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RESONANCE IN GOVERNED HYDRO PIPING SYSTEMS

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NOMENCLATURE

A = cross sectional area of pipe; constant

A_n = nozzle area

a = wave speed in pipe

B = constant

C = constant

C_d = discharge coefficient of nozzle

D = diameter of pipe

e = turbine efficiency

f = Darcy-Weisbach friction factor

g = acceleration due to gravity

H = complex function of x only

H_R = complex head at upstream end

H_S = complex head at downstream end

h = elevation of hydraulic gradeline

$\overline{h}$ = steady-state elevation of hydraulic gradeline

h' = fluctuation of hydraulic gradeline elevation

I = moment of inertia of rotating masses

$i = \sqrt{-1}$

K = number of harmonics

k = integer

ℓ = length of pipe

ℓ_1, ℓ_2 = linkage lengths in governor

$m = i\omega$

N = wheel speed

ΔN_a = displacement due to permanent droop

ΔN_c = displacement of dashpot cylinder

ΔN_m = manual speed setting

ΔN_p = displacement of dashpot piston

n = exponent on friction term

P = power of turbine

P_G = power absorbed by generator

p = pressure in dashpot

Q = complex function of x only

Q_R = complex discharge at upstream end

Q_S = complex discharge at downstream end

q = discharge

$\bar{q}$ = steady-state discharge

q' = discharge fluctuation

R = resistance coefficient; governor constant

R_1, R_1', R_3', R_g = governor constants

r = radius to centerline of buckets

r_2, r_4, r_4', r_g = governor constants

t = time

T = function of t only; torque; period

u = peripheral speed of centerline of buckets

V = fluid velocity leaving nozzle

X = function of x only

x = distance along pipe

Y = displacement of spool valve

z = hydraulic impedance

z_C = characteristic impedance

z_R = impedance, upstream end

z_S = impedance, downstream end

α = real part of γ

β = pure imaginary part of γ

ζ = vane angle, exit to buckets

Θ = governor arm angle

γ = complex number; unit weight of liquid

ρ = fluid density

φ_A = phase angle, nozzle area to head

φ_N = phase angle, speed to head

φ_q = phase angle, discharge to head

ω = circular frequency

I. INTRODUCTION

In calculating the stability of penstock systems under the control of a governor, the resonant periods of the complex piping systems have not been adequately taken into account. Gaden[1] and Rich[2] assumed the penstock system could be modeled by a simple pipe having the same theoretical period $\sum 4\ell/a$ as the actual penstock with its usual reductions in diameter. Borel[3] takes the pressure pipe and surge tank into account, but considers incompressible surge only. Wylie[4,5] has developed methods for finding the fundamental (apparent) period of a piping system and its various harmonics, for inviscid flow conditions.

In this paper equations for determination of impedance phase and magnitude are developed for analysis of complex piping systems with real fluids. Then equations for an impulse wheel and governor are developed, linearized, and programmed to determine phase and magnitude of impedance as well as other pertinent variables. A criteria for stability analysis is presented based on energy concepts and the real fluid impedance calculations.

Questions of stability are examined for two situations: waves on the reservoir upstream from the penstock when the unit is not operating; and series systems under control of a governor.

II. RESONANT FREQUENCIES OF VISCOUS FLUID PIPING SYSTEMS

The water hammer equations for continuity and equation of motion, in terms of discharge q and elevation of hydraulic gradeline h are, respectively

$$q_x + \frac{gA}{a^2} h_t = 0 \tag{1}$$

and

$$h_x + \frac{1}{gA} q_t + \frac{f}{2gDA^n} q^n = 0 \tag{2}$$

in which a is the speed of the pressure pulse wave, A is the pipe cross-sectional area; f is the Darcy-Weisbach friction factor for steady state conditions, D is the pipe diameter, n is the exponent of the velocity in the loss term, g is the acceleration of gravity, x is the distance from upstream end of pipe, and t is the time. The subscripts x and t denote partial differentiation.

These equations are solved for steady-oscillatory flow, following methods of transmission line theory.[6,7] The variables h and q are considered as

$$h = \bar{h} + h' \qquad q = \bar{q} + q' \tag{3}$$

with $\bar{h}$, $\bar{q}$ average steady-state values, and h' , q' variations from the average. By making these substitutions into Equations (1) and (2) and simplifying

$$q'_x + \frac{gA}{a^2} h'_t = 0 \tag{4}$$

and

$$h'_x + \frac{1}{gA} q'_t + Rq' = 0 \qquad (5)$$

in which

$$R = \frac{nf\bar{q}^{\,n-1}}{2gDA^n} \qquad (6)$$

is used as the linearized friction coefficient for turbulent flow.

Equations (4) and (5), by differentiation, may be written in terms of a single dependent variable, q', or h',

$$q'_{xx} = \frac{1}{a^2} q'_{tt} + \frac{gA}{a^2} Rq'_t \qquad (7)$$

or

$$h'_{xx} = \frac{1}{a^2} h'_{tt} + \frac{gA}{a^2} Rh'_t \qquad (8)$$

To solve Equation (8), let $h' = X(x)\,T(t)$, in which X is a function of x only and T is a function of t only. After substitution and rearrangement

$$\frac{X''}{X} = \frac{1}{a^2}\frac{T''}{T} + \frac{gA}{a^2} R \frac{T'}{T} = \gamma^2 \qquad (9)$$

in which $\gamma = \alpha + i\beta$ must be a complex constant. The equation for X in terms of γ yields

$$X = Ae^{\gamma x} + Be^{-\gamma x} \qquad (10)$$

with A and B constants of integration to be determined by conditions at inlet and outlet of pipe. By restricting the solution for T to the steady-oscillatory case, $T = C_1 e^{mt}$ with $m = i\omega$; $i = \sqrt{-1}$, and ω the angular frequency, substitution into Equation (9) yields

$$\gamma^2 = (\alpha + i\beta)^2 = \frac{\omega g}{a^2} \cdot A(iR - \frac{\omega}{gA}) \qquad (11)$$

After solving for α and β

$$\alpha = \sqrt{\frac{\omega g A}{a^2}} \cdot \left[\left(\frac{\omega}{gA}\right)^2 + R^2\right]^{1/4} \sin\left(\frac{1}{2} \tan^{-1} \frac{gAR}{\omega}\right) \qquad (12)$$

$$\beta = \sqrt{\frac{\omega g A}{a^2}} \left[\left(\frac{\omega}{gA}\right)^2 + R^2\right]^{1/4} \cos\left(\frac{1}{2} \tan^{-1} \frac{gAR}{\omega}\right) \qquad (13)$$

Since R and ω are always positive, the angle is in the first quadrant and α and β are real, positive numbers. These equations are easily solved by computer for each pipe at the frequency ω being considered.

The equation for h' is

$$h' = e^{i\omega t} (Ae^{\gamma x} + Be^{-\gamma x}) \qquad (14)$$

After combining this equation with Equation (4) to eliminate h' ,

$$q' = \frac{\omega g A}{i a^2 \gamma} e^{i\omega t} (Ae^{\gamma x} - Be^{-\gamma x}) \qquad (15)$$

The <u>hydraulic impedance</u> z(x) is the ratio of head fluctuation to discharge fluctuation h'/q'

$$z(x) = \frac{i\gamma a^2}{\omega g A} \frac{Ae^{\gamma x} + Be^{-\gamma x}}{Ae^{\gamma x} - Be^{-\gamma x}} \qquad (16)$$

The <u>characteristic impedance</u> z_C of a pipe is a function of ω , defined by

$$z_C = \frac{\gamma a^2}{i\omega g A} = \frac{a^2}{\omega g A} (\beta - i\alpha) \tag{17}$$

so

$$z(x) = - z_C \frac{Ae^{\gamma x} + Be^{-\gamma x}}{Ae^{\gamma x} - Be^{-\gamma x}} \tag{18}$$

A and B must now be expressed in terms of conditions at inlet and outlet of the pipe.

