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DYNAMIC RESPONSE OF THE 6-FOOT DIAMETER SHOCK TUBE TO A CONSTANT VELOCITY PRESSURE FRONT

by

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ABSTRACT

The problem of determining the dynamic effect of an internal nondecaying pressure front which moves with constant velocity parallel to the axis of a circular tube is treated. It was known that the equations for the problem of a beam resting on an elastic foundation are equivalent to those for a circular tube, after a simple replacement of constant terms. Thereafter, the equivalent beam problem is considered with both infinite and finite lengths. The finite beam is investigated with two types of boundary conditions: both ends simply supported, and both ends fixed. Viscous damping is considered for the infinite beam. The effects of shear and rotatory inertia are neglected in all cases. It is shown that the dynamic factors for the infinite and finite beams are nearly identical, and the solution for one circular tube is obtained on the basis of an equivalent infinite beam.

PUBLICATION REVIEW

This report has been reviewed and is approved.

Tyler A. Redfield
For and in the absence of
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LIST OF MAJOR NOTATIONS

E = Modulus of elasticity of the beam

I = Moment of inertia of the beam

M = Bending moment

R = Radius of the circular tube

P_o = Magnitude of the concentrated force

a = Cross sectional area of the beam

c = Viscous damping coefficient per unit length

g = Gravitational acceleration

h = Wall thickness of the circular tube

k = Spring constant per unit length of the beam

ℓ = Span of the finite beam

p = Intensity of the uniformly distributed force

t = Time

v = Velocity of the moving force

y = Transverse deflection of the beam

β = Damping factor (defined later)

γ = Weight of the beam per unit volume

σ = Fiber stress

μ = Poisson's ratio

INTRODUCTION

The purpose of this investigation is to determine the dynamic effect of an internal nondecaying pressure front which moves with constant velocity parallel to the axis of a circular tube. The differential equation of equilibrium and compatibility for a circular tube under the action of internal pressure with small deformation is:

$$D \frac{d^4y}{dx^4} + \frac{Eh}{R^2} y = p(x) \qquad \text{(Ref. 1)} \qquad (0.1)$$

where $D = \frac{Eh^3}{12(1-\mu^2)}$ and $p(x)$ = internal pressure. The differential equation of equilibrium and compatibility for a transversely loaded beam resting on elastic foundation, if the transverse deflection is small, has the same form as (0.1), i.e.

$$EI \frac{d^4y}{dx^4} + ky = p(x) \qquad \text{(Ref. 2)} \qquad (0.2)$$

These two equations—(0.1) and (0.2)—are entirely equivalent, if we set

$$EI = D = \frac{Eh^3}{12(1-\mu^2)} \text{ and } k = \frac{Eh}{R^2} \qquad (0.3)$$

Thus the solution of the problem of a beam resting on an elastic foundation provides also the related solution for a circular tube, if replacements $EI = \frac{Eh^3}{12(1-\mu^2)}$ and $k = \frac{Eh}{R^2}$ are made. Thus this paper deals not only with the dynamic effect of a beam resting on elastic foundation under forces moving with constant horizontal velocity, but also provides a solution for an internal pressure front moving down a circular tube.

Both the finite beam (Part I) and the infinite beam (Part II) are considered. Two kinds of boundary conditions—both ends simply supported and both ends clamped—are investigated in Part I. Viscous damping effect is considered in Part II.

In all cases, in calculating dynamic load amplification factor, we neglect the effects of shear and rotatory inertia. We also assume that the circular tube vibrates radial-symmetrically. We neglect the effect of the longitudinal stress wave, as its velocity, $v = \sqrt{gE/\gamma} = 16{,}150$ ft/sec, is almost ten times the velocity of the transverse wave propagated down the tube.

In Timoshenko's paper (Ref. 4), "Method of Analysis of Statical and Dynamical Stresses in Rail," he has derived a formula to calculate the deflection of a simply supported beam resting on elastic foundation under a moving force with constant velocity. His formula is identically the same as Formula (1.10) in this paper. Using this formula, we can find the maximum bending moment in the beam. One numerical example has been done by taking the summation of one hundred terms of the series. Using the same idea as his, we derived the formula of the deflection curve and bending moment for the beam with both ends clamped, but did not try to do any numerical computation. The series in this case is very slow in its convergence.

In Kenney's paper (Ref. 6), "Steady-State Vibrations of Beam on Elastic Foundation for Moving Load," he established a solution for the infinite beam resting on elastic foundation. His formula is used to find maximum bending moment both for concentrated load and uniformly distributed load. Dynamic load amplification factors based upon the derived maximum moment formula are computed with various velocities and wall thickness of the circular tube of 6-foot diameter.

PART I

BEAMS WITH FINITE LENGTH

1. Equation of Free Vibration.

Take the origin of the coordinate at the left end of the beam, then

$$EI \frac{\partial^4 y}{\partial x^4} + \frac{a\gamma}{g} \frac{\partial^2 y}{\partial t^2} + ky = 0 \qquad (1.1)$$

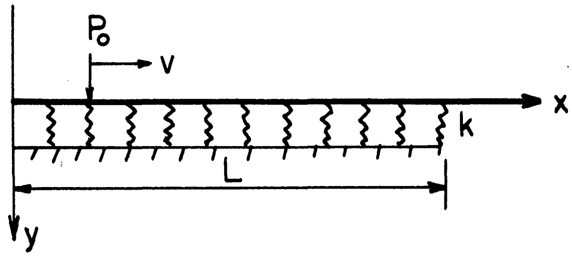


Fig. 1.1

Assume $y = X(x) \cdot \tau(t)$

By the method of separation of variables, we have

$$\frac{X^{iv}}{X} + \frac{k}{EI} = - \frac{a\gamma}{EIg} \cdot \frac{\tau''}{\tau} = \rho^2$$

Where ρ is a real arbitrary constant.

Now

$$\frac{X^{iv}}{X} + (\frac{k}{EI} - \rho^2) = 0$$

Put

$$\frac{k^2}{EI} - \rho^2 < 0 \text{ and } \alpha^4 = \rho^2 - \frac{k}{EI}$$

Where α is a real arbitrary constant too.

Then

$$\frac{X^{iv}}{X} = \alpha^4$$

and

$$X = A\cos \alpha x + B\sin\alpha x + C\cosh\alpha x + D\sinh\alpha x \qquad (1.2)$$

where A, B, C, and D are constants and determined by boundary conditions.

2. Case for a Beam with Both Ends Simply Supported.

Boundary conditions:

$$\begin{aligned} X(0) &= X(\ell) = 0 \\ X''(0) &= X''(\ell) = 0 \\ \therefore \quad A &= C = D = 0 \end{aligned} \qquad (1.3)$$

or $X = B \sin \alpha x$, $\sin \alpha\ell = 0$

$$\alpha\ell = n\pi, \qquad \alpha = \frac{n\pi}{\ell}$$

where $n = 1,2,3,\ldots.$

(1) From the above solution, we have the modal shapes of the simply supported beam for free vibration as the following:

$$X_n = B_n \sin \frac{n\pi x}{\ell} \qquad (1.4)$$

The modes are orthogonal.

(2) Norms of X and X"

$$\phi_n = \int_0^\ell X_n^2 \, dx = B_n^2 \int_0^\ell \sin^2 \frac{n\pi x}{\ell} \, dx = \frac{\ell}{2} B_n^2 \qquad (1.5)$$

$$\phi_n'' = \int_0^\ell (X_n'')^2 \, dx = B_n^2 \frac{n^4\pi^4}{\ell^4} \int_0^\ell \sin^2 \frac{n\pi x}{\ell} \, dx = \frac{n^4\pi^4}{2\ell^3} B_n^2 \qquad (1.6)$$

(3) Response due to a moving concentrated force P_o.

i. By the method of superposition of normal modes, we may assume the deflection curve of the beam as the following:

$$y = \sum_{n=1}^{\infty} q_n \sin \frac{n\pi x}{\ell} \quad \text{(Ref. 3)} \qquad (1.7)$$

where q_n is a function of time t only and q_n is known as the generalized displacement.

