THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

VALVE STROKING TO CONTROL WATERHAMMER

Victor L. Streeter

April, 1962

IP-560

TABLE OF CONTENTS

LIST OF FIGURES

VALVE STROKING TO CONTROL WATERHAMMER

Victor L. Streeter,[1] M. ASME

The usual approach to waterhammer is the analysis of a given situation, such as the closure of a valve in a system, or perhaps the failure of a pump. Friction is usually neglected, or at most lumped at one or more discreet points. Either numerical analysis(1) or graphical analysis(1,2) has been used, with some work(3,4,5) dealing with Bessels functions and linearized friction terms. This paper takes a completely different approach to the problem, considering the rate of valve motion and the pipe system as a unit, to be designed for arbitrary maximum pressure without backflow and without separation of fluid column.

At present valves are not on the market in a form so that they can be closed easily according to a special prescription. Development work is required, but would be well worth the effort owing to economies resulting from pipeline design, or in improving control system operation.

Laws, or relationships, are developed for the closing or opening of valves to control the flow throughout the pipe system, usually with uniform pipe flow during significant phases of valve operation. Friction is included, proportional to velocity squared, or to any selected power. The valve motion may be worked out easily

[1] Professor of Hydraulics, Civil Engineering Dept., The University of Michigan.

with slide rule or desk calculator, but the analysis of the systems have been carried out by high-speed digital computer (IBM 709) to prove out the effectiveness of the methods.

A simple first-order-accuracy method of solution of water-hammer has been worked out - first, based on the method of Streeter and Lai[6], and is used for most of the computer work in the study. The problem of closing a valve is first considered, with and without the effects of fluid friction. The next type of solution is the opening of a valve to control water hammer, and finally the closure of a valve in a system of twenty sections of pipe of different wall thicknesses is undertaken, including effects of friction.

WATER HAMMER EQUATIONS

In this section, starting with the equations of motion and the continuity equation applied to the fluid in a short pipe segment, the equations for water hammer with nonlinear friction are developed. From Figure 1, the equation of motion is applied to a fluid element, yielding

$$-\gamma AH_X dX - \tau_o \pi D dX = \frac{\gamma H}{g} dX \frac{dV}{dT}$$

in which γ is the specific weight of fluid, A the pipe cross section, H is head, τ_o the wall fluid shear stress, D the diameter of pipe, T the time, the distance measured downstream, and V the average velocity. g is the acceleration of gravity and variable subscripts indicate partial differentiation. Since $\tau_o = \rho f V^2/8$ with f the Darcy-Weisback friction factor, and with ρ the density, and $H_f = fLV^2/2gD$, with H_f the friction head loss in length L, the equation of motion reduces to

$$H_X + \frac{H_f}{L} + \frac{1}{g}(V_T + VV_X) = 0$$

the term VV_X may be shown to be small compared with V_T for pipes, and is neglected, leaving

$$H_X + \frac{H_f}{L} + \frac{V_T}{g} = 0 \qquad (1)$$

The continuity equation for the differential length dX is an expression that the net flow into the region per unit time must just equal the rate of compression of fluid volume plus the rate of expansion

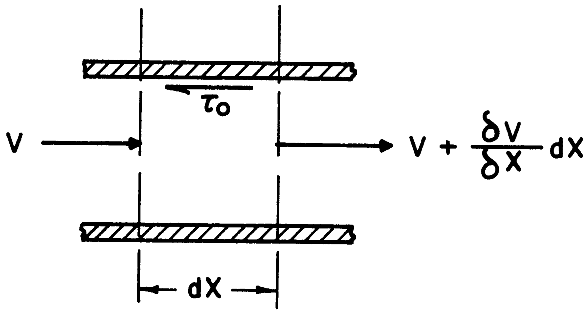

Figure 1. Element of Pipe.

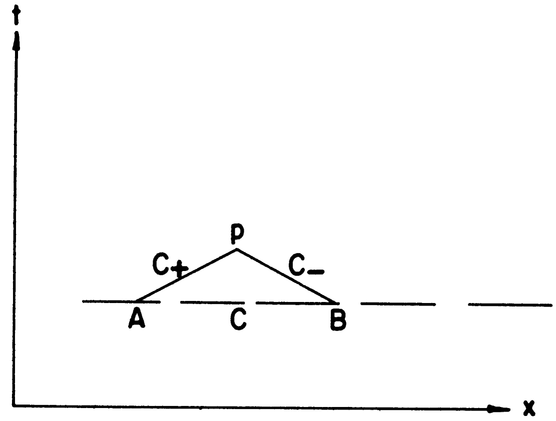

Figure 2. Characteristic Curve Elements.

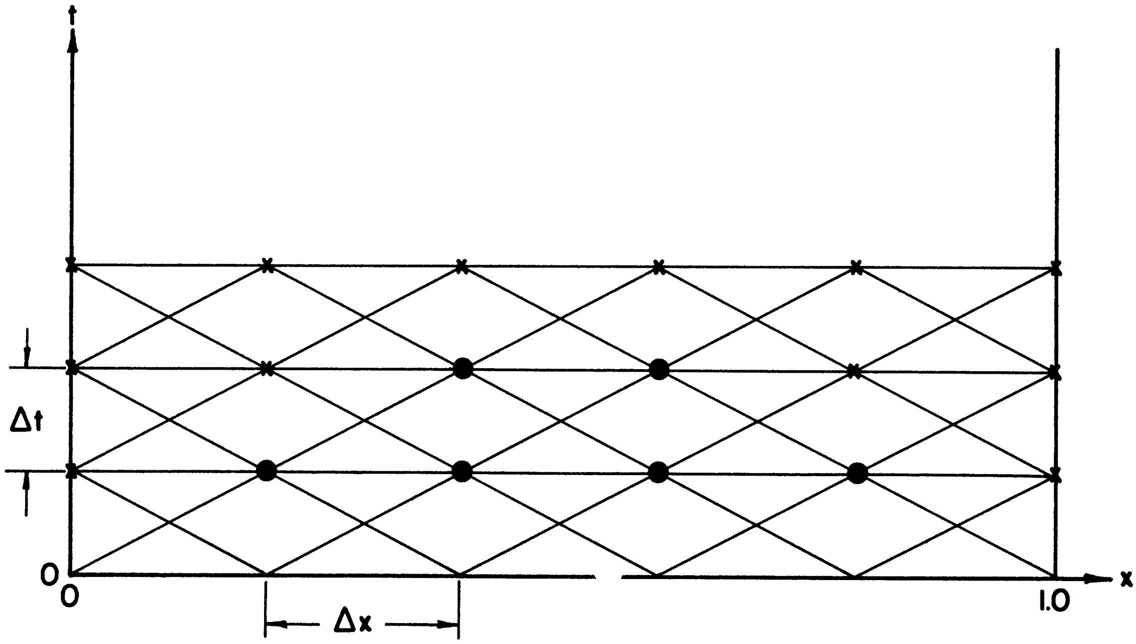

Figure 3. Specified Time and Distance Increments.

o Points Computed from Initial Conditions.

x Points Computed from Conditions at x = 0 or x = 1.

of volume within the elemental length. It becomes, Figure 1,

$$- AV_X dX = \frac{\gamma dH}{dT}\frac{A}{K} dX + \frac{\gamma dH}{dT}\frac{D}{2t'E}\frac{D}{2}\pi D dX$$

in which K is the bulk modulus of elasticity of the fluid, E the Young's modulus for the pipe wall material, and t' is the wall thickness. The first term on the right is the liquid compression rate and the final term is the expansion rate of pipe volume, assuming constant length of pipe segment dX. By making the substitution

$$a = \frac{\sqrt{K|\rho}}{\sqrt{1 + \frac{KD}{Et'}}}$$

which is the speed of sound through the fluid in the pipe, the continuity equation reduces to

$$V_X + \frac{g}{a^2}(VH_X + H_T) = 0$$

VH_X is small compared with H_T [in ratio $\sim (V/a)$], and may be neglected, yielding

$$V_X + \frac{g}{a^2} H_T = 0 \qquad (2)$$

Equations (1) and (2) are next made dimensionless by the following substitutions:

$$t = \frac{T}{2L/a}, \quad v = \frac{V}{V_o}, \quad h = \frac{H}{H_o}, \quad x = \frac{X}{L}$$

in which V_o is the steady-state velocity, L the pipe length and H_o is usually the steady state head at the control valve. The friction term H_f is first expressed in terms of H_{fo} the steady state friction

$H_f = H_{fo}\ v^2$. Dividing by H_o

$$hfo = \frac{H_{fo}}{H_o}$$

and

$$H_f = hfo\ H_o\ v|v|$$

in which the absolute valve sign is used to maintain the proper sign on the friction term for flow in either direction. If an exponential formula for friction loss is used, with n the exponent on the velocity term,

$$H_{fo} = hfo\ H_o\ v|v|^{n-1}$$

After making these substitutions in Equations (1) and (2)

$$h_x + hfo\ v|v| + \frac{aV_o}{gH_o}\ v_t = 0$$

and

$$h_t + 2\frac{aV_o}{gH_o}\ v_x = 0$$

The dimensionless ratio aV_o/gH_o is an important parameter in waterhammer (frequently called 2ρ) and is symbolized by B, hence

$$L_1 = h_x + \frac{B}{2}\ v_t + hfo\ v|v| = 0 \tag{3}$$

and

$$L_2 = h_t + 2B\ v_x = 0 \tag{4}$$

are the reduced forms of the equations of motion and continuity.

These two partial differential equations of the hyperbolic type are transformed by the methods of the theory of characteristics[7] into four ordinary differential equations. Combining the equations linearly with an unknown multiplier λ,

$$L_1 + \lambda L_2 = \lambda\left(\frac{h_x}{\lambda} + h_t\right) + \frac{B}{2}\left(4\lambda\, v_x + v_t\right) + hfo\, v|v| = 0$$

Values of λ are sought that will make this equation a total differential equation. Since

$$\frac{dh}{dt} = h_t + h_x \frac{dx}{dt} \qquad \frac{dv}{dt} = v_t + v_x \frac{dx}{dt}$$

the first quantity in parantheses is an exact differential if $dx/dt = 1/\lambda$ and the second quantity in parantheses is exact if $dx/dt = 4\lambda$. Equating these values of dx/dt

$$4\lambda^2 = 1$$

or

$$\lambda = \pm \frac{1}{2}$$

Hence the following four equations result, applying first $\lambda = +1/2$, then $\lambda = -1/2$,

$$\left.\begin{aligned} &\frac{1}{2}\frac{dh}{dt} + \frac{B}{2}\frac{dv}{dt} + hfo\, v|v| = 0 \\ &\frac{dx}{dt} = 2 \end{aligned}\right\} \text{Along } C_+ \qquad (5)$$

$$\left.\begin{aligned} &-\frac{1}{2}\frac{dh}{dt} + \frac{B}{2}\frac{dv}{dt} + hfo\, v|v| = 0 \\ &\frac{dx}{dt} = -2 \end{aligned}\right\} \text{Along } C_- \qquad (6)$$

The first equation is valid along the line or "characteristic line" given by $dt = dx/2$, and the third equation holds along the line $dt = -dx/2$, as shown in Figure 2.

By rewriting the equations as difference equations

$$v_p - v_A + \frac{1}{B}(h_p - h_A) + \frac{2}{B}\, hfo\, v_A|v_A|(t_p - t_A) = 0 \qquad (7)$$

$$t_p - t_A - \frac{1}{2}(x_p - x_A) = 0 \qquad (8)$$

$$v_p - v_B - \frac{1}{B}(h_p - h_B) + \frac{2}{B}\, hfo\, v_B|v_B|(t_p - t_B) = 0 \qquad (9)$$

$$t_p - t_B + \frac{1}{2}(x_p - x_B) = 0 \qquad (10)$$

Now, by taking equal x-increments, with the t-increments half as large, Figure 3, the solution can be built up from known conditions along a line t = constant, together with boundary conditions relating v and h at $x = 0$ and $x = 1$. If the pipe is divided into N increments of equal length, then $x_p - x_A = x_B - x_p = 1/N$, and $t_p - t_A = t_p - t_B = t_p - t_C = 1/2N$. By selecting a suitably large N, the friction term may be approximated by allowing v_A and v_B in it to be replaced by v_C which is known when v_p and h_p are to be calculated. This leaves only two equations, (7) and (9) to be solved for the two unknowns, as follows:

$$h_p = \frac{1}{2}(h_A + h_B) + \frac{B}{2}(v_A - v_B) \qquad (11)$$

$$v_p = \frac{1}{2}(v_A + v_B) + \frac{1}{2B}(h_A - h_B) - \frac{hfo}{BN}\, v_C|v_C| \qquad (12)$$

These two equations suffice for computation of all interior points when all values are known on the preceding time = constant line.

