values of $B$. This value corresponds to natural convection from an isothermal flat plate embedded in a porous medium as obtained using the Oseen linearization. A similarity (exact) solution to this problem was performed by Cheng and Mikowycz yielding $N_{u} = 0.888 R_{a}^{0.75}$. In the other extreme, as $A$ becomes very large, the overall heat transfer diminishes for all values of the wall conductivity and therefore the thermal communication between the pipe fluid and the porous material. In all cases, the counter-flow configuration yields a higher overall heat flux than the parallel-flow configuration. This effect, however, weakens as the parameter $B$ decreases, such that for values of $B < 0.1$ the overall heat transfer through the pipe is identical for both cases. Note that decreasing $B$ while keeping $A$ constant is equivalent to increasing the flow rate in the pipe.

**Conclusions**

In this technical Note, a simple yet reliable analysis was presented for the problem of counter-flow and parallel-flow convection in a vertical pipe surrounded by a porous material. Important results revealed interesting features of the temperature distribution of the pipe outer surface, of the mean fluid temperature in the pipe, and of the overall heat flux from the pipe to the surroundings. As the values of parameters $A$ and $B$ approach zero, the outer pipe surface approached an isothermal condition. A maximum was observed in the $\theta_{_{0}}$ distribution in the parallel-flow case. This maximum is more pronounced and occurs closer to the pipe inlet for larger values of $B$.

The overall heat flux through the pipe reaches a plateau as $A$ decreases. This plateau corresponds to natural convection from an isothermal vertical wall embedded in a porous medium. The counter-flow configuration yields higher overall heat transfer than for the parallel-flow configuration. This feature diminishes as the pipe flow rate is increased (or the parameter $B$ is decreased).

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**References**


**Entropy Production in Boundary Layers**

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**Introduction**

The interpretation of the contemporary problems of thermomechanics in terms of entropy production is lately receiving increased attention. Because of its size, no attempt will be made here to survey the literature (see, for example, Bejan[1-2] for applications involving heat transfer and Arpaci[3-4] and Arpaci and Selamet[5-6] for applications involving radiation and flames). The following brief review on the local entropy production is for later convenience.

The development of the entropy production in moving media requires the consideration of the momentum, energy, and entropy balances. The fundamental difference,

$$\frac{\text{Total energy}}{\text{(Momentum)}} - \text{(Entropy)} T$$

may be rearranged to yield

$$\rho \left( \frac{Du}{Dt} - T \frac{Ds}{Dt} + p \frac{Dv}{Dt} \right) = - \left( \frac{q_{i}}{T} \frac{dT}{dx} + \tau_{0} \frac{\partial q_{i}}{\partial x} + u - T s \right)$$

where $s_{0}$ is the rate of deformation. For a reversible process, all forms of dissipation vanish, and

$$\left( \frac{Du}{Dt} - T \frac{Ds}{Dt} + p \frac{Dv}{Dt} \right) = 0$$

which is the Gibbs Thermodynamic relation. For an irreversible process, Eq. (3) continues to hold provided the process can be assumed in local equilibrium. Then, the local entropy production is found to be

$$s = \frac{1}{T} \left( \frac{q_{i}}{T} \frac{dT}{dx} + \tau_{0} \frac{\partial q_{i}}{\partial x} + u - T s \right)$$

where the first term in brackets denotes the dissipation of thermal energy into entropy (lost heat), the second term denotes the dissipation of mechanical energy into heat (lost work), and the third term denotes the dissipation of any (except thermomechanical) energy into heat. When radiation is appreciable, $q_{i}$ denotes the total flux involving the sum of the conductive flux and the radiative flux

$$q_{i} = q_{c} + q_{r}$$

Neglecting contribution of viscous dissipation and assuming conductive and radiative heat fluxes to be in the transversal direction, Eq. (4) may be rearranged as

$$s = - \frac{1}{T} \left( q_{c} + q_{r} \frac{\partial T}{\partial y} \right)$$

Foregoing general considerations are applied below to a forced convection boundary layer.

