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A MODEL FOR RAIN EROSION

OF HOMOGENEOUS MATERIALS

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FOREWORD

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This report covers the work carried out during the period from June 1971 through May 1972.

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ABSTRACT

The behavior of homogeneous materials subjected to repeated impingements of liquid droplets is investigated. Based on fatigue theorems, a model is presented for describing both the incubation period n_i (i.e., the time elapsed before the mass loss of the material becomes appreciable), and the mass loss past the incubation period m. The parameters are established which govern the length of the incubation period and the subsequent mass loss rate, and simple algebraic expressions are developed relating n_i and m to the properties of the impinging droplets and the material. The limits of applicability of the model are also established.

The results obtained are compared to available experimental data. Reasonable agreement is found between the present results and the data, indicating that the model developed can be used to estimate the incubation period and the mass loss of the material.

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ILLUSTRATIONS

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NOMENCLATURE+

a_1-a_{10}	constants (dimensionless)
A	area (ft^2)
b	constant defined by Eq. (19), (dimensionless)
b_1	constant in Eq. (16), (dimensionless)
b_2	knee in the fatigue curve (see Fig. 4)
C	speed of sound (ft/sec)
d	diameter of the droplet (ft)
E	modulus of elasticity (lbf/ft^2)
f	number of stress cycles, (see Eq. 9)
F	force (lbf)
I	rain intensity (ft/sec)
m	mass eroded per unit area (lbm/ft^2)
m^*	dimensionless mass loss defined by Eq. (40)
n	number of drops impinging per unit area ($number/ft^2$)
n^*	number of drops impinging per site, (see Eq. (22), dimensionless)
n_a^*	characteristic life (dimensionless)
N	fatigue life (see Fig. 4), (dimensionless)
p	probability defined by Eq. (27), (dimensionless)
P	stress (lbf/ft^2)
q	drop density ($number/ft^3$)
r	distance (ft)

+ In this list the units are indicated in the English Engineering System (ft, sec, lbm and lbf). However, any consistant set of units may be used in the equations.

S	parameter defined by Eq. (21), (lbf/ft^2)
t	time (sec)
V	velocity of impact (ft/sec)
V_t	terminal velocity of a rain droplet (ft/sec)
W	weight loss due to erosion (lbf)

GREEK LETTERS

α	rate of mass loss (lbm/impact) (see Fig. 2b)
α^*	dimensionless rate of mass loss (see Eq. 33)
β	Weibull slope in Eq. (27), (dimensionless)
ν	Poisson's ratio (dimensionless)
ρ	density (lbm/ft^3)
θ	angle (radians)
σ	stress (lbf/ft^2)
σ_I	endurance limit (lbf/ft^2)
σ_u	ultimate tensile strength (lbf/ft^2)

SUBSCRIPTS

II	quantities associated with Region II
i	end of incubation period
f	upper limit of validity of model
L	liquid
s	solid

SECTION I

INTRODUCTION

Aircraft components such as radomes, leading edge surfaces, helicopter blades, and various structural members may experience heavy damage when subjected to repeated impingements of rain droplets. Liquid droplets may also cause significant damage to steam turbine blades. Owing to the severity of the problem numerous investigations have been concerned with the effects of liquid impingement on surfaces. Excellent reviews summarizing previous experimental and analytical work have been given, among others, by Eisenberg (Reference 13), Engel (Reference 15), Heymann (References 32 and 33), Heymann and Arcella (Reference 35) and Wahl (Reference 65). Many of the recent studies dealing with the problem are also described in the Proceedings of the First, Second and Third Rain Erosion Conferences (References 23-25).

The majority of previous studies on the subject of rain erosion have been experimental in nature, with the bulk of prior research concentrating on the measurement of an erosion parameter (e.g. total weight loss) of a sample subjected to specific conditions. Such empirical studies provide information on the behavior of a given material under a given condition, but fail to describe material behavior beyond the range of the experiments in which they were obtained. Recently, attempts have been made to derive analytical or semi-empirical formulae which describe the dependence of material damage on selected operating variables, such as impact velocity and droplet size (References 29, 32, 33, 35, 53 and 58). Although these studies shed light on some of the mechanisms which contribute to material damage, a satisfactory method has not yet been devised which is capable of correlating the existing data and generalizing the results obtained from a few experiments. The objective of this investigation is to develop a model which is consistent with experimental observations and which predicts quantitatively "erosion" of materials under previously untested conditions. It is evident, that such a model is needed for the selection of the proper materials and for the design of appropriate structures and components subject to severe liquid impingement.

The model proposed here is aimed at describing a) the "incubation period", i.e. the time elapsed before the mass loss of the material becomes appreciable, and b) the degradation of the material past the incubation period as manifested by its mass loss. The existence of an incubation period suggests that the damage produced in the material is the consequence of cumulative fatigue damage produced by the repeated impacts of the droplets. Therefore, the present model is based on fatigue concepts. The role of fatigue in rain erosion has been recognized in the past (References 9, 48, 49, 57 and 63), and attempts have been made to describe the material damage in terms of fatigue parameters (References 32, 33 and 51). However, expressions for the incubation period and for the subsequent material loss have not yet been derived.

SECTION II

THE PROBLEM

The problem investigated is the following. Spherical liquid droplets of constant diameter d impinge repeatedly upon a semi-infinite, homogeneous material (Fig. 1). The angle of incidence of the droplets θ, and the velocity of impact V are taken to be constant. The spatial distribution of the droplets is considered to be uniform. Thus, the number of droplets impinging on unit area in time t is

$$n = (V \cos\theta) q\, t \tag{1}$$

where q is the number of droplets per unit volume. Rain, falling with constant terminal velocity V_t, is usually characterized by a parameter I called "intensity", which is related to q by the expression

$$q = \frac{6}{\pi} \frac{I}{V_t d^3} \qquad (2)^*$$

In Eq. (2) I has the units of length/time. Values for V_t may be obtained from charts (Reference 27), or may be calculated from the empirical relationship (Reference 1)

$$V_t = 965 - 1030\, e^{-6d} \quad \text{(cm/sec)} \tag{3}$$

where d is in cm. Equations (1) and (2) may be combined to yield

$$n = \frac{6}{\pi} \frac{(V \cos\theta) I}{V_t d^3}\, t \tag{4}$$

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* Any consistent set of units may be used in Eq. (2). Customarily, I is expressed in in/hr, V_t in ft/sec and d in mm. For these units

$$q \cong 1250 \frac{I}{V_t d^3} \quad \text{(number/cu ft)}$$

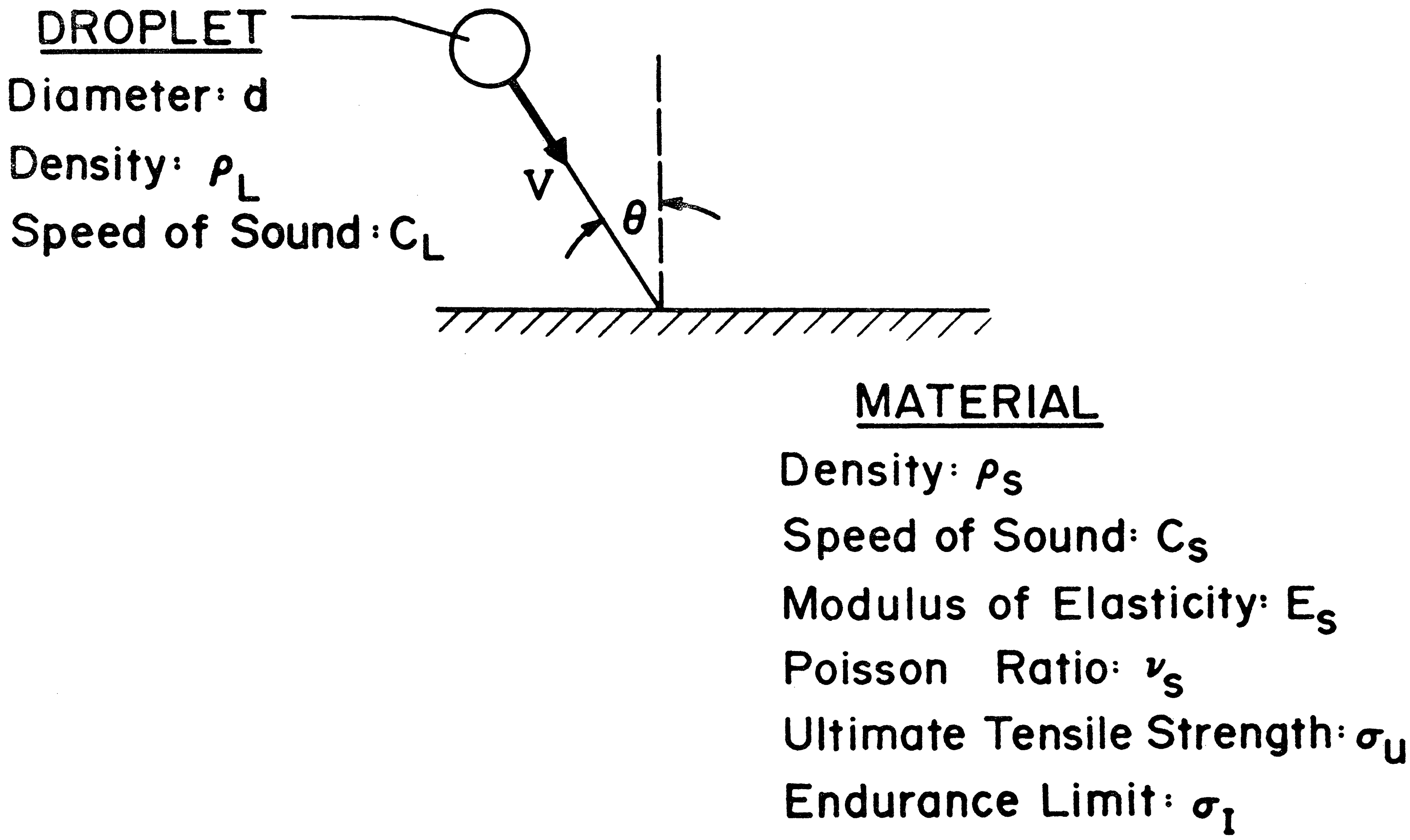

Figure 1. Droplet Impingement on a Homogeneous Material. Description of the Problem.

The impingement rate is assumed to be sufficiently low so that all the effects produced by the impact of one droplet diminish before the impact of the next droplet. This assumption is justified since, in practice, the time between subsequent impingements at a point is of the order of 10^{-2} sec, while the stresses produced in the material become negligible in about 10^{-6} sec (Reference 33).

