For a basic aircraft weight of 14,000 lbs., Mr. Rutowski's derivation gives the numerical results
\[ \Delta h/\Delta W = -(20,940/14,000) = -1.5 \text{ lb. per lb.} \]
\[ \Delta V/\Delta W = 0 \]

for altitudes above 35,000 ft. and all speeds. His formula was derived on the assumption that Eq. (1) can be rebalanced after a change in weight by an appropriate change in \( \rho \). He neglected the fact that any changes in \( \rho \) will act to unbalance Eq. (2) and that both Eqs. (1) and (2) must be satisfied for equilibrium during flight.

**References**


**Author's Reply**

Edward S. Rutowski
Douglas Aircraft Company, Inc.
October 29, 1954

Mrs. Birman claims that the subject paper1 neglects the thrust-drag equilibrium requirement in deriving a formula relating the change in gross weight of a jet-powered aircraft to the change in altitude during the cruise portion of the flight. Her claim, however, is not justified in that the thrust-drag equilibrium requirement is implicitly satisfied for a turbojet-powered aircraft flying at a constant attitude and at a constant Mach Number if the thrust is assumed to be directly proportional to the atmospheric density. This frequently made assumption, which was implicit in the derivation in the subject paper, is reasonably good for a turbojet at altitudes above 35,000 ft. in the isothermal layer since the turbojet is essentially a temperature-limited device. The engine characteristics used by Mrs. Birman apparently do not fit this assumption well, and hence her results differ from those in the subject paper.

If Mrs. Birman had chosen an engine with the characteristic of having a thrust directly proportional to the atmospheric density at a constant Mach Number above 35,000 ft., her results and those in the subject paper would have coincided. This may be seen by evaluating Mrs. Birman's Eqs. (8) and (9) for the case when \( T = K_x \).

Then, since

\[ \frac{\Delta T}{\Delta \rho} = \frac{T}{\rho} \]

Eq. (8) reduces to

\[ \Delta \rho/\Delta W = \frac{\rho}{W} \]

and Eq. (9) reduces to

\[ \Delta V/\Delta W = 0 \]

These equations now are equivalent to those in the subject paper and are therefore not the result of neglecting the thrust-drag equilibrium requirement as claimed by Mrs. Birman but rather the result of an implicit assumption for the thrust variation of a turbojet above 35,000 ft. Furthermore, despite Mrs. Birman's having demonstrated one example where it is not strictly valid, this approximation still has justification in my mind for the purposes of the analysis in the paper.

**References**


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**Forced Convection Through a Laminar Boundary Layer over an Arbitrary Surface with an Arbitrary Temperature Variation**

Myron Tribus† and John Klein‡
University of California, Los Angeles, and University of Michigan, Ann Arbor, Respectively
August 11, 1954

In a previous report the authors considered the general problem of forced convection in several systems when the wall temperature is nonuniform.1 Table 1 summarizes the results of that report and is reproduced here because several new entries have been made in the table and a number of typographical errors have been discovered in reference 1. In all cases, if the wall temperature is given, the heat flux is to be calculated from

\[ q(x) = \int_{\xi}^{x} h(\xi, x) dT_w(\xi) \]  

(1)

where

\[ q(x) = \text{heat flux from the wall, B.t.u./hr. ft.} \]

\[ h(\xi, x) = \text{an integrating kernel (see Table 1), B.t.u./hr. ft.} \]

\[ T_w(\xi) = \text{wall temperature, °F.} \]

\[ x = \text{distance along the wall, ft.} \]

\[ \xi = \text{dummy variable, ft.} \]

and if the heat flux is given, the wall temperature is calculated from

\[ T_w(x) - T_0 = \int_{\xi}^{x} g(\xi, x) q(\xi) d\xi \]  

(2)

where

\[ T_0 = \text{reference temperature of fluid, °F.} \]

\[ g(\xi, x) = \text{integrating kernel (see Table 1), °F. per B.t.u./hr. ft.} \]

Integral 1 is interpreted in the Stieltjes sense.

In this note we give explicit directions for using the fourth entry of Table 1 to compute convection from an arbitrary surface. The fluid properties are constant.** Aerodynamic heating is zero.

