WATERHAMMER ANALYSIS OF DISTRIBUTION SYSTEMS

Victor L. Streeter

October, 1966

IP-748
ACKNOWLEDGEMENTS

This study has been supported by NSF Grant GP-340 to the University of Michigan.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>vii</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>HARDY CROSS METHOD FOR BALANCING HEADS</td>
<td>3</td>
</tr>
<tr>
<td>ALGEBRAIC WATERHAMMER EQUATIONS</td>
<td>6</td>
</tr>
<tr>
<td>APPLICATION OF THE ALGEBRAIC METHOD TO A TYPICAL JUNCTION</td>
<td>8</td>
</tr>
<tr>
<td>DESCRIPTION OF COMPUTER PROGRAM AND EXAMPLE PROBLEM</td>
<td>11</td>
</tr>
<tr>
<td>SUMMARY AND CONCLUSIONS</td>
<td>17</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>17</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Simple circuit to which head balancing is applied</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>Pipe of length L and wave speed a, with positive flow direction shown by arrow</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>Junction 91 having four pipes and a valve outlet</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>Example system comprised of 45 pipes, 26 junctions, 20 simple circuits and 19 valved outlets. Junction numbers are encircled. Assumed positive flow direction shown by arrows</td>
<td>19</td>
</tr>
<tr>
<td>5</td>
<td>Computer program for balancing heads by Hardy Cross Method and for calculating transients by algebraic waterhammer equations</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>Input data for first read statement</td>
<td>21</td>
</tr>
<tr>
<td>7</td>
<td>Input data for seconds read statement</td>
<td>21</td>
</tr>
<tr>
<td>8</td>
<td>Transient solution for problem with data given in Figures 6 and 7</td>
<td>22</td>
</tr>
<tr>
<td>9</td>
<td>Transients caused by rupture at junction 21 when reservoir is at 200 feet</td>
<td>23</td>
</tr>
</tbody>
</table>
INTRODUCTION

The great speed of a modern large digital computer permits very complex piping systems to be analyzed for transient flow provided a method for describing the network can be adapted to computer usage. For most systems the friction may be adequately treated either as proportional to the square of the velocity, or to some exponential value of the velocity. In the algebraic method, the end boundary condition equations for the $C^+$ and $C^-$ characteristics\(^1\) developed from the method of characteristics equations are applied over the whole pipe length between two junctions. In effect this "lumps" the friction for the pipe as that given by the velocity at the opposite end of the pipe one wave-travel time previous to calculation of events at a junction. In case certain pipes are very long compared with others, any interior sections of those pipes may be considered as junctions. The combination of algebraic waterhammer equations plus indexing of the pipes leaving every junction permits a simple program to be prepared for complex systems. Each pipe is divided into a number $P_i$ of equal reaches such that the wave travel time $Dt$ for each reach is the same for every reach in the system. Hence $P_i \cdot Dt = L_i/a_i$ in which $L_i$ is the length of pipe $i$ and $a_i$ is the pulse wave speed in that pipe. This is a limitation on the method, but since $Dt$ may be selected so that a pipe has up to 10 reaches, suitable values of $P_i$ can usually be found for each pipe by minor adjustments of pipe lengths and of wave speeds. In general, wave speeds are not known with precision and small adjustments are permissible.

Steady-state initial conditions in the piping system must first be established before solving the transient problem. For very large systems
the steady-state heads and flows should be found separately to conserve storage for the transient solution. For the example given both the steady-state determination and the transient solution are combined. The program will handle head-balancing and transient solution for up to about 300 pipes. Many works have been published on steady-state solution of networks; however, for the purposes of illustrating the transient methods, the Hardy Cross method\textsuperscript{2} of balancing heads is used in this treatment.

The method of balancing heads is first discussed, then the algebraic equations are developed. They are next applied to a typical junction of a network. The computer program is described and then an example problem is given consisting of 20 simple circuits, 26 junctions and 45 pipes.
HARDY CROSS METHOD FOR BALANCING HEADS

The method for balancing heads, as developed by Hardy Cross, is discussed briefly with regard to its application to use by the computer. In this method, the elevation of hydraulic gradeline is known at one point in the system, and the inflows and outflows from the system are given. In addition the resistance properties of each pipe must be known. The head loss $H_i$ in pipe $i$ is given by

$$H_i = R_i Q_i |Q_i|^{n_i-1} \quad (1)$$

in which $R_i$ is the head drop over the length of pipe for unit discharge; $n_i$ is the exponent in the head-loss equation, $Q_i$ is the discharge. Each pipe in the network is numbered and a positive flow direction indicated in an arbitrary manner. If the flow changes direction, Equation (1) changes sign.