The pressure within the droplet varies both with position and with time. For simplicity, the pressure will be taken to be constant, its value being given by the water hammer pressure (Reference 34)

$$P = \frac{\rho_L C_L V \cos\theta}{1 + \dfrac{\rho_L C_L}{\rho_s C_s}} \tag{5}$$

where ρ is the density, and C the speed of sound. The subscripts L and s refer to the liquid and the solid, respectively. In terms of the modulus of elasticity E_s

$$C_s = \sqrt{E_s/\rho_s} \tag{6}$$

Although more accurate representation of the pressure is possible (Reference 34), the accuracies afforded by the use of Eq. (5) will suffice in the present analysis. Thus, the force imparted to the surface by each droplet is

$$F = P\,\frac{\pi d^2}{4} \tag{7}$$

The forces, created by the repeated droplet impacts, damage the material as manifested by the formation of pits and cracks on the surface, and by weight loss of the material. Experimental evidence indicates that under a wide range of conditions the weight loss W varies with time t as shown, schematically, in Fig. 2a. For some period of time, referred to as incubation period, the weight loss is insignificant. Between the end of the incubation period t_i and a time denoted by t_f the weight loss varies nearly linearly with time. After t_f the relationship between W and t becomes more complex. Here, we will be concerned only with the behavior of the material up to time t_f. In most practical situations the usefulness of the material does not extend beyond t_f.

It is advantageous to replace the total weight loss of the sample by the mass loss per unit area m, and the time by the number of droplets impinging upon unit area n. In terms of the parameters m and n, schematic representation of the data is given in Fig. 2b. It is now assumed that the data can be approximated by two straight lines as shown in Fig. 2b, i.e.

$$m = 0 \ , \qquad 0 < n_i \tag{8a}$$

$$m = \alpha \ (n-n_i) \ , \qquad n_i < n < n_f \tag{8b}$$

Thus, the material loss m produced by a certain number of impacts n, can be calculated once the incubation period n_i and the rate of subsequent mass loss (as characterized by the slope α) are known. Therefore, the problem at hand is to determine the parameters n_i, α, and n_f, the latter being the upper limit of validity of Eq. (8b).

It is noted here that the above model is valid only if there is an incubation period. It has been observed experimentally that under some conditions even one impact will result in appreciable damage, a situation in which $t_i \cong 0$. Problems of this type will be discussed in Section VII.

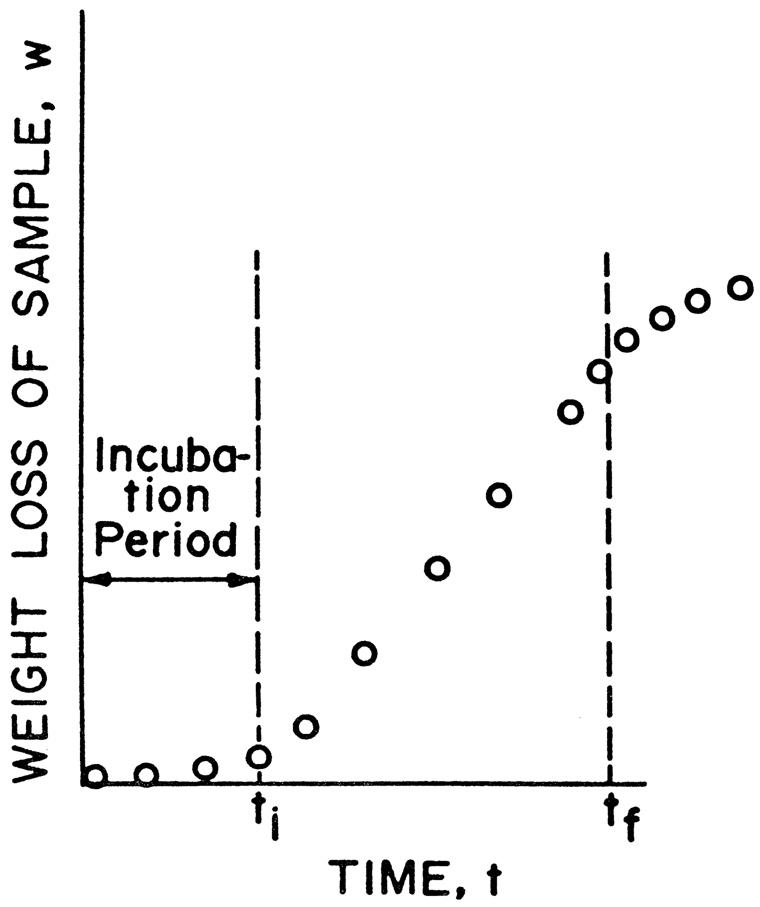

Figure 2a. Schematic of the Experimental Results.

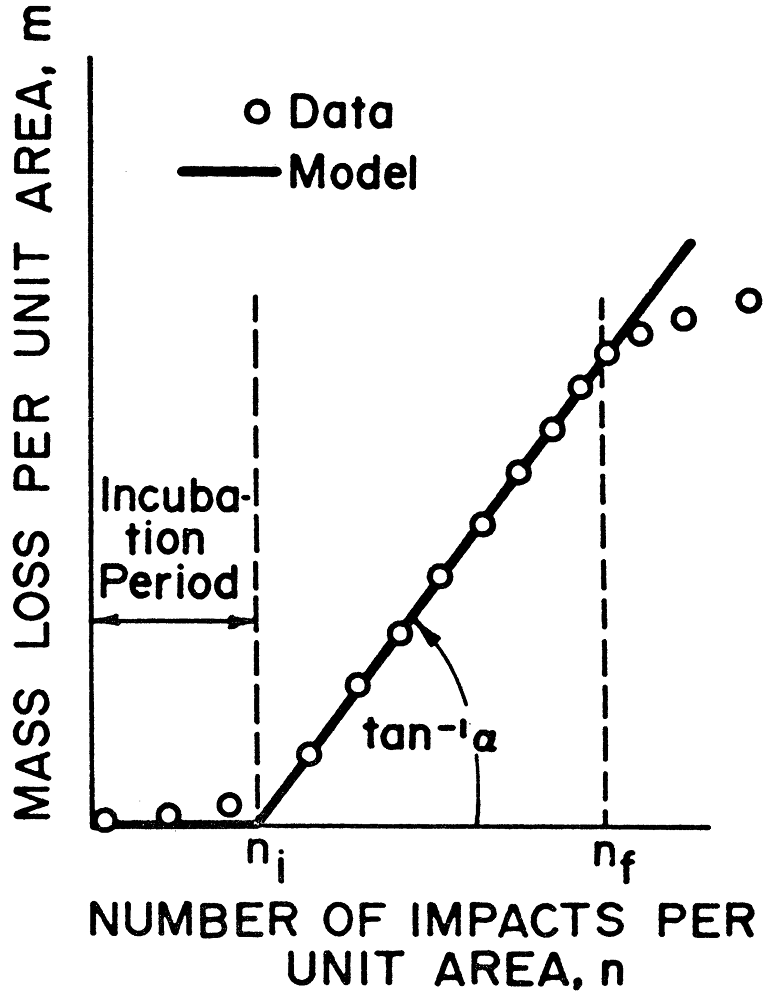

Figure 2b. The Solution Model.

SECTION III

INCUBATION PERIOD n_i

It has been recognized that fatigue plays an important role in the erosion process (References 9, 32, 33, 48, 49, 51, 57 and 63), particularly in the "early" stages of the process, corresponding to the incubation period. Therefore, it is expected that fatigue theorems established for the torsion and bending of bars might be applied, at least qualitatively, to materials subjected to repeated liquid impingement. The failures of bars undergoing repeated torsion or bending have been found to follow Miner's rule (Reference 50), which is expressed by the following equation

$$\frac{f_1}{N_1} + \frac{f_2}{N_2} + \ldots\frac{f_k}{N_k} = a_1 \tag{9}$$

where f_1, $f_2 \ldots f_k$ represent the number of cycles the specimen is subjected to specified overstress levels σ_1, $\sigma_2 \ldots \sigma_k$, and N_1, $N_2 \ldots N_k$ represent the life (in cycles) at these overstress levels, as given by the fatigue (σ vs N) curve. a_1 is a constant which in torsion and bending tests generally varies between 0.6 and 2.2.

Let us now consider one element at point B on the surface of the material as shown in Fig. 3. Each droplet impinging on the surface will create a stress at point B. Assuming that the force created by the droplet at its point of impact is a "point force", the stress at point B due to any one droplet is (Reference 64)

$$\sigma(r) = \frac{F(1-2\nu_s)}{2\pi r^2} \tag{10}$$

where ν_s is the Poisson ratio. Every droplet which falls at a radius r

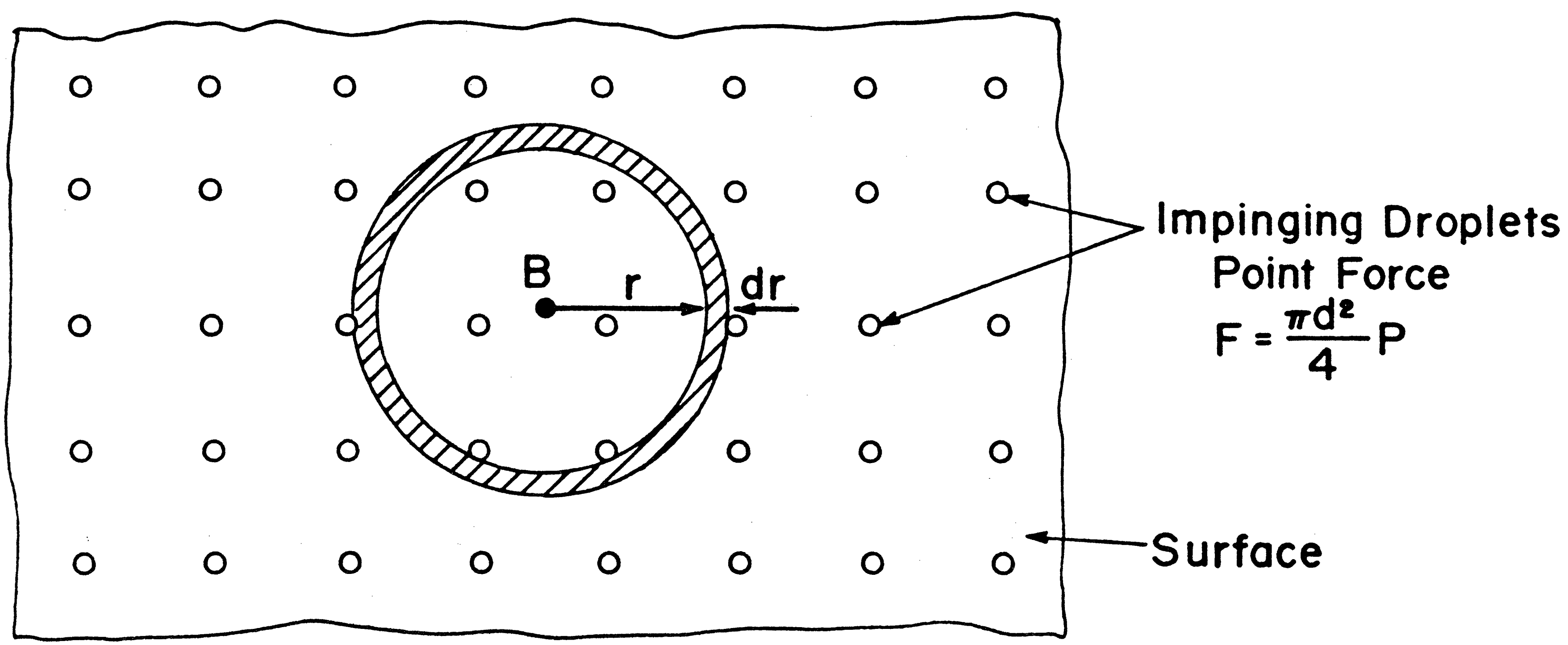

Figure 3. Force Distribution on the Surface.