Lagrange's equation will be used to find the response. Total strain energy in the beam:

$$V = \frac{EI}{2}\int_0^{\ell}\left(\frac{\partial^2 y}{\partial x^2}\right)^2 dx + \frac{k}{2}\int_0^{\ell} y^2 dx$$

$$= \frac{EI}{2}\sum_{n=1}^{\infty}\frac{n^4\pi^4}{2\ell^3}q_n^2 + \frac{k}{2}\sum_{n=1}^{\infty}\frac{\ell}{2}q_n^2$$

$$= \frac{EI\pi^4}{4\ell^3}\sum_{n=1}^{\infty} n^4 q_n^2 + \frac{k\ell}{4}\sum_{n=1}^{\infty} q_n^2$$

Total kinetic energy for the beam:

$$T = \frac{\gamma a}{2g}\int_0^{\ell}\left(\frac{\partial y}{\partial t}\right)^2 dx = \frac{\gamma a\ell}{4g}\sum_{n=1}^{\infty}\dot{q}_n^2$$

Lagrange's equation:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_n}\right) - \frac{\partial T}{\partial q_n} + \frac{\partial V}{\partial q_n} = Q_n \qquad (1.8)$$

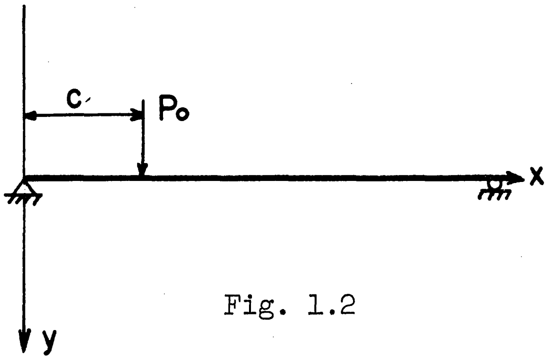


Fig. 1.2

where $\dot{q}_n = \frac{dq_n}{dt}$ and Q_n is the generalized force defined as follows:

We assume the deflection curve of the beam due to a moving force P_o to be

$$y = \sum_{n=1}^{\infty} q_n \sin\frac{n\pi x}{\ell} \qquad (1.9)$$

At time $t = 0$, P_o is at the origin and at $t = t_1$, P_o moves a distance c from the origin, thus $c = vt$.

Let δ_y be the change of deflection of the beam of c due to change of i-th generalized coordinate q_i, then

$$\delta_y = \delta q_i \sin\frac{i\pi c}{\ell} = \delta q_i \sin\frac{i\pi v t_1}{\ell}$$

The work done by the external force P_o on the beam due to δ_y will be

$$P_o \delta q_i \sin\frac{i\pi v t_1}{\ell} .$$

The generalized force times the change of i-th generalized displacement q_i will be equal to that amount of work. i.e.,

$$Q_i \delta q_i = P_o \delta q_i \sin \frac{i\pi v t_1}{\ell}$$

$$\therefore Q_i = P_o \sin \frac{i\pi v t_1}{\ell}$$

Insert V, T and Q_n into Lagrange's equation, we have

$$\ddot{q}_n + \frac{2g}{\gamma a \ell} \left(\frac{EI\pi^4}{2\ell^3} n^4 + \frac{k\ell}{2}\right) q_n = \frac{2gP_o}{\gamma a \ell} \sin \frac{n\pi v t}{\ell}$$

Let

$$\omega_n^2 = \frac{2g}{\gamma a \ell} \left(\frac{EI\pi^4}{2\ell^3} n^4 + \frac{k\ell}{2}\right)$$

The general solution of the differential equation will be

$$q_n = A_n \cos \omega_n t + B_n \sin \omega_n t + \frac{2g}{\gamma a \ell} \frac{P_o}{\omega_n} \cdot \int_0^t \sin \frac{n\pi v t_1}{\ell} \sin \omega_n (t - t_1) dt_1$$

Suppose that both initial displacement and velocity of the beam are zero, then

$$A_n = B_n = 0$$

$$q_n = \frac{2g}{\gamma a \ell} \frac{P_o}{\omega_n} \int_0^t \sin \frac{n\pi v t_1}{\ell} \sin \omega_n (t - t_1) dt_1$$

$$= \frac{2gP_o \ell}{\gamma a \omega_n (\ell^2 \omega_n^2 - n^2 \pi^2 v^2)} \left(\omega_n \sin \frac{n\pi v t}{\ell} - \frac{n\pi v}{\ell} \sin \omega_n t\right)$$

The term $n\pi v/\ell \sin\omega_n t$ due to free vibration will be gradually damped out, so we neglect it, and

$$q_n = \frac{2gP_o\ell}{\gamma a\omega_n(\ell^2\omega_n^2 - n^2\pi^2v^2)}\,\omega_n \sin\frac{n\pi vt}{\ell}$$

$$\therefore\ y = \sum_{n=1}^{\infty} q_n \sin\frac{n\pi x}{\ell}$$

$$= \frac{2P_o\ell^3}{EI\pi^4}\sum_{n=1}^{\infty}\frac{\sin\frac{n\pi x}{\ell}\sin\frac{n\pi vt}{\ell}}{n^4 + \frac{k\ell^4}{EI\pi^4} - \frac{n^2v^2\gamma a}{g}\cdot\frac{\ell^2}{EI\pi^2}} \quad \text{(Ref. 4)} \quad (1.10)$$

ii. Maximum stress in the beam.

Since the bending stress is linearly proportional to bending moment M, the maximum stress, will be determined by the maximum bending moment.

Now

$$M = -EI\frac{\partial^2 y}{\partial x^2}$$

$$\frac{\partial^2 y}{\partial x^2} = -\frac{2P_o\ell}{EI\pi^2}\sum_{n=1}^{\infty}\frac{\sin\frac{n\pi x}{\ell}\sin\frac{n\pi vt}{\ell}}{n^2 + \frac{k\ell}{EI\pi^4 n^2} - \frac{v^2\gamma a}{g}\cdot\frac{\ell^2}{EI\pi^2}}$$

For maximum value of $\frac{\partial^2 y}{\partial x^2}$, $\left|\sin\frac{n\pi x}{\ell}\sin\frac{n\pi vt}{\ell}\right| = 1$

$$\therefore M_{max} = -EI\left(\frac{\partial^2 y}{\partial x^2}\right)_{max} = \frac{2P_o\ell}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2 + \frac{k\ell^4}{EI\pi^4 n^2} - \frac{v^2\gamma a}{g}\cdot\frac{\ell^2}{EI\pi^2}} \quad n = 1,3,5,\ldots \quad (1.11)$$

iii. Numerical example.

Take a circular tube with radius R and wall thickness h as an example. Given data as following:

$E = 29 \times 10^3$ k/in.^{2}
$\mu = 0.3$
$h = 1.0$ in.
$R = 36$ in.
$\ell = 120$ in.
$\gamma = 0.284 \times 10^{-3}$ k/in.^{3}
$g = 386$ in./sec^2

From (0.3)

$$\text{Equivalent } k = \frac{Eh}{R^2} = 22.377 \text{ k/in.}^3$$

$$\text{Equivalent } I = \frac{h^3}{12(1-\mu^2)} = 0.0916 \text{ in.}^3$$

$$EI = 2655.6 \text{ k-in.}$$

For the static case, i.e., $v \to 0$, but $vt \neq 0$

$$M = \frac{2P_0 \ell}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2 + \frac{k\ell^4}{EI\pi^4 n^2}} \approx \frac{2P_0 \ell}{\pi^2} \sum_{n=1}^{199} \frac{1}{n^2 + \frac{k\ell^4}{EI\pi^4 n^2}}$$

$$= 1.108\ P_0,\ n = 1,3,5,\ldots.$$

For $v = 1600$ ft/sec

$$M = \frac{2P_0 \ell}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2 + \frac{k^2\ell^4}{EI\pi^4 n^2} - \frac{v^2\gamma a}{g} \cdot \frac{\ell^2}{EI\pi^2}}$$

$$\approx \frac{2P_0 \ell}{\pi^2} \sum_{n=1}^{199} \frac{1}{n^2 + \frac{k^2\ell^4}{EI\pi^4 n^2} - \frac{v^2\gamma a}{g} \cdot \frac{\ell^2}{EI\pi^2}}$$

$$= 1.690\ P_0,\ n = 1,3,5,\ldots.$$

Dynamic load amplification factor $F = \dfrac{1.690P_0}{1.108P_0} = 1.53$

(4) Deflection curve for uniformly distributed load.

