Abstract

Equivalent currents and incremental length diffraction coefficients are derived for an impedance half plane. These apply to a half plane with unequal face impedances and reduce to the corresponding incremental length diffraction coefficients and equivalent currents derived by Mitzner and Michaeli for the perfectly conducting edge.

INCREMENTAL LENGTH DIFFRACTION COEFFICIENTS FOR AN IMPEDANCE HALF PLANE

1 Introduction

In the context of GTD, the scattering by a complex body is represented as a sum of contributions from reflection and diffraction mechanisms [1,2]. The last are attributed to any abrupt surface derivatire discontinuity or possibly any abrupt changes in the material composition of the body. To permit the characterization of a specific discontinuity in surface curvature or material composition, it is necessary that an analytical treatment of an associated cononical geometry be found. Unfortunately, this has only been rigorously accomplished for a handful of geometries which primarily include metallic edges and wedges at normal and skew incidences [3-5], second order surface discontinuity [6,7], impedance wedges at normal incidence [8], thin dielectric and resistive edges at normal and skew incidences[9,10], impedance or material discontinuities in a plane [11,12] and the right- angled wedge at skew incidence with one of its faces perfectly conducting[13]. Solutions may be also found in the literature for creeping wave characterizations and for some non-generic configurations.

All of the aforementioned cononical solutions are applicable to straight edges formed by planar surfaces, whereas in practice the edges are curved and generally formed by non- planar surfaces. This prompted the development of uniform theories [2, 14] which allowed the treatment of practical edge configurations and removed the non-physical field discontinuities at the geometrical optics boundaries. Two different uniform theories were formalized in the seventies almost concurrently but among these the uniform theory of diffraction (UTD) introduced by Kouyoumjian and Pathak [2] has been most popular.

Aside from its geometrical complexity, the uniform GTD permits the treatment of practical geometries in a systematic manner. Nevertheless, the GTD and UTD are still at caustics and cannot handle finite length edges in a rigorous manner due to a lack of appropriate corner diffraction coefficients [15, 16]. The method of equivalent currents (MEC) was developed to overcome these difficiencies by replacing the diffraction effects with fictitious equivalent currents which are then integrated over the length of the (curved) edge to obtain the diffracted field. Original versions [17, 18] of such equivalent currents were not as accurate for computations away from the Keller cone. This was corrected by Michaeli [19,20] whose equivalent currents (EC) turned out to be identical to Mitzner's [21] incremental length diffraction coefficients (ILDCs) once the physical optics contributions to the diffracted field was removed from Michaeli's equivalent currents as noted by Knott [22]. Since Mitzner's ILDCs were derived in the context of the Physical Theory of Diffraction (PTD) developed by Ufimtsev [23], this comparison provided a rigorous connection between GTD (actually the MEC) and PTD. In practice when treating arbitrary surfaces, it is more convenient to compute the physical optics field (rather than the geometrical optics field) and supplement these with any edge diffracted field. Consequently, the ILDCs are more suited for practical implementation and can be readily derived from the ECs as noted above. The equivalent currents derived by Michaeli [17] and their similar versions proposed by other [24, 25] are applicable only to perfectly conducting edges. Nowadays, though, man-made vehicles are composed of non-metallic material and a need therefore exists to derive corresponding equivalent currents to characterize the diffraction by material edges and discontinuities. The simplest non-metallic edge for which a skew incidence solution has been derived is that formed by an impedance half plane [9, 10]. It is therefore instructive to first derive the ECs for this geometry and such is the subject of the paper. The procedure employed for this derivation parallels that adopted by Michaeli [19] for the perfectly conducting case and in the process we employ Sommerfeld's inversion theorem [26] as suggested by Pelosi etc [27]. Use of this inversion theorem is essential in obtaining a closed form expression for the equivalent edge currents.

In contrast to the ECs derived for the conducting wedge, those derived for the impedance edge are more involved because of the presence of electric and magnetic currents on the surface of the faces forming the edge. They are, nevertheless, given here in explicit form which is suited for computer implementation. As expected they involve the tangential spectral components (with reference to the Sommerfeld integral) of the surface fields. These are the components which must be obtained from a function theoretic solution of the cononical problem for the impedance half plane with equal face impedances they have been derived by Senior [9] (see also corrections given in [28]). For the half plane with unequal face impedances the spectral field components have been derived by Bucci and Franceschetti [10]. Unfortunately, the expressions given in [10] and are cumbersome to use and are recast here (Section III) in a convenient form suitable for computer implementation. In Section IV we present the ILDCs for the impedance half plane and discuss how these expressions can be used for edges formed by the surfaces of an impedance wedge.

2 Equivalent Current Expressions

Let us consider the impedance half plane, shown in figure 1, which is illuminated by the plane wave.

$$\begin{aligned} \mathbf{E}^i &= (\hat{x}e_x + \hat{y}e_y + \hat{z}e_z)\, e^{-ik(x\sin\beta_o\cos\phi_o + y\sin\beta_o\sin\phi_o + z\cos\beta_o)} \\ \mathbf{H}^i &= Y_o\,(\hat{x}h_x + \hat{y}h_y + \hat{z}h_z)\, e^{-ik(x\sin\beta_o\cos\phi_o + y\sin\beta_o\sin\phi_o + z\cos\beta_o)} \end{aligned} \tag{1}$$

in which $k = 2\pi/\lambda$ is the wave number and $Y_o = 1/Z_o$ is the free space intrinsic admittance. The half plane satisfies the boundary condition.

$$\hat{y} \times \hat{y} \times \mathbf{E} = -\eta_o Z_o \hat{y} \times \mathbf{H} \tag{2a}$$

on its upper face and the condition

$$\hat{y} \times \hat{y} \times \mathbf{E} = \eta_n Z_o \hat{y} \times \mathbf{H} \tag{2b}$$

on its lower face. Thus $\eta_{o,n}$ are the normalized surface impedances on the $\phi = 0$ and the $\phi = n\pi = 2\pi$ faces of the half plane, respectively.