produces the same stress at point B. Therefore, the number of cycles for which the material at point B is subjected to a given stress between σ and $\sigma + d\sigma$ is equal to the number of impacts on a dr wide annulus located at r (Fig. 3). During the incubation period the total number of impacts on the annulus is

$$f(r) = n_i 2\pi r dr \tag{11}$$

Therefore, f_1, $f_2 \ldots f_k$ in Eq. (9) are replaced by $f(r_1)$, $f(r_2) \ldots f(r_k)$, i.e.,

$$\frac{f(r_1)}{N_1} + \frac{f(r_2)}{N_2} + \ldots . \frac{f(r_k)}{N_k} = a_1 \tag{12}$$

Since r varies continuously from zero to infinity, Eqs. (11-12) mav be written as

$$\int_0^\infty \frac{n_i 2\pi rdr}{N} = a_1 \tag{13}$$

Using Eq. (10), rdr can be expressed in terms of σ

$$rdr = -\frac{1}{2\pi} \cdot \frac{F(1-2\nu_s)}{2\sigma^2} d\sigma \tag{14}$$

Equations (13) and (14), together with Eq. (7) yield

$$-\int_{\sigma_u}^{\sigma_I} \frac{n_i [P\frac{\pi d^2}{4}(1-2\nu_s)/2\sigma^2]}{N} d\sigma = a_1 \tag{15}$$

The lower and upper limits of the integrals have been changed to the ultimate tensile strength σ_u and the endurance limit σ_I, respectively. In order to perform the integration the fatigue life N must be known as a function of σ. For most materials the fatigue curve between σ_u and σ_I may be approximated by (Fig. 4)

$$N = b_1 \sigma^b \tag{16}$$

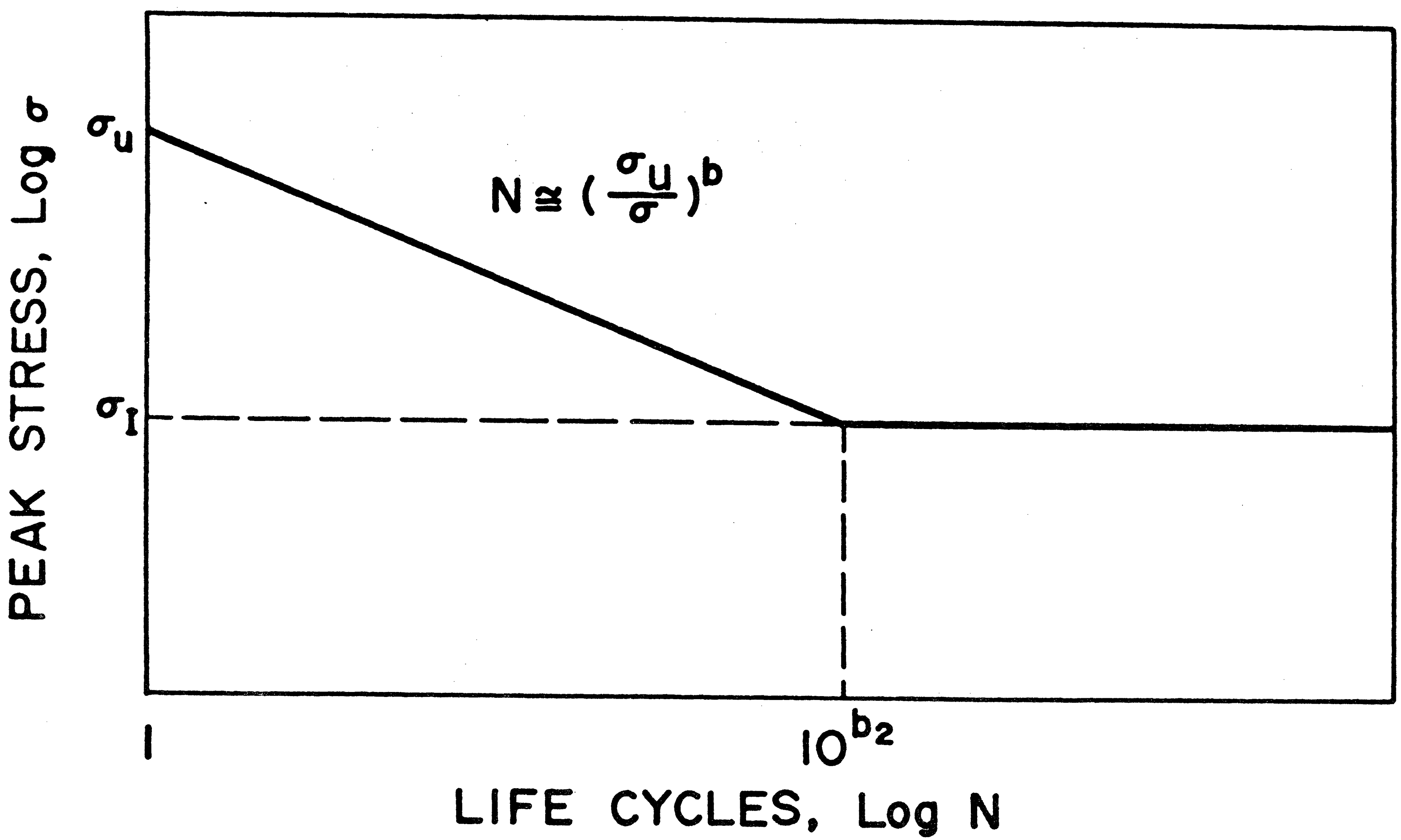

Figure 4. Idealized σ-N Curve.

where b and b_1 are constants. Equation (16) must satisfy the conditions

$$N = 1 \qquad \text{for } \sigma = \sigma_u \tag{17a}$$

$$N = 10^{b_2} \qquad \text{for } \sigma = \sigma_I \tag{17b}$$

In Eq. (17b), 10^{b_2} corresponds to the "knee" in the fatigue curve (Fig. 4). Equations (16) and (17) yield

$$N = \left(\frac{\sigma_u}{\sigma}\right)^b \tag{18}$$

$$b \equiv \frac{b_2}{\log_{10}\left(\frac{\sigma_u}{\sigma_I}\right)} \tag{19}$$

Substituting Eq. (18) into Eq. (15) and integrating we obtain

$$\frac{\pi d^2}{4} n_i \; P(1-2\nu_s)\frac{\sigma_u^{b-1}-\sigma_I^{b-1}}{2(b-1)\sigma_u^b} = a_1 \tag{20}$$

Introducing the definitions

$$S \equiv \frac{2\sigma_u(b-1)}{(1-2\nu_s)\left[1-\left(\frac{\sigma_I}{\sigma_u}\right)^{b-1}\right]} \cong \frac{2\sigma_u(b-1)}{1-2\nu_s} \tag{21}$$

$$n_i^* \equiv n_i \frac{\pi d^2}{4} \tag{22}$$

equation (20) becomes

$$n_i^* = a_1 \frac{S}{P} \tag{23}$$

Defining by "site" the surface area equal to the cross sectional area of one droplet, the number of sites per area A is $A/(\pi d^2/4)$. Since n_i is the number of impacts per unit area, n_i^* is the total number of impacts per "site". The minimum value of n_i^* is unity.

The parameter S characterizes the "strength" of the material. Thus, the number of impacts per sight needed to initiate damage is proportional to the ratio of the "strength" of the material S to the stress P produced by the impinging droplets. Such a dependence of n_i^* on S and P is reasonable, since the length of the incubation period is expected to increase with increasing S and with decreasing P. However, in view of the fact that Eq. (23) is based on the fatigue properties of materials in pure torsion and bending, one cannot expect a linear relationship to hold between n_i^* and S/P. In order to extend the range of applicability of Eq. (23), while retaining its major feature (namely the functional dependence of n_i^* on S/P) we write

$$n_i^* = a_1 \left(\frac{S}{P}\right)^{a2} \tag{24}$$

where both a_1 and a_2 are as yet undetermined constants. Since the form of the above functional relationship between n_i^* and S/P is arbitrary, its validity must be evaluated by plotting experimentally observed values of n_i^* versus S/P. The relationship will prove to be correct if on a log-log scale such a plot results in a straight line. The equation of this line would provide the constants a_1 and a_2.

