THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

WATERHAMMER ANALYSIS OF DISTRIBUTION SYSTEMS

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INTRODUCTION

The great speed of a modern large digital computer permits very complex piping systems to be analyzed for transient flow provided a method for describing the network can be adapted to computer usage. For most systems the friction may be adequately treated either as proportional to the square of the velocity, or to some exponential value of the velocity. In the algebraic method, the end boundary condition equations for the C^+ and C^- characteristics[1] developed from the method of characteristics equations are applied over the whole pipe length between two junctions. In effect this "lumps" the friction for the pipe as that given by the velocity at the opposite end of the pipe one wave-travel time previous to calculation of events at a junction. In case certain pipes are very long compared with others, any interior sections of those pipes may be considered as junctions. The combination of algebraic waterhammer equations plus indexing of the pipes leaving every junction permits a simple program to be prepared for complex systems. Each pipe is divided into a number P_i of equal reaches such that the wave travel time Dt for each reach is the same for every reach in the system. Hence $P_i\ Dt = L_i/a_i$ in which L_i is the length of pipe i and a_i is the pulse wave speed in that pipe. This is a limitation on the method, but since Dt may be selected so that a pipe has up to 10 reaches, suitable values of P_i can usually be found for each pipe by minor adjustments of pipe lengths and of wave speeds. In general, wave speeds are not known with precision and small adjustments are permissible.

Steady-state initial conditions in the piping system must first be established before solving the transient problem. For very large systems the steady-state heads and flows should be found separately to conserve storage for the transient solution. For the example given both the steady-state determination and the transient solution are combined. The program will handle head-balancing and transient solution for up to about 300 pipes. Many works have been published on steady-state solution of networks; however, for the purposes of illustrating the transient methods, the Hardy Cross method[2] of balancing heads is used in this treatment.

The method of balancing heads is first discussed, then the algebraic equations are developed. They are next applied to a typical junction of a network. The computer program is described and then an example problem is given consisting of 20 simple circuits, 26 junctions and 45 pipes.

HARDY CROSS METHOD FOR BALANCING HEADS

The method for balancing heads, as developed by Hardy Cross, is discussed briefly with regard to its application to use by the computer. In this method, the elevation of hydraulic gradeline is known at one point in the system, and the inflows and outflows from the system are given. In addition the resistance properties of each pipe must be known. The head loss H_i in pipe i is given by

$$H_i = R_i \, Q_i \, |Q_i|^{n_i - 1} \qquad (1)$$

in which R_i is the head drop over the length of pipe for unit discharge; n_i is the exponent in the head-loss equation, Q_i is the discharge. Each pipe in the network is numbered and a positive flow direction indicated in an arbitrary manner. If the flow changes direction, Equation (1) changes sign.

To start the balancing procedure which yields the correct flow in each pipe, flow is assumed through each pipe such that continuity is satisfied at every junction. This can be accomplished rapidly by starting at a junction with known inflow. Many pipes may be assumed to have zero flow, since it is not important that values close to the final correct values be assumed. Continuity must be exactly satisfied at every junction, however, because it is not upset or improved by subsequent steps in the balancing procedure.

The complex network is treated as a collection of simple circuits. In balancing a simple circuit, Equation (1) is applied to each pipe and the net head loss around the circuit is determined. In Figure 1 pipes are shown with their assumed flow directions; as Q's have been assumed for each pipe, the head drop around the system is given by

$$\sum_{i=1}^{4} H_i = \sum_{i=1}^{4} R_i \, Q_i \, |Q_i|^{n_i - 1} \qquad (2)$$

The clockwise direction may be assumed as positive; if $\Sigma H_i = 0$, then the heads are balanced and no correction to the flows are needed. When ΣH_i does not equal zero, a correction to the discharge ΔQ is calculated and applied to every pipe in the simple circuit in either the clockwise or the counterclockwise direction depending upon the sign of Equation (2). To obtain the correction ΔQ, Equation (1) is differentiated

$$\frac{\Delta H_i}{\Delta Q_i} = n_i \, R_i \, |Q_i|^{n_i - 1} \qquad (3)$$

and $\Delta H_i/\Delta Q_i$ is summed for every pipe in the simple circuit with all terms positive. Then the corrective flow ΔQ is given by

$$\Delta Q = \frac{\Sigma H_i}{\Sigma(\Delta H_i/\Delta Q_i)} = \frac{\Sigma R_i \, Q_i |Q_i|^{n_i - 1}}{\Sigma n_i \, R_i |Q_i|^{n_i - 1}} \qquad (4)$$

If the ΣH_i in the clockwise direction is positive, then the flow in each pipe should be decreased by ΔQ in the clockwise direction, e.g. in Figure 1 the flow in pipe 1 would be decreased by ΔQ and the flow in pipes 2,3, and 4 would be increased by the same ΔQ.

Each simple circuit is corrected in this manner, in any convenient sequence; then the procedure is repeated until the sum of the absolute values of ΔQ for all simple circuits becomes less than some predetermined tolerance. After the heads are balanced in all simple circuits, the elevation of hydraulic gradeline is calculated for each junction, starting with the junction with given head.

For determination of steady-state flows for different boundary conditions, other balancing techniques may be needed. The transient equations, developed in the next section, may also be used to find steady-state initial conditions. Whether this is an economical way depends upon the relative cost of engineering time to program the steady-state solution compared with cost of computer time. To do this, for example, all flows could be taken as zero, all heads equal to reservoir head, then the assumed outflows started instantaneously at time zero. Steady-state conditions will be approached after several round-trip wave travel times across the network.

ALGEBRAIC WATERHAMMER EQUATIONS

Solution of the partial differential equations of waterhammer by the method of characteristics[2] leads to two finite difference equations, one valid along the C^+ characteristic and the other valid along the C^- characteristic. When expressed in terms of discharge Q and elevation of hydraulic gradeline H, applied to the pipe of Figure 2, they take the form

$$C^+: \quad H_B(t) = H_A(t-\tfrac{L}{a}) - \frac{a}{gA}\left[Q_B(t) - Q_A(t-\tfrac{L}{a})\right] - \frac{f}{2D}\frac{a}{gA^2} Q_A(t-\tfrac{L}{a})\left|Q_A(t-\tfrac{L}{a})\right|^{n-1}\frac{L}{a} \tag{5}$$

$$C^-: \quad H_A(t) = H_B(t-\tfrac{L}{a}) + \frac{a}{gA}\left[Q_A(t) - Q_B(t-\tfrac{L}{a})\right] + \frac{f\,a}{2DgA^2} Q_A(t-\tfrac{L}{a})\left|Q_A(t-\tfrac{L}{a})\right|^{n-1}\frac{L}{a} \tag{6}$$

a is the wave speed, A the pipe cross sectional area, f the friction factor, L the pipe length, t the time, D the pipe diameter, and g the acceleration due to gravity. These equations are valid for any time t, in steady or unsteady flow, as long as the pipe remains filled with liquid. In general, each equation has two unknowns, Q_B and H_B at time t for Equation (5), and Q_A and H_A at time t for Equation (6).