In accordance with the method of equivalent currents (MEC), the entire scattering by the half plane can be thought as generated by filamentary electric and magnetic currents placed at the edge of the half plane. Referring to figure 3, and denoting these edge equivalent currents as I(ℓ) and M(ℓ), respectively, the Fresnel and far zone fields can be expressed as

$$\mathbf{E}^s \sim -ik \int_C \left[Z_o I(\ell')\hat{s} \times (\hat{s} \times \hat{t}) + M(\ell')\hat{s} \times \hat{t} \right] G(\mathbf{r}, \mathbf{r}')d\ell' \tag{3}$$

where $\hat{t}(=\hat{z})$ is the unit vector tangent to the edge, $d\ell'$ is the increment along the edge described by C,

$$G(\mathbf{r},\mathbf{r}') = \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi \mid \mathbf{r}-\mathbf{r}' \mid} \tag{4}$$

with $\mathbf{r}$ and $\mathbf{r}'$ denoting the observation and integration points, respectively, and

$$\hat{s} = \frac{\mathbf{s}}{s} = \frac{\mathbf{r}-\mathbf{r}'}{\mid \mathbf{r}-\mathbf{r}' \mid} \approx \hat{x}\cos\phi\sin\beta + \hat{y}\sin\phi\sin\beta + \hat{z}\cos\beta \tag{5}$$

To find expressions for $I(\ell')$ and $\mathrm{M}(\ell')$ we revert back to the original problem and note that the scattered field from the half plane can be written as

$$\begin{aligned}\mathbf{E}^s = -ik \Big\{ \int\int_S \left[Z_o \hat{s}\times\hat{s}\times\mathbf{j}_o(\mathbf{r}') + \hat{s}\times\mathbf{m}_o(\mathbf{r}')\right] G(\mathbf{r},\mathbf{r}')ds' \\ + \int\int_S \left[Z_o \hat{s}\times\hat{s}\times\mathbf{j}_n(\mathbf{r}') + \hat{s}\times\mathbf{m}_n(\mathbf{r}')\right] G(\mathbf{r},\mathbf{r}')ds' \Big\}\end{aligned} \tag{6}$$

where $\mathbf{j}_{o,n}(\mathbf{r}')$ and $\mathbf{m}_{o,n}(\mathbf{r}')$ denote the electric and magnetic currents over the upper (oth) or lower (nth) surface of the half plane. It is required that (3) and (6) must be equal and by following a procedure similar to that in [19], we deduce that

$$M(\ell) = \sum_{p=o,n} M_p(\ell) = \frac{1}{\sin^2\beta}\sum_{p=o,n}\left[(\hat{z}-\hat{s}\cos\beta)\cdot\mathbf{K}_p^m + \hat{z}\cdot(\hat{s}\times\mathbf{K}_p^e)\right] \tag{7}$$

$$Z_o I(\ell) = \sum_{p=o,n} Z_o I_p(\ell) = \frac{1}{\sin^2\beta}\sum_{p=o,n}\left[-\hat{z}\cdot(\hat{s}\times\mathbf{K}_p^m) + (\hat{z}-\hat{s}\cos\beta)\cdot K_p^e\right] \tag{8}$$

where $\mathbf{K}_p^m$ and $\mathbf{K}_p^e$ are given by the integrals

$$\mathbf{K}_p^m = \sin\beta_o \int_o^\infty \mathbf{m}_p e^{-ik\sigma\cos\gamma} d\sigma \tag{9a}$$

$$\mathbf{K}_p^e = Z_o \sin\beta_o \int_o^\infty \mathbf{j}_p e^{-ik\sigma\cos\gamma} d\sigma \tag{9b}$$

in which

$$\cos\gamma = \hat{\sigma}\cdot\hat{s} = \sin\beta\sin\beta_o\cos\phi + \cos\beta\cos\beta_o \tag{10}$$

implying that $\hat{\sigma}$ is a unit vector in the xz plane making an angle of β_o with respect to the z-axis.

Concentrating on the currents associated with the upper surface of the half plane we note that

$$\mathbf{m}_o = \mathbf{E}\times\hat{y}, \qquad \mathbf{j}_o = \hat{y}\times\mathbf{H} \tag{11}$$

and from the impedance boundary conditions (2a) we deduce that

$$\eta_o\mathbf{K}_o^e = \hat{y}\times\mathbf{K}_o^m \tag{12}$$

These relations imply that we can write $M_o(\ell)$ and $I_o(\ell)$ in terms of $\mathbf{K}_o^e$ or $\mathbf{K}_o^m$. Alternatively, we may choose to express $I_o(\ell)$ and $M_o(\ell)$ in terms of the x components of $\mathbf{K}_o^e$ and $\mathbf{K}_o^m$. Doing so, gives

$$\begin{bmatrix} Z_oI_o \\ M_o \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{\eta_o} - \frac{\sin\phi}{\sin\beta}\right) & -\cot\beta\cos\phi \\ \cot\beta\cos\phi & \left(\eta_o - \frac{\sin\phi}{\sin\beta}\right) \end{bmatrix} \begin{bmatrix} -K_{ox}^m \\ K_{ox}^e \end{bmatrix} \tag{13}$$

It remains to evaluate $K_{ox}^{e,m}$ in terms of known quantities found from a solution of the canonical problem. Combining (9) and (11), yields

$$K_{ox}^e = \sin\beta_o\int_0^\infty Z_oH_z e^{-ik\sigma\cos\gamma}d\sigma \tag{14a}$$

$$K_{ox}^m = -\sin\beta_o\int_0^\infty E_z e^{-ik\sigma\cos\gamma}d\sigma \tag{14b}$$

and by following the procedure employed in [19], we may now introduce the exact integral representations for the surface fields. From the impedance half plane solution we have [10]

$$\begin{bmatrix} E_z \\ Z_oH_z \end{bmatrix} = \frac{e^{ikz\cos\beta_o}}{2\pi i}\int_\Gamma \begin{bmatrix} \mathcal{E}_z(\alpha+\pi-\phi) \\ Z_o\mathcal{H}_z(\alpha+\pi-\phi) \end{bmatrix} e^{-ik\rho\sin\beta_o\cos\alpha}d\alpha \tag{15}$$

