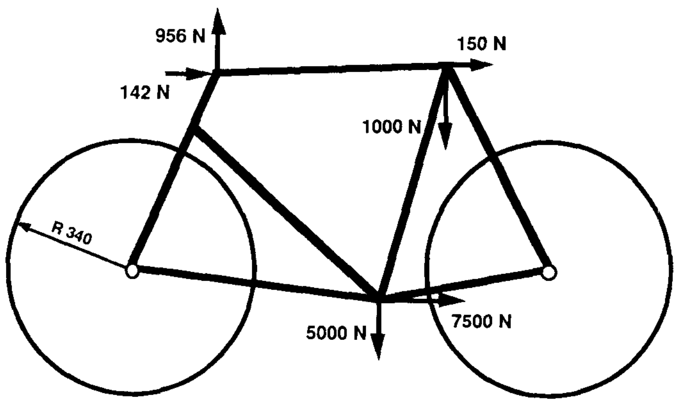


Figure 2. A typical bicycle frame loading conditions applied to it. See Fig 3 for dimensions and boundary conditions.

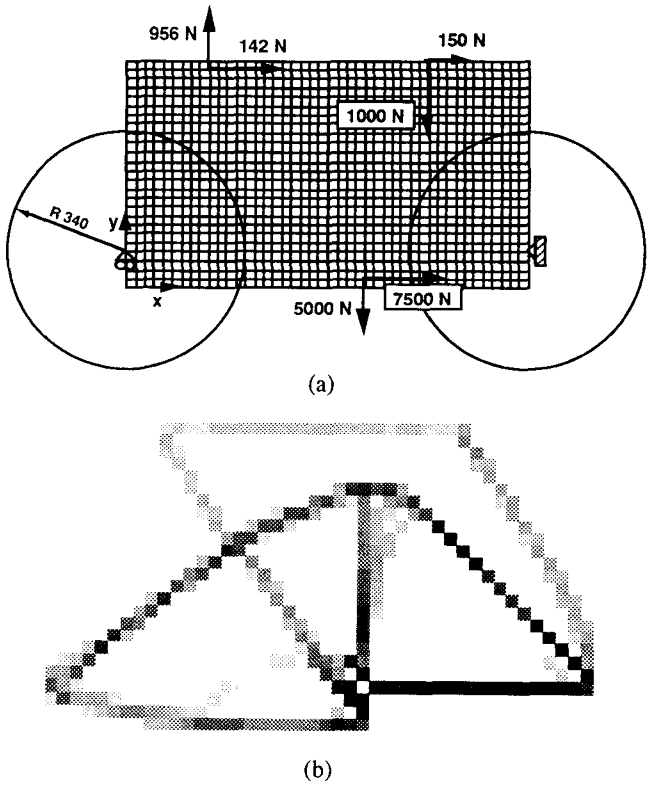


Figure 3. (a) Initial design model for the bicycle-frame problem, each element is a square with a size of 25 mm. The steering functionality is not taken into account. (b) Output of Phase I for the model given in Fig. 3(a), with a volume constraint of 10%.

Phase II: Figures 5(a) and 5(b) show the thresholded and smoothed images resulting from Fig. 4(b), respectively. The threshold value for this design is 0.12 and had to be entered manually. The threshold value of 0.35 calculated automatically according to the algorithm presented in Ref. [5] gave a disconnected topology.

A frame interpretation of the structure shown in Fig. 5(b) is given in Fig. 6. Numbers 1 through 7 denote the nodal numbers. The design represented here differs from the one obtained in Ref. [10].

Phase III: Detailed design optimization is performed using the capabilities of SAPOP [11] for skeletal structures. A frame model is used for this phase. The cross-sectional area of the elements (beams) is an annular tube, as it is commonly used in the design of bicycle frames. Since, due to limitations in SAPOP, only one design variable per cross-section is allowed, the thickness of the tube must be a function of the outer radius. In this case, the thickness is chosen to be a tenth of the outer radius of the annular cross-section. The initial design is the same cross-sectional area for every beam of 59.69 $mm^2$, corresponding to an outer radius of 10 mm. The material is assumed to be the same for all the beams and taken to have an allowable stress of 50 $N/mm^2$ for both tension and compression, and a density of 7800 $kg/m^3$. The optimization algorithm chosen for this particular problem is the generalized reduced gradient method (GREGA) [11].

