

There is a point of interest concerning the shock growth as $\epsilon \rightarrow 0$. The relation between shock radius and blast energy E is given by

$$r_s = \text{const.} \left(\frac{E}{\rho_\infty I_j(\gamma)} \right)^{1/(3+j)} t^k \tag{16}$$

where

$$I_j(\gamma) = \int_0^1 \left(\frac{P}{\gamma - 1} + \frac{RV^2}{2} \right) \lambda^{2+j} d\lambda \tag{17}$$

Using Eq. (10), a straightforward but somewhat lengthy calculation yields

$$I_j(\gamma) = \frac{1 - \epsilon(2 \ln 2 - 1) + O(\epsilon^2)}{(3 + j)^2(j + 1)} \epsilon^{-1} \tag{18}$$

Hence, $r_s = O(\epsilon^{1/(3+j)})$, which means that the shock speed tends to zero as $\epsilon \rightarrow 0$. Thus, one has the paradoxical case of a strong shock which does not expand. In view of the analogy between the blast wave and steady hypersonic flow, it is of interest to see what this implies. In this analogy, $x = U_\infty t$ and one can show that

$$\left(\frac{r_s}{d} \right) = \left[\frac{C_D}{4^{1+j} I_j(\gamma)} \right]^{1/(3+j)} \left(\frac{x}{d} \right)^k \tag{19}$$

d denotes the leading-edge thickness or body diameter and C_D the nose-drag coefficient.

The range of validity R of the analogy is limited below by the small-shock-angle condition ($\tan^2 \sigma \leq 1/A$, where $A \gg 1$) and above by the strong-shock assumption. Using Eqs. (18) and (19), the range is found to be

$$AC \epsilon^{1/(1+j)} \leq (x/d) \leq C(M_\infty/M_s^*)^{(3+j)/(1+j)} \epsilon^{1/(1+j)} \tag{20}$$

where

$$C = (2C_D)^{1/(1+j)} / (3 + j) \tag{21}$$

and M_s^* is the shock Mach number below which the shock can no longer be regarded as strong.

Therefore, R gets closer to the nose as ϵ decreases and is $O(\epsilon^{1/(1+j)})$, which means that the analogy fails completely in the limit $\epsilon = 0$. Moreover, there is another reason that the analogy fails. The flow field can be divided into two regions: in one region the streamlines have crossed the shock at points where σ is not small, in the other, at points where σ is small. Using Eq. (13) the thickness of the shock layer δ_j in which the blast-wave analogy is valid is found to be

$$\delta_j = \text{const.} [2/(1 + j)]\epsilon + O(\epsilon^2) \tag{22}$$

Hence, this thickness tends to zero as $\gamma \rightarrow 1$ and is twice as large for $j = 0$ as for $j = 1$.

REFERENCES

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² Hayes, W. D., and Probstein, R. F., *Hypersonic Flow Theory*, Academic Press, pp. 52-61, 1959.
³ Einbinder, H. A., *A Note on Blast-Wave Theory*, Journal of the Aero/Space Sciences, Vol. 26, No. 3, pp. 191-192, March, 1959.

Author's Reply

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 August 8, 1960

MR. BROCHER, in the above note, appears to credit me in my previous note¹ with more than I had intended. The sole purpose of this note was to demonstrate that to derive a Newtonian solution in the explosion case, it was necessary to retain the first-order term in the exponent of Eq. (13) as is confirmed by the

exact solution. Physically, this means that the process is required to be isentropic and not isothermal as Newtonian approach would suggest. The order of the approximation was not given in the note since, as Mr. Brocher demonstrates, the complete uniformly valid solution is more complicated. The terms neglected become significant, however, only in the region (r_0/r_s) of order e^{-1/ϵ^2} . Although it is undoubtedly true that the complete uniform behavior is given by retaining the complete form of the exponent in Eq. (12), it is difficult to see why the Newtonian theory would "demand" this.

I would also like to apologize to readers for an inexcusable error in my note which requires that ϵ should read 2ϵ in all equations except the exponent of Eq. (12). The following typographical errors should also be corrected:

- Eqs. (6) and (7): left-hand side should read p/ρ_∞
- Sentence following Eq. (7): insert factor ρ_∞ before parentheses
- Eq. (9): change p/p_s to read ρ/ρ_s
- Eq. (10a): change p_∞ to read ρ_∞

An Analogy Between Boundary-Layer Pressure Gradient and Chemical Reaction-Rate Effects