where Γ is the Sommerfeld contour shown in Fig. 2 and we remark that $\phi = 0$ and $\rho = x$ for observations on the upper surface of the half plane. For the moment we will postpone the definition of the spectral quantities $\mathcal{E}_z(\alpha)$ and $\mathcal{H}_z(\alpha)$. These are explicitly given in [10] and will be expressed later in a convenient form for computational purposes. However, before we substitute (15) into (14), it is necessary to introduce an alternative representation to (15) which will permit use of the Sommerfeld inversion theorem (see Appendix) [26] for a closed form evaluation of $\mathbf{K}^{e,m}$. This amounts to replacing $\mathcal{E}_z(\alpha)$ and $\mathcal{H}_z(\alpha)$ by some odd functions without affecting the outcome of the integration. Such a replacement is possible by exploiting the properties of Γ. As seen from figure 2, Γ is comprised of two contours symmetrically located with respect to the imaginary axis. Thus, $\mathcal{E}_z(\alpha+\pi)$ and $\mathcal{H}_z(\alpha+\pi)$ may be replaced by their odd parts

$$F_E(\alpha) = \frac{1}{2}\left[\mathcal{E}_z(\alpha+\pi) - \mathcal{E}_z(-\alpha+\pi)\right] \tag{16a}$$

$$F_H(\alpha) = \frac{Z_o}{2}\left[\mathcal{H}_z(\alpha+\pi) - \mathcal{H}_z(-\alpha+\pi)\right] \tag{16b}$$

without affecting the value of the integral in (15). It is convenient to re-express these only in terms of the spectra $\mathcal{E}_z(\alpha+\pi)$ and $\mathcal{H}_z(\alpha+\pi)$ by invoking an alternative form of the boundary conditions (2). From [12] we find that (2) can be rewritten as

$$\left(\frac{\partial}{\partial y} + ik\eta_o \sin^2\beta_o\right) Z_o H_z + \cos\beta_o \frac{\partial E_z}{\partial x} = 0 \tag{17a}$$

$$\left(\frac{\partial}{\partial y} + \frac{ik}{\eta_o}\sin^2\beta_o\right) E_z - Z_o \cos\beta_o \frac{\partial H_z}{\partial x} = 0 \tag{17b}$$

and when these conditions are applied to the integrals in (15), the resulting integrand must be even with respect to α. It then follows that

$$\begin{bmatrix} \sin\alpha + \frac{\sin\beta_o}{\eta_o} & \cos\alpha\cos\beta_o \\ -\cos\alpha\cos\beta_o & \eta_o\sin\beta_o + \sin\alpha \end{bmatrix} \begin{bmatrix} \mathcal{E}_z(+\alpha+\pi) \\ Z_o\mathcal{H}_z(+\alpha+\pi) \end{bmatrix} =$$

$$\begin{bmatrix} -\sin\alpha + \frac{\sin\beta_o}{\eta_o} & \cos\alpha\cos\beta_o \\ -\cos\alpha\cos\beta_o & \eta_o\sin\beta_o - \sin\alpha \end{bmatrix} \begin{bmatrix} \mathcal{E}_z(-\alpha+\pi) \\ Z_o\mathcal{H}_z(-\alpha+\pi) \end{bmatrix} \tag{18}$$

permitting us to rewrite (16) as

$$F_E(\alpha) = \frac{-\sin\alpha}{D_o}\left[(\eta_o \sin\beta_o - \sin\alpha)\mathcal{E}_z(\alpha+\pi)\right.$$

$$\left. - \cos\beta_o \cos\alpha Z_o \mathcal{H}_z(\alpha+\pi)\right] \tag{19a}$$

$$F_H(\alpha) = \frac{-\sin\alpha}{D_o}\left[\cos\beta_o \cos\alpha \mathcal{E}_z(\alpha+\pi)\right.$$

$$\left. + \left(\frac{\sin\beta_o}{\eta_o} - \sin\alpha\right) Z_o \mathcal{H}_z(\alpha+\pi)\right] \tag{19b}$$

where D_o is the determinant

$$D_o = 1 + \sin^2\beta_o \sin^2\alpha - \left(\eta_o + \frac{1}{\eta_o}\right)\sin\alpha \sin\beta_o \tag{20}$$

In accordance with the above analysis we may now express the surface fields as

$$\begin{bmatrix} E_z \\ Z_o H_z \end{bmatrix} = \frac{e^{ikz\cos\beta_o}}{2\pi i}\int_\Gamma \begin{bmatrix} F_E(\alpha) \\ F_H(\alpha) \end{bmatrix} e^{-ik\rho\sin\beta_o\cos\alpha} d\alpha \tag{21}$$

and when these are substituted into (14), upon setting $\rho = \sigma\sin\beta_o, z = \sigma\cos\beta_o$ we may invoke the Sommerfeld inversion theorem given in the Appendix to find

$$K^e_{ox} = \frac{2}{i\kappa\sin\alpha_o}F_H(\alpha_o) \tag{22a}$$

$$K^m_{ox} = \frac{-2}{i\kappa\sin\alpha_o}F_E(\alpha_o) \tag{22b}$$

where

$$\kappa = k\sin\beta_o \tag{23}$$

and

$$\alpha_o = \cos^{-1}\left\{-\frac{\cos\gamma - \cos^2\beta_o}{\sin^2\beta_o}\right\} \tag{24}$$

