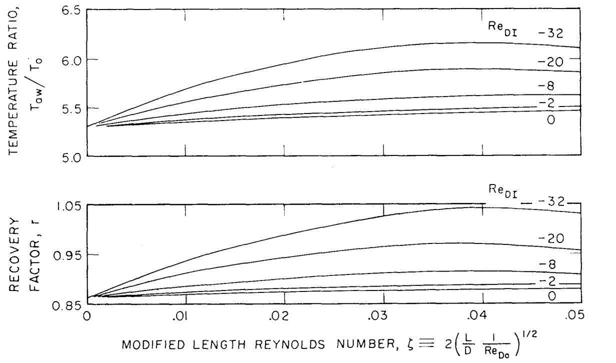

FIG. 2. Variation of temperature ratio and recovery factor along tube length for various values of the injection Reynolds Number, $Re_{DI} \equiv -Re_D\,(\rho u)_{\text{wall}}/(\rho W)_0$.

with an adverse pressure gradient in a tube, with an entrance Mach Number of 5, predicts a small rise in recovery factor, as shown in Fig. 2. At tube entrance, corresponding to the leading edge of a flat plate, the predicted increase in recovery factor is 0; slightly downstream, this increase varies from 0 to about 8 per cent at a value of ζ of 0.035 where validity of the solution becomes questionable.

A parallel experimental investigation of diffusion of helium into the laminar boundary layer of a supersonic flow of air in the entrance region of a tube has been under way for 3 years. Some experimental results[2] for uniform injection of helium into an air stream with an inlet Mach Number of about 4.8 and an inlet diameter Reynolds Number of 123,000 confirm the theoretical predictions given above. These experimental data show practically zero increase in recovery factor ratio with increasing values of injection Reynolds Number Re_{DI}, as long as the flow in the boundary layer remains laminar during injection.

REFERENCES

[1] Radbill, J. R., *Analytical Investigation of the Effects of a Diffusion Field on a Laminar Boundary Layer in Supersonic Flow*, Sc.D. thesis, Massachusetts Institute of Technology, Mechanical Engineering Department, Cambridge, Mass., January, 1958.

[2] Gouse, S. W., Jr., Brown, G. A., and Kaye, J., *Preliminary Results of Measurements of Effects of Diffusion of Helium into the Laminar Boundary Layer of a Supersonic Flow of Air in a Tube at Mach 5*, Readers' Forum, Journal of the Aero/Space Sciences, Vol. 26, No. 8, pp. 533–535, August, 1959.

A Semiempirical Relation for Laminar Separation*

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March 23, 1959

A THEORETICAL RELATION for the pressure rise to a laminar separation point in two-dimensional supersonic flow has been derived by Gadd.[2] For separation from a flat surface, this relation can be expressed as

$$(p_s - p_1)/p_1 \sim M_1^2/[(M_1^2 - 1)Re_1]^{1/4}$$

where $Re_1 = \rho_1 V_1 x_1/\mu_1$, x_1 is the distance from leading edge to beginning of boundary-layer interaction, and the subscript 1 refers to conditions just ahead of the interaction and outside the boundary layer. This equation can be simplified by substituting M_1^2 for $M_1^2 - 1$. (For $M_1 = 1.5$, this simplification changes the pressure rise by 16 per cent; for $M_1 = 2.5$, by only 4 per cent.) There results

$$(p_s - p_1)/p_1 \sim \sqrt{\chi_1}$$

where χ_1 is the hypersonic viscous-interaction parameter

$$\chi_1 = M_1^3/\sqrt{Re_1}$$

(This parameter has been found to be important in the problem of the pressure rise caused by turning of a hypersonic flow to allow for thickening of a laminar boundary layer. Its appearance in the laminar separation equation is not unreasonable since this also is a problem involving turning of a supersonic flow to follow a viscous layer.)

Experimental data on separated flows with completely laminar boundary and mixing layers have been obtained by Chapman, Kuehn, and Larson.[3] The values they obtained for the plateau pressure rise after laminar separation from a flat surface (plotted in Fig. 1 as a function of the hypersonic interaction parameter, χ_1) are well represented by the semiempirical equation

$$(p_2 - p_1)/p_1 = 1.27\sqrt{\chi_1} \tag{1}$$

for Mach Numbers above 1.3. It is seen that the plateau pressure rise varies linearly with $\sqrt{\chi_1}$, in agreement with the approximate equation for pressure rise to the separation point derived above from the theory of Gadd.