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THE PURPOSE OF THIS NOTE is to point out a qualitative similarity between the effect of a pressure gradient on the boundary-layer velocity profile $u(Y)$ shape, and the effect of a homogeneous chemical reaction-rate term on the shapes of the corresponding thermodynamic-state profiles [fractional atom concentration $\alpha_A(Y)$ and temperature distribution $T(Y)$]. Consider the following boundary-layer equations governing a dissociated binary-mixture flow in the immediate vicinity of a mass-transfer-free surface $Y = 0(u = v = 0)$:

Momentum

$$\left[\frac{\partial}{\partial Y} \left(\mu \frac{\partial u}{\partial Y} \right) \right]_{Y=0} = \frac{dp_e}{dX} \tag{1}$$

Atomic-Specie Mass

$$\left[\frac{\partial}{\partial Y} \left(\frac{\mu}{S_c} \frac{\partial \alpha_A}{\partial Y} \right) \right]_{Y=0} = - \left[\frac{\dot{W}_A}{\rho} \right]_{Y=0} \tag{2}$$

Energy

$$\left[\frac{\partial}{\partial Y} \left(\lambda \frac{\partial T}{\partial Y} \right) \right]_{Y=0} = \left[(h_A - h_M) \frac{\dot{W}_A}{\rho} \right]_{Y=0} - \left[\mu \left(\frac{\partial u}{\partial Y} \right)^2 \right]_{Y=0} - \left[\frac{\mu}{S_c} (c_{PA} - c_{PM}) \frac{\partial \alpha_A}{\partial Y} \frac{\partial T}{\partial Y} \right]_{Y=0} \tag{3}$$

where S_c is the Schmidt number, \dot{W}_A the net rate of atomic mass production by chemical reaction, and the subscripts A, M refer to atomic and molecular species, respectively. Now, Eq. (1) illustrates the well-known fact that an unfavorable pressure gradient ($dp_e/dx > 0$) tends to cause an inflection point [$(\partial^2 u / \partial Y^2)(0) > 0$] in $u(Y)$ at some $Y > 0$. The net atomic-reaction rate at the wall plays a similar role in Eq. (2) regarding the shape of the atomic-specie profile. Recombination-dominated chemical nonequilibrium ($\dot{W}_A < 0$) in the gas near the wall acts like the unfavorable pressure gradient in the momentum equation, in that it tends to cause an inflection point in $\alpha_A(Y)$ at some $Y > 0$. This chemical-reaction effect holds true for either

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a catalytic [$\alpha_A(0) = 0$] or noncatalytic [$(d\alpha_A/dY)(0) = 0$] wall. While the values of $\alpha_A(0)$ and $(d\alpha_A/dY)(0)$ are determined by the degree of catalytic activity on the wall surface and the integrated effect of the gaseous-reaction rates across the entire boundary layer, the value of $(\partial^2\alpha_A/\partial Y^2)(0)$ is directly influenced by the gaseous-reaction rates in the vicinity of the wall. Furthermore, since $\dot{W}_A/\rho \sim p_c^2(T/300)^{\omega-2}$ for dissociation-recombination in air^{1, 2} the tendency of $(\partial\alpha_A/\partial Y)$ to increase with Y for recombination-dominated reactions will be greater as the local pressure increases (being a maximum at the stagnation point), and will be most prominent at a given pressure for a recombination-rate temperature-dependence law that assumes $\omega > 2$ for $T_w > 300^\circ\text{K}$.

The effect of the reaction-rate term on the temperature profile near the wall (Eq. 3) is opposite to the effect on $\alpha_A(Y)$; an inflection point in $T(Y)$ will appear in the presence of *dissociation-dominated* chemical nonequilibrium in the gas adjacent to the wall. This qualitative analogy with a pressure-gradient effect proves useful in the interpretation of the effects of dissociation-recombination reactions on the thermodynamic-state profile behavior in the boundary layer.

REFERENCES

¹ Fay, J. A., and Riddell, F. R., *Theory of Stagnation Point Heat Transfer in Dissociated Air*, AVCO, Everett Research Report 1, April, 1957.
² Inger, G. R., *Chemical Non-Equilibrium in the Laminar Hypersonic Boundary Layer*, AF CSR TN-59-237, Report 7010-6 (ASTIA AD 212 007, Contract No. AF 49(638)-380), Bell Aircraft Corp., March, 1959.