To start the balancing procedure which yields the correct flow in each pipe, flow is assumed through each pipe such that continuity is satisfied at every junction. This can be accomplished rapidly by starting at a junction with known inflow. Many pipes may be assumed to have zero flow, since it is not important that values close to the final correct values be assumed. Continuity must be exactly satisfied at every junction, however, because it is not upset or improved by subsequent steps in the balancing procedure.

The complex network is treated as a collection of simple circuits. In balancing a simple circuit, Equation (1) is applied to each pipe and the net head loss around the circuit is determined. In Figure 1 pipes are shown with their assumed flow directions; as $Q$'s have been assumed for each pipe, the head drop around the system is given by

$$\sum_{i=1}^{4} H_i = \sum_{i=1}^{4} R_i Q_i |Q_i|^{n_i-1} \quad (2)$$
The clockwise direction may be assumed as positive; if \( \Sigma H_i = 0 \), then the heads are balanced and no correction to the flows are needed. When \( \Sigma H_i \) does not equal zero, a correction to the discharge \( \Delta Q \) is calculated and applied to every pipe in the simple circuit in either the clockwise or the counterclockwise direction depending upon the sign of Equation (2). To obtain the correction \( \Delta Q \), Equation (1) is differentiated

\[
\frac{\Delta H_i}{\Delta Q_i} = n_i R_i |Q_i|^{n_i-1}
\]

and \( \Delta H_i / \Delta Q_i \) is summed for every pipe in the simple circuit with all terms positive. Then the corrective flow \( \Delta Q \) is given by

\[
\Delta Q = \frac{\Sigma H_i}{\Sigma (\Delta H_i / \Delta Q_i)} = \frac{\Sigma R_i Q_i |Q_i|^{n_i-1}}{\Sigma n_i R_i |Q_i|^{n_i-1}}
\]

If the \( \Sigma H_i \) in the clockwise direction is positive, then the flow in each pipe should be decreased by \( \Delta Q \) in the clockwise direction, e.g. in Figure 1 the flow in pipe 1 would be decreased by \( \Delta Q \) and the flow in pipes 2, 3, and 4 would be increased by the same \( \Delta Q \).

Each simple circuit is corrected in this manner, in any convenient sequence; then the procedure is repeated until the sum of the absolute values of \( \Delta Q \) for all simple circuits becomes less than some predetermined tolerance. After the heads are balanced in all simple circuits, the elevation of hydraulic gradeline is calculated for each junction, starting with the junction with given head.

For determination of steady-state flows for different boundary conditions, other balancing techniques may be needed. The transient equations, developed in the next section, may also be used to find steady-state initial conditions. Whether this is an economical way depends upon the
relative cost of engineering time to program the steady-state solution compared with cost of computer time. To do this, for example, all flows could be taken as zero, all heads equal to reservoir head, then the assumed outflows started instantaneously at time zero. Steady-state conditions will be approached after several round-trip wave travel times across the network.
ALGEBRAIC WATERHAMMER EQUATIONS

Solution of the partial differential equations of waterhammer by the method of characteristics leads to two finite difference equations, one valid along the C\(^+\) characteristic and the other valid along the C\(^-\) characteristic. When expressed in terms of discharge \(Q\) and elevation of hydraulic gradeline \(H\), applied to the pipe of Figure 2, they take the form

\[
C^+: \quad H_B(t) = H_A\left(t - \frac{L}{a}\right) - \frac{a}{gA} \left[ Q_B(t) - Q_A\left(t - \frac{L}{a}\right) \right] - \frac{f}{2DgA} \frac{a}{Q_A\left(t - \frac{L}{a}\right)} Q_A\left(t - \frac{L}{a}\right) \left| \frac{n-1}{n} \frac{L}{a} \right|^n
\]

\[
C^-: \quad H_A(t) = H_B\left(t - \frac{L}{a}\right) + \frac{a}{gA} \left[ Q_A(t) - Q_B\left(t - \frac{L}{a}\right) \right] + \frac{f}{2DgA} \frac{a}{Q_B\left(t - \frac{L}{a}\right)} Q_B\left(t - \frac{L}{a}\right) \left| \frac{n-1}{n} \frac{L}{a} \right|^n
\]

\(a\) is the wave speed, \(A\) the pipe cross sectional area, \(f\) the friction factor, \(L\) the pipe length, \(t\) the time, \(D\) the pipe diameter, and \(g\) the acceleration due to gravity. These equations are valid for any time \(t\), in steady or unsteady flow, as long as the pipe remains filled with liquid.