Eq. (1) is essentially a local relationship between the plateau pressure rise and the location of separation for given free-stream conditions. In order to solve many problems of pure laminar separation, a second relationship between these two variables

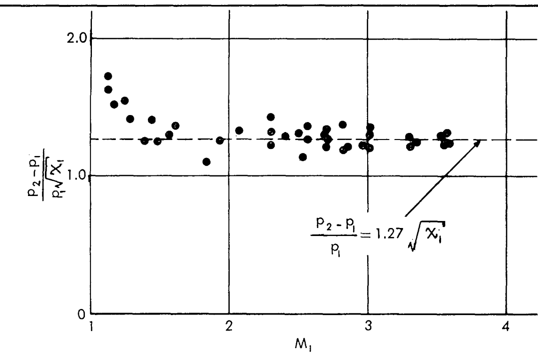

FIG. 1. Plateau pressure rise data from reference 3.

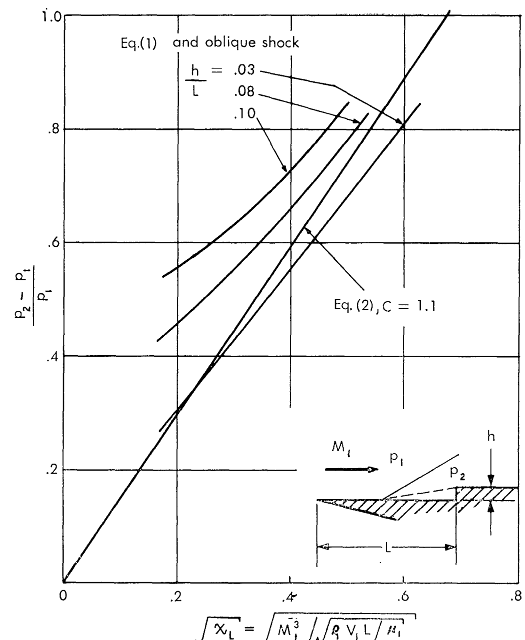

FIG. 2. Pure laminar separation ahead of step at $M_1 = 3.0$.

* This work was supported by the U.S. Air Force as part of Contract No. AF 33(616)-3573, monitored by Aeronautical Research Laboratory, WCLJD, WADC. A complete account is given in reference 1.

must be found. In the simple case of separation ahead of a step, the reattachment point is fixed, and the oblique-shock theory gives the required second relation between separation-point location and pressure rise. For most other problems, however, the reattachment point is not known. It is, therefore, necessary to find a relationship between reattachment-point location and pressure rise at reattachment. Further experimental work is needed to define such a relation.

An expression similar to the present semiempirical relation has been proposed by Guman.[4] His expression, however, uses a Reynolds Number based on the distance to the point where the disturbance would intersect the boundary layer if there were no interaction. It, therefore, gives the pressure rise for given flow conditions directly without requiring local expressions for separation-point or reattachment-point locations. An empirical constant is used to fit the equation to experimental data for a given body shape. Guman's equation can be written

$$(p_2 - p_1)/p_1 = C\gamma[(M_1^2 - 1)/M_1^2]^{1/4}\sqrt{\chi_L} \tag{2}$$

where C is the empirical constant. A comparison of Eqs. (1) and (2) is shown in Fig. 2 for the case of a forward-facing step at $M_1 = 3.0$.

References

[1] Amick, J. L., and Carvalho, G. F., *Low Reynolds Number Aerodynamics of Flapped Airfoils at Supersonic Speeds*, WADC TR 58-466, September, 1958.

[2] Gadd, G. E., *The Effects of Convex Surface Curvature on Boundary Layer Separation in Supersonic Flow*, Current Paper No. 289, British A.R.C., 1956.

[3] Chapman, D. R., Kuehn, D. M., and Larson, H. K., *Investigation of Separated Flows in Supersonic and Subsonic Streams with Emphasis on the Effect of Transition*, NACA TN 3869, 1957.

[4] Guman, W. J., *On the Plateau and Peak Pressure of Regions of Pure Laminar and Fully Turbulent Separation in Two-Dimensional Supersonic Flow*, Readers' Forum, Journal of the Aero/Space Sciences, Vol. 26, No. 1, p. 56, January, 1959.