Equations (14) and (15) may be written

$$h'(x, t) = H(x)e^{i\omega t} \tag{19}$$

$$q'(x, t) = Q(x)e^{i\omega t} \tag{20}$$

in which H and Q are complex functions of x only. For the inlet, let

$$x = 0, H(0) = H_R, Q(0) = Q_R \tag{21}$$

After substitution in Equations (14) and (15), eliminating A and B ,

$$H(x) = H_R \cosh \gamma x - Q_R z_C \sinh \gamma x \tag{22}$$

$$Q(x) = \frac{-H_R}{z_C} \sinh \gamma x + Q_R \cosh \gamma x \tag{23}$$

and

$$z(x) = \frac{H(x)}{Q(x)} = \frac{z_R - z_C \tanh \gamma x}{1 - (z_R/z_C) \tanh \gamma x} \tag{24}$$

in which $z_R = H_R/Q_R$. For known conditions at the pipe outlet, $x = \ell$, let

$$x_1 = \ell - x \ , \ H(\ell) = H_S \ , \ Q(\ell) = Q_S \ , \ z_S = H_S/Q_S \qquad (25)$$

hence

$$H(x) = H_S \cosh \gamma x_1 + z_C \, Q_S \sinh \gamma x_1 \qquad (26)$$

$$Q(x) = \frac{H_S}{z_C} \sinh \gamma x_1 + Q_S \cosh \gamma x_1 \qquad (27)$$

and

$$z(x) = \frac{z_S + z_C \tanh \gamma x_1}{1 + (z_S/z_C) \tanh \gamma x_1} \qquad (28)$$

Since z_R , z_S , z_C , and γ are complex, $z(x)$ is a complex number having a magnitude $|z(x)|$ and a phase angle φ_q , $z(x) = |z(x)| \, e^{i\varphi_q}$. The discharge lags the head by φ_q.

By analyzing a piping system, starting with known impedances, say, at reservoirs, deadends, or orifices, the impedance at some desired point in the system may be calculated for a given ω , when R, A, a, f, ℓ, n and $\bar{q}$ are known for each pipe. As a simple example, if a known sine wave variation of head is superposed on a steady flow at a point, and the discharge variation is desired at this point, the calculation of $z = |z| \, e^{i\varphi_q}$ yields the magnitude of discharge fluctuation.

$$h' = \Delta h \sin \omega t \ , \qquad q' = \Delta q \sin (\omega t - \varphi_q) \qquad (29)$$

$$\Delta q = \frac{\Delta h}{|z|}$$

For more complex periodic head variations, the head is expressed by a Fourier series by use of harmonic analysis. The impedance for each of the frequencies is calculated and q' is then found by adding the harmonics taking phase into account,

$$q' = \sum_{k=1}^{K} \Delta q_k \sin(k\omega t - \varphi_k) \qquad (30)$$

in which there are K harmonics and ω is the frequency of the fundamental period. Those frequencies yielding a high impedance are the resonant frequencies for the system.

III. EQUATIONS FOR IMPULSE TURBINE AND GOVERNOR SYSTEM

Since the principal aim of this paper is to correctly handle the pipeline transients, the turbine equations are limited to the impulse impeller, and variations in electrical transients are neglected.

A. Turbine

The power P produced by an impulse wheel is given by

$$P = \rho q e \, (V - u) \, u(1 - \cos \zeta) \tag{31}$$

in which ρ is the density, q the discharge, e the turbine efficiency, V the fluid velocity leaving the nozzle, u the peripheral speed of the centerline of the buckets and ζ the vane angle of the buckets. If the power absorbed by the generator is P_G, then the equation for determination of speed change of the wheel is (from $TN = IN\dot{N}$, with N the wheel speed in radians per second, T the net torque, and I the moment of inertia of rotating masses)

$$P - P_G = IN\dot{N} \tag{32}$$

$\dot{N}$ is the angular acceleration. From nozzle dynamics

$$V = C_d\sqrt{2gh} \quad , \quad q = C_d A_n \sqrt{2gh} \quad , \quad u = N\, r \tag{33}$$

in which C_d is the discharge coefficient of the nozzle, A_n is the nozzle area, h is the head at the base of the nozzle, and r is the radius of centerline of the buckets. By use of Equations (31) to (33)

$$\rho C_d A_n \sqrt{2gh} \; e(1 - \cos \zeta) \; N r \; (C_d \sqrt{2gh} - Nr) - P_G = I N \mathring{N} \qquad (34)$$

By assuming constant e for small discharge and speed changes (i.e., constant windage, bearing, splitter and friction losses), Equation (34) for steady operation becomes

$$\rho C_d A_{n_o} \sqrt{2gh_o} \; e(1 - \cos \zeta) \; N_o r \; (C_d \sqrt{2gh_o} - N_o r) - P_{G_o} = 0 \qquad (35)$$

After dividing Equation (34) by Equation (35)

$$\frac{A_n}{A_{n_o}} \sqrt{\frac{h}{h_o}} \; \frac{N}{N_o} \; \frac{C_d \sqrt{2gh} - N r}{C_d \sqrt{2gh_o} - N_o r} - \frac{P_G}{P_{G_o}} = \frac{I}{P_{G_o}} \; N_o^2 \; \frac{N}{N_o} \; \frac{\mathring{N}}{N_o} \qquad (36)$$

Let the superscript * indicate the dimensionless variable, e.g., $A_n^* = A_n/A_{n_o}$, $N^* = N/N_o$, $\mathring{N}^* = \mathring{N}/N_o$. For a small fluctuation $\Delta N = N - N_o$, and $\Delta N^* = N^* - 1$, etc. Equation (36) may be reduced to

$$\Delta A_n^* + \Delta N^* \left[1 - \frac{N_o r}{C_d \sqrt{2gh_o} - N_o r} \right] + \frac{1}{2} \; \Delta h^* \left[1 + \frac{C_d \sqrt{2gh_o}}{C_d \sqrt{2gh_o} - N_o r} \right] - \Delta P_G^*$$

$$= \frac{I \; N_o^2}{P_{G_o}} \; N^* \; \dot{N}^* \qquad (37)$$

Since N^* remains close to unity, let $N^* = 1$, and define

$$C_1 = 1 - \frac{N_o r}{C_d \sqrt{2gh_o} - N_o r} , \quad C_2 = \frac{1}{2} \left[1 + \frac{C_d \sqrt{2gh_o}}{C_d \sqrt{2gh_o} - N_o r} \right]$$

$$C_3 = \frac{I}{P_G} \; N_o^2 \qquad (38)$$

then

$$\Delta A^* + C_1 \Delta N^* + C_2 \Delta h^* = C_3 \Delta \dot{N}^* + \Delta P_G^* \tag{39}$$

is the linearized equation for the impulse turbine. When changes in generator loading are not under consideration $\Delta P_G^* = 1$.

