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VALVE STROKING FOR COMPLEX PIPING SYSTEMS

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by Victor L. Streeter,[1] F.A.S.C.E.

Synopsis

The theory for movement of valves in complex piping systems has been developed so that the transient ceases when the valve movement ceases. One pipe in the system is selected and the head-time relations worked out so that this flow is adjusted in a controlled manner not exceeding predetermined head limits. In a complex system, the flow changes are apportioned among the branches as a linear relation of initial to final steady-state flow in each pipe. The combination of these two procedures leads to values of head and flow at each control point in the system; hence the valves or other moving boundaries may be adjusted to cause the desired transient change to occur. Frictional effects are included in the analysis, which is based on the method of characteristics solution of the transient flow equations.

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Introduction

The analysis of transients in fluid piping systems has been carried out by arithmetic methods[2] since the turn of the century, by graphical methods since around 1930, and by digital computer methods starting around 1960. In general a change in boundary conditions is hypothesized, such as the closing of a valve in some manner, then the resulting transients are calculated. If they are too severe, other boundary changes are tried until a satisfactory solution has been found.

Valve stroking, on the other hand, is the design or synthesis approach wherein certain allowable pressure fluctuations are specified, and the changes in boundary conditions are calculated which cause the changes in flow to take place so that steady-state flow conditions are established upon cessation of boundary movements.

More specifically, in reducing the flow in a pipe from one steady-state value to another steady-state value (perhaps rest), the flow change is accomplished in three phases, the first and last phases of time duration one round-trip wave travel time, and the central phase of such duration that the change can be effected within the specified pressure limits. The head does not decrease below initial steady-state values and does not exceed the predetermined maximum.

In 1957 E. Ruus[3] studied the problem of optimum valve closure.

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[2]For a bibliography on waterhammer prior to 1954 the reader is referred to the references in "Waterhammer Analysis," by John Parmakian (Dover Book Company). pp. 143-145.

[3]"Bestimmung vor Schliessfunktionen, welche den kleinsten Wert des maximalen Druckstosses ergeben," by E. Ruus, thesis submitted to the Technical University of Karlsruhe in Germany in partial fulfillment of the requirements for the degree of Doctor of Engineering.

At about the same time[4] the author was studying a similar closure method. In 1963 the author[5] presented a study in which valve stroking was defined so that heads would not drop below steady-state during closure and so that the transient ceased when the valve movement ceased. This work applied to a single pipe with friction taken into account for opening or closing, and included a study of several reaches of pipe of constant diameter but with varying wall thickness (hence, varying wave speeds). The concept of adjusting the flow in a pipe from any unsteady situation to uniform flow with a constant inclined hydraulic gradeline in one round-trip wave travel time was presented for frictionless conditions.[6] This concept was then applied to valving during failure of power to a centrifugal pump to control the transients during reversal of the flow and runaway of the pump. In 1965 valve stroking was extended to series pipes for frictionless flow only.[7] These methods have now been superseded by more convenient and exact methods that take friction into account.

In 1966, E. Ruus[8] extended his studies to the closing of turbine gates.

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[4]Fluid Mechanics, by V. L. Streeter, McGraw-Hill Book Co., Inc., 2nd Edition, pp. 372-374, 1958.

[5]"Valve Stroking to Control Waterhammer," by V. L. Streeter, Journal of Hydraulics Division, ASCE, Vol. 89, No. HY2 Proc. Paper 3452, March, 1963, pp. 39-66.

[6]"Waterhammer Analysis of Pipelines", by V. L. Streeter, Journal of Hydraulics Division, ASCE, Vol. 90, No. HY4, Proc. Paper 3974, July, 1964, pp. 151-172.

[7]"Computer Solution of Surge Problems", V. L. Streeter, Symposium on Surges in Pipeline, Institution of Mechanical Engineers, London, Proc. 1965-66, Vol. 180, Part 3E.

[8]"Optimum Rate of Closure of Hydraulic Turbine Gates", by E. Ruus, presented at 1966 ASME-EIC, Fluids Engineering Conference, Denver, Colorado, April 25, 1966.

In his studies he holds the hydraulic gradient in its maximum adverse position until the fluid comes to rest; this causes a severe fluctuation to be set up in the penstock after gate closure.

The present status of valve stroking, the subject of this paper, includes series, branching, and parallel systems with detailed inclusion of friction. The application of these principles to the design of piping systems, whether industrial or hydro and water supply, should result in safer, more complete utilization of materials.

Examples of practical applications are:

(a) Closing a valve on the end of an oil pipeline from a storage reservoir to a barge. The valve is to close as rapidly as possible without exceeding allowable pipe stress and without column separation.

(b) Closing a valve on refueling of aircraft from underground supply pipes.

(c) Establishing flow in a penstock of a pumped-storage project when switching over from pumping to turbining.

(d) Control of a wholesale water supply system so that flow may be changed for certain clients without disturbing flow to other clients.

(e) Control of flow in reciprocating or intermittant cases such that steady-state flow is obtained immediately after the adjustment.

In this paper the theory is first developed for a single pipe with inclusion of friction, which forms the basis for extention of the method to complex systems. Series, branching and parallel systems are then considered followed by examination of several special boundary conditions. Experimental verification is then presented for several configurations of piping systems.

Valve Stroking Theory for Single Pipe with Friction

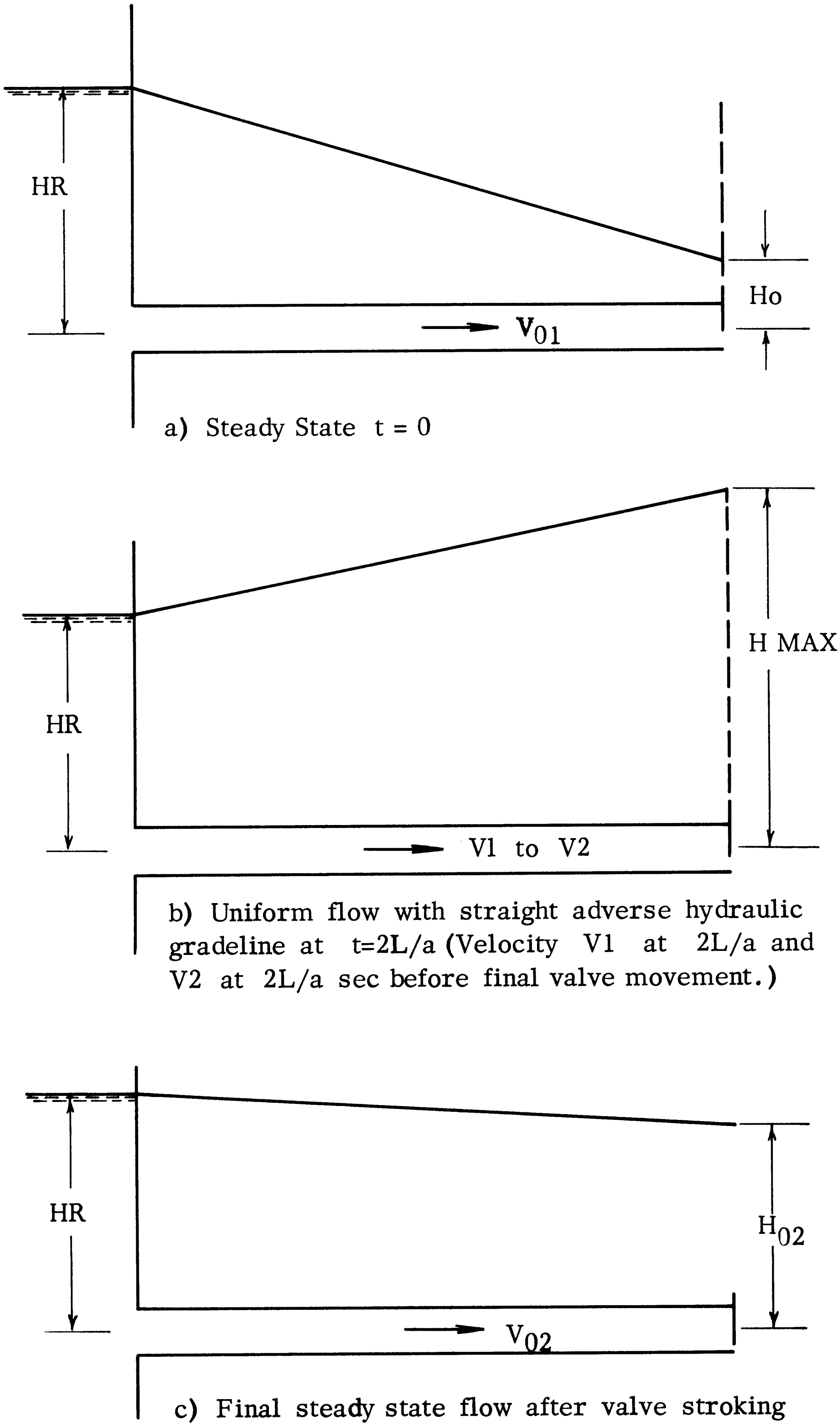
In this section a sequence of events that might take place in a single pipe are first hypothesized, based on results of earlier studies of valve stroking. It is then proved that conditions at the boundaries can be found that cause this sequence of events to take place, thereby proving existance of the hypothesized solution. The detailed discussion is based on a situation where the velocity is being reduced from V_{o1} to V_{o2} as shown in Fig. 1, by closure of a valve at the downstream end of the pipe. The maximum permitted head is HMAX. The relationships for increasing the velocity are very similar and in fact, are incorporated into the same computer program. Dimensionless presentations are used for convenience. Heads are divided by H_o the initial loss through the valve, velocities are divided by V_{o1} and time by 2L/a, the round-trip wave travel time for the pipe of length L and wave speed a, thus,

