Opportunities to Learn in and through Professional Development: An Analysis of Curriculum Materials

by

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Dedication

To my fantastic family!

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List of Abbreviations

COMET	Cases of Mathematics Instruction to Enhance Teaching
DMIMMO	Developing Mathematical Ideas: Making Meaning for Operations
EF	Educative Features
IIRP	Improving Instruction in Rational Numbers and Proportionality
ISBI	Implementing Standards-Based Mathematics Instruction
MTF	The Mathematical Tasks Framework
NCTM	National Council of Teachers of Mathematics
OTL	Opportunity to Learn
PLT	Professional Learning Task
QUASAR	The Quantitative Understanding: Amplifying Student Achievement and Reasoning project
RQ1	Research Question 1
RQ2	Research Question 2
RQ3	Research Question 3
RQ4	Research Question 4
TFRU	Teaching Fractions and Ratios for Understanding

Abstract

Professional development (PD) is key to improving mathematics teaching and learning in the middle grades. Many PD projects and initiatives have been undertaken across the United States. Yet, we know very little about what teachers learn in and through such PD, nor about what those who conduct the PD might be learning as well. This knowledge gap hinders efforts to design and deliver effective professional learning opportunities to mathematics teachers.

This study addresses this gap by analyzing a selection of commonly used PD curriculum materials to ascertain the opportunities they provide for middle school mathematics teachers to learn ideas central to improving their instructional practice. Specifically it focuses on teachers' opportunities to learn about: the mathematical topics of proportionality, rational numbers, and linear functions; using multiple representations of mathematical ideas; and using cognitively demanding mathematical tasks in instruction. Due to the important role that professional developers play in supporting teacher learning and their limited opportunities to learn to do so, this study also explores the ways the curriculum materials are designed to support the learning of professional developers – that is, the extent to which they are educative.

The study employed a survey to identify the PD curriculum materials most frequently used with middle school mathematics teachers in NSF-MSP projects across the country. Two analytic frameworks were developed and used to analyze the learning opportunities provided to teachers and professional developers in the curriculum materials. The analysis of teachers' learning opportunities revealed extensive opportunities to learn about proportionality, moderate opportunities to learn about rational numbers, and scarce opportunities to learn about linear functions; mixed opportunities to learn about using multiple representations, making strong connections between symbols, diagrams, and verbal descriptions, but little use of graphs; and mixed opportunities to learn about using cognitively demanding tasks in instruction, providing opportunities to reflect on the pedagogical issues associated with such use. The analysis of professional developers' learning opportunities revealed wide variance in the extent to which the materials are educative, with some materials offering very few opportunities and others offering extensive learning opportunities. The theoretical and practical implications of these findings are discussed.

Chapter I: The Research Problem

The Research Problem

In the past decade, there has been an increased focus on mathematics and the development of mathematical thinking. Policy documents, both national (National Council of Teachers of Mathematics, 2000) and international (UNESCO, 2002), call for *all* students to have opportunities to learn mathematics and develop their critical thinking and problem solving skills to a greater degree. In the United States, the National Council of Teachers of Mathematics (NCTM) in their *Principles and Standards* document (2000) calls not only for all students to have opportunities to learn mathematics, but also for students to learn mathematics in deeper ways. In addition to raising expectations for student learning, this document called for a new vision of mathematics teaching and learning in schools in the United States. The new vision espoused by this document is of classrooms in which students discuss and engage with conceptually rich and challenging tasks that allow them to develop their mathematical knowledge and problem-solving skills. In these envisioned classrooms, teachers would guide student learning through introducing concepts, using multiple representations, and encouraging mathematical discussions around these challenging mathematical tasks. In short, the vision is of mathematics instruction that reflects what is known about how to better support the learning of a diverse study body.

In order to achieve this vision for mathematics education in the United States, the NCTM states that a focus on *teachers* is needed. “Students’ understanding of mathematics, their ability to use it to solve problems, and their confidence in, and disposition toward, mathematics are all shaped by the teaching they encounter in school” (National Council of Teachers of Mathematics, 2000, pp. 16-17). Teachers’ mathematical knowledge for teaching and their instructional decisions shape the degree to which students learn as they engage with mathematical tasks (Henningsen & Stein, 1997; Hill, Rowan, & Ball, 2005). Yet, a disturbing percentage of mathematics teachers are un- or under-prepared to teach mathematics. The National Center for Education Statistics (National Center for Education Statistics, 1999) has reported that 18% of mathematics teachers of grades 7-12 did not even have as much as an undergraduate minor in mathematics and were teaching out of field.

In the 10 years since the release of the *Principles and Standards* document (2000), the mathematics teacher education community has focused on providing professional development. The professional development is aimed at both supporting unprepared teachers to do the work of mathematics teaching, and helping veteran teachers to make the shift to the mathematics teaching and learning envisioned in the Standards document. There is the common assumption that “to understand and embrace the reforms envisioned by NCTM… teachers need professional development and support” (Mewborn, 2003, p. 48). Thus, districts across the country have included professional development as an important element in their efforts to reform and improve instruction.

Though professional development is seen as key to improving instruction, professional development experiences have not traditionally provided teachers with rich or sustained learning opportunities. Teachers in the United States generally experience a patchwork of formal and informal learning opportunities within a fragmented professional development system (Wilson & Berne, 1999). As Ball and Cohen (1999) describe, “although a good deal of money is spent on staff development in the United States, most is spent on sessions and workshops that are often intellectually superficial, disconnected from deep issues of curriculum and learning, fragmented, and noncumulative” (pp. 3-4). Such professional development offerings seem inadequate to improve mathematics instruction. Sykes (1996) has called this inadequacy of professional development “the most serious unsolved problem for policy and practice in American education today” (p. 465) and Borko (2004) has described it as “a ‘serious unsolved problem’ for educational research as well” (p. 3). Though millions of dollars have been invested in such professional development programs across the country, little is known about the specific opportunities for teacher learning provided in these programs. In fact, “although there is a large body of literature on professional development, surprisingly little attention has been given to what teachers actually learn in professional development activities, that is, their content” (Garet, Porter, Desimone, Birman, & Yoon, 2001, p. 923). This lack of attention and knowledge about what teachers have an opportunity to learn in professional development represents a problematic gap in our knowledge as an educational community.

Additionally, in the past decade, new approaches to professional development have arisen. For example, practice-based professional development has become more prevalent. In practice-based professional development, the ongoing professional development of teachers is connected to the actual work of teaching. In sessions, artifacts of practice (e.g. student work samples and narratives of classroom episodes) are used to depict the work of mathematics teaching and open it up for collective inquiry and reflection (Smith, 2001). While there is some recent evidence that this type of professional development provides opportunities for teachers to learn about the mathematics they teach (Silver, Clark, Ghousseini, Charalambous, & Sealy, 2007), many questions remain about what teachers actually learn in this type of professional development. Not only has there been little research on the content of professional development but also "little research has been conducted on the relative efficacy of professional development activities that focus on different types of knowledge, skills, and teaching practices" (Garet, et al., 2001, p. 923). More research is needed to explore the opportunities for teacher learning afforded by the professional development currently underway in the country.

More ambitious student learning goals have created a push for improvement in mathematics instruction. Professional development is seen as a key element in such improvement of instruction. However, professional development has been perceived as providing inadequate learning opportunities to teachers (Sykes, 1996; Wilson & Berne, 1999) and little is known about the specific learning opportunities that *are* provided (Borko, 2004; Garet, et al., 2001). This represents a gap in the mathematics education community's knowledge about learning opportunities provided in professional development nationally. This gap is problematic as professional development is the means for preparing the 18% of teachers who are unprepared to do the work of mathematics teaching, and the central means for improving mathematics instruction generally. With these two lofty goals for teacher learning in professional development, it is important to build a clear understanding of the content in professional development to see if and how these goals can be accomplished. The purpose of this study is to close the gap in our knowledge and build up our understanding of teacher learning in professional development by investigating the learning opportunities provided within mathematics professional development in the United States.

Previous research suggests that a consideration of opportunities for teacher learning dictates attention to both the curriculum and pedagogy used in professional development:

> Understanding teacher learning includes attending to both the curriculum and pedagogy of professional development, to what teachers learn and how teachers are taught. (Wilson & Berne, 1999, p. 176)

> To effect what teachers might learn, one must consider the curriculum and pedagogy of professional development. (Ball & Cohen, 1999, p. 6)

Though it would be an effort beyond the scope of a dissertation study to observe professional development sessions throughout the country, it is possible to develop a first-order approximation of what teachers have opportunities to learn by examining another piece of the puzzle: the curriculum materials used in professional development. The purpose of this study is to consider *what* it is that mathematics teachers are having opportunities to learn in professional development in the United States. By considering a sample of widely used, publicly available, professional development curriculum materials designed for use with mathematics teachers, I can achieve this purpose by exploring what large numbers of teachers have opportunities to learn.