In general, each equation has two unknowns, \(Q_B\) and \(H_B\) at time \(t\) for Equation (5), and \(Q_A\) and \(H_A\) at time \(t\) for Equation (6).

For algebraic waterhammer these equations are simplified by collecting the constants.

\[
B_i = \frac{gA}{a_i} \quad R_i = \frac{f_i L_i}{2DgA_i^2}
\]

The term \(R_i\) is the same head loss coefficient used in balancing head in the Hardy Cross method. Each pipe is divided into \(P_i\) equal reaches such that

\[
P_i Dt = \frac{L_i}{a_i}
\]
with $Dt$ the time increment for the wave to travel through any reach of
the system. The discharge $Q$ for a pipe is triple subscripted $Q(J_1,J_2,J)$
in which $J_1$ is 2 if the downstream end of the pipe is designated and is 1
if the upstream end of the pipe is designated, $J_2$ is an integer representing
the pipe number, and $J$ is an integer representing the time

$$T = (J + JC)Dt$$

with $JC$ a constant integer. The junction heads are double subscripted,
$H(E,J)$ with $E$ the junction number and $J$ the time counter. In this notation,
Equations (5) and (6) become

$$C^+ : \quad Q(2,J_2,J) = Q(1,J_2,J-P(J_2)) + B(J_2) \left[ H(E_A,J-P(J_2))-R(J_2)Q(1,J_2,J-P(J_2)) \right]
\left[Q(1,J_2,J-P(J_2))^{n-1}\right] - B(J_2) \ H(E_B,J)$$

$$C^- : \quad Q(1,J_2,J) = Q(2,J_2,J-P(J_2)) - B(J_2) \left[ H(E_B,J-P(J_2))+R(J_2)Q(2,J_2,J-P(J_2)) \right]
\left[Q(2,J_2,J-P(J_2))^{n-1}\right] + B(J_2) \ H(E_A,J)$$

These two equations, which are available for each pipe at any time, are
used together with continuity at a junction, and a valve equation (if an
outlet is present) to calculate the elevation of hydraulic gradline at the
junction and the flow through each pipe into the junction.
APPLICATION OF THE ALGEBRAIC METHOD TO A TYPICAL JUNCTION

In Figure 3 junction 91 has four pipes connecting it with a valve outlet; two of the pipes, 7 and 13 have the sign convention for flow into the junction while the other two, 17 and 23 have the sign convention indicating flow out of the junction. The head $H(91, J)$ and the flows $Q(2, 7, J)$, $Q(2, 13, J)$, $Q(1, 17, J)$ and $Q(1, 23, J)$ are unknowns to be found at the current time. There is an equation for each pipe, Equation (10) for pipes 7 and 13 and Equation (11) for pipes 17 and 23 as well as the equation of continuity for the junction, so for no valve outlet five equations in five unknowns are at hand. With a valve, as shown, an additional equation for flow through the valve $QV(91, J)$ for given opening is available for determination of the additional unknown $QV(E, J)$.

In Equation (10), applied to pipe 7

$$Q(2, 7, J) = C1(7) - B(7)H(91, J)$$  \hspace{1cm} (12)

in which $C1(7)$ is comprised of the first two terms on the right-hand side of Equation (10), all being known from previous calculations. For pipe 17, Equation (11) applies and yields for flow into the junction

$$-Q(1, 17, J) = C1(17) - B(17)H(91, J)$$  \hspace{1cm} (13)

in which $C1(17)$ is the negative of the first two terms on the right-hand side of Equation (11), a known quantity.

For no valve at the junction the sum of the discharges into the junction must be zero. After summing up all the flows into the junction

$$\sum Q_i = 0 = \sum C1_i - H(91, J)\Sigma B_i$$ \hspace{1cm} (14)

Hence

$$H(91, J) = \frac{\Sigma C1_i}{\Sigma B_i}$$ \hspace{1cm} (15)

Then from Equations (12) and (13) the discharges are calculated.
If junction 91 of Figure 3 has an outlet to atmosphere controlled by a valve, then the net inflow to the junction through the four pipes must just equal the flow through the valve. It is convenient to express the valve opening by a dimensionless coefficient $\tau$. First by writing the orifice equation for the valve for steady-state

$$Q_{VV}(E) = (C_{D} A_{V})_0 \sqrt{2g \text{ HH}(E)}$$ (16)

in which, for convenience, the network is assumed to be in a horizontal plane and the elevation datum for hydraulic gradelines is the system elevation. In general for non-horizontal systems

$$Q_{VV}(E) = (C_{D} A_{V})_0 \sqrt{2g (\text{HH}(E) - \text{EL}(E))}$$ (16a)

in which $\text{EL}(E)$ is the junction elevation based on the same datum as the hydraulic gradelines. For any flow through the system

$$Q_{V}(E,J) = C_{D} A_{V} \sqrt{2g \text{ H}(E,J)}$$ (17)

or, for the more general case

$$Q_{V}(E,J) = C_{D} A_{V} \sqrt{2g (\text{H}(E,J) - \text{EL}(E))}$$ (17a)