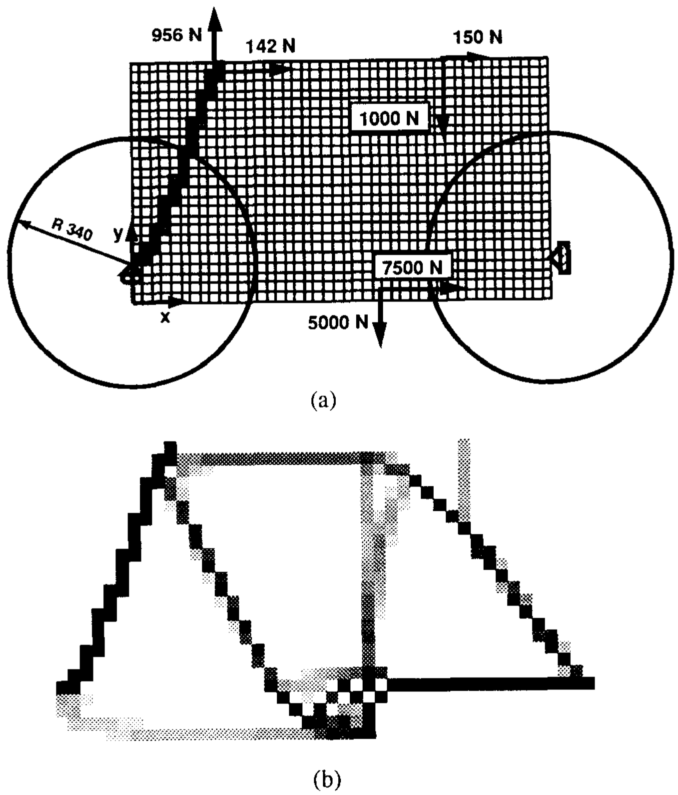


Figure 4. (a) Initial design model for the bicycle-frame problem, each element is a square with a size of 25 mm. The steering functionality is taken into account by setting the density of the dotted elements to 1 and declaring them as non-designable elements. (b) Phase I output for the model given in Fig. 3(a) with a volume constraint of 13%.

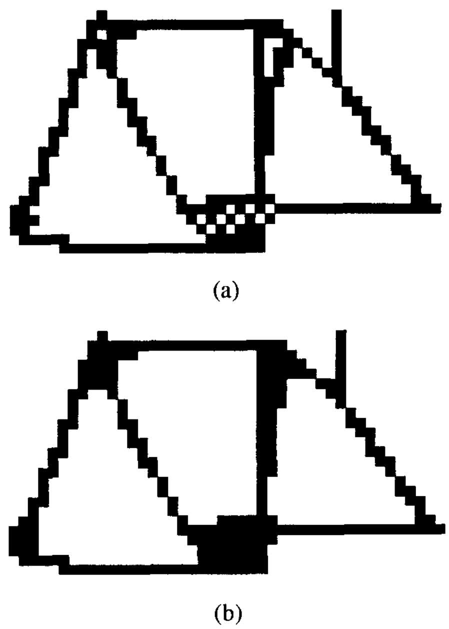


Figure 5. (a) Thresholded image of Fig. 4(a), threshold value is 0.12, (b) smoothed image of Fig. 5(a).