This study focuses on teachers of students in grades 6–8 because the middle school years are especially problematic for mathematics teaching and learning (Silver & Stein, 1996). “Preparation for teachers of the middle grades is often overlooked” (Hillen, 1996, pp. 7-8) with few courses targeted for mathematics at this level offered in teacher preparation programs (Conference Board of the Mathematical Sciences, 2001). This lack of preparation is troubling as it is during these years that students tend to have major difficulties with the mathematical topics addressed, such as proportionality (Beaton, et al., 1996; Post, Behr, & Lesh, 1988) and algebra (Kieran, 2006). In our current environment of highstakes testing, the consequences of students not gaining a firm grasp of these foundational middle school topics can be significant. They will only become more so as large school districts, such as Los Angeles Unified School District, institute high school exit exams in which the mathematical component is largely based on middle grade mathematical content. Incidentally, the Los Angeles Unified School District reported a 38% failure rate for the mathematics portion of that exam in the 2005-2006 year (Office of Communications, 2006). Based on the strong need for improved teaching and learning at the middle school level, professional development offered to middle school mathematics teachers is the focus of this study. Through a curricular analysis, I explore opportunities for learning provided in professional development curriculum materials designed for use with middle school teachers.

In order to teach mathematics effectively “teachers must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks" (National Council of Teachers of Mathematics, 2000, p. 17). However, mathematical content knowledge itself is not enough. Teachers need to know and be able to make use of both mathematical and pedagogical knowledge (Ball & Bass, 2000; Shulman, 1986). Recommendations abound for professional development to provide opportunities for teachers to develop both types of knowledge (Ball & Bass, 2003; Ben-Chaim, Keret, & Ilany, 2007; Loucks-Horsley & Matsumoto, 1999; Matos, Powell, Sztajn, Ejersbo, & Hovermill, 2009; Mewborn, 2003; Smith, 2001). Therefore, in my study, I investigate what opportunities the materials provide for teachers to deepen *both* their mathematical content knowledge and their pedagogical knowledge. I investigate the opportunities provided for teachers to develop their capacity to do the work of mathematics teaching.

Recalling that curriculum is only one piece of the instructional puzzle (Ball & Cohen, 1996; Jackson, 1992; Remillard, 2005), I also pay attention to the facilitation of professional development. Such attention is important because, while the materials may present opportunities for teachers to learn about various concepts, the professional developer who facilitates sessions shapes those opportunities much like teachers shape learning opportunities in the classroom (Henningsen & Stein, 1997; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). Because going into sessions across the country and observing the curriculum as it is set up and enacted by professional developers is impractical, one alternative is to examine how the authors of the materials advised that it be set up and enacted. It is possible to consider the features of the materials that developers have included to guide professional developers in how to conduct the professional development sessions. Such guidance is important to consider as often those acting as professional developers have received little training on how to do so and have few continued learning opportunities (Ball & Cohen, 1999). Additionally, across the country, professional developers vary widely in their academic backgrounds and levels of experience. They include university professors of mathematics education, teachers, district personnel, textbook representatives, and research mathematicians (Banilower, Boyd, Pasley, & Weiss, 2006). Given their diversity of prior training and limited opportunities for continued learning about the enactment of mathematics professional development, it seems important to consider to what extent the professional development curriculum materials were designed to promote professional developers' learning – the extent to which they are *educative curriculum materials* (Davis & Krajcik, 2005).

In the current educational context, with the push for increased student achievement and improved instructional practices in classrooms, the professional development of mathematics teachers is paramount. Professional development is one of the few sites where middle school mathematics teachers have opportunities to learn ideas central to improving their instructional practice. Though the learning opportunities provided in professional development are pivotal for instructional improvement, we simply do not know enough about what those opportunities are. This study attempts to address this problem by analyzing commonly used professional development curriculum materials to identify what opportunities to learn are being provided. Using analytic frameworks that I developed for this purpose, I analyze a sample of curriculum materials to investigate the learning opportunities provided for middle school mathematics teachers and the degree to which the materials are also educative – that is, were designed to promote the learning of professional developers about how to effectively facilitate professional development.

Research Questions

The main research question that this study addresses is: *To what extent and in what ways do professional development materials provide opportunities for middle school teachers to learn about mathematical content and pedagogy?* With this question, I investigate the exposure to and the quality of teachers' learning opportunities provided within professional development curriculum materials. The answer to this question will inform the research community's knowledge of what middle school mathematics teachers have opportunities to learn in professional development in order to meet the heightened expectations placed upon them to improve their instruction and increase student learning.

Within the main research question, I focus on investigating what opportunities are designed into materials for teachers to learn about ideas central to mathematics teaching and learning in the middle grades, such as *proportionality* (Lanius & Williams, 2003; Lesh, Post, & Behr, 1988), the *use of multiple representations* (Ball, 1993b; Brenner, Herman, Ho, & Zimmer, 2002; Lesh & Zawojewski, 2007), and the *use of cognitively demanding tasks* in instruction (Doyle, 1988; Henningsen & Stein, 1997). Therefore, the study addresses three sub-research questions:

RQ1. To what extent and in what ways do the professional development curriculum materials provide opportunities for teachers to learn *middle school mathematical content – specifically proportionality, rational numbers, and linear equations and functions?*

RQ2. To what extent and in what ways do the professional development curriculum materials provide opportunities for teachers to learn about *using multiple representations of mathematical ideas?*

RQ3. To what extent and in what ways do the professional development curriculum materials provide opportunities for teachers to learn about *using cognitively demanding mathematical tasks in instruction?*

Due to the importance that facilitation has on shaping teachers' opportunities to learn in professional development, a fourth research question about the features of the professional development curriculum materials that support facilitation also is investigated.

RQ4. To what extent and in what ways do the professional development curriculum materials *appear to be educative for professional developers?*

The research questions of the study are illustrated in Figure 1.1. In the following sections a brief rationale for the inclusion of each sub-question will be discussed.

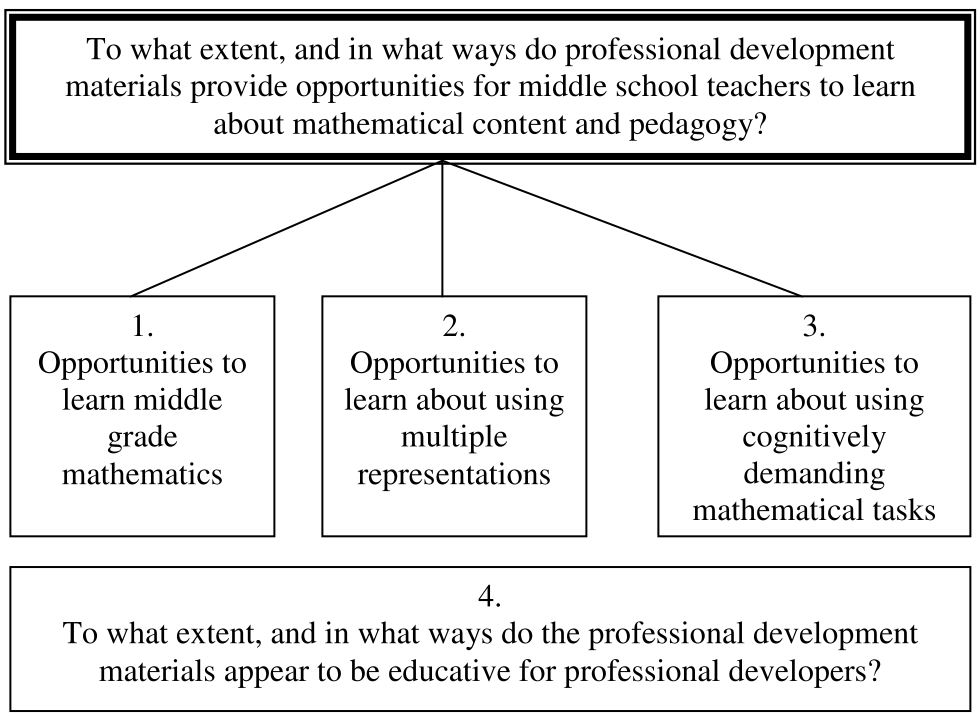


Figure 1.1. Research questions of the study.

