An Analysis of the Term Structure of Interest Rates

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I. Introduction

The most widely accepted model for predicting behavior in the yield curve for interest generating securities is the expectations theory, which predicts future interest rates of varying maturities based on current rates. Many authors have tested this theory against the data and found it fails to predict expected changes. In the following exposition, we reach a similar conclusion, but seek to understand the cause of the failure as well as comparing the relative performances under different specifications and during different time periods.

Throughout the relatively short history of macroeconomic thought there have been a few particularly revealing periods. In these periods, economic observers have had preconceived notions shattered and learned to make deeper connections between various economic relationships. One such revealing period — the Great Depression — changed the Federal Reserve into what it is today, especially in open market operations aimed at pegging the interest rate. In their paper "The Changing Behavior of the Term Structure of Interest Rates"[2], N. Gregory Mankiw and Jeffrey A. Miron analyze the effect this enlightenment has on the performance of the expectations theory. As they found, the presence of the Federal Reserve lowered the variance in the interest rate and the expectations theory did not perform as well after the inception of the Federal Reserve System. This paper updates their analysis in after the treasury bill took hold from 1959 to 1999 and frames it in terms of new enlightenments.

The stagflation of the 1970's realized a dearth of macroeconomic intuition; prior to the simultaneous occurrence of high unemployment and high inflation, Keynesian economists believed unemployment and inflation were countervailing market forces. This newfound realization brought about a policy change comparable to that of the Great Depression. This change manifested through the Federal Reserve Chairmanship of Paul Volcker, who was appointed in August of 1979. After Volcker's seat was vacated, another Federal Reserve dynamo, Alan Greenspan, was appointed. Using the three– and six–month treasury bill rates from the St. Louis Federal Reserve Bank, we break the current era into three regimes: the stagflation era (January 1959 to July 1979), the Volcker chairmanship (August 1979 to July 1987) and the Greenspan chairmanship (August 1987 to December 1999). Though the Volcker chairmanship exhibits a marked spike in variances of all time series, the expectations theory does not perform as well in this period. This seems to contradict the findings in [2], however through the following analysis it allows us to refine their conclusion. The volatile rates during the 1970s illuminate a point that the data from the turn of the century can't illustrate.

In Section II, we go into detail explaining the expectations theory and how it relates to our data. We follow up with an explanation of the nature of the data series in Section III as well as why we used them.

In Section IV, we perform several regressions on the data. In this initial section, we use the basic model implied by the expectations theory and find we can reject the theory with some statistical significance at both the monthly and quarterly level for the data. In reaction to this failure, in Section V we relax the specifications and perform a general regression of the change in the short term interest rate on all relevant interest rates. After again witnessing an apparent failure of the expectations theory in Section V, it is clear that the theory performs better during the latter part of the twentieth century and we seek an explanation.

Section VI is the platform for our addition to [2], principally through the presence of an additional explanatory factor. In Section VI, we analyze the relative size in volatility of the premium for exchanging from short to long term interest rates. Assumed to be constant by Mankiw and Miron, our data exhibit large volatility in this premium and we use this volatility to explain the difference in performance of the expectations theory.

Given the implication from the regressions that the expectations theory performs better in the later part of the data, we believe there may be a detectable date when predictability of the relationship changes. The final two sections, Sections VII and VIII, seek to find such a date using various methods. Our analysis is inspired by a similar analysis performed by Mankiw, Miron and Weil in [3]. We find the change was, as expected, statistically likely to have occurred during the period of high volatility in interest rates.

II. Expectations Theory of the Term Structure of Interest Rates

The expectations theory of the term structure of interest rates gives a way to compare portfolios of investments with different maturities. As Mankiw and Miron do, we use the theory to compare one– and two–period bills against each other in the same two–period span. Letting r_t be the yield on a one–period bill and R_t the yield on a two–period bill, the (linearized) expectations theory hypothesizes that

$$R_t = \theta + \frac{1}{2}(r_t + \mathrm{E}_t(r_{t+1})) \tag{1}$$

where E_t is the expectation operator at time t. This theory posits that the yield on a two–period bill is some premium θ plus the average of the yield on the one–period bill and the expected yield on the same bill one period ahead.