The n_i^* and S/P values deduced from all the available experimental data are shown in Fig. 5. The symbols used in this figure and the corresponding experimental conditions are identified in Table I. It must be pointed out that only those data could be included in Fig. 5 for which not only n_i but also the material properties (σ_u, σ_I, b_2, ν_s, E_s, ρ_s) were available. It is seen from Fig. 5 that all the data can be correlated by a straight line. The equation of this line, obtained by

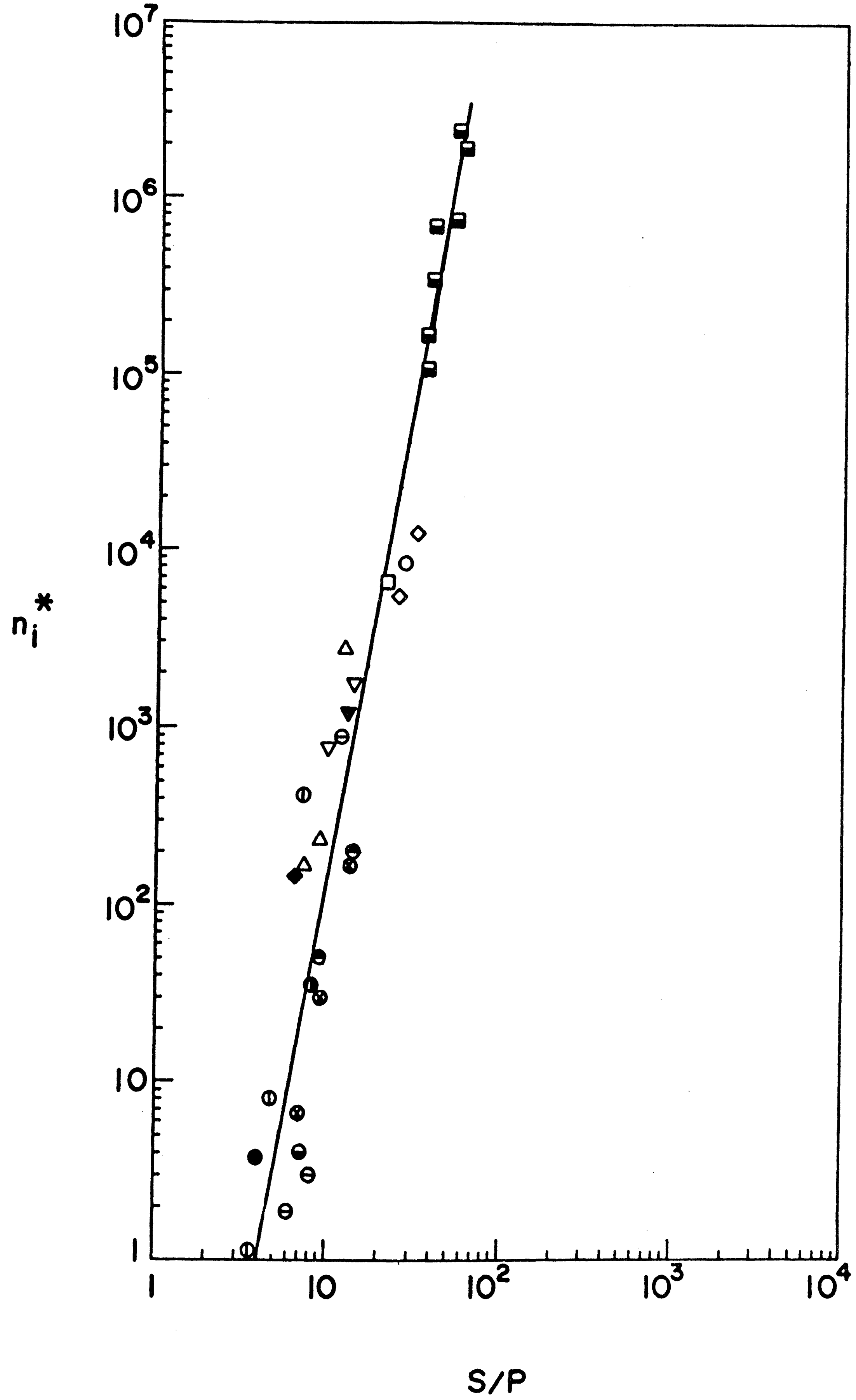

Figure 5. Incubation Period Versus S/P. Symbols for Data Defined in Table I. Solid Line: Best Fit to Data.

a least square fit of the data, is

$$n_i^* = 3.7\text{x}10^{-4}\ (S/P)^{5.7} \tag{25}$$

The excellent correlation in Fig. 5 lends support to the validity of the model.

As was discussed in Section II, the present model is valid only when the incubation time is greater than zero. This condition is met when $n_i^*>1$ or, according to Eq. 25, when $S/P>4.0$. Thus, an incubation period exists if

$$\begin{aligned} n_i^* &> 1 \\ S/P &> 4.0 \end{aligned} \tag{26}$$

The foregoing analysis (Eq. 25) can be applied only when the above conditions are satisfied. When S/P is equal to or less than 4.0 damage will occur even upon one impact per site (Section VII). This is most likely to occur at high impact velocities in which case P is high (since $P \sim V$) and S/P is low.

SECTION IV

RATE OF MASS REMOVAL

Beyond the incubation period, erosion of the surface of the material (as expressed in terms of mass loss) proceeds at a nearly constant rate as shown in Fig. 2b. In order to calculate this erosion rate, an analogy is drawn again between the behavior of the material upon which liquid droplets impinge, and the behavior of specimens subjected to torsion or bending fatigue tests. Experimental observations show that in the latter case the specimens do not all fail at once at some "minimum life", but their failure is scattered around a "characteristic life". For specimens in torsion and bending tests the probability that failure will occur between minimum life n_i and any arbitrary longer life n may be estimated from the Weibull distribution (Reference 67)

$$p = 1 - \exp\left[-\left(\frac{n-n_i}{n_a}\right)^{\beta}\right] \tag{27}$$

where n_a is the characteristic life corresponding to the 63.2 percent failure point and β is a constant (Weibull slope). For $(n-n_i)/n_a \ll 1$ Eq. (27) may be approximated by

$$p \cong \left(\frac{n-n_i}{n_a}\right)^{\beta} \tag{28}$$

The probability p can also be taken as the number of specimens that fail between n_i and n. If the material undergoing erosion due to liquid impingements is considered to be made up of many small "parts", then the amount of material eroded (mass loss) is proportional to p, i.e.

$$\frac{m}{\rho_s d} = a_3 \left(\frac{n-n_i}{n_a}\right)^{\beta} = a_3 \left(\frac{n^*-n_i^*}{n_a^*}\right)^{\beta} \tag{29}$$

In Eq. (29) m was nondimensionalized with respect to $\rho_s d$ in order to render the proportionality constant a_3 dimensionless. Equation (8b) is now rewritten in dimensionless form

$$\frac{m}{\rho_s d} = \frac{\alpha}{\pi \rho_s d^3/4} (n^* - n_i^*) \tag{30}$$

Equating Eqs. (29) and (30) we obtain

$$\frac{\alpha}{\pi \rho_s d^3/4} = a_3 \left(\frac{n^* - n_i^*}{n_a^*}\right)^{\beta} \tag{31}$$

According to Eq. (31) the mass loss rate α depends on the total number of impacts n. However, our model postulates a constant mass loss rate (i.e. α is independent of n, see Fig. 2b), at least when $n_i \leq n < n_f$. This requirement can be met by setting $\beta = 1$. Such a value for β is not unreasonable under high frequency loading (Reference 63). The characteristic life n_a is related to the minimum life n_i. This relationship may be expressed suitably as

$$n_a^* = a_4 n_i^{*a_5} \tag{32}$$

where a_4 and a_5 are constants. Introducing the dimensionless mass loss rate

$$\alpha^* \equiv \frac{\alpha}{\pi \rho_s d^3/4} \tag{33}$$

equations (31-33), together with the assumption $\beta = 1$ yield

$$\alpha^* = a_3 \frac{1}{(n_i^*)^{a_6}} \tag{34}$$

The new constant a_6 was introduced in place of βa_5.

The validity of Eq. (34) can be assessed (and the constants a_3 and a_6 can be determined) by plotting experimentally obtained values of α^* versus $(1/n_i^*)$. According to Eq. (34), on a log-log scale such a plot should result in a straight line. All the available experimental data are presented in such a manner in Fig. 6. As can be seen the data follows reasonably closely a straight line of the equation

$$\alpha^* = 0.023 \left(\frac{1}{n_i^*}\right)^{0.7} \tag{35}$$

indicating that the arguments leading to Eq. (35) were reasonable. The somewhat larger scatter of this data as compared to the data on the n_i^* vs S/P curve is due to the facts that a) n_i can be estimated more accurately from the available data than the slope of α, and b) all available data have a rather wide margin of error, but the value of n_i is less sensitive to these errors than the value of α.

Instead of plotting α^* versus $1/n_i^*$ as was done in Fig. 6, α^* could have been correlated directly with the parameter S/P by using the relationship between n_i^* and S/P given in Eq. (25). However, if α^* were plotted versus S/P then in this figure only those data could be included for which the material properties needed for calculating S were known. By plotting α^* against $1/n_i^*$ the use of these properties could be avoided; an advantage because for many of the data only n_i^* were known but not the material properties.

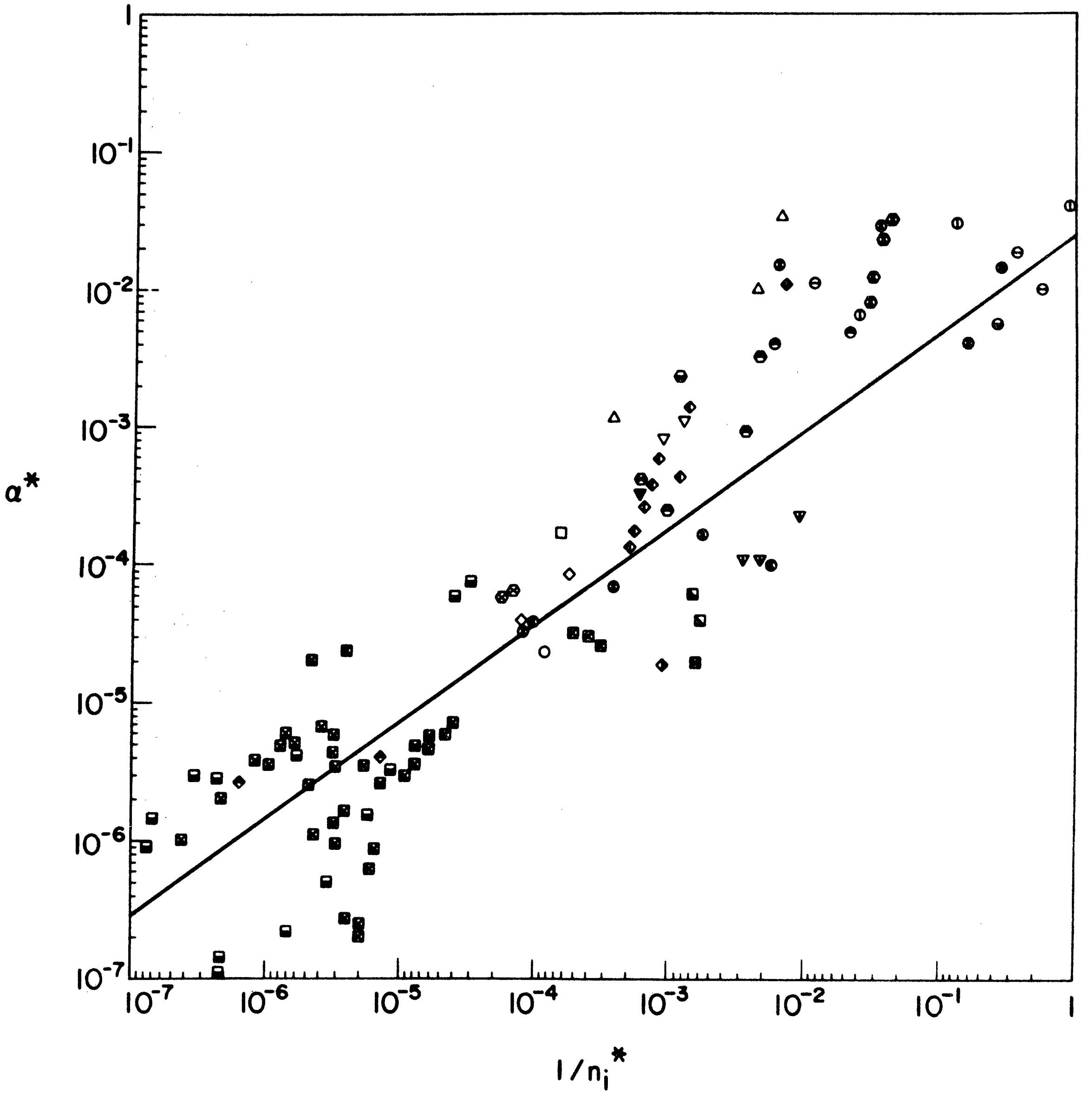

Figure 6. Rate of Erosion Versus the Inverse of the Incubation Period. Symbols for Data Defined in Table I. Solid Line: Best Fit to Data.