RQ1: Teachers' opportunities to learn about middle grade mathematics – proportionality, rational numbers, and linear equations and functions. Proportionality is a key concept in middle grade mathematics and has been argued to be the "one overarching concept with the potential to unite, relate, and clarify many important, complex middle grade topics into a cohesive theme" (Lanius & Williams, 2003, p. 392) as it underlies a wide range of mathematical topics (Shield & Dole, 2008). A proportion is a relationship between quantities wherein multiplication defines the relationship. For example, the relationship between the gross income of an employee who earns an hourly wage and their number of hours worked is proportional. In such a relationship the ratio is constant, for example, $15 an hour. It can be graphically represented as the slope of the line on a gross income/hours worked graph that passes through the origin. The common and crucial component of all proportional situations is the multiplicative relationship that exists among the quantities in the situation (Cramer, Post, & Currier, 1993).

Proportionality is the mathematical terrain wherein learners make the transition from additive to multiplicative reasoning – a shift that is necessary for algebra and higher mathematics. Understanding the concept of ratio is vital for making this shift as maintaining a constant ratio necessitates the use of multiplicative rather than additive reasoning (Sowder, Armstrong, et al., 1998). Multiplicative reasoning is called for throughout higher mathematics and proportionality is the topic where students are introduced to this type of thinking. Thus, Lesh, Post and Behr (1988) have described proportional reasoning as, "the capstone of children's elementary school arithmetic… the cornerstone of all that is to follow" (p. 94).

Proportionality is intertwined with two other important topics in the middle grades: *rational numbers* and *linear functions and equations*. Rational numbers are used throughout proportional situations and one can consider "the essential characteristics of proportional reasoning to involve reasoning about the holistic relationship between two rational expressions such as rates, ratios, quotients, and fractions" (Lesh, et al., 1988, p. 93). Further, ratios and fractions can be considered overlapping sets that appear in proportional situations. For example, 1 cup sugar : 2 cups flour would be considered a ratio only, 1cup sugar : 3 cups ingredients would be considered within the intersection as either ratio or fraction, and ½ cup sugar would be considered as only a fraction (Clark, Berenson, & Cavey, 2003). All of these examples may be present in a proportional problem and illustrate that the topics of proportionality and rational numbers are interconnected.

A proportional relationship ($y = mx$) can be considered to be a special case of a linear relationship ($y = mx+b$). While a linear relationship is not proportional in general, it also involves the multiplicative reasoning developed through engagement with proportional problems. The topic of proportionality acts as a cornerstone by being a vital part of the foundation of multiplicative reasoning upon which the topic of linear functions and equations rests.

These three mathematical topics are addressed across the middle grades in the *Principles and Standards* document (National Council of Teachers of Mathematics, 2000) and in the *Curricular Focal Points* document (National Council of Teachers of Mathematics, 2006) which identifies mathematical topics for grades K-8 that "should be considered as major instructional goals and desirable learning expectations" (p. 10). The current draft of the *Common Core State Standards* (2010), a document outlining what students should understand and be able to do at the various grade levels across the United States, also firmly places these three mathematical topics within the middle grades.

With these topics being a central focus in the middle grades and teachers being encouraged to help students build up conceptual understandings of these mathematical topics, it is a concern that mathematics teachers have been found to struggle with these same topics. Indeed, teachers have been found to have difficulty recognizing proportional situations, determining when to use additive or multiplicative reasoning (Cramer, et al., 1993; Hillen, 1996), helping students recognize and deal with multiplicative situations (Sowder & Philipp, 1995), and talking about rates conceptually (Simon & Blume, 1994; Thompson & Thompson, 1994, 1996). In addition to having struggles with proportionality, teachers have been found to lack conceptual understanding of the operations of multiplication and division on rational numbers (Ma, 1999) – operations that are needed in problems with proportional or linear relationships. Further, teachers have also been found to have an underdeveloped conceptual understanding of functions (Even, 1993). In order to support students to build strong conceptual understanding of these topics, teachers need to develop their own conceptual understandings of these mathematical topics.

Given the centrality of and focus upon proportionality, rational numbers and linear equations and functions in middle grade mathematics, the evidence of teacher struggle with these mathematical topics indicates a strong need for professional development that provides opportunities for teachers to develop their knowledge of these topics. Therefore, my study explores to what extent and in what ways the materials provide opportunities for teachers to learn about these mathematical concepts.

RQ2: Teachers' opportunities to learn about using multiple representations of mathematical ideas. The use of multiple representations is essential to meaning-making in mathematics as mathematical language is multi-semiotic by nature (Lemke, 2003). Mathematical language "as a discourse exploits the meaning potential of linguistic, symbolic, and visual systems of representation" (O'Halloran, 2003, p. 189). Symbolic and visual representations are intertwined parts of mathematical language and cannot be understood in isolation (Cuoco, 2001). The symbolic, visual, and linguistic types of representation together form one integrated meaning-making system. When students can use this meaning-making system well and move flexibly between representational forms, they are said to have developed *representational competence*. "Flexibility in moving across representations is a hallmark of competent mathematical thinking" (Brenner, et al., 2002, p. 214). Therefore, it is not surprising that representational competence has been found to be important for problem solving in mathematics and has been positively linked to student learning and achievement (Brenner, Herman, Ho, & Zimmer, 1999). As new technologies introduce new and dynamic representations of mathematical ideas and students are called upon to communicate mathematically to a greater degree, it can be seen that more and more that representational competence "is critical to enhancing the communication capability and conceptual flexibility that are important to the development of solutions to many real-life problem-solving situations" (Lesh & Zawojewski, 2007, pp. 791-792).

Students in the middle grades often struggle as they are introduced to more formalized ways of using symbols in algebra (Kaput, 1989). Helping students learn to use multiple representations in these grades is especially important as Kieran (2006) suggests that, "it is the interrelationships among the various mathematical representations themselves that are considered to support meaning building in algebra" (p. 33). Both the *Principles and Standards* (2000) document with its Communication and Representation Standards and the *Curriculum Focal Points* document (2006) call for middle school teachers to use multiple representations to help students learn mathematics.

Though teachers are called upon to help students to use multiple representations of mathematical ideas, U.S. textbooks often do not do a good job supporting this venture. In comparison with Japanese 7th grade textbooks, U.S. textbooks did not as often present nor explicitly link multiple representations of example problems (Mayer, Sims, & Tajika, 1995). The work of providing multiple representations and making explicit the connections between them, hence, falls upon teachers. This work is not easy. Choosing which representations to use when and for what purpose is a difficult task. Ball (1993b) refers to representing mathematical content as one of the main dilemmas of mathematics teaching. She states that, "constructing and orchestrating fruitful representational contexts… is inherently difficult and uncertain, requiring considerable knowledge and skill" (Ball, 1993a, p. 162). Though the work is difficult, using multiple representations to model mathematical ideas is a mathematical practice that teachers are called upon to support all students to use (Common Core State Standards Initiative, 2010).

Different representations can reveal different structural aspects of problems as they have different affordances (Brenner, et al., 2002). For example, graphs are well suited for displaying trends and dynamic processes (O'Halloran, 2003). Teachers must carefully consider which combination of representations will best support student learning as they engage with various mathematical tasks. Such consideration is necessary when engaging in the everyday work of mathematics teaching, such as the sharing of multiple student solutions. Teachers may struggle with how to order and connect student work that employs multiple representations and solution strategies (Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005).

As managing the use of multiple representations in the classroom is a major task in instruction, and representational competence is linked to student achievement, it is central to improving instruction and student learning. Since teachers struggle with the use of multiple representations, opportunities should be available in professional development curriculum materials for teachers to reflect upon, and learn to use more effectively the multiple representations that make up mathematical language (Lemke, 2003), and are key to meaning making in middle school mathematics (Kieran, 2006). In the study, therefore, I investigate the ways in which and the extent to which such opportunities are provided in professional development curriculum materials.

RQ3: Teachers' opportunities to learn about using cognitively demanding tasks in instruction. Mathematical tasks serve as the context for students' thinking during instruction and have different *cognitive demands* – require different cognitive processes to solve them (Doyle, 1988). Researchers in the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) educational reform project (Hillen, 1996), recognizing the centrality of tasks to mathematics teaching and student learning, studied the use of cognitively demanding tasks in urban middle school classrooms. Through their work they developed the Mathematical Tasks Framework (MTF) (see Figure 1.2) which provides a representation of how tasks change as they unfold in classrooms and influence student learning (Stein & Smith, 1998).