B. Governor Equations

The governing system of Borel,[3] shown schematically in Figure 1 is analyzed as a typical system with temporary droop provided by a dash-pot. In Figure 2 the flyballs of the governor are assumed to pivot as shown, and friction and gravity loads, except on the flyballs, are neglected. Let N_g be the angular velocity of flyballs, then $N_g = C_g N$. After taking moments about 0

$$m\, C_g^2\, N^2\, R_g^2 \sin\Theta \cos\Theta = mg\, R_g \sin\Theta \tag{40}$$

and

$$\cos\Theta = \frac{g}{R_g C_g^2\, N^2} \tag{41}$$

For steady-state conditions

$$\cos\Theta_o = \frac{g}{R_g C_g^2\, N_o^2} \tag{42}$$

The displacement of governor collar from steady state position is

$$C_4\, \Delta N = 2r_g\, (\cos\Theta_o - \cos\Theta) = \frac{2g\, r_g (N + N_o)}{R_g C_g^2\, N_o^2\, N^2} (N - N_o) \tag{43}$$

After linearizing

$$C_4 = \frac{4g\ r_g}{R_g C_g^2\ N_o^3} = \frac{4\ r_g}{N_o} \cos\ \Theta_o \tag{44}$$

In Figure 3, let Y = displacement of spool valve from closed position (neglecting overlap); then

$$Y = C_4\ \Delta N + \frac{\ell_1}{\ell_1 + \ell_2}\ (\Delta N_p - C_4\ \Delta N) \tag{45}$$

in which ΔN_p is the displacement of dashpot piston from steady-state position, and ℓ_1 , ℓ_2 are the linkage lengths shown. To nondimensionalize this equation, first divide by RN_o , with R to be determined.

$$Y^{**} = \frac{Y}{RN_o} = \frac{C_4}{R} \frac{\ell_2}{(\ell_1 + \ell_2)} \Delta N^* + \frac{\ell_1\ \Delta N_p}{(\ell_1 + \ell_2) RN_o} \tag{46}$$

To simplify the arithmetic, let

$$\frac{C_4\ \ell_2}{R(\ell_1 + \ell_2)} = 1\ , \quad N_{p_o} = N_o\ C_4\ \frac{\ell_2}{\ell_1}\ , \quad \Delta N_p^* = \frac{\Delta N_p}{N_{p_o}} \tag{47}$$

then

$$Y^{**} = \Delta N^* + \Delta N_p^* \tag{48}$$

To analyze the dashpot behavior, Figure 4, the force F in the piston rod is given by

$$F = K_s\ \xi = (p_2 - p_1)\ A_p \tag{49}$$

K_s is the spring constant, p_1 and p_2 the pressures in the cylinder, and A_p the piston area. The flow through the piston of the dashpot Q_p is assumed to be laminar

$$Q_p = \frac{C_v(p_2 - p_1)\pi D^4}{128 \mu \ell_o} = -\dot{\eta} \; A_p \qquad (50)$$

D is the diameter of opening through the piston, ℓ_o its length, μ the viscosity, and C_v a correction for end effects due to the short tube flow. η is the piston position; then

$$-\dot{\eta} = \dot{N}_C - \dot{N}_p = \frac{C_v K_s \pi D^4}{128 \mu \ell_o A_p^2} (N_p - N_a) \qquad (51)$$

using Equations (49) and (50) and $\xi = N_p - N_a$. Let

$$R_3' = \frac{128 \mu \ell_o A_p^2}{C_v K_s \pi D^4} \qquad (52)$$

yielding

$$\Delta N_p - \Delta N_a = R_3' (\Delta \dot{N}_C - \Delta \dot{N}_p) \qquad (53)$$

After dividing through by N_{p_o} to nondimensionalize

$$\Delta N_p^* - \Delta N_a^* = R_3' (\Delta \dot{N}_C^* - \Delta \dot{N}_p^*) \qquad (54)$$

The displacement of spool valve is taken proportional to the flow through it, which is proportional to the rate of change of position X of the area control of the nozzle,

$$\Delta \dot{X}^* = \frac{d}{dt}\left(\frac{X - X_o}{X_o}\right) = -\frac{1}{R_1} Y^{**} \qquad (55)$$

The nozzle area change is proportional to change in X for small displacements from steady state

$$\Delta A_n = r_2 \Delta X^* \tag{56}$$

The position of the dashpot cylinder is proportional to X, Figure 4,

$$\Delta N_C^* = r_4' \Delta X^* \tag{57}$$

and the position N_a of the permanent droop control, Figure 4, is given by

$$\Delta N_a^* = r_4 \Delta X^* - \Delta N_m^* \tag{58}$$

in which ΔN_m^* is the manual speed control.

By elimination of Y^{**}, ΔN_p^*, ΔX^*, ΔN_a^*, and ΔN_C^* in Equations (48), and (55) to (58)

$$\Delta \overset{\circ}{A}{}_n^*(R_1+R_3'r_4') = - r_4 \Delta A_n^* - r_2 \Delta N^* + r_2 \Delta N_m^* - R_3' r_2 \Delta \overset{\circ}{N}{}^* + R_3' R_1 \Delta \overset{\circ\circ}{A}{}_n^* \tag{59}$$

In practice $R_1 \ll R_3' r_4'$, so the $\Delta \overset{\circ\circ}{A}{}^*$ term may be dropped. By defining $R_1' = R_1 + R_3' r_4'$

$$\Delta \overset{\circ}{A}{}_n^* = - \frac{r_2}{R_1'} \left[\frac{r_4}{r_2} \Delta A_n^* + \Delta N^* + R_3' \Delta \overset{\circ}{N}{}^* - \Delta N_m^* \right] \tag{60}$$

Equations (39) and (60) for turbine and governor may be solved by assuming a steady-oscillatory sine wave variation of all variables at the selected frequency ω for the analysis. The following four equations define the magnitude and phase relations among the variables:

$$\Delta h^* = \Delta H \sin \omega t \tag{61}$$

$$\Delta q^* = \Delta Q \sin (\omega t - \varphi_q) \tag{62}$$

$$\Delta N^* = \Delta N \sin (\omega t - \varphi_N) \tag{63}$$

$$\Delta A_n^* = \Delta A \sin (\omega t - \varphi_A) \tag{64}$$

The substitution of Equations (61), (63), and (64) into Equations (39) and (60) permits the solution for $\Delta N/\Delta A_n$, $\Delta H/\Delta A_n$, φ_A and φ_N. Then by use of the nozzle equation from $q = C_d A_n \sqrt{2gh}$

$$\Delta q^* = \Delta A^* + \frac{1}{2} \Delta h^*$$

permits $\Delta Q/\Delta A$ and φ_q to be determined. A computer program has been written to effect these solutions.

IV. ENERGY RELATIONSHIPS

Whenever oscillatory flows and pressures exist in a system the energy input and output may differ from the steady state flow condition. If a net energy input exists due to the oscillations, that is, the energy in exceeds the energy out plus losses, the amplitudes will be magnified until an energy balance is reached or a failure occurs. This is the situation when resonance develops.

Assuming the oscillatory motion superposed upon a steady state condition or upon a zero flow condition in a system, the energy being added during each wave period, T, is

$$\int_0^T \gamma\, h'\, q'\, dt\ .$$

If sinusoidal waves are assumed the evaluation of this integral yields the energy in foot-pounds entering the system during each wave period.

$$E = \frac{\gamma\, h'\, q'\, T}{2} \cos \varphi_q \qquad (65)$$

For any given magnitude of oscillation, h' and q', the phase angle, φ_q, can be seen to be the important controlling parameter. At the input end of a system a phase angle in the first or fourth quadrant permits the addition of energy over the original steady flow condition. At the outflow terminal of a system a phase angle in the second or third quadrant allows a lower outflow of energy than the original steady flow condition. In each of these cases the energy level will be increasing (less losses) within the system if the other terminal of the system maintains the same energy relationship as in the steady flow condition, as is commonly the situation.

Using the impedance methods, phase angles can be evaluated for any particular system over a wide range of frequencies. The phase angle at the fundamental period of the exciting mechanism (or at the period of one of the predominate harmonics of the wave form) provides the necessary information to decide if an oscillating condition will develop or attenuate.