This corresponds to comparing two portfolios over the same two periods, one of which has the two–period bill and the other of which has a one–period bill in each period. The premium θ can be thought of as both a transaction or shoeleather cost of obtaining two bills instead of one and a gain from splitting the variance in half by rolling over investments in the short term. As the bills in this paper are held by banks as part of much larger portfolios the drop in variance is expected to dominate the shoeleather cost. Since the expectation $\mathrm{E}_t(r_{t+1})$ is a "best–prediction" for r_{t+1} given all current information, it is traditionally written

$$r_{t+1} = \mathrm{E}_t(r_{t+1}) + v_{t+1} \tag{2}$$

for some prediction error term v_{t+1} which is assumed to be independent of the expected value. Using this and the expectations hypothesis, the model can be rewritten as

$$\Delta r_t = r_{t+1} - r_t = -2\theta + 2(R_t - r_t) + v_{t+1}. \tag{3}$$

The terms in this equation — the change in the one–period rate, the difference between the two– and one–period rates, called the **spread** and the constant θ, called the **term premium** — are the focal point of the following analysis. The presence of the error term v_{t+1} in equation 3 gives a framework to test the hypothesis: the effect of the spread on the short rate can be analyzed through a regression.

III. Data

In [2], Mankiw and Miron first seek to fit the expectations theory of term structure to the interest rate data from the turn of the twentieth century and the ensuing wartime. They use three– and six–month time loans from New York banks, the established investment of the day, to test the expectations theory. During this era, the time loan was a prevalent investment, but it fell out of favor as the American economy became more sophisticated.

Initially authorized by Congress in 1929, the treasury bill did not become a prevalent investment until much later. The six–month bill was introduced in December of 1958, and the practice of exclusively issuing three–month bills was ended. As a result, the data used in this paper begins in January of 1959. The time–series used are the interest rates on three– and six–month treasury bills tabulated monthly obtained from the St. Louis Federal Reserve. Quarterly data are also used, where the rate from the last month in each quarter is used as a representative [1]. The data are divided into the mentioned regimes as follows: the pre–Volcker/stagflation regime (January 1959 to July 1979, 247 months), the chairmanship of Volcker (August 1979 to July 1987, 96 months) and Alan Greenspan's chairmanship through the twentieth century (August 1987 to December 1999, 149 months).

[1] The notation 1989:5 to refer to May of 1989 and 1981:IV to refer to the fourth quarter of 1981 is adopted. The fourth quarter is that containing October, November and December.

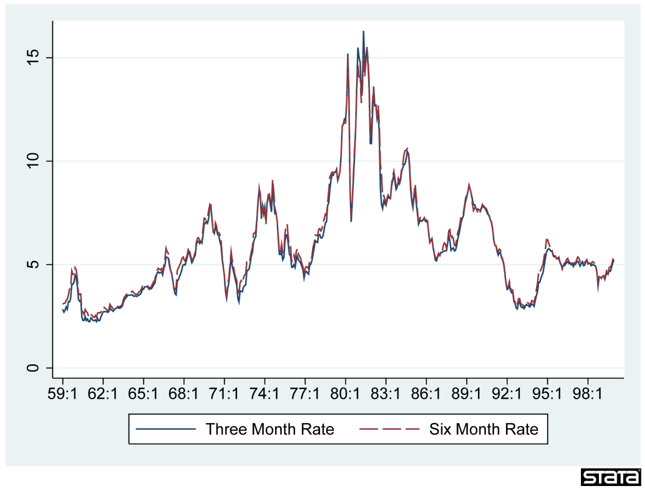


Figure 1: Graph of three– and six–month rates

IV. The Predictive Power of the Spread

Initially the theory is tested with the most basic specifications to see how well the data conforms to the expectations theory. As the period on a bill is three months and the data is at a monthly level, Δr_t is $r_{t+3} - r_t$. To account for any autocorrelation in the series Δr_t in the monthly case, the Newey–West standard errors with three lags are used.

Upon estimating equation 3 for the first regime, we find a coefficient on the spread with t–statistic 1.99 using the Newey–West standard errors. This is statistically different from zero at the 5% level. However, we can also reject the null hypothesis that the coefficient is two at the 0.5% level. The miniscule adjusted R–squared value of 0.024 coupled with the fact that the spread coefficient is statiscally different from two indicates this relationship has virtually no predictive power. In this regime, we find, as Mankiw and Miron did, that the "slope of the yield curve appears to contain no information about the path the short rate will follow."

