THE UNIVERSITY OF MICHIGAN

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CALCULATIONS OF SHIP HULL FORMS WITH ELECTRONIC DIGITAL COMPUTERS

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ABSTRACT

The author has calculated the hull forms mathematically induced by the following source distribution

$$m(\xi, \zeta) = f_1(\xi) \cdot f_2(\zeta)$$

$$f_1(\xi) = \alpha_1 \sin (\frac{\pi}{2} \xi) \qquad - 1 \leq \xi$$

$$f_2(\zeta) = 1 \qquad - t \leq \zeta \leq t$$

Two models were calculated; one is a fine model C-101 ($\alpha_1 = 0.4$, $t = 0.1$), the other is a full model C-201 ($\alpha_1 = 0.6$, $t = 0.1$). These models (L = 2.500m) were subjected to the tank experiments at the twoing tank of the Tokyo University.

I. INTRODUCTION

A very important problem in the study of wave-resistance theory is that of obtaining a hull form which satisfies the hull boundary conditions exactly.[1] To reduce the error arrising from the usual approximate hull boundary conditions of the linear wave-resistance theory in the case of a ship of finite beam, such a hull form becomes indeed a requirement of that theory. Furthermore, since the wave-resistance theory is based on the assumption of an ideal fluid, a surface satisfying the exact hull boundary conditions also provides a more correct form for the separation of the effects of viscosity on the wave-making characteristics of the hull. In addition it should lead to a more correct understanding of the relationship between hull form, wave-resistance and wave pattern.

For a given hull form the linear wave-resistance theory assumes that the hull can be represented by a distribution of singularities.

This distribution can be divided into three parts.

(i) A distribution of sources in the plane of symmetry of the ship of strength given by the slope of the hull surface.

(ii) An image system, with respect to the static free surface, of source distribution as given by (i).

(iii) A distribution of singularities distributed above the free-surface to satisfy the free-surface conditions.

The systems (i) and (iii) combined represent the double model of the given hull form moving in an infinite fluid region.

The assumption of the linear theory that the source strength at a point in the plane of symmetry of the ship is given by the slope of hull form at that point is only exact in the limit as the beam approaches zero. No simple method of correcting for the finite beam exists. In the attempt to satisfy the hull boundary conditions exactly one is therefore forced to consider the inverse problem namely, for a given source distribution what is the hull form that this distribution represents.[2]

The complete inverse problem, which includes the contribution from the singularity system (iii), becomes a very difficult one to solve. The secondary effect of system (iii) is relatively small at small Froude numbers, however, and in this paper only the source distribution representing a double model moving at constant speed in an infinite fluid will be considered.

For a hypothetical wall sided model of infinite draft the problem of determining the closed stream lines of the source distribution representing the hull, neglecting the free surface effects, is reduced to a two-dimensional case. It can be solved either by the method of conformal mapping or by solving a first degree integral equation.[1] In the case of finite draft, i.e., a three dimensional hull form, the direct problem of obtaining the singularity distribution which represents a given hull form becomes extremely difficult even if the free surface effects are neglected. Its solution involves integral equations of the second order. The inverse problem is considerably simpler, however, and this approach was therefore used by Inui and Eggers.[1,2]

This paper describes the solution of the inverse problem by means of electronic digital computers as applied to two different source distributions.

II. SOURCE DISTRIBUTION AND HULL FORM

Let us assume the singularity distribution to be of the form

$$m(\xi,\zeta) = f_1(\xi) \cdot f_2(\zeta) \tag{1}$$

where $m(\xi,\zeta)$ is the strength of a distributed source (flow per unit area per unit time) divided by the ship's speed V. Because a double model has to be considered, the region of source distribution is given by

$$-1 \leq \xi \leq 1 , \qquad -t \leq \zeta \leq t \tag{2}$$

where

$$\xi = \frac{x'}{L} , \qquad \zeta = \frac{z'}{2L} ,$$

the length of the ship being equal to 2L and the draft of the ship at bow and stern, $T_F = tL/2$.