V. PIPING SYSTEM EXAMPLES

A. Waves on Reservoir

A series penstock system leading from a reservoir to a turbine is shown in Figure 5. If the turbine valve is closed, it is possible for surface waves on the reservoir to set up pressure fluctuations in the pipe. Should the surface wave period be such that the corresponding phase angle from the piping system is at or near zero degrees, only a few consecutive cycles would be required to add enough energy to cause a severe oscillation.

The development of pressure fluctuations at the closed valve in the system identified as system b in Figure 5 are shown in Figure 6. These results were obtained using the characteristics solution[8] of the equations for water hammer, Equations (1) and (2), on the digital computer. A two-foot sinusoidal wave form was assumed at the reservoir surface. The wave period was selected to match the third harmonic of the piping system at which point the phase angle is zero.

Figure 7 shows a plot of the pressure head transfer function, calculated using impedance methods, vs. frequency. This transfer function is the ratio of the oscillatory head developed at the closed valve end to the oscillatory head applied at the reservoir. The fundamental, third, fifth and seventh harmonics are easily identified at the frequencies associated with the large transfer function.

B. Governed Turbine

With the turbine operating in the example shown in Figure 5, the piping system can be analyzed over a wide range of frequencies using the impedance concepts, yielding phase angle and impedance magnitudes at the valve. The governor can also be analyzed in order to evaluate the same parameters at the valve. If the impedance magnitudes are matched at a frequency where the phase angles of the two system components are in the second or third quadrant, an unstable operation will result.

In analyzing a given system with a particular governor setting, a graphical representation of phase angle and impedance modulus is helpful. For the example under consideration, these are shown in Figure 8. The upper curves represent, φ_q , the phase angle between the pressure and discharge at the valve, plotted against frequency. The lower curves show impedance modulus vs. frequency. In each section the solid line is information obtained from an analysis of the governor alone. The dashed lines represent the results of the impedance analysis of the piping system only.

Piping system b (Figure 5) represents a situation wherein the governor and complex piping system are matched in a stable zone of operation. That is, at the matched condition of impedance the governor phase angle is 285° which indicates stability. On the other hand, at the matched impedance magnitude point of system a, the governor phase angle is 257° indicating an instability.

Figure 9 displays the unstable operation of system a, Figure 5. These data were obtained from a digital computer analysis which simultaneously solved the linearized governor differential equations by Runge-Kutta Methods and the hydraulic transients in the piping system by the method of characteristics.

This same turbine, valve and governor setting on piping system b, Figure 5, is shown to be stable as exhibited in Figure 10. The initial disturbance introduced to the system is shown to die out quickly on the computer analysis.

The phase angle relationships at the valve can be observed in Figures 9 and 10. The unstable condition in Figure 9 shows $\varphi_q = 256°$, while the stable condition in Figure 10 shows $\varphi_q = 280°$.

C. Other Excitations

A reciprocating pump produces a periodic pulse superposed upon the mean flow through the pump. This periodic motion can be analyzed by harmonic analysis and φ_q can be determined from the system for each of the harmonics of the pump period. Energy considerations show that, on the suction side of the pump, φ_q in the vicinity of 180° at the predominate frequencies of the pump should be avoided. On the discharge side, φ_q in the vicinity of 0° at these same frequencies should be avoided. A comprehensive analysis of the entire system using impedance methods can be easily accomplished on the digital computer.

Any mechanism on the outflow end of a pipe line which will operate in a manner such that the head and discharge are approximately 180° out of phase can cause an instability. This fact has been well illustrated by a leaky seal on a large penstock valve at a Power Plant.(5)

In this case, on the high pressure side of the oscillation the leakage was shut off, at low pressure the seal opened allowing a higher flow. This provided an "ideal" exciter for a severe resonating condition.

VI. CONCLUSIONS

A method of stability analysis is presented for complex systems which are connected to various forms of exciters. The primary basis for the analysis is the impedance theory which is developed for viscous systems. Equations for the governed impulse wheel are also developed and placed in a form suitable for solution on the digital computer. The solution of these equations over a wide frequency range along with the impedance equations from the piping system yields the necessary information for the stability criteria. Confirmation of the analysis is provided using the characteristic solution on the digital computer.

Energy considerations are discussed and related to the impedance calculations as they apply to the development of resonance in practical pipe line systems.

The impedance theory enables the analyzer to take into account the complete detail of the piping system. Simplifying assumptions which approximate the system are no longer necessary.

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LIST OF ILLUSTRATIONS

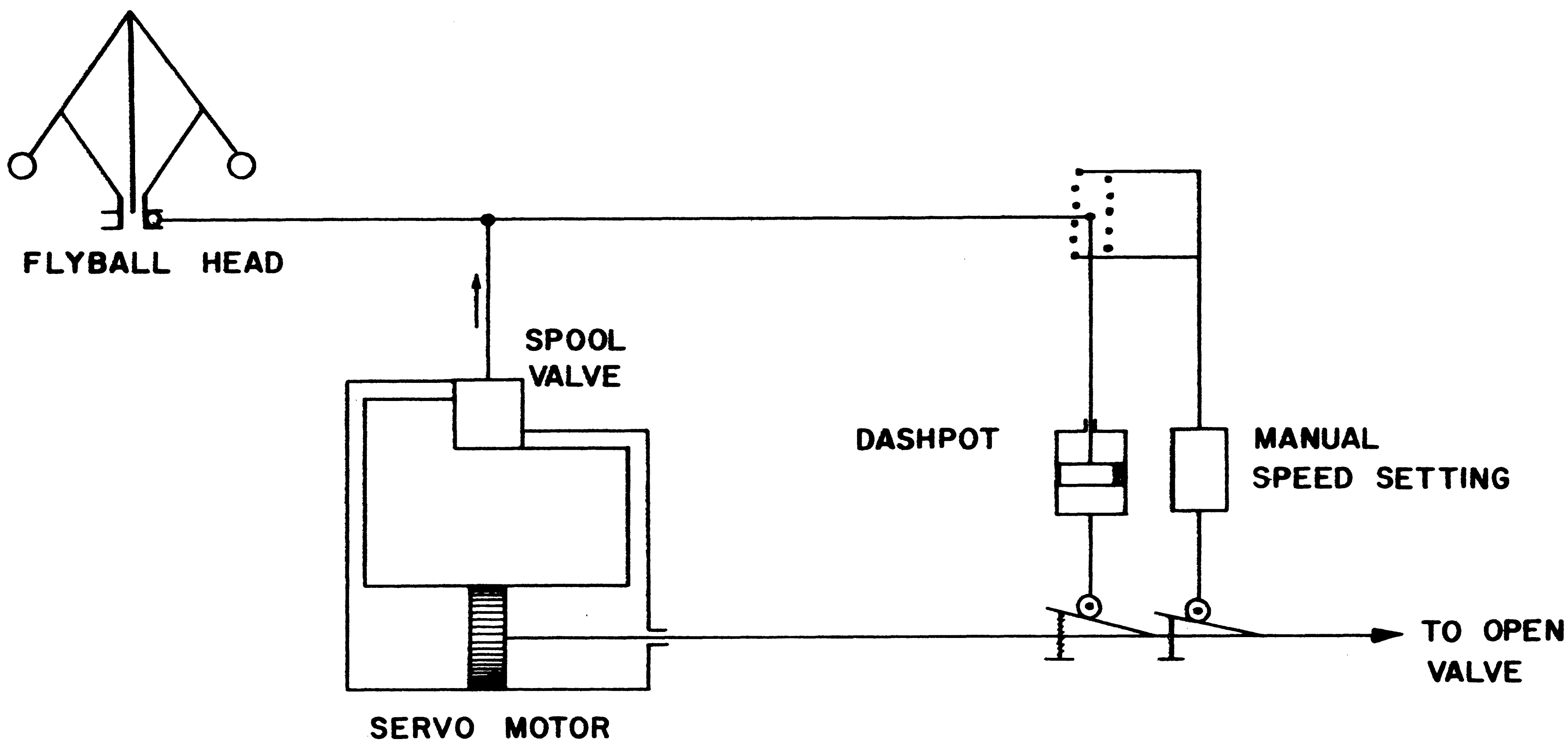


Figure 1. Schematic View of Governor System. (Borel[3])