$$x' = \frac{x}{L} \qquad t' = \frac{t}{2L/a} \qquad v = \frac{V}{V_{o1}} \qquad h = \frac{H}{H_o} \qquad B = \frac{a\, V_{o1}}{g\, H_o} \tag{1}$$

B is the dimensionless parameter relating changes of head Δh to changes in velocity Δv, ie., $\Delta h = B \Delta v$, when no reflections are present.

The concept of the solution is presented on dimensionless, independent variable plots with x' as abscissa, measured positive in the flow direction, and with t' as ordinate.

First Phase. For the first phase of valve closure, from t' = 0 to t' = 1 (t = 0 to t = 2L/a), it is assumed that the velocity is reduced to a uniform value V1 at t' = 1. In Fig. 2, the head and velocity h, v are required for x' = 1 over this time period so that the velocity V1 (yet



a) Steady State t = 0

b) Uniform flow with straight adverse hydraulic gradeline at t=2L/a (Velocity V1 at 2L/a and V2 at 2L/a sec before final valve movement.)

c) Final steady state flow after valve stroking

Figure 1. Initial, central phase, and final hydraulic gradeline for valve closure.

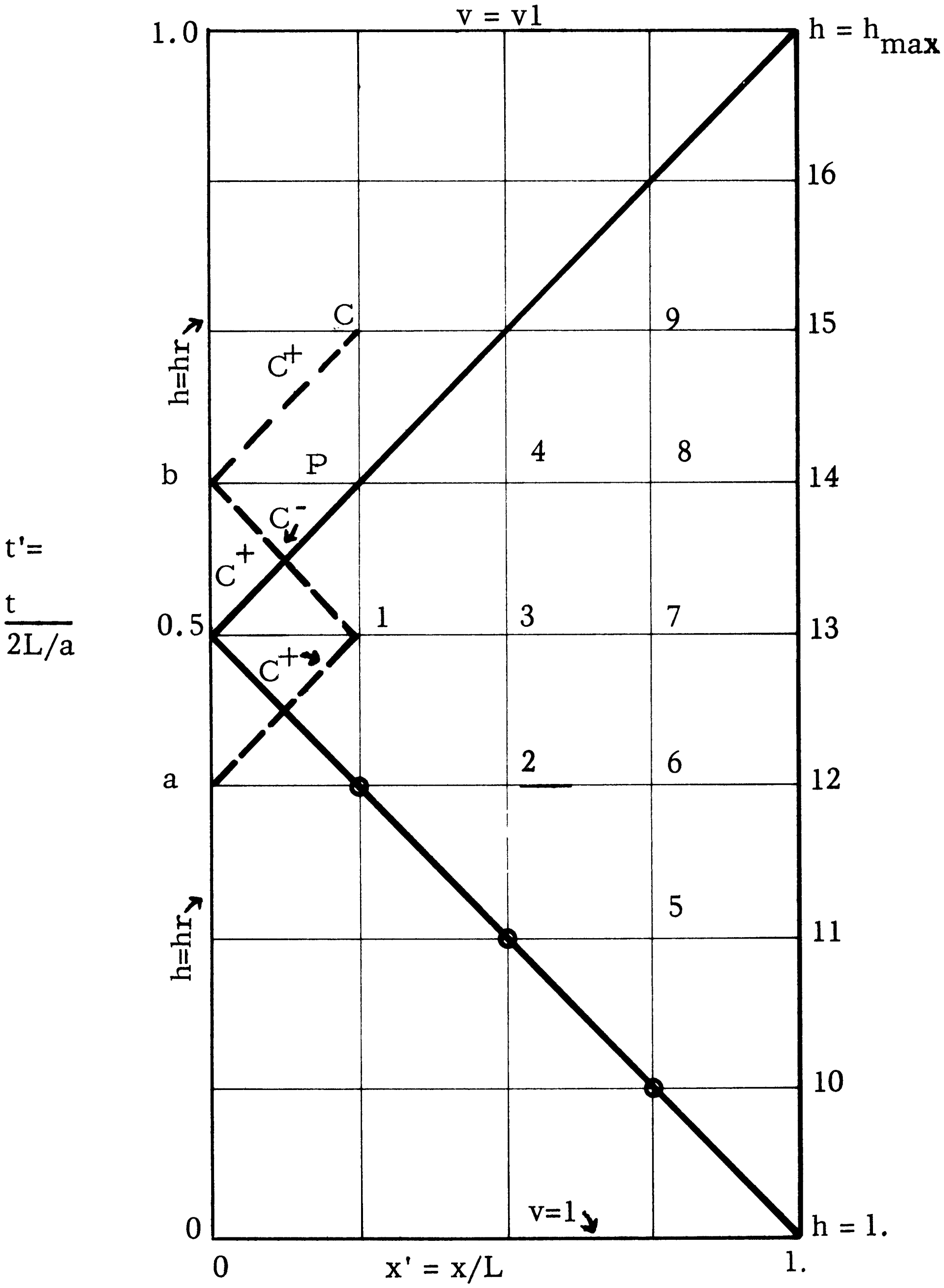


Figure 2. First phase of valve stroking as presented on a dimensionless x'-t' plot.

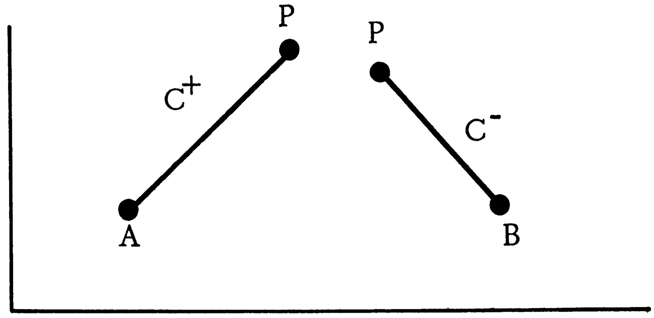


Figure 3. C^+ and C^- characteristics notation

undetermined) occurs over the whole pipe at $t' = 1$ and so that the hydraulic gradeline is straight. With h and v known the valve position $\tau = v/\sqrt{h}$ may be calculated as a function of time. Minor losses and entrance velocity-head changes are neglected in this presentation. At the upstream end of the pipe ($x' = 0$) $h = HR/H_o = h_r$ remains constant for all time. The pipe is divided into P reaches of equal length with P any convenient integer number. In Fig. 2, P has been selected as 4. Below and to the left of the lower diagonal line from $x' = 1$, $t' = 0$ to $x' = 0$, $t' = 0.5$ steady, known conditions prevail. To the left and above the diagonal line from $x' = 0$, $t' = 0.5$ to $x' = 1$, $t' = 1$ the velocity is assumed to be uniform (same throughout this portion of the pipe at any instant) and the hydraulic gradeline constant, with dimensionless change over a reach of $(h_{max} - h_r)/P$ in which $h_{max} = HMAX/H_o$. It should be noted that once the velocity is made uniform, and the hydraulic gradeline is constant, the velocity must remain uniform although it is undergoing a deceleration. It is now necessary to show that values of h, v can be found at $x' = 1$, $t' = 0$ to $t' = 1$ to produce these hypothesized conditions.

Only two equations are needed to develop the detailed information. They are the C^+ and C^- characteristic equations obtained from the method of characteristics.