Let p be the intensity

$$pdx = pvdt_1$$

$$dy = \frac{2pv\ell^3}{EI\pi^4} \sum_{n=1}^{\infty} \frac{\sin)\frac{n\pi x}{\ell} \sin \frac{n\pi vt_1}{\ell}}{n^4 + \frac{k\ell^4}{EI\pi^4} - \frac{n^2v^2\gamma a}{g} \cdot \frac{\ell^2}{EI\pi^2}} dt_1$$

$$\therefore y = \frac{2pv\ell^3}{EI\pi^4} \sum_{n=1}^{\infty} \int_0^t \frac{\sin \frac{n\pi x}{\ell} \sin \frac{n\pi vt_1}{\ell}}{n^4 + \frac{k\ell^4}{EI\pi^4} - \frac{n^2v^2\gamma a}{g} \cdot \frac{\ell^2}{EI\pi^2}} dt_1$$

$$= \frac{2p\ell^4}{EI\pi^5} \sum_{n=1}^{\infty} \frac{\sin \frac{n\pi x}{\ell}(1-\cos \frac{n\pi vt}{\ell})}{n\left[n^4 + \frac{k\ell^4}{EI\pi^4} - \frac{n^2v^2\gamma a}{g} \cdot \frac{\ell^2}{EI\pi^2}\right]} \tag{1.12}$$

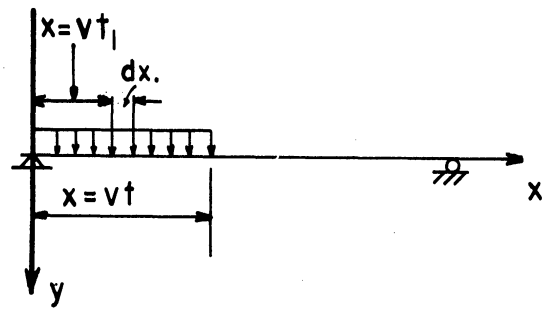


Fig. 1.3

3. Case for a Beam with Both Ends Clamped.

Boundary conditions:

$$X(0) = X(\ell) = 0$$
$$X'(0) = X'(\ell) = 0 \tag{1.13}$$

$$X = A\cos \alpha x + B\sin \alpha x + C\cosh \alpha x + D\sinh \alpha x$$

$$\therefore C = -A, \quad D = -B \tag{1.14}$$

$$A = -\frac{(\sin \alpha\ell - \sinh \alpha\ell)}{(\cos \alpha\ell - \cosh \alpha\ell)} B$$

$$A = +\frac{(\cos \alpha\ell - \cosh \alpha\ell)}{(\sin \alpha\ell + \sinh \alpha\ell)} B \tag{1.15}$$

$$\begin{vmatrix} (\cos \alpha\ell - \cosh \alpha\ell) & (\sin \alpha\ell - \sinh \alpha\ell) \\ (-\sin \alpha\ell - \sinh \alpha\ell) & (\cos \alpha\ell - \cosh \alpha\ell) \end{vmatrix} = 0$$

or

$$\cos \alpha_n\ell \cosh \alpha_n\ell - 1 = 0 \tag{1.16}$$

(1) Modal Shape

$$X_n = A_n\cos \alpha_n x + B_n\sin \alpha_n x + C_n\cosh \alpha_n x + D_n\sinh \alpha_n x$$

$$= A_n(\cos \alpha_n x - \cosh \alpha_n x) + B_n(\sin \alpha_n x - \sinh \alpha_n x)$$

$$n = 1,2,3,\ldots.$$

X_n's are orthogonal. (Ref. 5)

A_n, B_n and α_n have the following relations:

$$A_n = -\frac{(\sin \alpha_n\ell - \sinh \alpha_n\ell)}{(\cos \alpha_n\ell - \cosh \alpha_n\ell)} B_n$$

or

$$A_n = +\frac{(\cos \alpha_n\ell - \cosh \alpha_n\ell)}{(\sin \alpha_n\ell + \sinh \alpha_n\ell)} B_n$$

$$\cos \alpha_n\ell \cosh \alpha_n\ell = 1$$

$$\therefore \alpha_n\ell = 4.730,\ 7.853,\ 10.996,\ 14.137,\ 17.279,\ldots.$$ (Ref. 3)

(2) Norms of X_n and X_n''

$$\phi_n = \int_0^\ell X_n^2 dx = \frac{\ell}{4}(X_n'')^2 = A_n^2\ell \cdot \frac{\sin^2\alpha_n\ell \sinh^2\alpha_n\ell}{(\cos \alpha_n\ell - \cosh \alpha_n\ell)^2} \tag{1.17}$$

(Ref. 5)

$$\phi_n'' = \int_0^\ell (X_n'')^2 dx = \ell\alpha_n^4 A_n^2 \frac{\sin^2\alpha_n\ell \sinh^2\alpha_n\ell}{(\cos \alpha_n\ell - \cosh \alpha_n\ell)^2} \tag{1.18}$$

(3) Reponse due to P_o

Assume the deflection curve of the beam as the following:

$$y = \sum_{n=1}^{\infty} q_n(t) \cdot X_n(x)$$

From Lagrange's equation, we have

$$\ddot{q}_n + \frac{g}{\gamma a}\left(EI\frac{\phi_n''}{\phi_n} + k\right)q_n = \frac{g}{\gamma a\phi_n} Q_n$$

where $Q_n = P_o X_n(vt)$ is the generalized forced and defined in Section 2-(3).

Let

$$\omega_n^2 = \frac{g}{\gamma a}\left(EI\frac{\phi_n''}{\phi_n} + k\right)$$

Then the solution of the differential equation will be:

$$q_n = \frac{g}{\gamma a\phi_n}\cdot\frac{1}{\omega_n}\int_o^t Q_n \sin\omega_n(t-t_1)\,dt_1$$

if we assume the initial displacement and velocity of the beam to be zero. The smallest value of $\alpha\ell$ is 4.730, so we can put $A_n \approx - B_n$ from (1.15)

$$Q_n = P_o X_n(vt_1) \approx P_o A_n(\cos\alpha_n vt_1 - \sin\alpha_n vt_1 - e^{-\alpha_n vt_1})$$

$$\therefore q_n = \frac{g_n}{\gamma a\phi_n}\cdot\frac{P_o A_n}{\omega_n}\left\{\frac{\omega_n}{\omega_n^2-\alpha_n^2v^2}(\cos\alpha_n vt - \cos\omega_n t)\right.$$

$$- \frac{1}{\omega_n^2-\alpha_n^2v^2}(\omega_n \sin\alpha_n vt - \alpha_n v\sin\omega_n t) +$$

$$\left.\frac{1}{\omega_n^2+\alpha_n^2v^2}(\omega_n\cos\omega_n t - \alpha_n v\sin\omega_n t - \omega_n e^{-\alpha_n vt})\right\}$$