Since it is required that the hull form be closed the total source strength must be equal to zero, thus

$$\int_{-t}^{t} \int_{-1}^{1} m(\xi,\zeta)\, d\xi\, d\zeta = 0$$

Due to symmetry with respect to the L.W.L. it follows that

$$m(\xi,-\zeta) = m(\xi,\zeta) .$$

If the ship is stationary, and if the uniform flow is in the negative ξ direction

$$m(\xi,\zeta) \leq 0, \quad \xi > 0$$

$$m(\xi,\zeta) \leq 0, \quad \xi < 0$$

For fore-and-aft symmetry,

$$m(\xi,\zeta) = -m(\xi,\zeta)$$

For a source distribution of this type, the location of stagnation points at bow and stern ξ_f and ξ_a become:

$$\xi_f = 1 + \epsilon$$

$$\xi_a = -1 - \epsilon$$

Ship length is in reality 2 + 2ϵ, but ϵ is normally small and is neglected. Drafts at bow and stern becomes t + ϵ' by similar analogy, but ϵ' again can be neglected.

The velocity potential due to a source distribution as given by (1) is:

$$\phi(x,y,z) = -\frac{1}{4\pi} \int_{-t}^{t} \int_{-1}^{1} \frac{m(\xi,\zeta)d\xi\, d\zeta}{\sqrt{(x-\xi)^2 + y^2 + (z-\zeta)^2}} - x$$

and by definition the velocity components become

$$\{u,v,w\} = \text{grad}\ \phi$$

The velocity components can be substituted into the differential equation for the streamlines

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

and a stream line may thus be obtained by means of successive integration using the Runge-Kutta method. Four or five suitably chosen streamlines will normally define a hull form with sufficient accuracy. As for the initial values of the integral, $u(1 + \epsilon, 0, 0) = 0$ gives the stagnation point at the bow. The value of ϵ is determined by equating u to zero. It is preferable to start at $x = 1$, and since the flow around the bow is essentially two-dimensional the initial value of y may be determined from the two-dimensional case as outlined later. The initial value of z is optional within the depth of the singularity distribution. Actually the streamlines coverge closely toward the midship and a slight error in the starting value of y makes little difference in the tracing of the streamlines.

When the singularity distribution is uniform draftwise,

$$f_2(\zeta) = 1 \qquad (-t \leq \zeta \leq t)$$

the velocity components become

$$\left.\begin{aligned}
-4\pi u &= 4\pi - \int_{-t}^{t}\int_{-1}^{1} \frac{f_1(\xi)(x-\xi)\,d\xi\,d\zeta}{[(x-\xi)^2+y^2+(z-\zeta)^2]^{3/2}} \\
&= 4\pi + \int_{-1}^{1} \frac{f_1(\xi)(x-\xi)}{(x-\xi)^2+y^2}\left(\frac{z-t}{r} - \frac{z+t}{r'}\right) d\xi \\
-4\pi v &= -\int_{-t}^{t}\int_{-1}^{1} \frac{f_1(\xi)\,y\,d\xi\,d\zeta}{[(x-\xi)^2+y^2+(z-\zeta)^2]^{3/2}} \\
&= +\int_{-1}^{1} \frac{f_1(\xi)\,y}{(x-\xi)^2+y^2}\left[\frac{z-t}{r} - \frac{z+t}{r'}\right] d\xi
\end{aligned}\right\} \qquad (3)$$

$$- 4\pi w = - \int_{-t}^{t} \int_{-1}^{1} \frac{f_1(\xi)(z - \xi)\, d\xi\, d\zeta}{[(x - \xi)^2 + y^2 + (z - \zeta)^2]^{3/2}}$$

$$= - \int_{-1}^{1} f_1(\xi) \left(\frac{1}{r} - \frac{1}{r'}\right) d\xi$$

where

$$r = \sqrt{(x - \xi)^2 + y^2 + (z - t)^2}$$

$$r' = \sqrt{(x - \xi)^2 + y^2 + (z + t)^2}$$

For two dimensional flow, the streamline is given by

$$y = \frac{1}{2\pi} \int_{-1}^{1} f_1(\xi) \tan^{-1} \left(\frac{y}{x-\xi}\right) d\xi$$

III. COMPUTING METHOD AND RESULT

For the present computations, the functions of singularity distribution in (1) and (2) were chosen as follows.

$$f_1(\xi) = a_1 \sin\left(\frac{\pi}{2}\xi\right) \qquad -1 \leq \xi \leq 1$$

$$f_2(\zeta) = 1 \qquad -t \leq \zeta \leq t$$

TABLE I

Hull Type	A_1	t	Computor	digit, required time
C-101	0.4	0.1	HITAC - 301	10-decimal 12 digit, float. pt, 6-1/2 hours
C-201	0.6	0.1	BENDIX G-15D	10-decimal 7 digit, float. pt. 4 hours.