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Radiation Affected Forced Convection

Consider the effect of radiation on the forced convection boundary layer over a horizontal flat plate. For heat transfer studies, rather than velocity profiles, a good approximation of these profiles near boundaries is convenient. This approach, in the absence of radiation, is well known and has been studied extensively (see Curle for an early reference and Arpaci and Larsen for a later reference). Also, the extension of the approach to the limiting cases of \( Pr < 1 \) and \( Pr > 1 \) are discussed in Arpaci and Larsen. Since the case of \( Pr < 1 \) is for opaque fluids and has no application to radiation-affected problems and the case of \( Pr > 1 \) is known to approximate for all fluids with \( Pr > 1 \), here only the latter case is considered.

Replacing the longitudinal velocity by its tangent on the wall and using this velocity in the conservation of mass to determine the transversal velocity and including the radiation effect, the thermal energy balance gives

\[
\rho c_p \left[ y \left( \frac{\tau_w}{\mu} \right) \frac{\partial T}{\partial x} - \frac{1}{2} \left( \frac{\tau_w}{\mu} \right) \frac{\partial T}{\partial y} \right] = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_y}{\partial y} \tag{7}
\]

subject to (Lord and Arpaci)

\[
\frac{\partial q_y}{\partial y} = 4 \kappa_p \left[ (E_b - E_{bw}) - \frac{e_w}{2} (E_{bw} - E_{bw}) E_2 (r) \right] \tag{8}
\]

where \( \tau_w \) denotes the wall shear stress, \( \kappa_p \) the Planck mean absorption coefficient, \( E_2 \) the emissive power, \( e_w \) the wall emissivity, \( E_2 \) the second exponential integral, and \( r \) the optical thickness. The boundary conditions to be satisfied are

\[
T(0,y) = T_w, \quad T(x,0) = T_w, \quad T(x,\infty) = T_w \tag{9}
\]

A similarity variable including both conduction and radiation is not feasible because of intrinsic lack of similarity between conduction and radiation. However, the effect of thin gas radiation on conduction is small. This fact suggests the use of the similarity variable for conduction by which the radiation effect can be treated locally similar.

Introducing \( \eta = y/g(x) \) (see, for example, Arpaci and Larsen), into Eq. (7) leads to the equation satisfied by \( g(x) \),

\[
\left( \frac{\tau_w}{\mu} \right) \frac{\partial g}{\partial x} + \frac{3}{2} \eta^2 \frac{\partial \left( \frac{\tau_w}{\mu} \right)}{\partial x} = \alpha \tag{10}
\]

which readily gives

\[
g(x) = \frac{\left( \frac{\eta}{\alpha} \right)^{2/3} \left( \frac{\tau_w}{\mu} \right)^{1/2} dx}{\left( \frac{\tau_w}{\mu} \right)^{1/2}}
\]

and

\[
\eta = \left( \frac{\tau_w}{\mu} \right)^{1/2} \frac{y}{\alpha} \tag{11}
\]

In terms of Eqs. (10) and the approximation \( E_2 = \exp(-\sqrt{3} \tau) \), Eqs. (7) and (8) are combined to

\[
d^2\theta \left( \frac{d\theta}{d\eta} \right)^2 = 3 \frac{d\theta}{d\eta} \right) dx = \kappa_k \frac{\tau_w}{\mu} \frac{\partial g}{\partial y} \tag{12}
\]

subject to \( \theta(0) = 1 \) and \( \theta(\infty) = 0 \). Here, \( \chi = (k_p/k_R)^{1/2} \) is the weighted nongrayness, \( \kappa_k \) the Rosseland mean absorption coefficient and

\[
\theta = \frac{T - T_w}{T_w - T_0}, \quad \Theta^* = \frac{T^* - T_w^*}{T_w - T_0}, \quad \gamma = \sqrt{3} \kappa_M G \tag{13}
\]

and

\[
g = G \chi^{1/2}, \quad G = \left[ \frac{4 \chi^3}{0.332 U_w (U_w/\nu)^{1/2}} \right]^{1/3} \tag{14}
\]

In terms of \( \eta \) and \( \theta \), the conductive constitution becomes

\[
n = \frac{\kappa_k}{\kappa_k} \left( \frac{\partial \theta}{\partial \eta} \right) \left( \frac{\partial \theta}{\partial \eta} \right) \tag{15}
\]

where \( \eta \) and \( g \) are defined by Eqs. (10) and (12), respectively. Inserting \( T \), the thin gas radiative heat flux, and the conductive heat flux expressed by Eq. (13) into Eq. (6), the volumetric local entropy production is found as

\[
\frac{s}{\eta} = \left( \frac{\partial \eta}{\partial \theta} \right) \frac{\partial \eta}{\partial \theta} \left( \frac{T_w}{T_w - T_0} \right)^2 \tag{16}
\]