For algebraic waterhammer these equations are simplified by collecting the constants.

$$B_i = \frac{gA_i}{a_i} \qquad\qquad R_i = \frac{f_i L_i}{2gD_iA_i^2} \tag{7}$$

The term R_i is the same head loss coefficient used in balancing head in the Hardy Cross method. Each pipe is divided into P_i equal reaches such that

$$P_i\,Dt = \frac{L_i}{a_i} \tag{8}$$

with Dt the time increment for the wave to travel through any reach of the system. The discharge Q for a pipe is triple subscripted Q(J1, J2, J) in which J1 is 2 if the downstream end of the pipe is designated and is 1 if the upstream end of the pipe is designated, J2 is an integer representing the pipe number, and J is an integer representing the time

$$T = (J + JC)Dt \tag{9}$$

with JC a constant integer. The junction heads are double subscripted, H(E, J) with E the junction number and J the time counter. In this notation, Equations (5) and (6) become

$$C^{+}; \quad Q(2,J2,J) = Q(1,J2,J-P(J2)) + B(J2)\ [H(E_A,J-P(J2))-R(J2)Q(1,J2,J-P(J2))\ |Q(1,J2,J-P(J2))|^{n-1}] - B(J2)\ H(E_B,J) \tag{10}$$

$$C^{-}: \quad Q(1,J2,J) = Q(2,J2,J-P(J2)) - B(J2)\ [H(E_B,J-P(J2))+R(J2)Q(2,J2,J-P(J2))\ |Q(2,J2,J-P(J2))|^{n-1}] + B(J2)\ H(E_A,J) \tag{11}$$

These two equations, which are available for each pipe at any time, are used together with continuity at a junction, and a valve equation (if an outlet is present) to calculate the elevation of hydraulic gradeline at the junction and the flow through each pipe into the junction.

APPLICATION OF THE ALGEBRAIC METHOD TO A TYPICAL JUNCTION

In Figure 3 junction 91 has four pipes connecting it with a valve outlet; two of the pipes, 7 and 13 have the sign convention for flow into the junction while the other two, 17 and 23 have the sign convention indicating flow out of the junction. The head H(91,J) and the flows Q(2,7,J), Q(2,13,J), Q(1,17,J) and Q(1,23,J) are unknowns to be found at the current time. There is one equation for each pipe, Equation (10) for pipes 7 and 13 and Equation (11) for pipes 17 and 23 as well as the equation of continuity for the junction, so for no valve outlet five equations in five unknowns are at hand. With a valve, as shown, an additional equation for flow through the valve QV(91,J) for given opening is available for determination of the additional unknown QV(E,J).

In Equation (10), applied to pipe 7

$$Q(2,7,J) = C1(7) - B(7)H(91,J) \qquad (12)$$

in which C1(7) is comprised of the first two terms on the right-hand side of Equation (10), all being known from previous calculations. For pipe 17, Equation (11) applies and yields for flow into the junction

$$-Q(1,17,J) = C1(17) - B(17)\,H(91,J) \qquad (13)$$

in which C1(17) is the negative of the first two terms on the right-hand side of Equation (11), a known quantity.

For no valve at the junction the sum of the discharges into the junction must be zero. After summing up all the flows into the junction

$$\Sigma Q_i = 0 = \Sigma C1_i - H(91,J)\Sigma B_i \qquad (14)$$

Hence

$$H(91,J) = \frac{\Sigma C1_i}{\Sigma B_i} \qquad (15)$$

Then from Equations (12) and (13) the discharges are calculated.

If junction 91 of Figure 3 has an outlet to atmosphere controlled by a valve, then the net inflow to the junction through the four pipes must just equal the flow through the valve. It is convenient to express the valve opening by a dimensionless coefficient τ. First by writing the orifice equation for the valve for steady-state

$$QVV(E) = (C_D A_V)_0 \sqrt{2g\ HH(E)} \tag{16}$$

in which, for convenience, the network is assumed to be in a horizontal plane and the elevation datum for hydraulic gradelines is the system elevation. In general for non-horizontal systems

$$QVV(E) = (C_D A_V)_0 \sqrt{2g(HH(E) - EL(E))} \tag{16a}$$

in which EL(E) is the junction elevation based on the same datum as the hydraulic gradelines. For any flow through the system

$$QV(E,J) = C_D A_V \sqrt{2g\ H(E,J)} \tag{17}$$

or, for the more general case

$$QV(E,J) = C_D A_V \sqrt{2g\ (H(E,J) - EL(E))} \tag{17a}$$

After dividing Equation (17) by Equation (16)

$$QV(E,J) = \tau\ VCN(E) \sqrt{H(E,J)} = CV \sqrt{H(E,J)} \tag{18}$$

or dividing Equation (17a) by Equation (16a)

$$QV(E,J) = \tau\ VC(E) \sqrt{H(E,J) - EL(E)} \tag{18a}$$

The dimensionless valve coefficient τ is given by

$$\tau = \frac{C_D\ A_V}{(C_D\ A_V)_D} \tag{19}$$

in either case and the other constants are collected into

$$VCN(E) = \frac{QVV(E)}{\sqrt{HH(E)}} \tag{20}$$

or

$$VC(E) = \frac{QVV(E)}{\sqrt{HH(E) - EL(E)}} \tag{20a}$$

τ is unity for steady-state flow and goes to zero as the valve is closed. CV in equation (18) is τ VCN(E).

By writing the continuity equation including the valve

$$\Sigma Q_i - QV(91,J) = 0 = \Sigma C1_i - H(91,J)\Sigma B_i - CV\sqrt{H(91,J)} \qquad (21)$$

which yields a quadratic equation in H(91,J) only and may easily be solved.

The transient solution starts with steady-state at time t = 0, the time is incremented to Dt, then each junction is solved for heads and flows as indicated, in any order. Special boundary conditions are then solved, such as the flows from junctions of fixed hydraulic gradelines. The results may then be printed out, the time incremented by Dt and the procedure repeated, taking the valve openings as function of t, i.e. $\tau(E,J)$. Special boundary conditions such as one or more pumping stations may be included.

