is well known that, for two-dimensional, turbulent mixing between a jet and a dead region, the mixing zone grows linearly with a divergence angle of about 15° (this should be taken only as a rule of thumb). Now, if we calculate the free-streamline pattern for a plate airfoil with separation, we find the dead-water region to be roughly wedge shaped with its edge at the point of separation. If the wedge angle is greater than the angle of turbulent mixing, separation will be unaffected. If it is less, presumably the mixing will knit the free-stream flow back to the surface. An analytical solution, estimating the losses on an airfoil with frictionless surface but with this kind of separation and reattachment, would be most welcome. This would, undoubtedly, be difficult to obtain. However, in the case of the channel, even the crudest of investigations yields much useful information.

Taking account of the above-mentioned phenomenon of reattachment, let us return again to the diffuser. Let us say that separation has taken place and that the dead-water region satisfies the condition of constant pressure. Some distance past the separation point the streamlines will have become parallel. The angle bounding the dead region will be the angle of divergence of the diffuser (see Fig. 2).

If this angle is greater than the angle of mixing, reattachment will not take place. If it is less, separation will not take place. Whether or not this last conclusion is true, the angle of separation undoubtedly influences the behavior of the boundary layer; and, where separation has not taken place, the *virtual angle of separation* offers to be a factor of considerable influence. By *virtual angle of separation* is meant the angle at which the separated flow would leave the surface *if* separation took place at a given point. Any explanation of the behavior of the boundary layer must be in terms of factors within the boundary layer or in the immediately adjacent portions of the free stream. Confining surfaces some distance away cannot in themselves influence the boundary layer. They do, however, determine the virtual angle of separation which is a factor within the boundary layer itself. This angle is a point function, variable along the surface. Its influence should be of importance even after actual separation has occurred, especially when the dimensions of the separated region are small.

From these observations it appears that, in any study of the boundary layer in a channel, one should take cognizance of this angle as well as the surface velocity distribution and aspect ratio (this last factor influences secondary flow). Conversely, if two surfaces have the same pattern of velocity variation and of virtual-separation-angle variation, one would expect similar boundary-layer behavior, irrespective of the actual geometry. This last statement can be taken as a specification of the conditions for quasi-similitude. Once it becomes established that this angle does truly account for the influence of confining walls, it should be possible to generalize experimental measurements from a few typical channels to a variety of others. For example, it should be possible to predict, confidently, the behavior of a mixed-flow impeller from studies of a stationary channel modeled upon the factors mentioned above.

Two reservations should enter into the evaluation of the ideas presented above. First, the validity of virtual separation in explaining boundary-layer behavior need not necessarily depend upon the mechanisms proposed here. Secondly, when one attempts to compute the angle of separation, he may find, say, that at the separation point it is initially zero, gradually growing to a finite value. In an effort to bypass such local effects, the author has had some success in restricting the analysis to overall mass movements of the fluid rather than to local conditions. For example, in dealing with the velocity recovery on the surface of a vane in a turning cascade, the width of the separated zone in the plane of trailing edges and the distance of this line from the separation point were used to compute the angle. This procedure reduces the virtual-separation concept to nothing more than a yardstick for comparison purposes. As such, however, it still promises to be useful in the study of narrow turbomachine passages.

One cannot but feel that there is some more exact concept lying behind that which has been discussed here. Now that the need has been made apparent it may soon be forthcoming.

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Location of Mach Discs and Diamonds in Supersonic Air Jets†

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September 17, 1956

SYMBOLS

A_t = throat area of nozzle
A_e = exit area of nozzle
D_d = diameter of Mach disc
D_e = exit diameter of nozzle
Ma_e = exit Mach Number computed from Eq. (1)
P_a = atmospheric pressure
P_e = exhaust pressure
P_0 = upstream stagnation pressure
x = distance from nozzle exit to oblique shock-wave intersection point or distance from nozzle exit to Mach disc location
γ = ratio of heat capacities, assumed to be 1.4

INTRODUCTION

THE COMPLICATED shock and rarefaction patterns existing in supersonic jets have long been of fundamental interest in the study of fluid dynamics. It is well known that an overexpanded nozzle (exit pressure less than receiver pressure) leads to oblique shock waves at the exit of the nozzle which intersect and give the familiar "diamond" pattern. If the degree of overexpansion is great enough, this pattern is modified so as to terminate the oblique shocks with a normal shock—or the so-called Mach disc configuration. On the other hand, a sufficiently underexpanded nozzle will lead to a similar Mach disc because of the focusing of compression waves from the jet boundary. At extreme degrees of underexpansion, the oblique shocks and their reflection from the Mach disc are markedly curved, and the configuration is appropriately termed a shock bottle.