Explicit expressions for the equivalent edge currents I_o and M_o can now be readily obtained in terms of $\mathcal{E}_z(\alpha+\pi)$ and $\mathcal{H}_z(\alpha+\pi)$. By substituting (22) into (13) and making use of (19) we have

$$\begin{aligned} Z_o I_o &= -\frac{2i}{\kappa D_o}\Bigg[\Bigg\{\left(\frac{\sin\phi}{\sin\beta}-\frac{1}{\eta_o}\right)(\eta_o\sin\beta_o-\sin\alpha_o) \\ &+ \cot\beta\cos\beta_o\cos\phi\cos\alpha_o\Bigg\}\mathcal{E}_z(\alpha_o+\pi) \\ &+\Bigg\{\left(\frac{1}{\eta_o}-\frac{\sin\phi}{\sin\beta_o}\right)\cos\beta_o\cos\alpha_o \\ &+ \cot\beta\cos\phi\left(\frac{\sin\beta_o}{\eta_o}-\sin\alpha_o\right)\Bigg\}Z_o\mathcal{H}_z(\alpha_o+\pi)\Bigg] \end{aligned} \tag{25}$$

$$\begin{aligned} M_0 &= -\frac{2i}{\kappa D_o}\Bigg[\Bigg\{-\cot\beta\cos\phi(\eta_o\sin\beta_o-\sin\alpha_o) \\ &- \left(\eta_o-\frac{\sin\phi}{\sin\beta}\right)\cos\beta_o\cos\alpha_o\Bigg\}\mathcal{E}_z(\alpha_o+\pi) \\ &+\Bigg\{\cot\beta\cos\beta_o\cos\phi\cos\alpha_o \\ &- \left(\eta_o-\frac{\sin\phi}{\sin\beta}\right)\left(\frac{\sin\beta_o}{\eta_o}-\sin\alpha_o\right)\Bigg\}Z_o\mathcal{H}_z(\alpha_o+\pi)\Bigg] \end{aligned} \tag{26}$$

in which D_o is given by (20) with $\alpha=\alpha_o$. We remark that from (25) and (26) it is seen that I_o and M_o are dual quantities, as expected.

The equivalent edge currents I_n and M_n which are associated with the bottom face of the half plane can be obtained directly from I_o and M_o by letting $\beta_o\to\pi-\beta_o$, $\beta\to\pi-\beta$, $\phi_o\to 2\pi-\phi_o$, $\phi\to 2\pi-\phi$, $\mathcal{E}_z(\alpha_o+\pi)\to\mathcal{E}_z(\alpha_n-\pi)$ and $\mathcal{H}_z(\alpha_o+\pi)\to\mathcal{H}_z(\alpha_n-\pi)$ with

$$\cos\alpha_n=\cos\alpha_o,\quad \sin\alpha_n=\sqrt{1-\cos^2\alpha_n}=\sin\alpha_o \tag{27}$$

Doing so, we obtain

$$Z_o I_n=\frac{2i}{\kappa D_n}\Bigg[\Bigg\{\left(\frac{\sin\phi}{\sin\beta}+\frac{1}{\eta_n}\right)(\eta_n\sin\beta_o-\sin\alpha_o)$$

$$- \cot\beta\cos\beta_o\cos\phi\cos\alpha_o\Big\}\mathcal{E}_z(\alpha_n-\pi)+\Big\{\left(\frac{\sin\phi}{\sin\beta}+\frac{1}{\eta_n}\right)\cos\beta_o\cos\alpha_o$$

$$+\cot\beta\cos\phi\left(\frac{\sin\beta}{\eta_n}-\sin\alpha_o\right)\Big\}Z_o\mathcal{H}_z(\alpha_n-\pi) \tag{28a}$$

$$M_n=-\frac{2i}{\kappa D_n}\Big[\Big\{\cot\beta\cos\phi(\eta_n\sin\beta-\sin\alpha_o)$$

$$+\cos\beta_o\cos\alpha_o\left(\frac{\sin\phi}{\sin\beta}+\eta_n\right)\Big\}\mathcal{E}_z(\alpha_n-\pi)+\Big\{\cot\beta\cos\beta_o\cos\phi\cos\alpha_o$$

$$-\left(\frac{\sin\phi}{\sin\beta}+\eta_n\right)\left(\frac{\sin\beta}{\eta_n}-\sin\alpha_o\right)\Big\}Z_o\mathcal{H}_z(\alpha_n-\pi)\Big] \tag{28b}$$

where

$$D_n=1+\sin^2\beta_o\sin^2\alpha_o-\left(\eta_n+\frac{1}{\eta_n}\right)\sin\alpha_o\sin\beta_o$$

For the perfectly conducting half plane I_o and M_o reduce to

$$M_o=\frac{2i}{k\sin\beta_o\sin\alpha_o'}\frac{\sin\phi}{\sin\beta}Z_o\mathcal{H}_z(2\pi-\alpha_o') \tag{29a}$$

$$I_o=\frac{2i}{k\sin^2\beta_o}\mathcal{E}_z(2\pi-\alpha_o')$$

$$+\frac{2i}{k\sin\beta_o\sin\alpha_o'}\left(-\cot\beta_o\cos\alpha_o'+\cot\beta\cos\phi\right)Z_o\mathcal{H}_z(2\pi-\alpha_o') \tag{29b}$$

in which $\alpha_o'=\pi-\alpha_o$ and from [3, p.255]

$$\mathcal{E}_z(2\pi-\alpha_o')=\frac{1}{4}\frac{2\sin\frac{\phi_o}{2}}{\cos\frac{\pi-\alpha_o'}{2}-\cos\frac{\phi_o}{2}} \tag{30a}$$

$$\mathcal{H}_z(2\pi-\alpha_o')=-\frac{1}{4}\frac{2\sin\left(\frac{\pi-\alpha_o'}{2}\right)}{\cos\frac{\pi-\alpha_o'}{2}-\cos\frac{\phi_o}{2}} \tag{30b}$$

It is then readily concluded that these expressions are in agreement with those in [19] provided the definition for $\mu = \cos\alpha_o'$ given in [20, equ.(22)] is used. In the next section we present an explicit computationally efficient form for the Sommerfeld spectra associated with the impedance half plane.