After dividing Equation (17) by Equation (16)

$$Q_{V}(E,J) = \tau \text{VCN}(E) \sqrt{\text{H}(E,J)} = CV \sqrt{\text{H}(E,J)}$$ (18)

or dividing Equation (17a) by Equation (16a)

$$Q_{V}(E,J) = \tau \text{VC}(E) \sqrt{\text{H}(E,J) - \text{EL}(E)}$$ (18a)

The dimensionless valve coefficient $\tau$ is given by

$$\tau = \frac{C_{D} A_{V}}{(C_{D} A_{V})_D}$$ (19)

in either case and the other constants are collected into

$$\text{VCN}(E) = \frac{Q_{VV}(E)}{\sqrt{\text{HH}(E)}}$$ (20)

or

$$\text{VC}(E) = \frac{Q_{VV}(E)}{\sqrt{\text{HH}(E) - \text{EL}(E)}}$$ (20a)
\( \tau \) is unity for steady-state flow and goes to zero as the valve is closed. \( CV \) in equation (13) is \( \tau \) \( VCN(E) \).

By writing the continuity equation including the valve

\[
\Sigma q_1 - QV(91, J) = 0 = \Sigma c_{11} - H(91, J)EB_1 - CV \sqrt{H(91, J)}
\]

which yields a quadratic equation in \( H(91, J) \) only and may easily be solved.

The transient solution starts with steady-state at time \( t = 0 \), the time is incremented to \( Dt \), then each junction is solved for heads and flows as indicated, in any order. Special boundary conditions are then solved, such as the flows from junctions of fixed hydraulic gradelines. The results may then be printed out, the time incremented by \( Dt \) and the procedure repeated, taking the valve openings as function of \( t \), i.e. \( \tau(E, J) \). Special boundary conditions such as one or more pumping stations may be included.
DESCRIPTION OF COMPUTER PROGRAM AND EXAMPLE PROBLEM

A network, Figure 4, is used to illustrate the methods. It contains 20 simple circuits, 26 junctions, and 45 pipes. All inflow is at junction 26 where the elevation of hydraulic grade line is held at 500 feet by the presence of a reservoir. Ten outlets, numbered 1 to 10 discharge from the system to atmosphere through valves. These valves may be stroked in any manner, the t-time data being supplied as input data to the program. Since Hardy Cross balancing may be accomplished using relative values of flow and of pipe resistances, the inflow is taken as 100 and the desired steady-state outflow from the ten outlets is given as input data that must add to exactly 100. Each pipe is numbered in any manner with the pipe number shown at its midpoint. An arrow is drawn along each pipe to establish a positive flow direction. The relative pipe resistances R(1), . . . R(45) are given in the input data. They may be the actual resistances, from Equation (1), calculated from Darcy Weisbach friction factors or from the various exponential formulas. The exponent may vary for each pipe as well as the resistance coefficient.

The steady-state flows through each pipe and the junction head determinations are first considered. By starting at junction 26 flows are assumed through each pipe such that continuity is satisfied at each junction. These are shown in the input data as QQ(1) = 60., 40., 10., etc. listing the flow through each pipe from 1 through 45. The apportioning of flows is rapidly accomplished and does not need to be close to the final answer for the method to converge. One may, for example, select flows such that 28 of the lines are initially assumed to have no flow, and still satisfy continuity at every junction.
The computer program, written in MAD compiler language (Michigan Algorithm Decoder), Figure 5, contains two read statements. The first read statement, No. 001 inputs circuit information for the Hardy Cross steady-state balancing. The input data for this statement is shown in Figure 6. The first part of the program, statements 001 through 007 solves for the steady-state initial conditions. Statement 002 is used in locating integers in the series \(X(1) = 1, 2, 2, \ldots\) which describes the simple circuits. For example \(G(4)\) is the number of integers needed to describe the 3-sided circuits, \(G(5)\) equals \(G(4)\) plus the number of integers needed to describe the 4-sided circuits, etc.. Statement 003 sets all exponents equal to the given exponent \(NN\) in the head loss term. Each \(NN(I)\) may be placed in storage directly with input data if they vary.