Two hull forms were computed for the values of parameters as given in Table I. For both hulls, 4 streamlines were traced starting at the following points

i) $x = 1,\ z = 0,\ y = \epsilon_1$ (waterplane)

ii) $x = 1,\ z = 0.05,\ y = \epsilon_2$

iii) $x = 1,\ z = 0.08,\ y = \epsilon_3$

iv) $x = 1,\ y = 0,\ z = 0.1 + \epsilon_4$ (Keel line)

The y and z coordinates were evaluated at the following values of x:

$x = 0.9975,\ 0.995,\ 0.9925,\ 0.99,\ 0.8,\ 0.6,\ 0.4,\ 0.2,\ 0$

The velocity components were also evaluated at these points.

The initial values of ϵ_1, ϵ_2, ϵ_3 and ϵ_4 are determined according to the integral equation for streamlines of two two-dimensional flow. When these values are two small for the computer, however, suitable modified values might be needed. For the computed cases the following values were used.

C-101: $\epsilon_1 = \epsilon_2 = \epsilon_3 = 1.2 \times 10^{-10}$, $\epsilon_4 = 10^{-10}$

C-201: $\epsilon_1 = \epsilon_2 = \epsilon_3 = 1.69 \times 10^{-6}$, $\epsilon_4 = 1.45 \times 10^{-6}$

Since computation methods were different for the two hull forms, each method will be explained separately.

3.1 Hull Form C-101

It is observed that the integrands of Equations (3) have sharp peaks or steepness in the vecinity of the bow and stern. At these points the streamlines pass close to the singularities. Proper care must be taken in the computations to account for these features. In detail the extreme values were found to be located as follows:

u: peaks at $\xi = x \pm y$, equal to zero at $\xi = x$

v: peaks at $\xi = x$

w: peaks at $\xi = x$

Numerical integration was performed by the method of Legendre-Gauss. The region of ξ was subdivided into sub-regions of various lengths so that maximum accuracy could be obtained, i.e., coordinates were closely spaced in regions where the integrands displayed sharp variations. Figure 2 shows the sub-division of the region $-1 \leq \xi \leq 1$ used in the computations of the velocity component u.

To check the final programming a linear distribution of sources ($f_1(\xi) = a_1 \xi$), was tried. Professor Inui of the Tokyo University had independently computed the streamlines for this form previously.

A complete agreement up to 3-digits was found to exist. Computation results of Hull Type C-101 are given in Tables II and III.

3.2 Hull Type C-201

The integrals to be computed in this case are exactly the same as in the case of C-101, but for the regions of close proximity of the singularities, an analytical treatment was employed.* When

$$\xi = 1, \ \sin\left(\frac{\pi}{2}\xi\right) \backsimeq 1.$$

For $0 \ll \epsilon \ll 1$, (3) can be changed to:

$$\int_{1-\epsilon}^{1} \frac{\sin(\frac{\pi}{f}\xi)(x-\xi)}{(x-\xi)^2+y^2}\left(\frac{z-0.1}{r} - \frac{z+0.1}{r'}\right) d\xi \backsimeq \int_{1-\epsilon}^{1} \frac{x-\xi}{(x-\xi)^2+y^2}\left(\frac{z-0.1}{r} - \frac{z+0.1}{r'}\right) d\xi$$

$$= \left[\ln \frac{(x-\xi)^2+y^2}{\left(\sqrt{(x-\xi)^2+y^2+(z-0.1)^2}+(0.1-z)\right)\left(\sqrt{(x-\xi)^2+y+(z+0.1)^2}+(0.1+z)\right)}\right]_{\xi=1-\epsilon}^{\xi=1}$$

$$\int_{1-\epsilon}^{1} \sin\left(\frac{\pi}{2}\xi\right)\left(\frac{1}{r} - \frac{1}{r'}\right) d\xi = \int_{1-\epsilon}^{1} = \left(\frac{1}{r} - \frac{1}{r'}\right) d\xi$$

$$= \left[\ln \frac{z-0.1+\sqrt{(x-\xi)^2+(z-0.1)^2}}{z+0.1+\sqrt{(x-\xi)^2+(z+0.1)^2}}\right]_{\xi=1-\epsilon}^{\xi=1}$$