Fig. 1 Dimensionless temperature versus similarity variable.
In terms of Eq. (20), the wall heat flux from Eq. (17) yields

$$q_w^R = \epsilon_w (E_{bw} - E_{bw0}) \left( 1 - \frac{3}{4} \tau_a \right)$$

This relation apparently excludes the effect of conduction. To include this effect, reconsider the conditions given by Eq. (18), and in place of Eq. (19), now utilize the wall balance of the thermal energy

$$k \frac{d^2 T}{dy^2} \bigg|_w = \frac{dq_w^R}{dy} \bigg|_w$$

which in terms of Eq. (8) may be rearranged to give

$$k \frac{d^2 T}{dy^2} \bigg|_w = 4\kappa (1 - \frac{\alpha T}{2}) (E_{bw} - E_{bw0})$$

Also, from the (linearized) Stefan-Boltzmann law

$$\frac{dE_b}{d\tau^2} \bigg|_w = 12\chi (1 - \frac{\alpha T}{2}) (E_{bw} - E_{bw0})$$

where \( \Theta = 4\kappa T'_{bw}/3kT'_{bw}\kappa_M \). Then, the polynomial approximation subject to Eqs. (18) and (25) yields

$$E_b = E_{bw} - E_{bw0} = \frac{1}{2} \left[ -\left( 3 + 1/2 \frac{\Theta}{\tau_a} \right) \frac{T}{\tau_a} + \frac{\Theta}{\tau_a} \left( \frac{T}{\tau_a} \right)^2 \right]$$

$$+ \left( 1 - \frac{\Theta}{\tau_a} \right) \left( \frac{T}{\tau_a} \right)^3$$

where \( \Theta = 12\chi (1 - \epsilon_w/2) T'_{bw} \). In terms of Eq. (26), Eq. (17) results in

$$q_w^R = \epsilon_w (E_{bw} - E_{bw0}) \left[ 1 - \tau_a \left( \frac{3}{4} \left( 1 - \frac{\epsilon_w}{2} \right) r_{sp}\Phi \right) \right]$$

which shows the explicit effect of conduction on the radiative heat flux. However, for the thin gas radiation, \( \tau_a \Theta \ll 1 \), and, to first order, the explicit effect of conduction on the radiation flux is negligible, and Eq. (27) reduces to Eq. (21), which is the upper limit of the radiative flux obtained from strict radiative considerations. Now, in terms of this flux, the total heat transfer becomes

$$q_w = -k \frac{\partial T}{\partial y} \bigg|_w + \epsilon_w (E_{bw} - E_{bw0}) \left( 1 - \frac{3}{4} \tau_a \right)$$

where, after neglecting the effect of thin gas radiation on the thermal boundary layer, \( \tau_a = \kappa_M \Delta = \kappa_M \delta_{visc}/Pr_{visc}^{1/3} \). From approximate studies on viscous boundary layers, \( \delta = 5.0\kappa/Re_{visc}^{1/3} \), and \( \tau_a = 5.0\kappa/Re_{visc}^{1/3} \). Also, from thermal boundary-layer studies,