If we neglect the terms due to free vibration, then

$$q_n = \frac{g}{\gamma a\phi_n} P_o A_n\left\{\frac{1}{\omega_n^2-\alpha_n^2v^2}(\cos\alpha_n vt - \sin\alpha_n vt) - \frac{e^{-\alpha_n vt}}{\omega_n^2+\alpha_n^2v^2}\right\}$$

$$y = \sum_{n=1}^{\infty} q_n(t)\cdot X_n(x)$$

$$= \frac{gP_o}{\gamma a}\sum_{n=1}^{\infty}\frac{(\cos\alpha_n\ell - \cosh\alpha_n\ell)^2}{\ell\sin^2\alpha_n\ell\,\sinh^2\alpha_n\ell}\,\circ$$

$$\left\{\frac{1}{\omega_n^2-\alpha_n v^2}(\cos\alpha_n vt - \sin\alpha_n vt) - \frac{e^{-\alpha_n vt}}{\omega_n^2+\alpha_n^2v^2}\right\}\circ$$

$$\left\{\cos\alpha_n x - \cosh\alpha_n x - \sin\alpha_n x + \sinh\alpha_n x\right\} \qquad (1.19)$$

where α_n's are roots of the equation $\cos \alpha_n \ell \cosh \alpha_n \ell = 1$ and

$$\omega_n^2 = \frac{g}{\gamma a}\left(EI \frac{\phi_n''}{\phi_n} + k\right)$$

$$= \frac{g}{\gamma a}\left(EI\alpha_n^4 + k\right)$$

$$\frac{\partial^2 y}{\partial x^2} = -\frac{gP_o}{\gamma a} \sum_{n=1}^{\infty} \frac{(\cos \alpha_n \ell - \cosh \alpha_n \ell)^2 \alpha_n^2}{\ell \sin^2 \alpha_n \ell \ \sinh^2 \alpha_n \ell} \cdot$$

$$\left\{\frac{1}{\omega_n^2 - \alpha_n^2 v^2} (\cos \alpha_n vt - \sin \alpha_n vt) - \frac{e^{-\alpha_n vt}}{\omega_n^2 + \alpha_n^2 v^2}\right\} \cdot$$

$$\left\{\cos \alpha_n x + \cosh \alpha_n x - \sin \alpha_n x + \sinh \alpha_n x\right\} \qquad (1.20)$$

$$M = -EI \frac{\partial^2 y}{\partial x^2}$$

(4) Deflection Curve for Uniformly Distributed Force p.

Use the same process as that in Section 2-(4), to get the deflection curve.

PART II

BEAMS WITH INFINITE LENGTH (REF. 6)

1. General Equation of Motion.

$$EI \frac{\partial^4 y}{\partial x^4} + \frac{a\gamma}{g} \frac{\partial^2 y}{\partial t^2} + C \frac{\partial y}{\partial t} + ky = P_o(x,t) \qquad (2.1)$$

We transpose the y-axis so that it is attached to the moving force P_o which moves with constant velocity v.

Let x_1 be horizontal coordinate in the new system

$$x_1 = x - vt$$

then (2.1) becomes

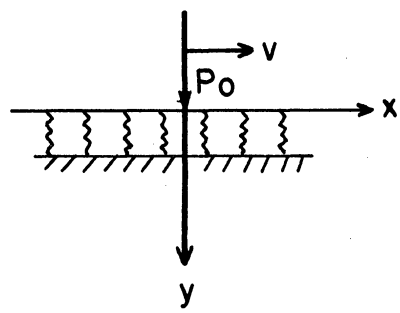


Fig. 2.1

$$EI \frac{\partial^4 y}{\partial x_1^4} + \frac{a\gamma v^2}{g} \frac{\partial^2 y}{\partial x_1^2} - Cv \frac{\partial y}{\partial x_1} + ky - 2\gamma v \frac{\partial^2 y}{\partial x_1 \partial t} + \frac{a\gamma}{g} \frac{\partial^2 y}{\partial t^2} + C \frac{\partial y}{\partial t} = P_o(x_1,t) \qquad (2.2a)$$

Put $x_1 = x$ in (2.2a), since x_1 is a dummy variable.

As this is an infinite beam of indefinite extent both fore and aft of the moving load, the deflected shape will not change with respect to the moving coordinate axis after the applied load p moves on the beam for a suitable time. All the time derivatives are zero and the equation for steady state response will be:

$$EI \frac{d^4 y}{dx^4} + \frac{a\gamma v^2}{g} \frac{d^2 y}{dx^2} - Cv \frac{dy}{dx} + ky = 0 \qquad (2.2b)$$

Put

$$\lambda = (k/4EI)^{\frac{1}{4}} \qquad (2.3)$$

$$C_{cr} = 2\left(k\frac{\gamma}{ag}\right)^{\frac{1}{2}}, \quad v_{cr} = \left(\frac{4kEIg^2}{a^2\gamma^2}\right)^{\frac{1}{4}} \tag{2.4}$$

$$\Theta = v/v_{cr}, \quad \beta = C/C_{cr} \tag{2.5}$$

The equation becomes

$$y^{iv} + 4(\Theta\lambda)^2 y'' - 8\Theta\beta\lambda^3 y' + 4\lambda^4 y = 0 \tag{2.6}$$

and is subjected to the following boundary conditions.

$$\begin{aligned}
&\text{(a)} \lim_{\epsilon\to 0} [y(0+\epsilon) - y(0-\epsilon)] = 0 \quad \text{(deflection continuous)}\\
&\text{(b)} \lim_{\epsilon\to 0} [y'(0+\epsilon) - y'(0-\epsilon)] = 0 \quad \text{(slope continuous)}\\
&\text{(c)} \lim_{\epsilon\to 0} [y''(0+\epsilon) - y''(0-\epsilon)] = 0 \quad \text{(curvature continuous)}\\
&\text{(d)} \lim_{\epsilon\to 0} [y'''(0+\epsilon) - y'''(0-\epsilon)] = -\frac{P_o}{EI} \quad \text{(shear discontinuous due to concentrated load).}
\end{aligned} \tag{2.7}$$

The solution of equation (2.6) with boundary conditions shown in (2.7) is

$x < 0$

$$y = \frac{P_o\lambda}{2k}\left[\frac{\eta e^{\eta\lambda x}}{\eta^4 + (\eta\Theta)^2 + \frac{1}{2}(\Theta\beta/\eta)^2}\right] \cdot \left\{-\frac{(\Theta\beta/\eta+\eta^2)\sin(2\Theta^2+\eta^2-2\Theta\beta/\eta)^{\frac{1}{2}}\lambda x}{\eta(2\Theta^2+\eta^2-2\Theta\beta/\eta)^{\frac{1}{2}}} + \cos(2\Theta^2+\eta^2-2\Theta\beta/\eta)^{\frac{1}{2}}\lambda x\right\} \tag{2.8a}$$

$x > 0$

$$y = \frac{P_o\lambda}{2k}\left[\frac{\eta e^{-\eta\lambda x}}{\eta^4 + (\eta\Theta)^2 + \frac{1}{2}(\Theta\beta/\eta)^2}\right] \cdot \left\{-\frac{(\Theta\beta/\eta-\eta^2)\sin(2\Theta^2+\eta^2+2\Theta\beta/\eta)^{\frac{1}{2}}\lambda x}{\eta(2\Theta^2+\eta^2+2\Theta\beta/\eta)^{\frac{1}{2}}} + \cos(2\Theta^2+\eta^2+2\Theta\beta/\eta)^{\frac{1}{2}}\lambda x\right\} \tag{2.8b}$$

where η is the positive real root of the equation

$$\eta^6 + 2\theta^2\eta^4 + (\theta^4-1)\eta^2 - \theta^2\beta^2 = 0 \tag{2.9}$$

if θ is less than 1.