DESCRIPTION OF COMPUTER PROGRAM AND EXAMPLE PROBLEM

A network, Figure 4, is used to illustrate the methods. It contains 20 simple circuits, 26 junctions, and 45 pipes. All inflow is at junction 26 where the elevation of hydraulic gradeline is held at 500 feet by the presence of a reservoir. Ten outlets, numbered 1 to 10 discharge from the system to atmosphere through valves. These valves may be stroked in any manner, the τ-time data being supplied as input data to the program. Since Hardy Cross balancing may be accomplished using relative values of flow and of pipe resistances, the inflow is taken as 100 and the desired steady-state outflow from the ten outlets is given as input data that must add to exactly 100. Each pipe is numbered in any manner with the pipe number shown at its midpoint. An arrow is drawn along each pipe to establish a positive flow direction. The relative pipe resistances R(1). . . R(45) are given in the input data. They may be the actual resistances, from Equation (1), calculated from Darcy Weisbach friction factors or from the various exponential formulas. The exponent may vary for each pipe as well as the resistance coefficient.

The steady-state flows through each pipe and the junction head determinations are first considered. By starting at junction 26 flows are assumed through each pipe such that continuity is satisfied at each junction. These are shown in the input data as QQ(1) = 60., 40., 10., etc. listing the flow through each pipe from 1 through 45. The apportioning of flows is rapidly accomplished and does not need to be close to the final answer for the method to converge. One may, for example, select flows such that 28 of the lines are initially assumed to have no flow, and still satisfy continuity at every junction.

The computer program, written in MAD compiler language (Michigan Algorithm Decoder), Figure 5, contains two read statements. The first read statement, No. 001 inputs circuit information for the Hardy Cross steady-state balancing. The input data for this statement is shown in Figure 6. The first part of the program, statements 001 through 007 solves for the steady-state initial conditions. Statement 002 is used in locating integers in the series X(1) = 1,2,2 . . . which describes the simple circuits. For example G(4) is the number of integers neededto describe the 3-sided circuits, G(5) equals G(4) plus the number of integers needed to describe the 4-sided circuits, etc.. Statement 003 sets all exponents equal to the given exponent NN in the head loss term. Each NN(I) may be placed in storage directly with input data if they vary.

Statement 004 is a compound iteration statement that solves Equations (2), (3), and (4) then corrects the flows, checks to determine if the tolerance has been met, and continues to balance all the circuits until it has gone through them Z times or until the desired tolerance is reached. Statement 005 converts the relative R's by multiplying by SS, and converts the relative discharges to actual discharges QQ(I) by multiplying by SF. Statement 006 calculates the heads at each junction of the system.

The indexing system for specifying the circuits starts with X(1) = 1,2,2,12, 11, 3. These first 6 integers specify the 3-sided circuit comprised of pipes 12, 11, and 3. The first number 1 indicates flow in pipe 12 is clockwise, the seconds number 2 indicates the flow in pipe 11 is counterclockwise and the third number 2 indicates the flow in pipe 3 is counterclockwise. In the second line of statement 004 starting J = 0,1, etc., W determines the clockwise or counterclockwise term, and Y the pipe number. QP collects certain terms that are used in the next two terms, DH collects ΣH_i and HD is $\Sigma\ dH_i/dQ_i$. Then DQ the correction is calculated, and the absolute DQ's summed for all circuits. Finally the Q's are corrected for each circuit. This is continued until the tolerance is satisfied or Z is reached. In accomplishing the balancing, statement 004 takes up all 3-sided circuits in the order given by the data in X(1) = ..., then continues to the 4-sided circuits, etc.

Statement 006 uses the indexing given by the data for XX(1) = 1,1, 12,5,1,14 . . . H = 500 is given as elevation of hydraulic gradeline at junction 26. With reference to Figure 4, the first 3 numbers of XX(1) indicate in order, the pipe number, the flow direction (1 if toward the next junction, 2 if toward the previous junction) and the junction at which the elevation of hydraulic gradeline is being computed. Similarly 5,1,14 gives indexing information for calculation of head at junction 14, etc. By judicious selection of order of junctions, all elevations of hydraulic gradelines are calculated and the final one is junction 26 which checks the program. In statement 006, L is the junction number, Y is the sign on the head loss term and Z is the pipe number.

The print results statement, 007, prints out the final steady-state discharges through each pipe and the elevation of hydraulic gradeline at each junction.

The second read statement, 008, provides the additional data needed for the transient calculation. These data are shown in Figure 7. Statement 009 sets initial high values for minimum HGL elevation and low values for maximum HGL elevations, which are subsequently adjusted to their proper values. Statement 010 is similar to statement 002, now acting for the 2nd set of X(I) data read in. It computes the number of integers in

X(I), i.e. G(2) = 0 is given in the data, G(3) is the number of junctions with 2 pipes times the number of integers needed to describe the junction. For a junction having two pipes 8 numbers are needed. X(1) = 10,1,7,14,1, 8,15,2 states, in order, junction 10, 1 means a valve is present (0 for no valve), pipe 7, connected to junction 14 has flow (1) into junction 10; pipe 8, connected to junction 15 has flow (2) out of junction 10. The indexing data for all other junctions having two pipes follow in any order; then all the junctions having three pipes are listed etc. Hence, G(4) represents the number of all the integers listed in X(I) up to the beginning of the listing for 4-pipe junctions.

Statement 011 gives constants needed to handle the valves. DTA(I) is the spacing, in time, between data points in the tau-time data TA(I,0) = 1., . . . listed in Figure 7. VCN(I) calculates Equation (20).

Since the algebraic waterhammer method needs values of H and Q, L_i/A_i seconds earlier than the current calculation, it is necessary to place in core storage the steady-state heads and Q's for the previous JJ increments of DT, where JJ is the largest P_i for any pipe, plus one. Statement 012 accomplishes this objective for discharges, and statement 013 does the same for heads.

Statements 014, 015, and 016 sets the headings for the transient output. Statement 017 is the heart of the transient program, as it, together with the internal functions Pt. and AT., calculates transient heads and discharges at each junction. Jt is the number of time steps DT that may be taken without exceeding storage requirements. Statements 018 through 022 move the calculations back in the storage space and causes 017 to be executed again and again until the maximum time TM of the transient is obtained.

In reading statement 017, each opening parentheses starts an iterative through statement; each closing parentheses to one of these statements is the end of the iteration. J represents the time steps, M is used to denote the number of pipes at junction under consideration and K locates data in X(I). A is the $\Sigma C1_i$ in Equation (14) and C is ΣB_i in the same equation. E denotes junction number. L is the counter in proceeding through all the pipes at a junction, from 1 to M, used in determining A and C. C1(L) is the first two terms on the right hand side of Equation (10) or the negative of the corresponding terms of Equation (11). The iteration through L for a single junction closes just before AT. in line number 4. AT.(T) is the internal function statements 026 through 041 that performs a parabolic interpolation for τ and then calculates the head, and the discharge through the valves at the junction. The next L iteration calculates the discharges through the pipes at the junction. The M, K, and L loops close here, then the PT. internal function solves for special boundary conditions and prints out results.