Comparing this result to that in the other regimes with Table I, we have similar findings: at the 5% level all spread coefficients are statistically different from two. The adjusted R^2 also remains small save in the regime with Alan Greenspan. As in the first regime, in the second regime and over the entire sample, the relationship in equation 3 gives no predictive power.

However, during the third regime, a p–value near 5% and an $\overline{R}^2$ an order of magnitude larger than the other values indicate that under Greenspan, the expectations theory explains the data better than in the other regimes. The coefficient is nearly twice the size of the coefficient in the first regime and three times the size of that in the second regime.

Table I (Monthly Data)

	Dependent variable: $r_{t+3} - r_t$			
	Stagflation	Volcker	Greenspan	All
Period	59:1–79:7	79:8–87:7	87:8–99:12	59:1–99:12
Constant	−0.067	−0.185	−0.153	−0.107
	(0.105)	(0.297)	(0.063)	(0.096)
$R_t - r_t$	0.728	0.508	1.228	0.750
	(0.365)	(0.592)	(0.369)	(0.352)
$\bar{R}^2$	0.024	−0.003	0.183	0.023
$D.W.$	0.472	0.575	0.349	0.530
p–value at 2	0.001	0.013	0.038	0.000

To ground this result, we also run the same specifications at the quarterly level. Again we have similar findings in the first regime and the whole sample, indicating the expectations theory fails here. We find a quite large p–value (over 30%) in the second regime, but the coefficient is still less than 1. The large p–value is likely caused by the small sample size; Volcker was chairman for 32 quarters. In the second regime, the $\overline{R}^2$ value is negative, indicating this p–value lends little predictive power.

Table II (Quarterly Data)

	Dependent variable: $r_{t+1} - r_t$			
	Stagflation	Volcker	Greenspan	All
Period	59:I–79:III	79:IV–87:III	87:IV–99:IV	59:I–99:IV
Constant	0.063	−0.308	−0.144	−0.120
	(0.125)	(0.437)	(0.072)	(0.111)
$R_t - r_t$	0.225	0.878	1.229	0.816
	(0.469)	(1.076)	(0.362)	(0.393)
$\bar{R}^2$	−0.010	−0.011	0.180	0.020
$D.W.$	1.479	2.513	1.550	2.290
p–value at 2	0.000	0.305	0.038	0.003

In the quarterly regression, we again we find a p–value near 5% and a spread coefficient (relatively) near two in the Greenspan regime. This coupled with an $\overline{R}^2$ an order of magnitude larger than any other solidifies our suspicion. Though we can't strongly accept any hypotheses in the third regime, it is clear that the theory works better here. This rise in predictability may have been caused by Greenspan's steady hand. During his time, the technology boom was occurring and inflation was ever low and relatively stable. Even given a tiny bump in the early '90s, Greenspan's decisions were usually in line with market expectations (the market "trusted" him).

V. Evidence on Predictability

In order to make concrete the difference between the Greenspan regime and the other time periods we seek a testable explanation. As in [2], the indication is that the higher coefficient is caused by greater variance in the predictable changes in the short rate. In order to get a handle on these variances, we allow a reduced form model on our data. We regress the change in the short rate on both the long and short rates and one lag of each.

Given this relaxed specification, the $\overline{R}^2$ never exceeds 0.06 in the regimes where the theory has failed up to now, so no predictable changes are exhibited. In the first two regimes, the p–value on the F–statistic can not even reject the null hypothesis that all coefficients are zero; again predictability is negligible. However, in the third regime and the whole sample, we can reject the same null hypothesis, with the rejection on the whole sample likely caused by the Greenspan regime.