3 The Sommerfeld Spectra for the Impedance Half Plane

From [10] we find that

$$\mathcal{E}_z(\alpha+\pi) = -\frac{\sin\beta_o}{(1-\sin^2\beta_o\sin^2\alpha)}$$

$$\cdot\ [\sin\alpha\cos\beta_o\mathcal{E}_y(\alpha+\pi) + \cos\alpha Z_o\mathcal{H}_y(\alpha+\pi)] \qquad (31a)$$

$$Z_o\mathcal{H}_z(\alpha+\pi) = -\frac{\sin\beta_o}{1-\sin^2\beta_o\sin^2\alpha}$$

$$\cdot\ [\sin\alpha\cos\beta_o Z_o\mathcal{H}_y(\alpha+\pi) - \cos\alpha\mathcal{E}_y(\alpha+\pi)] \qquad (31b)$$

where the spectra $\mathcal{E}_y(\alpha+\pi)$ and $\mathcal{H}_y(\alpha+\pi)$ are associated with the y component of the corresponding fields. They can be conveniently expressed as

$$\mathcal{E}_y(\alpha+\pi) = \frac{1}{(1-\sin^2\beta_o\sin^2\phi_o)}$$

$$\left[U_s(\alpha+\pi,\phi_o;\beta_o,\frac{1}{\eta_o},\frac{1}{\eta_n})e_y - V_s(\alpha+\pi,\phi_o;\beta_o,\frac{1}{\eta_o},\frac{1}{\eta_n})h_y\right] (32a)$$

$$Z_o\mathcal{H}(\alpha+\pi) = \frac{1}{(1-\sin^2\beta_o\sin^2\phi_o)}$$

$$\cdot\ [U_s(\alpha+\pi,\phi_o;\beta_o,\eta_o,\eta_n)h_y + V_s(\alpha+\pi,\phi_o;\beta_o,\eta_o,\eta_n)e_y] (32b)$$

where

$$U_s(\alpha+\pi,\phi_o;\beta_o,\eta_o,\eta_n) = \left\{\left[\cos^2\beta_o + \sin^2\beta_o\cos\phi_o\cos\alpha\right]\left[\frac{\frac{1}{2}\sin\frac{\phi_o}{2}}{\cos\frac{\alpha}{2}-\cos\frac{\phi_o}{2}}\right]\right.$$

$$\left. + \sin\beta_o\cos\beta_o\sin\frac{\phi_o}{2}\left[M_2 + M_4\cos\frac{\alpha}{2}\right]\right\}\frac{\psi(\alpha+\pi,\theta^{\eta_o},\theta^{\eta_n})}{\psi(\pi-\phi_o,\theta^{\eta_o},\theta^{\eta_n})} \quad (33)$$

$$V_s(\alpha+\pi,\phi_o;\beta_o,\eta_o,\eta_n) =$$

$$-\sin\beta_o\cos\beta_o\sin\frac{\phi_o}{2}\left[M_1 + M_3\cos\frac{\alpha}{2}\right]\frac{\psi(\alpha+\pi,\theta^{\eta_o},\theta^{\eta_n})}{\psi(\pi-\phi_o,\theta^{\eta_o},\theta^{\eta_n})} \quad (34)$$

In these,

$$M_1 = \frac{4}{\Delta}\left(a_1^+\cos\frac{\phi_o}{2} - a_2^+\right) \quad (35)$$

$$M_2 = \frac{i}{\Delta}\left[a_1^+\left\{a_1^-\cos\frac{\phi_o}{2} + a_2^-\right\} - a_2^+\left\{a_1^- + a_3^-\cos\frac{\phi_o}{2}\right\}\right] \quad (36)$$

$$M_3 = -\frac{4}{\Delta}\left(a_3^+\cos\frac{\phi_o}{2} - a_1^+\right) \quad (37)$$

$$M_4 = -\frac{i}{\Delta}\left[a_3^+\left\{a_1^-\cos\frac{\phi_o}{2} + a_2^-\right\} - a_1^+\left\{a_1^- + a_3^-\cos\frac{\phi_o}{2}\right\}\right] \quad (38)$$

$$\Delta = (a_1^+)^2 - a_2^+ a_3^+ \quad (39)$$

$$a_1^\pm = f_-(\alpha_\Delta) \pm f_-(\alpha_\Delta^*) \quad (40)$$

$$a_2^\pm = f_+(\alpha_\Delta)\sin\frac{\alpha_\Delta}{2} \pm f_+(\alpha_\Delta^*)\sin\frac{\alpha_\Delta^*}{2} \quad (41)$$

$$a_3^{\pm} = \frac{f_+(\alpha_\Delta)}{\sin\frac{\alpha_\Delta}{2}} \pm \frac{f_+(\alpha_\Delta^*)}{\sin\frac{\alpha_\Delta^*}{2}} \tag{42}$$

$$f_\pm(\alpha) = \frac{\psi(\pi-\phi_o,\theta_{\eta_o},\theta_{\eta_n})}{\psi(\pi-\phi_o,\theta^{\eta_o},\theta^{\eta_n})}\left[\frac{\psi(\alpha,\theta^{\eta_o},\theta^{\eta_n})}{\psi(\alpha,\theta_{\eta_o},\theta_{\eta_n})} \pm \frac{\psi(-\alpha,\theta^{\eta_o},\theta^{\eta_n})}{\psi(-\alpha,\theta_{\eta_o},\theta_{\eta_n})}\right] \tag{43}$$

$$\alpha_\Delta = \pi/2 + i\ell n\,(\tan\beta_o/2) \tag{44}$$

and

$$\sin\theta_{\eta_{o,n}} = \frac{\eta_{o,n}}{\sin\beta_o}, \qquad \sin\theta^{\eta_{o,n}} = \frac{1}{\eta_{o,n}\sin\beta_o} \tag{45}$$

Also,

$$\psi(\alpha,\theta^+,\theta^-) = \psi_\pi\left(\alpha+\frac{3\pi}{2}-\theta^+\right)\psi_\pi\left(\alpha-\frac{3\pi}{2}+\theta^-\right)\cdot$$
$$\cdot\psi_\pi\left(\alpha+\frac{\pi}{2}+\theta^+\right)\psi_\pi\left(\alpha-\frac{\pi}{2}-\theta^-\right) \tag{46}$$

in which $\psi_\pi(\alpha)$ is the Maliuzhinets [8] function and simple algebraic expressions for this have been given in [29].