$$Nu^e \equiv 0.629(-d\Theta/d\eta_{bw})Re_{visc}^{1/3}Pr_{visc}^{1/3}$$

which, for the pure conduction case

$$(-d\Theta/d\eta_{bw})^K = 0.538$$

gives

$$Nu_{bw}^K = 0.339 Re_{visc}^{1/2} Pr_{visc}^{1/3}$$

and \( \tau_a = \frac{5}{3} \tau_x/Nu_x^K \)
Thus
\[
\frac{\text{Nu}_s}{\text{Nu}_s^K} = \frac{(-\partial \theta/\partial y|_w) + \frac{3}{4} \epsilon \text{Pr} \left( \frac{\tau_{\text{vis}}}{\text{Nu}_s^K} \right) \left( 1 - \frac{5}{4} \frac{\tau_{\text{vis}}}{\text{Nu}_s^K} \right)}{(-k \partial \theta/\partial y|_w) + \frac{3}{4} \epsilon \text{Pr} \left( \frac{\tau_{\text{vis}}}{\text{Nu}_s^K} \right) \left( 1 - \frac{5}{4} \frac{\tau_{\text{vis}}}{\text{Nu}_s^K} \right)}
\]
and the local thermal entropy production on the wall is
\[
s^*_{\text{w}} = -\frac{1}{T^2_w} \left( q^*_{\text{x}} + q^*_{\text{y}} \right) \left( \frac{\partial T}{\partial y} \right)_w \quad (31)
\]
Introducing a wall local entropy production number, \( \Pi_s = s^*_{\text{w}} x^2/k \), Eq. (31) may be arranged as
\[
\Pi_s = \left( 1 - \frac{T^*_{\text{w}}}{T_w} \right)^2 \left( 1 + q^*_{\text{x}} \frac{\partial T}{\partial y} \right)_w \quad (32)
\]
With the definition of local Nusselt number
\[
\text{Nu}_s = \frac{q^*_{\text{x}}}{T^*_{\text{w}}} = \frac{q^*_{\text{y}}}{T^*_{\text{w}}} = \frac{(\partial T/\partial y)_w}{(T_w - T^*_{\text{w}})_w} \quad (33)
\]
Eq. (32) may finally be expressed as
\[
\Pi_s = \left( 1 - \frac{T^*_{\text{w}}}{T_w} \right)^2 \left( 1 + q^*_{\text{x}} \frac{\partial T}{\partial y} \right)_w \text{Nu}_s^2 \quad (34)
\]
Concluding Remarks
The radiation-affected forced convection over a flat plate is investigated in terms of thin gas. The distribution of entropy production within and outside the radiation-affected thermal boundary layer is evaluated. The retained nonlinearity of temperature in the entropy production leads to an extremum in this production within the boundary layer rather than on the boundary.

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Attenuating Thin Gas

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Introduction

A DERIVATION of the monochromatic intensity balance (transfer equation) under the influence of emission, absorption, and scattering is available in the literature. The following brief review is for later convenience.

The monochromatic transfer equation integrated over the frequency domain gives
\[
l \frac{\partial I}{\partial x_j} = \kappa I_o + \frac{\sigma_\text{I}}{4\pi} \int P(l_i, l_j) I(l_i) \, d\Omega - \beta I
\]
where \( I \) is the intensity, \( I_o \) its equilibrium state, \( \kappa \) the absorption coefficient, \( \sigma_\text{I} \) the scattering coefficient, \( \Omega \) the solid angle, and \( P(l_i, l_j) \) the phase function that satisfies
\[
\frac{1}{4\pi} \int P(l_i, l_j) \, d\Omega' = 1
\]
l being the direction of the optical energy balance and \( l_i \) the direction of the scattering.

The first specular moment of Eq. (1) yields the radiative energy balance
\[
\frac{\partial q^R}{\partial x_j} = 4\pi E_r + \frac{\sigma_\text{I}}{4\pi} \int P(l_i, l_j) I(l_i) \, d\Omega - \beta q^R
\]
where \( q^R = I_g l / |l| \) is the radiative heat flux in the \( x_i \) direction, \( E_r = \pi I_o \) the equilibrium blackbody emissive power, and \( J = |l| / |l| \) the specular integrated intensity. In view of Eq. (2) and
\[
\int_{l_i} P(l_i, l_j) \, d\Omega = \int_{l_i} P(l_i, l_j) I(l_i) \, d\Omega = 4\pi J
\]
Eq. (3) may be rearranged as
\[
\frac{\partial q^R}{\partial x_j} = \kappa (4E_r - J)
\]
The second specular moment of the transfer equation leads to the radiative momentum balance
\[
\frac{\partial \Pi_{lj}}{\partial x_j} = \frac{\sigma_\text{I}}{4\pi} \int P(l_i, l_j) \Pi_{lj} \, d\Omega - \beta q^R
\]
where \( \Pi_{lj} \) is related to the radiative stress \( r^R \) by
\[
r^R = \frac{1}{c} \int_{l_i} l_i l_j \, d\Omega = \frac{1}{c} \Pi_{lj}
\]
c being the velocity of light.

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