For the example problem, arbitrary valve closures have been selected for all ten of the valves, rapid enough so that a rather violent pressure fluctuation occurs. The computer output is shown in Figure 8 - - - it has been selected in an arbitrary manner to illustrate heads and flows at various junctions. To print out all the results would take excessive computer time. The maximum and minimum HGL elevations, however, are printed out for all junctions. Although calculations are made for each 0.1 sec the results are shown for 1.0 sec intervals. The core storage has been moved back after each ten time steps. This, however, is not necessary with the small number of pipes used. The program, with the dimension declarations shown (statement 025) is suitable for a 250-pipe system with up to 20 valve-controlled outlets. An approximate formula for JT, the number of steps that may be retained in storage is

$$JT = \frac{18,500 - (14JP + 5\ JU + 17\ JV)}{2\ JP + JU + JV} \qquad (22)$$

in which JP is the number of pipes, JU the number of junctions and JV the number of valves. This is based on the 32K - core storage of the IBM 7090.

If more detailed information is desired on the transient conditions within a given pipe, any section between two reaches (or all sections) may be considered to be a valveless junction. In fact, column separation and subsequent rejoining could be calculated for the pipe. Computer execution time for larger systems would be roughly proportional to the number of pipes in the system. The example problem required 81 sec time, from which estimates may be made for larger systems and longer duration of the transient.

Figure 9 shows the transient heads resulting from a pipe rupture at junction 21. For this calculation the head at junction 21 was set equal to 0. after steady-state conditions were established with the reservoir at 200. feet. Minimum heads were restricted to - 33. feet. Since these periods of vapor formation lasted only a short time, up to 1 $\frac{1}{2}$ sec., very little column separation would occur and high pressures would not be obtained.

SUMMARY AND CONCLUSIONS

A procedure has been developed for calculating hydraulic transients in complex distribution systems containing up to two or three hundred pipes with multiple valved outlets. The complete computer program, in compact MAD compiler language is presented. The basis of the waterhammer analysis is the use of algebraic waterhammer equations applied over each pipe of the system, thereby calculating flow at each end of the pipe and head at each junction at time increments of the order of one-tenth the wave travel time in the longest pipe.

REFERENCES

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2. "Analysis of Flow in Networks of Conduits or Conductors," by Hardy Cross, University Illinois, Bull. 286, November, 1946.

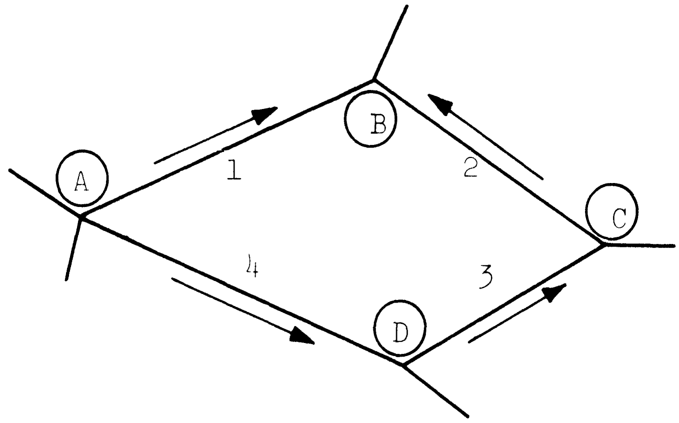

Figure 1. Simple circuit to which head balancing is applied.

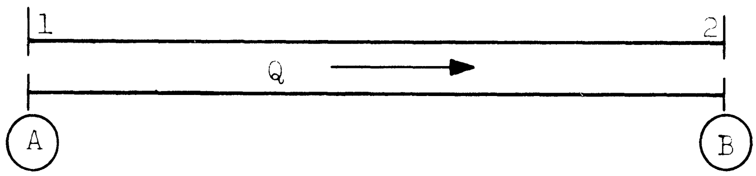

Figure 2. Pipe of length L and wave speed a, with positive flow direction shown by arrow.

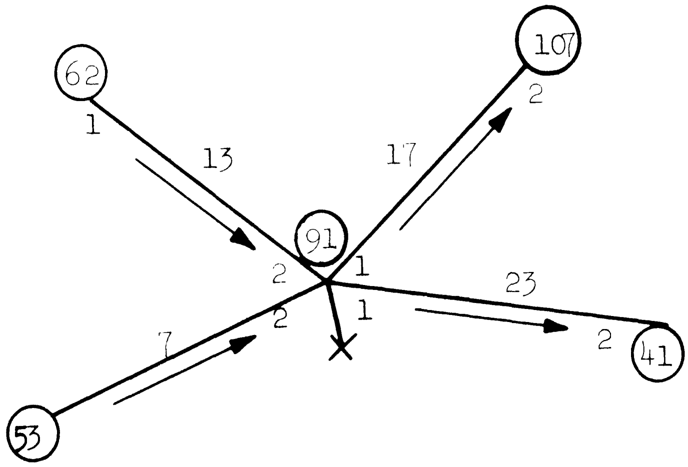

Figure 3. Junction 91 having four pipes and a valve outlet.

