where

$$M_0 = V_0/a_0, \qquad M_R = R\omega/A_0$$

and ω is the actual rotational velocity of the propeller.

In order to apply Eq. (9) to finite disc loadings, it is easily shown that we must multiply it by Eq. (1) and replace M_0 by M_1 by means of Eq. (2) in order to get the higher order influence of the induced axial inflow ($\delta_1 V_0$), considered uniform across the disc. However, the question now arises whether this higher order term is justified in comparison with other neglected effects. For example, by using the same method as used in deriving Eq. (2.8) on page 194 of reference 2, it may be shown that for subsonic flow the first-order effect, due to the decreased enthalpy produced by the slipstream rotation, may be expressed as

$$\delta_1 = (1/2)\delta_3 \; [1 + \delta_1(M_0^2/M_R^2)] \qquad (10)$$

Another term, also of the same order, is found to be that produced by the subsonic compressibility effect of the semi-infinite helical trailing vortex system accompanying the infinite number of propeller blades. This term may be evaluated to the first order by combining Eq. 43 of reference 3 with the procedure used in deriving Eq. (89) of reference 4 to show that

$$\delta_1 = \frac{1}{2}\,\delta_3 \left(1 - \frac{\delta_1}{2\sqrt{1 - M_0^2}}\right) \qquad (11)$$

for light disc loadings and $M_0 \rightarrow 0$.

The important fact is now seen that Eqs. (1) and (11) have exactly the opposite influence on Eq. (9) as have Eqs. (2) and (10). Consequently, Eq. (9) is probably applicable to heavier disc loads in its present simplest form. The effect of varying inflow or varying chord can be shown to be a higher order effect that tends to cancel, leaving Eq. (9) as the ratio of the compressibility effect for any propeller.

Eqs. (10) and (11) prove that the axial-momentum theory is not justified by itself alone. The question still arises, however, as to the limitations of the actuator disc concept in subsonic flow. For example, it is easily shown that the so-called isentropic "ideal propeller" used in reference 1 is not a valid assumption for larger disc loadings. For Eq. (8) of reference 1 it is stated that the term due to $\rho_3 < \rho_0$ exists because of the propeller profile drag and such additional losses. This, however, is not the case, since Eq. (8) of reference 1 corresponds to the required increase of entropy that *must* coexist with the discontinuous pressure rise assumed for the actuator disc. Therefore, one cannot assume both the thin actuator disc and $\rho_3 = \rho_0$ as in reference 1. Actually, Eqs. (1) through (6), the first-order effect, should provide a better approximation than reference 1. It is interesting to note that Eq. (4) approaches the correct limit for the mass-rate of flow for sonic conditions ($M_0 \rightarrow 1$)—namely,

$$m_* = \rho_0 V_0 A = \rho_0 V_0 \pi R^2 \qquad (12)$$

The first-order effect of $\rho_3 < \rho_0$ may be easily obtained from the energy equation and the first-order approximation for power required (TV_1) as

$$\delta_1 = \frac{1}{2}\,\delta_3 \left\{1 + \frac{[(\rho_0/\rho_3) - 1]}{\left(\dfrac{\gamma - 1}{2}\right) M_0^2\, \delta_3^2}\right\} \qquad (13)$$

showing that the effect of $\rho_3 < \rho_0$ cannot be neglected unless

$$[(\rho_0/\rho_3) - 1] << (\delta_3)^2 \approx (2\delta_1)^2$$

References

[1] Vogeley, Arthur W., *Axial-Momentum Theory for Propellers in Compressible Flow*, N.A.C.A. T.N. No. 2164, July, 1951.

[2] Durand, W. F., *Aerodynamic Theory*, Vol. IV, pp. 169–360.

[3] Theodorsen, Theodore, *Theory of Propellers*, McGraw-Hill Book Company, Inc., New York, 1948.

[4] Tsien, Hsue-Shen, and Lees, L., *The Glauert-Prandtl Approximation for Subsonic Flows of a Compressible Fluid*, Journal of the Aeronautical Sciences, Vol. 12, No. 2, pp. 186–187, April, 1945.

Theory of Heat Transfer Through a Laminar Boundary Layer

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The Readers' Forum note by H. Schuh (Journal of the Aeronautical Sciences, Vol. 20, No. 2, p. 146, February, 1953) appears to cover the same ground as the paper by M. J. Lighthill on "Contributions to the Theory of Heat Transfer Through a Laminar Boundary Layer" (*Proc. Roy. Soc. London*, Series A, Vol. 202, pp. 359–377, 1950).

The Lighthill results are somewhat more general than those given by Schuh.

Errata—"Diffuser Efficiency of Free-Jet Supersonic Wind Tunnels at Variable Test Chamber Pressure"†

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March 9, 1953

In the reference article,† the following errors were made:

(1) Eq. (29) should read

$$M_{2\alpha,\,2\beta}^* = \frac{\left(\dfrac{2\gamma}{\gamma + 1} - \dfrac{1}{f'}\dfrac{\gamma - 1}{\gamma + 1}\right) M_1^{*2} + \dfrac{1}{f'}}{2M_1^*} \pm \sqrt{\left[\frac{\left(\dfrac{2\gamma}{\gamma + 1} - \dfrac{1}{f'}\dfrac{\gamma - 1}{\gamma + 1}\right) M_1^{*2} + \dfrac{1}{f'}}{2\,M_1^*}\right]^2 - 1}$$

(2) On page 379, the fifth line following Eq. (42), η_α must read η_β.

† Journal of the Aeronautical Sciences, Vol. 19, No. 6, June, 1952.