Readers' Forum

BRIEF REPORTS of investigations in the aeronautical sciences and discussions of papers published in the JOURNAL will be presented in this special department. The publication will be completed approximately 6 to 8 weeks after receipt of the material. No proof will be sent to the authors. The Editorial Committee does not hold itself responsible for the opinions expressed by the correspondents. Contributions should not exceed 800 words in length.

Supersonic Airfoils Simplified

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REFERENCE IS MADE to the simplified "Approach to Supersonic Airfoil Theory" of W. E. Strohmeyer and D. R. Gero in the August issue, as corrected in the October issue (JOURNAL OF THE AERONAUTICAL SCIENCES). The following simpler and still more direct approach has been found particularly well suited to undergraduate instruction because of the easy visualization of the geometrical concepts involved. The customary symbols are employed, as noted in the accompanying sketch.

It is assumed as usual that the general flow conditions at the given airfoil section are known and two-dimensional in character. The figure shows a simple example at $M = \sqrt{2}$; a thin, flat plate moving horizontally through the air at a small angle of attack α. Consider what has happened since the *LE* was one chord length back of where the *TE* is now. At that moment, pressure impulses traveled out from the *LE'* in all directions, relative to the air at a rate of V/M ft. per sec.; and, in the time required for the wing to go a distance $2c$, will have reached the circumference of a circle of radius $c\sqrt{2}$. Similar circles could be drawn from all other points through which the *LE* has passed, defining a zone limited by a common tangent line (the so-called Mach line) in this case at 45° to the direction of motion, as shown.

Consider the air now as in motion relative to the airfoil's lower surface. By construction, the pressure and deflection of air (because of its reaction on the airfoil surface) are confined to the region above and to the right of the line *LE-A* extended, because only within that zone is it possible for a disturbance to be transmitted. A particle of air striking directly under the *LE* will be deflected out of its original path through the angle α, but will have no immediate effect on particles still farther below. Particles of air which have gone before, however, have made it possible for all the air in the triangle *LE-TE-A* to be similarly deflected and, for negligible changes in density, it is uniformly so deflected. Furthermore, due to the weak oblique shock wave and small deflection, the longitudinal component of speed in this triangle is still approximately V.

The total section of air (normal to the direction of motion) acted upon over a unit of span is then *TE-A* or approximately c; the mass of air acted upon is ρVc slugs per sec., and the downward velocity imparted to it is $V\alpha$. The rate of change of momentum per sec., constituting the normal force for the lower surface (lbs. per ft. of span), is

$$F_l = \rho V^2 c\alpha = 2qc\alpha \quad (1)$$

The surface pressure per unit area is then F_l/c, and the gage pressure coefficient

$$p/q = F_l/cq = 2\alpha \quad (2)$$

Similar analysis holds for the upper surface, the pressure there being numerically equal but negative. Thus the section lift coefficient (substantially equal to the normal force coefficient):

$$C_l = (F_l - F_u/cq) = 4\alpha \quad (3)$$

It must be noted that Eqs. (2) and (3) are valid for the assumed $M = \sqrt{2}$ only. However, it is clear from the same principles that in any case the length of the line *TE-A*, and hence the reacting force, will be proportional to $\tan\mu$, which in this case happened to be unity. As $\tan\mu = 1/\sqrt{M^2 - 1}$, the final equations for pressure and lift (for $\alpha \ll \mu$) become:

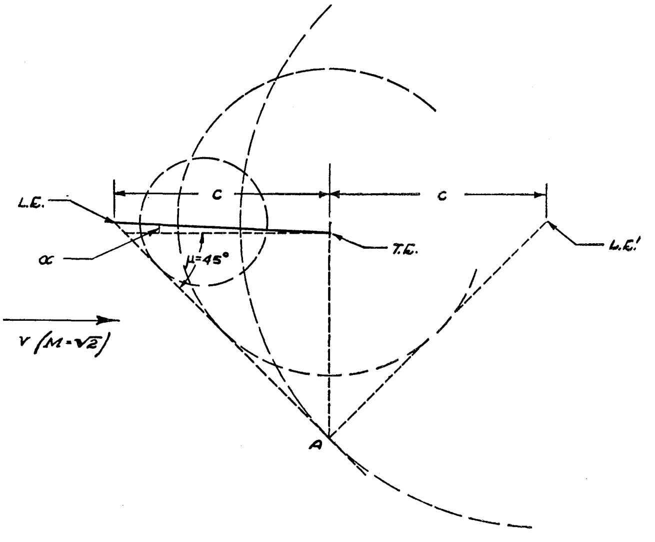
$$p/q = (2\alpha/\sqrt{M^2 - 1}) \quad (4)$$

