THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

HYDRAULIC TRANSIENTS CAUSED BY RECIPROCATING PUMPS

V. L. Streeter
E. B. Wylie

May, 1966

IP-741

ACKNOWLEDGEMENTS

The work described in this paper has been supported by Research Grant GP-340 of the National Science Foundation to the University of Michigan. The Union Pump Company, Battle Creek, Michigan, supplied the triplex pump used in the experimental program and T. P. Propson performed the actual testing.

TABLE OF CONTENTS

LIST OF FIGURES

Figure | Page

NOMENCLATURE

A	Cross sectional area of pipe.
A_p	Area of piston.
a	Speed of pressure pulse.
C_1 , C_2	Constants of integration.
C_v	Volume displacement coefficient.
D	Inside diameter of pipe.
f	Friction factor.
g	Gravitational acceleration.
H	Elevation of hydraulic gradeline above a fixed datum.
$\overline{H}$	Steady-state elevation of hydraulic gradeline.
$H(x)$	Non time-varying complex number for head.
H_{sub}	Complex number notation for instantaneous head at a point. The subscript designates the location.
h'	Fluctuation of hydraulic gradeline elevation.
i	$\sqrt{-1}$
K	Bulk modulus of elasticity of the liquid.
L	Length of pipeline.
M	Number of harmonics.
m	Integer which identifies a particular harmonic.
NPSH	Net positive suction head.
n	Exponent on friction term.
Q	Discharge.
$\overline{Q}$	Steady-state discharge.
$Q(x)$	Non time-varying complex number for discharge.
Q_{sub}	Complex number for instantaneous discharge at a point. The subscript designates the location.

q' Discharge fluctuation.

R Resistance coefficient per unit length; crank arm radius.

S Length of connecting rod.

t Time.

V Average velocity of fluid at a cross section.

Ꝟ Volume.

ꝟ Volume fluctuation.

x Distance along pipe in downstream direction; position of piston.

Z Hydraulic impedance, Complex ratio of pressure head to discharge.

Z_c Characteristic impedance.

α Real part of γ .

β Pure imaginary part of γ .

γ Complex number, $\gamma = \alpha + i \beta$

θ Angular position of crank arm on suction stroke.

θ_m Phase angle of the pressure head fluctuation.

φ Angular position of crank arm on discharge stroke.

φ_q Impedance phase angle.

ψ_m Phase angle for the mth harmonic.

ω' Angular velocity of crank shaft.

ω Angular freguency.

1. Introduction.

Reciprocating pumps, by the nature of their kinematics, produce certain periodic variations in flow into and out of the pump. These fluctuations may be expressed in harmonics of the pumping cycle for purposes of analysis. When one of the flow harmonics coincides with the fundamental or an harmonic of the suction or discharge piping system, severe pressure and flow pulsations may develop. In this paper the kinematics and dynamics of the flow into and out of the pump are first considered, followed by the development of methods of analysis of the piping systems, the pump, and a few appurtenances to reduce fluctuations. The inclusion of fluid compressibility adds considerably to the severity of the predicted flow pulsation produced by the pump and yields a better agreement between theory and actual conditions. This factor is not adequately taken into account in the literature.(1)* Experimental results from the suction side of a triplex pump are given which confirm the method of analysis.

---

* Number in parentheses designate references at end of paper.

2. Kinematics and Dynamics of Flow Through Reciprocating Pumps.

To illustrate the analysis of the steady-oscillatory flow in reciprocating pumps, one piston of the pump is shown in Figure 1, having a crank radius R and a connecting rod length of S . The displacement from fully extended position is given by x . Two check valves are utilized for each piston as shown. In this analysis the valves are assumed to work perfectly, i.e. to close the instant the flow starts to reverse. For more than one piston to a crank shaft. They are assumed to be spaced at equal angles.

For the suction stroke the displacement of piston is given by

$$x = R(1 - \cos\theta) + S(1 - \cos\alpha) \tag{1}$$

but since $R\sin\theta = S\sin\alpha$

$$x = R(1 - \cos\theta) + S(1 - \sqrt{1 - (\tfrac{R}{S})^2 \sin^2\theta}) \tag{2}$$

After differentiating with respect to t to find the velocity $\dot{x}$ of the piston

$$\dot{x} = \omega' R \sin\theta \quad 1 + \frac{R\cos\theta}{\sqrt{S^2 - R^2\sin^2\theta}} \tag{3}$$

ω' is the angular velocity of crank shaft. When pumping the discharge due to displacement of the piston is given by $\dot{x} A_p$ with A_p the cross-sectional area of piston. Owing to the compressibility of the liquid being pumped, the piston does not draw fluid into the chamber over the 180° crank position. For example, on the suction side at the end of the discharge stroke (x = 0), the liquid trapped between valves is at the discharge pressure. Before the suction valve can open it is necessary to decompress the trapped