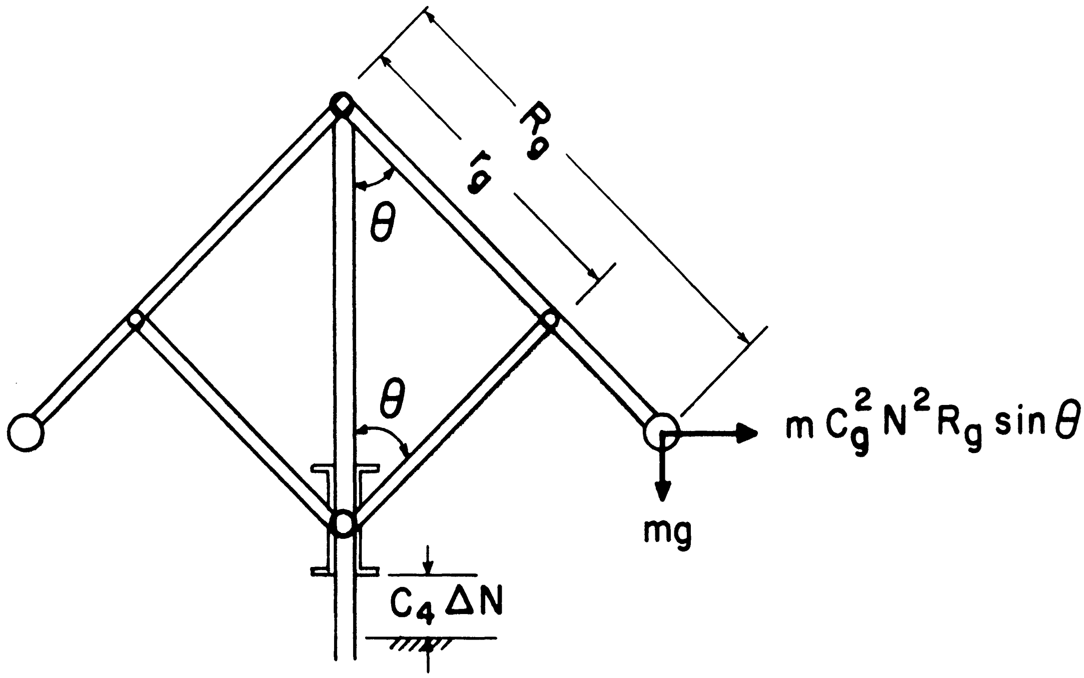


Figure 2. Flyball Governor

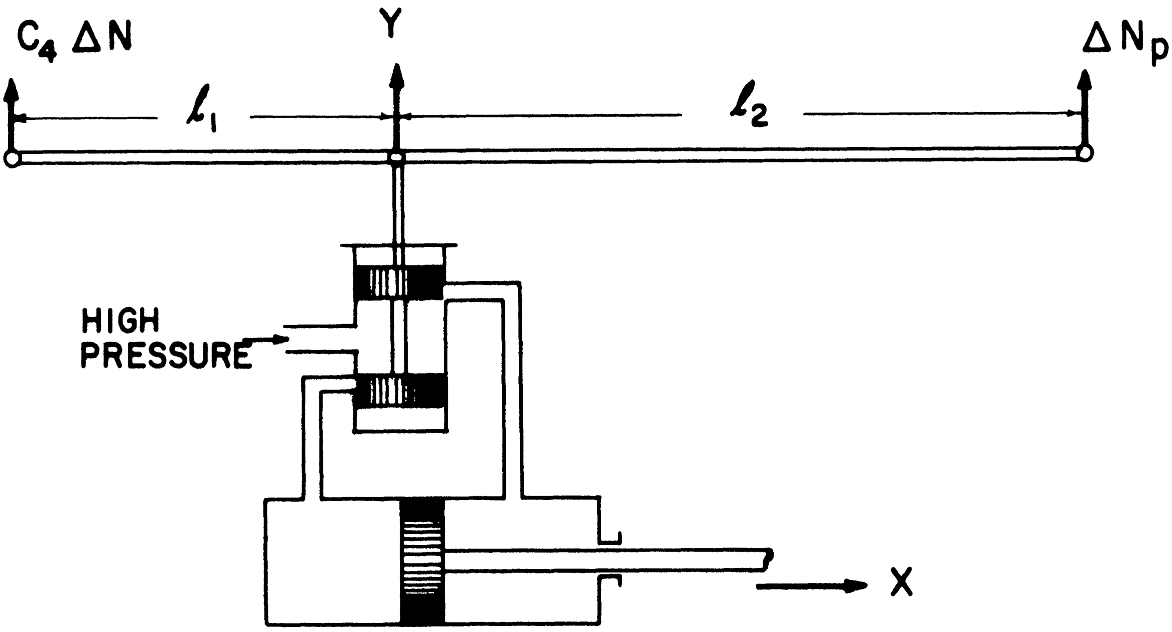


Figure 3. Spool-Valve Linkage to Flyball and Dashpot.