For computation Equations (11) and (12) are written in subscript notation. The pipe is divided into N reaches of equal length, with velocities and heads to be computed at N + 1 sections, v_o, v_1, ... v_N etc. With v and h known at these sections at time t, at time t + 1/2N

$$h_{pi} = \frac{1}{2}(h_{i-1} + h_{i+1}) + \frac{B}{2}(v_{i-1} - v_{i+1}) \qquad i = 1,2, \ldots N-1 \qquad (13)$$

$$v_{pi} = \frac{1}{2}(v_{i-1} + v_{i+1}) + \frac{1}{2B}(h_{i-1} - h_{i+1}) - \frac{hfo}{BN} v_i |v_i| \qquad (14)$$

$$i = 1,2, \ldots N-1$$

The computation of h_{po}, v_{po}, h_{pN}, v_{pN} depends upon the boundary conditions, and are considered next.

Boundary Conditions

At x = 0, Equation (9) applies and yields one equation in the two unknowns v_{po} and h_{po}. From the boundary condition another relation is required. Consider, for example, that x = 0 is the junction of a reservoir and the pipe. Then h_{po} is the head due to the reservoir and provides the needed condition. Perhaps the flow entering is known as a function of time. Then from continuity v_{po} can be found to provide the needed condition.

At x = 1, Equation (7) applies, and an extra condition is needed to solve for the two unknowns. For example if a valve is located at x = 1 and discharges into the atmosphere, the gate equation is

$$Q = AV = C_D A_G \sqrt{2gH_{p1}}$$

For steady state conditions

$$Q = AV_o = (C_D A_G)_o \sqrt{2gH_o}$$

in which $(C_D A_G)_o$ is the product of gate opening by discharge coefficient for the steady state situation. Dividing the first equation by the second equation

$$\frac{V}{V_o} = \frac{C_D A_G}{(C_D A_G)_o} \sqrt{\frac{h_{pl}}{H_o}}$$

or, in dimensionless notation

$$v_{pl} = \tau \sqrt{h_{pl}}$$

in which τ is the dimensionless area of gate opening modified by the appropriate discharge coefficient. With τ known as a function of t, the second required condition is available.

VALVE CLOSURE

The problem to be solved is how to close a valve at the downstream end of a pipe leading from a reservoir in a minimum time without exceeding a predetermined pressure and without reducing the pressure below steady state conditions. The hydraulic grade line is to stay within DEF of Figure 4. Friction is neglected in the first case and an exact solution to the problem is found. Graphically the closure is to take place according to Figure 5. The valve is to be closed in such a way that in unit dimensionless time the head h has increased linearly from 1 to h_m and the velocity has reduced linearly from 1 to $1-(h_m-1)/B$, where $h_m = H_{max}/H_o$. From the graphical construction it will be noted that at t = 1 the velocity is uniform in the pipe, i.e., A_1, B_1, C_1 (subscript 1 refers to time t = 1) are on the same ordinate of Figure 5. The head also varies uniformly from h = 1 at the reservoir to h_m at the valve. This uniform velocity is now maintained by reducing the velocity at twice the initial velocity reduction, or at the rate

$$\frac{dv}{dt} = -2 \frac{h_m-1}{B}$$

while the head is maintained at $h = h_m$ at the valve. This uniform deceleration continues until $v = (h_m-1)/B$, then the closure takes the form permitting the velocity to reduce uniformly to zero at the

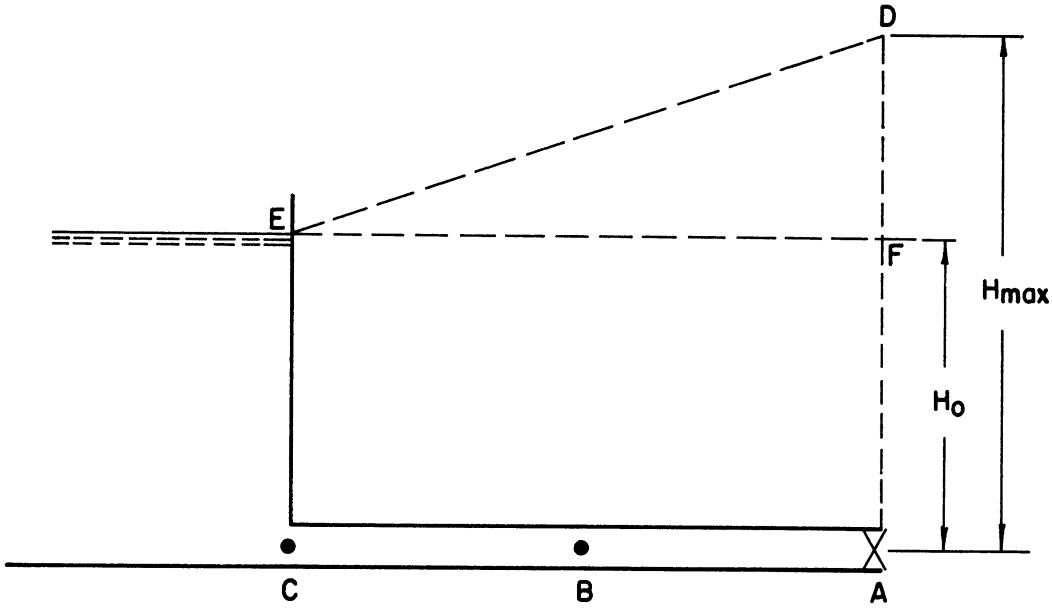

Figure 4. Valve Closure to Avoid Heads Greater Than H_{max} or Less than H_o, Friction Neglected.

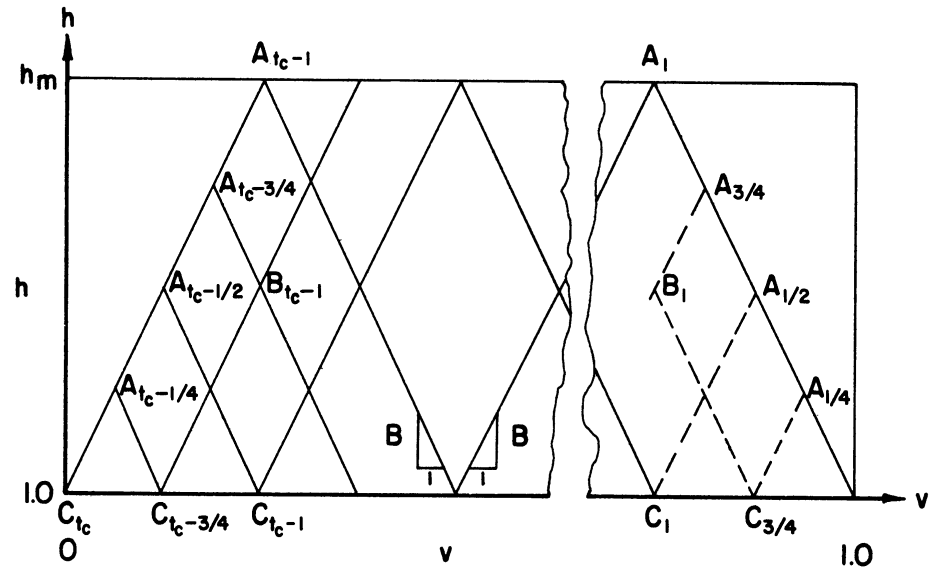

Figure 5. Graphical Solution of Frictionless Valve Closure.

gate in $\Delta t = 1$, while the head likewise reduces uniformly at the gate from h_m to 1 in the same time. Under these conditions at time of closing t_c the velocity is zero throughout the pipe and the head is uniformly at $h = 1$ throughout the pipe, as shown in the graphical solution.

Since $v = \tau\sqrt{h}$ must hold at all times at the gate, τ may be found as a function of t in terms of only two parameters, h_m and B. Equations are now developed for the three phases of the closure.

For $0 < t \leq 1$

$$v = 1 - \frac{h_m-1}{B}\,t$$

$$h = 1 + (h_m-1)t$$

and

$$\tau = \frac{v}{\sqrt{h}} = \frac{B - (h_m-1)t}{B\sqrt{1 + (h_m-1)t}} \qquad 0 < t \leq 1 \qquad (15)$$

For $1 < t \leq t_c - 1$, the velocity reduction rate is

$$\frac{dv}{dt} = -2\,\frac{h_m-1}{B}$$

and the velocity reduces from $1-(h_m-1)/B$ to $(h_m-1)B$, or by the amount $1 - 2(h_m-1)/B$. Hence the time for this intermediate phase is

$$-\frac{1 - 2(h_m-1)/B}{dv/dt} = \frac{B}{2(h_m-1)} - 1$$

and the complete closure time t_c is 2 plus this value, or

$$t_c = 1 + \frac{B}{2(h_m-1)} \qquad (16)$$

The expression for dv/dt may be integrated for $t = 1$, $v = 1-(h_m-1)/B$,

$$v = 1 - \frac{h_m-1}{B}(2t-1)$$

and the head remains constant at h_m. Then

$$\tau = \frac{B - (h_m-1)(2t-1)}{B\sqrt{h_m}} \qquad 0 < t \leq t_c - 1 \qquad (17)$$

For $t_c - 1 < t \leq t_c$, the final closing phase

$$v = \frac{h_m-1}{B}(t_c-t)$$

$$h = 1 + (h_m-1)(t_c-t)$$

and

$$\tau = \frac{(h_m-1)(t_c-t)}{B\sqrt{1 + (h_m-1)(t_c-t)}} \qquad (18)$$

The minimum closing time for these results is $t = 2$, or $h_m = 1 + B/2$, which is the maximum permitted head.

The proof has been demonstrated by the general graphical solution as the equations were developed. It may easily be checked out for any given case numerically or by computer solution, with the aid of Equations (13) and (14) of the characteristic method, with hfo = 0. The upstream boundary condition is $h_p = 1$, and from Equation (9).

$$v_{po} = v_1 + \frac{1}{B}(1 - h_1)$$

At the valve, h_p in Equation (7) is replaced by v_p^2/τ^2 and the equation solved for v_p, with hfo = 0,

$$v_p = \frac{B\tau^2}{2}\left[\sqrt{1 + \frac{4}{B\tau^2}(v_{n-1} + \frac{h_{n-1}}{B})} - 1\right] \qquad (19)$$

$$h_p = \frac{v_p^2}{\tau^2} \qquad (20)$$

For $\tau = 0$, Equation (7) yields $v_p = 0$, $h_p = h_{N-1} + Bv_{N-1}$. The computer solution for a frictionless case is given in Figure 6. The nature of the closure is shown in Figure 7 in the plot of τ vs t.

Valve Closure with Friction

When pipe friction is taken into account, the solution is not exact as in the previous case, but the heads and velocities may be maintained within one per cent of the desired values. Figure 8 shows the situation with EF the steady state hydraulic grade line, and ED the maximum pressure line. The flow is to come to rest with hydraulic grade line EG.

Basically, the same solution is used as in the frictionless case, except for a friction correction:

$$B\frac{dv}{dt} = -[h_m\text{-}1\text{-}hfo(1 - v^2)] \quad 0 < t \leq 1, \quad t_c - 1 < t \leq t_c \qquad (21)$$

and

$$B\frac{dv}{dt} = -2[h_m\text{-}1\text{-}hfo(1 - v^2)] \quad 1 < t \leq t_c - 1 \qquad (22)$$

The head variation at the valve is somewhat more complicated than before, B = 40.000000, HFO = .000000, HM = 6.000000, N = 20

S = .125000, SS = .125000, V2 = .875000, V3 = .125000

TC = 5.000000, DVDT1 = -.250000, DVDT2 = -.250000, DVDT3 = -.250000

DIMENSIONLESS VELOCITIES AND HEADS

TIME	TAU	X=	0	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95	1.0
.000	1.0000	H=	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		V=	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
.150	.7418	H=	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.13	1.25	1.38	1.50	1.63	1.75
		V=	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.99	.99	.99	.98	.98
.300	.6087	H=	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.13	1.25	1.38	1.50	1.63	1.75	1.88	2.00	2.13	2.25	2.38	2.50
		V=	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.99	.99	.99	.98	.98	.98	.97	.97	.97	.97	.96
.450	.5235	H=	1.00	1.00	1.00	1.13	1.25	1.38	1.50	1.63	1.75	1.88	2.00	2.13	2.25	2.38	2.50	2.63	2.75	2.88	3.00	3.13	3.25
		V=	1.00	1.00	1.00	1.00	.99	.99	.99	.98	.98	.98	.97	.97	.97	.97	.96	.96	.96	.95	.95	.95	.94
.600	.4825	H=	1.00	1.25	1.50	1.75	2.00	2.13	2.25	2.38	2.50	2.63	2.75	2.88	3.00	3.13	3.25	3.38	3.50	3.63	3.75	3.87	4.00
		V=	.97	.97	.97	.97	.97	.97	.97	.97	.96	.96	.96	.95	.95	.95	.94	.94	.94	.93	.93	.93	.92
.750	.4156	H=	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.63	3.75	3.88	4.00	4.13	4.25	4.38	4.50	4.63	4.75
		V=	.94	.94	.94	.94	.94	.94	.94	.94	.94	.94	.94	.93	.93	.93	.92	.92	.92	.92	.91	.91	.91
.900	.3784	H=	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.13	5.25	5.37	5.50
		V=	.90	.90	.90	.90	.90	.90	.90	.90	.90	.90	.90	.90	.90	.90	.90	.90	.90	.90	.89	.89	.89
1.050	.3521	H=	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		V=	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86
1.200	.3363	H=	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		V=	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82
1.350	.3215	H=	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		V=	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79
1.500	.3062	H=	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		V=	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75
1.650	.2909	H=	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		V=	.71	.71	.71	.71	.71	.71	.71	.71	.71	.71	.71	.71	.71	.71	.71	.71	.71	.71	.71	.71	.71
1.800	.2756	H=	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		V=	.67	.67	.67	.67	.67	.67	.67	.67	.67	.67	.67	.67	.67	.67	.67	.67	.67	.67	.67	.67	.67
1.950	.2603	H=	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		V=	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64
2.100	.2449	H=	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		V=	.60	.60	.60	.60	.60	.60	.60	.60	.60	.60	.60	.60	.60	.60	.60	.60	.60	.60	.60	.60	.60
2.250	.2296	H=	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		V=	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56