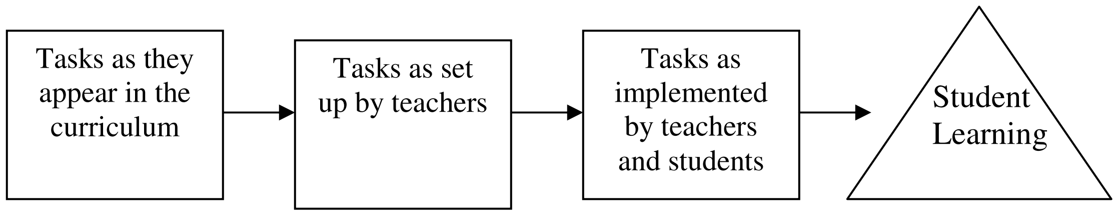


Figure 1.2. The mathematical tasks framework. Adapted from (Stein, Grover, & Henningsen, 1996, p. 459).

After categorizing mathematical tasks into four levels of cognitive demand (Collopy, 2003), QUASAR researchers found that selection and implementation of tasks impacted student learning. Student learning gains were greatest when mathematical tasks with high levels of cognitive demand were implemented well (Silver & Stein, 1996). This finding supports the call by the National Council of Teachers of Mathematics (2000) for students to be given access to more challenging mathematical tasks. Evidence suggests that students in U.S. classrooms rarely have access to such cognitively demanding tasks (Hiebert, et al., 2003; Stigler & Hiebert, 1999).

The QUASAR researchers found that mathematical tasks with high-levels of cognitive demand were difficult for teachers to use. Since cognitively demanding tasks often do not have a specified solution path, students come up with diverse and unexpected solution strategies that teachers must then interpret, manage, and capitalize upon to support learning in the moment. Dealing with a wide range of solution strategies and student thinking is challenging for teachers. It was observed that often teachers transformed these high-level tasks into less-demanding tasks during instruction. Using a selection of data from the QUASAR project, Henningsen & Stein (1997) identified classroom-based factors in instruction that supported or inhibited high-level thinking and reasoning among students. For example, instructional factors (such as scaffolding and sustained press for explanation and meaning) were associated with maintaining student engagement at a high-level of reasoning and factors (such as inappropriate amount of time and removal of challenging aspects of the task) were found to be associated with a decline in the level of student engagement. Even when teachers use instructional practices that support high-level thinking as students are engaged with cognitively demanding tasks, they face the challenge of facilitating productive mathematical discussions that bring together students' diverse solution strategies and thinking. Facilitating such discussions around cognitively demanding tasks has been found to be challenging for teachers, especially for novice teachers (Stein, Engle, Smith, & Hughes, 2008).

Research findings on the centrality of tasks, the positive effects of cognitively demanding tasks on student learning, and scarcity of use in US classrooms, support the need for more mathematics teachers to use and implement well cognitively demanding tasks. The evidence that the use of cognitively demanding tasks is both difficult for teachers and beneficial for students' learning indicates a need for teacher learning on how to effectively use such tasks in the classroom. Therefore, my study includes attention to how professional development curriculum materials provide teachers with opportunities to learn about how to use cognitively demanding tasks in their instruction.

RQ4: Educative features of the professional development curriculum materials. The facilitation of professional development is a complex task. There are a multitude of challenges in supporting teacher learning in professional development settings. For example, the task of creating and supporting professional learning communities of teachers that engage in collective inquiry into mathematics teaching and learning is a challenging endeavor (Stein, Smith, & Silver, 1999). Individuals who act as facilitators of professional development, *professional developers*, have great variance in their level of training and experience (Banilower, et al., 2006) and few opportunities for continued training (Ball & Cohen, 1999). In fact, there is more demand for professional developers than the professional development infrastructure can handle, so more and more teachers are being called upon to lead professional development and become professional developers working with their peers (Elliott, et al., 2009). In the face of the imbalance between demand for professional developers and novices' lack of training for this work, veteran professional developers are also being asked to learn new ways of conducting professional development. Based on the finding that more effective professional development is sustained and coherent (Garet, et al., 2001), professional developers are being asked to provide more long-term professional development that often requires a different skill set than providing the short-term workshops or courses of the past (Stein, et al., 1999). Given this context, there is a clear need for professional developers to have access to ongoing opportunities to learn to support teacher learning in changing professional development contexts. It would be greatly beneficial if professional developers could be provided opportunities to learn about supporting teacher learning in professional development through the use of their professional development curriculum materials.

Curriculum materials used in K-12 settings with students that are designed to also promote teachers' learning are referred to as *educative curriculum materials* (Davis & Krajcik, 2005; Schneider & Krajcik, 2002). In my study of professional development curriculum materials, the "teacher" in this setting is the professional developer. *Educative professional development curriculum materials* would be materials designed to support the learning of both mathematics teachers and professional developers in professional development settings.

One benefit of curriculum materials is that they can provide learning opportunities on a large scale (Schneider & Krajcik, 2002). Thus, educative professional development curriculum materials could provide much needed opportunities for continued learning to large numbers of professional developers across the country. Given this potential, the study's fourth research question investigates the ways and extent to which the sampled professional development curriculum materials were designed to promote the learning of professional developers.

Significance of the Study

The mathematics professional development landscape in the United States is relatively uncharted. Districts and programs across the country operate largely independently of each other and the educational community generally does not have a clear picture of what mathematics teachers are having opportunities to learn. By analyzing professional development curriculum materials that are commonly used with large numbers of teachers in districts spread across the country and, being publicly available, have the potential to be used with thousands more, this study provides important information on the opportunities for learning that mathematics teachers engaging with these materials can have. Thus, the study contributes to charting out the landscape of mathematics professional development to allow for its further analysis.

This study identifies the learning opportunities provided in the sample of four widely used professional development curricula for mathematics teachers. Such identification can support professional developers by informing their decisions about the selection and sequencing of professional development curricula and professional learning tasks that they use in professional development. Additionally this study investigates the degree to which the professional development curriculum materials were designed to be educative and provide learning opportunities to professional developers. The inclusion of such investigation is significant as the concept of educative curricula has previously been limited to the analysis of K-12 curricula. This study extends this concept and opens up the possibility of examining the learning opportunities provided in professional development curriculum materials to both mathematics teachers and professional developers.

One area that this study directly addresses is the alignment between what teachers are being called upon to do in their instruction (National Council of Teachers of Mathematics, 2000) and what they are being prepared to do in professional development. We cannot expect teachers to change their instruction in the ways envisioned in current reform if we do not provide support and opportunities for them to learn to enact this new form of instruction (Elmore, 2004). The results of my study illustrate the level to which the sampled professional development curriculum materials support teachers in developing deep knowledge of key middle school mathematical topics and in using multiple representations and cognitively demanding tasks in their instruction – skills needed to teach mathematics effectively.

This dissertation study sheds light on the very important, but under-explored, area in mathematics education: professional development. Its findings assist teacher educators' and professional developers' selection of curriculum materials by providing information on opportunities to learn within them; and allow for an examination of the alignment between what mathematics teachers are being called upon to do in their instruction (National Council of Teachers of Mathematics, 2000) and what they are being prepared to do. The study can act as a stepping stone for future research as its methods and analytic frameworks can be used to further explore professional development curriculum materials and their connection to teacher learning. This study is significant because it addresses, through an analysis of curriculum materials, a serious unsolved problem in mathematics education: learning in and through professional development.

Dissertation Overview

The dissertation is organized into five chapters. In this first chapter I have introduced the problem being researched and provided an argument for why this study is both necessary and potentially beneficial. In the Chapter 2 I lay out the terrain to be explored and the theoretical foundation that is being built upon in the study. I discuss: how teacher learning is conceptualized in professional development settings; the nature and function of professional development curriculum; the concept of opportunity to learn in regard to curriculum materials; professional learning tasks in curriculum materials; and, how the concept of educative curricula can be extended from the K-12 into the professional development space. The chapter illustrates how the study builds upon previous knowledge and research to present new ideas for exploration.

The methodology of the study is explained in Chapter 3. I provide a rationale for the study design and explain its connection to the research questions. Along with a description of the development process for the two analytical frameworks that were used in the study, detailed descriptions of the methods of data collection and analysis are provided. Finally, issues around validity and reliability are discussed, along with the limitations of the study.