In this computation, $\epsilon = 10^{-2}$ ($\sin\frac{\pi}{2}(1-10^{-2}) = 0.999$), and $-1 \leq \xi \leq 1 - 10^{-2}$ was suitably divided to apply numerical integration by Simpson's rule. The result is shown in Tables IV and V.

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* Because of this treatment the precedure of calculation for C-201 is not suitable for a more arbitrary distribution of the singularity.

Two sets of hull lines, C-101 and C-201, each developed from four streamlines as obtained in the above are given in Figure 3. Also their off sets are given in Tables VI and VII.

IV. CONCLUSION

Hull types C-101 and C-201 were constructed into models of 2.5 meters, and were tested for resistance and wave observation at Tokyo University Towing Tank. Also, to these models, suitable bulbs were fitted at bow and stern, and Waveless Hull Form studies based on the interference cancellation of bulb wave and main hull wave were conducted.[3]

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Table 2 Coordinates of Calculated Stream-Lines for Model C–101

Unit : $L = 20$

x	Square Station Number		L.W.L.	Stream–Line A		Stream–Line B		Keel Line
			y	y	z	y	z	z
10	0	10	·000	·000	·500	·000	·800	1.000
9.9	1/20	s19/20	·025	·025	·508	·024	·815	1.030
9.0	1/2	9 1/2	·201	·196	·564	·167	·937	1.174
8.0	1	9	·362	·343	·634	·258	1.067	1.306
6.0	2	8	·608	·552	·758	·362	1.263	1.492
4.0	3	7	·773	·685	·848	·423	1.391	1.619
2.0	4	6	·868	·759	·902	·456	1.465'	1.693
0.	5		·900	·783	·920	·467	1.490	1.718

Table 3 Velocity on Calculated Stream–Lines for Medel C–101

Unit : Uniform Flow $u_0 = 1$

x	L.W.L.		S–L A			S–L B			K.L.	
	u	v	u	v	w	u	v	w	u	w
10	·0573	·1001	·0674	·1001	·0451	·0984	·1001	·1026	·3405	·6723
9.9	·7995	·1860	·8107	·1852	·0448	·8375	·1825	·0958	·9493	·2442
9.0	·9601	·1682	·9632	·1593	·0659	·9676	·1131	·1365	·9762	·1369
8.0	·9930	·1452	·9930	·1295	·0683	·9924	·0704	·1177	·9927	·1136
6.0	1.0174	·1032	1.0157	·0844	·0549	1.0128	·0393	·0803	1.0119	·0794
4.0	1.0286	·0664	1.0263	·0521	·0368	1.0288	·0235	·0512	1.0215	·0508
2.0	1.0343	·0326	1.0317	·0251	·0184	1.0280	·0112	·0251	1.0264	·0250
0.	1.0361	·0000	1.0333	·0000	·0000	1.0297	·0000	·0000	1.0279	·0000

Table 4 Coordinates of Calculated Stream–Lines for Model C–201

Unit : L = 20

x	Square Station Number		L.W.L.	Stream–Line A		Stream–Line B		Keel Line
			y	y	z	y	z	z
10	0	10	·002	·002	·500	·002	·800	1.000
9.9	1/20	9 19/20	·041	·041	·509	·038	·817	1.040
9.0	1/2	9 1/2	·295	·283	·590	·230	·987	1.256
8.0	1	9	·515	·478	·688	·340	1.155	1.418
6.0	2	8	·834	·739	·853	·470	1.397	1.660
4.0	3	7	1.048	·905	·968	·547	1.554	1.848
2.0	4	6	1.164	·994	1.033	·590	1.644	1.936
0.	5		1.206	1.025	1.057	·606	1.677	1.968