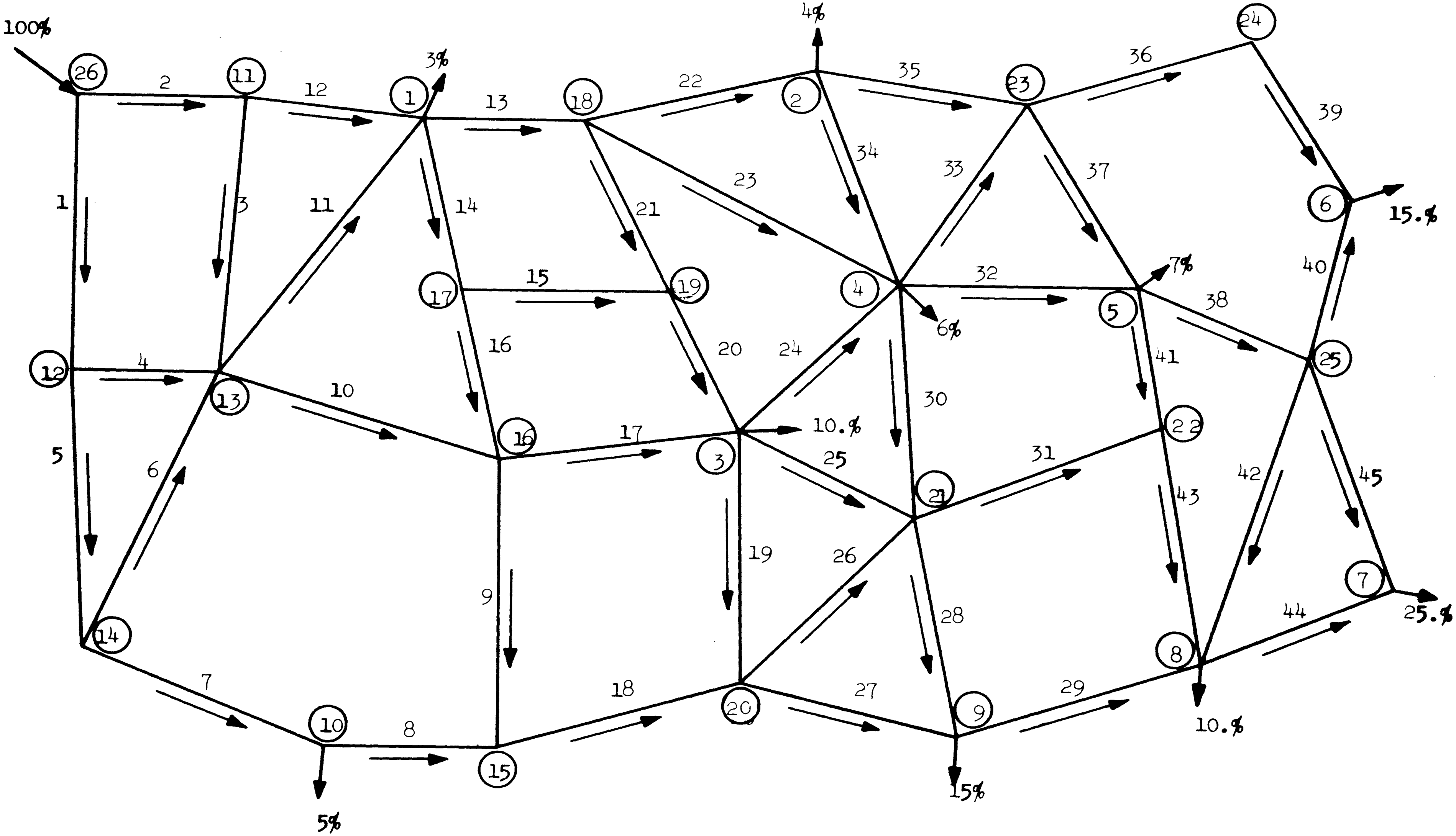

Figure 4. Example system comprised of 45 pipes, 26 junctions, 20 simple circuits and 10 valved outlets. Junction numbers are encircled. Assumed positive flow direction shown by arrows.

```
HERE          READ AND PRINT DATA                                                    *001
              (I=4,1,I.G.5,G(I)=G(I-1)+2*(I-1)*N(I-1))                               *002
              SPRAY.(NN,NN(1)...NN(JP))                                              *003
             CALCULATION OF STEADY STATE HEADS AND FLOWS
              (M=0,1,M.G.Z.OR.QD.L.TOL,QD=0.,(L=3,1,L.G.N ,U=2*L,V=N(L)*U,(         *004
            2 I=1,U,I.G.V,DH=0.,HD=0.,(J=0,1,J.E.L,W=X(G(L)+I+J+L),Y=X(G(L)          *004
            3 +I+J),QP=R(W)*.ABS.QQ(W).P.(NN(W)-1.),DH=DH+S(Y)*QQ(W)*QP,HD=         *004
            4 HD+NN(W)*QP),DQ=DH/HD,QD=QD+.ABS.DQ,(J=L,1,J.E.U,W=X(G(L)+I+J         *004
            5 -L),Y=X(G(L)+I+J),QQ(Y)=QQ(Y)-S(W)*DQ))))                             *004
              (I=1,1,I.G.JP,R(I)=R(I)*SS,QQ(I)=QQ(I)*SF)                             *005
              (I=1,3,I.G.JXX,L=XX(I+2),Y=S(XX(I+1)),Z=XX(I),                         *006
            2 HH(L)=H-Y*R(Z)*QQ(Z)*.ABS.QQ(Z).P.(NN(Z)-1.),H=HH(L))                  *006
              PRINT RESULTSM,TM,SS,SF,TOL,QQ(1)...QQ(JP),R(1)...R(JP),HH(1)...HH(JU) *007
              READ AND PRINT DATA                                                    *008
              (I=1,1,I.E.JU,HMIN(I)=H,HMAX(I)=0.)                                    *009
              (I=3,1,I.G.6,G(I)=G(I-1)+N(I-1)*(3*I-1))                               *010
              (I=1,1,I.G.JV,DTA(I)=TC(I)/DTV,VCN(I)=QVV(I)*SF/SQRT.(HH(I)))          *011
              (J2=1,1,J2.G.JP,(J1=1,1,J1.G.2,(J=1,1,J.G.JJ,Q(J1,J2,J)=QQ(J2          *012
            2 ))))                                                                   *012
              (J=1,1,J.G.JJ,(J1=1,1,J1.G.JU,H(J1,J)=HH(J1)))                         *013
              PRINT COMMENT$1    HEADS AT SELECTED JUNCTIONS FOR INDICATED           *014
            2 TIMES. ALSO DISCHARGES AT DOWNSTREAM END OF SELECTED PIPES$            *014
              PRINT COMMENT$0  J   TIME  HJ1   HJ2   HJ3   HJ4   HJ5   HJ6           *015
            2   HJ7   HJ8   HJ9  HJ10  HJ11  HJ13  HJ14  HJ19  HJ21   Q1             *015
            3  Q8    Q23   Q31   Q38$                                                *015
              PRINT COMMENT$0$                                                       *016
FOX           (J=J7,1,J.G.JT,T=(J+JC)*DT,(M=2,1,M.G.N,J2=2+3*M,J3=N(M)*J2,          *017
            1 (K=1,J2,K.G.J3,A=0.,C=0.,E=X(G(M)+K),(L=1,1,L.G.M,V=K+3*L,W=X          *017
            2 (G(M)+V+1),Y=X(G(M)+V-1),Z=X(G(M)+V),D=Q(W,Y,J-P(Y)),C1(L)=D*          *017
            3 S(W)+B(Y)*(H(Z,J-P(Y))-S(W)*R(Y)*D*.ABS.D.P.(NN(Y)-1.)),A=A+C          *017
            4 1(L),C=C-B(Y)),AT.(T),(L=1,1,L.G.M,V=K+3*L,F=K(X(G(M)+V+1)),Y          *017
            5 =X(G(M)+V-1),Q(F,Y,J)=S(F)*(B(Y)*H(E,J)-C1(L))))),PT.(T))              *017
              (J=1,1,J.G.JJ,(V=1,1,V.G.JU,H(V,J)=H(V,J+ZZ)))                         *018
              (I=1,1,I.G.JP,(V=1,1,V.G.2,(J=1,1,J.G.JJ,Q(V,I,J)=Q(V,I,J+ZZ)          *019
            2 )))                                                                    *019
              (J=1,1,J.G.JJ,(V=1,1,V.G.JV,QV(V,J)=QV(V,J+ZZ)))                       *020
              JC=JC+ZZ                                                               *021
              J7=JJ+1                                                                *022
              TRANSFER TO FOX                                                        *023
              INTEGER I,J,J1,J2,J3,J7,JC,U,V,W,Y,Z,N,P,XX,E,XI,JR,JP,JU,JV,JXX,      *024
            2 JJ,JT,ZZ,K,K1,K2,M,L,G,F,X                                             *024
              DIMENSION X(2000),(QVV,VCN,DTA,TC,C1,K)(20),Q(2*250*20),H(135*20),     *025
            2 QV(20*20),(P,B,R,NN)(250),TA(20*(0...11)),N(9),S(2),XX(420),           *025
            3 HH(135),QQ(250),(HMAX,HMIN)(250),G(10)                                 *025
              INTERNAL FUNCTION  AT.(TT)                                             *026
              WHENEVER X(G(M)+K+1).E.1.AND.TT.L.TC(E)                                *027
              I=TT/DTA(E)+1                                                          *028
              TH=(TT-I*DTA(E))/DTA(E)                                                *029
              CV=VCN(E)*(TA(E,I)+.5*TH*(TA(E,I+1)-TA(E,I-1)+TH*(TA(E,I+1)+T          *030
            2 A(E,I-1)-2.*TA(E,I))))                                                 *030
              WHENEVER CV.LE.0.,TRANSFER TO FX1                                      *031
              H(E,J)=(.5*(CV/C).P.2-A/C)*(1.-SQRT.(1.-(A/(C*(.5*(CV/C).P.2-          *032
            2 A/C))).P.2))                                                           *032
              QV(E,J)=CV*SQRT.(H(E,J))                                               *033
              OTHERWISE                                                              *034
FX1           WHENEVER X(G(M)+K+1).E.1,QV(E,J)=0.                                    *035
              H(E,J)=-A/C                                                            *036
              END OF CONDITIONAL                                                     *037
              WHENEVER H(E,J).L.HMIN(E),HMIN(E)=H(E,J)                               *038
              WHENEVER H(E,J).G.HMAX(E),HMAX(E)=H(E,J)                               *039
              FUNCTION RETURN                                                        *040
              END OF FUNCTION                                                        *041
              INTERNAL FUNCTION PT.(TT)                                              *042
              H(JU,J)=HO                                                             *043
              (I=1,1,I.G.2,D=Q(2,I,J-P(I)),Q(1,I,J)=D+B(I)*(HO-H(13-I,J-P(I          *044
            2 ))-R(I)*D*.ABS.D.P.(NN(I)-1.)))                                        *044
              WHENEVER J/U*U.E.J,PRINT FORMAT$1H ,I3,F6.1,15F6.0,5F6.2*$,J+JC,T,(V=1,1,V.  *045
            2 G.11,H(V,J)),H(13,J),H(14,J),H(19,J),H(21,J),Q(2,1,J),Q(2,8,J          *045
            3 ),Q(2,23,J),Q(2,31,J),Q(2,38,J)                                        *045
              WHENEVER T.G.TM                                                        *046
              PRINT RESULTS HMAX(1)...HMAX(JU-1),HMIN(1)...HMIN(JU-1),DT,DTV,JT,JXX  *047
              TRANSFER TO HERE                                                       *048
              END OF CONDITIONAL                                                     *049
              FUNCTION RETURN                                                        *050
              END OF FUNCTION                                                        *051
              END OF PROGRAM                                                         *052
```