We remark that the edge diffracted fields can be conveniently written as

$$E_y^d \sim \frac{e^{i(\kappa\rho-\pi/4)}}{\sqrt{2\pi\kappa\rho}}\left(1-\sin^2\beta_o\sin^2\phi_o\right)^{-1}$$
$$\cdot\left[U\left(\phi,\phi_o;\beta_o,\frac{1}{\eta_o},\frac{1}{\eta_n}\right)e_y - V\left(\phi,\phi_o;\beta_o,\frac{1}{\eta_o},\frac{1}{\eta_n}\right)h_y\right] \tag{47a}$$

$$ZH_y^d \sim \frac{e^{i(\kappa\rho-\pi/4)}}{\sqrt{2\pi\kappa\rho}}\left(1-\sin^2\beta_o\sin^2\phi_o\right)^{-1}\left[U\left(\phi,\phi_o;\beta_o,\eta_o,\eta_n\right)h_y\right.$$
$$\left.+V\left(\phi,\phi_o;\beta_o,\eta_o,\eta_n\right)e_y\right] \tag{47b}$$

where

$$V(\phi,\phi_o;\beta_o,\eta_o,\eta_n) = U_s(-\phi,\phi_o;\beta_o,\eta_o,\eta_n)$$

$$- U_s(2\pi-\phi,\phi_o;\beta_o,\eta_o,\eta_n) \tag{48a}$$

$$U(\phi,\phi_o;\beta_o,\eta_o,\eta_n) = V_s(-\phi,\phi_o;\beta_o,\eta_o,\eta_n)$$

$$- V_s(2\pi-\phi,\phi_o;\beta_o,\eta_o,\eta_n) \tag{48b}$$

and these are in a form compatible to that given by Senior [9] for the case of a half plane having equal face impedances ($\eta_o = \eta_n$). The solution procedure followed by Senior [9] is completely independent of that employed by Bucci and Franceschetti [10]. It is therefore of interest to compare (47) with the diffracted fields given in [9] when $\eta_o = \eta_n = \eta$. This should provide some validation of (47), particularly since these expressions differ (only in the sign at two locations) from those given in [10, equ.(82)].

To compare the U and V functions given above to those derived in [9, 28] for the half plane of equal face impedances we proceed as follows. First the U and V functions for the half plane with equal face impedances given in [9] are rewritten as

$$U(\phi,\phi_o;\beta_o,\eta) = \left[\frac{\cos^2\beta_o - \sin^2\beta_o\cos\phi\cos\phi_o}{(\cos\phi+\cos\phi_o)}\left(1-2\eta\sin\beta_o\cos\frac{\phi}{2}\cos\frac{\phi_o}{2}\right)\right.$$

$$\left.+\sin\beta_o\cos\beta_o\cdot\left\{i\frac{h_-(\alpha_\Delta,\alpha_\Delta^*,\eta)}{h_+(\alpha_\Delta,\alpha_\Delta^*,\eta)} - i2\eta\sin\beta_o\cos\frac{\phi}{2}\cos\frac{\phi_o}{2}\frac{p_-(\alpha_\Delta,\alpha_\Delta^*,\eta)}{p_+(\alpha_\Delta,\alpha_\Delta^*,\eta)}\right\}\right]$$

$$\cdot\, K_+(-\kappa\cos\phi;\beta_o,\eta)\,.K_+(-\kappa\cos\phi_o;\beta_o,\eta) \tag{49a}$$

$$V(\phi,\phi_o;\beta_o,\eta) = -V\left(\phi_o,\phi;\beta_o,\frac{1}{\eta}\right) =$$

$$\sin\beta_o\cos\beta_o\left[\eta\sin\beta_o\frac{4\cos\frac{\phi}{2}}{h_+\left(\alpha_\Delta,\alpha_\Delta^*,\frac{1}{\eta}\right)} - \frac{2\cos\frac{\phi_o}{2}}{p_+\left(\alpha_\Delta,\alpha_\Delta^*,\frac{1}{\eta}\right)}\right]$$

$$\cdot\, K_+(-\kappa\cos\phi;\beta_o,\eta)\,K_+\left(-\kappa\cos\phi_o;\beta_o,\frac{1}{\eta}\right) \tag{49b}$$

in which

$$p_{\pm}\left(\alpha_{\Delta}, \alpha_{\Delta}^{*}, \eta\right) = g\left(\alpha_{\Delta}, \eta\right) \sin \frac{\alpha_{\Delta}}{2} \pm g\left(\alpha_{\Delta}^{*}, \eta\right) \sin \frac{\alpha_{\Delta}^{*}}{2} \tag{50}$$

$$h_{\pm}\left(\alpha_{\Delta}, \alpha_{\Delta}^{*}, \eta\right) = \frac{g(\alpha_{\Delta}, \eta)}{\sin \frac{\alpha_{\Delta}}{2}} \pm \frac{g(\alpha_{\Delta}^{*}, \eta)}{\sin \frac{\alpha_{\Delta}^{*}}{2}} \tag{51}$$

$$g(\alpha, \eta) = \frac{1}{\eta} \frac{\psi(\alpha, \theta_{\eta}, \theta_{\eta})}{\psi(\alpha, \theta^{\eta}, \theta^{\eta})} \tag{52}$$

$$K_{+}\left(-\kappa \cos \alpha; \beta_{o}, \eta\right) = \frac{\left\{\psi_{\pi}\left(\frac{\pi}{2}\right)\right\}^{4}}{2\sqrt{\eta} \sin \beta_{o}} \frac{\cos \frac{\alpha}{2}}{\psi\left(\alpha, \theta^{\eta}, \theta^{\eta}\right)} \tag{53}$$

where $\sin \theta^{\eta} = \frac{1}{\eta \sin \beta_o}$ and α_{Δ} was defined in (44). These were obtained from those in [9, 28] by replacing the parameter γ employed in [9, 28] with its explicit form in terms of the split functions.