Statement 004 is a compound iteration statement that solves Equations (2), (3), and (4) then corrects the flows, checks to determine if the tolerance has been met, and continues to balance all the circuits until it has gone through them \(Z\) times or until the desired tolerance is reached. Statement 005 converts the relative \(R\)'s by multiplying by \(SS\), and converts the relative discharges to actual discharges \(QQ(I)\) by multiplying by \(SF\). Statement 006 calculates the heads at each junction of the system.

The indexing system for specifying the circuits starts with \(X(1) = 1, 2, 2, 12, 11, 3\). These first 6 integers specify the 3-sided circuit comprised of pipes 12, 11, and 3. The first number 1 indicates flow in pipe 12 is clockwise, the seconds number 2 indicates the flow in pipe 11 is counterclockwise and the third number 2 indicates the flow in pipe 3 is counterclockwise. In the second line of statement 004 starting \(J = 0, 1\), etc., \(W\) determines the clockwise or counterclockwise term, and \(Y\) the pipe
number. QP collects certain terms that are used in the next two terms, DH collects \( \Sigma H_1 \) and HD is \( \Sigma dH_i/dQ_i \). Then \( DQ \) the correction is calculated, and the absolute \( DQ \)'s summed for all circuits. Finally the Q's are corrected for each circuit. This is continued until the tolerance is satisfied or \( Z \) is reached. In accomplishing the balancing, statement 004 takes up all 3-sided circuits in the order given by the data in \( X(1) = \ldots \), then continues to the 4-sided circuits, etc.

Statement 006 uses the indexing given by the data for \( XX(1) = 1,1,12,5,1,14 \ldots \) \( H = 500 \) is given as elevation of hydraulic gradeline at junction 26. With reference to Figure 4, the first 3 numbers of \( XX(1) \) indicate in order, the pipe number, the flow direction (1 if toward the next junction, 2 if toward the previous junction) and the junction at which the elevation of hydraulic gradeline is being computed. Similarly 5,1,14 gives indexing information for calculation of head at junction 14, etc.

By judicious selection of order of junctions, all elevations of hydraulic gradelines are calculated and the final one is junction 26 which checks the program. In statement 006, \( L \) is the junction number, \( Y \) is the sign on the head loss term and \( Z \) is the pipe number.

The print results statement, 007, prints out the final steady-state discharges through each pipe and the elevation of hydraulic gradeline at each junction.

The second read statement, 008, provides the additional data needed for the transient calculation. These data are shown in Figure 7. Statement 009 sets initial high values for minimum HGL elevation and low values for maximum HGL elevations, which are subsequently adjusted to their proper values. Statement 010 is similar to statement 002, now acting for the 2nd set of \( X(I) \) data read in. It computes the number of integers in
X(I), i.e. G(2) = 0 is given in the data, G(3) is the number of junctions with 2 pipes times the number of integers needed to describe the junction. For a junction having two pipes 8 numbers are needed. \( X(1) = 10, 1, 7, 14, 1, 8, 15, 2 \) states, in order, junction 10, 1 means a valve is present (0 for no valve), pipe 7, connected to junction 14 has flow (1) into junction 10; pipe 8, connected to junction 15 has flow (2) out of junction 10. The indexing data for all other junctions having two pipes follow in any order; then all the junctions having three pipes are listed etc. Hence, G(4) represents the number of all the integers listed in X(I) up to the beginning of the listing for 4-pipe junctions.

Statement 011 gives constants needed to handle the valves. DTA(I) is the spacing, in time, between data points in the tau-time data TA(I,0) = 1, ..., listed in Figure 7. VCN(I) calculates Equation (20).

Since the algebraic waterhammer method needs values of H and Q, \( L_1/A_1 \) seconds earlier than the current calculation, it is necessary to place in core storage the steady-state heads and Q's for the previous JJ increments of DT, where JJ is the largest \( P_i \) for any pipe, plus one. Statement 012 accomplishes this objective for discharges, and statement 013 does the same for heads.