The relationship between the time rate of mass loss ($\partial m/\partial t$) and the impact velocity V can now be established. For a rain of constant diameter and intensity impinging upon a given material, α may be expressed as

$$\alpha = \frac{\pi \rho_s d^3}{4} \alpha^* = \frac{\pi \rho_s d^3}{4} \left[0.023\left(\frac{1}{n_i^*}\right)^{0.7}\right] \sim P^4 \sim V^4 \qquad (36)$$

Noting, that $\partial n/\partial t$ = Vq cosθ (see Eq. 1), we may write

$$\frac{\partial m}{\partial t} = \frac{\partial m}{\partial n}\frac{\partial n}{\partial t} = \alpha V q \cos\theta \sim \alpha V \sim V^5 \qquad (37)$$

This result is consistent with the experimental observation that the time rate of mass loss varies approximately with the 5th power of the impact velocity.

SECTION V

TOTAL MASS LOSS

In Figures 5 and 6 only those data could be included for which the variation of the mass (or weight loss) with time was known. There are numerous data available where such complete information is unavailable, but where the mass loss is given at one instant of time. A comparison of the present model with the latter "single point" results would be desirable, since agreement with such "single" data points would lend confidence to the validity of the model. To facilitate such a comparison Eq. (8b) is rewritten in dimensionless form

$$m^* = \alpha^* \, (n^* - n_i^*) \tag{38}$$

or

$$\frac{m^*}{\alpha^*} = n^* - n_i^* \tag{39}$$

where the dimensionless mass loss is defined as

$$m^* \equiv \frac{m}{\rho_s d} \tag{40}$$

According to our model, expressed by Eq. (39), all data should correlate on a m^*/α^* versus $(n^*-n_i^*)$ plot. Such correlation is presented in Fig. 7. This figure includes all existing data known to us in which droplets impinge continuously on a homogeneous material (see Table I). The results of experiments in which a jet impinges upon the surface (References 5, 6, 36 and 37) were not included in this figure. As can be seen, the agreement between data and the theoretical line given by Eq. (39) is quite reasonable, particularly in view of the large errors inherent in most of the experimental data.

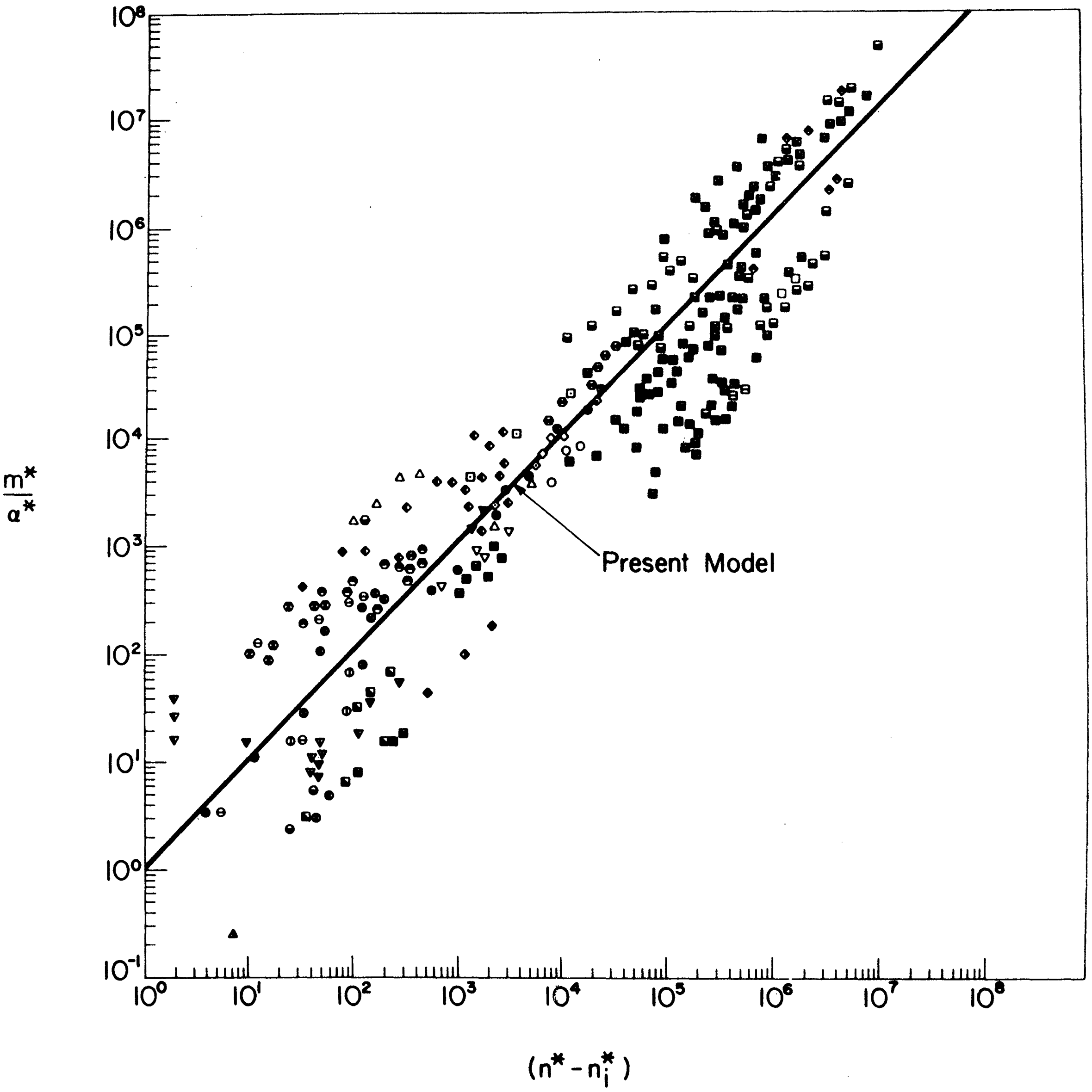

Figure 7. Comparison of Present Model (Solid Line, Eq. 38) with Experimental Results. Symbols for Data Defined in Table I.

SECTION VI

UPPER LIMIT OF APPLICABILITY OF MODEL

As was stated in Section II, the model proposed is valid only as long as the mass loss varies linearly either with time t or with the number of impacts n. Consequently, the upper limit of t (or n) must be established beyond which the present model cannot be applied. An estimate of this limit n_f was made by observing that for many of the data given in Figs. 5, 6, and 7 the number of impacts n was as high as $3n_i$. Up to this point the data obtained at higher values of n did not show any systematic deviation from the data obtained at lower n values. It was concluded, therefore, that the model is valid at least up to $n_f = 3n_i$, i.e.

$$n_f \leqslant 3n_i \tag{41}$$

or, in dimensionless form

$$n_f^* \leqslant 3n_i^* \tag{42}$$

SECTION VII

NO INCUBATION PERIOD

The model discussed thus far can be applied only if there is an incubation period or, as shown previously, if

$$n_i^* > 1$$
$$S/P > 4 \qquad (26)$$

The domain in which the above conditions are satisfied will be referred to as Region I, as illustrated in Fig. 8. The domain corresponding to zero incubation period will be referred to as Region II. One would wish also to extend the model developed for Region I to Region II. To accomplish this, data of the form shown in Fig. 9 would be needed, i.e. the mass loss would have to be measured as a function of time (or number of impacts) for different materials of known properties. Unfortunately, to date such data have not yet been obtained. Therefore, one can only hypothesize that the data in Region II have the general trend shown in Fig. 9. If this trend is correct then, similarly to Region I, the data may be approximated by two straight lines (Fig. 9). Accordingly, the mass loss may be written as

$$m^* = m_{II}^* + \alpha_{II}^* (n^* - 1) \qquad (43)$$

where m_{II}^* is the mass loss caused by one impact per sight, and α_{II}^* is the rate of mass loss beyond $n^* = 1$. Intuitively, one might argue that both m_{II}^* and α_{II}^* should be directly proportional to the force of the impact (i.e. the pressure P), and inversely proportional to the "strength"