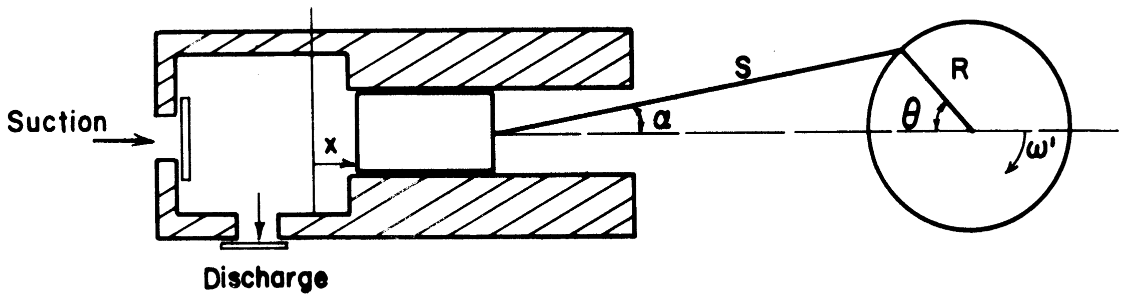

Figure 1. Displacement of Piston in Reciprocating Pump on Suction Stroke.

liquid, which requires an angular displacement of, say, Θ_o . At this angle the piston has acquired a velocity $\dot{x}(\Theta_o)$ which imparts a sudden drop in suction intake pressure as the inflow commences. A similar problem arises on the discharge stroke, since the fluid must be compressed to discharge pressure before the discharge valve opens. The greater the discharge pressure, the greater is Θ_o and $\dot{x}(\Theta_o)$; hence the worse the pressure and flow fluctuations.

Equations (1), (2) and (3) apply to the suction inflow of a single piston between the angles Θ_o and π radians. Θ_o depends upon the volume of liquid between valves, expressed as number of piston displacement volumes C_V , the discharge pressure p_o and the bulk modulus of elasticity K of the liquid, for given piston area. The linear displacement x_o at which pumping starts is

$$x_o = 2\ C_V\ R\ \frac{p_o}{K} \tag{4}$$

for the suction stroke. C_V is greater by one for the discharge stroke. With x_o known, Θ_o may be found from Equation (2) by trial.

The number of pistons in the pump also greatly changes the pattern and severity of flow fluctuations. In Figure 2 calculated flows over a cycle (one revolution divided by number of pistons) are shown for four pumps, using water as liquid, with trapped volume $C_V = 3.6$. By plotting the flow from time of maximum flow, the curves coincide except for those regions where a piston is decompressing the liquid. Two pressures are shown to indicate how the fluctuations become more severe as the discharge pressure rises. The total piston displacement per revolution is the same in all cases shown. With low discharges pressures, for example, the triplex flow

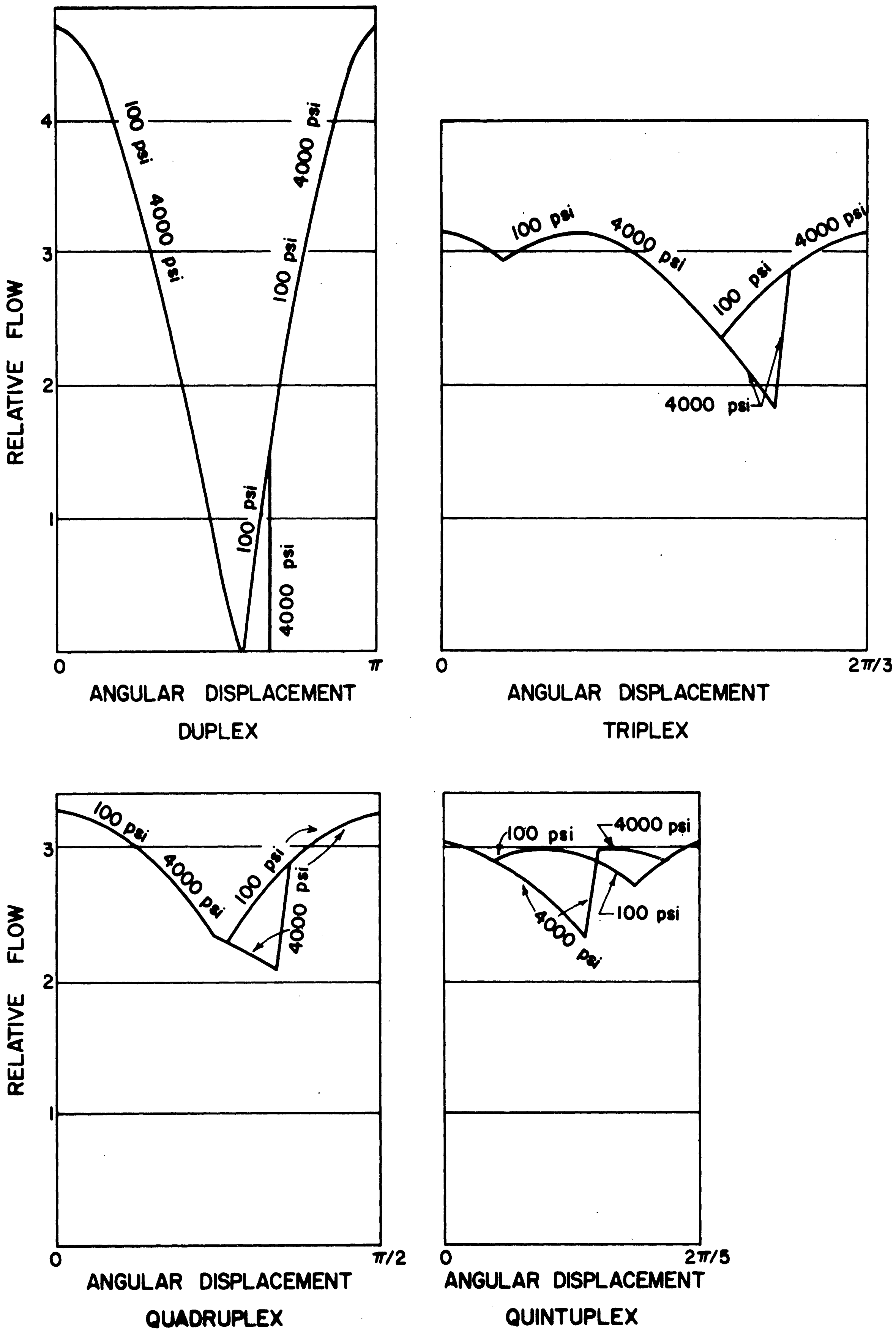

Figure 2. Effect of Number of Pistons and Pressure on Flow into Reciprocating Pump.

is well represented by 4 to 8 terms in an harmonic series, but with high pressures as many as 24 harmonics are needed to describe the flow.