Equation (17) of reference 3 when solved for h_P yields

$$C^+: \qquad h_P = h_A - B(v_P - v_A) - 2\, h_{fo}\, v_A |v_A| \Delta t' \qquad (2)$$

$\Delta t'$ is the dimensionless time for the wave to travel from A to P, Fig. 3, and h_{fo} is the dimensionless steady-state ($v_A = 1$) head loss over the whole pipe. Since $\Delta t' = 1/2P$, by letting $h_f = h_{fo}/P$, the C^+ characteristic equation for one reach is

C^+: $h_P = h_A - B(v_P - v_A) - h_f\, v_A|v_A|$ (3)

If the equation is to be applied over more than one reach, then h_f must be multiplied by this number of reaches. Similarly the C^- characteristic equation is

C^-: $h_P = h_B + B(v_P - v_B) + h_f\, v_B|v_B|$ (4)

If exponential friction is to be used the friction term becomes $h_f\, v_A|v_A|^{n-1}$ for Eq. (3), with n the exponent of the velocity. For tabulated friction, as obtained by steady-state measurements, the friction term may be indicated as $h_f\,(v_A)$ and is evaluated by parabolic interpolation from the tabulated values for equal velocity increments. In general, when considering valve stroking, the absolute value signs are not needed as the velocity is not permitted to reverse.

The next step in developing the procedure for the first phase is to find the value of v for each horizontal line of Fig. 2 between t' = 0.5 and t' = 1., to the left of the diagonal. By writing Eq. (3) from x' = 0, t' = 0.5 to point P, remembering that $h_p - h_A = (h_m - h_r)/P$

$$v_P = 1. - \frac{1}{B}\left(\frac{h_m - h_r}{P} + h_f\right) \qquad (5)$$

as $v_a = 1.$

Now, since $v_b = v_P$ for uniform flow, the same equation is applied from b to c of Fig. 2 for the same hydraulic gradeline,

$$v_c = v_b - \frac{1}{B}\left(\frac{h_m - h_r}{P} + h_f\, v_b^2\right) \qquad (6)$$

This process is repeated for each horizonal line in turn until t' = 1., which then yields v_1. Now h and v are known for all points on and to the left of the two diagonal lines.

The value of h and v at point 1 may now be found by writing the C^+ equation [Eq. (3)] from a to 1 and the C^- equation [Eq. (4)] from 1 to b

$$C^+: \quad h_1 - h_r + B(v_1 - 1) + h_f = 0 \tag{7}$$

$$C^-: \quad h_r - h_1 - B\,(v_b - v_1) - h_f\, {v_1}^2 = 0 \tag{8}$$

The quadratic in v_1 obtained by adding the equations may be solved to yield v_1, then Eq. (7) will give h_1. If tabulated or exponential friction is used, an iteration procedure is utilized to find v.

In exactly this same manner v and h at the other numbered points are found which yields the necessary information at the valve so that it may be stroked to yield the hypothesized flow and pressure relations for phase one.

Second Phase. After the straight adverse hydraulic gradeline has been established, along with the uniform flow $v = v_1$ throughout the pipe at $t' = 1$, the hydraulic gradeline is maintained constant and flow continues to reduce uniformly due to both the adverse gradient and friction. Equation (6) is used repeatedly along the short C^+ diagonals, Fig. 4 until the calculated velocity v is less than the final desired velocity v_{o2}. The time for the complete valve stroking sequence can now be determined as follows:

The dimensionless time step is $\Delta t' = 1/2P$ with P the number of reaches of pipe. The time t' may then be expressed as

$$t' = J \Delta t' \tag{9}$$

with J an integer; hence at the beginning of the second phase $J = 2P$. When the first value of v is calculated that is less than v_{o2} [i.e. v(J)], an

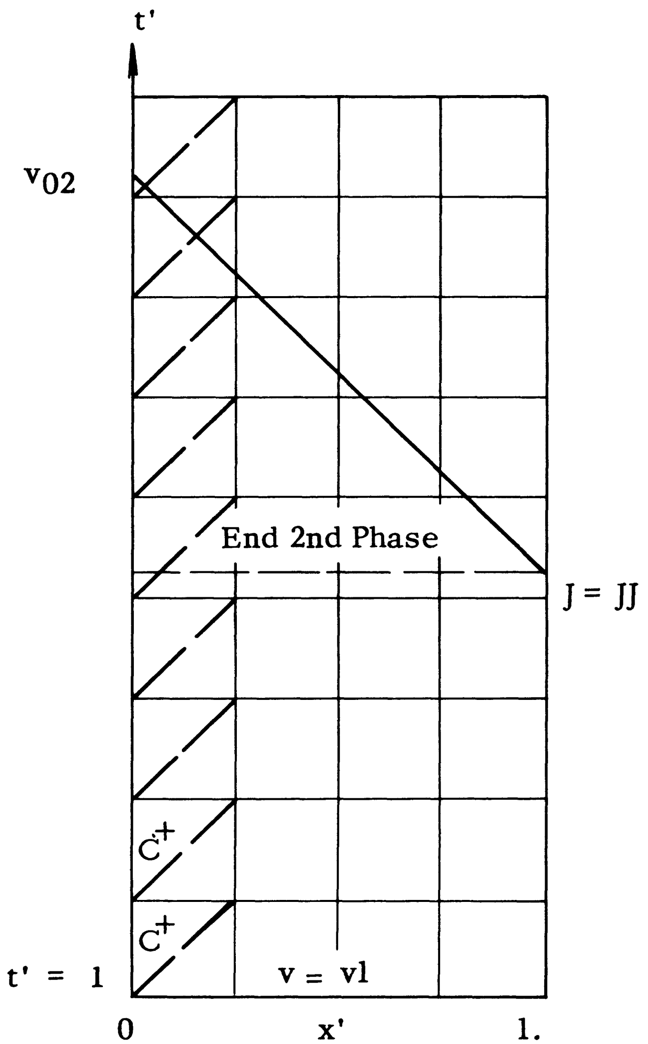


Figure 4. Second phase of valve stroking.

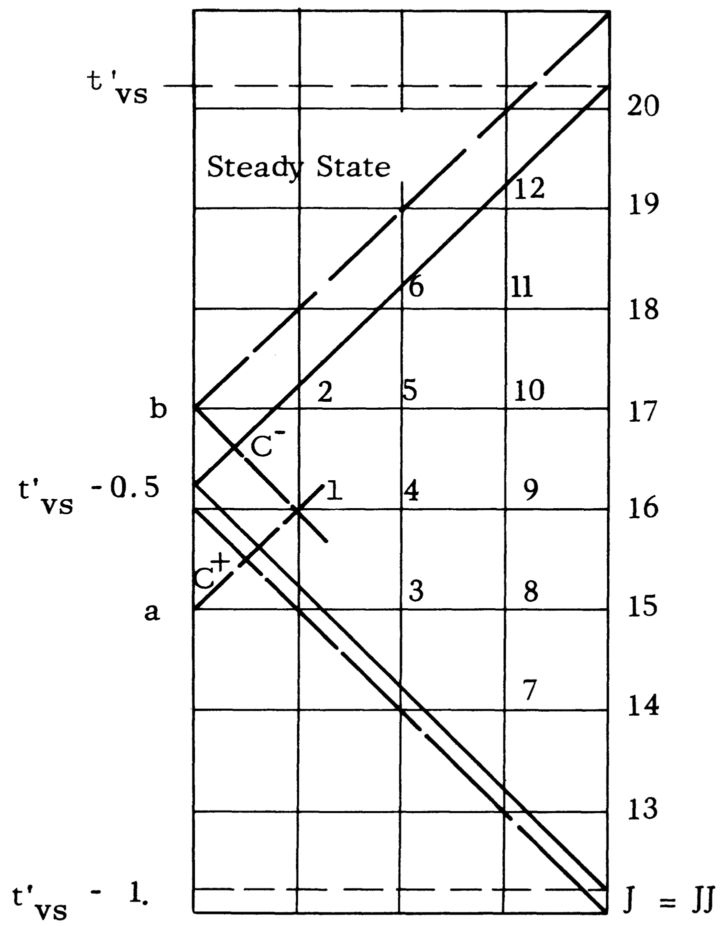


Figure 5. Third phase of valve stroking.

interpolation is made for the time t' of valve stroking.

$$t'_{vs} = 0.5 + \left[\frac{v_{o2} - v\ (J-1)}{v\ (J) - v(J-1)} + J - 1\right] \Delta t' \qquad (10)$$

The actual time in seconds is t'_{vs} (2L/a). It is shown in the discussion of the third phase that final steady-state conditions can be established L/a seconds after the velocity in the pipe at the reservoir end reaches its final desired valve.

The second phase ends one round-trip wave travel time before t'_{vs}. Let JJ be the integer number of time steps to the last calculated values in the second phase, Fig. 4,

$$JJ = \frac{t'_{vs} - 1}{\Delta t'} \qquad (10a)$$

The right hand side of Eq.(10a) is truncated to its next lower integer value.