2.400	.2145	H=1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		V= .52	.52	.52	.52	.52	.52	.52	.52	.52	.52	.52	.52	.52	.52	.52	.52	.52	.52	.52	.52	.52
2.550	.1990	H=1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		V= .49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49
2.700	.1837	H=1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		V= .45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45
2.850	.1684	H=1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		V= .41	.41	.41	.41	.41	.41	.41	.41	.41	.41	.41	.41	.41	.41	.41	.41	.41	.41	.41	.41	.41
3.000	.1531	H=1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		V= .38	.38	.38	.38	.38	.38	.38	.38	.38	.38	.38	.38	.38	.38	.38	.38	.38	.38	.38	.38	.38
3.150	.1378	H=1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		V= .34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34
3.300	.1225	H=1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		V= .30	.30	.30	.30	.30	.30	.30	.30	.30	.30	.30	.30	.30	.30	.30	.30	.30	.30	.30	.30	.30
3.450	.1072	H=1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		V= .26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26
3.600	.0919	H=1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		V= .23	.23	.23	.23	.23	.23	.23	.23	.23	.23	.23	.23	.23	.23	.23	.23	.23	.23	.23	.23	.23
3.750	.0765	H=1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		V= .19	.19	.19	.19	.19	.19	.19	.19	.19	.19	.19	.19	.19	.19	.19	.19	.19	.19	.19	.19	.19
3.900	.0612	H=1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		V= .15	.15	.15	.15	.15	.15	.15	.15	.15	.15	.15	.15	.15	.15	.15	.15	.15	.15	.15	.15	.15
4.050	.0495	H=1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.63	5.75
		V= .11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.12	.12
4.200	.0447	H=1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.13	4.25	4.38	4.50	4.63	4.75	4.88	5.00
		V= .08	.08	.08	.08	.08	.08	.08	.08	.08	.08	.08	.08	.08	.08	.08	.08	.09	.09	.09	.10	.10
4.350	.0394	H=1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.63	2.75	2.88	3.00	3.13	3.25	3.38	3.50	3.63	3.75	3.88	4.00	4.13	4.25
		V= .04	.04	.04	.04	.04	.04	.04	.04	.04	.05	.05	.05	.06	.06	.06	.07	.07	.07	.08	.08	.08
4.500	.0334	H=1.00	1.13	1.25	1.38	1.50	1.63	1.75	1.88	2.00	2.13	2.25	2.38	2.50	2.63	2.75	2.88	3.00	3.13	3.25	3.38	3.50
		V= .00	.00	.01	.01	.01	.02	.02	.02	.03	.03	.03	.03	.04	.04	.04	.05	.05	.05	.06	.06	.06
4.650	.0264	H=1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.13	1.25	1.38	1.50	1.63	1.75	1.88	2.00	2.13	2.25	2.38	2.50	2.63	2.75
		V= .00	.00	.00	.00	.00	.00	.00	.00	.01	.01	.01	.02	.02	.02	.03	.03	.03	.03	.04	.04	.04
4.800	.0177	H=1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.13	1.25	1.38	1.50	1.63	1.75	1.88	2.00
		V= .00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.01	.01	.01	.02	.02	.02	.03
4.950	.0056	H=1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.13	1.25
		V= .00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.01
5.000	.0000	H=1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		V= .00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
5.025	.0000	H=1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		V= .00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00

Figure 6. Computer Solution for Valve Closure with Fluid Friction Neglected.

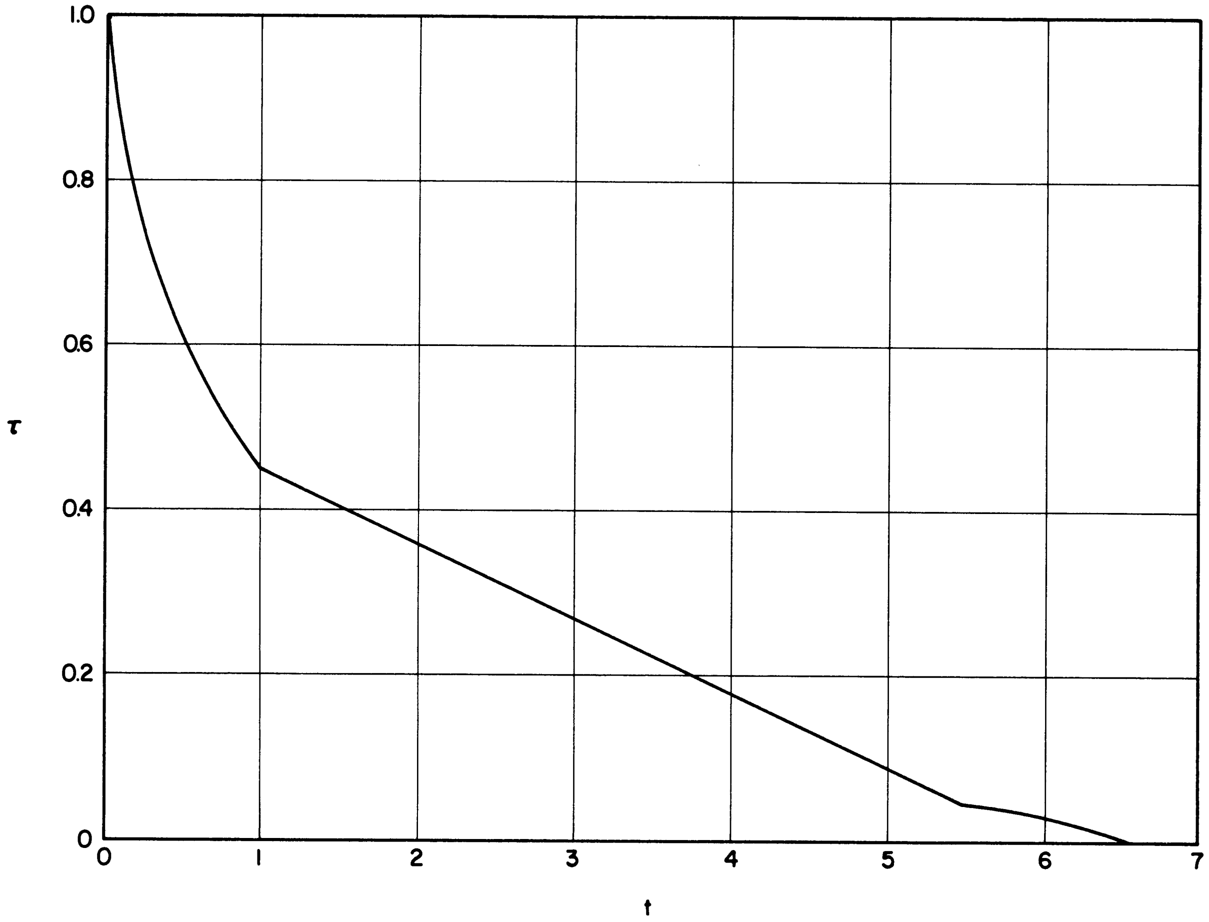

Figure 7. Typical Valve Closing Relation B = 30, h_m = 4, hfo = 0.4.

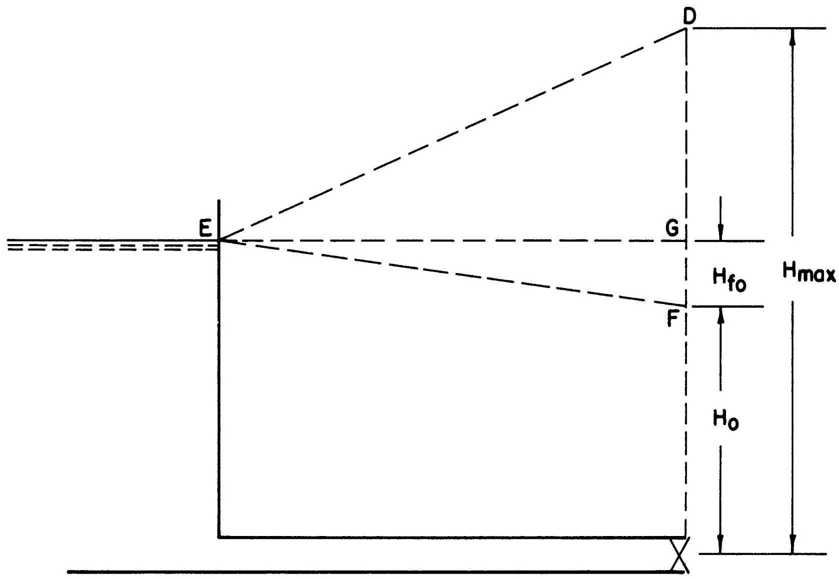

Figure 8. Valve Closure Notation for Case When Friction is taken into Account.

$$h = 1 + (h_m - 1)t \qquad 0 < t \leq 1 \qquad (23)$$

$$h = h_m \qquad 1 < t \leq t_c - 1 \qquad (24)$$

$$h = 1 + hfo - (1 + hfo - h_m)(t_c - t)$$

$$t_c - 1 < t \leq t_c \qquad (25)$$

The friction term hfo in the velocity expressions is modified by $(1 - v^2)$ and approximate expressions are used for v in these terms. For convenience, let

$$s = \frac{h_m - 1}{B} \qquad ss = \frac{h_m - 1 - hfo}{B}$$

For $0 < t \leq 1$

$$v \simeq 1 - st$$

as in the frictionless case, and

$$1 - v^2 \simeq 2\,st - s^2t^2$$

With this value of $1 - v^2$ substituted into Equation (21) it may be integrated for the initial conditions $t = 0$, $v = 1$, to yield

$$v = 1 - \frac{t}{B}\,[h_m - 1 - hfo(1 - s\,t/3)s\,t] \qquad (26)$$

Then, using Equation (23)

$$\tau = \frac{v}{\sqrt{h}} = \frac{B - t\,[h_m - 1 - hfo\ s\ t(1 - s\,t/3)]}{B\sqrt{1 + (h_m - 1)t}}$$

$$0 < t \leq 1 \qquad (27)$$

At $t = 1$ τ has the value

$$\tau_1 = \frac{B - h_m + 1 + hfo\ s(1 - s/3)}{B\sqrt{h_m}} \qquad t = 1$$

The final phase $t_c - 1 < t \leq t_c$ is next taken up as some of the results are useful for the second phase friction correction. The velocity is approximated by

$$v \approx ss(t_c - t) \qquad t_c - 1 < t < t_c$$

This value inserted into the right hand side of Equation (21) and integrated for $v = 0$, $t = t_c$, yields

$$Bv = (h_m - 1 - hfo)(t_c - t) + \frac{ss^2}{3} hfo\,(t_c - t)^3 \qquad (28)$$

By using this value of v with Equation (25) for head

$$\tau = \frac{(h_m - 1 - hfo)(t_c - t) + ss^2 hfo\,(t_c - t)^3/3}{B\sqrt{1 + hfo - (1 + hfo - h_m)(t_c - t)}}$$

$$t_c - 1 < t \leq t_c \qquad (29)$$

For $1 < t \leq t_c - 1$ for $t = 1$, from Equation (26)

$$v_2 = 1 - \frac{h_m - 1 - hfo\,(1 - s/3)s}{B} \qquad t = 1$$

and from Equation (28)

$$v_3 = \frac{h_m - 1 - hfo(1 - ss^2/3)}{B} \qquad t = t_c - 1$$

From Equation (22) it is observed that v reduces almost linearly from v_2 to v_3. This phase is broken into three equal velocity increments

$$\Delta v = \frac{v_2 - v_3}{3}$$

and the average velocity for each of these time periods is used in the friction correction

$$v_{11} = v_2 - \Delta v/2$$

$$v_{22} = v_{11} - \Delta v$$

$$v_{33} = v_{22} - \Delta v$$

Then, from Equation (22)

$$\left.\frac{dv}{dt}\right|_1 = -2\,[h_m - 1 - hfo(1 - v_{11}^2)]/B$$

$$\left.\frac{dv}{dt}\right|_2 = -2\,[h_m - 1 - hfo(1 - v_{22}^2)]/B$$

$$\left.\frac{dv}{dt}\right|_3 = -2\,[h_m - 1 - hfo(1 - v_{33}^2)]/B$$

The time for each of these subphases is

$$\Delta t_1 = \frac{\Delta v}{\left.\frac{dv}{dt}\right|_1}$$

$$\Delta t_2 = \frac{\Delta v}{\left.\frac{dv}{dt}\right|_2}$$

$$\Delta t_3 = \frac{\Delta v}{\left.\frac{dv}{dt}\right|_3}$$

and the closing time is

$$t_c = 2 + \Delta t_1 + \Delta t_2 + \Delta t_3 \qquad (30)$$

To obtain τ for the intermediate phase

$$\tau = \tau_1 + \left.\frac{dv}{dt}\right|_1 \frac{t-1}{\sqrt{h_m}} \qquad 1 < t \leq t_1 \tag{31}$$

$$\tau_2 = \tau_1 + \left.\frac{dv}{dt}\right|_1 \frac{\Delta t_1}{\sqrt{h_m}} \qquad t = 1 + \Delta t_1 = t_1$$

$$\tau = \tau_2 + \left.\frac{dv}{dt}\right|_2 \frac{t-t_1}{\sqrt{h_m}} \qquad t_1 < t < t_2 \tag{32}$$

$$\tau_3 = \tau_2 + \left.\frac{dv}{dt}\right|_2 \frac{\Delta t_2}{\sqrt{h_m}} \qquad t = t_1 + \Delta t_2 = t_2$$

$$\tau = \tau_3 + \left.\frac{dv}{dt}\right|_3 \frac{t-t_2}{\sqrt{h_m}} \qquad t_2 < t \leq t_c - 1 \tag{33}$$