Numerous investigators[1-4] have devoted effort to experimental and analytical studies of these phenomena. However, most of these investigations have been restricted to sonic nozzles and relatively low-pressure ratios (stagnation pressure to receiver pressure). Kelber and Jarvis[5] have utilized the method of characteristics to analyze the flow from a nozzle operating

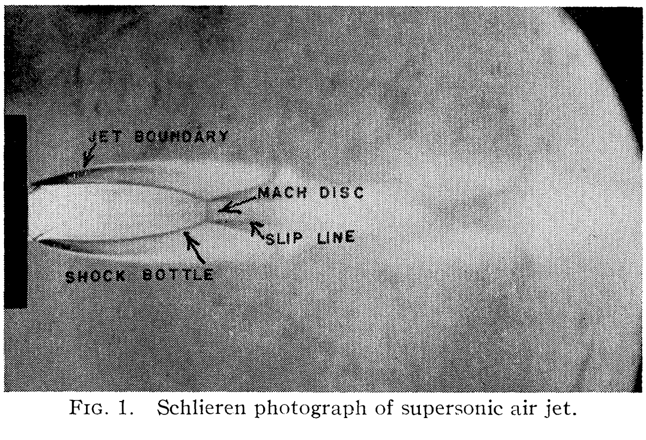

FIG. 1. Schlieren photograph of supersonic air jet.

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† This research was sponsored by the Office of Ordnance Research, U.S. Army. Discussions of this experimental program with Profs. T. C. Adamson, R. B. Morrison, and H. E. Bailey of the Aeronautical Engineering Department are gratefully acknowledged.

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under extremely high pressures. Such a technique fails to indicate the presence of a Mach disc so that the phenomenological considerations for locating the disc are in question. Weir[6, 7] has reported information on the variation of disc location with pressure ratio for airflow through two-dimensional slits.

The influential parameters in determining the Mach disc location in a supersonic jet would appear to be the ratio of driving pressure to receiver pressure, the nozzle exit Mach Number, and the nozzle divergence angle. Information of this type is meager and limited in the range of variation. Accordingly, the initial portion of this research contract was devoted to a systematic experimental investigation of the effect of these variables. These results have been used as an aid to the theoretical study which is still in progress. This paper is devoted solely to the experimental results.

EXPERIMENTAL APPARATUS AND PROCEDURE

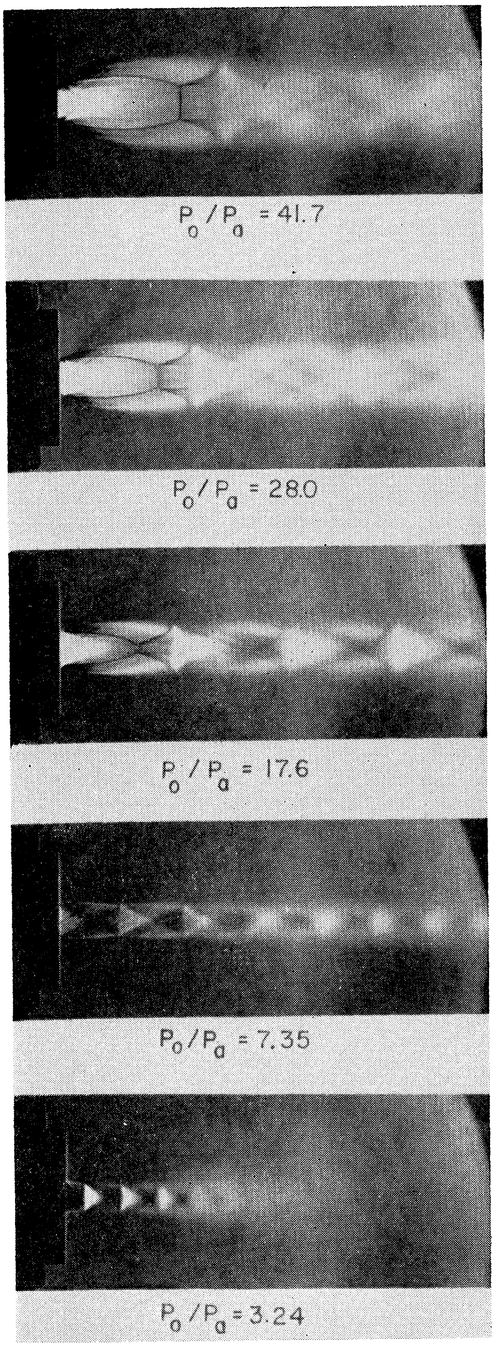
Air was obtained from a 2,000-psia storage system with a total volume of 170 cu.ft. The air used thus had a very low absolute humidity owing to the high storage pressure. The air leaving the storage system passed through two dome-control valves in parallel, through the test nozzle, and then discharged into the atmosphere. The pressure upstream of the test nozzle was maintained constant by the dome-control valves, which were operated by adjusting nitrogen pressure above the diaphragm. Any pressure desired from 0 to 600 psig could be maintained by this system.

