Boundaries of Fluid Mechanics

Keeve M. Siegel
Research Associate, Aeronautical Research Center, University of Michigan, Ann Arbor, Mich.
November 14, 1949

In October's Aeronautical Engineering Review, Mr. Roberts points out a new criterion suggested by H. W. Liepmann to distinguish the realms of fluid mechanics. In the literature there already are criteria that attempt to define the regions of fluid mechanics. This note compares these criteria.

**Boundary Between Continuum Analysis and Slip Flow**

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Donaldson</td>
<td>$M\lambda/l = 0.04$</td>
</tr>
<tr>
<td>(2) Tsien</td>
<td>$\lambda/\delta = M/\sqrt{R}$, $R \gg 1$</td>
</tr>
<tr>
<td>(3) Schultz, Spencer, and Reifman</td>
<td>$\lambda/\delta = 0.43(M/\sqrt{R}) \approx M/2\sqrt{R}$</td>
</tr>
<tr>
<td>(4) Roberts</td>
<td>$R/M^3 = 100$</td>
</tr>
</tbody>
</table>

We can compare these criteria in two ways: first,

$$M = \frac{M(v/\alpha)}{\rho l/\mu} = \frac{M \mu}{M \lambda \mu} \approx \frac{M}{l}$$

In the region of the atmosphere under consideration $T/T_0$ can be considered unity in an order of magnitude analysis.

$$\frac{M^3}{R^3} \approx \frac{(0.438 \frac{T}{T_0})^{1.56}}{1.2} = \frac{M \lambda 1}{1.2}$$

Second, an extension of Donaldson's Appendix A,2

$$\frac{M^3}{R^3} = \frac{M^3}{\rho l/\mu} = \frac{M^3}{\mu l/0.73\lambda a} = \frac{M \lambda}{0.73 \lambda a} \approx \frac{3 M \lambda}{4 l}$$
Let us use $M^2/R = (M/l)^{(1/2)}$ to compare the criteria.

1. Donaldson:
   $$M/l = 0.04$$

2. Tsien:
   $$M/l = 0.0002$$

3. Schultz, Spencer, and Reifman:
   $$M/l = 0.02$$

4. Roberts:
   $$M/l = 0.02$$

It thus appears that in any order of magnitude discussion that criteria (1), (3), and (4) are equivalent.

Lo has superimposed Tsien's criteria on a graph of $M$ versus $\log R$ for the flight of V-2 missile No. 21, fired on March 7, 1947. We have added the other criteria to this curve. We note in Fig. 1 that there is a spread of 131,000 ft. between application of these criteria.

**Different Criteria for the Lower Bound of Free Molecule Flow**

(A) Roberts:
   $$R/M^2 = 0.01$$
   $$M/\sqrt{R} = 10$$

(B) Schultz, Spencer, and Reifman:
   $$\lambda/\delta = 10$$
   $$\lambda/\delta = 0.438(M/\sqrt{R})$$

(C) Tsien:
   $$\lambda/\delta = \lambda/l = M/R = 10$$

One notes that, in the intended region of application of criterion (B), it was thought that $R \gg 1$, while criterion (C) implied that $R \ll 1$. Thus we can make no real comparison between these criteria.

Of course, we do not know which criteria are the best, since this answer must be determined by experiment.

**References**