Statements 014, 015, and 016 sets the headings for the transient output. Statement 017 is the heart of the transient program, as it, together with the internal functions Pt. and AT., calculates transient heads and discharges at each junction. \( J_t \) is the number of time steps DT that may be taken without exceeding storage requirements. Statements 018 through 022 move the calculations back in the storage space and causes 017 to be executed again and again until the maximum time TM of the transient is obtained.
In reading statement 017, each opening parentheses starts an iterative through statement; each closing parentheses to one of these statements is the end of the iteration. J represents the time steps, M is used to denote the number of pipes at junction under consideration and K locates data in X(I). A is the $\Sigma C_l_i$ in Equation (14) and C is $\Sigma B_i$ in the same equation. E denotes junction number. L is the counter in proceeding through all the pipes at a junction, from 1 to M, used in determining A and C. Cl(L) is the first two terms on the right hand side of Equation (10) or the negative of the corresponding terms of Equation (11). The iteration through L for a single junction closes just before AT in line number 4. AT(T) is the internal function statements 026 through 041 that performs a parabolic interpolation for $\tau$ and then calculates the head, and the discharge through the valves at the junction. The next L iteration calculates the discharges through the pipes at the junction. The M, K, and L loops close here, then the PT internal function solves for special boundary conditions and prints out results.

For the example problem, arbitrary valve closures have been selected for all ten of the valves, rapid enough so that a rather violent pressure fluctuation occurs. The computer output is shown in Figure 8 - - - it has been selected in an arbitrary manner to illustrate heads and flows at various junctions. To print out all the results would take excessive computer time. The maximum and minimum HGL elevations, however, are printed out for all junctions. Although calculations are made for each 0.1 sec the results are shown for 1.0 sec intervals. The core storage has been moved back after each ten time steps. This, however, is not necessary with the small number of pipes used. The program, with the dimension declarations shown (statement 025) is suitable for a 250-pipe
system with up to 20 valve-controlled outlets. An approximate formula for

\[ JT = \frac{18,500 - (14JP + 5JU + 17JV)}{2JP + JU + JV} \]  

(22)

in which JP is the number of pipes, JU the number of junctions and JV the
number of valves. This is based on the 32K - core storage of the IBM 7090.

If more detailed information is desired on the transient condi-
tions within a given pipe, any section between two reaches (or all sections)
may be considered to be a valveless junction. In fact, column separation
and subsequent rejoining could be calculated for the pipe. Computer execu-
tion time for larger systems would be roughly proportional to the number
of pipes in the system. The example problem required 81 sec time, from
which estimates may be made for larger systems and longer duration of
the transient.

Figure 9 shows the transient heads resulting from a pipe rupture
at junction 21. For this calculation the head at junction 21 was set equal
to 0, after steady-state conditions were established with the reservoir
at 200. feet. Minimum heads were restricted to - 33. feet. Since these
periods of vapor formation lasted only a short time, up to 1 1/2 sec., very
little column separation would occur and high pressures would not be ob-
tained.
SUMMARY AND CONCLUSIONS

A procedure has been developed for calculating hydraulic transients in complex distribution systems containing up to two or three hundred pipes with multiple valved outlets. The complete computer program, in compact MAD compiler language is presented. The basis of the waterhammer analysis is the use of algebraic waterhammer equations applied over each pipe of the system, thereby calculating flow at each end of the pipe and head at each junction at time increments of the order of one-tenth the wave travel time in the longest pipe.

REFERENCES

Figure 1. Simple circuit to which head balancing is applied.

Figure 2. Pipe of length L and wave speed c, with positive flow direction shown by arrow.