The values of τ in terms of t are now completely specified for the time range $0 \leq t \leq t_c$. To check the validity of the friction corrections, several cases have been run on the computer. One case is shown in Figure 9. Since the example of Figure 9 is in dimensionless form, it may apply to any size pipe line carrying turbulent flow. Two cases are discussed: V_o = 8 ft/sec, H_o = 35 ft, D = 48 in., L = 3128 ft, f = 0.018, and a = 4225 ft/sec. These values yield B = 30, h_m = 4, and hfo = 0.4. The time of closure is $t_c 2L/a$ = 6.50 x 2 x 3128/4225 = 9.62 sec. The maximum head is 140 ft. which occurs at the gate. For closure in less than 2L/a = 1.48 sec. the maximum head is 1049 ft. followed by vapor pressure and column separation on return of the negative wave. The second case is V_o = 30 ft/sec, H_o = 124.2 ft, D = 0.5 in., L = 6.17 ft f = 0.024 and a = 4000 ft/sec. The same values of B, h_m and hfo result. Closing time is 0.02 sec., and 2L/a = 0.003 sec. The maximum head by

B = 30.000000, HFO = .400000, HM = 4.000000, N = 20

S = .100000, SS = .086667, V2 = .901289, V3 = .086700

TC = 6.501476, DVDT1 = -.188961, DVDT2 = -.179841, DVDT3 = -.174653

DIMENSIONLESS VELOCITIES AND HEADS

TIME	TAU	X=	0	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95	1.0
.0001	.0000	H=	1.40	1.38	1.36	1.34	1.32	1.30	1.28	1.26	1.24	1.22	1.20	1.18	1.16	1.14	1.12	1.10	1.08	1.06	1.04	1.02	1.00
		V=	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
.150	.8180	H=	1.40	1.38	1.36	1.34	1.32	1.30	1.28	1.26	1.24	1.22	1.20	1.18	1.16	1.14	1.12	1.18	1.23	1.28	1.34	1.39	1.45
		V=	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.99	.99	.99	.99
.300	.7038	H=	1.40	1.38	1.36	1.34	1.32	1.30	1.28	1.26	1.24	1.29	1.35	1.40	1.46	1.51	1.57	1.62	1.68	1.73	1.79	1.84	1.90
		V=	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.99	.99	.99	.99	.98	.98	.98	.98	.97	.97
.450	.6231	H=	1.40	1.38	1.36	1.41	1.47	1.52	1.58	1.63	1.69	1.74	1.80	1.85	1.91	1.96	2.02	2.07	2.13	2.18	2.24	2.29	2.35
		V=	1.00	1.00	1.00	1.00	1.00	.99	.99	.99	.99	.98	.98	.98	.98	.97	.97	.97	.97	.96	.96	.96	.96
.600	.5620	H=	1.40	1.53	1.66	1.79	1.91	1.97	2.02	2.08	2.13	2.19	2.24	2.30	2.35	2.41	2.47	2.52	2.58	2.63	2.69	2.74	2.80
		V=	.98	.98	.98	.98	.98	.98	.98	.97	.97	.97	.97	.96	.96	.96	.96	.95	.95	.95	.95	.94	.94
.750	.5135	H=	1.40	1.53	1.66	1.79	1.92	2.04	2.17	2.30	2.43	2.56	2.69	2.75	2.80	2.86	2.91	2.97	3.03	3.08	3.14	3.19	3.25
		V=	.95	.95	.95	.95	.95	.95	.95	.95	.95	.95	.95	.95	.95	.94	.94	.94	.94	.93	.93	.93	.93
.900	.4736	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.44	2.57	2.70	2.82	2.95	3.08	3.21	3.34	3.47	3.53	3.59	3.64	3.70
		V=	.92	.92	.92	.92	.92	.92	.92	.92	.92	.92	.92	.92	.92	.92	.92	.92	.92	.92	.92	.91	.91
1.050	.4459	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.44	2.57	2.70	2.83	2.96	3.09	3.22	3.35	3.48	3.61	3.74	3.87	4.00
		V=	.89	.89	.89	.89	.89	.89	.89	.89	.89	.89	.89	.89	.89	.89	.89	.89	.89	.89	.89	.89	.89
1.200	.4317	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.45	2.58	2.71	2.84	2.97	3.10	3.23	3.35	3.48	3.61	3.74	3.87	3.99
		V=	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86	.86
1.350	.4176	H=	1.40	1.53	1.66	1.79	1.92	2.06	2.19	2.32	2.45	2.57	2.70	2.83	2.96	3.09	3.22	3.35	3.48	3.61	3.73	3.86	3.99
		V=	.83	.83	.83	.83	.83	.83	.83	.83	.83	.83	.83	.83	.83	.83	.83	.83	.83	.83	.83	.83	.83
1.500	.4034	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.44	2.57	2.69	2.82	2.95	3.08	3.21	3.34	3.47	3.60	3.73	3.86	3.99
		V=	.80	.80	.80	.81	.81	.81	.81	.81	.81	.81	.81	.81	.81	.81	.81	.81	.81	.81	.81	.81	.81
1.650	.3892	H=	1.40	1.53	1.66	1.78	1.91	2.04	2.17	2.30	2.43	2.56	2.69	2.82	2.95	3.08	3.21	3.34	3.47	3.60	3.73	3.86	3.99
		V=	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78
1.800	.3751	H=	1.40	1.53	1.66	1.78	1.91	2.04	2.17	2.30	2.43	2.55	2.68	2.81	2.94	3.07	3.20	3.33	3.46	3.59	3.72	3.86	3.99
		V=	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75
1.950	.3609	H=	1.40	1.53	1.66	1.79	1.92	2.04	2.17	2.30	2.43	2.56	2.69	2.82	2.95	3.08	3.21	3.33	3.46	3.59	3.72	3.85	3.98
		V=	.72	.72	.72	.72	.72	.72	.72	.72	.72	.72	.72	.72	.72	.72	.72	.72	.72	.72	.72	.72	.72
2.100	.3467	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.44	2.57	2.70	2.83	2.96	3.09	3.22	3.34	3.47	3.60	3.73	3.86	3.99
		V=	.69	.69	.69	.69	.69	.69	.69	.69	.69	.69	.69	.69	.69	.69	.69	.69	.69	.69	.69	.69	.69
2.250	.3325	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.44	2.57	2.70	2.83	2.96	3.09	3.22	3.35	3.48	3.61	3.74	3.87	4.00
		V=	.67	.67	.67	.67	.67	.67	.67	.67	.67	.67	.67	.67	.67	.67	.67	.67	.66	.66	.66	.66	.66

2.400	.3184	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.44	2.57	2.70	2.83	2.96	3.09	3.22	3.35	3.48	3.61	3.75	3.88	4.01
		V=	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64	.64
2.550	.3047	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.44	2.57	2.71	2.84	2.97	3.10	3.23	3.36	3.49	3.62	3.75	3.88	4.00
		V=	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61
2.700	.2912	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.44	2.58	2.71	2.84	2.97	3.09	3.22	3.35	3.48	3.61	3.74	3.87	4.00
		V=	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58
2.850	.2777	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.44	2.57	2.70	2.83	2.96	3.09	3.22	3.35	3.48	3.61	3.74	3.87	4.00
		V=	.55	.55	.55	.55	.55	.55	.55	.55	.55	.55	.55	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56
3.000	.2643	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.44	2.57	2.69	2.82	2.95	3.08	3.21	3.34	3.47	3.60	3.73	3.86	3.99
		V=	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53
3.150	.2508	H=	1.40	1.53	1.66	1.79	1.91	2.04	2.17	2.30	2.43	2.56	2.69	2.82	2.95	3.08	3.21	3.34	3.47	3.60	3.73	3.86	3.99
		V=	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50
3.300	.2373	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.30	2.43	2.56	2.69	2.82	2.95	3.08	3.21	3.34	3.47	3.60	3.73	3.86	3.99
		V=	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47
3.450	.2238	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.44	2.57	2.69	2.82	2.95	3.08	3.21	3.34	3.47	3.60	3.73	3.86	3.99
		V=	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45
3.600	.2103	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.44	2.57	2.70	2.83	2.96	3.09	3.22	3.35	3.48	3.61	3.74	3.87	4.00
		V=	.42	.42	.42	.42	.42	.42	.42	.42	.42	.42	.42	.42	.42	.42	.42	.42	.42	.42	.42	.42	.42
3.750	.1968	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.44	2.57	2.70	2.83	2.96	3.09	3.22	3.35	3.48	3.61	3.74	3.87	4.00
		V=	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39
3.900	.1833	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.44	2.57	2.70	2.83	2.96	3.09	3.22	3.36	3.49	3.62	3.75	3.88	4.01
		V=	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37
4.050	.1701	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.44	2.58	2.71	2.84	2.97	3.10	3.23	3.36	3.49	3.62	3.75	3.88	4.01
		V=	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34	.34
4.200	.1570	H=	1.40	1.53	1.66	1.79	1.92	2.06	2.19	2.32	2.45	2.58	2.71	2.84	2.97	3.10	3.23	3.36	3.49	3.62	3.75	3.88	4.01
		V=	.31	.31	.31	.31	.31	.31	.31	.31	.31	.31	.31	.31	.31	.31	.31	.31	.31	.31	.31	.31	.31
4.350	.1439	H=	1.40	1.53	1.66	1.79	1.93	2.06	2.19	2.32	2.45	2.58	2.71	2.84	2.97	3.09	3.22	3.35	3.48	3.61	3.74	3.87	4.00
		V=	.29	.29	.29	.29	.29	.29	.29	.29	.29	.29	.29	.29	.29	.29	.29	.29	.29	.29	.29	.29	.29
4.500	.1308	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.44	2.57	2.70	2.83	2.96	3.09	3.22	3.35	3.48	3.61	3.74	3.87	4.00
		V=	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26	.26
4.650	.1177	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.17	2.30	2.43	2.56	2.69	2.82	2.96	3.09	3.22	3.35	3.48	3.61	3.74	3.87	4.00
		V=	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24
4.800	.1046	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.43	2.56	2.69	2.82	2.95	3.08	3.21	3.34	3.47	3.60	3.73	3.86	3.99
		V=	.21	.21	.21	.21	.21	.21	.21	.21	.21	.21	.21	.21	.21	.21	.21	.21	.21	.21	.21	.21	.21
4.950	.0915	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.44	2.57	2.70	2.82	2.95	3.08	3.21	3.34	3.47	3.60	3.73	3.86	3.99
		V=	.18	.1	.18	.18	.18	.18	.18	.18	.18	.18	.18	.18	.18	.18	.18	.18	.18	.18	.18	.18	.18
5.100	.0784	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.44	2.57	2.70	2.83	2.96	3.09	3.22	3.35	3.47	3.60	3.73	3.86	3.99
		V=	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16
5.250	.0653	H=	1.40	1.53	1.66	1.79	1.92	2.05	2.18	2.31	2.44	2.57	2.70	2.83	2.96	3.09	3.22	3.35	3.48	3.61	3.74	3.87	4.00
		V=	.13	.13	.13	.13	.13	.13	.13	.13	.13	.13	.13	.13	.13	.13	.13	.13	.13	.13	.13	.13	.13