The results of the study are presented in Chapter 4. I share the learning opportunities for teachers that were identified in the curricula and describe the extent to which they focused on the topics under consideration in research questions RQ1-RQ3. The educative features found to be present in the curricula are also described.

The final chapter, Chapter 5, consists of a discussion of the study's findings and concluding remarks. I discuss the identified learning opportunities for mathematics teachers and professional developers and share several implications for research and practice. Additionally, I suggest avenues for future research to follow up on this study. Finally I make concluding remarks about how this study affects our understanding of teacher learning and instructional improvement in the United States in and through professional development.

Chapter II: Theoretical Foundations

Overview

At its core, this is a study of teacher learning. It is specific to the learning of middle school mathematics teachers in professional development settings. Since investigating teacher learning "includes attending to both…what teachers learn and how teachers are taught" (Wilson & Berne, 1999, p. 176), in this study I focus on *what* teachers are provided opportunities to learn by examining professional development curriculum materials. Additionally, I explore the ways in which the curriculum materials are designed to provide professional developers with opportunities to learn how to support teacher learning. The purpose of this chapter is to describe to the reader some of the theoretical foundations upon which this study is built.

The literature review presented in this chapter is divided into four sections. The first describes the conceptualization of teacher learning that is used in this study. The second describes how professional development can act as a site for such learning to occur. The third section describes my conceptualization and use of the terms *curriculum materials*, *opportunity to learn*, and *professional learning task* in the study. I describe how I understand and define these terms and how each is related to teacher learning in professional development settings. Finally I explain the concept of *educative curriculum materials* and how that concept can be extended into professional development spaces.

Teacher Learning: The Development of Teachers' Capacity to Do the Work of Mathematics Teaching

In the past, the emphasis of reform was on changing the curriculum materials that entered classrooms. The current emphasis is on changing what occurs within classrooms through an emphasis on both curriculum materials and instructional practices around their use. In mathematics education, the goal of the current reform is to enact changes in the teaching practices used in U.S. mathematics classrooms to better support student learning of mathematics. Instead of teaching procedures through the use of many simplistic tasks, the goal is for teachers to build up both students' conceptual and procedural knowledge by using challenging tasks and fostering students' capacity to engage in mathematical practices such as participating in productive discussions of mathematical ideas (National Council of Teachers of Mathematics, 2000). As a consequence of the desire for such significant changes, for teachers "the kind of learning that will be required has been described as transformative, that is, as requiring wholesale changes in deeply held beliefs, knowledge, and habits of practice" (Stein, et al., 1999, p. 238). Such a requirement is based on the assumption that there is a close relationship between teachers' learning and their classroom practices.

Many researchers have conceptualized and researched the connection between teacher learning and their classroom practices and found that teachers' learning does influence their classroom practices (Freeman & Johnson, 2005; Mewborn, 2003; Shulman, 1986; Sowder, Philipp, Armstrong, & Schappelle, 1998; Thompson, 1984). Thus teacher learning is important for changes in classroom teaching practices to occur. Remillard (2000) has suggested that "accomplishing pedagogical change is likely to require opportunities for teachers to examine and expand their mathematical and pedagogical understandings" (p. 332). The conceptualization of teacher learning that will be used in this study is one that encompasses changes in teachers' knowledge, skills, beliefs and practices around mathematical content and pedagogy.

Such a conception of teacher learning can be described as the development of teachers' capacity to do the work of mathematics teaching. This concept of teacher learning is closely tied to classroom practices. In the literature, *teacher capacity* has been described as the knowledge, skills, and dispositions that teachers use to do the challenging work of mathematics teaching (Grant, 2008). Similarly, *mathematical knowledge for teaching* has been conceived to consist of the mathematical knowledge, skills, and habits of mind that are entailed in the work in which teachers engage, inside and outside the classroom (Ball & Bass, 2003). I have mentioned mathematical knowledge for teaching (MKT) because Hill, Rowan and Ball (2005) found that teachers' measured MKT was positively related to student achievement in first and third grades after controlling for key student and teacher level covariates. Thus, their study suggests that increasing teachers' capacity would not only have effects on their classroom practices but these effects could positively affect student learning. The reason I favor using the term *teacher capacity* is that it can be conceptualized more in terms of practices than knowledge. Grossman, McDonald, Hammerness and Ronfeldt (2008) "conceptualize teacher capacity as less about distinct forms of knowledge and belief than about classroom practices, emphasizing the importance of integrating theoretical principles with the enactment of practice in the preparation of teachers" (p. 243). With this view, they suggest that increasing teachers' capacity to successfully do the work of mathematics teaching "requires the development of both conceptual and practical tools" (p. 245).

Thus, in order to support teachers' learning they should be provided with opportunities to learn new ways to both think about and perform the classroom practices involved in the work of mathematics teaching. Ball and Forzani (2009) have argued for making classroom practices the core of teacher education. I agree with this argument and suggest that in order to develop teachers' capacity to do the work of mathematics teaching, the classroom practices involved in mathematics teaching should be the focus of teacher learning activities.

Teacher learning occurs in much the same way that any learning occurs: in a process over time wherein teachers use their prior knowledge, beliefs, and experiences to actively build new understandings (Cochran-Smith & Demers, 2008). This process of learning is long term, non-linear, and can be supported by a combination of conditions for teacher learning, such as long-term reflection about mathematics instruction within a community of teachers (Hoban, 2002) and opportunities to deepen understandings of the mathematics they teach in the context of the experiences and cognitive abilities of their students (Darling-Hammond & Ball, 1998).

Based on the literature that conceptualizes teachers' learning as closely tied to their classroom practices, in this study I consider teacher learning as the development of teachers' capacity to do the work of mathematics teaching. With this view, activities to support teachers' learning over time should be built around classroom mathematics instruction. Opportunities should be provided for teachers to analyze and reflect upon classroom instruction while developing their knowledge of mathematical content, student thinking, and pedagogy in this context. As this learning process is long-term and closely tied to classroom teaching, on-going professional development with practicing teachers can provide a space for such learning to occur.

Professional Development as a site for Teacher Learning

Teachers can develop their capacity to do the work of mathematics teaching by learning about mathematics instruction in professional development. Mathematics instruction can be conceived in a variety of ways, but one useful and oft-cited description is the instructional triangle in which “instruction consists of interactions among teachers and students around content, in environments” (Cohen, Raudenbush, & Ball, 2003, p. 122). As can be seen in Figure 2.1, in the context of the mathematics classroom, the content being interacted with is mathematics.

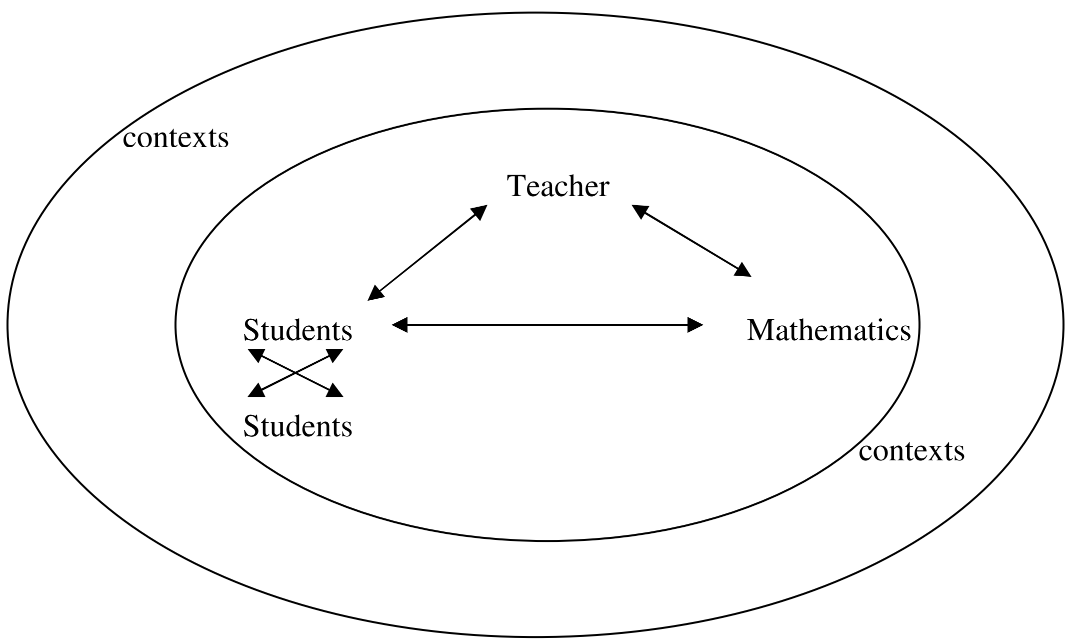


Figure 2.1. The instructional triangle. Adapted from (Cohen, et al., 2003, p. 124).