Table III: Reduced Form Forecasting Equations (Quarterly Data)

	Dependent variable: $r_{t+1} - r_t$			
	Stagflation	Volcker	Greenspan	All
Period	59:I–79:III	79:IV–87:III	87:IV–99:IV	59:I–99:IV
Constant	0.281	1.956	0.114	0.383
	(0.272)	(1.621)	(0.253)	(0.267)
r_t	−0.076	0.296	0.008	−0.530
	(0.559)	(1.224)	(0.548)	(0.433)
r_{t-1}	0.665	−0.103	−0.875	−0.024
	(0.522)	(1.147)	(0.438)	(0.431)
R_t	0.320	−0.789	0.322	0.322
	(0.525)	(1.342)	(0.487)	(0.457)
R_{t-1}	−0.922	0.368	0.507	0.157
	(0.511)	(1.223)	(0.437)	(0.450)
$\bar{R}^2$	0.045	0.059	0.267	0.052
$D.W.$	1.871	2.213	2.050	2.062
F–statistic	1.95	1.48	5.37	3.20
p–value	0.111	0.235	0.001	0.015

As Mankiw and Miron postulate, the variances in the change in the short rate, as well as the innovation of the above regression are directly related to the predictability of the expectations theory. Using the regressions in Table III, the residual in each gives an estimate of v_{t+1} in equation 3. Unraveling the theory, this quantity is the forecast innovation from equation 2; subtracting r_t from this equation we find

$$r_{t+1} - r_t = \Delta r_t = \mathrm{E}_t \Delta r_t + v_{t+1} = \mathrm{E}_t(r_{t+1} - r_t) + v_{t+1} \tag{4}$$

since $\mathrm{E}_t(r_t) = r_t$ is just the observed value. The assumption of an independent innovation implies

$$\mathrm{Var}(\Delta r_t) = \mathrm{Var}(\mathrm{E}_t \Delta r_t) + \mathrm{Var}(v_{t+1}). \tag{5}$$

We compile the relevant variances in each regime in Table IV below. Using the above, the third row in the table (listing $\mathrm{Var}(\mathrm{E}_t \Delta r_t)$) is just the difference of the first two rows.

Table IV: Relevant Variances

Period	Stagflation 59:I–79:III	Volcker 79:IV–87:III	Greenspan 87:IV–99:IV	All 59:I–99:IV
$\mathrm{Var}(\Delta r_t)$	0.448	5.459	0.203	1.337
$\mathrm{Var}(\Delta r_t - \mathrm{E}\Delta r_t)$	0.411	4.476	0.137	1.243
$\mathrm{Var}(\mathrm{E}_t\Delta r_t)$	0.037	0.983	0.067	0.094
$\mathrm{Var}(R_t - r_t)$	0.025	0.154	0.027	0.052

Viewing the table, the variances in the Volcker regime are by far the largest, however all the results up to this point have indicated the expectations theory fails to predict the data in this era. On impact, this appears to refute the findings of Mankiw and Miron, who say "The predictability of the short rate appears the major determinant of the success of the expectations theory." However, it enables us to frame their result in its full glory. Upon closer inspection, a new explanatory factor arises: the variance in the term premium.

VI. Term Premium

Allowing θ to vary, equation 1 becomes

$$2(R_t - r_t) = 2\theta_t + [r_t + \mathrm{E}_t(r_{t+1})] - 2r_t = 2\theta_t + \mathrm{E}_t\Delta r_t. \tag{6}$$

Hence under the original assumption of no variance in the term premium, the variance in the spread is completely determined by that of the predictable variation. If, on the other hand, predictable variation is assumed to have no variance, then the spread variance is determined by variance in the term premium. It is clear there is a tradeoff in the variance of the spread between the predictable variation and the term premium.

Letting $\mathrm{Var}(\theta_t) = \sigma_\theta^2$, $\mathrm{Var}(\mathrm{E}_t\Delta r_t) = \sigma_E^2$ and $\mathrm{Cov}(\mathrm{E}_t\Delta r_t, \theta_t) = \rho\sigma_E\sigma_\theta$, the estimate $\widehat{\beta}$ of the spread coefficient converges[2] to

$$\mathrm{plim}(\widehat{\beta}) = \frac{2\sigma_E^2 + 4\rho\sigma_E\sigma_\theta}{\sigma_E^2 + 4\rho\sigma_E\sigma_\theta + 4\sigma_\theta^2}. \tag{7}$$

Under the assumption $\sigma_\theta^2 = 0$ (which forces $\rho = 0$ as well) we have $\mathrm{plim}(\widehat{\beta}) = 2$. However, as σ_θ^2 increases, the term $4\rho\sigma_E\sigma_\theta + 4\sigma_\theta^2$ in the denominator rises by more than the term $4\rho\sigma_E\sigma_\theta$ in the numerator, biasing $\mathrm{plim}(\widehat{\beta})$ downwards.