2. Case Without Damping ($\beta = 0$).

From (2.9)

$$\eta = (1-\theta^2)^{\frac{1}{2}}$$

(1) Deflection curve.

$$y = \frac{P_o\lambda}{2k}\left[\frac{e^{-(1-\theta^2)^{\frac{1}{2}}|\lambda x|}}{(1-\theta^2)^{\frac{1}{2}}}\right] \cdot$$

$$\left[\frac{(1-\theta^2)^{\frac{1}{2}}}{(1+\theta^2)^{\frac{1}{2}}} \sin (1+\theta^2)^{\frac{1}{2}}|\lambda x| + \cos(1+\theta^2)^{\frac{1}{2}}\lambda x\right] \tag{2.10}$$

(2) Bending moment.

$$M = - EI \frac{d^2y}{dx^2}$$

$$= \frac{P_o}{4\lambda} \cdot \frac{e^{-(1-\theta^2)^{\frac{1}{2}}|\lambda x|}}{(1-\theta^2)^{\frac{1}{2}}} \cdot$$

$$\left[\frac{-(1-\theta^2)^{\frac{1}{2}}}{(1+\theta^2)^{\frac{1}{2}}} \sin(1+\theta^2)^{\frac{1}{2}}|\lambda x| + \cos(1+\theta^2)^{\frac{1}{2}}\lambda x\right] \tag{2.11}$$

Maximum moment occurs at $x = 0$ and

$$M_{max} = \frac{P_o}{4\lambda} \cdot \frac{1}{(1-\theta^2)^{\frac{1}{2}}} \tag{2.12}$$

(3) Dynamic load amplification factor.

For static case

$$M_{max} = \frac{P_o}{4\lambda} \qquad \text{(Ref. 1)} \tag{2.13}$$

Dynamic load amplification factor

$$F = \frac{1}{(1-\theta^2)^{\frac{1}{2}}} \qquad (2.14)$$

(4) Uniformly distributed load.

Let O be the point where maximum moment occurs, and x_1, x_2 be the distances from each end of the uniformly distributed load to point O.

From (2.12)

$$dM = \frac{p}{4\lambda} \cdot \frac{e^{-(1-\theta^2)^{\frac{1}{2}}\lambda x}}{(1-\theta^2)^{\frac{1}{2}}}$$

$$\left[\frac{(1-\theta^2)^{\frac{1}{2}}}{(1+\theta^2)^{\frac{1}{2}}} \sin(1+\theta^2)^{\frac{1}{2}}\lambda x + \cos(1+\theta^2)^{\frac{1}{2}}\lambda x\right] dx$$

$$M = \int_{x_1}^{x_2} \cdot \frac{p}{4\lambda} \cdot \frac{e^{-(1-\theta^2)^{\frac{1}{2}}\lambda x}}{(1-\theta^2)^{\frac{1}{2}}} \left[\frac{(1-\theta^2)^{\frac{1}{2}}}{(1+\theta^2)^{\frac{1}{2}}} \sin(1+\theta^2)^{\frac{1}{2}}\lambda x + \cos(1+\theta^2)^{\frac{1}{2}}\lambda x\right] dx$$

$$= \frac{p}{4\lambda^2(1-\theta^4)^{\frac{1}{2}}} \left[e^{-(1-\theta^2)^{\frac{1}{2}}\lambda x_2} \sin(1+\theta^2)^{\frac{1}{2}}\lambda x_2 - e^{-(1-\theta^2)^{\frac{1}{2}}\lambda x_1} \sin(1+\theta^2)^{\frac{1}{2}}\lambda x_1\right]$$

As x_2 becomes large, $e^{-(1-\theta^2)^{\frac{1}{2}}\lambda x_2}$ approaches zero if $\theta \neq 1$, we neglect it

$$M = -\frac{p}{4\lambda^2} \cdot \frac{e^{-(1-\theta^2)\lambda x_1}}{(1-\theta^4)^{\frac{1}{2}}} \cdot \sin(1+\theta^2)^{\frac{1}{2}}\lambda x_1 \qquad (2.14)$$

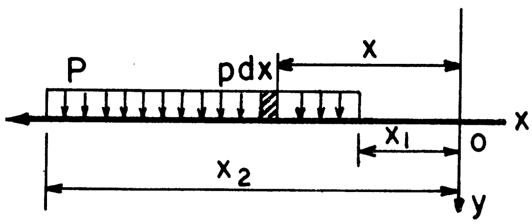


Fig. 2.2

For maximum moment

$$\frac{dM}{dx_1} = 0, \text{ or } \tan(1+\theta^2)^{\frac{1}{2}}\lambda x_1 = \frac{(1+\theta^2)^{\frac{1}{2}}}{(1-\theta^2)^{\frac{1}{2}}}$$

For the static case ($\theta = 0$), maximum moment occurs at $\tan \lambda x_1 = 1$, or $\lambda x_1 = \frac{\pi}{4}$

$$M_{max} = \frac{p}{4\lambda^2} e^{-\frac{\pi}{4}} \sin \frac{\pi}{4} \qquad (2.15)$$

Dynamic load amplification factor =

$$\frac{1}{(1-\theta^4)^{\frac{1}{2}}} \frac{\left[e^{-(1-\theta^2)\lambda x_1} \sin(1+\theta^2)^{\frac{1}{2}}\lambda x_1\right]}{\left[e^{-\frac{\pi}{4}} \sin \frac{\pi}{4}\right]} \qquad (2.16)$$

where λx_1 is determined by the formula

$$\tan(1+\theta^2)^{\frac{1}{2}}\lambda x = \frac{(1+\theta^2)^{\frac{1}{2}}}{(1-\theta^2)^{\frac{1}{2}}}$$

(5) Graphs of dynamic load amplification factors versus velocity.

These are shown both for concentrated load and uniformly distributed load on the following pages. Since the dynamic load amplification factor depends on θ, or v/v_{cr} where

$$v_{cr} = \left(\frac{4kEIg^2}{h^2\gamma^2}\right)^{\frac{1}{4}}$$

we should use v_{cr} as a parameter. For a circular tube with radius R and wall thickness h, equivalent k and I will be:

$$k = \frac{Eh}{R^2}, \quad I = \frac{h^3}{12(1-\mu^2)}$$

v_{cr} depends on h only if E, μ, and R are constants, so the graphs are drawn with h as a parameter.

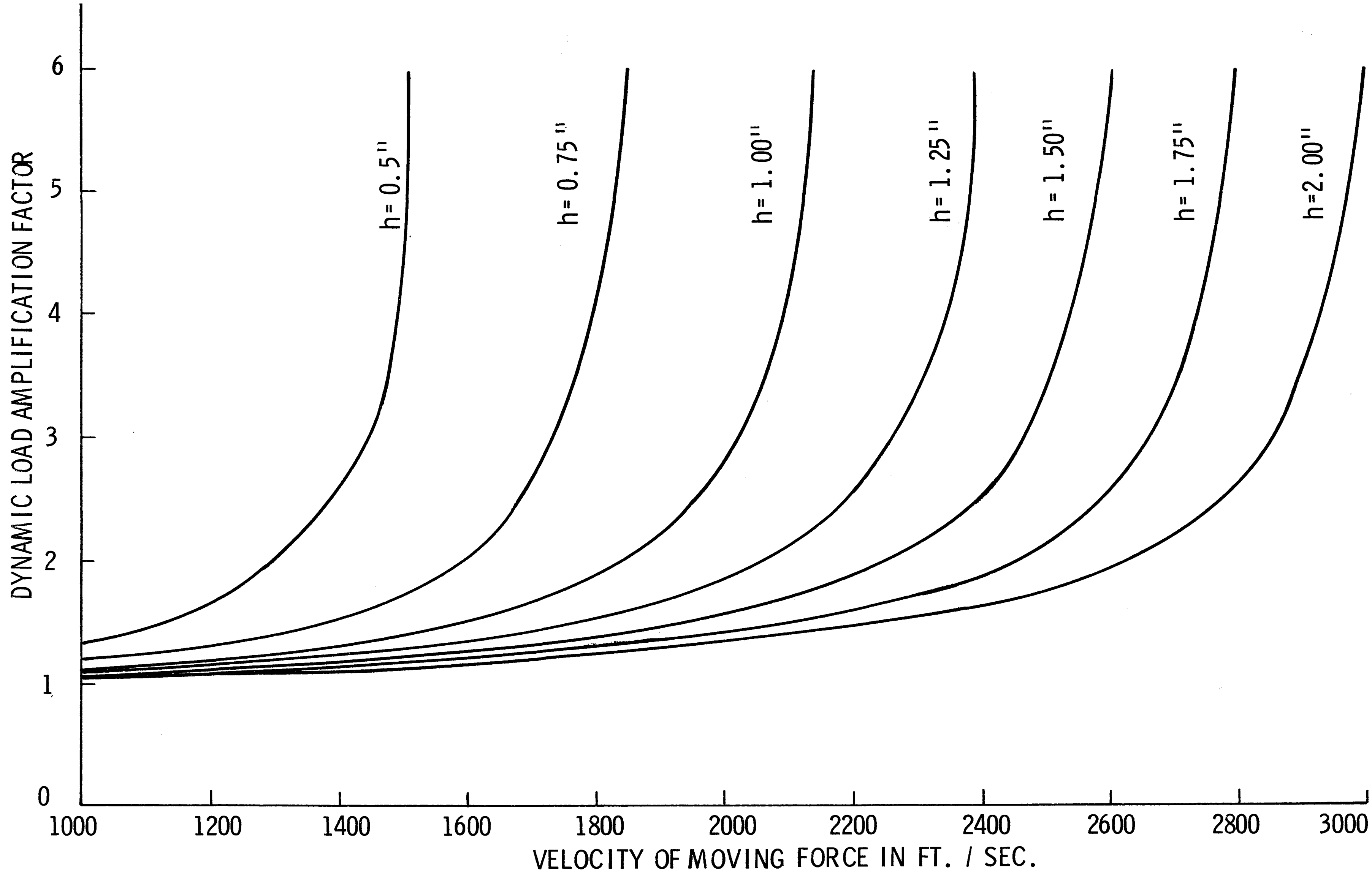
We use $E = 29 \times 10^3$ k/in.^{2}, $\mu = 0.3$, and R = 36 in. to plot all curves.