Third Phase. During the final phase, of duration one round-trip (calculated from J = JJ to J = JJ + 2P + 1), the final desired conditions are hypothesized, and then it is shown that values of h and v at the valve can be found that permits this solution. Conditions have been calculated and are known below the diagonal dashed line, Fig. 5, from x' = 1, J = JJ and cannot be affected by subsequent valve movements (after JJ x Δt'). Final steady-state is hypothesized for conditions above the other diagonal dashed line; hence, the velocity is everywhere v_{o2} there and the pressure is known at all points because the final steady-state hydraulic gradeline is known. The problem now is to calculate v and h at points 13 to 20 for these conditions. By starting at point 1 (Fig. 5) and by writing the C^+ and C^- characteristic equations (3) and (4) from a to 1 and from 1 to b

$$C^+: \quad h_1 - h_a + B(v_1 - v_a) + h_f\, v_a^2 = 0 \tag{11}$$

$$C^-: \quad h_b - h_1 - B(v_b - v_1) - h_f\, v_1^2 = 0 \tag{12}$$

By adding the equations a quadratic in v_1 is obtained; by substitution in Eq. (11) h_1 is then calculated.

By taking the remaining numbered points 2, 3, 4, - - 20 in order v and h are calculated for each point in turn. This proves that a solution can be found for the downstream boundary so that the final hypothesized steady-state conditions are realized. Now by calculating $\tau = v/\sqrt{h}$ for each time increment at $x' = 1$, the valve stroking is completely prescribed.

If it is desired to increase the flow in a pipe, a minimum allowable head is selected for the downstream end of the pipe that must be less than the final steady-state head. The procedures for finding the solution are then similar to the case for reducing the flow. In one round-trip wave travel time the flow is made uniform with the hydraulic gradeline straight and from the reservoir to the minimum selected head at the valve. Figures 3 to 5 apply except h_{max} becomes h_{min}. The two methods are so similar that the same computer program may be utilized. It is listed in the appendix. Figure 6 shows the solution obtained for reducing the flow in a pipe; Fig. 7 is a confirmation of the method by the method of characteristics solution.

SOLUTION BY ALGEBRAIC METHODS

J	TIME	VA(J)	VB(J)	HA(J)	TAU
0	.0000	1.0000	1.0000	1.0000	1.0000
1	.0625	.9401	1.0000	1.1448	.8786
2	.1250	.8870	1.0000	1.2729	.7862
3	.1875	.8409	1.0000	1.3979	.7112
4	.2500	.7989	1.0000	1.5108	.6499
5	.3125	.7619	1.0000	1.6226	.5981
6	.3750	.7286	1.0000	1.7228	.5551
7	.4375	.6996	1.0000	1.8221	.5183
8	.5000	.6728	1.0000	1.9119	.4866
9	.5625	.6492	.9202	2.0020	.4589
10	.6250	.6271	.8483	2.0843	.4343
11	.6875	.6073	.7629	2.1677	.4125
12	.7500	.5882	.7233	2.2448	.3926
13	.8125	.5711	.6686	2.3235	.3746
14	.8750	.5544	.6177	2.3966	.3581
15	.9375	.5393	.5698	2.4714	.3431
16	1.0000	.5246	.5246	2.5413	.3291
17	1.0625	.4816	.4816	2.5413	.3021
18	1.1250	.4405	.4405	2.5413	.2764
19	1.1875	.4012	.4012	2.5413	.2517
20	1.2500	.3634	.3634	2.5413	.2280
21	1.3125	.3322	.3271	2.5290	.2069
22	1.3750	.3127	.2920	2.4913	.1981
23	1.4375	.2939	.2581	2.4545	.1876
24	1.5000	.2757	.2252	2.4180	.1773
25	1.5625	.2580	.1934	2.3823	.1671
26	1.6250	.2408	.1624	2.3470	.1572
27	1.6875	.2241	.1322	2.3123	.1474
28	1.7500	.2079	.1029	2.2780	.1378
29	1.8125	.1921	.0833	2.2443	.1283
30	1.8750	.1768	.0833	2.2108	.1189
31	1.9375	.1618	.0833	2.1779	.1096
32	2.0000	.1472	.0833	2.1452	.1005
33	2.0625	.1330	.0833	2.1131	.0915
34	2.1250	.1191	.0833	2.0811	.0825
35	2.1875	.1055	.0833	2.0497	.0737
36	2.2500	.0922	.0833	2.0184	.0649
37	2.3125	.0833	.0833	1.9975	.0590

Figure 6. Computer Results, in Dimensionless Form, for Reducing the Velocity from 1.2 ft/sec to 0.1 ft/sec in 4,000 ft of 0.9 i.d. Copper Tubing. H_{Res} = 80 ft, H_o = 39.35 ft, and H_{max} = 100. P = 8. Tabulated Steady-State Head Losses Were Used. The Valve is Located at A and the Reservoir at B . Wave Speed 2,550 ft/sec. The dimensionless Time of Closure is 2.2927.

TIME	TAU	X/L=	0.	.25	.5	.75	1.
.0000	1.0000	H=	2.0330	1.7748	1.5165	1.2583	1.0000
		V=	1.0000	1.0000	1.0000	1.0000	1.0000
.1250	.7562	H=	2.0330	1.7748	1.5165	1.2583	1.2729
		V=	1.0000	1.0000	1.0000	1.0000	.8870
.2500	.6499	H=	2.0330	1.7748	1.5165	1.5055	1.5108
		V=	1.0000	1.0000	1.0000	.8976	.7989
.3750	.5551	H=	2.0330	1.7748	1.7404	1.7234	1.7228
		V=	1.0000	1.0000	.9073	.8168	.7286
.5000	.4866	H=	2.0330	1.9773	1.9399	1.9197	1.9119
		V=	1.0000	.9161	.8333	.7516	.6728
.6250	.4343	H=	2.0330	2.1601	2.1212	2.0972	2.0843
		V=	.8483	.8483	.7729	.6989	.6271
.7500	.3926	H=	2.0330	2.1601	2.2872	2.2608	2.2448
		V=	.7233	.7233	.7233	.6551	.5882
.8750	.3581	H=	2.0330	2.1601	2.2872	2.4142	2.3966
		V=	.6177	.6177	.6177	.6177	.5544
1.0000	.3291	H=	2.0330	2.1601	2.2872	2.4142	2.5413
		V=	.5246	.5246	.5246	.5246	.5246
1.1250	.2764	H=	2.0330	2.1601	2.2872	2.4142	2.5413
		V=	.4405	.4405	.4405	.4405	.4405
1.2500	.2280	H=	2.0330	2.1601	2.2872	2.4142	2.5413
		V=	.3634	.3634	.3634	.3634	.3634
1.3750	.1981	H=	2.0330	2.1601	2.2872	2.4142	2.4913
		V=	.2920	.2920	.2920	.2920	.3127
1.5000	.1773	H=	2.0330	2.1601	2.2872	2.3658	2.4180
		V=	.2252	.2252	.2252	.2453	.2757
1.6250	.1572	H=	2.0330	2.1601	2.2401	2.2946	2.3470
		V=	.1624	.1624	.1818	.2113	.2408
1.7500	.1378	H=	2.0330	2.1143	2.1708	2.2254	2.2780
		V=	.1029	.1218	.1506	.1793	.2079
1.8750	.1189	H=	2.0330	2.0466	2.1032	2.1580	2.2108
		V=	.0833	.0929	.1209	.1489	.1768
2.0000	.1005	H=	2.0330	2.0241	2.0372	2.0921	2.1452
		V=	.0833	.0833	.0926	.1200	.1472
2.1250	.0325	H=	2.0330	2.0241	2.0152	2.0278	2.0811
		V=	.0833	.0833	.0833	.0924	.1191
2.2500	.0549	H=	2.0330	2.0241	2.0152	2.0064	2.0184
		V=	.0833	.0833	.0833	.0833	.0922
2.3750	.0590	H=	2.0330	2.0241	2.0152	2.0064	1.9975
		V=	.0833	.0833	.0833	.0833	.0833

Figure 7. Configuration of Valve Stroking Method by the Characteristics Method Solution with HA(J) of Figure 6 being used as Boundary Condition at the Downstream End of the Pipe.

Extentions of Valve Stroking to Complex Systems

In this section configurations of piping systems are considered in which the flow originates from a reservoir and terminates with valves, dead ends, or reservoirs at the downstream ends of the system. Figure 8 represents such a system. There are two controlled outlets, at J1 and at the terminus of pipe 4 and a reservoir at the terminus of pipe 3 where the head is held constant. By adjusting the two valves in the proper manner the transients may be controlled in the system and will disappear when valve motion ceases. With initial and final flows through the valves given, and by using v^2 friction throughout, the initial and final steady flows through each pipe may be accurately calculated by solution of a quadratic equation. By considering first a case in which the total flow through the two valves is to be reduced, a maximum head at J1 is selected (it must be somewhat above the final steady-state head at J1) which determines the duration of the valve stroking.