Figure 3. Junction 91 having four pipes and a valve outlet.
Figure 4. Example system comprised of 45 pipes, 26 junctions, 20 simple circuits and 10 valved outlets. Junction numbers are encircled. Assumed positive flow direction shown by arrows.
Figure 5. Computer program for balancing heads by Hardy Cross Method and for calculating transients by algebraic waterhammer equations.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>.00</td>
<td>171</td>
<td>151</td>
<td>154</td>
<td>151</td>
<td>147</td>
<td>141</td>
<td>141</td>
<td>143</td>
<td>148</td>
<td>168</td>
<td>188</td>
<td>191</td>
<td>175</td>
<td>176</td>
<td>162</td>
<td>164</td>
<td>165</td>
<td>157</td>
<td>157</td>
<td>152</td>
<td>0</td>
<td>147</td>
<td>148</td>
<td>144</td>
</tr>
<tr>
<td>.50</td>
<td>171</td>
<td>151</td>
<td>154</td>
<td>151</td>
<td>147</td>
<td>141</td>
<td>141</td>
<td>143</td>
<td>148</td>
<td>168</td>
<td>188</td>
<td>191</td>
<td>175</td>
<td>176</td>
<td>162</td>
<td>164</td>
<td>165</td>
<td>157</td>
<td>157</td>
<td>152</td>
<td>0</td>
<td>147</td>
<td>148</td>
<td>144</td>
</tr>
<tr>
<td>1.00</td>
<td>171</td>
<td>151</td>
<td>115</td>
<td>77</td>
<td>147</td>
<td>141</td>
<td>141</td>
<td>143</td>
<td>149</td>
<td>168</td>
<td>188</td>
<td>191</td>
<td>175</td>
<td>176</td>
<td>164</td>
<td>165</td>
<td>157</td>
<td>157</td>
<td>129</td>
<td>0</td>
<td>-33</td>
<td>148</td>
<td>144</td>
<td>142</td>
</tr>
<tr>
<td>1.50</td>
<td>171</td>
<td>141</td>
<td>90</td>
<td>68</td>
<td>100</td>
<td>141</td>
<td>141</td>
<td>147</td>
<td>176</td>
<td>188</td>
<td>191</td>
<td>175</td>
<td>176</td>
<td>165</td>
<td>144</td>
<td>144</td>
<td>120</td>
<td>121</td>
<td>78</td>
<td>0</td>
<td>-33</td>
<td>119</td>
<td>144</td>
<td>142</td>
</tr>
<tr>
<td>2.00</td>
<td>145</td>
<td>98</td>
<td>67</td>
<td>92</td>
<td>80</td>
<td>141</td>
<td>117</td>
<td>71</td>
<td>57</td>
<td>153</td>
<td>188</td>
<td>191</td>
<td>175</td>
<td>166</td>
<td>167</td>
<td>128</td>
<td>110</td>
<td>99</td>
<td>82</td>
<td>0</td>
<td>20</td>
<td>93</td>
<td>114</td>
<td>97</td>
</tr>
<tr>
<td>2.50</td>
<td>131</td>
<td>87</td>
<td>76</td>
<td>60</td>
<td>96</td>
<td>118</td>
<td>116</td>
<td>57</td>
<td>41</td>
<td>116</td>
<td>173</td>
<td>186</td>
<td>155</td>
<td>160</td>
<td>92</td>
<td>93</td>
<td>100</td>
<td>107</td>
<td>71</td>
<td>44</td>
<td>0</td>
<td>67</td>
<td>80</td>
<td>102</td>
</tr>
<tr>
<td>3.00</td>
<td>118</td>
<td>90</td>
<td>40</td>
<td>61</td>
<td>73</td>
<td>111</td>
<td>104</td>
<td>85</td>
<td>42</td>
<td>118</td>
<td>158</td>
<td>173</td>
<td>136</td>
<td>139</td>
<td>73</td>
<td>93</td>
<td>69</td>
<td>91</td>
<td>64</td>
<td>47</td>
<td>0</td>
<td>41</td>
<td>79</td>
<td>85</td>
</tr>
<tr>
<td>3.50</td>
<td>102</td>
<td>81</td>
<td>40</td>
<td>46</td>
<td>70</td>
<td>100</td>
<td>98</td>
<td>63</td>
<td>38</td>
<td>106</td>
<td>163</td>
<td>158</td>
<td>108</td>
<td>111</td>
<td>71</td>
<td>73</td>
<td>89</td>
<td>77</td>
<td>42</td>
<td>44</td>
<td>0</td>
<td>-8</td>
<td>69</td>
<td>73</td>
</tr>
<tr>
<td>4.00</td>
<td>110</td>
<td>64</td>
<td>64</td>
<td>41</td>
<td>68</td>
<td>98</td>
<td>94</td>
<td>54</td>
<td>52</td>
<td>89</td>
<td>158</td>
<td>136</td>
<td>92</td>
<td>90</td>
<td>87</td>
<td>67</td>
<td>77</td>
<td>63</td>
<td>64</td>
<td>43</td>
<td>0</td>
<td>-24</td>