From Graph 1, for h = 1 in., v = 1600 ft/sec dynamic load amplification factor F = 1.51. In comparing that with the simply supported beam calculated in Part I, F = 1.53, dynamic load factors both for simple beam and infinite beam are almost the same.

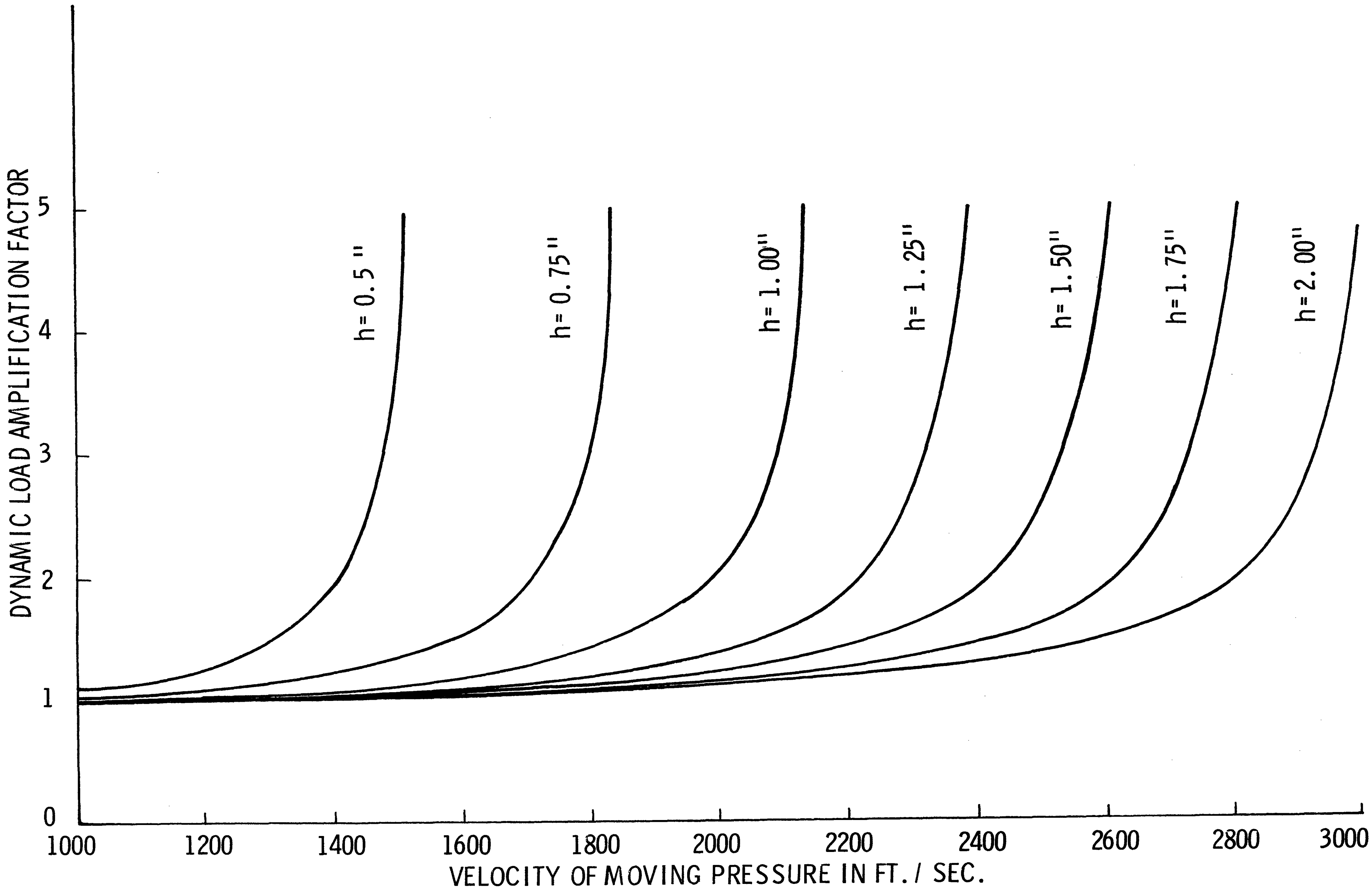
3. Case with Damping.

(1) Deflection curve for steady state response due to concentrated moving force P_o.

From (2.8), we get



Graph 1. Dynamic load amplification factors for a moving force distributed radially along a circular tube of radius R = 36 in.



Graph 2. Dynamic load amplification factors for a moving uniformly distributed internal pressure acting on a circular tube of radius R = 36 in.

i. $x < 0$

$$y = \frac{P_o\lambda}{2k}\left[\frac{\eta e^{\eta\lambda x}}{\eta^4 + (\eta\theta)^2 + \frac{1}{2}(\theta\beta/\eta)^2}\right] \cdot$$

$$\left\{- \frac{(\theta\beta/\eta+\eta^2)\sin(2\theta^2+\eta^2-2\theta\beta/\eta)^{\frac{1}{2}}\lambda x}{\eta(2\theta^2+\eta^2-2\theta\beta/\eta)^{\frac{1}{2}}} + \cos(2\theta^2+\eta^2-2\theta\beta/\eta)^{\frac{1}{2}}\lambda x\right\}$$

ii. $x > 0$

$$y = \frac{P_o\lambda}{2k}\left[\frac{\eta e^{-\eta\lambda x}}{\eta^4 + (\eta\theta)^2 + \frac{1}{2}(\theta\beta/\eta)^2}\right] \cdot$$

$$\left\{- \frac{(\theta\beta/\eta-\eta^2)\sin(2\theta^2+\eta^2+2\theta\beta/\eta)^{\frac{1}{2}}\lambda x}{\eta(2\theta^2+\eta^2-2\theta\beta/\eta)} + \cos(2\theta^2+\eta^2+2\theta\beta/\eta)\lambda x\right\}$$

where η is the positive real root of the equation

$$\eta^6 + 2\theta^2\eta^4 + (\theta^4-1)\eta^2 - \theta^2\beta^2 = 0$$

(2) Bending moment.