The valve stroking procedures on a single pipe are now applied to pipe 1 for its change in flow for the specified maximum head at J1, which yields the head at J1 for all time of the transient as well as the flow into J1 from pipe 1 for the complete transient.

A basic assumption is now made regarding the distribution of the flow change coming into J1. Let QF represent final steady conditions and QS represent initial steady conditions, with Q the value of the transient flow. The change of flow in pipe 2 is taken to be proportional to the change in flow in pipe 1. Hence for flow out of J1 into pipe 2

$$\frac{QS(2) - Q(2)}{QS(1) - Q(1)} = \frac{QS(2) - QF(2)}{QS(1) - QF(1)} \qquad (13)$$

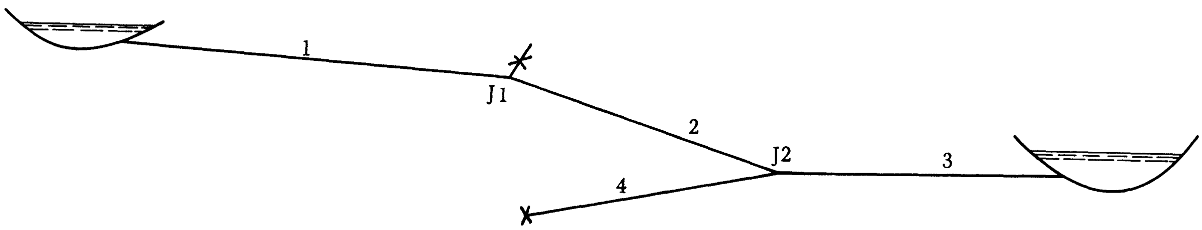


Figure 8. Branching and series pipe system with flow originating from a reservoir.

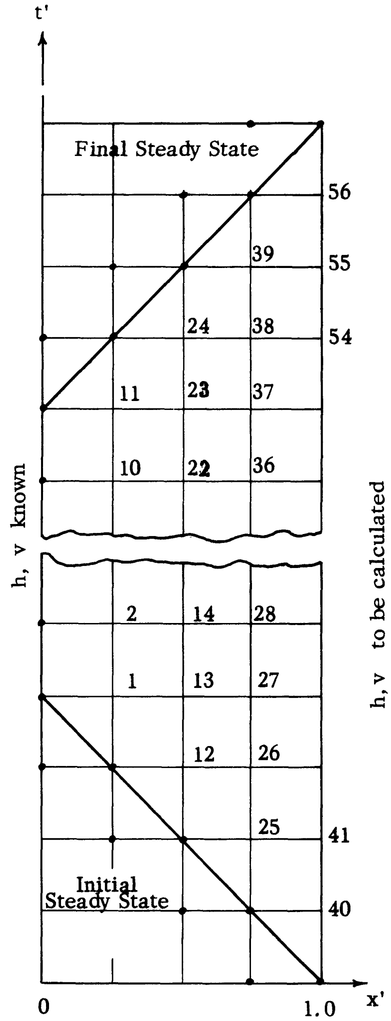


Figure 9. Method of progressive solution to find h, v at x'=1 when h, v at x'=0 is known.

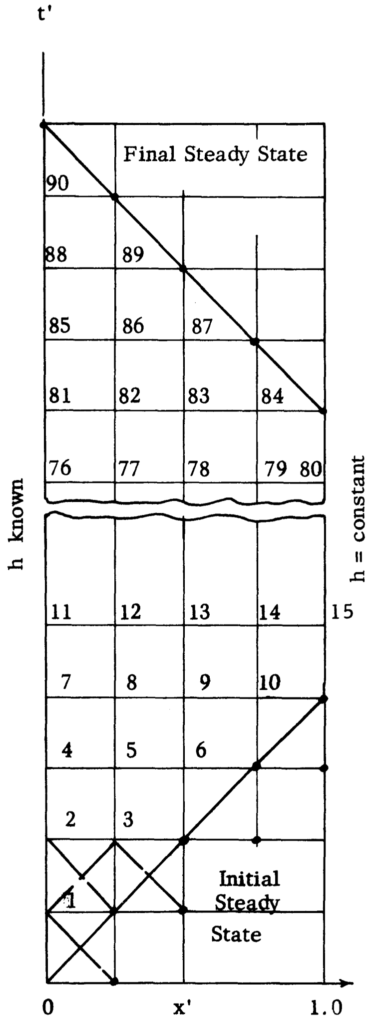


Figure 10. Procedure for calculating flow through a pipe when head is known at each end.

Since all quantities are known except Q(2) it is completely determined for the duration of the transient. With Q(2) known

$$Q_{J1} = Q(1) - Q(2) \qquad (14)$$

Q_{J1} is the flow out of the valve at J1.

With the flow and head known at the upstream end of a pipe for all time, the flow and head at the downstream end of the pipe may be calculated. Figure 9 illustrates the procedure. With values of h and v known at all points enclosed by a circle, by use of the C^+ and C^- equations applied at points 1, 2, ..., in order, h and v are evaluated at the downstream end of the pipe for the complete transient. Therefore, the head and flow into J2 are known.

With the head known at J2 for all time, and the head constant at the reservoir downstream from pipe 3, the flow into and out of pipe 3 may be calculated for the transient by the procedure shown in Fig. 10. By selecting the points in order 1, 2, 3, - -, enough information is always available to calculate the unknowns at the point. At points 1 and 2, v is the only unknown in the C^- characteristic equation. At point 3 the C^+ and C^- equations are used to find v_3 and h_3. By continuing this procedure v is found for each end of the pipe. Hence by continuity the flow leaving J2 by pipe 4 is found. With h and v known for the upstream end of pipe 4, h and v may be calculated for the downstream end of the pipe by the method outlined in Fig. 9.

Now to find valve movements

$$\tau_{j1} = \frac{v_{j1}}{h_{j1}} \qquad \tau_4 = \frac{v_4}{h_4} \qquad (15)$$

v_{j1} is the dimensionless velocity through the valve and v_4 is the dimensionless velocity through the valve on pipe 4 with h_4 the head at the downstream end of pipe 4. These τ equations assume free outflow from the valves.

The motion of the valves with time have now been fully specified to cause the transient to occur. If the duration of transient should be decreased a larger value of H_{max} at J1 may be taken. There are certain limitations to valve stroking. A maximum head at J1 cannot be assumed that would cause the transient in pipe 1 to occur in less than 4L/a. For certain dimensions of systems other limitations in rapidity of stroking may show up, such as the case where pipes 1, 2, and 4 are short and pipe 3 is long.

In all pipes considered it is assumed that the same time increment Δt is utilized for all calculations, i.e.,

$$\Delta t = \frac{L_1}{a_1 P_1} = \frac{L_2}{a_2 P_2} \quad \text{etc.} \tag{16}$$

In order to make P_1, P_2 - - P_n suitable integers this may require a slight adjustment of the wave speeds or the lengths of the pipes. Wave speeds are probably not known with great precision in most piping systems.

For changes in the system of Fig. 8 causing an increase in flow through pipe 1 a minimum head at J1 is selected that is below the final steady-state head and the same procedures are applied to find the required valve movements.

A parallel piping system has been studied to find the valve movement needed to bring the flow to rest in a system consisting of a reservoir, a pipe, 2 pipes in parallel followed by a pipe containing a valve. Due to the complexity of the equations, characteristic equations are written over whole lengths rather than reaches, but the time increment is still based on the pipe leaving the reservoir, i.e. 1/2P. After the upstream pipe transients are specified, five equations are needed to solve for velocity in each of the parallel pipes at each end and for the head at the downstream junction. An interesting feature of the parallel system is that a flow circulation is set up in the loop when flow is stopped, due to the unequal momentum originally in the system.

When one member of a branching system has a dead end, in general it is not possible to cause the flow in this line to terminate its fluctuations at the instant the valve motion ceases.

Special Boundary Conditions

Situations arise where it would be desirable to have valve stroking when there is no reservoir at one end of a system. Two examples are considered, first a pipe with a centrifugal pump upstream and a valve downstream, and second, a pipe with a moveable diaphragm upstream and an orifice downstream. The object is to first hypothesize a set of conditions such that the transient is under the control of the designer and ends when valve or boundary motion ceases; then to demonstrate that the desired solution exists.

Centrifugal Pump. To avoid cumbersome solutions with systems of many nonlinear equations, the characteristic equations are written for the whole pipe which in effect lumps the friction as the value at the earlier time L/a.