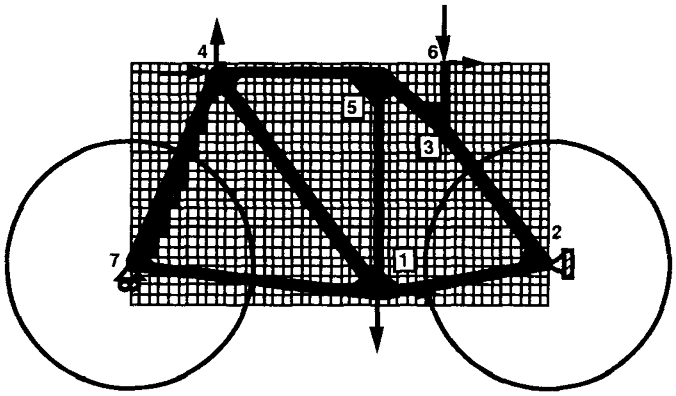


Figure 6. A frame approximation to the bicycle-frame example.

The following two optimization cases are examined. The first case represents sizing optimization only, where the design variables are the cross-sections of the beams. The final design for this case weighs 4.29 kg. The second case includes both sizing and geometry optimization for the frame. Design variables for this case are both cross-sections of the beams and the coordinates of those nodes that do not carry any loads nor have any boundary conditions applied to them. For this example these are Nodes 3 and 5, as shown in Fig. 6. Although the geometric changes compared to the previous case (sizing optimization only) are nominal, the optimal design for this case weighs 2.25 kg.

## 4. THREE-DIMENSIONAL ACTIVITIES IN ISOS

The underlying theory for three-dimensional activities of Phases I (topology generation) and II (image processing) of ISOS have been extensively discussed in References [12] and [13], respectively. Here, only a brief review is provided by solving a three-dimensional example. The algorithm used here for Phase II activities converts a spatial enumeration scheme for solid representation into a CSG representation; consult Ref. [13] for further detail.

This example has been suggested and solved previously in Ref. [12] with different density constraints. The initial design model is shown in Fig. 7, along with the loading and boundary conditions.

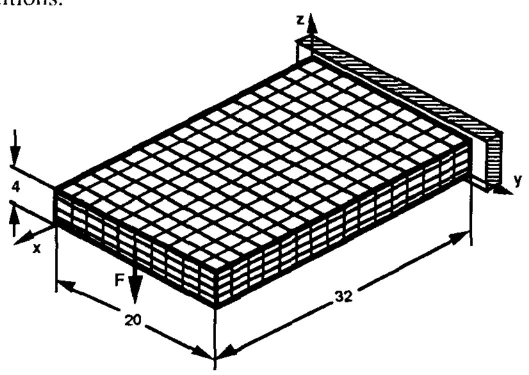


Figure 7. Initial design domain and boundary and loading conditions.

The homogenization output for a solid-to-void ratio of 1:1 is shown in Fig. 8. The resulting structure is a sandwich structure. The threshold value to generate the binary image is 0.5 and its choice is not critical for the outcome of the algorithm. The upper and lower layers are identical and are visible in Fig. 8. A cut through the middle of the structure parallel to the xy-plane is given in Fig. 9 which shows the two identical middle layers.

The output of the algorithm is given in Fig. 10. Figure 10(a) shows the primitives in an axonometric view. Figure 10(c) shows the extracted CSG tree representation for that object. It is basically the difference of the block representing the initial design domain, B1, and the union of the Holes 1, 2, 3, 4, and object OM (whose top view is shown in Fig. 9). Hole OM cannot be approximated by a single primitive and becomes the difference of B2, as shown in Figures 9 and 10(b), and the union of Regions 5, 6, 7, and 8 that are approximated by corresponding triangular blocks.

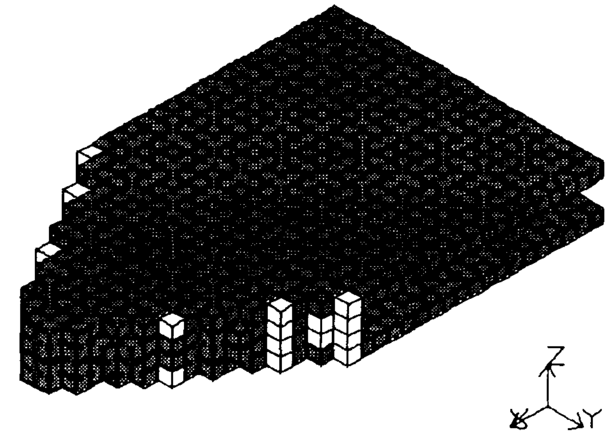


Figure 8. Homogenization output for the model of Fig. 7 (solid-to-void ratio is 1:1).