$$M = - EI\frac{d^2y}{dx^2}$$

i. $x < 0$

$$M = -\frac{P_o}{4\lambda}\left[\frac{\eta e^{\eta\lambda x}}{\eta^4 + (\eta\theta)^2 + \frac{1}{2}(\theta\beta/\eta)^2}\right] \cdot \left\{\frac{-\eta^3 - \theta^2\eta + \theta\beta + \theta^3\beta/\eta^2 - \theta^2\beta^2/\eta^3}{(2\theta^2+\eta^2-2\theta\beta/\eta)^{\frac{1}{2}}} \cdot\right.$$

$$\left.\sin(2\theta^2+\eta^2-2\theta\beta/\eta)^{\frac{1}{2}}\lambda x - (\eta^2+\theta^2)\cos(2\theta +\eta -2\theta\beta/\eta)^{\frac{1}{2}}\lambda x\right\}$$

ii. $x > 0$

$$M = -\frac{P_o}{4\lambda}\left[\frac{\eta e^{-\eta\lambda x}}{\eta^4 + (\eta\theta)^2 + \frac{1}{2}(\theta\beta/\eta)^2}\right] \cdot \left\{\frac{\eta^3 + \theta^2\eta + \theta\beta + \theta^3\beta/\eta^2 + \theta^2\beta^2/\eta^3}{(2\theta^2+\eta^2+2\theta\beta/\eta)^{\frac{1}{2}}} \cdot\right.$$

$$\left.\sin(2\theta+\eta^2+2\theta\beta/\eta)^{\frac{1}{2}}\lambda x - (\eta^2+\theta^2)\cos(2\theta^2+\eta^2+2\theta\beta/\eta)^{\frac{1}{2}}\lambda x\right\}$$

(3) Numerical example with damping ratio $\beta = 0.02$.

i. Dynamic load factor for $\Theta = 1$, the most critical case.

$x < 0$

$$M = -\frac{P_o}{4\lambda}\left[\frac{\eta e^{\eta\lambda x}}{\eta^4 + \eta^2 + \frac{1}{2}(\beta/\eta)^2}\right]\cdot\left\{\frac{-\eta^3 - \eta + \beta + \beta/\eta^2 - \beta^2/\eta^3}{(2+\eta^2-2\beta/\eta)^{\frac{1}{2}}}\right.$$

$$\left.\sin(2+\eta^2-2\beta/\eta)^{\frac{1}{2}}\lambda x - (\eta^2+1)\cos(2+\eta^2-2\beta/\eta)^{\frac{1}{2}}\lambda x\right\}$$

$x > 0$

$$M = -\frac{P_o}{4\lambda}\left[\frac{\eta e^{-\eta\lambda x}}{\eta^4 + \eta^2 + \frac{1}{2}(\beta/\eta)^2}\right]\cdot\left\{\frac{\eta^3 + \eta + \beta + \beta/\eta^2 + \beta^2/\eta^3}{(2+\eta^2+2\beta/\eta)^{\frac{1}{2}}}\cdot\right.$$

$$\left.\sin(2+\eta^2+2\beta/\eta)^{\frac{1}{2}}\lambda x - (\eta^2+1)\cos(2+\eta^2+2\beta/\eta)^{\frac{1}{2}}\lambda x\right\}.$$

where η is the real positive root of

$$\eta^6 + 2\eta^4 - \beta = 0, \quad \beta = 0.02$$

By Newton's method, we find

$$\eta = 0.1185$$

$$x < 0, \quad M = 5.14\,\frac{P_o}{4\lambda}$$

$$x > 0, \quad M = -5.44\,\frac{P_o}{4\lambda}$$

Dynamic load amplification factor $F = 5.44$.

ii. Dynamic load factor for $\Theta < 1$.

$$\eta^6 + 2\Theta^2\eta^4 + (\Theta^4-1)\eta^2 - \Theta^2\beta^2 = 0$$

If

$$\theta = 0.5, \quad \eta \approx 0.866 = (1-\theta^2)^{\frac{1}{2}} .$$

Since $\eta = (1-\theta^2)^{\frac{1}{2}}$ for $\beta = 0$, here η is the same as that without damping. The dynamic load amplification factor does not change from that obtained for the case without damping. For

$$\theta = 0.8, \quad \eta \approx 0.600 = (1-\theta^2)^{\frac{1}{2}}$$

$$\theta = 0.9, \quad \eta \approx 0.436 = (1-\theta^2)^{\frac{1}{2}}$$

$$\theta = 0.95, \quad \eta \approx 0.323 = (1-\theta^2)^{\frac{1}{2}}$$

Once η is determined the dynamic load factor can be computed. For $\theta \leqslant 0.95$, η is the same for both cases with and without damping, so that the dynamic load amplification factor for both cases are equal.

CONCLUSION

From the example of a circular tube with $h = 1.0$ in., $R = 36$ in., and $v = 1,600$ ft/sec, the dynamic load factor for an equivalent beam with length $\ell = 120$ in. is 1.53 and for an equivalent infinite beam is 1.51. Those two factors are practically the same. The deflection y of an infinite beam resting on an elastic foundation is a function including $e^{-(1-\theta^2)|\lambda x|}$ as a multiplier. As $|x|$ increases, $e^{-(1-\theta^2)|\lambda x|}$ decreases very rapidly, so the deflection at a short distance away from the loading point is nearly equal to zero. This is equivalent to the beam's having a support there. For this reason the simply supported, or even the clamped beam (zero end slopes), may be treated as an infinite beam, insofar as effects of end conditions on support interruptions are concerned.

Once we are allowed to treat the finite beam as an infinite beam, then we can use the graphs on pages 18-19 to find the closely approximate dynamic load amplification factors for the finite beam. The exact solution for a finite beam derived in Part I is in a form of infinite series which converges very slowly, so we can ignore it.

For ordinary structural members β varies from 0.005 to 0.03. From the end of this paper, we know that the dynamic amplification factors with these small damping ratios do not deviate much from the case without damping except for θ approaching 1. As the velocity of the moving force approaches the critical, the maximum dynamic load factor is 5.15 for $\beta = 0.02$.

The actual internal pressure is a decaying wave. The calculation of the response of the circular tube due to such a decaying wave would be complex. For an approximate estimation of the maximum bending moment in the circular tube caused by a decaying pressure wave, we can use an equivalent rectangular wave pulse instead of a decaying wave to calculate the static bending moment in the tube and multiply it by the dynamic load amplification factor from Graph II on page 19. If the duration of the shock wave is large in comparison with the time required for the wave front to travel through one segment of the tube, we can use the maximum intensity of the decaying wave pressure as that of the rectangular wave pressure. This is the case for our problem. Since the velocity of the wave front is from 1,000 ft/sec and the maximum length of the circular tube section in the test region is 10 ft it takes less than 1/100 sec to travel over the whole length; while the duration of the shock wave is 1/10 sec. Within 1/100 sec the intensity of the decaying wave pressure will not change much.