It is clear from the above that a large value of σ_θ^2 will deter predictability. For our regimes, the variance of the term premium is calculated in Table V.

[2]This calculation is performed in the Appendix.

Table V: Term Premium Variance

Period	Stagflation 59:I–79:III	Volcker 79:IV–87:III	Greenspan 87:IV–99:IV	All 59:I–99:IV
$\text{Var}(R_t - r_t)$	0.025	0.154	0.027	0.052
$\text{Var}(R_t - \frac{1}{2}(r_t + r_{t+1}))$	0.131	1.383	0.045	0.344
$\text{Var}(\frac{1}{2}(r_{t+1} - \text{E}_t r_{t+1}))$	0.103	1.119	0.034	0.311
$\text{Var}(\theta_t)$	0.029	0.264	0.011	0.033

Using equations 1 and 2 together, we find:

$$R_t = \theta_t + \frac{1}{2}(r_t + \text{E}_t r_{t+1}) = \theta_t + \frac{1}{2}(r_t + r_{t+1}) + \frac{1}{2}(\text{E}_t r_{t+1} - r_{t+1}) \tag{8}$$

which implies

$$R_t - \frac{1}{2}(r_t + r_{t+1}) = \theta_t + \frac{1}{2}(\text{E}_t r_{t+1} - r_{t+1}). \tag{9}$$

Again using independence of the forecast innovation $v_{t+1} = r_{t+1} - \text{E}_t r_{t+1}$, the variance of the term premium (the final row in the table) is computed as the difference of the two rows above it.

The large movement in the term premium causes the explanation to change. The results of Mankiw and Miron required this variance near zero; in the regimes they analyzed the variance never exceeds 0.07 and they say explicitly "While it appears that the variance of the term premium is not constant, the variation is much smaller than the variation in the variance of predicted changes in the short rate." When this variance is large, the spread coefficient estimate is biased downward; the term premium captures predictability from the spread. The expectations theory fails in the Volcker regime because the high predictable variance in the short rate is offset by the high variance in the term premium.

On the other hand, in the stagflation and Greenspan regimes, the variance in the term premium is small. In this setting, the conclusions of Mankiw and Miron can be applied. The variance of the predictable change in the short rate is nearly twice as large in the Greenspan regime as in the stagflation regime. This attests to the theory put forth in [2] and explains the higher predictability in the expectations theory under Greenspan.

VII. Simple Regime Switching

The results above indicate there may be some concrete change occurring in the predictability of the expectations theory. To test for such a break or switch in regimes, many methods have been produced. The **Quandt Likelihood Ratio (QLR)** test is one such basic test.

Let $D_t(\tau)$ be the step function which is zero up to period τ and 1 after τ. Using this, the hypothesis of no structural break at time τ can be tested with the model

$$r_{t+3} - r_t = [\alpha_0 + \beta_0 D_t(\tau)] + \alpha_1(R_t - r_t) + \beta_1(R_t - r_t)D_t(\tau) + v_{t+3} \tag{10}$$

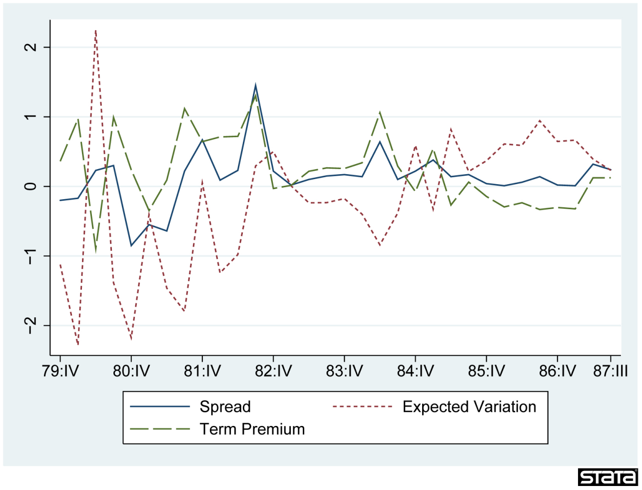


Figure 2: Spread, Expected Variation and Term Premium under Volcker

with null hypothesis $\beta_0 = \beta_1 = 0$ (the model is based on equation 3). For a fixed value of τ, there is an F–statistic for this null hypothesis. From [1], allowing τ to vary over the middle 70% of periods in the sample, the QLR statistic is the maximum of these F–statistics and the period in which it occurs is an estimator for break date. From Table 14.6 in [1] in our model with two restrictions, the 10% critical value for this statistic is 5. However, with a maximum F–statistic