Figure 5. Computer program for balancing heads by Hardy Cross Method and for calculating transients by algebraic waterhammer equations.

INPUT DATA FOR STEADY STATE FLOW AND HEAD DETERMINATION

```
QQ(I)= 60.0 40.0 10.0 30.0 30.0 10.0 20.0 15.0 15.0 35.0 15.0 30.0 20.0 22.0 12.0 10.0 30.0 30.0  5.0 16.0  4.0 10.0  6.0
       11.0 20.0 10.0 25.0 15.0 25.0   .0 15.0  8.0  3.0   .0  6.0  9.0   .0  6.0  9.0  6.0 -5.0   .0 10.0 25.0   .0

 R(I)=  1.0  1.4 15.0  7.0  6.0  8.0  5.0  6.0  6.5  3.0  9.0  4.0  6.0  5.5 10.0  8.0  4.5  4.0 20.0  5.0 25.0  7.0 10.0
        8.0  3.0  7.0  2.5  5.0  3.2 20.0  7.0 12.0 25.0 22.0 15.0 12.0 18.0 14.0  9.0  8.0 10.0  8.0  7.5  2.3  6.5

 X(I)=  1  2  2 12 11  3  1  2  2  4  6  5  1  1  2 22 34 23  1  1  2 24 30 25  1  2  2 25 26 19  1  1  2 26 28 27  1
        2  2 35 33 34  1  1  2 33 37 32  1  2  2 45 44 42  1  1  2  2  2  3  4  1  1  1  2  2 13 21 15 14  1  1  2  2
       15 20 17 16  1  1  2  2 17 19 18  9  1  2  2  2 23 24 20 21  1  1  2  2 32 41 31 30  1  1  2  2 31 43 29 28  1
        1  2  2 38 42 43 41  1  1  1  2 11 14 16 10  1  1  1  2  2  6 10  9  8  7  1  1  2  2  2 36 39 40 38 37

XX(I)=   1   1  12   5   1  14   7   1  10   8   1  15  18   1  20  27   1   9  29   1   8  44   1   7  45   2
        25  40   1   6  39   2  24  36   2  23  37   1   5  41   1  22  31   2  21  30   2   4  34   2   2  22
         2  18  21   1  19  20   1   3  17   2  16  16   2  17  14   2   1  11   2  13   3   2  11   2   2  26

N(3)=9,9,2,NN=2.00,Z=16,N=5,TOL=.10,JP=45,S(1)=1,S(2)=-1,JXX=78,HH(26)=500,SS=.01,SF=.60,JU=26,H=500,G(3)=0,QD=2
```

Figure 6. Input data for first read statement.