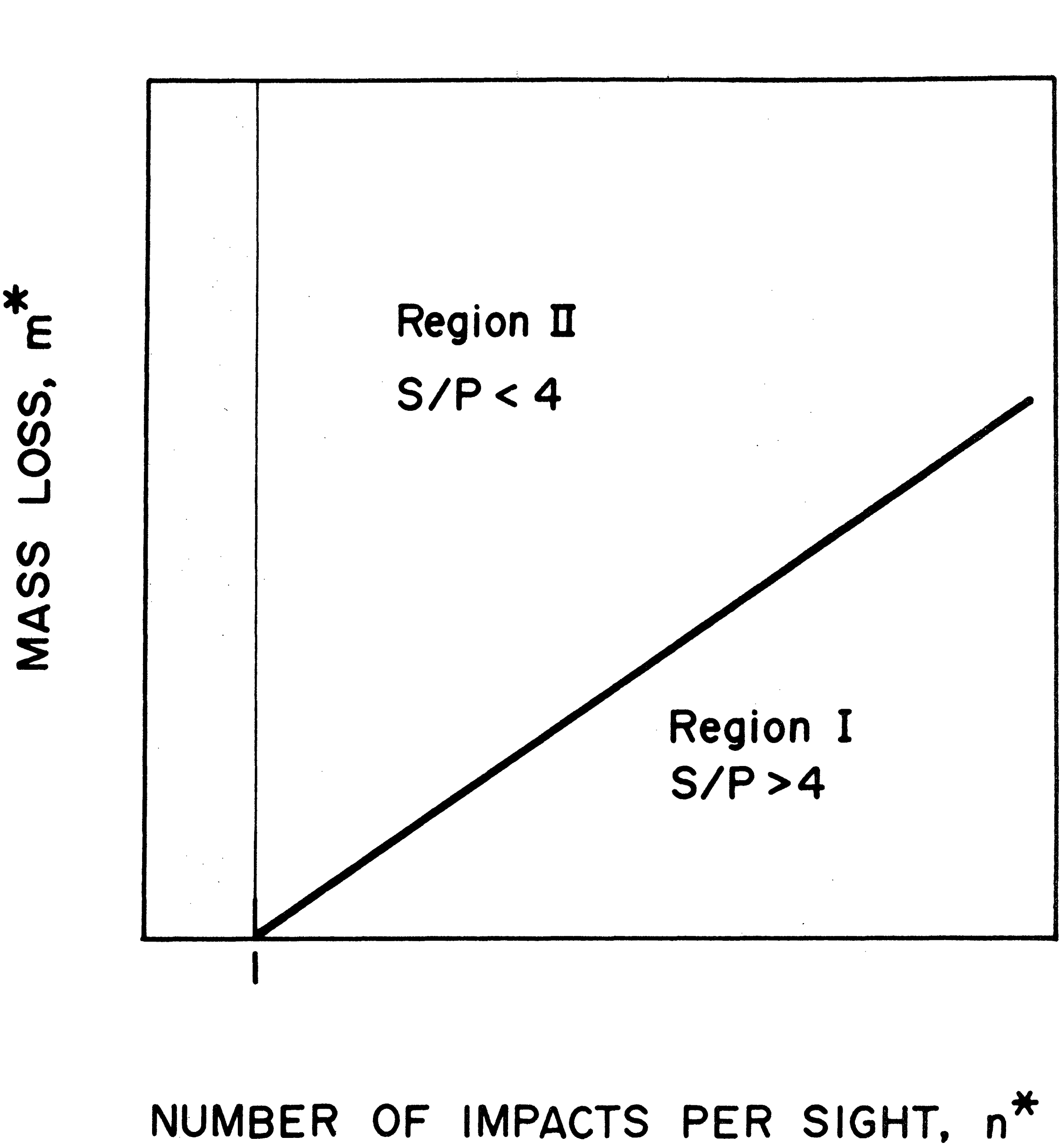

Figure 8. Limits of Region I.

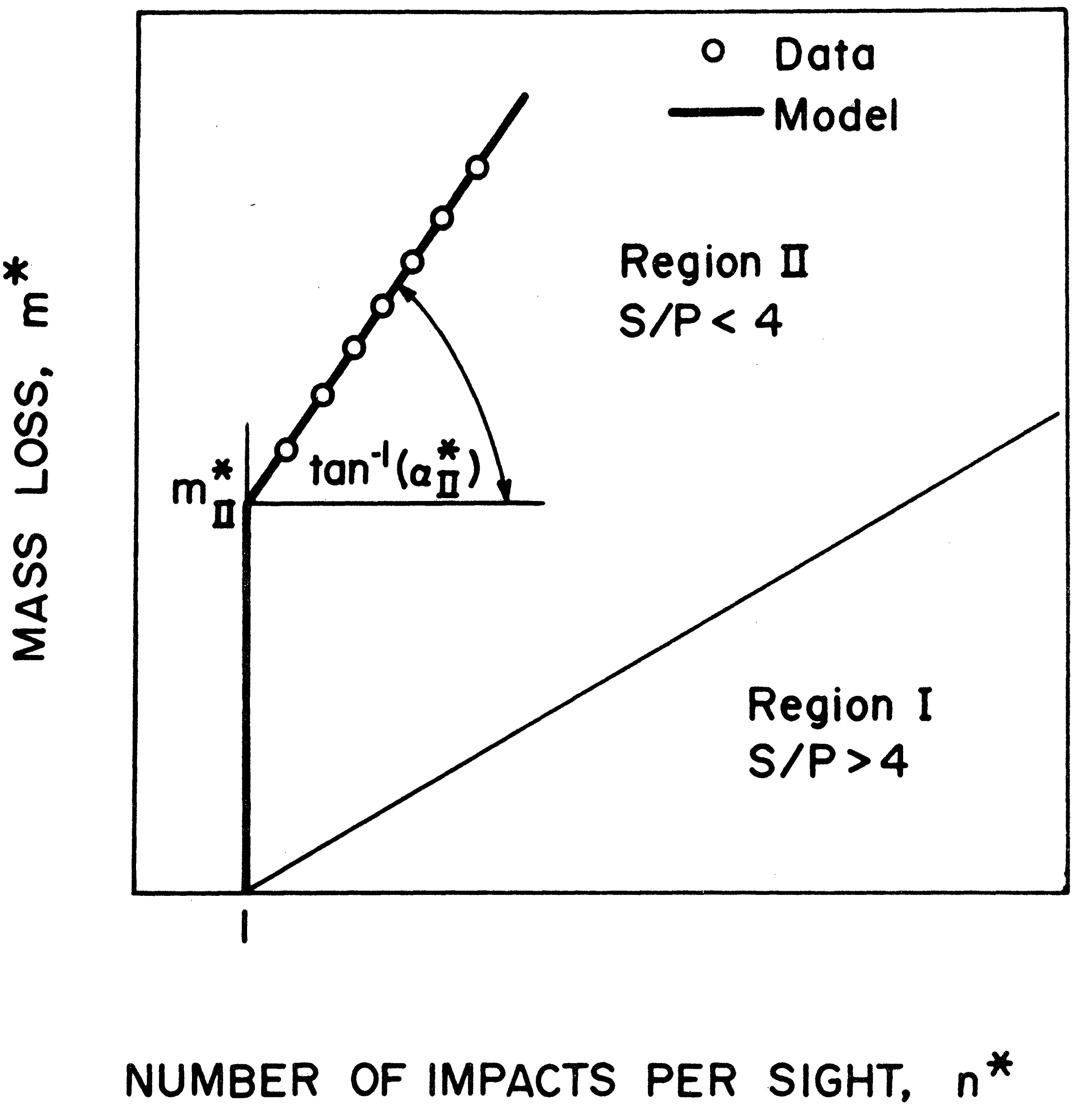

Figure 9. Schematic of Experimental Results in Region II, and Description of Model.

of the material S. Thus, m_{II}^{*} and α_{II}^{*} are expected to be of the form

$$m_{II}^{*} = a_7 \left(\frac{S}{P}\right)^{a_8}$$
$$\alpha_{II}^{*} = a_9 \left(\frac{S}{P}\right)^{a_{10}} \tag{44}$$

The validity of Eqs. (43)-(44), and the constants a_7-a_{10} could be assessed only by comparing these expressions to data, similarly as was done for the model in Region I. However, such comparisons will have to await the measurements of the appropriate data.

SECTION VIII

SUMMARY

In this report a model has been developed which can be used to estimate the incubation time and the mass loss of homogeneous materials subjected to repeated impingements of liquid droplets. The following results were obtained

a) Incubation Period

$$n_i^* = 3.7\text{x}10^{-4} \left[\frac{2\sigma_u(b-1)\left(1 + \frac{\rho_L C_L}{\rho_s C_s}\right)}{(1-2\nu_s)(\rho_L C_L V\cos\theta)} \right]^{5.7} \quad \left(\frac{\text{no. of impacts}}{\text{site}}\right) \tag{45}$$

or

$$n_i = \frac{4.71\text{x}10^{-4}}{d^2} \left[\frac{2\sigma_u(b-1)\left(1 + \frac{\rho_L C_L}{\rho_s C_s}\right)}{(1-2\nu_s)(\rho_L C_L V\cos\theta)} \right]^{5.7} \quad \left(\frac{\text{no. of impacts}}{\text{unit area}}\right) \tag{46}$$

or

$$t_i = \frac{4.71\text{x}10^{-4}}{qV\cos\theta d^2} \left[\frac{2\sigma_u(b-1)\left(1 + \frac{\rho_L C_L}{\rho_s C_s}\right)}{(1-2\nu_s)(\rho_L C_L V\cos\theta)} \right]^{5.7} \quad (\text{time}) \tag{47}$$

b) Rate of Mass Loss

$$\alpha^* = 5.75 \left[\frac{(1-2\nu_s)\,\rho_L C_L V\cos\theta}{2\sigma_u (b-1)\left(1 + \frac{\rho_L C_L}{\rho_s C_s}\right)} \right]^4 \tag{48}$$

or

$$\alpha = 4.51\,\rho_s d^3 \left[\frac{(1-2\nu_s)\,\rho_L C_L V\cos\theta}{2\sigma_u (b-1)\left(1 + \frac{\rho_L C_L}{\rho_s C_s}\right)} \right]^4 \quad \left(\frac{\text{mass loss}}{\text{impact}}\right) \tag{49}$$

c) Total Mass Loss

$$m^* = \alpha^* (n^* - n_i^*) \tag{50}$$

or

$$m = \alpha (n - n_i) \quad \left(\frac{\text{mass loss}}{\text{unit area}}\right) \tag{51}$$

Equations (46), (49) and (51) yield the mass loss per unit area in time t

$$m = 4.51\,\rho_s d^3 \left[\frac{(1-2\nu_s)\,\rho_L C_L V\cos\theta}{2\sigma_u (b-1)\left(1 + \frac{\rho_L C_L}{\rho_s C_s}\right)} \right]^4 \left\{ (q\ t\ V\cos\theta) - \frac{4.71\times10^4}{d^2} \left[\frac{2\sigma_u (b-1)\left(1 + \frac{\rho_L C_L}{\rho_s C_s}\right)}{(1-2\nu_s)(\rho_L C_L V\cos\theta)} \right]^{5.7} \right\} \tag{52}$$

The foregoing results are subject only to the following two constraints

a) incubation time must be greater than zero ($t_i > 0$), a requirement satisfied by the condition

$$\frac{S}{P} = \frac{2\sigma_u (b-1)\left(1 + \frac{\rho_L C_L}{\rho_s C_s}\right)}{(1-2\nu_s)(\rho_L C_L V\cos\theta)} > 4.0 \tag{53}$$

b) total time elapsed must be less than three times the incubation period, i.e.

$$\begin{aligned} t &< 3\ t_i \\ n &< 3\ n_i \\ n^* &< 3\ n_i^* \end{aligned} \tag{54}$$

or

$$\frac{3}{2}\ \frac{(V\cos\theta) It}{V_t d^3} < 1.11\text{x}10^{-3} \left[\frac{2\sigma_u (b-1)\left(1 + \frac{\rho_L C_L}{\rho_s C_s}\right)}{(1-2\nu_s)(\rho_L C_L V\cos\theta)} \right]^{5.7} \tag{55}$$

A tentative model has also been suggested for those problems where constraint (a) (Eq. 53) is not satisfied (see Section VII). Owing to lack of data the validity of this model could not yet be assessed.

