$$\tilde{C}_D = \left(\frac{18}{\gamma + 1}\right)^{1/3} \frac{F(11/6, 4/3)}{F(11/6, 2/3)}$$

where

$$F(a, b) = \int_0^\infty \sum_{n=1}^{\infty} (-1)^n n^a e^{-n\pi\tau/\delta}\tau^{b-1}\, d\tau$$
$$= (\delta/\pi)^b \Gamma(b) \sum (-1)^n n^{a-b}$$
$$= (\delta/\pi)^b \Gamma(b)\, (2^{a-b+1} - 1)\, \zeta(b - a)$$

where ζ is Riemann's Zeta function. The successive transformations are justified only if $b > 0$ and $b > a$, but the final result is valid without the restriction $b > a$ because of analytic continuation. Finally, after some simple transformation, the reduced drag coefficient of the wedge is obtained in the form

$$\tilde{C}_D = (\gamma + 1)^{1/3}\delta^{-5/3}\, C_D = \frac{2^{5/6}3^{5/3}}{7}\, \frac{2^{3/2} - 1}{2^{13/6} - 1}\, \frac{\zeta(3/2)}{\zeta(13/6)} \sec\frac{\pi}{12} = 1.488$$

Somewhat similar calculation based on the simple approximation $k(\tau) = 0$, which has been adopted in the author's previous paper,[4] gives

$$\tilde{C}_D = 2 \times 3^{2/3}\, \frac{2^{7/6} - 1}{2^{11/6} - 1}\, \frac{\zeta(7/6)}{\zeta(11/6)} \tan\frac{\pi}{12} = 1.944$$

These two values are to be compared with Guderley-Yoshihara's numerical result

$$\tilde{C}_{DGY} = 1.75$$

and Cole's analytical result

$$\tilde{C}_{DC} = 4\left(\frac{\pi}{3}\right)^{1/3} \frac{1 - 2^{1/3}}{[\Gamma(2/3)]^2}\, \zeta\left(\frac{2}{3}\right) = 1.67$$

The agreement is surprisingly good. This is probably due to the fact that the flow condition downstream of the corners of a finite wedge has only a little influence on the pressure over its front part. This seems to indicate that the "dead-air" theory might be useful for the consideration of nose drag of bodies at high subsonic speeds.

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Note on the Conditions at a Sharp Leading Edge in a Supersonic Stream*

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March 20, 1952

In a recent article, Bardsley[1] has described the flow configuration in the vicinity of the sharp leading edge of an airfoil at an angle of attack of 15° at a Mach Number of 1.965. The configuration observed is shown in Fig. 1. In the region above the wedge where inviscid theory predicts only a Prandtl-Meyer expansion wave, a weak shock was found preceding this wave. Bardsley explained the phenomenon by considering only the bluntness of the wedge. It was shown that flow separation and boundary-layer growth did not contribute to the formation of the leading shock under these conditions.

However, recent shock-tube experiments suggest that this configuration is by no means unique for a sharp airfoil at an angle of attack in a supersonic stream; rather it seems to be the limiting case for high Mach Numbers. At high subsonic Mach Numbers, the upper surface of an airfoil at an angle of attack is covered by a detached, turbulent boundary layer—that is, the airfoil is in a state of stall. At higher Mach Numbers this boundary layer becomes thinner, and at some transonic value the flow reattaches to the rear portion of the airfoil. The region of separated flow shrinks rapidly as the Mach Number is raised further and presumably ceases to exist at some value below 1.965.

Fig. 2 is a shock-tube shadowgraph of the transonic flow in nitrogen about a double-wedge airfoil at a Mach Number of 0.85. In this case the length of the separated boundary layer, or the compressibility bubble, is approximately 1/8 in.

A photograph showing the flow configuration at a higher Mach Number is presented in Fig. 3. This picture is a shadowgraph of the flow in carbon-tetrachloride vapor at a Mach Number of 1.65 about a sharp 12° wedge at an angle of attack of 15°. Carbon-tetrachloride vapor was used in order to obtain relatively high Mach Number without sacrificing resolution.

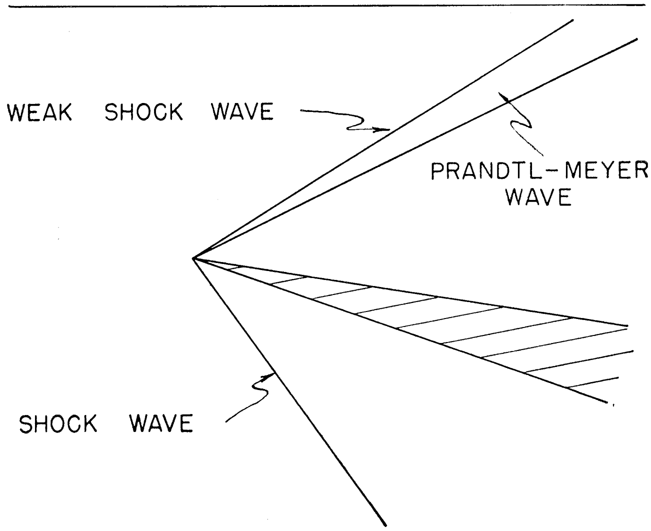

Fig. 1. Supersonic flow at the tip of a wedge, as observed by Bardsley.

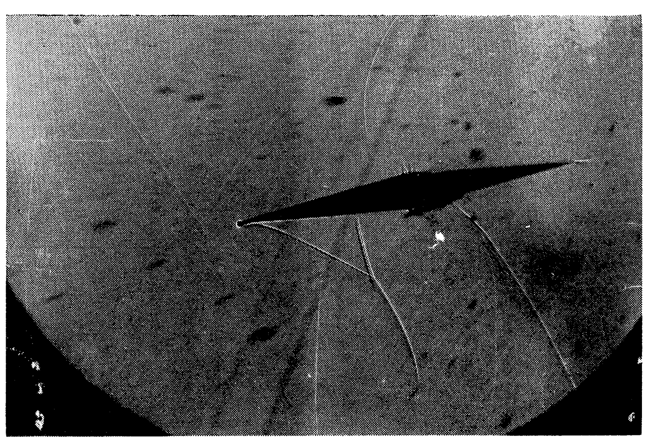

Fig. 2. Transonic flow showing a large compressibility bubble.

* The author wishes to express his gratitude to Robert N. Hollyer, Jr., for his active assistance in this research program and to the Office of Naval Research for financial support furnished through contract N6-ONR-232 TO IV.

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