INPUT DATA FOR ALGEBRAIC WATERHAMMER SOLUTION

```
P(I)= 7 4 7 5 6 8 6 5 6 5 9 6 5 5 7 4 5 6 5 5 4 5 6 9 6 7 6 5 6 7 9 6 6 5 5 7 6 7 5 5 4 7 6 9 9

B(I)= .040 .040 .012 .020 .020 .039 .016 .012 .008 .024 .010 .027 .020 .014 .012 .026 .021 .020 .033 .011 .001 .011 .010
      .008 .014 .006 .019 .010 .016 .015 .021 .008 .004 .002 .006 .007 .003 .006 .006 .008 .001 .004 .009 .013 .006

TC(I)=  7.0  5.0  6.0 16.0  8.0  9.0 15.0 12.0 10.0 10.0     QVV(I)=  3.0  4.0 10.0  6.0  7.0 15.0 25.0 10.0 15.0  5.0

TA(1,0)=1.00 .90 .80 .70 .60 .50 .40 .30 .20 .10 .00 .00    TA(2,0)=1.00 .93 .85 .76 .66 .55 .43 .32 .20 .07 .00 .00
TA(3,0)=1.00 .91 .82 .73 .60 .48 .37 .27 .15 .04 .00 .00    TA(4,0)=1.00 .80 .62 .46 .32 .20 .10 .05 .01 .00 .00 .00
TA(5,0)=1.00 .90 .80 .70 .60 .50 .40 .30 .20 .10 .00 .00    TA(6,0)=1.00 .93 .85 .76 .66 .55 .43 .32 .20 .07 .00 .00
TA(7,0)=1.00 .91 .82 .73 .60 .48 .37 .27 .15 .04 .00 .00    TA(8,0)=1.00 .80 .62 .46 .32 .20 .10 .05 .01 .00 .00 .00
TA(9,0)=1.00 .93 .85 .76 .66 .55 .43 .32 .20 .15 .10 .00   TA(10,0)=1.00 .90 .80 .70 .60 .50 .40 .30 .20 .10 .00 .00

2 PIPES X(I)= 10  1  7 14  1  8 15  2 24  0 36 23  1 39  6  2  6  1 39 24  1 40 25  1  7  1 44  8  1 45 25  1

3 PIPES 11  0  2 26  1  3 13  2 12  1  2 12  0  1 26  1  4 13  2  5 14  2 14  0  5 12  1  6 13  2  7 10  2
        15  0  8 10  1  9 16  1 18 20  2 17  0 14  1  1 15 19  2 16 16  2 19  0 21 18  1 15 17  1 20  3  2
         9  1 27 20  1 28 21  1 29  8  2  2  1 22 18  1 34  4  2 35 23  2 22  0 41  5  1 31 21  1 43  8  2

4 PIPES  1  1 12 11  1 11 13  1 14 17  2 13 18  2 16  0 10 13  1 16 17  1  9 15  2 17  3  2 20  0 19  3  1 18 15  1 26 21
         2 27  9  2 18  0 13  1  1 21 19  2 23  4  2 22  2  2 23  0 35  2  1 33  4  1 36 24  2 37  5  2  5  1 37 23  1 32
         4  1 41 22  2 38 25  2  8  1 29  9  1 43 22  1 42 25  1 44  7  2 25  0 38  5  1 42  8  2 45  7  2 40  6  2

5 PIPES 13  0  4 12  1  3 11  1  6 14  1 11  1  2 10 16  2  3  1 17 16  1 20 19  1 19
        20  2 24  4  2 25 21  2 21  0 25  3  1 26 20  1 30  4  1 31 22  2 28  9  2

6 PIPES  4  1 23 18  1 34  2  1 24  3  1 30 21  2 32  5  2 33 23  2

K(1)=2,K(2)=1,U=10,TM=30,DT=.1,N=6,N(2)=4,9,8,3,1,JV=10,JT=20,ZZ=10,J7=10,H0=500,G(2)=0,DTV=10JJ=,10,JC=-10
```

Figure 7. Input data for second read statement.

HEADS AT SELECTED JUNCTIONS FOR INDICATED TIMES. ALSO DISCHARGES AT DOWNSTREAM END OF SELECTED PIPES

J	TIME	HJ1	HJ2	HJ3	HJ4	HJ5	HJ6	HJ7	HJ8	HJ9	HJ10	HJ11	HJ13	HJ14	HJ19	HJ21	Q1	Q8	Q23	Q31	Q38
0	.0	471	451	454	451	447	441	441	443	448	468	488	475	476	457	451	30.39	9.84	7.47	7.67	5.46
10	1.0	474	469	463	464	472	472	468	469	464	478	489	476	478	461	455	30.39	9.92	7.34	7.55	5.44
20	2.0	486	499	481	491	507	522	520	509	494	484	491	482	482	476	481	30.29	9.89	7.20	7.15	5.39
30	3.0	504	540	519	532	562	600	575	556	536	503	499	495	493	507	529	29.95	9.69	6.85	6.78	5.24
40	4.0	531	592	564	593	625	676	632	618	597	533	508	518	516	553	586	29.05	9.21	6.28	6.41	4.99
50	5.0	567	645	624	647	695	764	722	686	661	578	523	549	547	598	644	27.34	8.62	5.57	6.00	4.78
60	6.0	598	666	673	706	761	856	830	770	734	625	536	585	583	658	709	24.57	7.75	4.68	5.39	4.35
70	7.0	621	720	719	753	833	952	931	861	805	664	544	614	616	693	770	20.89	6.89	3.34	4.59	3.79
80	8.0	642	768	757	813	897	1052	1025	926	864	691	554	629	630	719	828	16.58	5.86	1.94	3.70	3.05
90	9.0	657	815	794	853	927	1072	1081	966	883	699	557	633	630	755	864	12.16	4.60	.21	2.77	2.18
100	10.0	675	829	803	861	938	980	1086	968	901	701	565	641	633	779	870	7.74	3.11	-1.42	1.89	1.59
110	11.0	671	774	792	828	845	872	1029	950	897	697	566	650	643	751	866	3.12	1.52	-2.77	1.11	1.17
120	12.0	628	695	764	743	772	790	959	842	803	705	549	640	645	744	794	-1.58	.19	-3.99	.75	.60
130	13.0	596	636	677	679	708	743	822	757	730	653	538	608	620	672	701	-5.85	-.37	-5.25	.54	.24
140	14.0	568	625	586	609	624	675	690	635	614	562	529	555	550	559	587	-8.53	-.94	-5.56	-.24	-.16
150	15.0	512	545	480	527	557	554	522	500	478	483	500	478	478	503	508	-8.90	-1.15	-5.48	-.37	-.20
160	16.0	441	425	409	433	421	415	357	394	403	391	477	430	419	404	405	-7.18	-1.05	-5.35	-.29	-.01
170	17.0	395	329	337	303	279	251	288	290	330	376	458	399	399	338	322	-4.21	-1.23	-4.85	-.07	.04
180	18.0	369	247	282	240	184	146	203	220	232	383	449	395	408	322	224	-.89	-.10	-3.98	.08	.16
190	19.0	372	247	254	206	173	119	128	146	201	329	457	405	404	274	210	2.03	1.15	-2.80	.21	.30
200	20.0	376	288	257	245	195	171	90	144	179	343	453	384	394	281	203	5.46	2.32	-1.51	.77	.62
210	21.0	381	333	291	284	270	225	173	192	233	360	452	374	373	320	252	9.31	3.64	-.35	1.11	1.07
220	22.0	409	341	346	340	320	323	326	319	331	336	469	404	390	345	331	12.96	3.94	.38	1.39	1.23
230	23.0	437	407	411	403	415	427	471	462	430	439	474	443	449	423	422	15.36	4.01	.74	1.20	1.07
240	24.0	491	498	503	506	517	524	567	541	527	521	494	499	509	492	517	15.99	4.32	.58	.97	.98
250	25.0	565	600	613	622	632	652	599	616	622	567	528	560	556	591	616	14.82	4.07	.27	.75	.74
260	26.0	624	704	683	698	720	715	678	682	681	637	547	603	597	697	694	11.97	3.29	-.17	.85	.92
270	27.0	647	727	730	756	752	769	781	777	754	674	557	633	632	696	759	8.16	2.63	-.65	1.06	1.05
280	28.0	622	710	735	751	792	803	854	842	824	684	549	635	641	710	788	3.59	1.76	-1.60	.80	.70
290	29.0	607	685	732	728	771	848	886	844	773	664	537	608	613	714	768	-.34	.84	-3.04	.32	.48
300	30.0	609	688	683	717	750	795	819	778	756	611	541	582	570	660	749	-3.43	-.23	-4.17	.03	-.12