$$C_l = (4\alpha/\sqrt{M^2 - 1}) \quad (5)$$

In Eq. (4), α is considered negative when in a direction to produce negative p. The corresponding drag (neglecting surface friction) is then simply the rearward component of the normal force, or approximately in this case:

$$C_{dw} = C_l\alpha = (4\alpha^2/\sqrt{M^2 - 1}) \quad (6)$$

Here the subscript w indicates that it is wave drag, because the actual energy loss making up this element of the drag occurs in two oblique shock waves, one below the leading edge (practically coincident with the line *LE-A*) and a similar one above the trailing edge.



The same principles and the same result, expressed in Eq. (4) hold for *any* small increment of surface in two-dimensional flow. Hence, aerodynamic forces on finitely thick or curved surfaces complying with these general conditions can be found by simple summation or integration of the pressure times each increment of area involved. For a cambered airfoil of finite thickness with sharp *LE* and *TE*, C_l integrates to the same form as Eq. (5) if α is taken as the angle of attack of a straight line joining the *LE* and *TE*.

Corrections for sweep or tip loss are negligible for small *LE* sweep angle (up to about 10°) and for tips cut back at approximately the Mach angle. The theory becomes increasingly inaccurate at $M \to 1$, and in any case where the distance from the given airfoil section to the tip is less than $c/\sqrt{M^2 - 1}$. Subject to these and other minor qualifications, there is no induced angle in the sense employed in low-speed airfoil theory, and no direct effect of aspect ratio. Thus wave drag pertains to thickness as well as to lift (angle-of-attack), and can be superimposed from results of separate computations. From test data thus far available, frictional drag appears to be of the same order of magnitude as under subsonic conditions.

Unsteady Flow Theory in Dynamic Stability

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In a paper delivered at a recent I.A.S. meeting,[1] I. C. Statler concluded that "unsteady flow effects are not important in the calculation of the dynamic stability and response of high-speed aircraft." With this broad conclusion, we cannot agree, and we wish to point out (what we regard as) certain fallacies in both the interpretation of the experimental data and the theoretical arguments presented in his paper. At the same time, we wish to modify some earlier predictions of our own.[3]

In calculating the dynamic stability derivatives, we may generally assume that the reduced frequency $k(= \omega c/2U)$ is less than 0.1. It follows that we may neglect terms of order k or higher in the aerodynamic forces in phase with displacements or accelerations, since they are to be compared with aerodynamic terms of order, zero, and the very large mechanical inertia terms, respectively. In the case of the aerodynamic forces in phase with the velocities, however, we must consider all terms of order k (*including terms of order* $k \log k$), and it follows that the first-order effects of unsteady flow considerations will manifest themselves only in the damping terms. Moreover, in a system which is sufficiently underdamped to exhibit marked resonances, damping exerts only a second-order effect on the response of the system away from resonance, which will be governed by displacement or "spring" forces at lower frequencies and inertia forces at higher frequencies, whereas the magnitude of the resonant response is critically dependent on damping. Hence, the effects of discrepancies between the predictions of steady and unsteady flow theory as to the dynamic response of a system must be inferred from its behavior in the vicinity of resonance. Examination of Statler's results reveals that this is indeed the case, albeit he draws his conclusions from the behavior at the higher frequencies.