Heat Transfer to a Vaporizing, Ablating Surface

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March 23, 1959

The determination of the heat transfer to an ablating surface is an extremely complex one, which has been treated theoretically by many authors—e.g., Lees.[1] The exact solution for an arbitrary material involves knowledge, available for very few materials, of surface temperature, surface heat of reaction, vapor species, reaction rates of species with boundary-layer gas constituents, transport properties of vapor, gas constituents at the edge of the boundary layer, and many other properties. However, the results of available theories for injection of simple gases point the way to a simple engineering approach as well as a straightforward experiment for determining the quantities needed for this approach.

Implicitly bound up in reference 1 is a relation between q, the energy-transfer rate per unit area to a surface with mass addition, and q_0, the zero mass addition heat-transfer rate, where both q and q_0 are determined for the same surface temperature, T_w. This relation is

$$q/q_0 = f(B) \tag{1}$$

where

$B \equiv m_v/\rho_e u_e C_{H0}$
$C_{H0} \equiv q_0/\rho_e u_e(H_e - H_{w0})$
m_v = mass rate of vapor injection per unit area
ρ_e, u_e, H_e = gas density, velocity and specific stagnation enthalpy at edge of boundary layer, respectively
H_{w0} = gas specific enthalpy at wall with no mass injection

This is the same general form used by Rubesin and Pappas[2] for gas injection into a turbulent boundary layer on a flat plate. They also used a similar formula for shear force—namely,

$$\tau/\tau_0 = f_1(B_1) \tag{2}$$

where $B_1 \equiv m_v/(\rho_e u_e C_f/2)$

It is obvious from physical considerations that $f(0) = f_1(0) = 1$. Eq. (1), after substitution of C_{H0}, can be expanded to give

$$q/q_0 = 1 - \eta[m_v(H_e - H_{w0})]/q_0 + \ldots + \tag{3}$$

or neglecting higher-order terms

$$q = q_0 - \eta(H_e - H_{w0})m_v \tag{4}$$

Thus, the term $\eta(H_e - H_{w0})m_v$ gives the lowering of energy-transfer rate to the wall due to mass injection rate, m_v. A similar relation for shear stress is

$$\tau = \tau_0 - \eta_t u_e m_v \tag{5}$$

Scala and Sutton,[3] made calculations for air injection into air with Sc = Pr = 1 over a wide range of wall enthalpies. The author found[4] that these calculations could be correlated by $\eta = 0.8$, $\eta_t = 0.67$ over a very wide range of injection rates, indicating that the neglecting of higher-order terms was essentially correct. Baron,[5] made extensive calculations at various Schmidt and Prandtl Numbers for light gases as well as air. Bade,[6] found that these results could be put into the form of Eq. (4) and that $\eta \sim M^{-1/3}$ where M is the molecular weight of the injected vapor. The variation is probably a function of specific heat as well as molecular weight for complex molecules. For this data $\eta_{\text{air}} = 0.74$. The available calculations for injection into a turbulent flow indicate that η is of the order of 0.4 to 0.5 of the laminar case.

The energy transfer rate into the wall is given by

$$q_w = q - q_r - m_v\lambda_v \tag{6}$$

where q_r = net rate at which energy is radiated away from the surface
λ_v = latent heat of vaporization to produce vapor species actually present at surface

The above holds for either solid or liquid surfaces.

Most of the complexities of the boundary-layer flow can be lumped into the one constant η. The simple rules set forth for estimation of η are difficult to apply for arbitrary wall surfaces due to uncertainty in knowing the vapor species at the surface. However, the η for laminar flow may be found experimentally by considering steady-state ablation at a blunt-body stagnation point. At extended times under constant external conditions an ablating surface reaches a condition where the mass rate of ablation, heat-transfer rates, and temperature distribution in the material are constant with time. This is the condition of steady-state ablation.

For this case q_w is

$$q_w = m_v \int_{T^0}^{T_w} c\,dT = m_v c_a(T_w - T^\circ) \tag{7}$$

for a subliming solid,

$$q_w = mE \int_{T^0}^{T_w} c\,dT = mEc_a(T_w - T^\circ) \tag{8}$$

for a vaporizing glassy material, and

$$q_w = m\left[E\int_{T_M}^{T_w} c_L\,dT + \lambda_M + \int_{T^0}^{T_M} c_s\,dT\right] \tag{9}$$

for a solid with a discrete melting point, T_M. T° represents the

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