F–statistics

Period	F	Period	F
1980:10	3.364	1981:2	3.396
1980:11	3.305	1981:3	3.238
1980:12	4.007	1981:4	3.447
1981:1	4.410	1981:5	3.447

of 4.41, the null hypothesis of no break can't be rejected. As in previous sections, this is no deterent: we allow the reduced form forecast equation with quarterly data. Now the test is on the model

$$r_{t+1} - r_t = [\alpha_0 + \beta_0 D_t(\tau)] + \alpha_1 r_t + \beta_1 r_t D_t(\tau) + \alpha_2 r_{t-1} + \beta_2 r_{t-1} D_t(\tau) + \alpha_3 R_t + \beta_3 R_t D_t(\tau) + \alpha_4 R_{t-1} + \beta_4 R_{t-1} D_t(\tau) + v_{t+1} \quad (11)$$

with null hypothesis $\beta_0 = \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$.

The extreme spike of 4.76 in the fourth quarter of 1979 indicates this F–statistic is the maximum. With five restrictions, the 1% critical value is 4.53 so we can reject the null hypothesis of no statistical break. The QLR estimate of 1979:IV occurs in the quarter when Volcker took office.

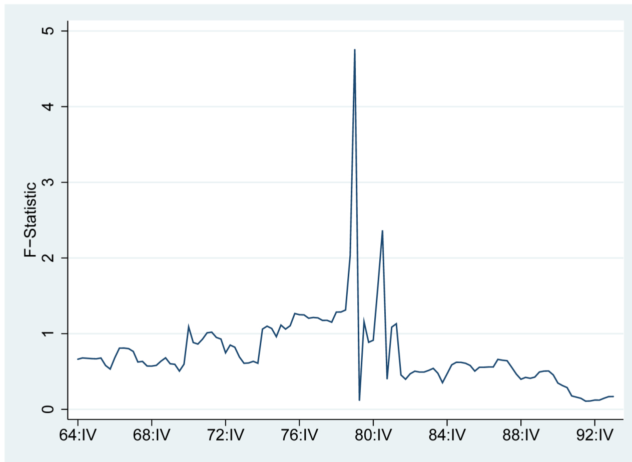


Figure 3: F–statistic from reduced form QLR

VIII: Step Switching

In Section IV of [3] Mankiw, Miron and Weil attempt to detect a date for a given regime switch. One technique they employ to test this uses the concept of **step switching**. This method quantifies a change in predictability by using maximum likelihood estimation while varying a **switch date**, a date at which there is a regime switch. In [3], the short rate is used to test for a regime switch. The choice of using the short rate instead of the expectations theory model is based on historical evidence more than economic theory. However, by 1959 the three–month treasury bill was a well–established investment. With the introduction of the six–month treasury bill in 1959, the interplay between the two bills can be an equally useful indicator.

The step switching model applied to predictability based on the expectations hypothesis can be applied in the following way. Given a switch at time T_s and a sample with length T, the model assumes two regimes:

$$\begin{aligned} \Delta r_{t+1} &= \kappa_0 + \rho_0(R_t - r_t) + v_{t+1} \qquad & t = 1, 2, \ldots, T_s - 1 \\ \Delta r_{t+1} &= \kappa_1 + \rho_1(R_t - r_t) + v_{t+1} \qquad & t = T_s, T_s + 1, \ldots, T \end{aligned}$$

with the assumption or normal errors with variances σ_0^2, σ_1^2 in each respective regime[3]. For each possible break date T_s, the maximum likelihood estimates for each of the parameters gives the maximum likelihood. As suggested by Goldfeld and Quandt, the break date estimate for T_s will be that which has the highest *maximum* likelihood[4].