Table 5 Velocity on Calculated Stream-Lines for Model C-201
Unit : Uniform Flow $u_0 = 1$

x	L.W.L. u	L.W.L. v	S-L A u	S-L A v	S-L A w	S-L B u	S-L B v	S-L B w	K.L. u	K.L. w
10	·367	·155	·390	·150	·051	·430	·150	·103	·162	·857
9.9	·743	·258	·756	·258	·058	·791	·252	·128	·875	·212
9.0	·947	·230	·951	·212	·094	·958	·135	·179	·963	·169
8.0	·989	·192	·990	·165	·093	·990	·087	·148	·991	·138
6.0	1.022	·137	1.020	·108	·072	1.018	·050	·099	1.015	·124
4.0	1.037	·082	1.034	·063	·046	1.031	·030	·063	1.029	·061
2.0	1.044	·040	1.041	·030	·023	1.038	·014	·031	1.035	·030
0.	1.047	·001	1.044	·001	·008	1.040	·004	·011	1.037	·011

Table 6 Off-Set for Model C-101
Unit : $L=20$

	Half Breadth y											Height of Keel Line
	L.W.L.	1	2	3	4	4½	5	5½	6	6½	7	
	0	0.24	0.48	0.72	0.96	1.08	1.20	1.32	1.46	1.58	1.68	
0 10												1.000
¼ 9¾	·105	·105	·104	·101	·086	·032						
½ 9½	·203	·202	·200	·188	·166	·120						1.174
¾ 9¼	·284	·284	·280	·268	·232	·186	·100					
1 9	·366	·366	·355	·337	·298	·254	·180					1.306
1½ 8½	·498	·497	·487	·461	·410	·374	·300	·196				
2 8	·608	·607	·592	·562	·508	·466	·408	·316	·160			1.492
2½ ½	·697	·692	·674	·642	·586	·546	·488	·404	·280	·016		
3 7	·773	·767	·746	·712	·657	·614	·560	·484	·376	·204		1.619
4 6	·868	·861	·840	·804	·745	·702	·648	·574	·475	·344	·088	1.693
5	·900	·893	·868	·832	·774	·729	·678	·606	·512	·388	·184	1.718

Table 7 Off-Set for Model C-201
Unit : $L=20$

	Half Breadth y													Height of Keel Line
	L.W.L.	1	2	3	4	4½	5	5½	6	6½	7	7½	8	
	0	0.24	0.48	0.72	0.96	1.08	1.20	1.32	1.46	1.58	1.68	1.80	1.92	
0 10														1.000
¼ 9¾	·166	·166	·163	·157	·122	·089								
½ 9½	·298	·296	·290	·277	·236	·194	·108							1 256
¾ 9¼	·408	·405	·394	·378	·334	·290	·221	·092						
1 9	·515	·510	·498	·474	·421	·378	·312	·209						1.418
1½ 8½	·686	·681	·669	·637	·573	·526	·462	·382	·261					
2 8	·834	·830	·815	·773	·703	·656	·597	·526	·433	·296				1.660
2½ 7½	·955	·949	·933	·893	·826	·784	·725	·653	·562	·447	·282			
3 7	1.047	1.040	1.020	·976	·908	·868	·814	·739	·652	·544	·412	·212		1.848
4 6	1.161	1.129	1.121	1.085	1.018	·972	·917	·852	·773	·674	·548	·390	·120	1.936
5	1.206	1.194	1.170	1.125	1.060	1.012	·962	·898	·816	·720	·600	·451	·232	1.968

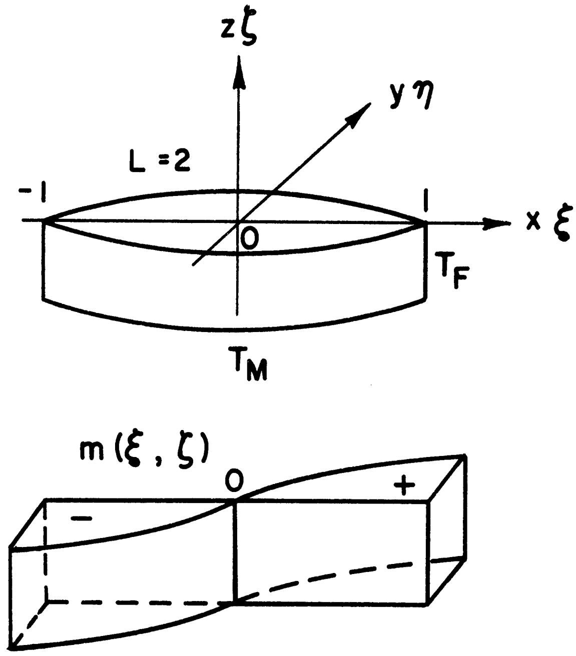


Figure 1.