In order to investigate quantitatively the influence of nozzle geometry, four axially symmetric nozzles with rounded inlets, throat diameters of 0.250 in., and conical diverging angles of 10°, 15°, 20°, and 30° (total angle) were designed to be connected to the 1-in. pipe downstream of the control valves. The Mach Number at the exit was 3.5 in all cases, as computed by the isentropic flow relation

$$\frac{A_t}{A_e} = Ma_e\left[\frac{(\gamma + 1)/2}{1 + [(\gamma - 1)/2]\, Ma_e^2}\right]^{(\gamma+1)/2(\gamma-1)} \qquad (1)$$

Schlieren photographs were obtained of the flow issuing from these nozzles at upstream stagnation pressures from 55 to 615 psia in 50-psi increments. A conventional schlieren system with a 6-in. field of view was used, the pictures being recorded on 8- by 10-in. film.

In order to investigate the influence of the exit Mach Number on the flow, the above-mentioned nozzles were modified. After each series of runs at one Mach Number, the nozzle was shortened until the exit diameter agreed with that given by the area ratio required for the next Mach Number. The Mach Numbers tested were 3.5, 3.0, 2.5, and 2.0.


$P_o/P_a = 41.7$

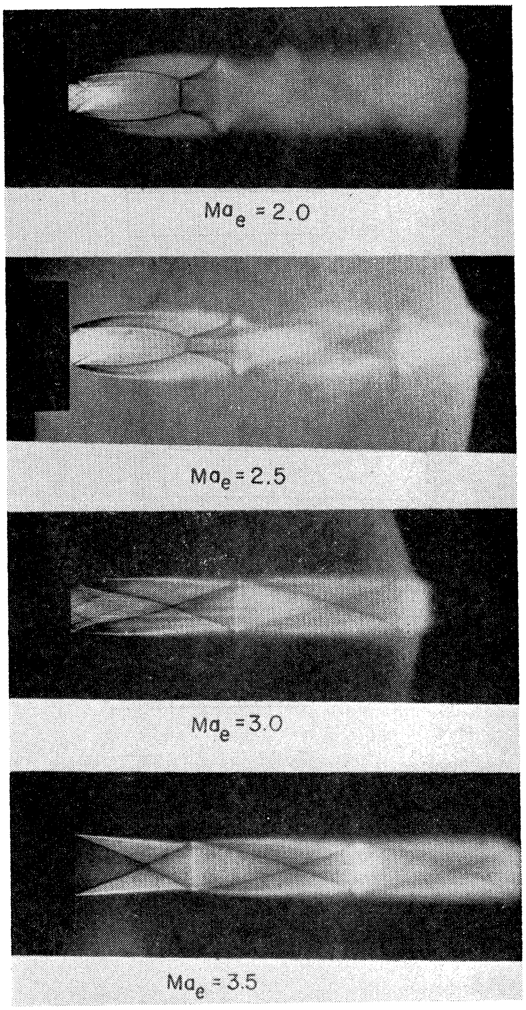
$P_o/P_a = 28.0$

$P_o/P_a = 17.6$

$P_o/P_a = 7.35$

$P_o/P_a = 3.24$

FIG. 2. Effect of pressure ratio on flow pattern. Exit Mach Number = 2.0. Divergence angle = 10°.


$Ma_e = 2.0$

$Ma_e = 2.5$

$Ma_e = 3.0$

$Ma_e = 3.5$

FIG. 3. Effect of exit Mach Number on flow pattern. Pressure ratio = 41.7. Divergence angle = 10°.