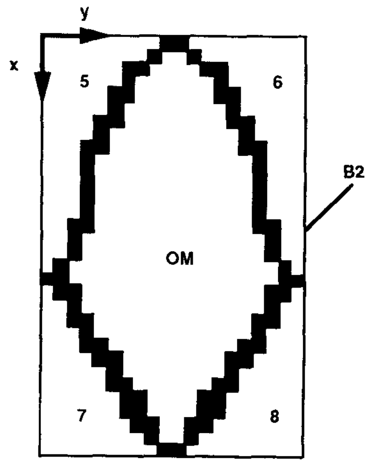


Figure 9. A section of the structure shown in Fig. 8 where the cutting plane is parallel to xy-plane and at z = 2.

## 5. ISOS AND EIGENFREQUENCY PROBLEMS

As explained earlier, the homogenization method seeks to maximize the stiffness of a structure by generating the optimum topology and rough shape for a specified amount of material. The output from the homogenization method is represented in the form of density arrays that are converted to a higher level of representation in Phase II [5,7]. The following important issues associated with this design process are discussed below.

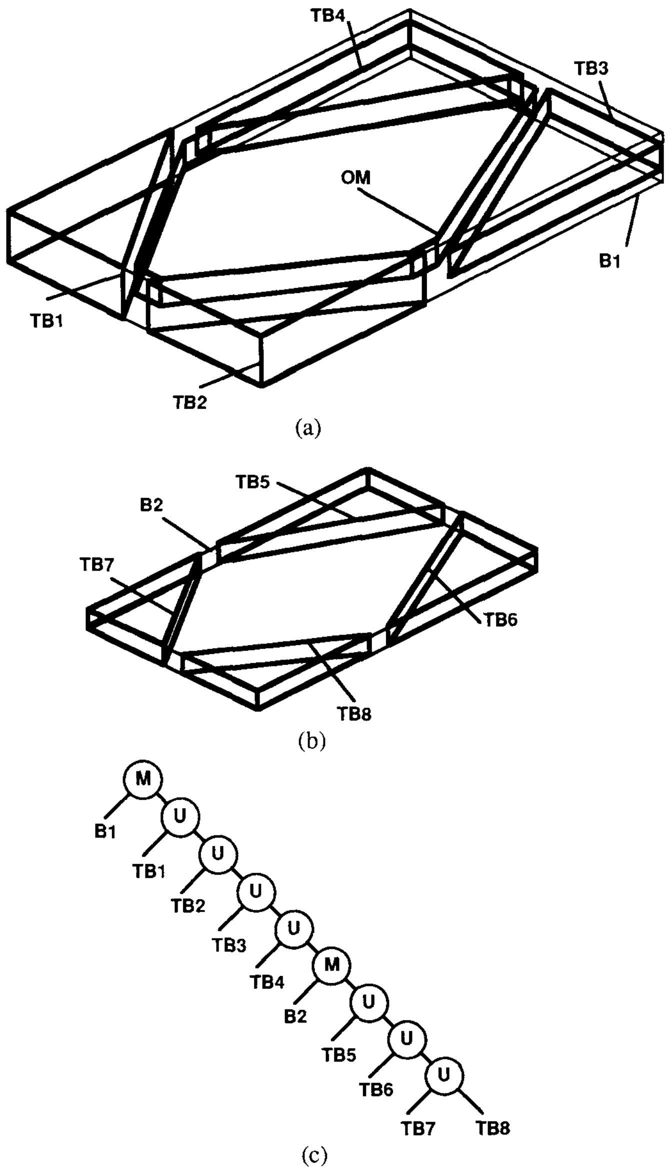


Figure 10. (a) Primitives approximating the structure, (b) primitives approximating the hole in the middle of the structure, referred to as OM, (c) CSG tree of the structure.