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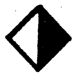
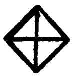
TABLE I

DESCRIPTION OF DATA AND SYMBOLS USED
IN FIGURES 5, 6 and 7.

Symbol	Material	Velocity (ft/sec)	Diameter of Drop (mm)	Author
△	Perspex	730	1.9	Fyall et al (1957)
		1180	1.9	Schmitt et al (1970)
◭		1395	2.0	King (1965)
▽	Alkathane 2	585-730	1.9	Fyall et al (1957)
▼	Q.S.-4 Polyethylene	730	1.9	Fyall et al (1957)
	Polyphenylene Oxide	535-3720	1.9	Schmitt et al (1970)
	Cast Urethane	730	1.8	Morris et al (1972)
	Polypropylene	980-1470	1.2	King (1967)
▶	Teflon	1180	1.9	Schmitt (1970)
○	Aluminum Alloy, D.T.D. 423B	730	1.9	Fyall et al (1957)
●	1100-0 Aluminum	1120	1.8	Morris & Wahl (1970)
◐	1145-H19 Aluminum	1120-2240	1.8	Morris & Wahl (1970)
◓	2024-T6 Aluminum	1120-2240	1.8	Morris & Wahl (1970)
⦶	5052-0 Aluminum	1120	1.8	Morris & Wahl (1970)
⊖	6061-T6 Aluminum	1120-2240	1.8	Morris & Wahl (1970)

Symbol	Material	Velocity (ft/sec)	Diameter of Drop (mm)	Author
⊗	7075-T6 Aluminum	1120-2240	1.8	Morris & Wahl (1970)
		1120		Morris et al (1972)
⊗	Aluminum (Pure)	820-980	1.2	King (1967)
⊗	Aluminum	1650-1420	1.2	Rieger (1965)
⊗	Aluminum Alloys	1340	1.2	Hoff et al (1969)
□	Magnesium Alloy D.T.D. 259	730	1.9	Fyall et al (1957)
◇	Copper Alloy B.S. 1433	585-730	1.9	Fyall et al (1957)
◆	Copper (Electrolytic)	1120	1.8	Morris & Wahl (1970)
◨	Nickel	1000	0.866	Engel et al (1971)
◩		1120	1.8	Morris et al (1972)
⬓	Cobalt-Chromium Alloys	1020	0.66	Baker et al (1966)
⊟	Iron	1000	0.866	Engel et al (1971)
⊠	Steels	1020	0.66	Baker et al (1967)
⊠		1455	0.64	Herbert (1965)
⊠		1120	1.8	Morris et al (1972)

Symbol	Material	Velocity (ft/sec)	Diameter of Drop (mm)	Author
	Titanium Alloys	1020	0.66	Baker et al (1967)
		1120	1.8	Morris et al (1972)
		1340	1.2	Hoff et al (1965)
	Tantalum	1000	0.866	Engel et al (1971)
	Udimet 700	1000	0.866	Engel et al (1971)
	Magnesia Ceramic	1340	1.2	Hoff (1965)
	Zirconia	1340	1.2	Hoff (1965)
	Alumina Ceramic	1340	1.2	Hoff (1965)
	Spinell	1340	1.2	Hoff (1965)
	Glass	1340	1.2	Hoff (1965)

APPENDIX: LITERATURE SURVEY

In this section a brief summary is given of the existing analytical and experimental investigations on the response of homogeneous materials subjected to the repeated impingement of liquid droplets. For convenience, the available information is divided into groups as shown in Table A-I. The present summary will follow the pattern of this table. It is emphasized that only homogeneous materials are included here. Composite and laminated materials are not discussed.

A-I. REVIEWS

One of the earliest reviews dealing with the subject of rain erosion of materials was given by Engel in 1953 (Reference 15). A considerable amount of work has been done in the years following this review. Excellent summaries of the later work are given by Eisenberg (Reference 13), Heymann (References 32,33), Heymann and Arcella (Reference 35) and Wahl (Reference 65).

It is noted that many of the results of the latest investigations were reported at the First, Second and Third rain erosion conferences (References 23-25). Comprehensive articles dealing with the subject may also be found in two special publications of the American Society for Testing and Materials (References 11 and 21).

A-II. EXPERIMENTAL INVESTIGATIONS

By far the largest number of investigations utilized experimental techniques to determine the behavior of materials subjected to liquid impingement. A comprehensive summary of the materials tested and the ranges of parameters covered in past experiments are given in Table A-II.

As can be seen from Tables A-I and A-II the experiments can be categorized as 1) single impact studies, 2) in-flight tests, 3) whirling arm tests, 4) rocket sled tests, and 5) jet experiments.

A-II-1. Single Impact Studies

The response of materials subjected to the impact of a single droplet was studied by Engel (Reference 17), Fyall (Reference 28) and Bowden and Brunton (Reference 7). It is difficult to apply these results to problems where multiple impingement occurs, the latter being the situation which is encountered commonly in practice.

A-II-2. In-Flight Tests

In-flight tests, in which the speciman is mounted on or forms part of an airplane flying through rainstorms are expensive and provide only limited control of the experimental conditions. For these reasons, only a few such tests have been conducted most notably by the United States Air Force (Reference 47) and by Cornell Aeronautical Laboratory (Reference 55).

A-II-3. Whirling Arm Tests

In the whirling arm experiments the model is placed on an arm rotating in a simulated rain fall. This technique has been employed successfully to study behavior of materials both in the United States and abroad. In the United States, whirling arm experiments have been performed by Schmitt (Reference 60), Morris and Wahl (Reference 53) and Engel and Almo (Reference 20); in England by Baker and his coworkers (References 2 and 3), by King (References 44 and 45) and by Fyall, King and Strain (Reference 26). In Germany such experiments have been performed at Dornier Systems (References 31,32,38,39,40,41,46,56). The whirling arm technique is advantageous in that it provides reasonably good control over the experimental parameters of interest. However, the impact velocities that can be achieved are limited. The maximum Mach number attained thus far with this type of apparatus is 2.2.

A-II-4. Rocket Sled Tests

Impact velocities of up to 4292 ft/sec (Mach Number $\sim$4.0) may be achieved in rocket sled tests. In this type of experiment the specimens are fastened on a sled propelled by a rocket and moving through an artificial rain field. Rocket sled tests for homogeneous materials have been conducted at Holloman Air Force Base, New Mexico, by Schmitt and his coworkers (References 58,59,60,61), and by Engel (Reference 19).

A-II-5. Jet Experiments

Continuous jets of liquids have also been used to simulate the impingement of droplets on surfaces. This is generally achieved by mounting the test specimen on a wheel rotating with its axis parallel to the jet. Such experiments have been performed by Beckwith and Marriott (References 5 and 6), Hobbs (Reference 36) and Hobbs and Brunton (Reference 37).

A-III. ANALYTICAL INVESTIGATIONS

The damage incurred by materials subjected to liquid impingement depends upon the force imparted by the droplet to the material and on the material properties.

The simplest expression for the pressure at the liquid-solid interface is obtained by assuming that the pressure is equal to the water hammer pressure caused by the impingement of an infinitely long liquid cylinder on a rigid solid. Formulae for the pressure, including the elastic behavior of materials, were developed by Engel (References 14 and 15) and by Heymann (Reference 34). The dependence of the pressure on time and position was considered by Huang (Reference 42).

Experimental evidence indicates that, in addition to the normal stress (i.e. the pressure discussed above), shear stresses are also present between the liquid and the solid. An estimate of the magnitude of these stresses was made by Beal, Lapp and Wahl (Reference 4), who showed that, in most cases, the magnitude of the shear stress is negligible.

The foregoing studies provide only the stresses (normal and tangential) at the surface. However, the prime interest is not in these stresses but in the behavior of materials, as expressed in terms of some convenient "erosion" parameter, such as the mass loss, incubation time etc. It has been observed that under many conditions the mass loss is related to the fatigue properties of the materials. Based on this information Heymann (Reference 32) and Thiruvengadam, Rudy and Ganasekaran (Reference 63) established qualitative relationships between the mass loss rate and time. Using fatigue concepts Mok (Reference 51) derived an integral equation which, under specific conditions, can be solved numerically to relate the incubation time to the maximum erosion rate.

Owing to the large number of parameters a complete analytical solution to the problem has not yet been achieved. Attempts have been made to establish empirical and semi-empirical relationships between the various parameters describing the problem. One of the first of these attempts was made by Engel (Reference 16) who used dimensional analysis to find the dimensionless groups which govern the depth of pit formation.

Parametric studies, aimed at correlating data, were also done by practically every investigator who performed experiments (see Table A-II). One of the most comprehensive correlations of existing data was given by Heymann (Reference 33).