On the discharge side of the pump, Figure 3, the displacement x' is given by

$$x' = R(1 - \cos\varphi) - S\left(1 - \sqrt{1 - \left(\frac{R}{S}\right)^2 \sin^2\varphi}\right) \qquad (5)$$

and the piston velocity $\dot{x}'$

$$\dot{x}' = \omega' R \sin\varphi \left(1 - \frac{R\cos\varphi}{\sqrt{S^2 - R^2\sin^2\varphi}}\right) \qquad (6)$$

Since the trapped volume is larger by one piston displacement, $2 A_p R$, on the discharge side, the angle φ_o at which pumping starts is greater then Θ_o for the suction side. Hence the sudden jump in discharge is more severe.

If violent pressure fluctuations occur on the suction side of the pump, causing cavitation and incomplete filling of the displaced volume, then the angle φ_o for start of pumping is much greater and the discharge fluctuations become worse.

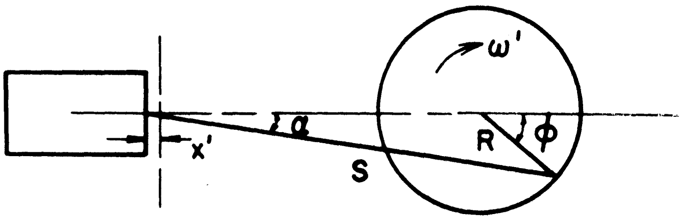

Figure 3. Displacement of Piston on Discharge Stroke.

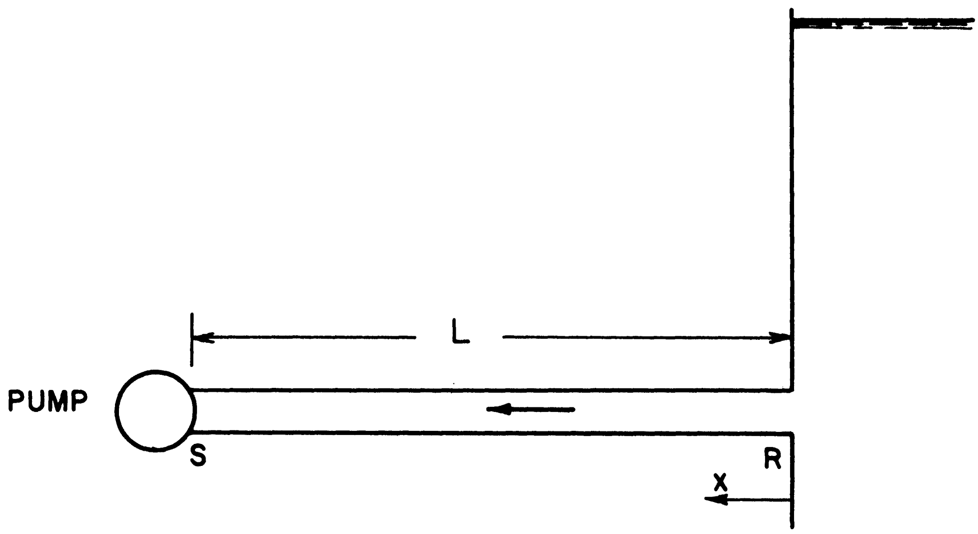

Figure 4. Suction Pipe From Reservoir.

3. <u>Impedance Method for Piping System.</u>

Whenever a mechanism is present on a system which creates a periodic fluctuation in discharge or pressure head, the impedance method[2,3], of analysis is very useful. Reciprocating pumps produce such an oscillation so the impedance theory as applied to real fluid systems is developed and used for analysis in typical systems. The complex impedance is petered from the piping system at the exciting frequencies. It is then combined with the periodic flow pattern from the pump to yield the pressure fluctuations.

The equations of water hammer, continuity and equation of motion, in terms of discharge Q and elevation of hydraulic gradeline H are, respectively

$$Q_x + \frac{gA}{a^2} H_t = 0 \tag{7}$$

and

$$H_x + \frac{1}{gA} Q_t + \frac{1}{2gDA^n} Q^n = 0 \tag{8}$$

in which a is the speed of the pressure pulse wave, A is the pipe cross-sectional area, f is the friction factor for steady state conditions, D is the pipe diameter, n is the exponent of the velocity in the loss term, x is the distance from upstream end of pipe, Figure 4, and t is the time. The subscripts x and t denote partial differentiation.