First, this optimization simplifies the design process, and as explained earlier, cannot be used directly as a design tool. Other performance criteria such as the maximum stress, maximum displacement, vibration analysis as well as various geometrical restrictions associated with manufacturing and assembly requirements must be considered to obtain a more realistic and practical structure.

Second, some interpretation simplifications and variances are introduced inevitably in Phase II due to manufacturing considerations and modeling approximations. The simplified topology and shape may not represent the optimum structure any more. Therefore, a design refinement must be preformed to obtain the *true* optimal structure.

A detailed design optimization process, that follows Phase II, helps overcome the mentioned shortcomings of ISOS. In this detailed optimization process, additional design constraints can be included for practical applications, and the optimal structure can be recovered from the modeling errors introduced in earlier design phases. In section 6, two design examples are presented to demonstrate the capabilities of ISOS for eigenfrequency response of structures. But, first some theoretical background on sensitivity analysis is provided.

perturbation in the design variables for every optimization iteration. In order to reduce the computational time, an efficient re-analysis method must be employed.

For static problems at a given design point, $\mathbf{d}$, the following system of finite element equations must be solved:

$$\mathbf{K}(\mathbf{d})\mathbf{u} = \mathbf{f}(\mathbf{d}) \tag{1}$$

A standard (direct) Gaussian elimination method, such as Crout method, is applied with skyline storage structure, and LU decomposition of the stiffness matrix, $\mathbf{K}(\mathbf{d})$, is obtained to compute its inverse, i.e., $\mathbf{K}^{-1}(\mathbf{d})$. This inverse matrix is used to precondition the matrices $\mathbf{K}(\mathbf{d}+\Delta\mathbf{d})$ and $\mathbf{K}(\mathbf{d}-\Delta\mathbf{d})$ and to solve the following perturbated problems.

$$\mathbf{K}(\mathbf{d}+\Delta\mathbf{d})\mathbf{u} = \mathbf{f}(\mathbf{d}+\Delta\mathbf{d}) \tag{2}$$
$$\mathbf{K}(\mathbf{d}-\Delta\mathbf{d})\mathbf{u} = \mathbf{f}(\mathbf{d}-\Delta\mathbf{d}) \tag{3}$$

Since design perturbation $\Delta\mathbf{d}$ is in general small, say 0.1%~0.01% of $\mathbf{d}$, the stiffness matrices $\mathbf{K}(\mathbf{d}+\Delta\mathbf{d})$ and $\mathbf{K}(\mathbf{d}-\Delta\mathbf{d})$ do not deviate much from the reference matrix $K(\mathbf{d})$. This proximity means that the preconditioned perturbated problems can effectively be solved by any iterative method, such as the conjugate gradient method [9].

The above iterative pre-conditioning method is also applicable to eigenvalue problems. A typical eigenvalue problem for structural problems can be formulated as follows.

$$\mathbf{A\Phi} = \mathbf{B\Phi\Lambda} \tag{4}$$

where the columns in $\mathbf{\Phi}$ are the eigenvectors and $\mathbf{\Lambda}$ is a diagonal eigenvalue matrix. For vibration problems, $\mathbf{A}$ and $\mathbf{B}$ are the stiffness and mass matrices, respectively. Whereas for linearized buckling analysis, $\mathbf{A}$ and $\mathbf{B}$ are the geometric and the linear stiffness matrices, respectively. This eigenvalue problem can be solved by the subspace iteration method [14]. The subspace iteration method first solves $\mathbf{X}_{k+1}$ with a starting iteration vector, $\mathbf{X}_k$,

$$\mathbf{A}\mathbf{X}_{k+1} = \mathbf{B}\mathbf{X}_k \tag{5}$$

in order to project the operators $\mathbf{A}$ and $\mathbf{B}$. The above equation is similar to static problems by treating $\mathbf{B}\mathbf{X}_k$ as loading vectors in Equations (1), (2), and (3). Therefore, this iterative pre-conditioning method can also be applied to eigenvalue problems.