The method consists of first solving the momentum equation by using the velocity distributions of Hartree2 and the "patching" technique of Eckert.3 The energy equation is then solved using the method of Lighthill.4 The shear stresses obtained from the momentum equation are modified according to the method of Tifford5 before being used in the Lighthill integrating kernel (line 4, Table 1). The results of two typical calculations are shown in Figs. 1 and 2, where the experimental results of Giedt6 and Drake7 are given. We refer to the method as the H.E.L.T. method (Hartree, Eckert, Lighthill, Tifford). A study of five references1–6 will provide the background for following these instructions for making the calculations such as are demonstrated in Figs. 1 and 2. For a given surface it is presumed the pressure distribution is known and either the heat flux or wall temperature are prescribed along the surface, with the unspecified quantity to be determined.

(1) From the pressure distribution a calculation is made and a graph prepared showing \( u(x) \) and \( du/dx \), where \( u(x) = \text{velocity just outside the boundary layer in feet per second, and } x = \text{distance along the surface in feet.} \)

* This work was done as a part of the Icing Research Program of the Engineering Research Institute at the University of Michigan and was sponsored by the Aeronautical Research Laboratory of the Wright Aeronautical Development Center.

† Associate Professor of Engineering.

‡ Research Assistant.

** See reference 18 for extension with variable properties.
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**TABLE**

INTEGRATING KERNELS FOR

Leading edge

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**Fig. 1.** Comparison of calculated and measured heat transfer from a uniformly heated circular cylinder in cross flow.**

**Fig. 2.** Comparison of calculated and measured heat transfer from a uniformly heated elliptical cylinder in cross flow.
(2) Starting from the stagnation point, it will be found that for a certain distance the graph of \( u(x) \) versus \( x \) is a straight line. In this region the momentum thickness \( \delta_0 \), in feet, is given by:
\[
\delta_0 = 0.292 x (R_e)^{-1/5}
\]
where the Reynolds modulus, \( R_e \), is formed from the local velocity, \( u(x) \), and the distance \( x \) from the stagnation point. In this region the "wedge parameter," \( m = 1 \) (see reference 2).

(3) Downstream of this region it is necessary to solve the following equations (see reference 3):
\[
\begin{align*}
\frac{d\delta^2}{dx} &= 1 - m \frac{\delta^2}{u} \frac{du}{dx} \\
\frac{\delta^2}{v} &= \frac{2m}{1 + m} F(m)\gamma (x)
\end{align*}
\]
\( v \) = kinematic viscosity in feet per second. \( F(m) \) = function obtained from references 2 and 3. Thus, when the velocity is no longer linear in \( x \), we use the known values of \( \delta^2 \), \( du/dx \), to calculate \( F(m) \) and from the graph, Fig. 3, we find \( m \). From Eq. (1) then, a new value of \( \delta \) is computed for a position \( Ax \) downstream. In this way a tabular set of values of \( \delta(x) \), \( m(x) \), \( F(m) \), etc., is prepared. Eckert\(^1\) suggests the isoline method of solution.

(4) The wall shear stress is calculated from the equation
\[
\tau_0 = \mu \frac{du}{dy} y = \mu u(x) F(m) f^\prime(0, m) \delta(x)
\]
Fig. 4 gives a graph of \( f^\prime(0, m) \).

(5) Before substituting this shear stress in the Lighthill integrating kernels (line 4, Table 1), the correction of Tifford is applied, which may be written:
\[
\tau_0 (\text{effective}) = \tau_0 \text{(Hartree)} \left[ 1 - \frac{4}{3} \frac{2m}{1 + m} F(m) f^\prime(0, m) P_r^{-1/4} \right]
\]
\( P_r \) = Prandtl modulus of the fluid, dimensionless.

This is a quasi-empirical correction to Lighthill’s approximation for the more complicated velocity distribution in the boundary layer.

(6) Values of \( \tau_0 \) (effective) are then substituted into the integrating kernels of Table 1, line 4, and the integrations performed graphically. Table 2 gives numerical values for \( F(m), f^\prime(0, m) \).

**References**