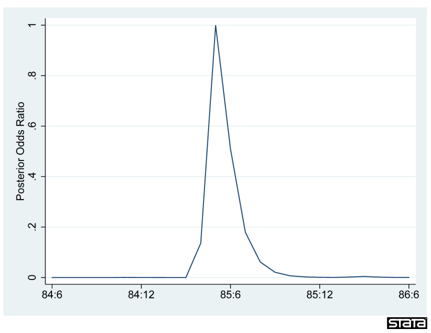
[3]This allowance for different variances makes this test more flexible than the Chow test and allows for the computation of posterior odds ratios.

[4]This is a maximum of maximums; you didn't read it wrong.

Assuming all switch dates are equally likely, the switch date estimate can be compared to the others through a posterior odds ratio. Under the normality assumption, the log likelihood is given by

$$\log(L) = -\frac{T}{2}\log(2\pi) - \frac{1}{2}(T_s - 1)\log(\sigma_0^2) - \frac{1}{2}(T - T_s + 1)\log(\sigma_1^2) - \frac{1}{2}\sum_{t=1}^{T_s-1}\frac{v_{t+1}^2}{\sigma_0^2} - \frac{1}{2}\sum_{t=T_s}^{T-T_s+1}\frac{v_{t+1}^2}{\sigma_1^2}. \quad (12)$$

For fixed T_s, as the regimes are assumed independent, the v_{t+1} can be estimated using the MLE estimates for the coefficients, which are the OLS estimates in each regime (1 to $T_s - 1$ and T_s to T). Estimating the model as such, Figure 4 shows that with both the monthly and quarterly data, the maximum occurs in the second quarter of 1985. From the odds ratios nearby we can declare with much confidence that the switch date occurred within a few months of May 1985.



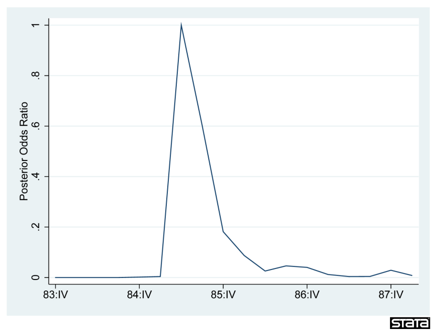


Figure 4: Posterior Odds Ratio for exp. theory and reduced form, resp.

This date is years ahead of the QLR estimate of 1979:IV. This curiosity is caused by the large interest rates in the early '80s as well as erratic behavior. Viewing the short and long rates during the Volcker regime with a vertical line drawn in May 1985 we see this is when the treasury bill rates begin to settle down again. The large gap between the fourth quarter of 1979 and the second quarter of 1985 is caused by the fact that we allowed variances to differ across regimes using the step switching model and the hypothesis of a single error required them to be the same using QLR. Therefore, the large variance during the early '80s pushed the date forward to account for the difference in variance.

IX. Conclusion

Using updated Treasury data, we have been able to verify and amplify the results of [2]. As they found, so we also find the random component induced by forecast error causes a failure in the test for predictability under the expectations hypothesis. However, the relative success of predictability under the model

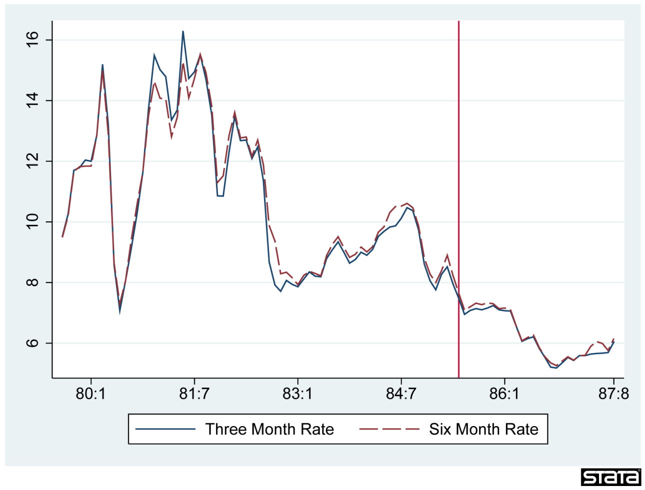


Figure 5: R_t and r_t during Volcker regime

was demonstrated to be tied to high variance in the forecast error during the Greenspan regime. In analyzing the term premium, the amplification occurred; the data provided us with a means to exploit theory that Mankiw and Miron developed but never used. The absence of high variation in the term premia analyzed in [2] did not allow analysis of the effects of said high variation. However, the highly volatile Volcker regime provided data that failed predictability for reasons other than low variance in predictable variation. In this setting, the theory of Mankiw and Miron performed as expected and explained the relative failure during chairman Volcker's time at the helm.