These equations are solved for steady-oscillatory flow, following methods of transmission line theory.[1,4] The variables H and Q are considered as

$$H = \overline{H} + h' \quad , \quad Q = \overline{Q} + q' \tag{9}$$

with $\bar{H}$, $\bar{Q}$ average steady-state values, and h' , q' variations from the average. By making these substitutions into Equations (7) and (8) and simplifying

$$q'_x + \frac{gA}{a^2} h'_t = 0 \tag{10}$$

and

$$h'_x + \frac{1}{gA} q'_t + Rq' = 0 \tag{11}$$

in which

$$R = \frac{nf\bar{Q}^{n-1}}{2gDA^n} \tag{12}$$

is used as the linearized friction coefficient for the turbulent flow.

By restricting the solution of Equations (10) and (11) to the steady oscillatory case, the resulting equation[4] for the oscillatory head is

$$h'(x,t) = H(x)e^{i\omega t} = e^{i\omega t} (C_1 e^{\gamma x} + C_2 e^{-\gamma x}) \tag{13}$$

where ω is any given angular frequency, $i = \sqrt{-1}$, C_1 and C_2 are constants of integration to be determined by use of the pipe boundary conditions, and γ is a complex constant, $\gamma = \alpha + i\beta$, commonly referred to as the propagation constant. The value of γ is determined to be

$$\gamma = \frac{\omega g}{a^2} A(i R - \frac{\omega}{gA}) \tag{14}$$

from which

$$\alpha = \sqrt{\frac{\omega gA}{a^2}} \left[(\frac{\omega}{gA})^2 + R^2 \right]^{1/4} \sin(\frac{1}{2} \tan^{-1} \frac{gAR}{\omega}) \tag{15}$$

$$\beta = \sqrt{\frac{\omega g A}{a^2}} \left[\left(\frac{\omega}{gA}\right)^2 + R^2 \right]^{1/4} \cos \left(\frac{1}{2} \tan^{-1} \frac{gAR}{\omega}\right) \qquad (16)$$

The solution for the oscillatory discharge is obtained by substituting Equation (13) into Equation (10) and integrating

$$q'(x,t) = Q(x)e^{i\omega t} = \frac{\omega g A}{i a^2 \gamma} e^{i\omega t} (C_1 e^{\gamma x} - C_2 e^{-\gamma x}) \qquad (17)$$

The <u>hydraulic impedance</u> $Z(x)$ is the ratio of head fluctuation to discharge fluctuation h'/q'

$$Z(x) = \frac{i\gamma a^2}{\omega g A} \frac{C_1 e^{\gamma x} + C_2 e^{-\gamma x}}{C_1 e^{\gamma x} - C_2 e^{-\gamma x}} \qquad (18)$$

The <u>characteristic impedance</u> Z_c of a pipe is a function of ω , defined

$$Z_c = \frac{\gamma a^2}{i\omega g A} = \frac{a^2}{\omega g A} (\beta - i\,\alpha) \qquad (19)$$

so

$$Z(x) = - Z_c \frac{C_1 e^{\gamma x} + C_2 e^{-\gamma x}}{C_1 e^{\gamma x} - C_2 e^{-\gamma x}} \qquad (20)$$

Conditions at the pipe inlet and outlet are now introduced to evaluate the integration constants, C_1 and C_2 . For the inlet, let

$$x = 0, \quad H(o)\, e^{i\omega t} = H_R \,, \quad Q(o)e^{i\omega t} = Q_R \qquad (21)$$

After substitution in Equations (13) and (17), eliminating C_1 and C_2 ,

$$h'(x,t) = H(x)e^{i\omega t} = H_R \cosh \gamma x - Q_R Z_c \sinh \gamma x \qquad (22)$$

$$q'(x,t) = Q(x)e^{i\omega t} = - \frac{H_R}{Z_c} \sinh \gamma x + Q_R \cosh \gamma x \qquad (23)$$

and

$$Z(x) = \frac{H(x)e^{i\omega t}}{Q(x)e^{i\omega t}} = \frac{Z_R - Z_C \tanh \gamma x}{1 - (Z_R/Z_C)\tanh\gamma x} \tag{24}$$

in which $Z_R = H_R/Q_R$. For known conditions at the pipe outlet, $x = L$, let

$$x_1 = L - x \ , \ H(L) = H_S \ , \quad Q(L) = Q_S \ , \quad Z_S = H_S/Q_S \tag{25}$$

then

$$Z(x) = \frac{Z_S + Z_C \tanh \gamma x_1}{1 + (Z_S/Z_C)\tanh\gamma x_1} \tag{26}$$

Since Z_R , Z_S , Z_C , and γ are complex, $Z(x)$ is a complex number having a magnitude $|Z(x)|$ and a phase angle φ_q , $Z(x) = |Z(x)|e^{i\varphi_q}$. The head leads the discharge by φ_q .

By analyzing a piping system, starting with known impedances, say, at reservoirs, deadends, valves, or orifices, the impedance at some desired point in the system may be calculated for a given ω , when R , A, a, f, L, n and $\overline{Q}$ are known for each pipe. As a simple example, if a known sine wave variation of discharge is superposed on a steady flow at a point, say $q' = \Delta Q \sin \omega t$, the head variation can easily be calculated at this point. The hydraulic impedance is evaluated at the point from the pipeline at the particular frequency ω and, since $h' = Z(x)q'$,

$$\Delta H = \Delta Q \, |Z| \quad , \quad h' = \Delta H \sin(\omega t + \varphi_q) \tag{27}$$

For more complex discharge variations as from the reciprocating pump, the discharge is expressed by a Fourier series. The impedance for each of the frequencies is calculated and h' is then found by adding the harmonics taking phase into account.