This model of instruction can be extended to professional development spaces (Nipper & Sztajn, 2008). In professional development the interactions occur between teacher educators (professional developers) and teachers around content in environments (see Figure 2.2). However, in professional development the content is not just mathematics, rather it is the mathematics instruction itself that occurs in classrooms.

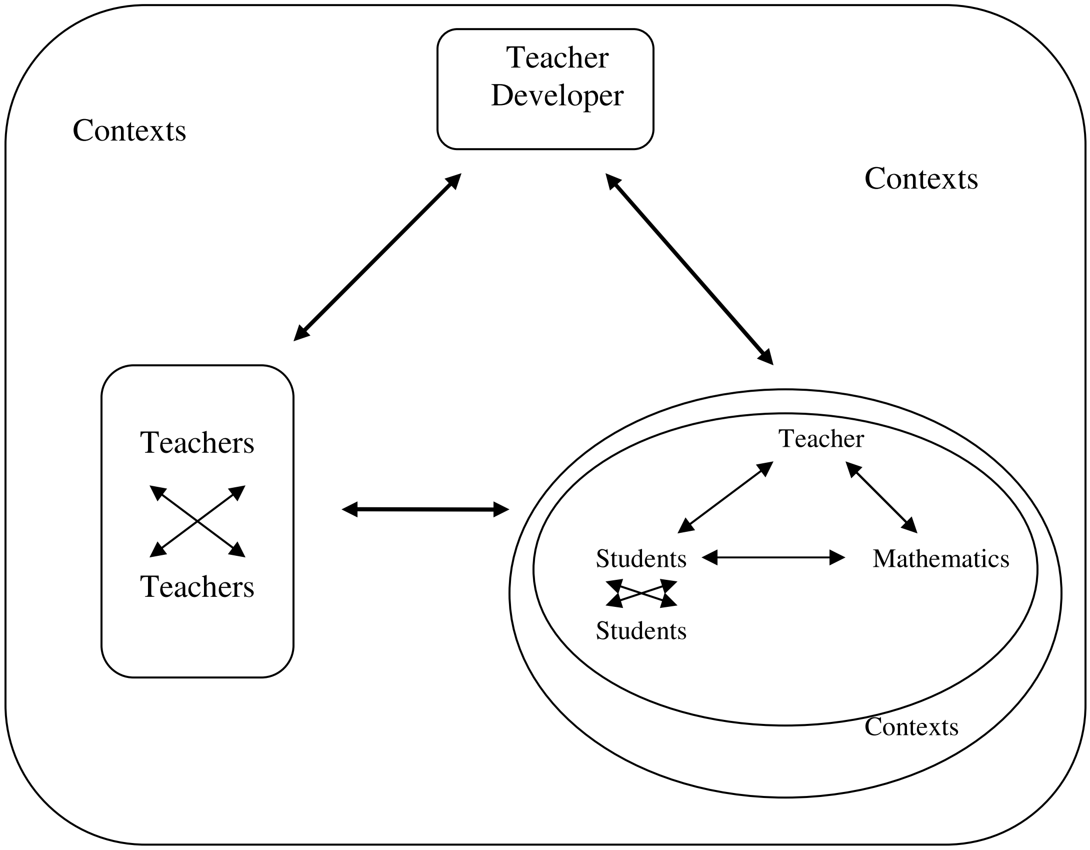


Figure 2.2. The instructional triangle in professional development spaces. Adapted from (Nipper & Sztajn, 2008, p. 337).

In professional development, the teaching and learning of mathematics is the content with which teachers and teacher educators engage. Thus, professional development provides a site for teachers to learn about mathematics teaching and learning.

Unfortunately, in this country, teacher learning has been a patchwork of formal and informal opportunities to learn because “we have no professional development system” (Wilson & Berne, 1999, p. 197). We do not have a national curriculum for professional development nor do we have general guidelines of what teachers should learn as they engage in professional development. What have traditionally been offered to teachers have been short-term, fragmented and noncumulative workshops and courses (Ball & Cohen, 1999).

Research on effective professional development has called for programs designed more in line with how teachers learn: programs that offer ongoing, long-term opportunities for learning that are coherent with other learning opportunities (Garet, et al., 2001). Such programs would provide opportunities for teachers to learn *in* and *from* practice by linking their learning opportunities with what occurs in classrooms. For example, the collective analysis of a narrative case of mathematics instruction (Merseth, 1996; Mousley & Sullivan, 1997) can be followed by teachers using the mathematical task from the case in their own classroom, and that could then be followed by group reflection on what was learned (Silver, et al., 2007). Programs could also provide opportunities for teachers to engage in inquiry about mathematics instruction and to reflect (Zaslavsky, 2009) upon images of reform-oriented mathematics teaching and learning. Thus, professional development programs can act as sites for teachers to learn in and from practice by providing opportunities for teachers to analyze and reflect upon the mathematics teaching of others and themselves (Matos, et al., 2009).

Professional development programs can promote teacher learning further by providing opportunities for teachers' learning within professional learning communities (Ball & Cohen, 1996, 1999; Mewborn, 2003; Schneider & Krajcik, 2002; Wilson & Berne, 1999). Professional learning communities have been defined as groups of teachers engaged in professional endeavors oriented around the work of teaching, in which teachers are specifically focused on "learning with and from colleagues" (Westheimer, 2008, p. 757). Such communities offer individual teachers collegial support, alternative ideas about instruction, and a space for discussion and reflection that can support their learning (Hoban, 2002; Stein, et al., 1999; Westheimer, 2008). One thing that has been described as the glue that can hold together a professional learning community is a set of common activities (Stein, et al., 1999). A well-designed professional development curriculum could provide such a set of common activities that encompass a number of conditions that are conducive for teacher learning, such as focus on specific instructional practices proven to support student learning (Desimone, Porter, Garet, Yoon, & Birman, 2002).

Professional development is an ideal site for teachers' learning as it can support teacher learning that is centered on mathematics instruction in a variety of contexts: their own, that of their colleagues in a professional learning community, and that of others whose instruction is depicted for collective inquiry. Well-designed professional development curriculum materials closely tie teachers' learning to their classroom practices and provide opportunities for teachers to develop their capacity to do the work of mathematics teaching.

Curriculum in Professional Development

Curriculum materials are both a central facet of instruction (Valverde, et al., 2002) and source of great confusion (Jackson, 1992) because the construct of *curriculum* is ambiguous and not well conceptualized (Remillard, 2005). Since the term curriculum has a myriad of meanings, I must clarify that in this study I define the term *curriculum materials* only to mean those published resources and guides used by teachers and professional developers in professional development contexts; along with any additional physical resources specified in the text. I have chosen to conceive of curriculum materials as the printed resources or tools that teachers and professional developers use to create the enacted or implemented curriculum (Remillard, 2005; Travers & Westbury, 1989): what happens as teachers, professional developers, curriculum materials and the local context interact in professional development sessions. Thus, I share the conceptualization of Valverde and colleagues (2002) that curriculum materials communicate the intended curriculum, reflect the particular vision of mathematics instruction and teacher learning that the authors hold, act as tools and resources for teachers and professional developers, and serve as mediators between the intended and the implemented or enacted curriculum.

Curriculum materials are one resource within the professional development system. A useful analogy to describe their role is that of sheet music to musicians (Brown, 2002). The sheet music represents the intended song. It reflects the intentions of its composer, defines the notes that potentially will be played, and often includes guidance on how they should be played, such as *allegro* which indicates the notes should be played in an uptempo or lively way. Musicians use the sheet music to create sound and melodies and often include their own interpretations to what is written. Even with the modifications that different musicians make to the sheet music, the main melody is usually maintained and the listener can recognize the song. Thus, the sheet music is central to what is potentially played and greatly determines what the listener has the opportunity to hear. Similarly, curriculum materials make up the intended curriculum and directly shape the opportunities to learn that teachers and professional developers will have as they engage with them.