5.400	.0522	H=1.40	[illegible]	1.66	1.79	[illegible]	2.05	2.18	2.31	2.44	2.57	2.70	2.83	2.96	3.09	3.22	3.35	3.48	3.61	3.74	3.88	4.01
		V= .10	[illegible]	.10	.10	.10	.10	.10	.10	.10	.10	.10	.10	.10	.10	.10	.10	.10	.10	.10	.10	.10
5.550	.0419	H=1.40	[illegible]	1.66	1.79	[illegible]	2.05	2.18	2.31	2.44	2.57	2.71	2.84	2.97	3.10	3.23	3.36	3.49	3.62	3.75	3.82	3.88
		V= .08	[illegible]	.08	.08	[illegible]	.08	.08	.08	.08	.08	.08	.08	.08	.08	.08	.08	.08	.08	.08	.08	.08
5.700	.0372	H=1.40	[illegible]	1.66	1.79	[illegible]	2.05	2.19	2.32	2.45	2.58	2.71	2.84	2.97	3.04	3.11	3.17	3.24	3.30	3.37	3.43	3.49
		V= .05	[illegible]	.05	.05	[illegible]	.05	.05	.05	.05	.05	.05	.05	.05	.05	.06	.06	.06	.06	.07	.07	.07
5.850	.0321	H=1.40	[illegible]	1.66	1.79	[illegible]	2.06	2.19	2.26	2.32	2.39	2.45	2.52	2.58	2.65	2.72	2.78	2.85	2.91	2.98	3.04	3.11
		V= .03	[illegible]	.03	.03	[illegible]	.03	.03	.03	.03	.03	.03	.04	.04	.04	.04	.05	.05	.05	.05	.05	.06
6.000	.0264	H=1.40	1.47	1.54	1.60	[illegible]	1.73	1.80	1.86	1.93	2.00	2.06	2.13	2.19	2.26	2.32	2.39	2.45	2.52	2.59	2.65	2.72
		V= .00	[illegible]	.00	.01	.01	.01	.01	.02	.02	.02	.02	.02	.03	.03	.03	.03	.03	.04	.04	.04	.04
6.150	.0200	H=1.40	[illegible]	1.40	1.40	1.40	1.40	1.40	1.47	1.54	1.60	1.67	1.73	1.80	1.87	1.93	2.00	2.06	2.13	2.19	2.26	2.32
		V=-.00	-.00	-.00	-.00	-.00	-.00	-.00	.00	.00	.01	.01	.01	.01	.02	.02	.02	.02	.02	.03	.03	.03
6.300	.0126	H=1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.47	1.54	1.60	1.67	1.73	1.80	1.86	1.93
		V=-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	.00	.00	.01	.01	.01	.01	.02	.02
6.450	.0036	H=1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.47	1.53
		V=-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	.00	.00	.00
6.500	.0001	H=1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.39	1.40
		V=-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	.00
6.525	.0000	H=1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.39	1.39	1.39	1.39
		V=-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	-.00	.00

**** ALL INPUT DATA HAVE BEEN PROCESSED.
AT LOC 12125

Figure 9. Computer Solution for Valve Closure with Fluid Friction Taken Into Account.

controlled closing is 496.8 ft. If an undamped solonoid valve were used that closed in less than 0.003 sec. maximum head would be 124.2 + 4000 x 30/32.2 = 3854 ft, and vaporization would occur with column separation.

It is interesting to compare the two cases with uniform closures in the same time, 6.5 x 2L/a, as the controlled closures. From Allievi charts[8], neglecting friction, the maximum head is 245 ft in the first case and is 869 ft in the second case, or 75% greater than for controlled closure.

It should be emphasized that a high-speed digital computer is not needed to determine the closure as a function of time. The three parameters h_m, hfo, and B are determined from the specific problem, then τ as a function of time is found from the formulas. The computer was used to demonstrate the accuracy of the friction corrections. Friction corrections could be handled in many ways, to even greater accuracy if desired. Since friction coefficients are usually not known within $\pm 5\%$, the corrections used here should be adequate.

The shortest time of controlled closure for no head reduction below H_O is 4L/a sec, with $h_m = 1 + B/2$. In the cases given this yields $h_m = 16$. For any closure in less than 2L/a sec $h_m = B$, or 30 for the cases cited.

ESTABLISHMENT OF FLOW

By opening a valve at the downstream end of a pipe leading from a reservoir in a controlled manner, the flow may be established smoothly without undesirable pressure fluctuations. The procedure is similar to the case of controlled valve closing, and is discussed here with friction effects included.

The principle of opening is best illustrated from the graphical solution, as shown in Figure 10. h_m is the minimum head in this example. The valve is to open so that v and h follow the path A_o A_1 A_{to-1} A_{to}. When A_{to-1} is below the parabola $\tau = 1$, or $v = \sqrt{h}$, as shown, the gate must open to a value greater than $\tau = 1$, its final steady state position for velocity V_o through the pipe with head H_o across the valve.

The friction correction is similar to that for valve closure,

$$B \frac{dv}{dt} = 1 + hfo(1 - v^2) - h_m \qquad 0 < t \leq 1, \; t_o - 1 < t \leq t_o \qquad (34)$$

t_o is the time to establish flow. For the intermediate phase

$$B \frac{dv}{dt} = 2[1 + hfo\,(1 - v^2) - h_m] \qquad 1 < t \leq t_o - 1 \qquad (35)$$

v is approximated for the friction correction term. Let

$$ss = \frac{1 + hfo - h_m}{B} \qquad\qquad s = \frac{1 - h_m}{B}$$

Then, for the first phase $v \approx ss\ t$, and

$$v = \frac{t}{B}\,[1 + hfo\,(1 - \frac{ss^2}{3}\,t^2) - h_m]$$

$$h = 1 + hfo - (1 + hfo - h_m)\,t$$

and

$$\tau = \frac{v}{\sqrt{h}} = \frac{t[1 + hfo(1 - ss^2t^2/3) - h_m]}{B\sqrt{1 + hfo - (1 + hfo - h_m)t}} \qquad 0 \leq t \leq 1 \qquad (36)$$

Hence

$$\tau_1 = \frac{1 + hfo(1 - ss^2/3) - h_m}{B\sqrt{h_m}} \qquad t = 1$$

$$v_1 = \frac{1 + hfo(1 - ss^2/3) - h_m}{B} \qquad t = 1$$

For the third, or final, phase $v \simeq 1 + s(t - t_o)$

$$1 - v^2 \simeq - s\,(t - t_o)[2 + s(t - t_o)]$$

and from Equation (34), for $v = 1$ when $t = t_o$

$$B(v - 1) = (t - t_o)\,\{1 - h_m - hfo\; s(t - t_o)[1 + s(t - t_o)/3]\}$$

Since

$$h = 1 + (1 - h_m)(t - t_o) \qquad t_o - 1 < t \leq t_o$$

$$\tau = \frac{B + (t-t_o)\,\{1-h_m-hfo\; s(t-t_o)\;[1 + s(t-t_o)/3]\}}{B\sqrt{1 + (1-h_m)(t - t_o)}} \qquad t_o-1 < t \leq t_o \qquad (37)$$

The velocity at $t = t_o - 1$ is

$$v_2 = 1 - \frac{1 - h_m + hfo\; s(1 - s/3)}{B}$$

which is used for the intermediate opening phase. Let

$$\Delta v = v_2 - v_1$$

$$v_{11} = v_1 + \frac{\Delta v}{2}$$

$$v_{12} = v_{11} + \Delta v$$

$$v_{13} = v_{12} + \Delta v$$

Then, from Equation (35)

$$\left.\frac{dv}{dt}\right|_1 = \frac{2}{B}\,[1 + hfo(1 - v_{11}^2) - h_m]$$

$$\left.\frac{dv}{dt}\right|_2 = \frac{2}{B}\,[1 + hfo(1 - v_{12}^2) - h_m]$$

$$\left.\frac{dv}{dt}\right|_3 = \frac{2}{B}\,[1 + hfo(1 - v_{13}^2) - h_m]$$

Now, let

$$\Delta t_1 = \frac{\Delta v}{\left.\frac{dv}{dt}\right|_1}, \qquad \Delta t_2 = \frac{\Delta v}{\left.\frac{dv}{dt}\right|_2}, \qquad \Delta t_3 = \frac{\Delta v}{\left.\frac{dv}{dt}\right|_3}$$

and

$$t_1 = 1 + \Delta t_1, \quad t_2 = t_1 + \Delta t_2\,, \quad t_3 = t_2 + \Delta t_3 \quad .$$

The complete time of opening is

$$t_o = 2 + \Delta t_1 + \Delta t_2 + \Delta t_3$$

The intermediate opening relationships are then

$$\tau = \tau_1 + \left.\frac{dv}{dt}\right|_1 \frac{t-1}{\sqrt{h_m}} \qquad 1 < t \leq t_1 \qquad (38)$$

$$\tau = \tau_2 + \left.\frac{dv}{dt}\right|_2 \frac{t-t_1}{\sqrt{h_m}} \qquad t_1 < t \leq t_2 \qquad (39)$$

$$\tau = \tau_3 + \left.\frac{dv}{dt}\right|_3 \frac{t-t_2}{\sqrt{h_m}} \qquad t_2 < t \leq t_3 \qquad (40)$$

in which

$$\tau_2 = \tau_1 + \frac{dv}{dt}\bigg|_1 \frac{\Delta t_1}{\sqrt{h_m}} \qquad t = t_1$$

$$\tau_3 = \tau_2 + \frac{dv}{dt}\bigg|_2 \frac{\Delta t_2}{\sqrt{h_m}} \qquad t = t_2$$

In this case the initial hydraulic grade line is EG (Figure 8) and is designed to stay within GEF. Figure 11 is the computer solution of an example of valve opening.

When $B > 1 + \sqrt{h_m}$, the valve opens to

$$\tau = \frac{B - 1 + h_m}{B\sqrt{h_m}}$$

at point $A_{t_o - 1}$ of Figure 10, which yields a τ greater than 1. The valve then closes to the valve $\tau = 1$ during the final opening phase. h_m should be less than 1 and must be greater than 0. When $B < 1 + \sqrt{h_m}$ the maximum valve opening is $\tau = 1$ at $t = t_o$.

Controlled Closure for Variable Thickness Pipe

This example of controlled closure is applied to a penstock of variable wall thickness, depending upon the head imposed by water hammer. The thickness of pipe wall is first selected for each of twenty sections of the same length on the basis of maximum stress of 10,000 psi, subject only to a minimum wall thickness of 1 in. Figure 12 shows a profile of the penstock, along with the maximum desired hydraulic grade line.

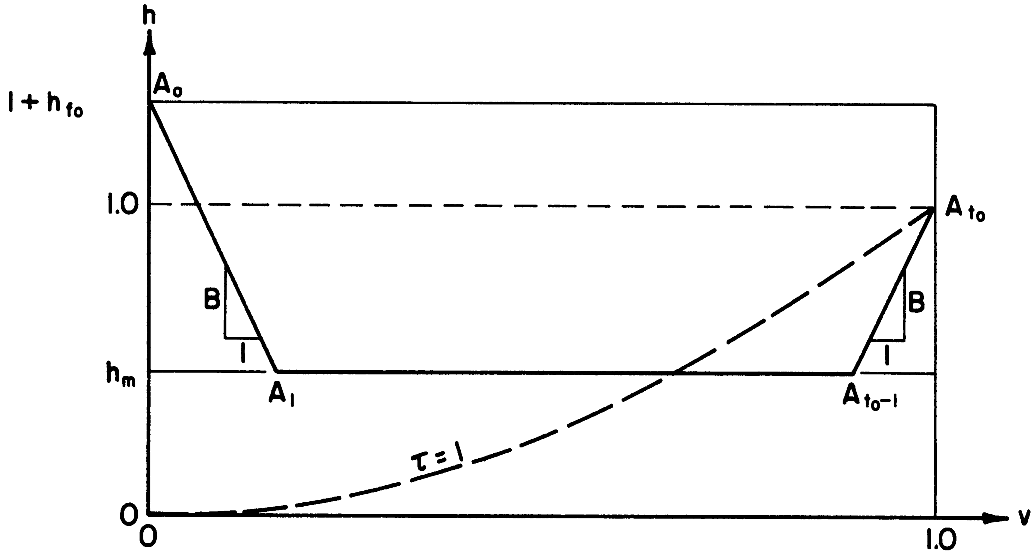

Figure 10. Graphical Representation of Path of v and h at the Gate for Controlled Opening with Friction.