We must now show that (49) reduce to those given by (33)-(45) when $\eta_o = \eta_n = \eta$. To do so we note that $\psi(\alpha, \theta, \theta) = \psi(-\alpha, \theta, \theta)$ and for $\eta_o = \eta_n = \eta$ the function $f_{\pm}(\alpha)$ simplifies to

$$f_{+}(\alpha) = 2 \frac{\psi\left(\pi - \phi_{o}, \theta_{\eta}, \theta_{\eta}\right) \psi\left(\alpha, \theta^{\eta}, \theta^{\eta}\right)}{\psi\left(\pi - \phi_{o}, \theta^{\eta}, \theta^{\eta}\right) \psi\left(\alpha, \theta_{\eta}, \theta_{\eta}\right)} \quad , \tag{54}$$

$$f_{-}(\alpha) = 0$$

where $\sin \theta_{\eta} = \eta / \sin \beta_{o}$. Consequently,

$$M_{1} = 2\eta \frac{\psi\left(\pi - \phi_{o}, \theta^{\eta}, \theta^{\eta}\right)}{\psi\left(\pi - \phi_{o}, \theta_{\eta}, \theta_{\eta}\right)} \frac{1}{h_{+}\left(\alpha_{\Delta}, \alpha_{\Delta}^{*}, \frac{1}{\eta}\right)} \tag{55}$$

$$M_{2} = -i \cos \frac{\phi_{o}}{2} \frac{p_{-}\left(\alpha_{\Delta}, \alpha_{\Delta}^{*}, \eta\right)}{p_{+}\left(\alpha_{\Delta}, \alpha_{\Delta}^{*}, \eta\right)} \tag{56}$$

$$M_{3} = 2\eta \cos \frac{\phi_{o}}{2} \frac{\psi\left(\pi - \phi_{o}, \theta^{\eta}, \theta^{\eta}\right)}{\psi\left(\pi - \phi_{o}, \theta_{\eta}, \theta_{\eta}\right)} \frac{1}{p_{+}\left(\alpha_{\Delta}, \alpha_{\Delta}^{*}, \frac{1}{\eta}\right)} \tag{57}$$

and

$$M_4 = -i\frac{h_-(\alpha_\Delta, \alpha_\Delta^*, \eta)}{h_+(\alpha_\Delta, \alpha_\Delta^*, \eta)} \tag{58}$$

In addition, from [30] we find that

$$\psi(\pi - \phi \mp \pi, \theta^{\eta_o}, \theta^{\eta_n}) = \frac{\{\psi_\pi(\pi/2)\}^8}{4\psi(\pi - \phi, \theta^{\eta_o}, \theta^{\eta_n})}$$
$$\left[B_1 + B_2\left\{\sin\left(\frac{\pi \pm \phi}{2}\right) \mp \sin\frac{\phi}{2}\right\} + \sin\left(\frac{\phi \pm \pi}{2}\right)\sin\frac{\phi}{2}\right] \tag{59}$$

where

$$B_1 = \cos\left(\frac{\pi - 2\theta^{\eta_o}}{4}\right)\cos\left(\frac{\pi - 2\theta^{\eta_n}}{4}\right) - \cos^2\left(\frac{\pi}{4}\right) \tag{60}$$

$$B_2 = \frac{1}{2\sin\frac{\pi}{4}}\left\{\cos\left(\frac{\pi - 2\theta^{\eta_o}}{4}\right) - \cos\left(\frac{\pi - 2\theta^{\eta_n}}{4}\right)\right\} \tag{61}$$

and when $\eta_o = \eta_n = \eta, \psi(\pi - \phi \mp \pi, \theta^{\eta_o}, \theta^{\eta_n})$ simplifies to

$$\psi(\pi - \phi \mp \pi, \theta^\eta, \theta^\eta) = \frac{\left\{\psi_\pi\left(\frac{\pi}{2}\right)\right\}^8}{4\psi(\pi - \phi, \theta^\eta, \theta^\eta)}\left[\frac{1}{2\eta\sin\beta_o} + \sin\left(\frac{\phi \pm \pi}{2}\right)\sin\frac{\phi}{2}\right] \tag{62}$$

Using (55)-(58) and (62) in (33) and (34), it is then readily shown that the U and V functions given in (48) reduce to those in (49) when $\eta_o = \eta_n = \eta$.

4 Fringe Equivalent Currents

In PTD implementations the total scattered field is obtained as the sum of the physical optics fields and fringe wave contribution. The last is computed by integrating the fringe wave equivalent currents along the edge or surface discontinuity given by

$$\begin{aligned} Z_o I_{o,n}^f &= Z_o I_{o,n} - Z_o I_{o,n}^{PO} \\ M_{o,n}^f &= M_{o,n} - M_{o,n}^{PO} \end{aligned} \tag{63}$$

In these I_o, n and $M_{o,n}$ are the edge equivalent current expressions (26), whereas $I_{o,n}^{PO}$ and $M_{o,n}^{PO}$ are the corresponding edge equivalent currents which result when the current on the faces forming the edge is set to its physical optics value. They can thus be obtained from (13) in conjunction with (9) upon setting

$$\mathbf{m}_{on} \rightarrow \mathbf{m}_{on}^{P.O.} = \pm\left[\mathbf{E}^i + \mathbf{E}^r\right] \times \hat{y} \quad \mathbf{j}_{on} \rightarrow \mathbf{j}_{on}^{PO} = \pm\hat{y} \times \left(\mathbf{H}^i + \mathbf{H}^r\right) \qquad (64)$$

where $(\mathbf{E}^r, \mathbf{H}^r)$ denote the reflected fields from the corresponding face. As can be expected, $\mathbf{j}_o^{PO}$ and $\mathbf{M}_o^{PO}$ are set to zero when the upper face is not illuminated and the same holds for $\mathbf{j}_n$ and $\mathbf{M}_n$ when the lower is not illuminated.