## Design Sensitivity

One of the most important steps in the detailed design optimization is sensitivity analysis, providing the essential information on the changes in the objective and constraint functions with respect to small perturbations in the design variables. Obtaining sensitivities involves the calculation of the derivatives of the objective and constraint functions. For large structures, design sensitivity analysis requires the largest computational time within the overall optimization process.

The calculation of design sensitivities can be preformed by a discretized approach which uses the finite-element model to capture the structural behavior. The following three different computational schemes are commonly used in the discretized approach: the analytical method, the semi-analytical method, and the finite-difference approximation. Although the analytical and the semi-analytical methods have drawn much attention from researchers, the most widely applicable method remains the finite-difference approximation.

The finite-difference approximation is relatively simple to implement, but requires extensive function evaluations. The objective and constraint functions must be evaluated for every perturbated model, requiring a finite-element analysis for each

## 6. DESIGN EXAMPLES FOR EIGENFREQUENCY PROBLEMS

### Example 1: Cantilever Beam

Consider a short cantilever beam subjected to a vertical load in the middle of the right end, as shown in Fig. 11. The entire left side of the boundary is clamped. The optimum topology generated by the homogenization method is shown in Fig. 12, and its boundaries are shown in Fig. 13. Since the first eigenmode of the structure is the bending mode, a new optimization problem can be formulated to maximize the minimum eigenfrequency. The volume constraint is imposed as an equality constraint. The design variables are the locations of the boundary nodes. The initial design, shown in Fig. 14, has its first eigenfrequency at 29.6 Hz, and weighs 1.51 kg. The final design, shown in Fig. 15, has its first eigenfrequency at 31.1 Hz, and weighs 1.51 kg. The first eigenfrequency is increased only by 4.9%, indicating the fact that the shape generated in Phase I is nearly optimal. This is due to the fact that the first mode of the structure is its bending mode and the optimum topology which generated by the homogenization method is subject to the bending load.

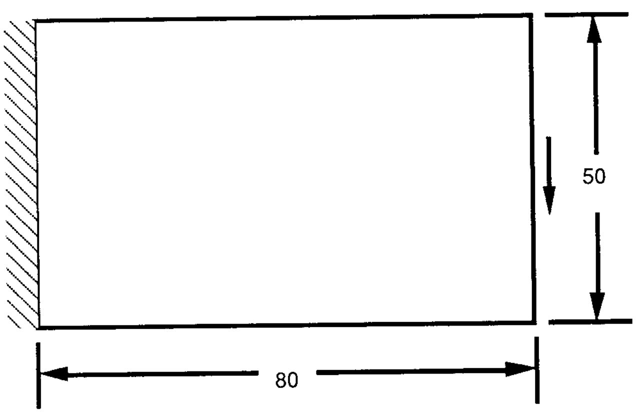


Figure 11. Initial design domain and loading and boundary specifications for a cantilever beam.

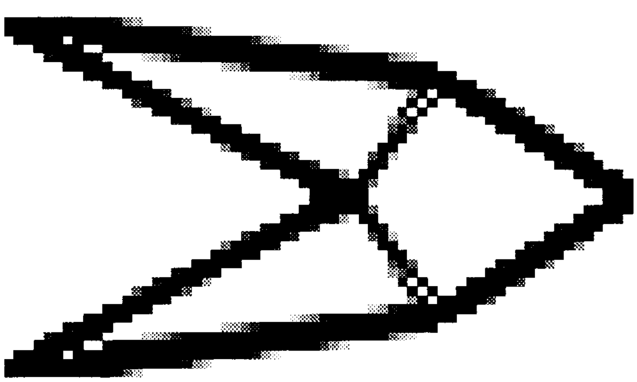

Figure 12. The optimum topology resulting from Phase I (homogenization method).