<td>68</td>
<td>91</td>
</tr>
<tr>
<td>4.50</td>
<td>99</td>
<td>56</td>
<td>39</td>
<td>49</td>
<td>58</td>
<td>99</td>
<td>93</td>
<td>51</td>
<td>30</td>
<td>76</td>
<td>164</td>
<td>135</td>
<td>96</td>
<td>94</td>
<td>64</td>
<td>83</td>
<td>66</td>
<td>77</td>
<td>86</td>
<td>57</td>
<td>0</td>
<td>-18</td>
<td>60</td>
<td>98</td>
</tr>
<tr>
<td>5.00</td>
<td>110</td>
<td>67</td>
<td>50</td>
<td>43</td>
<td>55</td>
<td>90</td>
<td>80</td>
<td>55</td>
<td>39</td>
<td>75</td>
<td>153</td>
<td>147</td>
<td>107</td>
<td>92</td>
<td>51</td>
<td>62</td>
<td>108</td>
<td>76</td>
<td>43</td>
<td>29</td>
<td>0</td>
<td>51</td>
<td>65</td>
<td>79</td>
</tr>
<tr>
<td>5.50</td>
<td>119</td>
<td>70</td>
<td>38</td>
<td>45</td>
<td>54</td>
<td>81</td>
<td>74</td>
<td>57</td>
<td>27</td>
<td>65</td>
<td>165</td>
<td>170</td>
<td>114</td>
<td>115</td>
<td>47</td>
<td>89</td>
<td>74</td>
<td>78</td>
<td>64</td>
<td>37</td>
<td>0</td>
<td>44</td>
<td>62</td>
<td>68</td>
</tr>
<tr>
<td>6.00</td>
<td>108</td>
<td>68</td>
<td>39</td>
<td>50</td>
<td>52</td>
<td>76</td>
<td>71</td>
<td>51</td>
<td>33</td>
<td>90</td>
<td>168</td>
<td>175</td>
<td>136</td>
<td>129</td>
<td>63</td>
<td>92</td>
<td>72</td>
<td>85</td>
<td>67</td>
<td>41</td>
<td>0</td>
<td>22</td>
<td>54</td>
<td>71</td>
</tr>
<tr>
<td>6.50</td>
<td>128</td>
<td>69</td>
<td>51</td>
<td>43</td>
<td>54</td>
<td>75</td>
<td>72</td>
<td>43</td>
<td>31</td>
<td>100</td>
<td>161</td>
<td>173</td>
<td>149</td>
<td>149</td>
<td>81</td>
<td>83</td>
<td>93</td>
<td>81</td>
<td>50</td>
<td>33</td>
<td>0</td>
<td>-28</td>
<td>64</td>
<td>56</td>
</tr>
<tr>
<td>7.00</td>
<td>118</td>
<td>70</td>
<td>40</td>
<td>45</td>
<td>51</td>
<td>72</td>
<td>66</td>
<td>38</td>
<td>23</td>
<td>113</td>
<td>179</td>
<td>180</td>
<td>145</td>
<td>161</td>
<td>69</td>
<td>111</td>
<td>96</td>
<td>88</td>
<td>81</td>
<td>51</td>
<td>0</td>
<td>21</td>
<td>62</td>
<td>67</td>
</tr>
<tr>
<td>7.50</td>
<td>124</td>
<td>74</td>
<td>65</td>
<td>46</td>
<td>46</td>
<td>68</td>
<td>61</td>
<td>37</td>
<td>29</td>
<td>126</td>
<td>172</td>
<td>185</td>
<td>144</td>
<td>148</td>
<td>97</td>
<td>98</td>
<td>124</td>
<td>86</td>
<td>75</td>
<td>53</td>
<td>0</td>
<td>31</td>
<td>67</td>
<td>69</td>
</tr>
<tr>
<td>8.00</td>
<td>134</td>
<td>75</td>
<td>65</td>
<td>50</td>
<td>51</td>
<td>62</td>
<td>57</td>
<td>39</td>
<td>39</td>
<td>130</td>
<td>166</td>
<td>178</td>
<td>147</td>
<td>143</td>
<td>112</td>
<td>110</td>
<td>120</td>
<td>94</td>
<td>83</td>
<td>62</td>
<td>0</td>
<td>45</td>
<td>67</td>
<td>60</td>
</tr>
<tr>
<td>8.50</td>
<td>131</td>
<td>80</td>
<td>61</td>
<td>57</td>
<td>50</td>
<td>62</td>
<td>54</td>
<td>46</td>
<td>34</td>
<td>129</td>
<td>182</td>
<td>176</td>
<td>146</td>
<td>143</td>
<td>92</td>
<td>121</td>
<td>103</td>
<td>93</td>
<td>99</td>
<td>74</td>
<td>0</td>
<td>26</td>
<td>63</td>
<td>70</td>
</tr>
<tr>
<td>9.00</td>
<td>134</td>
<td>78</td>
<td>73</td>
<td>53</td>
<td>54</td>
<td>62</td>
<td>55</td>
<td>34</td>
<td>41</td>
<td>115</td>
<td>172</td>
<td>168</td>
<td>135</td>
<td>139</td>
<td>93</td>
<td>99</td>
<td>119</td>
<td>94</td>
<td>82</td>
<td>57</td>
<td>0</td>
<td>-16</td>
<td>62</td>
<td>59</td>
</tr>
</tbody>
</table>

**Figure 9. Transients caused by rupture at junction 21 when reservoir is at 200 feet.**