In addition, we were able to estimate a date after which the implications of the expectations theory model changed. Viewing Figure 5, it is clear the date we found occurs after the highly elevated interest rates of the early '80s begin to settle down. This is in concert with our refinement of [2]. The failed predictability in the Volcker regime even given the high variance in predictable variation was a result of an artificially high variance caused by high volatility in all time series during that era. Hence, as the elevated interest rates fell, the predictability implied by the expectations theory returns to a form recognizable in the context of [2].

References

[1] J.H. Stock and M.W. Watson, *Introduction to Econometrics* 2nd Ed., (The Addison-Wesley Series in Economics, 2007)

[2] N. Gregory Mankiw and Jeffrey A. Miron, "The Changing Behavior of the Term Structure of Interest Rates", *The Quarterly Journal of Economics* **Vol. 101 No. 2** (May, 1986) 211–228

[3] N. Gregory Mankiw, Jeffrey A. Miron, David N. Weil, "The Adjustment of Expectations to a Change in Regime: A Study of the Founding of the Federal Reserve", *The American Economic Review* **Vol. 77 No. 3** (June, 1987) 358–374

Appendix

In the regression

$$Y_t = \alpha + \beta X_t + \varepsilon_t,$$

the estimated coefficients give

$$Y_t = \widehat{\alpha} + \widehat{\beta} X_t + \widehat{\varepsilon_t}$$

with $\widehat{\varepsilon_t}$ satisfying independence assumptions in the sample. Taking the sample covariance of this fitted equation with X_t using the fact that $\text{cov}(X_t, \widehat{\varepsilon_t}) = 0$ by construction we have

$$\text{cov}(Y_t, X_t) = \text{cov}(\widehat{\alpha} + \widehat{\beta} X_t + \widehat{\varepsilon}_t, X_t) = \widehat{\beta}\text{cov}(X_t, X_t).$$

Hence,

$$\text{plim}(\widehat{\beta}) = \frac{\text{Cov}(Y_t, X_t)}{\text{Var}(X_t)} = \frac{2\text{Cov}(Y_t, 2X_t)}{\text{Var}(2X_t)}$$

where $\text{Cov}(\cdot)$ denotes the population covariance.

For the expectations theory regression we have $Y_t = \Delta r_t$ and $X_t = R_t - r_t$. Under the theory, as in equations 4 and 6 we have $\Delta r_t = \text{E}_t(\Delta r_t) + v_{t+1}$ and $2X_t = 2(R_t - r_t) = \text{E}_t \Delta r_t + 2\theta_t$. Under assumption of independent forecast errors this yields

$$\begin{aligned}
\text{Cov}(Y_t, 2X_t) &= \text{Cov}(\text{E}_t \Delta r_t + v_{t+1}, \text{E}_t \Delta r_t + 2\theta_t) \\
&= \text{Cov}(\text{E}_t \Delta r_t, \text{E}_t \Delta r_t + 2\theta_t) \\
&= \sigma_E^2 + 2\rho\sigma_E\sigma_\theta
\end{aligned}$$

and

$$\begin{aligned}
\text{Var}(2X_t) &= \text{Var}(\text{E}_t \Delta r_t + 2\theta_t) \\
&= \sigma_E^2 + 2\text{Cov}(\text{E}_t \Delta r_t, 2\theta_t) + 2^2\sigma_\theta^2 \\
&= \sigma_E^2 + 4\rho\sigma_E\sigma_\theta + 4\sigma_\theta^2
\end{aligned}$$

forcing

$$\text{plim}(\widehat{\beta}) = \frac{2(\sigma_E^2 + 2\rho\sigma_E\sigma_\theta)}{\sigma_E^2 + 4\rho\sigma_E\sigma_\theta + 4\sigma_\theta^2}.$$