B =	6.000000,	HFO =	.400000,	HM =	.200000		
S =	.133333,	SS =	.200000,	V2 =	.199111,	V3 =	.858173
TO =	3.848170,	DVDT1 =	.387273,	DVDT2 =	.362738,	DVDT3 =	.325334

DIMENSIONLESS VELOCITIES AND HEADS

TIME	TAU	X=	0	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95	1.0
.000	.0000	H=	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40
		V=	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
.100	.0177	H=	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.37	1.34	1.31	1.28
		V=	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.01	.01	.02
.200	.0371	H=	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.37	1.34	1.31	1.28	1.25	1.22	1.19	1.16
		V=	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.01	.01	.02	.02	.03	.03	.04
.300	.0588	H=	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.37	1.34	1.31	1.28	1.25	1.22	1.19	1.16	1.13	1.10	1.07	1.04
		V=	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.01	.01	.02	.02	.03	.03	.04	.04	.05	.05	.06
.400	.0833	H=	1.40	1.40	1.40	1.40	1.40	1.37	1.34	1.31	1.28	1.25	1.22	1.19	1.16	1.13	1.10	1.07	1.04	1.01	.98	.95	.92
		V=	.00	.00	.00	.00	.00	.00	.01	.01	.02	.02	.03	.03	.04	.04	.05	.05	.06	.06	.07	.07	.08
.500	.1117	H=	1.40	1.37	1.34	1.31	1.28	1.25	1.22	1.19	1.16	1.13	1.10	1.07	1.04	1.01	.98	.95	.92	.89	.86	.83	.80
		V=	.00	.00	.01	.01	.02	.02	.03	.03	.04	.04	.05	.05	.06	.06	.07	.07	.08	.08	.09	.09	.10
.600	.1453	H=	1.40	1.34	1.28	1.22	1.16	1.13	1.10	1.07	1.04	1.01	.98	.95	.92	.89	.86	.83	.80	.77	.74	.71	.68
		V=	.04	.04	.04	.04	.04	.04	.05	.05	.06	.06	.07	.07	.08	.08	.09	.09	.10	.10	.11	.11	.12
.700	.1867	H=	1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.89	.86	.83	.80	.77	.74	.71	.68	.65	.62	.59	.56
		V=	.08	.08	.08	.08	.08	.08	.08	.08	.08	.08	.09	.09	.10	.10	.11	.11	.12	.12	.13	.13	.14
.800	.2405	H=	1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.65	.62	.59	.56	.53	.50	.47	.44
		V=	.12	.12	.12	.12	.12	.12	.12	.12	.12	.12	.12	.12	.12	.12	.13	.13	.14	.14	.15	.15	.16
.900	.3171	H=	1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.41	.38	.35	.32
		V=	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16	.16	.17	.17	.18
1.000	.4452	H=	1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.38	.32	.26	.20
		V=	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20
1.100	.5318	H=	1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.38	.32	.26	.20
		V=	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24	.24
1.200	.6184	H=	1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.38	.32	.26	.20
		V=	.28	.28	.28	.28	.28	.28	.28	.28	.28	.28	.28	.28	.28	.28	.28	.28	.28	.28	.28	.28	.28
1.300	.7050	H=	1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.38	.32	.26	.20
		V=	.32	.32	.32	.32	.32	.32	.32	.32	.32	.32	.32	.32	.32	.32	.32	.32	.32	.32	.32	.32	.32
1.400	.7916	H=	1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.38	.32	.26	.20
		V=	.35	.35	.35	.35	.35	.35	.35	.35	.35	.35	.35	.35	.35	.35	.35	.35	.35	.35	.35	.35	.35
1.500	.8782	H=	1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.38	.32	.26	.20
		V=	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39	.39
1.600	.9630	H=	1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.38	.32	.26	.20
		V=	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43
1.700	1.0441	H=	1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.38	.32	.26	.20
		V=	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47
1.800	1.1252	H=	1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.38	.32	.26	.20
		V=	.50	.50	.50	.50	.50	.50	.50	.50	.50	.51	.51	.51	.51	.51	.50	.50	.50	.50	.50	.50	.50
1.900	1.2063	H=	1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.38	.32	.26	.20
		V=	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54
2.000	1.2875	H=	1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.38	.32	.26	.20
		V=	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58	.58

2.1001.3686	H=1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.38	.32	.26	.20
	V= .61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61
2.2001.4474	H=1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.38	.32	.26	.20
	V= .65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65
2.3001.5202	H=1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.38	.32	.26	.20
	V= .68	.68	.68	.68	.68	.68	.68	.68	.68	.68	.68	.68	.68	.68	.68	.68	.68	.68	.68	.68	.68
2.4001.5929	H=1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.38	.32	.26	.20
	V= .71	.71	.71	.71	.71	.71	.71	.71	.71	.72	.72	.72	.72	.72	.72	.72	.71	.71	.71	.71	.71
2.5001.6656	H=1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.38	.32	.26	.20
	V= .75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75	.75
2.6001.7384	H=1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.38	.32	.26	.20
	V= .78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78	.78
2.7001.8111	H=1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.38	.32	.26	.20
	V= .81	.81	.81	.81	.81	.81	.81	.81	.81	.81	.81	.81	.81	.81	.81	.81	.81	.81	.81	.81	.81
2.8001.8839	H=1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.38	.32	.26	.20
	V= .84	.84	.84	.84	.84	.84	.84	.84	.84	.84	.84	.84	.84	.84	.84	.84	.84	.84	.84	.84	.84
2.9001.7622	H=1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.50	.44	.38	.32	.28	.24
	V= .87	.87	.87	.87	.87	.87	.87	.87	.87	.87	.87	.87	.87	.87	.87	.87	.87	.87	.87	.87	.87
3.0001.5534	H=1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.74	.68	.62	.56	.52	.48	.44	.40	.36	.32
	V= .90	.90	.90	.90	.90	.90	.90	.90	.90	.90	.90	.90	.90	.90	.90	.90	.90	.89	.89	.89	.88
3.1001.4132	H=1.40	1.34	1.28	1.22	1.16	1.10	1.04	.98	.92	.86	.80	.76	.72	.68	.64	.60	.56	.52	.48	.44	.40
	V= .93	.93	.93	.93	.93	.93	.93	.93	.93	.93	.93	.93	.92	.92	.92	.91	.91	.91	.90	.90	.90
3.2001.3114	H=1.40	1.34	1.28	1.22	1.16	1.10	1.04	1.00	.96	.92	.88	.84	.80	.76	.72	.68	.64	.60	.56	.52	.48
	V= .96	.96	.96	.96	.96	.96	.96	.96	.95	.95	.95	.94	.94	.94	.93	.93	.93	.92	.92	.92	.91
3.3001.2335	H=1.40	1.34	1.28	1.24	1.20	1.16	1.12	1.08	1.04	1.00	.96	.92	.88	.84	.80	.76	.72	.68	.64	.60	.57
	V= .99	.99	.99	.98	.98	.98	.97	.97	.97	.96	.96	.96	.95	.95	.95	.94	.94	.94	.93	.93	.93
3.4001.1718	H=1.40	1.38	1.36	1.32	1.28	1.24	1.20	1.16	1.12	1.08	1.04	1.00	.96	.92	.88	.84	.80	.76	.72	.68	.65
	V=1.00	1.00	1.00	1.00	.99	.99	.99	.98	.98	.98	.97	.97	.97	.97	.96	.96	.96	.95	.95	.95	.94
3.5001.1214	H=1.40	1.38	1.36	1.34	1.31	1.29	1.27	1.23	1.19	1.15	1.12	1.08	1.04	1.00	.96	.92	.88	.84	.80	.76	.73
	V=1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.99	.99	.99	.98	.98	.98	.97	.97	.97	.96	.96	.96
3.6001.0794	H=1.40	1.38	1.36	1.34	1.32	1.30	1.28	1.25	1.23	1.21	1.19	1.16	1.12	1.08	1.04	1.00	.96	.92	.88	.84	.81
	V=1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.99	.99	.99	.98	.98	.98	.97	.97
3.7001.0439	H=1.40	1.38	1.36	1.34	1.32	1.30	1.28	1.26	1.24	1.22	1.20	1.18	1.15	1.13	1.11	1.08	1.04	1.00	.96	.92	.88
	V=1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.99	.99	.99	.98	.98
3.8001.0133	H=1.40	1.38	1.36	1.34	1.32	1.30	1.28	1.26	1.24	1.22	1.20	1.18	1.16	1.14	1.12	1.10	1.08	1.06	1.04	1.00	.96
	V=1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.99
3.8251.0063	H=1.40	1.38	1.36	1.34	1.32	1.30	1.28	1.26	1.24	1.22	1.20	1.18	1.16	1.14	1.12	1.10	1.08	1.06	1.04	1.02	.98
	V=1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
3.8501.0000	H=1.40	1.38	1.36	1.34	1.32	1.30	1.28	1.26	1.24	1.22	1.20	1.18	1.16	1.14	1.12	1.10	1.08	1.06	1.04	1.02	1.00
	V=1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
3.8751.0000	H=1.40	1.38	1.36	1.34	1.32	1.30	1.28	1.26	1.24	1.22	1.20	1.18	1.16	1.14	1.12	1.10	1.08	1.06	1.04	1.02	1.00
	V=1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Figure 11. Computer Solution for Controlled Opening of Valve with Friction. The Computations were made for $\Delta t = .025$, with Every Fourth Calculation Printed.

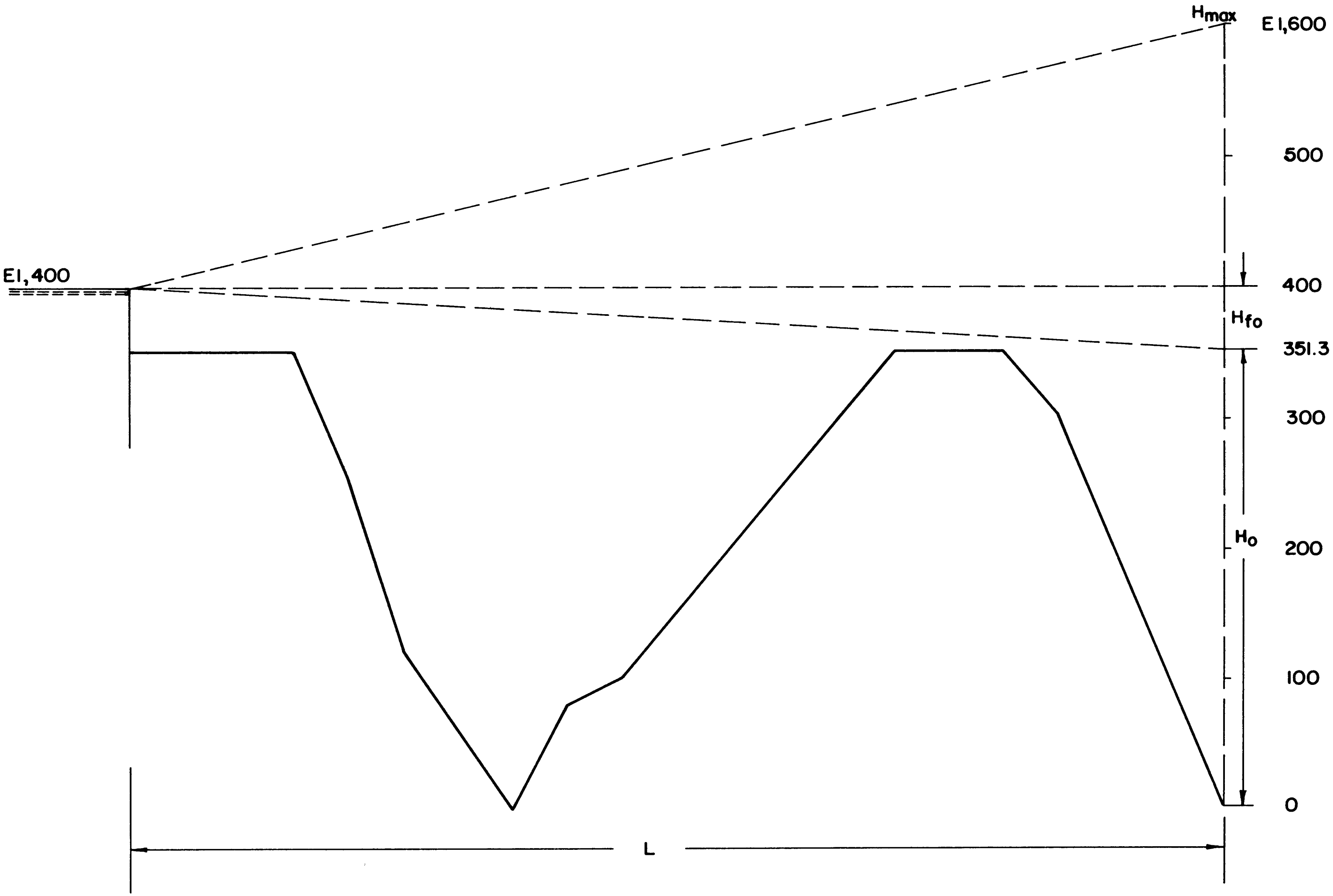

Figure 12. Profile of Pipeline of Variable Thickness.

The speed of sound wave in each section was computed by

$$a_i = \frac{\sqrt{K/\rho}}{\sqrt{1 + \frac{KD}{Et'_i}}}$$

in which t'_i is the thickness of the i-th section. Then for N sections, the mean wave speed is

$$\bar{a} = \frac{N}{\sum_{i=1}^{i=N} \frac{1}{a_i}}$$

The constant B is taken as $\sum B_i/N$. With these values of a and B to replace the variable values, the valve closure is handled as if it were a constant thickness pipe.

To determine how good this valve closure is, a computer program was written using the proper value of a and B for each of the twenty sections. As the speed of wave varies from one section to another, the time increment Δt must be small enough to give a convergent solution for the maximum wave speed. It is

$$\Delta t = \frac{\bar{a}}{2Na_{max}}$$

Since this time is not appropriate for the other sections, i.e., the characteristic line does not pass through, say p and A of Figure 2, an interpolation process is used. The computer solution, Figure 13, demonstrates that the valve closure is quite adequate to bring the flow to rest at time of closure.