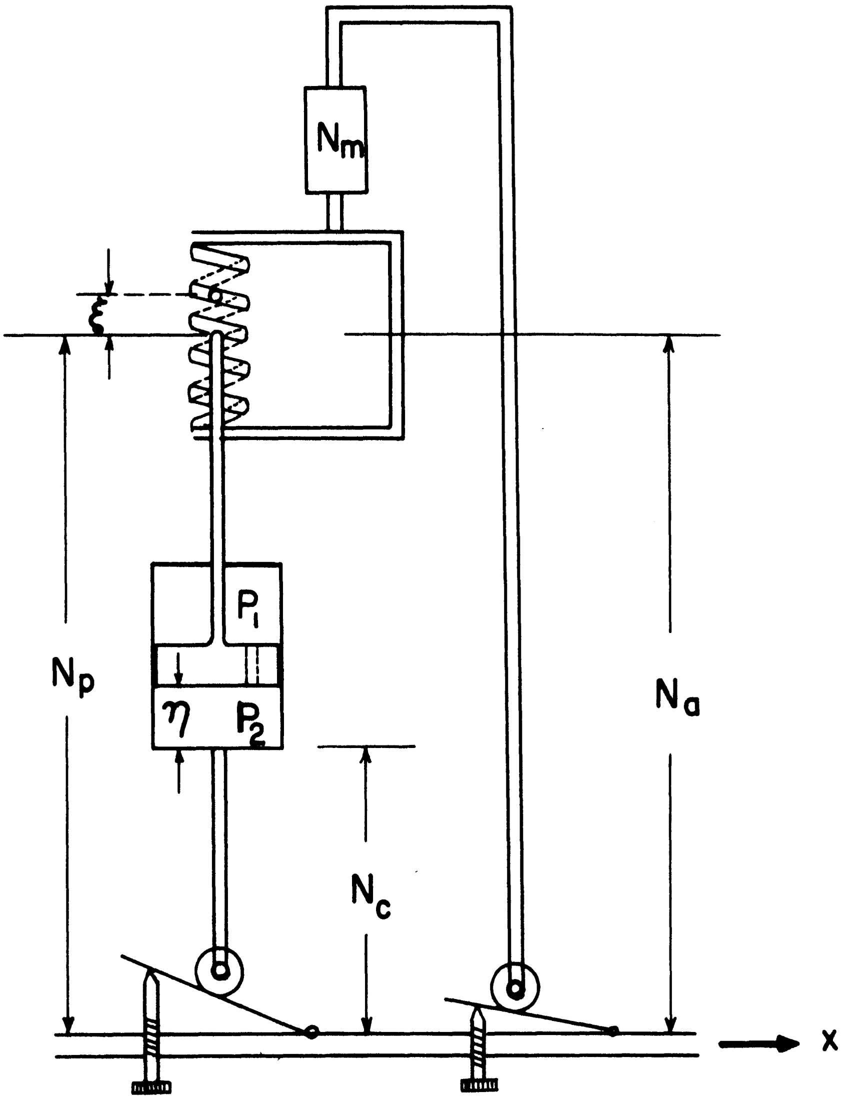
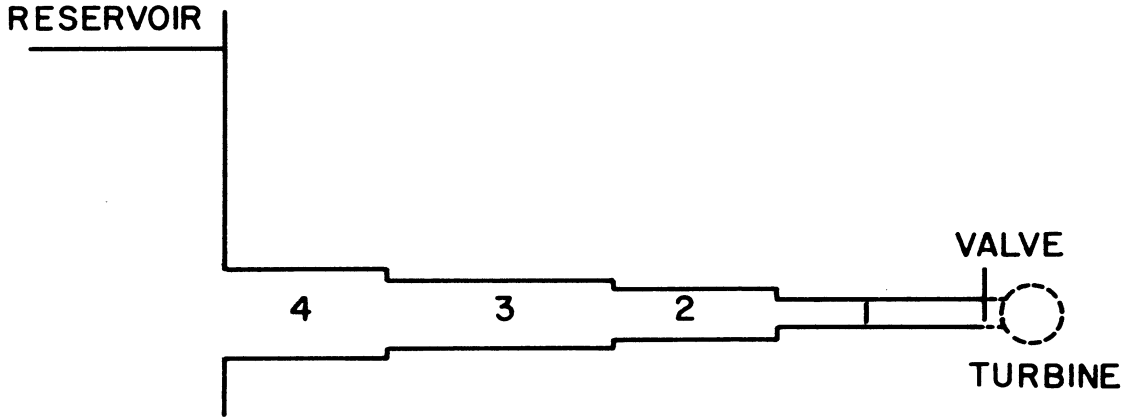


Figure 4. Dashpot Assembly



Pipe	4	3	2	1
Diameter	5.	4.5	4.	3.5
Wave Speed(ft/sec)	3600.	3800.	4000.	4200.
System a Length(ft)	1080.	1520.	1600.	1260.
System b Length(ft)	900.	950.	1200.	1050.

Turbine & Governor Constants

	R_1=.06	C_1 = .077
H_o=2590 ft.	r_2=1.	C_2 = 1.462
Q_o=107.5 cfs	r_4=.02	C_3 = 5.03
Speed: 600 rpm	r_4= .2	
I=19500 slug-ft^{-2}	R_3= 2.	