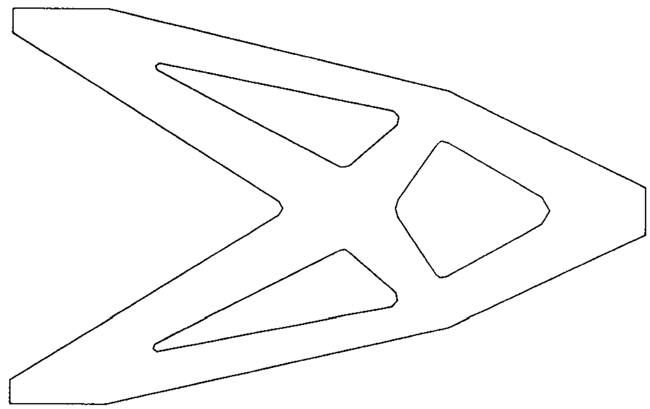

Figure 13. The extracted boundaries of the optimum topology.

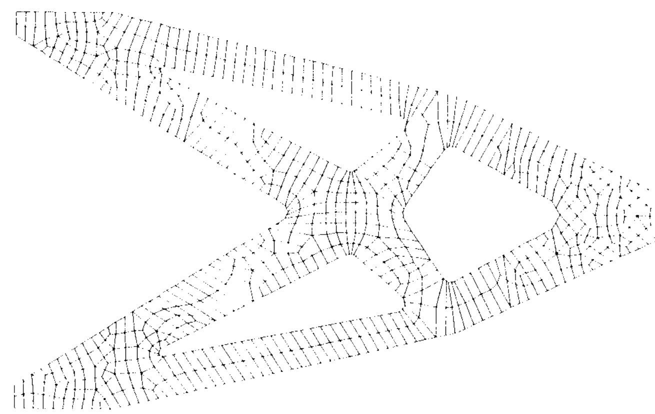

Figure 14. The initial finite-element model (input to Phase III).

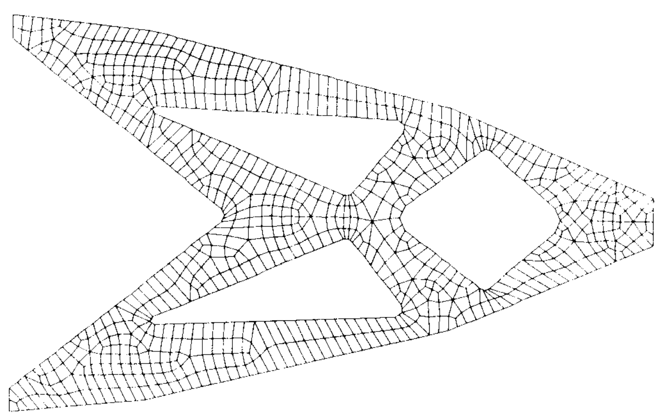

Figure 15. The final finite-element model (output from Phase III).

### Example 2: Long Beam

Figure 16 shows the design domain for a long beam whose both ends are clamped. Such beams can be used to model, for example, bridges. The different topologies, shown in Fig. 17, are the outputs of the homogenization method (Phase I) for different volume constraints [15]. Homogenization (Phase I) has maximized the first eigenfrequency subject to the specified volume constraints. Based on these images, a simplified shape with one center hole and two side arcs is generated, as shown in Fig. 18. Because of the symmetry of the structure, only half of the beam is modeled for the detailed design optimization process. The structure shown in Fig. 18 has an initial mass of 1.35 kg, and its first eigenfrequency is 27.5 Hz. In the detailed design optimization process, the upper and lower bounds of, respectively, 1.45 and 1.25 kg are imposed on the weight of the design. Additionally, some appropriate geometry constraints are imposed for every design variable representing the shape of the structure. The final design, shown in Fig. 19, has its lowest eigenfrequency at 29.0 Hz and weighs 1.43 kg. Thus, a relative increase of 5.5% in the lowest eigenfrequency is obtained.

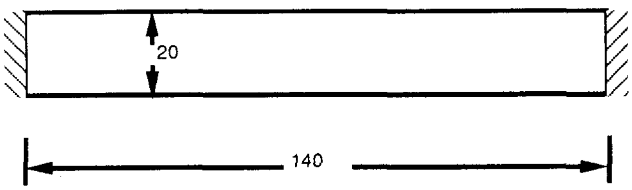


Figure 16. Initial design domain and boundary specifications for a long beam.