It is believed the examples cited show the general method of application of the principles of valve stroking to control water hammer. The method may be applied to other situations than the ones discussed.

```
E = 4.320000E 09,          K = 4.320000E 07,          RHO =      1.935000,          S = 1.440000E 06
N =             20,        D =      20.000000,        ZRES =   400.000000,          F =        .016000
HMAX =    600.000000,      LHOR = 1.960000E 04,       G =       32.200000,          VO =     14.000000

L = 2.000000E 04
          X(1)...X(20)
1.000000E 03   2.000000E 03   3.000000E 03   4.000000E 03   5.000000E 03   6.000000E 03   7.000000E 03   7.200000E 03
9.000000E 03   1.000000E 04   1.100000E 04   1.200000E 04   1.220000E 04   1.320000E 04   1.420000E 04   1.500000E 04
1.700000E 04   1.800000E 04   1.900000E 04   1.960000E 04
          Z(1)...Z(20)
3.500000E 02   3.500000E 02   3.500000E 02   2.500000E 02   1.200000E 02   6.000000E 01    .000000E 00   8.000000E 01
1.000000E 02   1.500000E 02   2.000000E 02   2.500000E 02   3.000000E 02   3.500000E 02   3.500000E 02   3.500000E 02
3.000000E 02   2.000000E 02   1.000000E 02    .000000E 00
          THICK(1)...THICK(20)
8.333000E-02   8.333000E-02   8.333000E-02   8.333000E-02   1.432284E-01   1.736048E-01   2.039812E-01   1.702493E-01
1.695428E-01   1.523237E-01   1.351045E-01   1.178853E-01   9.713392E-02   8.333000E-02   8.432991E-02   8.786205E-02
1.183268E-01   1.660107E-01   2.136946E-01   2.596125E-01
          A(1)...A(20)
2.562452E 03   2.562452E 03   2.562452E 03   2.562452E 03   3.052280E 03   3.220891E 03   3.357499E 03   3.204033E 03
3.200433E 03   3.106806E 03   3.000172E 03   2.877372E 03   2.701536E 03   2.562452E 03   2.573244E 03   2.610417E 03
2.880755E 03   3.182164E 03   3.395926E 03   3.551144E 03
          B(1)...B(20)
3.171352E 00   3.171352E 00   3.171352E 00   3.171352E 00   3.777574E 00   3.986251E 00   4.155321E 00   3.965387E 00
3.960932E 00   3.845057E 00   3.713084E 00   3.561103E 00   3.343485E 00   3.171352E 00   3.184708E 00   3.230714E 00
3.565291E 00   3.938322E 00   4.202879E 00   4.394980E 00

AMAX =   3551.143890,      ABAR =   2901.160583,      BB =       3.634092,          TH =        .408483
DELT =       .020424,      HM =        1.707921,      HFO =       .138614,          TIME =    13.787586
HO =     351.304348
S =          .194800,      SS =         .156657,      V2 =        .812148,          V3 =        .156969
TC =        3.966751,      DVDT1 =    -.351010,       DVDT2 =    -.331226,          DVDT3 =    -.318719
```

DIMENSIONLESS VELOCITIES AND HEADS

TIME	TAU	X=	0	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95	1.0
.0001	.000	H=	1.14	1.13	1.12	1.12	1.11	1.10	1.10	1.09	1.08	1.08	1.07	1.06	1.06	1.05	1.04	1.03	1.03	1.02	1.01	1.01	1.00
P=0		V=	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
.082	.957	H=	1.14	1.13	1.12	1.12	1.11	1.10	1.10	1.09	1.08	1.08	1.07	1.06	1.06	1.05	1.04	1.03	1.03	1.03	1.04	1.05	1.06
P=0		V=	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.99	.99	.99
.163	.917	H=	1.14	1.13	1.12	1.12	1.11	1.10	1.10	1.09	1.08	1.08	1.07	1.06	1.06	1.05	1.05	1.06	1.07	1.08	1.09	1.11	1.12
P=0		V=	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.99	.99	.98	.98	.97	.97
.245	.879	H=	1.14	1.13	1.12	1.12	1.11	1.10	1.10	1.09	1.08	1.08	1.08	1.08	1.09	1.09	1.10	1.11	1.12	1.13	1.15	1.16	1.18
P=0		V=	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.99	.99	.98	.98	.97	.97	.96	.96	.95
.327	.845	H=	1.14	1.13	1.12	1.12	1.11	1.10	1.10	1.10	1.11	1.11	1.12	1.13	1.14	1.15	1.16	1.17	1.18	1.19	1.20	1.22	1.23
P=0		V=	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.99	.99	.99	.98	.98	.97	.97	.96	.95	.95	.95	.94	.94
.408	.812	H=	1.14	1.13	1.13	1.12	1.12	1.13	1.14	1.15	1.16	1.17	1.18	1.19	1.20	1.21	1.22	1.23	1.24	1.25	1.26	1.27	1.29
P=0		V=	1.00	1.00	1.00	1.00	1.00	.99	.99	.99	.98	.98	.97	.97	.96	.96	.95	.94	.94	.93	.93	.93	.92
.490	.781	H=	1.14	1.15	1.15	1.16	1.17	1.18	1.19	1.20	1.22	1.23	1.24	1.25	1.26	1.27	1.28	1.29	1.30	1.31	1.32	1.33	1.35
P=0		V=	.99	.99	.99	.99	.98	.98	.97	.97	.97	.96	.96	.95	.95	.94	.94	.93	.92	.92	.91	.91	.91

.572	.752	H=	1.14	1.16	1.18	1.20	1.21	1.23	1.24	1.26	1.27	1.28	1.30	1.31	1.32	1.33	1.34	1.35	1.36	1.37	1.38	1.39	1.40
P=0		V=	.97	.97	.97	.97	.96	.96	.96	.95	.95	.95	.94	.94	.93	.93	.92	.92	.91	.90	.90	.90	.89
.654	.724	H=	1.14	1.16	1.19	1.21	1.23	1.26	1.28	1.30	1.32	1.34	1.35	1.36	1.38	1.39	1.40	1.41	1.42	1.43	1.44	1.45	1.47
P=0		V=	.94	.94	.94	.94	.94	.94	.94	.94	.93	.93	.93	.92	.92	.91	.91	.90	.89	.89	.89	.88	.88
.735	.698	H=	1.14	1.16	1.19	1.21	1.23	1.26	1.29	1.32	1.35	1.37	1.40	1.41	1.43	1.44	1.45	1.46	1.47	1.48	1.50	1.51	1.53
P=0		V=	.91	.91	.91	.91	.91	.91	.91	.91	.91	.91	.91	.90	.90	.89	.89	.88	.88	.87	.87	.87	.86
.817	.673	H=	1.14	1.16	1.18	1.21	1.23	1.26	1.29	1.32	1.35	1.38	1.41	1.44	1.46	1.48	1.50	1.51	1.52	1.54	1.55	1.57	1.58
P=0		V=	.88	.88	.88	.88	.88	.88	.88	.88	.88	.88	.88	.88	.88	.88	.87	.87	.86	.86	.85	.85	.85
.899	.649	H=	1.14	1.16	1.18	1.21	1.23	1.26	1.29	1.32	1.36	1.39	1.42	1.45	1.48	1.50	1.52	1.54	1.56	1.58	1.60	1.62	1.64
P=0		V=	.85	.85	.85	.85	.85	.85	.85	.85	.85	.85	.85	.85	.85	.85	.85	.85	.84	.84	.84	.83	.83
.980	.627	H=	1.14	1.16	1.19	1.21	1.23	1.26	1.30	1.33	1.36	1.39	1.42	1.45	1.48	1.51	1.53	1.56	1.58	1.60	1.63	1.66	1.68
P=0		V=	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.82	.81	.81
1.062	.605	H=	1.14	1.16	1.19	1.21	1.24	1.27	1.30	1.33	1.37	1.40	1.43	1.46	1.48	1.51	1.53	1.56	1.58	1.61	1.64	1.67	1.70
P=0		V=	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79	.79
1.144	.583	H=	1.14	1.16	1.19	1.22	1.24	1.27	1.30	1.34	1.37	1.40	1.43	1.46	1.49	1.51	1.53	1.56	1.58	1.61	1.64	1.67	1.70
P=0		V=	.76	.76	.76	.76	.76	.76	.76	.76	.76	.76	.76	.76	.76	.76	.76	.76	.76	.76	.76	.76	.76
1.225	.561	H=	1.14	1.16	1.19	1.22	1.24	1.27	1.30	1.34	1.37	1.40	1.43	1.46	1.48	1.51	1.53	1.56	1.58	1.61	1.64	1.67	1.70
P=0		V=	.73	.73	.73	.73	.73	.73	.73	.73	.73	.73	.73	.73	.73	.73	.73	.73	.73	.73	.73	.73	.73
1.307	.539	H=	1.14	1.16	1.19	1.21	1.24	1.27	1.30	1.33	1.37	1.40	1.43	1.46	1.48	1.51	1.53	1.56	1.58	1.61	1.64	1.67	1.71
P=0		V=	.70	.70	.70	.70	.70	.70	.70	.70	.70	.70	.70	.70	.70	.70	.70	.70	.70	.70	.70	.70	.70
1.389	.517	H=	1.14	1.16	1.19	1.21	1.23	1.26	1.30	1.33	1.36	1.39	1.42	1.45	1.48	1.51	1.53	1.56	1.58	1.61	1.64	1.67	1.71
P=0		V=	.67	.67	.67	.67	.67	.67	.67	.67	.67	.68	.68	.68	.68	.68	.68	.68	.68	.68	.68	.68	.68
1.471	.495	H=	1.14	1.16	1.19	1.21	1.23	1.26	1.29	1.33	1.36	1.39	1.42	1.45	1.48	1.51	1.53	1.56	1.58	1.61	1.64	1.68	1.71
P=0		V=	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65
1.552	.473	H=	1.14	1.16	1.19	1.21	1.23	1.26	1.29	1.33	1.36	1.39	1.42	1.45	1.48	1.50	1.53	1.55	1.58	1.61	1.64	1.68	1.71
P=0		V=	.62	.62	.62	.62	.62	.62	.62	.62	.62	.62	.62	.62	.62	.62	.62	.62	.62	.62	.62	.62	.62
1.634	.451	H=	1.14	1.16	1.19	1.21	1.24	1.26	1.30	1.33	1.36	1.39	1.42	1.45	1.48	1.50	1.53	1.55	1.58	1.61	1.64	1.67	1.71
P=0		V=	.59	.59	.59	.59	.59	.59	.59	.59	.59	.59	.59	.59	.59	.59	.59	.59	.59	.59	.59	.59	.59
1.716	.431	H=	1.14	1.16	1.19	1.21	1.24	1.27	1.30	1.33	1.36	1.39	1.42	1.45	1.48	1.50	1.53	1.55	1.58	1.61	1.64	1.67	1.71
P=0		V=	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56
1.797	.410	H=	1.14	1.16	1.19	1.21	1.24	1.27	1.30	1.33	1.36	1.39	1.42	1.45	1.48	1.51	1.53	1.55	1.58	1.60	1.63	1.67	1.70
P=0		V=	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53
1.879	.389	H=	1.14	1.16	1.19	1.21	1.24	1.27	1.30	1.33	1.36	1.40	1.43	1.45	1.48	1.51	1.53	1.55	1.58	1.60	1.63	1.67	1.70
P=0		V=	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51
1.961	.369	H=	1.14	1.16	1.19	1.21	1.24	1.27	1.30	1.33	1.36	1.39	1.42	1.45	1.48	1.51	1.53	1.55	1.58	1.61	1.64	1.67	1.70
P=0		V=	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48
2.042	.348	H=	1.14	1.16	1.19	1.21	1.24	1.27	1.30	1.33	1.36	1.39	1.42	1.45	1.48	1.51	1.53	1.55	1.58	1.61	1.64	1.67	1.71
P=0		V=	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45
2.124	.327	H=	1.14	1.16	1.19	1.21	1.24	1.27	1.30	1.33	1.36	1.39	1.42	1.45	1.48	1.51	1.53	1.56	1.58	1.61	1.64	1.67	1.71
P=0		V=	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43	.43
2.206	.306	H=	1.14	1.16	1.19	1.21	1.23	1.26	1.30	1.33	1.36	1.39	1.42	1.45	1.48	1.51	1.53	1.56	1.58	1.61	1.64	1.67	1.71
P=0		V=	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40
2.288	.286	H=	1.14	1.16	1.19	1.21	1.24	1.26	1.30	1.33	1.36	1.39	1.42	1.45	1.48	1.51	1.53	1.56	1.58	1.61	1.64	1.68	1.71
P=0		V=	.38	.38	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37	.37

```
2.369 .266 H= 1.14 1.16 1.19 1.21 1.24 1.27 1.30 1.33 1.36 1.39 1.42 1.45 1.48 1.51 1.53 1.56 1.58 1.61 1.64 1.67 1.71
      P=0  V=  .35  .35  .35  .35  .35  .35  .35  .35  .35  .35  .35  .35  .35  .35  .35  .35  .35  .35  .35  .35  .35

2.451 .246 H= 1.14 1.16 1.19 1.21 1.24 1.27 1.30 1.33 1.36 1.39 1.42 1.45 1.48 1.51 1.53 1.56 1.58 1.61 1.64 1.67 1.71
      P=0  V=  .32  .32  .32  .32  .32  .32  .32  .32  .32  .32  .32  .32  .32  .32  .32  .32  .32  .32  .32  .32  .32

2.533 .226 H= 1.14 1.16 1.19 1.21 1.24 1.27 1.30 1.33 1.37 1.40 1.43 1.46 1.48 1.51 1.53 1.56 1.58 1.61 1.64 1.67 1.71
      P=0  V=  .30  .30  .30  .30  .30  .30  .30  .30  .30  .30  .30  .30  .30  .30  .30  .30  .30  .30  .30  .30  .30

2.614 .206 H= 1.14 1.16 1.19 1.21 1.24 1.27 1.30 1.33 1.37 1.40 1.43 1.45 1.48 1.51 1.53 1.56 1.58 1.61 1.64 1.67 1.71
      P=0  V=  .27  .27  .27  .27  .27  .27  .27  .27  .27  .27  .27  .27  .27  .27  .27  .27  .27  .27  .27  .27  .27

2.696 .186 H= 1.14 1.16 1.19 1.21 1.24 1.27 1.30 1.33 1.36 1.39 1.42 1.45 1.48 1.51 1.53 1.56 1.58 1.61 1.64 1.67 1.71
      P=0  V=  .24  .24  .24  .24  .24  .24  .24  .24  .24  .24  .24  .24  .24  .24  .24  .24  .24  .24  .24  .24  .24

2.778 .166 H= 1.14 1.16 1.19 1.21 1.24 1.27 1.30 1.33 1.36 1.39 1.42 1.45 1.48 1.51 1.53 1.56 1.58 1.61 1.64 1.68 1.71
      P=0  V=  .22  .22  .22  .22  .22  .22  .22  .22  .22  .22  .22  .22  .22  .22  .22  .22  .22  .22  .22  .22  .22

2.859 .146 H= 1.14 1.16 1.19 1.21 1.24 1.27 1.30 1.33 1.36 1.39 1.42 1.45 1.48 1.51 1.53 1.56 1.58 1.61 1.64 1.68 1.71
      P=0  V=  .19  .19  .19  .19  .19  .19  .19  .19  .19  .19  .19  .19  .19  .19  .19  .19  .19  .19  .19  .19  .19

2.941 .126 H= 1.14 1.16 1.19 1.21 1.24 1.27 1.30 1.33 1.36 1.39 1.42 1.45 1.48 1.51 1.53 1.56 1.58 1.61 1.64 1.68 1.71
      P=0  V=  .17  .17  .17  .17  .17  .17  .17  .17  .17  .17  .17  .17  .17  .17  .17  .17  .17  .17  .17  .17  .17

3.023 .114 H= 1.14 1.16 1.19 1.21 1.24 1.27 1.30 1.33 1.36 1.39 1.42 1.45 1.48 1.51 1.53 1.56 1.58 1.61 1.64 1.66 1.67
      P=0  V=  .14  .14  .14  .14  .14  .14  .14  .14  .14  .14  .14  .14  .14  .14  .14  .14  .14  .14  .14  .14  .15

3.104 .106 H= 1.14 1.16 1.19 1.21 1.24 1.27 1.30 1.33 1.36 1.39 1.42 1.45 1.48 1.51 1.53 1.55 1.56 1.57 1.59 1.60 1.62
      P=0  V=  .11  .11  .11  .11  .11  .11  .11  .11  .11  .11  .11  .11  .11  .11  .11  .12  .12  .12  .13  .13  .13

3.186 .097 H= 1.14 1.16 1.19 1.21 1.24 1.27 1.30 1.33 1.36 1.40 1.42 1.45 1.47 1.48 1.49 1.50 1.52 1.53 1.54 1.56 1.57
      P=0  V=  .09  .09  .09  .09  .09  .09  .09  .09  .09  .09  .09  .09  .09  .10  .10  .10  .11  .11  .12  .12  .12

3.268 .088 H= 1.14 1.16 1.19 1.21 1.24 1.27 1.30 1.33 1.36 1.38 1.39 1.41 1.42 1.43 1.45 1.46 1.47 1.48 1.50 1.52 1.53
      P=0  V=  .06  .06  .06  .06  .06  .06  .06  .06  .06  .07  .07  .08  .08  .08  .09  .09  .10  .10  .10  .11  .11

3.350 .079 H= 1.14 1.16 1.19 1.21 1.23 1.26 1.28 1.29 1.31 1.33 1.34 1.36 1.37 1.39 1.40 1.41 1.43 1.44 1.46 1.47 1.49
      P=0  V=  .04  .04  .04  .04  .04  .04  .04  .05  .05  .05  .06  .06  .07  .07  .07  .08  .08  .09  .09  .09  .10

3.431 .070 H= 1.14 1.16 1.17 1.19 1.20 1.22 1.23 1.25 1.26 1.28 1.29 1.31 1.33 1.34 1.35 1.37 1.38 1.40 1.41 1.43 1.45
      P=0  V=  .01  .01  .02  .02  .02  .03  .03  .03  .04  .04  .04  .05  .05  .06  .06  .07  .07  .07  .08  .08  .08

3.513 .060 H= 1.14 1.14 1.14 1.15 1.16 1.17 1.19 1.20 1.22 1.23 1.25 1.26 1.28 1.29 1.31 1.32 1.33 1.35 1.36 1.38 1.40
      P=0  V=  .00  .00  .01  .01  .01  .01  .02  .02  .02  .03  .03  .03  .04  .04  .05  .05  .06  .06  .06  .07  .07

3.595 .050 H= 1.14 1.14 1.14 1.14 1.14 1.14 1.15 1.16 1.18 1.19 1.21 1.22 1.23 1.25 1.26 1.27 1.28 1.30 1.31 1.33 1.35
      P=0  V=  .00  .00  .00  .00  .00  .01  .01  .01  .01  .01  .02  .02  .02  .03  .03  .04  .04  .05  .05  .05  .06

3.676 .040 H= 1.14 1.14 1.14 1.14 1.14 1.14 1.14 1.15 1.15 1.16 1.17 1.18 1.19 1.20 1.21 1.22 1.24 1.25 1.26 1.28 1.30
      P=0  V=  .00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .01  .01  .01  .02  .02  .03  .03  .03  .04  .04  .05

3.758 .029 H= 1.14 1.14 1.14 1.15 1.15 1.15 1.15 1.15 1.15 1.15 1.15 1.15 1.15 1.16 1.17 1.18 1.19 1.20 1.22 1.23 1.25
      P=0  V=  .00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .01  .01  .01  .02  .02  .03  .03  .03

3.840 .018 H= 1.14 1.14 1.14 1.14 1.15 1.15 1.15 1.15 1.15 1.15 1.14 1.14 1.14 1.14 1.14 1.15 1.15 1.16 1.17 1.19 1.21
      P=0  V= -.00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .01  .01  .01  .01  .01  .02  .02

3.921 .007 H= 1.14 1.14 1.14 1.14 1.14 1.14 1.14 1.14 1.14 1.14 1.14 1.14 1.14 1.14 1.14 1.14 1.14 1.14 1.15 1.16 1.18
      P=0  V= -.00 -.00 -.00 -.00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .01  .01  .01

                P =           1,        TCLOS =      3.982701

4.003 .000 H= 1.14 1.14 1.14 1.14 1.13 1.13 1.13 1.13 1.13 1.13 1.13 1.13 1.14 1.14 1.14 1.14 1.15 1.15 1.16 1.16 1.16
      P=1  V= -.00 -.00 -.00 -.00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .00  .00 -.00 -.00  .00
8**** ALL INPUT DATA HAVE BEEN PROCESSED.
      AT LOC 13565
```