In two recent discussions in these columns, we have presented the first-order (in frequency) results for two-dimensional, thin airfoils in subsonic flow.[2, 3] (The first-order results for supersonic flow had been given earlier, for both the two- and three-dimensional cases.[4, 5]) These results indicate that the damping moment due to rotary motion (i.e., the *total* component of moment in phase with the angular velocity) is of order $k \log k$ for sufficiently small k, so that the pitch-damping derivative becomes infinite as $\log k$ when k approaches zero, indicating two-dimensional, unsteady flow considerations to be increasingly important (in determining the *damping per cycle*) as k is decreased, at least if strip theory is used, as in Statler's calculations. Moreover, the results of reference 3 indicate that this logarithmic term requires a compressibility correction factor of $(1 - M^2)^{-3/2}$ rather than the usual $(1 - M^2)^{-1/2}$. Unfortunately, as shown by Jones[6] and Reissner,[7] strip theory is completely inadequate for the treatment of unsteady flow effects at extremely low frequencies. Indeed, the reduction of Reissner's results for small k reveals that a term $k \log (k\, AR)$, where AR is the aspect ratio, must be subtracted from the term $k \log k$ in the two-dimensional result, whence the term $\log k$ in the pitch-damping derivative is replaced by the considerably smaller (in magnitude) term $\log AR$, and there is no singularity at $k = 0$. (There are, of course, additional aspect ratio corrections.) It follows that the numbers (predicting a large reduction in damping) cited by us in reference 3 are far from realistic and that Statler's use of strip theory cannot be a valid basis for any general conclusion. Nevertheless, an investigation of a conventional, subsonic configuration of a low aspect ratio tail aft of a straight wing based on the results of references 3 and 7, reveals that the only important unsteady flow effect appears in the "complex downwash lag" at the tail, in agreement with Statler's conclusion. (This investigation was carried out as a part of a more general study being conducted at the North American Aerophysics Laboratory.) However, this conclusion cannot be extended to such configurations as the canard or the flying wing or to any configuration in supersonic flow without further analysis, and, in particular, it appears that unsteady flow considerations will exercise a profound effect on the pitch damping of a flying wing in either subsonic or supersonic flow. Moreover, unsteady flow effects, particularly the complex sidewash correction, may be expected to exercise a pronounced effect on the lateral damping characteristics of a vertical fin, as implied by the tendency of many high-speed, jet aircraft (with fins designed on the basis of steady flow theory) to "snake." Thus, while we should agree with Statler's results for the particular configuration which he considered, we cannot agree with his conclusion that this happy state of affairs can be anticipated generally.

As previously pointed out,[2, 3] one of the errors in applying steady flow theory to unsteady flows is the failure of the former to include the term $\rho(\partial\phi/\partial t)$ in the Euler equation for the pressure. This same objection must be made to Statler's analysis of the oscillating fuselage. The potential given by the Munk theory[8] for a slender body of revolution is indeed valid for nonsteady flow, but the lift per unit length on a body of revolution is

$$\frac{\partial L(x, t)}{\partial x} = \rho U^2 \left(\frac{\partial}{\partial x} + \frac{1}{U}\frac{\partial}{\partial t}\right)\left[\frac{S(x)w(x, t)}{U}\right] \tag{1}$$

where $w(x, t)$ is the instantaneous downwash (positive down), and $S(x)$ is the cross-sectional area at x. (We hope to discuss this situation more extensively in a future paper.) The operation of time differentiation on that part of $w(x, t)$ in phase with the displacement evidently gives rise to a damping force. If q is the angular velocity, l the length, and S_0 the base area, the damping derivative due to an oscillation about a point a aft of the nose, is found to be given by

$$\partial M/[\partial(ql/U)] = -2(\rho U^2/2)S_0 l \left(1 - \frac{a}{l}\right)^2 \tag{2}$$

The result obtained by Statler (which, incidentally, retains some terms not justified by slender body theory) is appropriate to flight along a curved flight path at constant angle of attack, but not to the case of oscillation about a straight flight path. Indeed, it is the confusion of these two situations which has been primarily responsible for the failure to account correctly for the effects of unsteady flow in the calculation of dynamic stability derivatives.

We do not wish to appear unduly critical of Statler's paper and remark that the errors we have discussed are extant in much of the antecedent work on the same subject. There has been a good deal of specious reasoning relative to nonstationary processes in moving reference frames; in many cases this may not have led to serious error, but it may be recalled that Zeno's sophistry with respect to time-dependent phenomena did not, in fact, prevent Achilles from catching the tortoise nor the arrow from catching Achilles.

References

[1] Statler, I. C., *Dynamic Stability at High Speeds from Unsteady Flow Theory*, New York, N.Y. (January 24–27, 1949); I.A.S. Preprint 187.

[2] Miles, J. W., *Quasi-Stationary Thin Airfoil Theory*, Journal of the Aeronautical Sciences, Vol. 16, No. 7, p. 440, July, 1949.

[3] Miles, J. W., *Quasi-Stationary Airfoil Theory in Compressible Flow*, Journal of the Aeronautical Sciences, Vol. 16, No. 8, p. 509, August, 1949.