Figure 5. Series Piping System

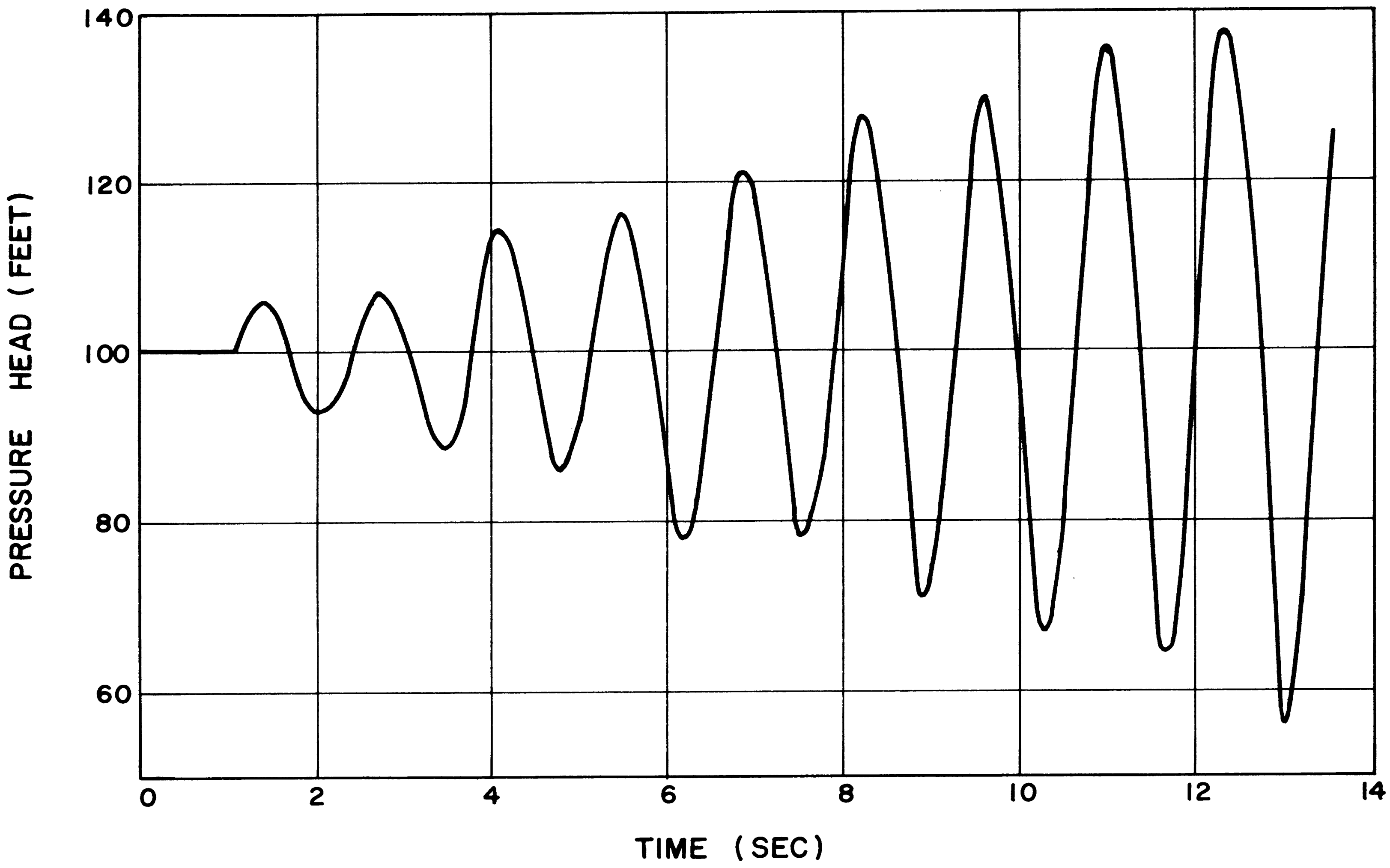


Figure 6. Pressure Oscillations at Closed Valve

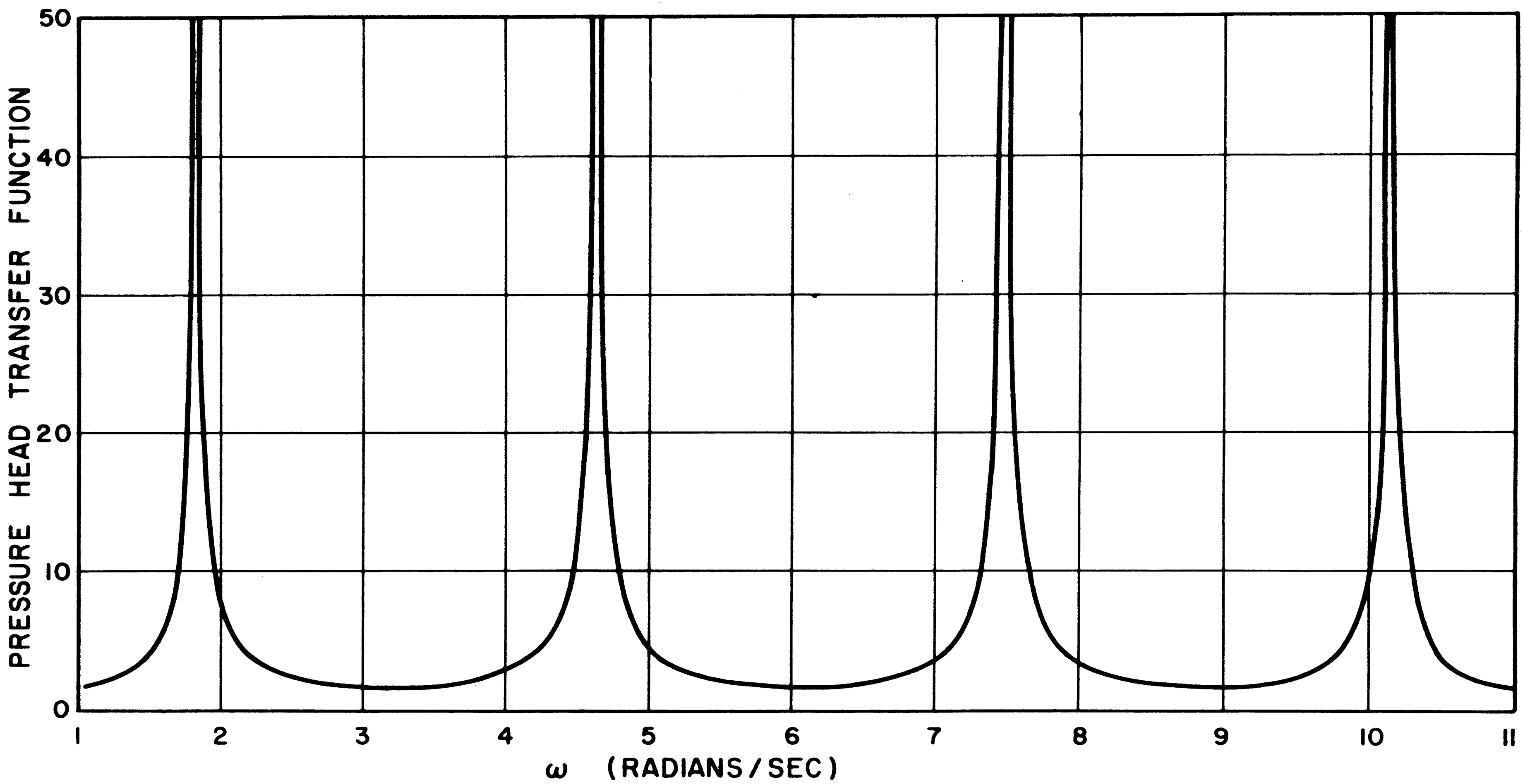


Figure 7. Transfer Function, Closed Series System Connected to a Reservoir

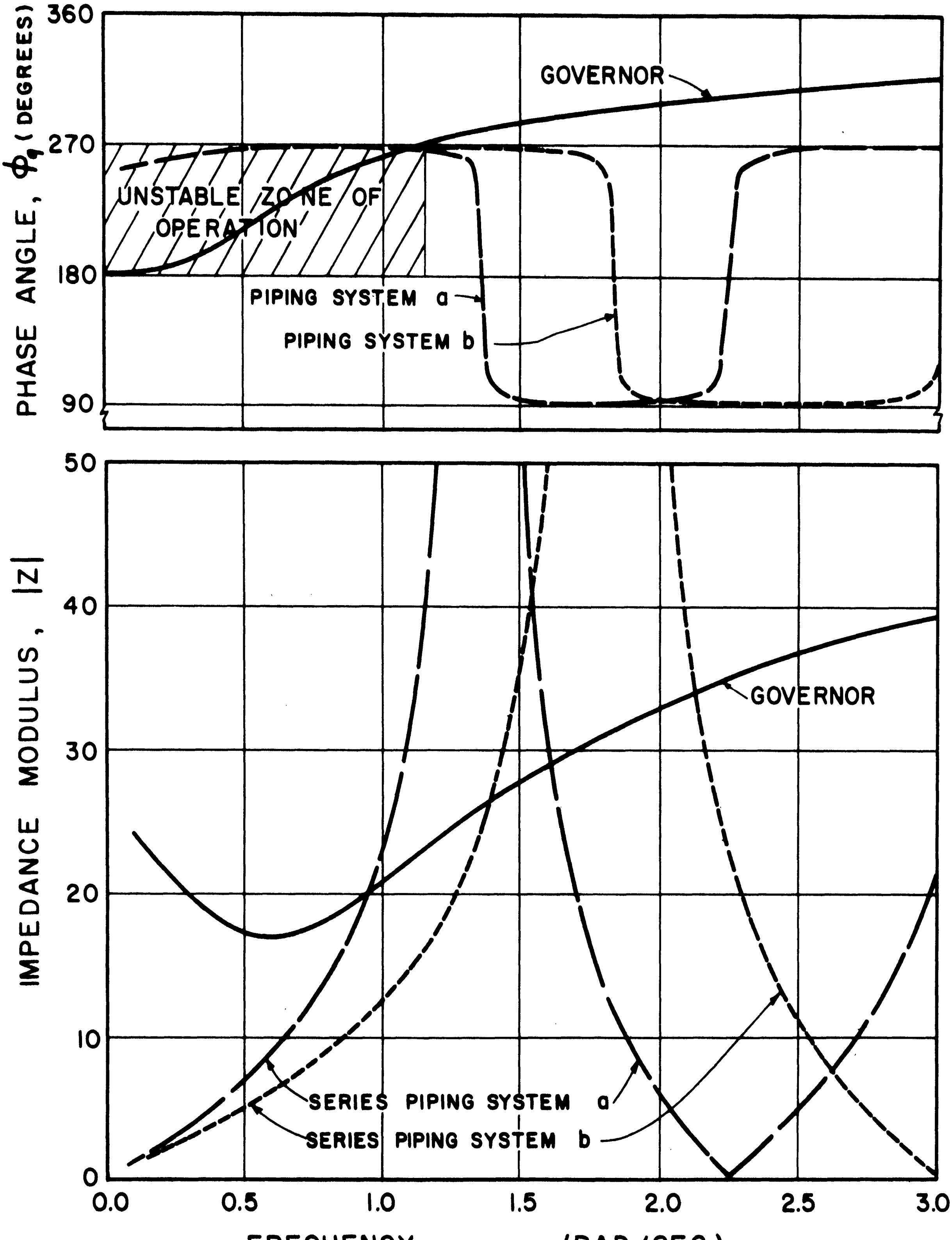