HMAX(1)...HMAX(25)

6.793659E 02	8.374125E 02	8.031355E 02	8.611248E 02	9.460591E 02	1.078578E 03	1.092752E 03	9.740511E 02
9.284932E 02	7.085588E 02	5.690587E 02	5.860337E 02	6.504819E 02	6.472526E 02	7.667943E 02	7.247586E 02
7.241734E 02	7.648816E 02	7.793333E 02	8.328438E 02	8.785914E 02	9.280301E 02	9.269794E 02	1.032449E 03
1.049448E 03							

HMIN(1)...HMIN(25)

3.679537E 02	2.351517E 02	2.497660E 02	2.063129E 02	1.686469E 02	1.189392E 02	8.494810E 01	1.250823E 02
1.785323E 02	3.248854E 02	4.485489E 02	4.194394E 02	3.716610E 02	3.725363E 02	2.670132E 02	3.165466E 02
3.183864E 02	2.925298E 02	2.681525E 02	2.352643E 02	1.987157E 02	1.505149E 02	1.651205E 02	1.191523E 02
1.279425E 02							

Figure 8. Transient solution for problem with data given in Figures 6 and 7.

HEADS AT EACH JUNCTION FOR RUPTURE AT JUNCTION NUMBER JI

TIME	J1	J2	J3	J4	J5	J6	J7	J8	J9	J10	J11	J12	J13	J14	J15	J16	J17	J18	J19	J20	J21	J22	J23	J24	J25
.00	171	151	154	151	147	141	141	143	148	168	188	191	175	176	162	164	165	157	157	152	0	147	148	144	142
.50	171	151	154	151	147	141	141	143	109	168	188	191	175	176	162	164	165	157	157	152	0	147	148	144	142
1.00	171	151	115	77	147	141	141	143	109	168	188	191	175	176	162	164	165	157	157	129	0	-33	148	144	142
1.50	171	141	90	68	100	141	141	76	73	168	188	191	175	176	139	144	144	120	121	78	0	-33	119	144	142
2.00	145	98	67	92	80	141	117	71	57	153	188	191	166	176	81	127	128	110	99	82	0	20	93	114	97
2.50	131	87	76	60	96	118	116	57	41	116	173	186	155	160	92	93	100	107	71	44	0	67	80	102	83
3.00	118	90	40	61	73	111	104	85	42	118	158	173	136	139	73	93	69	91	64	47	0	41	79	85	89
3.50	102	81	40	46	70	100	98	63	38	106	163	158	108	111	71	73	89	77	42	44	0	-8	69	73	101
4.00	110	64	64	41	68	98	94	54	52	89	158	136	92	90	87	67	77	63	64	43	0	-24	68	91	105
4.50	99	56	39	49	58	99	93	51	30	76	164	135	96	94	64	83	66	77	86	57	0	-18	60	98	73
5.00	110	67	50	43	55	90	80	55	39	75	153	147	107	92	51	62	108	76	43	29	0	51	65	79	56
5.50	119	70	38	45	54	81	74	57	27	65	165	170	114	115	47	89	74	78	64	37	0	44	62	68	61
6.00	108	68	39	50	52	76	71	51	33	90	168	175	136	129	63	92	72	85	67	41	0	22	54	71	71
6.50	128	69	51	43	54	75	72	43	31	100	161	173	149	149	81	83	93	81	50	33	0	-28	64	56	80
7.00	118	70	40	45	51	72	66	38	23	113	179	180	145	161	69	111	96	88	81	51	0	21	62	67	58
7.50	124	74	65	46	46	68	61	37	29	126	172	185	144	148	97	98	124	86	75	53	0	31	67	69	38
8.00	134	75	65	50	51	62	57	39	39	130	166	178	147	143	112	110	120	94	83	62	0	45	67	60	53
8.50	131	80	61	57	50	62	54	46	34	129	182	176	146	143	92	121	103	93	99	74	0	26	63	70	58
9.00	134	78	73	53	54	62	55	34	41	115	172	168	135	139	93	99	119	94	82	57	0	-16	62	59	69

HMIN(1)...HMIN(25)

9.860570E 01	5.591483E 01	2.799770E 01	3.828903E 01	4.432028E 01	6.004538E 01	5.405500E 01	2.899357E 01
2.109700E 01	6.113739E 01	1.525240E 02	1.260319E 02	9.209563E 01	8.483140E 01	4.031638E 01	5.863247E 01
5.948784E 01	6.307012E 01	4.021127E 01	2.046601E 01	.000000E 00	-3.300000E 01	5.025888E 01	4.737748E 01
3.773393E 01							

Figure 9. Transients caused by rupture at junction 21 when reservoir is at 200 feet.