With reference to Fig. 11, a value of h_m at $t' = 1$, $x' = 1$ is selected; by writing the C^+ characteristic from $t' = 0.5$, $x' = 0$ to $t' = 1$, $x' = 1$, the velocity v_1 is found. The hypothesis is made that flow is uniform at $t' = 1$ and that the hydraulic gradeline is straight. The points on $t' = 1$ are located by this assumption. The pump h-v relation permits h at $x' = 0$, $t' = 1$ to be found. Now by writing the appropriate C^+ or C^- characteristic equations for 1/4, 1/2, or 3/4 of the whole pipe, the values of h and v_o for

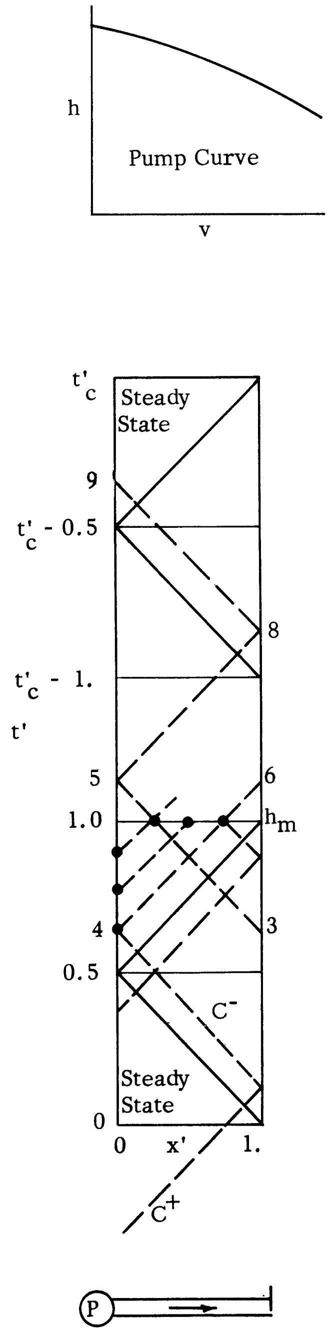


Figure 11. Procedure for Valve Stroking for Centrifugal Pump.

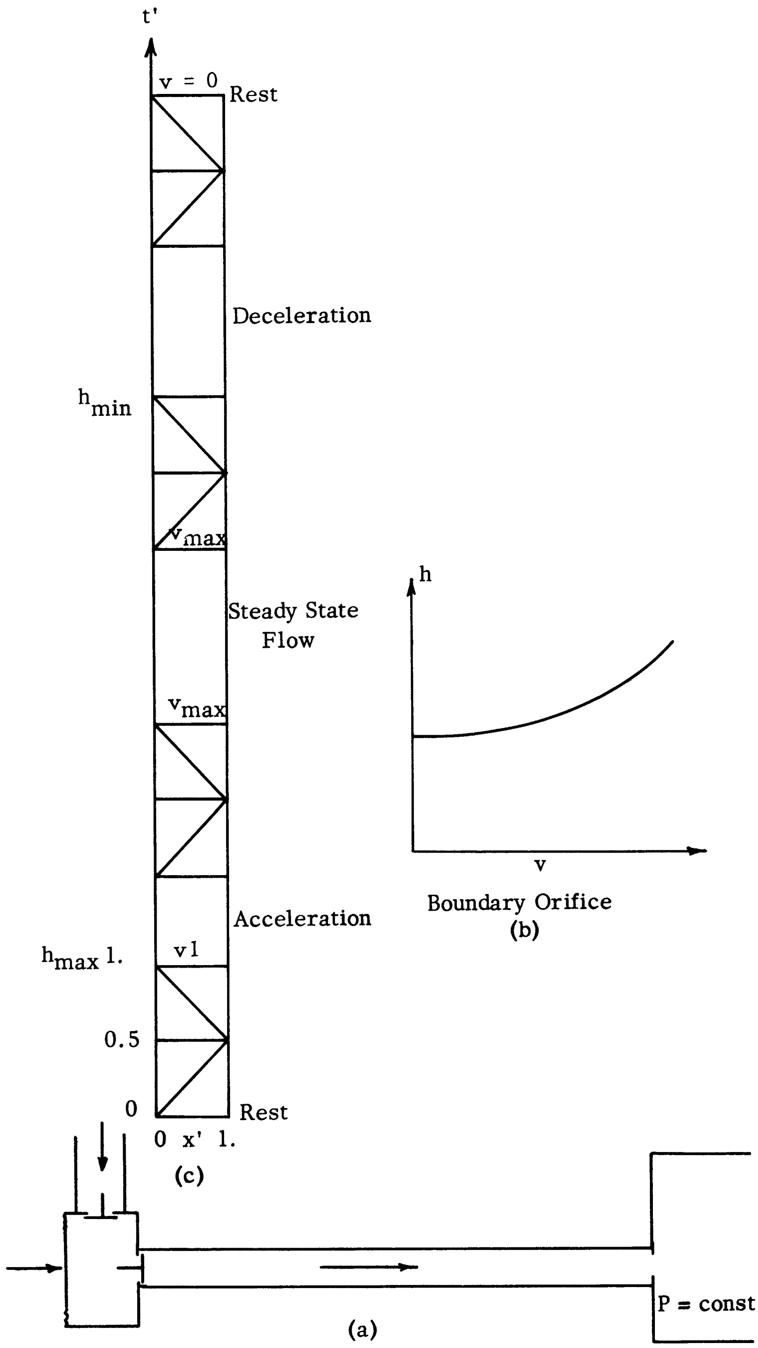


Figure 12. Diaphragm pump with Schematic Solution for Valve Stroking.

each of the circled points of the chart, Fig. 11, at $x' = 0$ between $t' = 0.5$ and $t' = 1$ are determined.

With these points and the initial steady-state values, points at $x' = 1$ for $t' = 0$ to $t' = 1$ are found from the C^+ and C^- characteristics for the pipe or portions of the pipe, as indicated by the dashed lines.

For the central phase, to find v and h at points 5 and 6, the assumption is made that $v_5 = v_6$. By writing the C^- equation from 3 - 5 and by use of the pump curve h_5 and v_5 are found; then by the C^+ equation 4 - 6 h_6 may be found as $v_6 = v_5$ is known. This procedure continues until the desired velocity is reached at $x' = 0$, which is labeled $t_c' - 0.5$. Along $x' = 0$ from $t_c' - 0.5$ to t_c' final steady-state conditions are known and by use of the C^+ and C^- characteristics, such as 5 - 8 and 8 - 9, h_8 and v_8 are found. This completely determines h and v at the valve for all time of the transient. The procedure applies for both increasing and decreasing the flow depending on the selection of h_m and calculation of final steady-state.

Diaphragm Pump. This case is of interest when a volitile liquid is to be pumped. Figure 12a shows one set-up where liquid is being pumped through the pipe into the receiver through an orifice at the downstream end of the pipe. A diaphragm is to be driven by a cam in such a manner that the transients in the pipe are under the control of the designer. The head discharge relation for the orifice is shown in Fig. 12b. Figure 12c shows the complete sequence of events for the liquid to start from rest and be brought up to steady-state discharge, then be brought back to rest at the end of the stroke. All procedures involved in the solution of Fig. 12c have already been discussed, since this problem is very similar to the one in Fig. 11.

The velocity at $x' = 0$ is determined for all time of the transient; hence, knowing the ratio of effective diaphragm area to pipe area, the motion of the diaphragm on its forward stroke is completely specified. The complete course of the transient is under control. The preselected h_{min} is the minimum pressure at any time during the transient and the selected h_{max} is not exceeded.

Experimental Verification of Valve Stroking

Although many of the various valve stroking cases have been verified by computer simulation using the method of characteristics solution for the appropriate boundary conditions, it was deemed desirable to actually demonstrate the methods in piping systems. These experiments were performed on various configurations by valve stroking using low-pressure, pneumatic-servo systems to stroke the valve.

The procedures were as follows:

1. Calculate the τ-time curve for the desired transient using tabulated steady-state friction for the system.
2. From a calibration of the valve, which produces a relation between stem position and τ, the stem position versus time is calculated.
3. A template is prepared relating valve stem position to time; it is then mounted in a turntable.
4. A motion transmitter converts the template height to an air pressure signal which is transmitted to a valve positioner.
5. The valve positioner places the valve (micro-flute) in the position called for by the template.