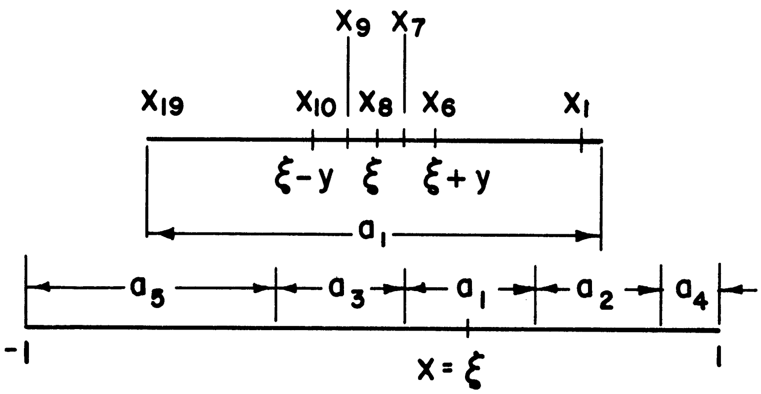
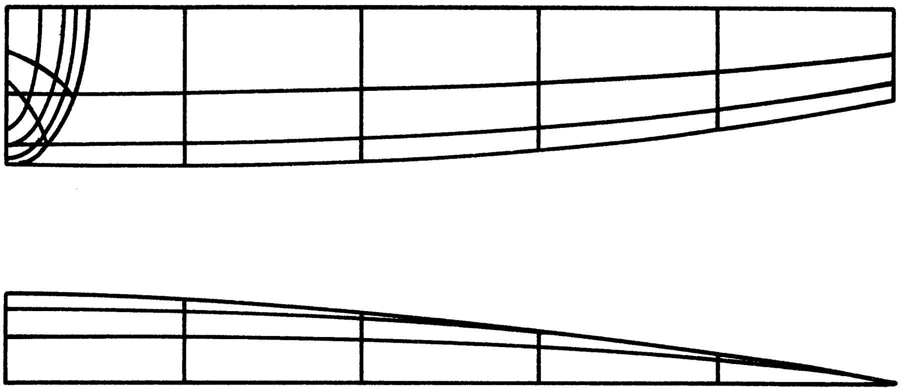


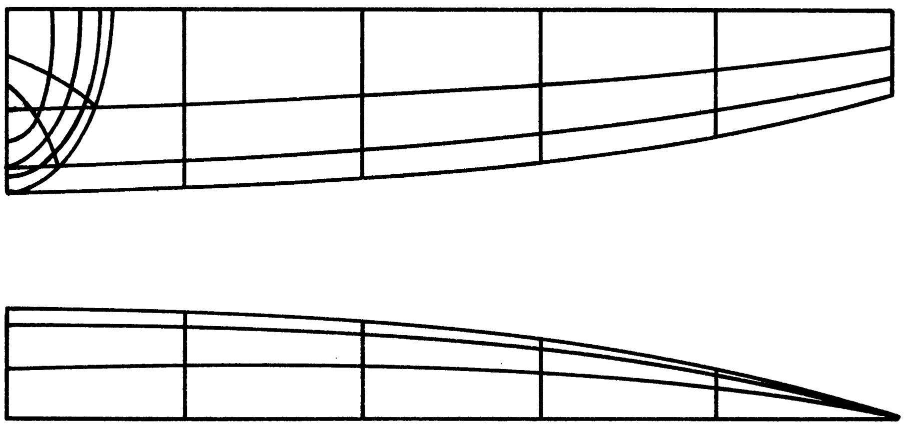
Figure 2.

C -101



B/L=0.090 T_F/L = 0.050 T_M/L = 0.0859

C - 201



B /L=0.121 T_F /L= 0.050 T_M/ L=0.0984

Figure 3.

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