Figure 8. Impedance Diagram for Stability Analysis

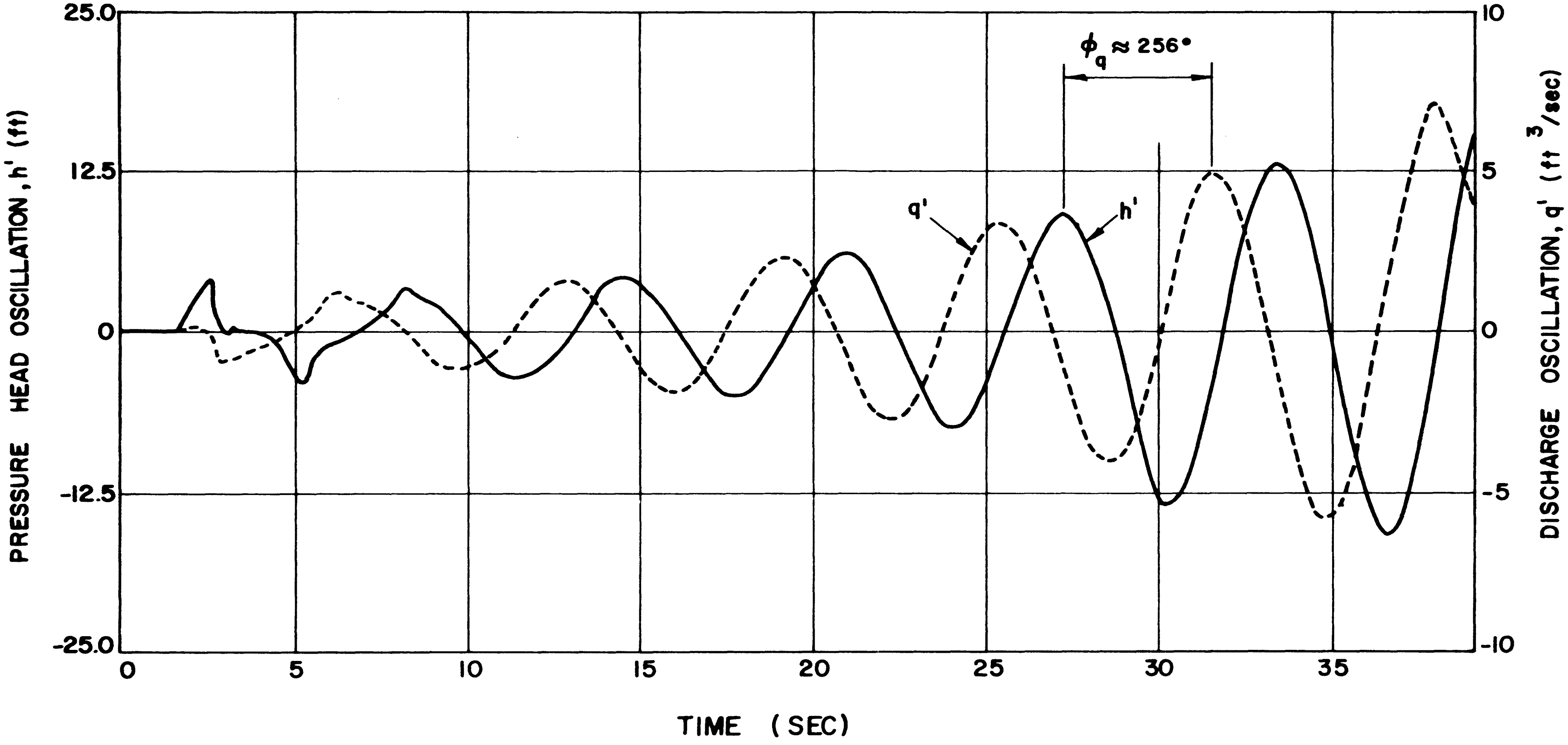


Figure 9. **Pressure Head** and Discharge Oscillations for Unstable Governor

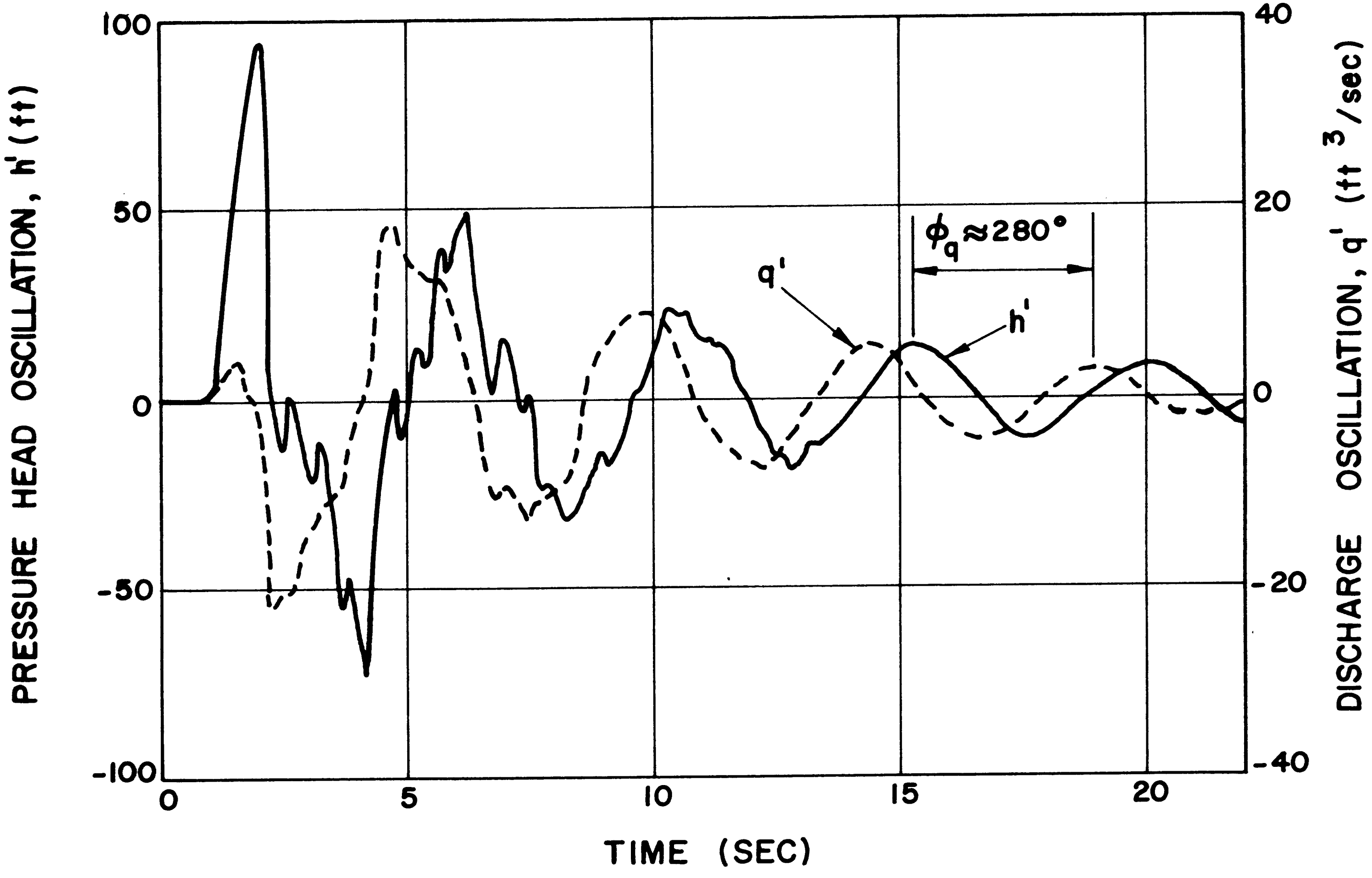


Figure 10. Pressure Head and Discharge Oscillations for Stable Governor