On applying the boundary condition (2a) we can readily obtain $(\mathbf{E}^r, \mathbf{H}^r)$ from which we find that

$$\mathbf{m}_o^{PO} = -2\Gamma \sin\phi_o \left\{\hat{x}\left[(\sin\phi_o + \eta_o \sin\beta_o)\, e_z - \cos\beta_o \cos\phi_o h_z\right]\right.$$
$$\left.-\hat{z}\eta_o \left[\cos\beta_o \cos\phi_o e_z + \left(\sin\phi_o + \frac{1}{\eta_o}\sin\beta_o\right) h_z\right]\right\} e^{-ik[x\sin\beta_o\cos\phi_o - z\cos\beta_o]} \qquad (65)$$

where

$$\Gamma = \frac{\eta_o}{(1 + \eta_o \sin\beta_o \sin\phi_o)(\eta_o + \sin\beta_o \sin\phi_o)} \qquad (66)$$

When (65) is substituted into 9(a), upon setting $x = \sigma\sin\beta_o, z = \sigma\cos\beta_o$ and integrating we obtain (lower limit only)

$$K_{ox}^m = -\eta_0 K_{oz}^e =$$

$$-\frac{2\Gamma\sin\phi_o}{ik\sin\beta_o\left[\cos\alpha_o' + \cos\phi_o\right]}\left[(\sin\phi_o + \eta_o\sin\beta_o)\, e_z - \cos\beta_o cos\phi_o h_z\right] (67a)$$

$$K_{ox}^e = \frac{1}{\eta_o} K_{oz}^m =$$

$$\frac{2\Gamma\sin\phi_o}{ik\sin\beta_o\left[\cos\alpha_o' + \cos\phi_o\right]}\left[\cos\beta_o\cos\phi_o e_z + \left(\sin\phi_o + \frac{1}{\eta_o}\sin\beta_o\right) h_z\right] (67b)$$

where again $\alpha_o' = \pi - \alpha_o$ with α_o as given in (24). The corresponding physical optics equivalent currents I_o^{PO} and M_o^{PO} are now obtained from (13), we have

$$Z_o I_o^{PO} = -\left(\frac{1}{\eta_o} - \frac{\sin\phi}{\sin\beta}\right) K_{ox}^m - \cot\beta\cos\phi K_{ox}^e \tag{68}$$

$$M_o^{PO} = -\cot\beta\cos\phi K_{ox}^m + \left(\eta_o - \frac{\sin\phi}{\sin\beta}\right) K_{ox}^e \tag{69}$$

and these can be shown to reduce to the known expressions for the perfectly conducting case. The physical optics equivalent current for the lower face can be obtained from (68) and (69) upon letting $\beta_0 \rightarrow \pi - \beta_0, \phi_o \rightarrow 2\pi - \phi_o, \beta \rightarrow \pi - \beta$ and $\phi \rightarrow 2\pi - \phi$.

5 Appendix: Sommerfeld's Inversion Theorem [26]

Let

$$f(x) = \frac{1}{2\pi i}\int_\Gamma e^{\mu x\cos\alpha} F(\alpha) d\alpha \tag{A1}$$

where $| arg(\mu) | \leq \pi/2$ and $f(x)$ satisfies the inequality

$$| f(x) | < | A |^{-1+a} e^{b|x|}$$

for A, a and b positive real numbers. Also, $0 < | x | < \infty$ and $f(x)$ is analytic in this region. Then, there exists one and only odd function $F(\alpha)$ which is regular on Γ and within Γ (except possibly at infinities), and which satisfies the inequality $| F(\alpha) | < A_1 e^{(1-a_1)|Im(\alpha)|}$. This function is represented by the integral

$$F(\alpha) = -\frac{\mu\sin\alpha}{2}\int_o^\infty f(x) e^{-\mu x\cos\alpha} dx \tag{A2}$$

and for this function $a_1 = a$, whereas A_1 is positive real.

Substituting $(A1)$ into $(A2)$ yields

$$F(\alpha_o) = -\frac{\mu\sin\alpha_o}{4\pi i}\int_o^\infty \int_\Gamma F(\alpha) e^{\mu x(\cos\alpha - \cos\alpha_o)} d\alpha dx$$

and for the particular case of interest $\mu = -ik$

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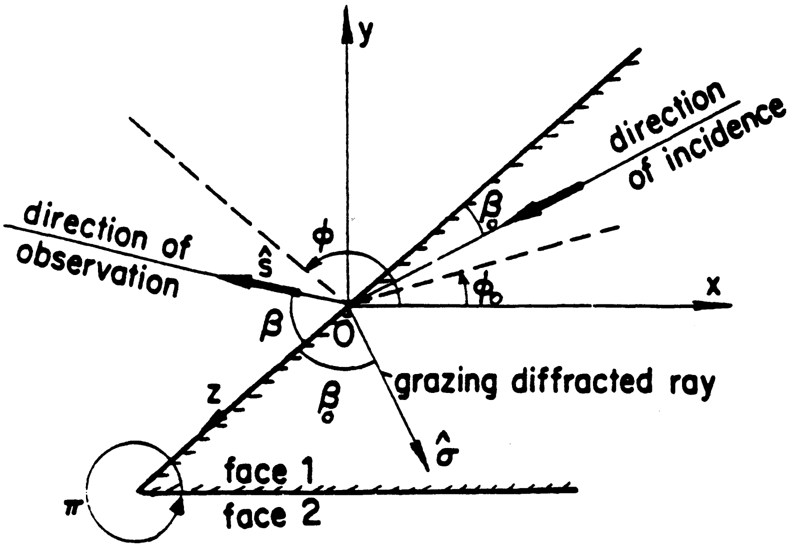

Figure. 1. Edge Geometry and angle definitions

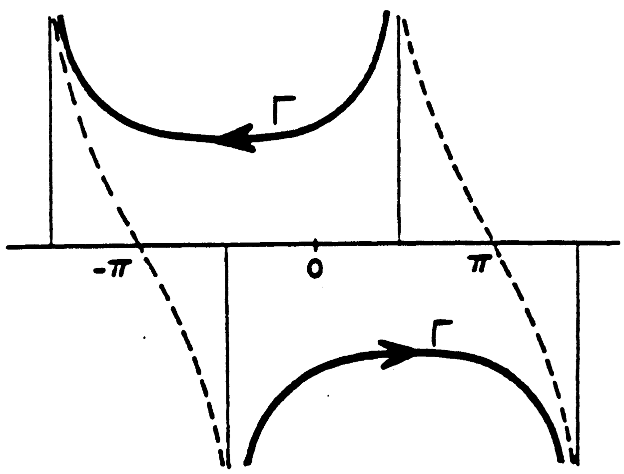

Figure 2. The integration contour Γ in the Complex plane