Curriculum materials used in professional development are designed to help teachers learn about mathematics instruction. As learning about mathematics instruction involves learning about and from the interactions of teachers and students around mathematics in classroom environments, such materials have to address many facets of mathematics instruction. Professional development curriculum materials, especially those for practice-based professional development, may be designed to support the learning of mathematical content, pedagogy, and student learning simultaneously (Ponte, et al., 2009) or may focus upon specific mathematical concepts or instructional practices (Lamon, 2005).

The concept of opportunity to learn. Researchers have long recognized the utility of analyzing textbooks to assess opportunities for learning given the central role that textbooks play in schooling (Ball & Cohen, 1996). In several large-scale international studies, textbook analyses have been undertaken to gain an understanding of the learning opportunities provided to students around the world (Kaiser, Luna, & Huntley, 1999; Stigler & Hiebert, 1999; Valverde, et al., 2002). These studies conducted by the International Association for the Evaluation of Educational Achievement (IEA) have defined opportunity to learn as “whether or not…students have had an opportunity to study a particular topic or learn how to solve a particular type of problem presented on the test” (Burstein, 1993, p. xxxiii). In these studies the concept of opportunity to learn was used as a means of ensuring validity on cross-national assessments. In the years since the term and concept of opportunity to learn has been taken up by others and has come to have a variety of meanings. For policy makers it has taken on a much broader meaning than just curriculum content (McDonnell, 1995). For example, in one piece of legislation, the Goals 2000: Educate America Act, opportunity for learning standards were defined as "the criteria for, and the basis of, assessing the sufficiency or quality of the resources, practices, and conditions necessary at each level of the educational system (schools, local educational agencies, and States) to provide all students with an opportunity to learn the material in voluntary national content standards or State content standards" (Pub L. No. 103-227, 3[7]).

Across the board, the concept of *opportunity to learn* (OTL) includes attention to both the amount of exposure and quality of exposure that learners have for a particular topic (Wang, 1998). In this study of curriculum materials, I define opportunity to learn to mean whether or not and in what ways teachers have an opportunity to study, reflect upon, and learn to use particular topics and practices that present themselves in the work of mathematics teaching. Thus, I consider both the exposure to and the quality of opportunities that teachers are provided to learn about mathematics teaching and learning.

Professional learning tasks. The major role of curriculum materials is the provision of tasks to serve as the context of thinking and learning (Doyle, 1988). In professional development, these tasks can act as shared activities of collective inquiry into mathematics instruction around which professional discourse can occur (Ball & Cohen, 1999). However, designing tasks for teachers that incorporate multiple high-quality features proven to support teacher learning is challenging and "requires a substantial amount of lead time and planning" (Garet, et al., 2001, p. 935). Professional development curriculum materials can reduce the planning work of professional developers by providing such well-designed tasks.

Because the term *task* can be used to refer to any number of activities for a variety of individuals, I will use the term *professional learning task* (PLT) to refer to tasks specifically used in professional development and teacher education settings and designed to promote teacher learning (Ball & Cohen, 1999; Silver, 2009; Smith, 2001; Stein, Smith, & Silver, 2001). A useful definition for professional learning tasks is that they are "activities that are situated in and organized around components and artifacts of instructional practice that replicate or resemble the work of teaching" (Silver, 2009, p. 245). Examples of such artifacts of practice are mathematical tasks, narrative cases, samples of student work, and video cases (Smith, 2001). A PLT opens up the work of teaching for investigation and inquiry.

I conceive of all PLTs as having two main components that are illustrated in Figure 2.3: (1) a mathematical task and/or (2) a link to practice component.

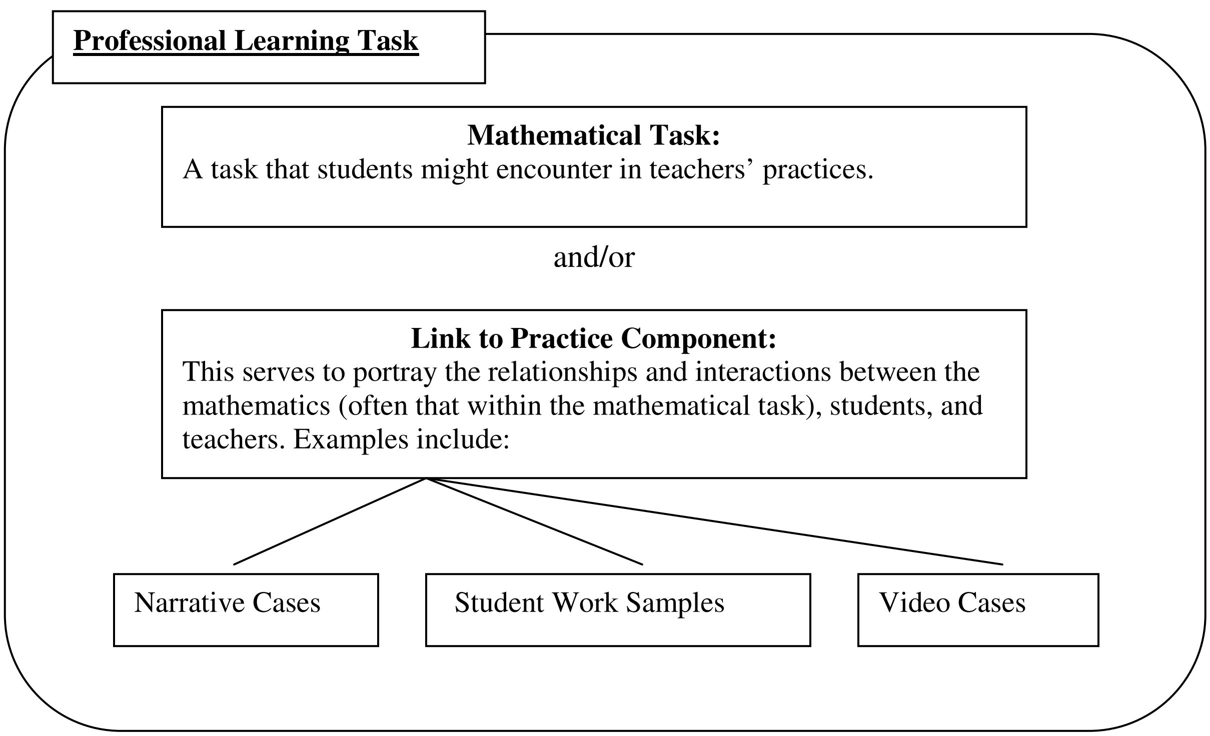


Figure 2.3. The components of a professional learning task.

The mathematical task focuses teachers' attention on the content being addressed in mathematics instruction: mathematics (See Figure 2.1). Often the mathematical task used is a task that students might encounter in the teachers' classrooms. The analysis of such mathematical tasks gives teachers the opportunity to revisit the mathematics that they teach and as well as learn more about mathematical topics' conceptual underpinnings and connections to other topics (Ferrini-Mundy, Burrill, & Schmidt, 2007; Ponte, et al., 2009; Sowder, Armstrong, et al., 1998; Zaslavsky & Leikin, 2004) in the context of their students' cognitive abilities (Darling-Hammond & Ball, 1998). The opportunity to revisit the mathematics they teach is desirable because while teachers generally have strong procedural knowledge of mathematics, they often "lack a conceptual understanding of the ideas that underpin the procedures" (Mewborn, 2003, p. 47). The mathematical task component of professional learning tasks serves to focus attention on teachers' learning of content knowledge – a feature of teacher learning (Darling-Hammond & Ball, 1998) and effective professional development (Garet, et al., 2001).

The link to practice component can appear in a number of different ways. The purpose of this component of the professional learning task is to open mathematics teaching and learning for collective inquiry. The link to practice component portrays the ways that students engage the mathematics within tasks and the ways teachers and students interact with each other and with the mathematics within tasks in classrooms through the use of artifacts from classroom instruction. The use of artifacts of practice has been found to be a powerful means of linking what is discussed in professional development to the work of mathematic teaching (Borko, 2004). Ball and Cohen (1999) "emphasize the importance of situating professional discussion in concrete tasks or artifacts of practice, because they ground the conversation in ways that are virtually impossible when the referents are remote or merely rhetorical" (p. 17). Ponte et al (2009) encourage the use of "tasks that embody authentic aspects of instructional practice and that allow teachers to access, utilize, and develop knowledge of mathematics content, pedagogy, and student learning simultaneously" (p. 191). Thus, they encourage the use of PLTs that open up the instructional triangle for collective inquiry by teachers and professional developers in professional development (Figure 2.2).