The computations to evaluate the impedance at a point in a single pipeline and at a particular frequency are quite involved. However, with the aid of a digital computer the equations are easily and rapidly handled for complicated systems involving many pipelines.

4. Suction Piping.

The hydraulic impedance at the suction flange of the pump in terms of frequency is found for the suction piping system, starting at reservoirs, dead ends, etc. and working to the suction flange. Independently, a harmonic analysis of the suction flow at the pump is carried out for as many harmonics, M, as are deemed necessary. The flow harmonics, when multiplied by the hydraulic impedance for the same frequency, result in an expression for head variation. When the head fluctuations for each harmonic are added for a given time, the head change from mean value is found.

The most common suction line would normally be a pipe of constant diameter connected to a reservoir and to the suction flange. At the reservoir, $\Delta H = 0$, and the hydraulic impedance is zero; hence, Figure 4, $Z_R = 0$. The impedance at S is given in general by Equation (24). For $Z_R = 0$ this reduces to

$$Z_S = - Z_C \tanh (\gamma L) \tag{28}$$

The complex numbers Z_S must be calculated for each of the M harmonics. The number of a particular harmonic is identified by m, its angular frequency by $m\omega$ where ω is the frequency of the fundamental (i.e. $3\,\omega'$ for a triplex pump).

The complex discharge fluctuation for the m-th harmonic, from the harmonic analysis is given by

$$q'_m = \Delta Q_m \sin(m\omega t + \psi_m) = \Delta Q_m \; e^{i\left(m\omega t + \psi - \frac{\pi}{2}\right)} \tag{29}$$

in which ΔQ_m is the amplitude of the m-th flow fluctuation and ψ_m is its phase angle. Since $Z_S(m)\; q'_m = h'_m$, the sinusoidal head fluctuation in complex form for the m-th harmonic is

$$h'_m = \Delta H_m e^{i(m\omega t + \Theta - \frac{\pi}{2})} = Z_S(m)\, q'_m \qquad (30)$$

From $Z_S(m)\, q'_m$, the magnitude of the head fluctuation of the m-th harmonic is $\Delta H_m = |Z_S(m)|\, \Delta Q_m$ and its phase angle, $\Theta_m = \psi_m + \varphi_m$. The angle φ_m is the impedance phase angle determined from the piping system for the m-th harmonic.

For M harmonics the head-time curve for a cycle at the pump suction flange then becomes

$$H = \overline{H} + \sum_{m=1}^{M} \Delta H_m \sin(m\,\omega\, t + \Theta_m) \qquad (31)$$

For a complex suction piping arrangement, which may consist of series or parallel pipes, branching pipes, or include a standpipe or more than one reservoir, the hydraulic impedance is calculated at the pump suction flange starting at the extremeties of the suction piping system.(3) With the impedance and complex flow known for each harmonic of the flow, the calculation is carried out to find the steady-oscillatory head as for a single suction pipe.

If the pressure approaches vapor pressure anywhere in the suction piping system or within the pump the behavoir of the pump changes and the impedance results are not valid.

The net positive suction head NPSH is frequently used in the specification of minimum suction conditions for a pump. It is defined for turbomachinery by

$$\mathrm{NPSH} = \frac{V_e^2}{2g} \quad \frac{p_a - p_v - \gamma H_S}{\gamma} = h_a - h_v - H_S \qquad (32)$$

in which point e , Figure 5, is the point of lowest pressure in the suction system, usually within the pump; h_a is atmospheric pressure expressed in ft of fluid flowing (abs.); h_v is vapor pressure in ft of fluid (abs.); and H_s is distance of suction reservoir below critical point e.

Since the pressure fluctuations for a given reciprocating pump depend on its speed, its discharge pressure, and upon the exact geometry of the suction piping system, tests to determine the maximum value of H_s for operation with no impairment of efficiency or without objectionable noise and damage must be made for each speed and set of internal conditions within the pump. However, it is likely that NPSH can be determined quite satisfactorily from a minimum of tests at various speeds on a specific pump; then, by use of the computer and the impedance relations for the suction piping system plus the kinematics and dynamics of the suction flow, these results can be extended to other discharge pressures and suction piping systems.

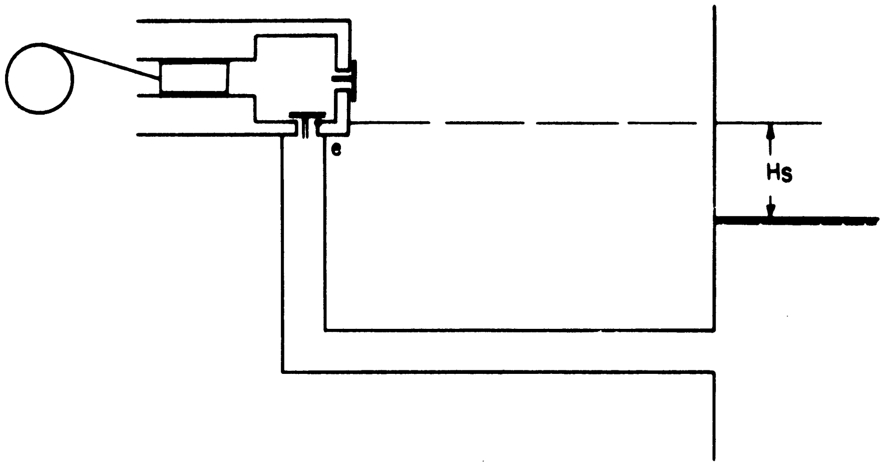

Figure 5. Suction Pipe for Definition of NPSH.