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APPENDIX I

1. Calculate maximum bending stress for a tube of radius 36 in. under static pressure p = 100 psi

$$M = \frac{p}{2\lambda^2}$$

$$\sigma = \frac{6M}{h^2} = \frac{6\,\frac{p}{2\lambda^2}}{h^2} = \frac{3p}{h^2\lambda^2}$$

$$\lambda^2 = \sqrt{\frac{3(1-\mu^2)}{h^2R^2}} = \frac{\sqrt{3(1-\mu^2)}}{hR}, \quad \mu = 0.3$$

$$\therefore \sigma = \frac{3p\cdot hR}{h^2\sqrt{3(1-\mu^2)}} = \frac{\sqrt{3}\,pR}{h\sqrt{1-\mu^2}} = \frac{6,550}{h}$$

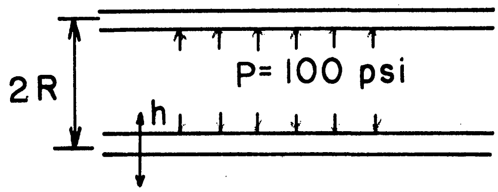

Fig. A-1

Wall thickness h(in.)	1/2	3/4	1	1-1/4	1-1/2	1-3/4	2
Max. bending stress σ(psi)	13,000	8,730	6,550	5,240	4,370	3,750	3,270

2. Possible maximum stress in 1 in. wall tube if shock front velocity coincides with the critical velocity.

i.e., $v = 2145$ ft/sec

Ass. $\beta = 0.02$, $p = 100$ psi

Dynamic load amplification factor F = 5.44

Static stress due to bending, from Section 1 in Appendix I:

$$\sigma = 6,550 \text{ psi}$$

Dynamic stress

$$\sigma_1 = 5.44 \times 6,550 = 35,700 \text{ psi}$$

Londitudinal stress, assume no dynamic amplification factor involved

$$\sigma_2 = \frac{100\pi \cdot 36^2}{2\pi \cdot 36.1} = 1800 \text{ psi}$$

Total maximum possible stress

$$\sigma = \sigma_1 + \sigma_2 = 37{,}500 \text{ psi}$$

APPENDIX II

Transient Response
(Time required to build up 90% of the maximum deflection)

1. For moving coordinate γ = mass of the beam per unit length

$$EI\frac{\partial^4 y}{\partial x^4} + \gamma v^2 \frac{\partial^2 y}{\partial x^2} - Cv\frac{\partial y}{\partial x} + ky - 2\gamma v\frac{\partial^2 y}{\partial x \partial t} + \gamma\frac{\partial^2 y}{\partial t^2} + C\frac{\partial y}{\partial t} = p(x,t)$$

Let

$$y = Y(x)\cdot\tau(t)$$

where

$$Y = \frac{p\lambda}{2k}\left[\frac{\eta}{\eta^4 + (\eta\theta)^2 + \frac{1}{2}(\theta\beta/\eta)^2}\right], \text{ for } x = 0$$

$$Y' = -\frac{p\lambda^2}{2k}\left[\frac{\theta\beta}{\eta(\eta^4+(\eta\theta)^2 + \frac{1}{2}(\theta\beta/\eta)^2}\right], \text{ for } x = 0$$

$$\because\ EI\frac{d^4Y}{dx^4} + \gamma v^2\frac{d^2Y}{dx^2} - Cv\frac{dY}{dx} = P_o(x_1t), \text{ for } x = 0$$

$$\therefore\ -2\gamma vY'\tau' + \gamma Y\tau'' + CY\tau' = 0$$

$$\tau'' + \frac{CY-2\gamma vY'}{\gamma Y}\tau' = 0$$

$$\tau'' + \left(\frac{C}{\gamma} - 2v\frac{Y'}{Y}\right)\tau' = 0$$

$$\frac{Y'}{Y} = -\frac{\lambda\theta\beta}{\eta^2}, \text{ for } x = 0$$

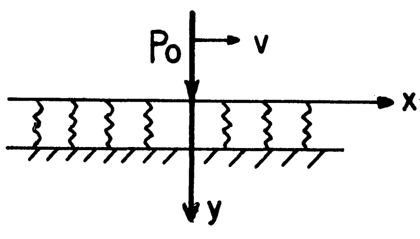

Fig. A-2

Boundary conditions:

$$\tau(0) = 0, \quad \lim_{t\to\infty}\tau(t) = 1$$

Let

$$\nu = \frac{C}{\gamma} + 2v\frac{\lambda\theta\beta}{\eta^2}$$

$$\tau'' + \nu\tau' = 0$$

Boundary conditions:

$$\tau(0) = 0, \quad \lim_{t\to\infty} \tau(t) = 1$$

Complementary function:

$$m^2 + \nu m = 0$$

$$m = -\nu, 0$$

$$\therefore \tau = A + Be^{-\nu t}$$

when

$$t = 0, \quad \tau = 0$$

$$A + B = 0 \text{ or } A = -B$$

$$t \to \infty, \quad \tau = 1$$

$$\therefore A = 1$$

$$\therefore \tau = 1 - e^{-\nu t}$$

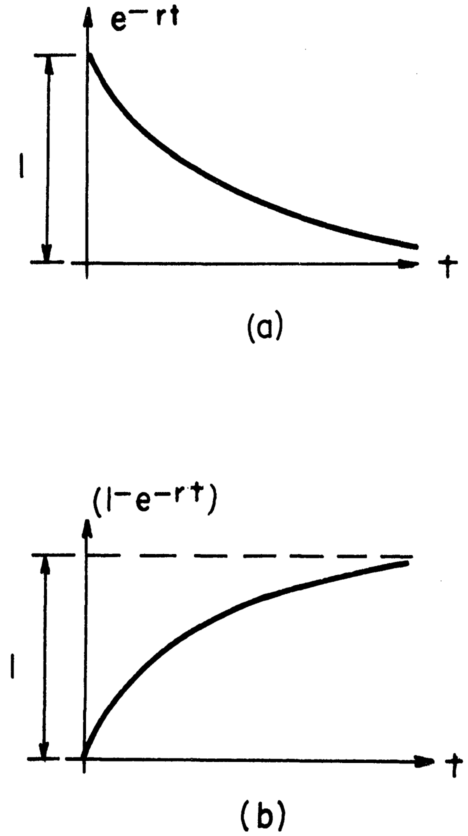

Fig. A-3

Investigate ν

$$\nu = \frac{C}{\gamma} + 2v\,\frac{\lambda\theta\beta}{\eta^2}$$

$$\frac{C}{\gamma} = \frac{1}{\gamma}\cdot\frac{C}{C_{cr}}\cdot C_{cr} = \frac{C_{cr}}{\gamma}\,\beta = \frac{2(k\gamma)^{\frac{1}{2}}}{\gamma}\,\beta = 2\beta\sqrt{\frac{k}{\gamma}}$$

$$2v\,\frac{\lambda\theta\beta}{\eta^2} = 2\,\frac{\lambda\theta\beta}{\eta^2}\cdot\frac{v}{v_{cr}}\cdot v_{cr}$$

$$= 2\,\frac{\lambda\theta^2\beta}{\eta^2}\left(\frac{4kEI}{\gamma}\right)^{\frac{1}{4}}$$

$$= 2\,\frac{\lambda\theta^2\beta}{\eta^2}\left(\frac{4EIk^2}{R\gamma}\right)^{\frac{1}{4}}$$

$$= \frac{2\theta^2\beta}{\eta^2}\sqrt{\frac{k}{\gamma}}$$

$$\therefore\quad \nu = 2\beta\sqrt{\frac{k}{\gamma}}\cdot\left[1 + \left(\frac{\theta}{\eta}\right)^2\right]$$

2. Numerical computation

For

$$h = 1\text{in},\qquad \theta = 1$$

$$\beta = 0.02,\qquad \eta = 0.1185$$

$$\gamma = \frac{0.284}{386}\,\frac{\text{lb-sec}^2}{\text{in.}^3} = \frac{0.284}{386{,}000}\,\frac{\text{K-sec}^2}{\text{in.}^3}$$

$$k = 22.38\ \text{K/in.}^3$$

$$\therefore \quad \nu = 2 \times 0.02 \sqrt{\frac{22.38 \times 386{,}000}{0.284}} \times \left[1+2\left(\frac{1}{0.1185}\right)^2\right]$$

$$= 0.04 \times 5520 \times 72$$

$$= 15{,}900$$

$$\tau = 1 - e^{-\nu t}$$

For

$$\tau = 0.9$$

$$0.1 = e^{-\nu t}$$

$$\therefore t = \frac{2.3}{15{,}900} = 1.45 \times 10^{-4} \text{ sec}$$

Distance travelled $= 2{,}398 \times 1.45 \times 10^{-4}$

$$= 0.347 \text{ ft}$$

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