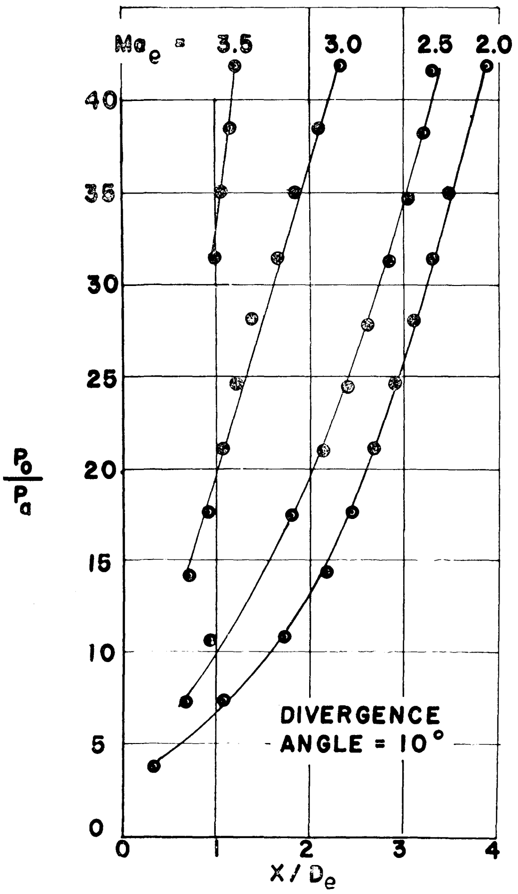


FIG. 4. Variation of shock-wave intersection location with stagnation pressure—10° nozzle.

RESULTS

A schlieren photograph of the flow issuing from a supersonic nozzle is presented in Fig. 1. The jet boundaries as indicated by the optical technique and the curved shock waves extending from the nozzle exit to the normal shock wave are clearly visible. Less visible are the weak oblique shock waves in the region between the nozzle exit and the Mach disc which are due to imperfections in the nozzle.

In Fig. 2, a series of schlieren photographs showing the influence of the pressure ratio on the flow pattern in the axially symmetric jet is presented. Fig. 3 presents four photographs of the flow issuing from nozzles with the same divergence angle (10°) and pressure ratio ($P_0/P_a = 42$) but at exit Mach Numbers of 2, 2.5, 3, and 3.5.

Measurements of the distance of the Mach disc or shock-wave intersection point from the nozzle exit (x) were made from the schlieren negatives. Fig. 4 presents the results of these measurements in graphic form for the 10° nozzle. The plots for all the angles are very similar. The ratio (x/D_e) is plotted versus pressure ratio (P_0/P_a) for the 10° divergent angle nozzle in Fig. 4, with exit Mach Number as a parameter.

DISCUSSION

As mentioned, the exit Mach Numbers presented were computed assuming isentropic flow. It is realized that there is some departure from isentropy in the nozzle, as evidenced by the schlieren photographs.

Figure 4 indicates that the exit Mach Number, in addition to the pressure ratio, is a most influential variable in determining the Mach diamond or disc location. The effect of the divergence angle of the nozzle is relatively small over most of the pressure range.

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Effect of Thermal Resistance of Joints Upon Thermal Stresses

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September 24, 1956

RECENTLY, Griffith and Miltonberger[1] have considered the effect of the thermal resistance of a skin-stringer joint upon the thermal stresses produced by a constant heat source on the skin [Fig. 1(b)]. By dividing the cross section of the joint [Fig. 1(b)] into 15 sections and using an electronic differential analyzer, they calculated the thermal stresses for several values of the parameters for heat transfer to the skin, heat transfer across the joint, and skin geometry. Also, Schuh[2] has solved numerically the equivalent problem [Fig. 1(a)] for no joint thermal resistance and has constructed curves for the maximum stresses in the web against the skin heat-transfer parameter $q = L(h/kh_s)^{1/2}$, where h is skin heat-transfer coefficient (B.t.u./hour ft.^{2} F.), k is coefficient of thermal conductivity (B.t.u./hour ft. F.), and L and h_s are defined in Fig. 1(a).

The results given in references 1 and 2 indicate that it may be possible to obtain an approximate analytical solution for the thermal stresses in the multiweb beam or skin-stringer element

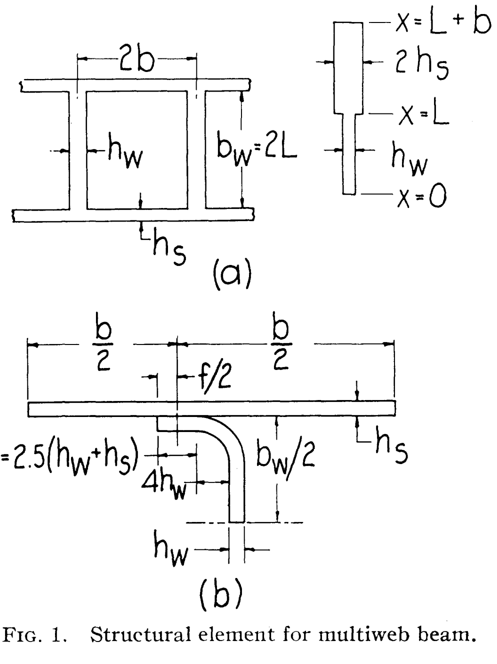

FIG. 1. Structural element for multiweb beam.