Figure 13. Computer Solution for Variable Thickness Penstock. The Actual Closure Time is 13.79 sec, and the Dimensionless Time in Increment for Calculations 0.0204 with Each Fourth Calculation Printed.

Effect of Errors in Valve Stroking

Analytical equations have been worked out for valve stroking to control the resulting water hammer. The question arises as to the effect of errors in following the analytical relations. To check on this, one valve closure case was deliberately thrown in error by changing every τ by the factor

$$1 + 0.05 \sin\left(\frac{4\pi t}{t_c}\right)$$

which causes a variable error up to $\pm$ 5% in the valve area. For the case of closure: B = 30, hfo = 0.4, h_m = 4., the maximum head at the valve became 4.42 in place of the design value of 4.0. At the pipe midpoint the maximum head was 2.95 in place of 2.70. The velocity remained remarkably uniform in the pipe (within 1%). During the final stages of closing the velocity took on the value v = - .01 in the upstream half of the pipe, but went to zero (within 1%) at time of closing. This demonstrates that it is unnecessary to achieve great accuracy in the closing to obtain highly beneficial results in reducing water hammer fluctuations.

SUMMARY AND CONCLUSIONS

A first order computer method for constant thickness pipe has been derived in this paper. Methods have been developed for closing valves and for opening valves so that the resulting water hammer is predetermined. To demonstrate the effectiveness of the valve motion relations several problems have been programmed using the first order method with includes the effects of friction.

The design of a pipe line of variable thickness was undertaken by using averaged values of wave speed and the parameter aV_o/gH_o. After determining the relations for closing a valve to control water hammer in this situation, its accuracy was checked out by computer.

Finally, a check was made on the effect of inaccurably closing a valve by deliberately changing the curve by a variable amount up to $\pm$ 5%. This increased the head of the valve about 10%, but no appreciable backflow or reflections occured.

In conclusion, new opening and closing relationships have been developed that reduce stresses in pipes, and take out vibration and reversal of flow due to water hammer. The next step is the development of actual valves that permit these improvements to be realized.

ACKNOWLEDGMENTS

The author wishes to express his appreciation to the University of Michigan Computing Center for use of their facilities, and to the Industry Program of the College of Engineering for preparing the manuscript and illustrations.

ABSTRACT

A method for calculating water hammer based on the theory of characteristics has been developed for first order accuracy, using a high-speed digital computer. The operation of valves in such a manner that the resulting water hammer is controlled has been studied in the remainder of the paper. Two cases of valve closing have been examined: one for a single pipe and the other for a pipe with thickness changing from section to section. Valve opening has also been studied. The controlled flows have certain phases of acceleration or deceleration in which the flow is uniform and the hydraulic grade line steady. Friction has been taken into account in the valve stroking, and checked out by the computer method.

REFERENCES

1. Parmakian, J. "Waterhammer Analysis," Prentice-Hall, Inc. New York, 1955.

2. Angus, R. W. "Water-hammer Pressures in Compound and Branched Pipes," Trans. ASCE, vol. 104 (1939) 340-401.

3. Rich, George R. "Water-hammer Analysis by the Laplace-Mellin Transformation," Trans. ASME, 1945.

4. Sarlat, I. M., and T. L. Wilson, "Surge Pressures in Liquid Transfer Lines," ASME Paper No. 61-WA-79, presented at annual meeting Nov. 26 - Dec. 1, 1961.

5. Wood, F. M. "The Application of Heavisides Operational Calculus of the Solutions of Problems in Water Hammer," Trans. ASME. 59, Paper Hyd-59-15 (Nov. 1937) 707-713.

6. Streeter, V. L., and Chintu Lai, "Water Hammer with Fluid Friction," Proc. ASCE, J. of the Hydraulics Div., April 1962.

7. Lister, M. The Numerical Solution of Hyperbolic Partial Differential Equations by the Method of Characteristics, in "Mathematical Methods for Digital Computers," ed. by A. Ralston and H. S. Wilf, New York: John Wiley and Sons, Inc. 1960.

8. Rich, George R. "Hydraulic Transients" New York: McGraw-Hill Book Company, Inc. 1951.

NOTATION

a	Speed of sound wave in pipe
A	Pipe cross sectional area
A_G	Area of valve opening
B	Dimensionless parameter aV_o/gH_o
C_d	Discharge coefficient
D	Diameter of pipe
E	Young's modulus of elasticity
f	Darcy-Weisbach friction factor
g	Acceleration of gravity
h	Dimensionless head H/H
h_m	Dimensionless maximum (or minimum) head, H_{max}/H_o or H_{min}/H_o
hfo	Dimensionless steady state friction, H_{fo}/H_o
H	Head in pipe
H_f	Head loss due to friction in length L
K	Bulk modulus of elasticity of liquid
L	Length of pipe
L_1	Label for equation of motion
L_2	Label for continuity equation
n	Exponent on velocity term
N	Number of equal pipe reaches
Q	Discharge through valve
s	Dimensionless parameter
ss	Dimensionless parameter

t	Dimensionless time $T/(2L/a)$
t_c	Time of closure, dimensionless
t_o	Time of opening, dimensionless
t'	Pipe wall thickness
v	Dimensionless velocity V/V_o
V	Velocity in pipe
x	Dimensionless pipe distance X/L
X	Distance along pipe
γ	Specific weight of liquid
λ	Unknown multiplier
ρ	Mass density of liquid
τ	Dimensionless gate opening, $C_DA_G/(C_DA_G)$.
τ_o	Fluid shear stress at pipe wall