## 7. CONCLUSIONS

The integrated structural optimization system (ISOS), described in this paper, offers a new capability for structural design. The ability to generate topologies on a rigorous analytical foundation has opened the way for integrating several tools from different disciplines, such as structural mechanics, manufacturing, computer vision, expert systems, and mathematical optimization. A unique attribute of this system is its capability to allow examination of design constraints in many different domains. Most of the activities of ISOS for two-dimensional structures have been automated. Only nominal user interaction is required, once the initial model is set up for topology-generation (Phase I). Extending the capabilities of ISOS to three-dimensional structures has been initiated and remains a challenging task for all the disciplines mentioned above. Extensions of the capabilities of ISOS to structural dynamics problems has been successful and seems promising.

### ACKNOWLEDGEMENTS

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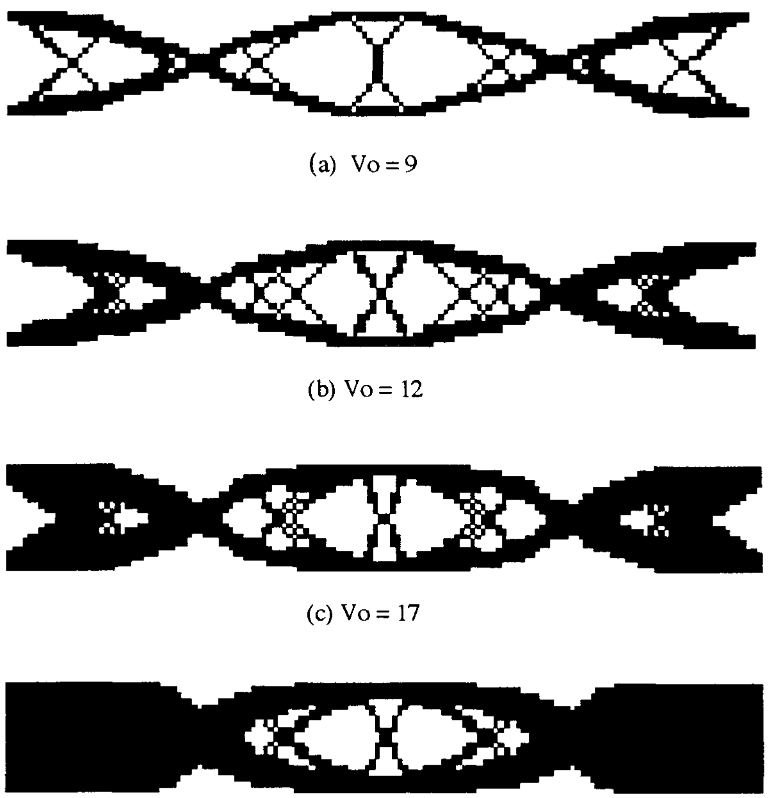

(a) Vo = 9

(b) Vo = 12

(c) Vo = 17

(d) Vo = 22

Figure 17. The optimum topology images from the homogenization method (Phase I) for different volume constraints [15].

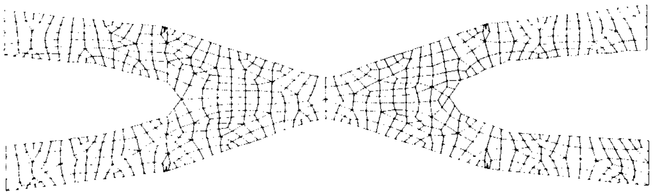

Figure 18. The initial finite-element model (input to Phase III).

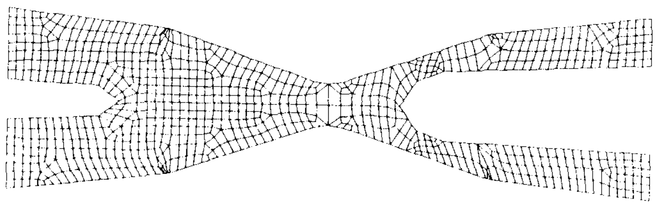

Figure 19. The final finite-element model (output from Phase III).

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