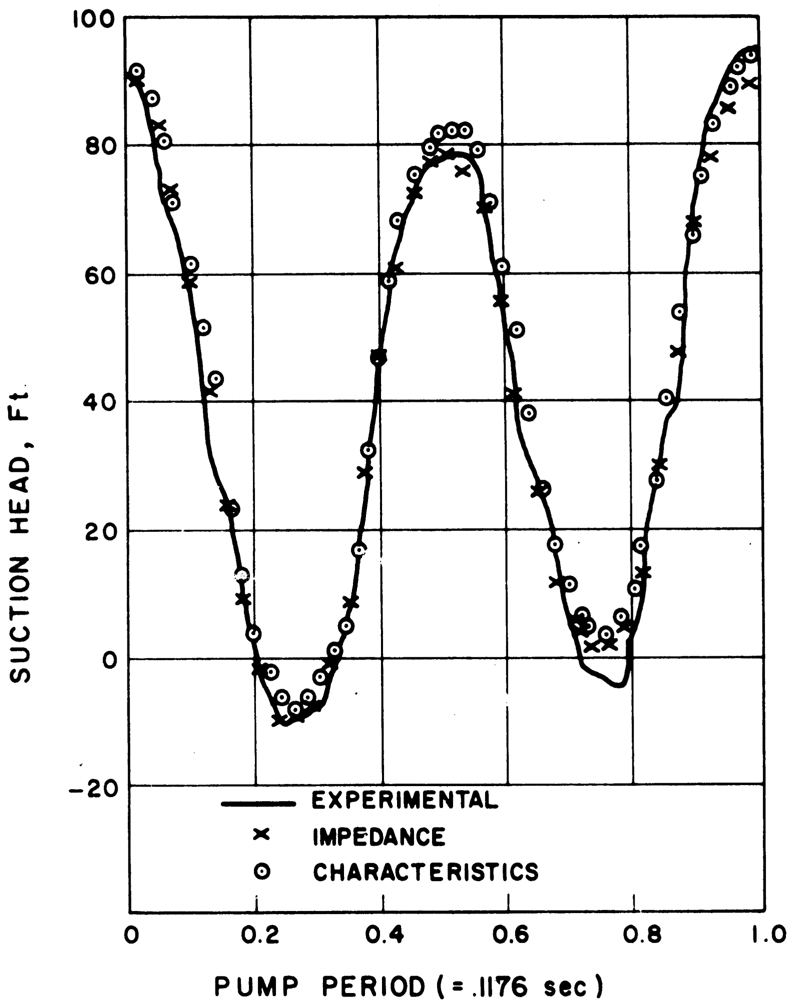

Figure 6. Comparison of Suction Flange Pressure Fluctuations as Calculated by the Characteristics Method (100th Cycle), the Real Fluid Impedance Method, and Laboratory Measurements for L = 56 ft, R = 0.135 ft S = 0.708 ft, p_o = 80 psi, A = 0.0218 ft θ = 3.2° C = 3.6, 170 RPM, for a Triplex Pump.

5. Discharge Piping System.

The flow as a function of time over the pump cycle may be quite accurately predicted by using Equation (6) with compressibility effects included. A harmonic analysis of the flow is performed when an impedance analysis is desired. If the system is such that expressions for impedances at the extremeties of the piping system can be obtained, then the impedance at the discharge flange can be calculated. With known flow magnitude and phase for each harmonic, and with known impedance magnitude and phase for the same harmonics, the head fluctuations may be calculated as outlined for the suction side. Transfer functions which relate pressure oscillations at one location to pressure head at another location, may be used to find pressure fluctuations throughout the system.

When the discharge piping system has components with nonlinear features that should not be linearized, then the characteristics methods[5,6] may be utilized. Nonlinear ordinary differential equations may be written for the elements (e.g. when Coulomb friction is included) and solved simultaneously with the characteristics method using Runge-Kutta techniques. Depending upon the magnitude of the fluctuation, several cycles may be required to obtain the steady-oscillatory fluctuations.

6. Experimental Confirmation.

A triplex pump, supplied by the Union Pump Company, Battle Creek, Michigan was set up in the fluids engineering laboratory of The University of Michigan. The 3" diameter suction pipe, supplied from a constant head reservoir, was 56 ft. long, Figure 4. A short length of discharge pipe was fitted with an adjustable relief valve to maintain the mean discharge pressure. A "Dynisco" transducer was placed in the suction pipe near the suction flange, and pressure time curves were photographed on an oscilliscope for various rotational speeds and relief valve settings. The system was programmed on the IBM7090 using the characteristics method[5,6] to study the flow and pressure fluctuations; and, independently, it was programmed for use of the real fluid impedance system. In general, operation of the system is quite smooth, except when a speed is selected which causes one of the flow harmonics to have the period $4L/a$. One of the severe, but not cavitating, speeds (170 RPM) was studied, and results of the three determinations are shown in Figure 6.

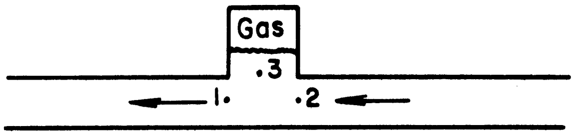

Figure 7. Accumulator, Neglecting Inertia.