TABLE II

CLASSIFICATION OF THE LITERATURE

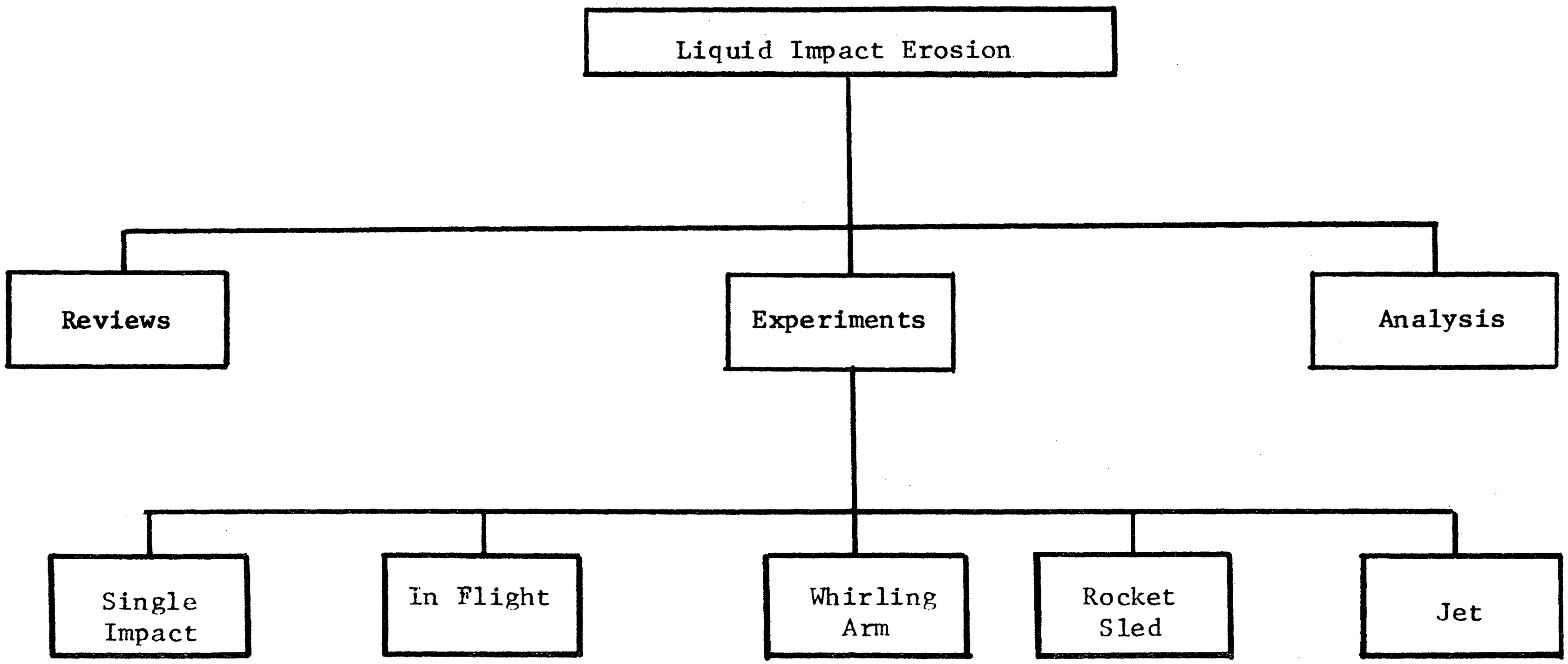


TABLE III

SUMMARY OF EXPERIMENTAL DATA ON EROSION OF

HOMOGENEOUS MATERIALS SUBJECTED TO LIQUID IMPINGEMENT

Investigator	Material	Drop or Jet Dia., (mm)	Impact Vel. (ft/sec)	Intensity (Inch/hr)	Parameter Studied	Remarks
Baker et al (1966)	Cobalt-chromium alloys	0.66	1020	Not Given	Mass Loss	Results given in terms of mass loss vs. mass of water impacted
Baker et al (1967)	Cobalt-chromium alloys, turbine blade steels, high strength steels, titanium alloys, cemented tungsten, titanium carbides	0.66	1020	Not Given	Mass Loss	Results given in terms of mass loss vs. mass of water impacted
Beckwith and Marriot (1966)	Chromium tungsten alloy	0.398	1030	(Jet)	Mass Loss	

Investigator	Material	Drop or Jet Dia., mm	Impact Vel. (ft/sec)	Intensity (Inch/hr)	Parameter Studied	Remarks
Beckwith and Marriot (1967)	Austenitized steel	0.398	1030-1400	(Jet)	Mass Loss	
Bowden and Brunton (1961)	Polymethyl-methacrylat, natural rubber, polyvinyl chloride, plate glass, stainless steel	1.5 to 6.0	3940	(Jet)	Diameter of ring crack	High speed photographs used. Concluded that for velocities above 1650 ft/sec. a liquid mass (water) behaves in a compressible manner
Bowden and Field (1964)	Glasses, hard polymers	3.0	3940	(Jet)	Development of fracture is followed by high speed photography	
Brunton (1967)	Perspex, duralumin	3.0	2620	(Jet)	Pit depth and photographs	

Investigator	Material	Drop or Jet Dia., mm	Impact Vel. (ft/sec)	Intensity (Inch/hr)	Parameter Studied	Remarks
DeCorso (1964)	Titanium alloys, chromium steel, stellite, and copper	0.127 to 1.52	4600	(Jet)	Maximum Depth	Different fluids: water, trichloroethylene, mercury, transformer oil, salt water, alcohol, ether and benzene
Engel (1959)	1110-0 aluminum, 2024-0 aluminum, copper, lead, steel, soft iron, and zinc	1 to 2	695-2445	Single Impact	Pit Depth	Theoretical relation for pit depth is derived and compared with experimental results
Engel (1971)	1110-aluminum	1.9	1635 to 4202	2.5	Microscopic examination	Tests conducted on rocket sled
Engel and Almo (1971)	Zinc, iron tantalum, nickel, and udimet 700	0.866	1000	Not mentioned	Mass Loss	
Field (1967)	Aluminum, brass, perspex and glass	2 to 3	2115-2460	(Jet)	Photographic	Conclusion: Change in surface profile in excess of 1000A°, can be significant in acting as sites for erosion damage

Investigator	Material	Drop or Jet Dia., mm	Impact Vel. (ft/sec)	Intensity (Inch/hr)	Parameter Studied	Remarks
Fyall, et al (1957)	Perspex, polymers, copper, aluminum, magnesium alloys, stainless steel, brass	1.9	981-1640	1 to 3	Mass Loss	
Fyall (1967)	Perspex,	2-3	1000	Single Impact	Area of cracking	High speed photography, photomicrography and profilimetry used
Herbert (1965)	Ferritic and austenitic chrome steel, nimonic 90	1.2	1455	$1x10^{-5}$ gr/cm^3	Mass Loss	
Herbert (1967)	Low carbon steels and high alloy steels with different heat treatments	0.52 to 2.1	1340	$3x10^{-6}$ to $1.2x10^{-5}$ gr/cm^3	Mass Loss	
Hobbs (1966)	Brass, aluminum, steel and nimonic	1.6	33-660	(Jet)	Mass Loss	

Investigator	Material	Drop or Jet Dia.,mm	Impact Vel. (ft/sec)	Intensity (Inch/hr)	Parameter Studied	Remarks
Hobbs and Brunton (1965)	Steel and cast iron	1.6	660	Jet	Mass Loss	
Hoff (1965)	Glass and ceramics	1.2	1340	1×10^{-5} gr/cm^3	Mass Loss	
Hoff (1968)	Glass, plexiglas	1.2	220-466	1.2×10^{-6} to 1.5×10^{-6} gr/cm^3	MDPR $\sin^2\theta$	1) MDPR -Mean depth of penetration rate. 2) Review of other data and attempted correlation between angleθ, velocity V and MDPR
Hoff, et al (1965)	Aluminum and titanium alloys	1.2	1340	1.2×10^{-5} gr/cm^3	Erosion depth	Effect of pressure and temperature also studied
Hoff and Langbein (1967)	Glass, magnesium oxide, aluminum oxide, plexiglass, aluminum pure iron	0.5 to 2.1	981-1310	1.2×10^{-5} gr/cm^3	Mass eroded after 12 min. exposure	Energy approach considered. Resistance to erosion $= \frac{\text{(energy of liquid)}}{\text{(volume of eroded material)}}$
King (1965)	Perspex	2.0	1395	1	Mass Loss	Testing by use of rocket propelled vehicles is discussed

Investigator	Material	Drop or Jet Dia., mm	Impact Vel. (ft/sec)	Intensity (inch/hr)	Parameter Studied	Remarks
King (1967)	Perspex, polypropylene, pure aluminum, aluminum alloy, copper alloy and stainless steel	1.2	820-1475	1 to 8	Mass Loss	
Langbein (1965)	Plastic	1.2	1345	2.5×10^{-6} gr/cm^3	Depth of erosion and mass loss	
Morris (1969)	Aluminum	1.8	730-1120	1	Micrographs and photographs	
Morris and Wahl (1970)	Aluminum alloys, titanium, copper, coatings, laminated plastics and ceramics	1.8	1120-2240	1	Mass Loss	
Morris et al (1972)	Ceramic coatings, polymeric coatings, ductile metals (aluminum, nickel, steel and titanium) and solid elastometers	1.8	730-1120	1	Mass Loss	

Investigator	Material	Drop or Jet Dia., mm	Impact Vel (ft/sec)	Intensity (Inch/hr)	Parameter Studied	Remarks
Rieger (1965)	Aluminum, berilium copper	1.2	650-1420	1×10^{-5} gr/cm^3	Mass Loss	Also results of increase in hardness and depth of erosion after a fixed time interval given
Schmitt et al (1967)	Isotropic ceramics, sandwich ceramics, plastic laminates, electroplated plastics, inorganic laminates, ceramic and elastomeric coated laminates, glasses, thermo-plastics, sandwich plastics, metals	1.9	1614 to 3048	2.5	Mass loss and MDP (MDP = mean depth of penetration = $\frac{\text{(mass loss)}}{\text{(density x area)}}$)	a) Only one point available after total time of test. b) Properties of some materials not specified.

Investigator	Material	Drop or Jet Dia., mm	Impact Vel. (ft/sec)	Intensity (Inch/hr)	Parameter Studied	Remarks
Schmitt and Krabill (1970)	Isotropic ceramics, sandwich ceramics, Plastic laminates, ceramic coated laminates, glasses, sandwich plastics, bulk plastics, metals metal coated laminates, cast resins and fibre composites.	1.9	1614 to 3040	2.25 to 2.50	Mass Loss and MDP	Rocket sled tests
Schmitt (1971)	Bulk and reinforced plastics	1.8	735 to 880	1	Mass Loss	
Tatnall et al (1967)	Furnace epoxy laminate, plexiglas, teflon, fused silica	1.9	1110-2240	2.5	Mass Loss	Rocket sled tests
Thiruvengadam et al (1968,1969)	316 Stainless steel, 1110-0 aluminum, nickel, titanium	0.78	133-522	Jet	Threshold impact velocity at 10^6 cycles	Fatigue endurance limit also determined

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13. ABSTRACT

The behavior of homogeneous materials subjected to repeated impingements of liquid droplets is investigated. Based on fatigue theorems, a model is presented for describing both the incubation period n_i (i.e. the time elapsed before the mass loss of the material becomes appreciable), and the mass loss past the incubation period m. The parameters are established which govern the length of the incubation period and the subsequent mass loss rate, and simple algebraic expressions are developed relating n_i and m to the properties of the impinging droplets and the material. The limits of applicability of the model are also established.

The results obtained are compared to available experimental data. Reasonable agreement is found between the present results and the data, indicating that the model developed can be used to estimate the incubation period and the mass loss of the material.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Rain Erosion Erosion Mechanism Fatigue Model Incubation Period						