A Simple Method of Estimating the Vortex Intensity

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SYMBOLS

- $A(D)$ = calibration function
- $a_1 a_2$ = constants in equation (11)
- c = wing chord
- D = distance from the trailing vortex origin
- $k_1 k_2$ = constants of variation in equation (7)
- n = r.p.m.
- p = pressure
- p_0 = pressure in the vortex-core center
- $p_p(r)$ = total pressure in distance r
- Δp_p = $p_p(r_0) - p_p(0)$
- r = coordinate from the core center

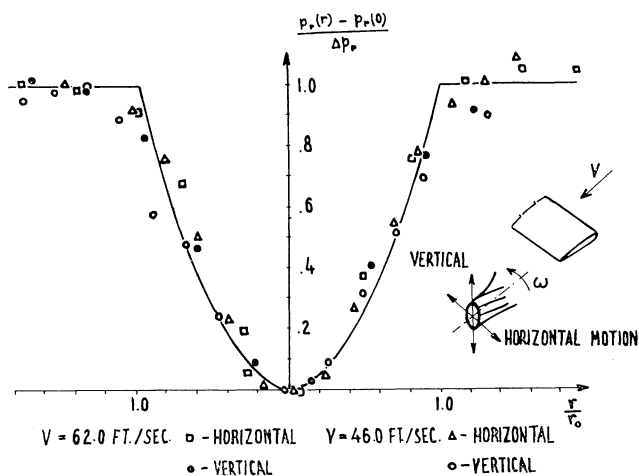


FIG. 1. Dimensionless total-pressure distribution in vortex. Comparison of Eq. (5) with experimental values.

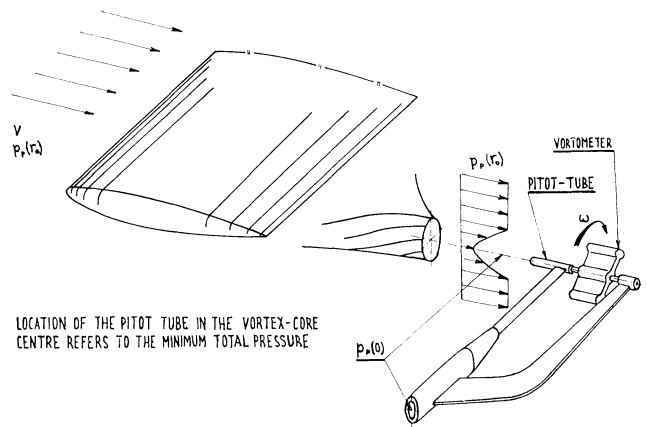


FIG. 2.

- r_0 = radius of vortex core
- t = time
- V = free-stream velocity
- V_i, V_c = velocity outside and inside the vortex core
- Γ = vortex intensity
- $\gamma = \Gamma/Vc$ = nondimensional vortex intensity
- ν = kinematic viscosity
- ρ = density
- σ = surface bounded by integral curve
- ω = angular velocity

INTRODUCTION

TO DETERMINE THE PATTERN and intensity of a trailing vortex behind a wing or any other fairing body, the whole velocity field¹ or the angular velocity of the vortex field² has to be measured. These methods require much time. Therefore, they have no practical value in the verification of assumptions and theories based on the necessary simplifications of actual complex phenomena. The ideas set out below have been found very useful in overcoming this difficulty.

PRINCIPLE OF THE METHOD

As shown in reference 2, a "core" is formed in the center of the potential vortex flow in a real fluid. From a mathematical viewpoint, this can be simulated by assuming the existence of a vortex sheet on the vortex core surface in the ideal fluid. According to reference 3, it can be said that the fluid inside the core behaves like a solid body rotating at a constant angular velocity while the fluid outside the vortex core has the characteristics of the ideal fluid. Thus the velocities, inside and outside the vortex core, can be expressed as

$$V_c = (\Gamma/2\pi r_0^2)r, \quad V_i = (\Gamma/2\pi r) \quad (1)$$

Assuming that the fluid in the vortex core is a rigid body, the equilibrium equation for the element of the core can be easily expressed. The tangent stress need not be taken into account because there is no relative radial motion of fluid inside the core. Considering the fact that velocities on the core surface should be equal, the static pressure in the core will be as follows:

$$p = p_0 + (\rho\omega^2/2)r^2 \quad (2)$$

The total pressure will be

$$p_p(r) = p + (\rho/2)(V^2 + r^2\omega^2) \quad (3)$$

By substituting Eq. (2) into Eq. (3), the following result is obtained:

$$p_p(r) = (\rho V^2/2) + \rho r^2\omega^2 + p_0 \quad (4)$$

The distribution of total pressure along the radius can be verified by measurements. For this purpose, the following nondimensional term has been found practical:

$$[p_p(r) - p_p(0)]/\Delta p_p = (r/r_0)^2 \quad (5)$$

It will be seen in Fig. 1 that, for practical purposes, the experimental data and the plot of term (5) are more or less identical.