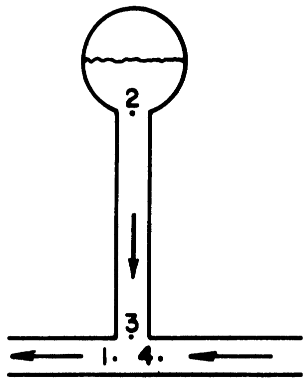

Figure 8. Accumulator with
Inertia and Friction.

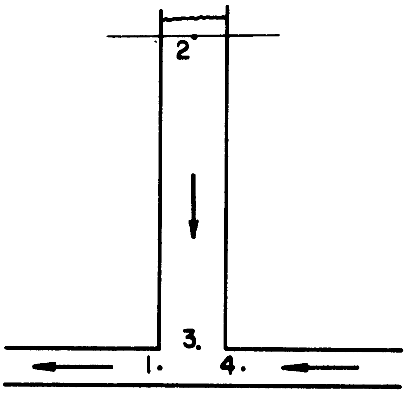

Figure 9. Notation for Impedance
Analysis of a Standpipe.

7. Reduction of Pressure Fluctuations in Reciprocating Pump Systems.

There are many ways to improve the performance of a reciprocating pump system, varying from design changes in the pump to changes in arrangement, sizes and length of piping components, and inclusions of appurtenances to absorb certain fluctuations.

Pump design changes. Since the kinematics of the pumping cycle is relatively fixed, and economics usually does not permit the use of slower and larger units, one big design improvement is the reduction of volume between suction and discharge valves. Not only will this reduce suction and discharge flow fluctuations, but it will reduce stresses within the pump body. Another possibility of reducing the shock due to the compressibility of the liquid is to build in internal accumulators that would precompress the liquid on the start of the discharge stroke and decompress it on the start of the suction stroke. For the analysis discussed in this paper, perfect valve function has been assumed. To the extent that the valve delays in closing or in opening the pump behavior is worsened.

Suction piping system. In general, waterhammer effects are related to changes in velocity; the greater dV/dt is at a point, the greater the pressure change. By increasing the diameter of the suction-piping, fluctuations in pressure are reduced. This one change may reduce noise and cavitation effects. The length of the piping system may also be critical. If a single suction-line is employed, its fundamental period is $4L/a$, i.e., four times the wave travel time over its length. If one of the periods of the flow harmonics ($T = 2\pi/m\omega$ where ω is the angular frequency of the fundamental of the flow fluctuation) happens to coincide closely with the pipe period, severe pressure fluctuations occur. The greater the magnitude of the flow harmonic, the greater the pressure fluctuation. For example, a suction line 34 ft long might be quite satisfactory for an installation; but one 30 ft long would give trouble. For complex suction piping systems an impedance analysis, will reveal the fundamental and harmonic periods. Coincidence of a harmonic period of the system and one of the pump flow may cause serious pressure fluctuations, depending upon the magnitude of the flow harmonic.

Close attention should therefore be given to the proper size and length of suction piping systems. In addition, certain corrective devices may be added to the system, to eliminate particular pressure fluctuations, or to give broad reduction of fluctuations over a wide frequency range. A dead-end stub(7) may be added to the system and quite completely eliminate a disturbance of a given frequency. Combinations of resistances, chokes, and accumulators may yield broad improvement. There are several commercial devices that apparently act to reduce fluctuations of the higher harmonics by causing a partial phase shift. The introduction of piping having a different wave speed, such as rubber hose, may be very effective. Standpipes at or near the suction flange may be very effective when long suction lines are required. Any of these devices which act on a one-dimensional basis can be analyzed by impedance or characteristics methods.

Discharge piping system. The same remarks regarding the suction system apply for the most part to the discharge system, except that cavitation is not usually a problem. Before attempting to improve the discharge system it is very important that the suction system avoids any effects of cavitation. If the suction stroke does not completely fill the cylinder, very high fluctuations occur on the discharge side. Owing to the generally high pressures on the discharge side, the cost of reducing pressure fluctuations is greater. Accumulators must be selected so that inertial effects do not reduce their effectiveness.

Impedance analysis of inertialess, frictionless accumulator.

Consider the accumulator shown in Figure 7 which has substantially no inertia and no friction. If it is attached to the suction piping system and Z_2 , the hydraulic impedance, is known, then Z_1 , may be calculated as follows:

The head fluctuation h' at points 1, 2, and 3 is the same, in magnitude and phase, and the gas volume fluctuation is always 180° out of phase with the head. From continuity

$$q_1' = q_2' + \dot{\forall} \tag{33}$$

in which q_1' and q_2' are the flow fluctuations and $\dot{\forall}$ the time rate of change of gas volume in the accumulator for the harmonic. By substitution of the impedances

$$Z_1 = \frac{h'}{q_1'} \quad , \quad Z_2 = \frac{h'}{q_2'} \tag{34}$$

$$\frac{1}{Z_1} = \frac{1}{Z_2} + \frac{\dot{\forall}}{h'}$$

Now, by assuming polytropic expansion or compression of the gas volume

$$H \forall^n = \text{constant} \tag{35}$$

with H the absolute head, $\forall$ the gas volume, and n the polytropic exponent. By writing

$$H = \overline{H} + \Delta H \qquad \forall = \overline{\forall} + \Delta\forall$$

substituting into Equation (35) and linearizing

$$\frac{\Delta\forall}{\Delta H} = - \frac{\overline{\forall}}{n\overline{H}} \tag{36}$$