The pressure at the valve is measured by a "Dynisco" transducer and

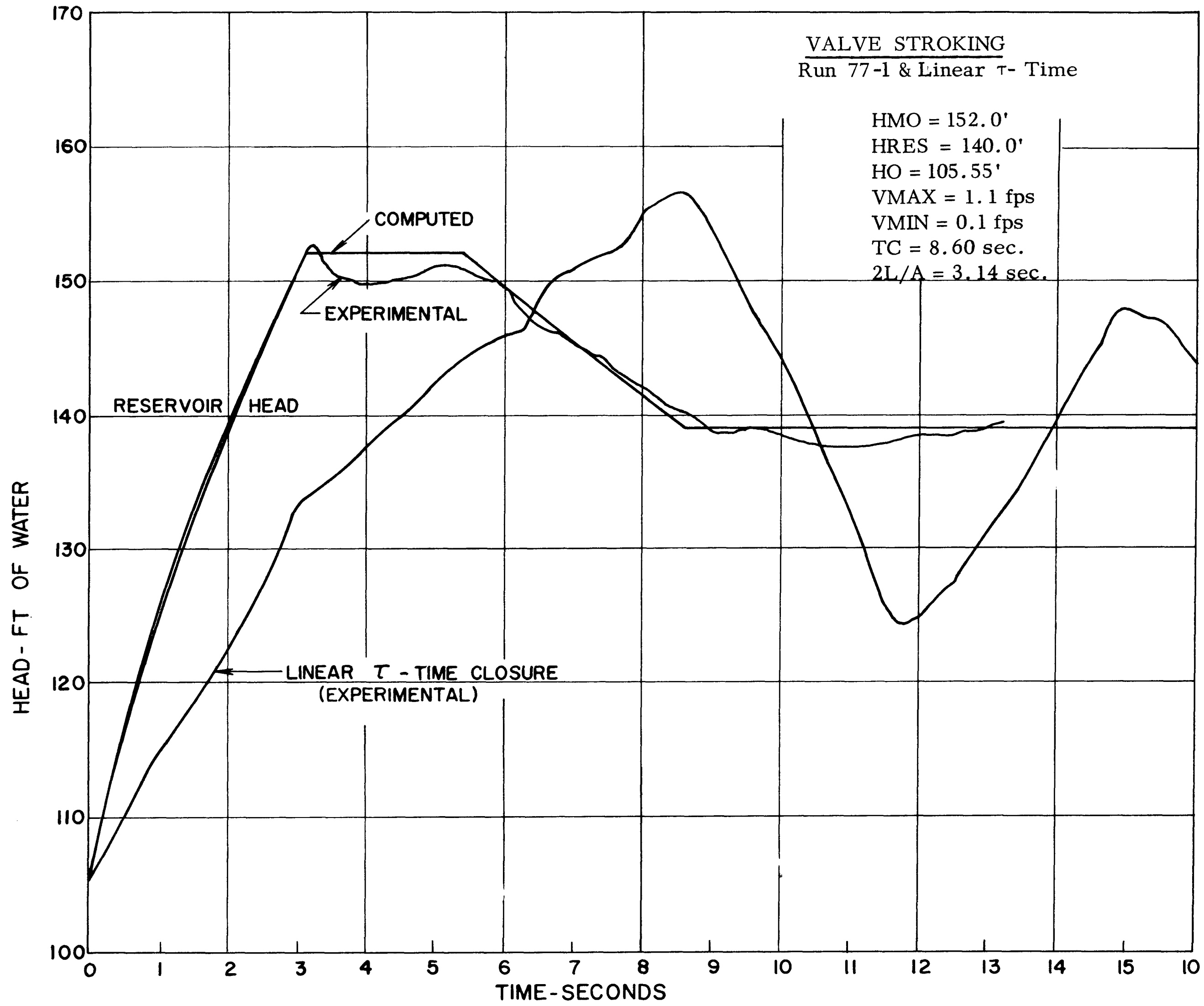


Figure 13. Comparison of Experimental Valve Stroking with Theory, for 4,000 feet of 0.95 inch i.d. copper Tubing. Also Experimental Head-Time Curve for Same Steady State Conditions with τ Reduced Linearly to Final Steady State in the Same Time Period.

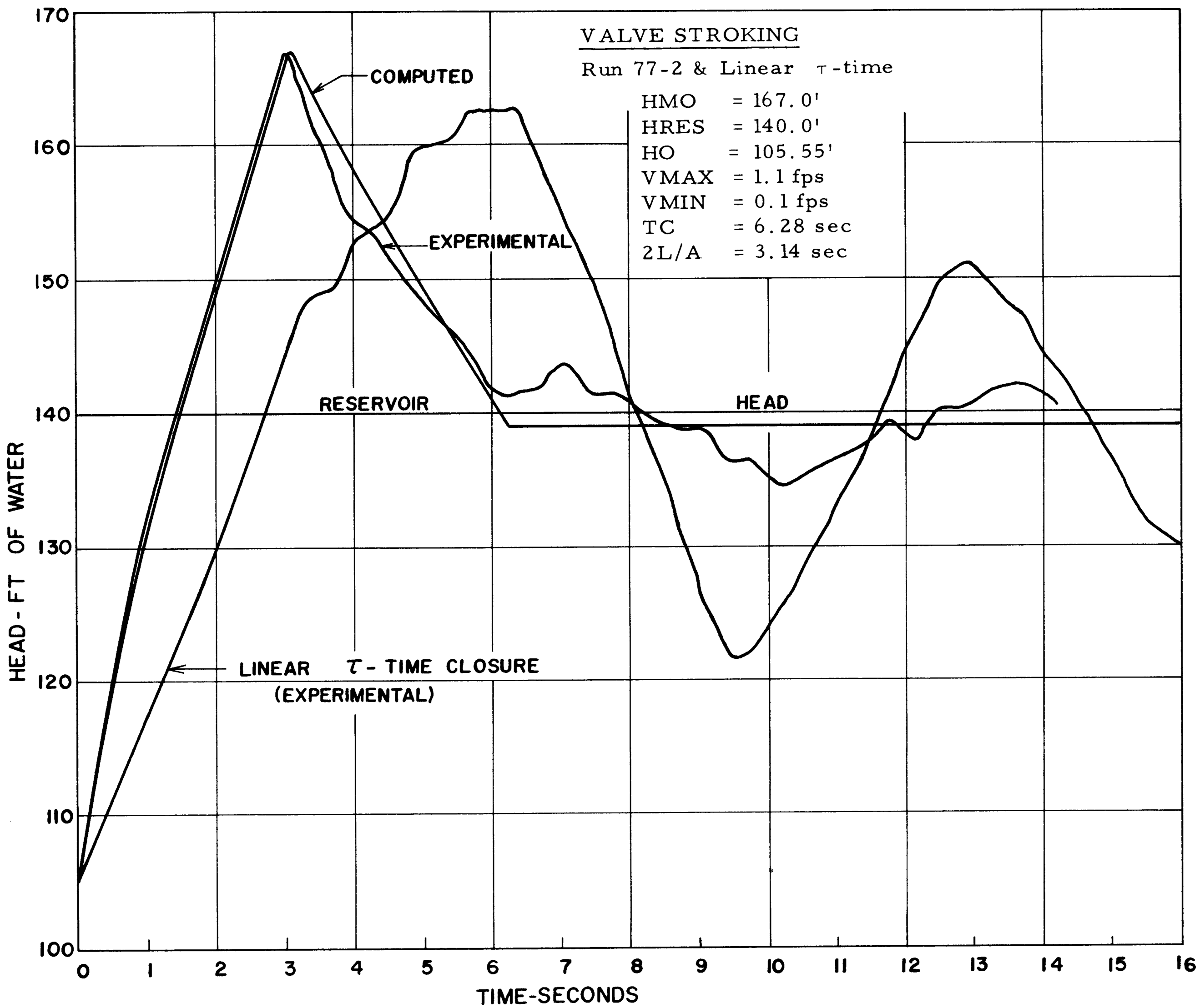


Figure 14. Comparison of Experimental Valve Stroking With Theory for Case of Minimum Valve Stroking Time ($t_c = 4L/a$) . Linear Closure Experimental Results for Same Time Are Also Shown, which Indicate the Transient Condition Remaining After Valve Movement.

recorded on a paper strip recorder along with a trace showing the stem position of the valve.

Figure 13 - 14 shows results of the tests plus cases of linear valve closure.

Summary and Conclusions

The theory for control of transients in liquid piping systems has been developed for various configurations and boundary conditions, starting with a single pipe with reservoir upstream and valve downstream, then proceeding to complex series and branching systems, parallel systems and then to special boundary conditions such as centrifugal pumps, diaphragm pumps and orifices. Friction has been included in each case by use of waterhammer equations. Experimental verification was given for two cases involving long lines. including establishment of flow, series and branching systems.

Design methods are now available for control of a very wide range of transient flow problems in piping systems by proper valve movements or by suitable change of end boundary conditions.

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Graduate students, Joel Caves, Werner Zielke, James Cassan, and Michael Stoner aided in the experimental studies under the direction of E. B. Wylie, Associate Prof. of Civil Engineering.