After defining the head fluctuation for the harmonic as

$$h'_m = \Delta H_m e^{i(\Theta_m - \frac{\pi}{2} + m\omega t)} \qquad (37)$$

Then the volume fluctuation $\forall$ is given by

$$\forall_m = \Delta\forall_m e^{i(\Theta_m + \frac{\pi}{2} + m\omega t)} \qquad (38)$$

since it is 180° out of phase with the head. ΔH_m is the magnitude of head fluctuation $|h'_m|$ and $\Delta\forall_m$ is the magnitude of volume fluctuation $|\forall_m|$. After taking the derivative of $\forall$ with respect to time

$$\dot{\forall}_m = im\omega \; \Delta \forall_m e^{i(\Theta_m + \frac{\pi}{2} + m\omega t)} \qquad (39)$$

and

$$\frac{\dot{\forall}_m}{h'_m} = - im\omega \frac{\Delta\forall_m}{\Delta H_m} = \frac{im\omega\bar{\forall}}{n\bar{H}} \qquad (40)$$

by use of Equation (36). Now, using Equation (34)

$$\frac{1}{Z_1} = \frac{1}{Z_2} + \frac{im\omega\bar{\forall}}{n\bar{H}}$$

and

$$Z_1 = \frac{Z_2}{1 + \frac{im\omega\bar{\forall}Z_2}{n\bar{H}}} \qquad (41)$$

With Z_1 calculated for each harmonic, the head at 1 may be calculated if the flow at 1 is known.

A small accumulator at the suction flange can be effective in reducing fluctuations. For example, computer calculations show that an air volume of 0.3 cu ft. reduces the head fluctuation from about 100 ft (Figure 6) to 0.05 ft and an accumulator of 0.01 cu ft air volume reduces the over all head fluctuation to 1.4 ft.

<u>Impedance analysis of accumulator with friction and inertia.</u> If an accumulator has appreciable inertia and frictional effects are to be taken into consideration it may be represented as in Figure 8. The hydraulic impedance at 2 is

$$Z_2 = \frac{h'_{2m}}{\dot{V}_m} = - \frac{in\bar{H}}{m\omega\bar{V}} \qquad (42)$$

from Equation (40). The impedance at 3 is found by considering the length 2-3 of pipe to be so short that the general impedance equation [Equation (24)] may be lumped. Then with the impedance at 3 and 4 known the impedance at 1 is found for a branching system. For the lumped system,

$$Z_3 = Z_C \frac{Z_2(1 + i\ \alpha\beta L^2) - Z_C L\ (\alpha + i\beta)}{Z_C(1 + i\ \alpha\beta L^2) - Z_2 L\ (\alpha + i\beta)} \qquad (43)$$

<u>Impedance analysis of a standpipe.</u> In Figure 9 a standpipe on a suction line is shown. The impedance at 2, close to the surface is obtained as follows: Let S be the submergence at 2; then for a harmonic

$$S_2 = h'_{2m} = \triangle S_m e^{i\ (\Theta_{2m} + m\omega t - \frac{\pi}{2})} \qquad (44)$$

The flow q'_{2m} is

$$q'_{2m} = \dot{S}_{2m} A = A\ im\omega \triangle S_m e^{i\ (\Theta_{2m} + m\omega t - \frac{\pi}{2})} \qquad (45)$$

From Equations (44) and (45)

$$Z_2 = \frac{h'_{2m}}{q'_{2m}} = - \frac{i}{m\omega A} \qquad (46)$$

With Z_2 known, Z_3 and then Z_1 may be found for known Z_4 .

Although only a few cases of design to avoid severe fluctuations have been discussed, the general methods of analysis of systems have been presented.

8. Conclusions.

An analysis is presented for the study of hydraulic transients produced by reciprocating pumps. Although the frictional term is linearized in the impedance theory, the linearization applies only to the oscillatory components of flow and not to the steady flow. Thus the linearized friction assumption is an acceptable approximation. Various design possibilities are suggested for noisy and/or rough operating conditions, and analyses are given for these components. Experimental confirmation is provided to validate the theory presented.

It can be concluded that most practical reciprocating pump installations can be correctly analyzed. When an undesirable condition appears in an installation or is uncovered in a design, several possible remedial measures are available.

REFERENCES

1. Waller, E. J., Prediction of Pressure Surges in Pipelines by Theoretical and Experimental Methods. Publication No. 101, Oklahoma State University, Stillwater, June, 1958.

2. Streeter, V. L., and Wylie, E. B., "Resonance in Governed Hydro Piping Systems," Proc. of International Symposium on Waterhammer in Pumped Storage Projects, ASME, Chicago, 1965.

3. Wylie, E. B., "Resonance in Pressurized Piping Systems," Journal of Basic Eng., Trans. ASME, Vol. 87, Series D, No 4, Dec. 1965, pp 960-966.

4. Moore, R. K., Traveling-Wave Engineering, McGraw-Hill Book Company, New York, 1960.

5. Streeter, V. L., and Lai, C., "Water-Hammer Analysis including Fluid Friction," Journal of the Hydraulics Division, ASCE, 88, No. HY3 (May, 1962), 79-112.

6. Streeter, V. L., "Computer Solution of Surge Problems," Symposium on Surges in Pipelines, Institution of Mechanical Engineering, Proc. 1965-66, Vol. 180, Part 3E.

7. Waller, E. J., Pressure Surge Control in Pipeline Systems. Publication No. 102, Oklahoma State University, Stillwater, Jan. 1959.