APPENDIX

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          VALVE STROKING FOR SINGLE PIPE WITH RESERVOIR UPSTREAM.TABULA
          TED FRICTION.FRICTION CALCULATED AT R AND S FOR P SECTIONS AS
          IN CHARACTERISTICS PROGRAM.CHARACTERISTICS CHECK.
          D'LESS CLOSURE FROM 1. TO VMIN,OR OPENING FROM VMIN TO 1.
             IJ=0 TO CLOSE,AND IJ=1 TO OPEN.
           INTEGER I,J,PP,P,N,U,NN,JJJ,PJ,K,M,II,PPP,IJ
           DIMENSION (H,HP,V,VP,HH)(20),(HA,VA,VB)(600),(W,Y,WW,YY)((0...20)*(
         2 0...20)),TAB(600),(F,FP)(20)
HERE       READ AND PRINT DATA
           PRINT RESULTS NN,DV,A,K,    G,HRES,VMIN,P,HMO ,JJJ,PJ,HH...HH(NN)
         2 ,VMAX,IJ
           JJJ=0
           INTERNAL FUNCTION HF.(VV)
           V7=VV*VMAX
           M=V7/DV
           TH=(V7-M*DV)/DV
           WHENEVER M.E.0
           M=1
           TH=TH-1.
           END OF CONDITIONAL
           HF1=HH(M)+.5*TH*(HH(M+1)-HH(M-1)+TH*(HH(M+1)+HH(M-1)-2.*HH(M)
         2 ))
           WHENEVER JJJ.E.2
           FUNCTION RETURN HF1/HOP
           OTHERWISE
           JJJ=2
           FUNCTION RETURN HF1
           END OF CONDITIONAL
           END OF FUNCTION
           FH=HF.(1.)
           HO=HRES-FH
           HOP=HO*P
           HFO=FH/HO
           DT=1./(2.*P)
           HR=HRES/HO
           HM=HMO /HO
           B=A*VMAX/(G*HO)
           VMN=VMIN/VMAX
           PRINT RESULTS FH,HO,HFO,    DT,HR,B,VMN,HM
           FHM=HF.(VMN)
           WHENEVER IJ.E.0
           VST=1.
           HOO=1.
           VSP=VMN
           HV2=HF.(VSP)
           OTHERWISE
           VST=VMN
           HOO=HR-P*FHM
           VSP=1.
           HV2=HFO/P
           END OF CONDITIONAL
           THROUGH AB1,FORI=0,1,I.G.P
           THROUGH AB1,FOR J=0,1,J.G.P-I
```

```
          W(I,J)=VSI
AB1       Y(I,J)=((P-I)*HR+I*H00)/P
          HRM=(HR-HM)/P
          THROUGH AB2,FOR J=P+1,2,J.G.2*P
          W(0,J)=W(0,J-1)+(HRM-HF.(W(0,J-1)))/B
AB2       W(0,J+1)=W(0,J)+(HRM-HF.(W(0,J)))/B
          THROUGH AB3,FOR J=P,1,J.G.2*P
          THROUGH AB3,FOR I=0,1,I.G.J-P
          W(I,J)=W(0,J)
AB3       Y(I,J)=HR-HRM*I
          THROUGH AB4,FOR I=1,1,I.G.P
          THROUGH AB4,FOR J=P+1-I,1,J.G.P+I-1
          HFF=HF.(W(I-1,J-1))
          W(I,J)=.5*(W(I-1,J+1)+W(I-1,J-1)+(Y(I-1,J-1)-Y(I-1,J+1)))/B
          THROUGH AB5,FOR II=0,1,II.G.5
AB5       W(I,J)=.5*(W(I-1,J+1)+W(I-1,J-1)+(Y(I-1,J-1)-Y(I-1,J+1)-HFF+
        2  HF.(W(I,J)))/B)
AB4       Y(I,J)=Y(I-1,J-1)-B*(W(I,J)-W(I-1,J-1))-HFF
          THROUGH AB6,FOR J=0,1,J.G.2*P
          VB(J)=W(0,J)
          VA(J)=W(P,J)
AB6       HA(J)=Y(P,J)
          J=2*P+1
AA7       VB(J)=VB(J-1)+(HRM-HF.(VB(J-1)))/B
          WHENEVER IJ.E.0.AND.VB(J).L.VSP,TRANSFER TO AA9
          WHENEVER IJ.E.1.AND.VB(J).G.VSP,TRANSFER TO AA9
          HA(J)=HM
          VA(J)=VB(J)
          J=J+1
          TRANSFER TO AA7
AA9       PP=J-P-1
          TC=1.+(PP+(VB(PP+P)-VSP)/(VB(PP+P)-VB(J)))/(2.*P)
          PRINT RESULTS TC,PP,VB(J),VB(PP+P)
          THROUGH AC1,FOR I=0,1,I.G.P
          THROUGH AC1,FOR J=0,1,J.G.P-I
          YY(I,J)=HR-HRM*I
AC1       WW(I,J)=VB(PP+J)
          THROUGH AC2,FOR J=P+1,1,J.G.2*P+1
          THROUGH AC2,FOR I=0,1,I.G.J-P-1
          YY(I,J)=HR-HV2*I
AC2       WW(I,J)=VSP
          THROUGH AC4,FOR I=1,1,I.G.P
          THROUGH AC4,FOR J=P+1-I,1,J.G.P+I
          HFF=HF.(WW(I-1,J-1))
          WW(I,J)=WW(I-1,J-1)
          THROUGH AC3,FOR II=0,1,II.G.5
AC3       WW(I,J)=.5*(WW(I-1,J+1)+WW(I-1,J-1)+(YY(I-1,J-1)-YY(I-1,J+1)+
        2 HF.(WW(I,J))-HFF)/B)
AC4       YY(I,J)=YY(I-1,J-1)-B*(WW(I,J)-WW(I-1,J-1))-HFF
          PPP=PP+2*P+1
          THROUGH AC5,FOR J=PP+1,1,J.G.PPP
          VA(J)=WW(P,J-PP)
          HA(J)=YY(P,J-PP)
AC5       VB(J)=WW(0,J-PP)
          HA(PPP+1)=HA(PPP)
          I=J-1
          PRINT COMMENT$1    SOLUTION BY ALGEBRAIC METHODS$
          PRINT COMMENT$0              J                   TIME
        2 VA(J)              VB(J)               HA(J)              T
        3 AU$
```

```
            PRINT COMMENT$0$
            THROUGH AA11,FOR J=0,1,J.G.I
            TAU(J)=VA(J)/SQRT.(HA(J))
AA11        PRINT FORMAT$1H ,I12,5F20.4*$,J,J*DT,VA(J),VB(J),HA(J),TAU(J)
            WHENEVER PJ.E.0,TRANSFER TO HERE
            N=P
            J=TC/DT+2
            H=HR
            HP=HR
            FFG=HF.(VST)
            THROUGH AA51,FOR I=0,1,I.G.N
            F(I)=FFG
            V(I)=VST
AA51        H(I)=HR-I*FFG
            T=0.
            TAU=V(N)/SQRT.(H(N))
            U=0
            PRINT COMMENT$1   TIME     TAU     X/L=  0.        .25       .5
          2          .75        1.$
AA61        PRINT FORMAT$1H0,F7.4,F8.4,3H H=, 5F10.4/S16,3H V=, 5F10.4*
          2 $,T,TAU(U),H,H(2),H(4),H(6),H(8),V,V(2),V(4),V(6),V(8)
AA71        T=T+DT
            U=U+1
            WHENEVER U.G.J,TRANSFER TO HERE
           INTERIOR POINTS
            THROUGH AA81,FOR I=1,1,I.E.N
            VP(I)=.5*(V(I-1)+V(I+1)+(H(I-1)-H(I+1)-F(I-1)-F(I+1))/B)
            HP(I)=H(I-1)-B*(VP(I)-V(I-1))-F(I-1)
AA81        FP(I)=HF.(VP(I))
           UPSTREAM BOUNDARY CONDITION
            VP=V(1)+(HR-H(1)-F(1))/B
            FP=HF.(VP)
           DOWNSTREAM BOUNDARY CONDITION
            HP(N)=HA(U)
            VP(N)=V(N-1)+(H(N-1)-HP(N)-F(N-1))/B
            TAU(U)=VP(N)/SQRT.(HP(N))
            FP(N)=HF.(VP(N))
            THROUGH AA91,FOR I=0,1,I.G.N
            F(I)=FP(I)
            V(I)=VP(I)
AA91        H(I)=HP(I)
            WHENEVER U/K*K.E.U,TRANSFER TO AA61
            TRANSFER TO AA71
            END OF PROGRAM
```