Research supports the use of artifacts of practice. Studies have demonstrated that analysis of narrative and video cases provides opportunities for collective inquiry (Harrington & Garrison, 1992); the discussion of pedagogical issues central to the work of teaching , such as management of multiple student solutions (Silver, et al., 2005); and teacher learning *in* the profession (Mousley & Sullivan, 1997). The collective examination of student work has also been found to be a promising practice that promotes teacher learning (Westheimer, 2008). Analysis of artifacts of practice can help teachers anticipate student responses to math tasks – one of the five practices for effectively using student responses during whole class discussions – because they provide extensive information about student responses to particular mathematical tasks (Stein, et al., 2008).

The link to practice component of professional learning tasks is also the space in which authors can situate teacher learning in the types of instructional practices that reformers wish to encourage (Ball & Cohen, 1999; Wilson & Berne, 1999), practices that are aligned with state and national standards and research on effective instruction (Garet, et al., 2001). In this way, professional learning tasks can both open up mathematics instruction for reflection and inquiry and also provide new images of what such instruction can be.

In this study teacher learning is conceptualized as the development of teachers' capacity to do the work of mathematics teaching. Professional development provides the space for the development of teachers' capacity by centering teacher learning on mathematics instruction. The PLTs in professional development curriculum materials, through a combination of their mathematical task(s) and link to practice components, provide teachers with opportunities to learn about mathematics instruction: the mathematics, the teachers, the students, and the interactions between them in authentic classroom contexts. The PLTs provide opportunities for teacher learning that is closely tied to their own classroom practices. It is hypothesized that teacher learning that is closely tied to classroom practices is more likely to result in the improvement of classroom instruction and, hence, positively influence student learning (Freeman & Johnson, 2005; Mewborn, 2003). This study analyzes a sample of widely-used professional development curriculum materials to determine what opportunities teachers are provided to learn important mathematical content and pedagogy called for in their work teaching middle grade mathematics. This study identifies what learning opportunities are being provided to middle school mathematics teachers to develop their capacity to do specific aspects of their work teaching mathematics.

Supporting Professional Developers: Educative Professional Development Curriculum Materials

Recalling the analogy of curriculum materials as sheet music wherein musicians' knowledge, skills, and dispositions influence their decisions about how to play the music, one can consider how the capacity of teachers and professional developers influence how they interact with and learn from what is presented in curriculum materials. The relationship between professional developers and curriculum materials is especially important to consider as professional developers use curriculum materials to set up the planned curriculum that is then experienced by teachers (Remillard, 2005) and, in doing so, shape teachers' learning opportunities (Morris, 2003).

In Remillard's (2005) review of key concepts on classroom teachers' use of mathematics curricula, a framework is presented that illustrates how the teacher and curriculum are in a participatory relationship in which teacher characteristics (such as knowledge, beliefs, capacity, perceptions and identity) and curriculum characteristics (such as representations, structures, voice and look) influence how the teacher interprets and uses curriculum materials to create the planned curriculum. This framework can apply to professional development settings as well, wherein professional developers use their knowledge, beliefs, and dispositions to interpret and perhaps change professional development curriculum materials before using them with teachers. In their report summarizing what can be learned from a decade of mathematics and science professional development, Banilower and colleagues (2006) found that the capacity of professional developers significantly influenced the way curriculum materials were used and the quality of sessions provided to teachers. They found that more skilled professional developers were "able to convey a vision for effective instruction; connect content, pedagogy, and materials; and create a professional development culture that supported rigorous investigation and inquiry" (p. 35). Professional developers with less capacity struggled with supporting teacher learning in these ways.

Due to its influence on the learning opportunities provided to teachers, the capacity of professional developers is also considered in this study. Due to the high demand for professional development, a wide range of individuals with varied capacities to do the work serve as professional developers (Banilower, et al., 2006) with few of them having had any preparation for the work (Zaslavsky & Leikin, 2004). Additionally with the need for professional development outstripping the supply of professional developers more and more teachers are being asked to serve as professional developers or teacher leaders (Elliott, et al., 2009). However, there are few opportunities are available for continued learning in how to effectively support teachers' learning (Ball & Cohen, 1999) and little attention has been given in research on *what* or how professional developers learn (Elliott, et al., 2009; Even, 2008). Since professional developers' capacity to support teacher learning shapes the learning opportunities provided in professional development sessions, it is important to support professional developers learning. This study can shed light on what professional developers can learn from using professional development curriculum materials by investigating the extent to which the materials are designed to support their learning as well as teachers – that is, to what extent the materials can be described as educative.

The concept of *educative curriculum materials* has, thus far, been used to refer to K-12 curriculum materials used by teachers with students. In that setting, they have been defined as "curriculum materials designed to address teacher learning as well as student learning" (Schneider & Krajcik, 2002, p. 221). Such curriculum materials are designed to both build teachers' knowledge about specific instructional strategies but also develop general knowledge that they can use flexibly in new situations. There are several high-level guidelines for the design of educative curriculum materials (Ball & Cohen, 1996). Specific to the activities in the curriculum, the curriculum materials should help teachers anticipate how students will think about and react to different activities and should support teachers to relate the units in the curriculum (Collopy, 2003; Davis & Krajcik, 2005). Generally, they should also be designed to support teachers' learning of subject matter, to make transparent the authors' pedagogical judgments, and to develop teachers' pedagogical design capacity, that is, teachers' ability to adapt the curriculum materials to achieve particular instructional aims (Ball & Cohen, 1996; Davis & Krajcik, 2005). Curriculum materials designed for use in K-12 classrooms that meet these guidelines and support the learning of both teachers and students can be described as educative.

I argue that within the learning context of professional development, curriculum materials can also be designed to be educative. In this study, I define *educative professional development curriculum materials* to mean curriculum materials designed to support the learning of both professional developers and teachers in professional development settings. Such materials are especially needed at this time because they can serve as one of the means of providing much needed opportunities for continued learning to professional developers on a large scale.

The transformative changes envisioned for mathematics teachers' practices (National Council of Teachers of Mathematics, 2000) and findings about features of effective professional development (Garet, et al., 2001) mean that professional development, too, must be transformed. Recommendations abound for professional development to shift from short-term, fragmented workshops and courses to long-term, coherent programs that support teachers' learning in professional learning communities (Garet, et al., 2001; Stein, et al., 1999; Westheimer, 2008; Wilson & Berne, 1999). Not only are there calls to change the structuring of professional development, but also there are calls for the facilitation of professional development to model the reform-oriented practices that teachers are supposed to learn, such as facilitating mathematical discussions (Ball & Cohen, 1999; Mewborn, 2003; Zaslavsky & Leikin, 2004). All of these proposed changes means that "if professional developers are to be effective in supporting the transformation of teachers, they, too, must undergo shifts in their knowledge, beliefs, and habits of practice that are more akin to transformation than to tinkering around the edges of their practice" (Stein, et al., 1999, p. 262). Given the scarcity of opportunities that professional developers have to continue their learning, educative professional development curriculum materials are a critical and promising means of supporting professional developers' learning of how to better facilitate professional development.

Unlike educative curriculum materials using in K-12 settings that are designed using the abundant literature on teacher learning, educative professional development curriculum materials, at present, have a weak literature base. There are few pieces of literature on what mathematics professional developers need to know and be able to do to meet the current needs of the work. In fact, in Even's (2008) review of the existing research on this topic the focus was on the literature that was missing. As more research is directed towards the work and knowledge requirements of mathematics teacher educators and professional developers, such as the recent dissertation by Deborah Zoft (2007) that identifies features of mathematical knowledge needed for teaching teachers, then the content to be addressed in educative professional development curriculum materials will become more research based, defined, and better tied to the work of facilitating professional development with mathematics teachers.

Chapter III: Methodology and Research Procedures

Overview

Little attention in research on professional development has focused on what teachers actually learn (Garet, et al., 2001). The purpose of this study is to address this gap in knowledge and explore the opportunities provided to middle school teachers in professional development to learn ideas central to improving their instruction. Due to the centrality of curriculum materials in instruction (Valverde, et al., 2002), an exploration of learning opportunities can be accomplished through the analysis of the curriculum materials (Ball & Cohen, 1996; Kaiser, et al., 1999). Therefore, in this study I analyze a sample of widely used, publicly available, professional development curriculum materials in order to determine what large numbers of mathematics teachers have opportunities to learn. Additionally, due to the importance of facilitation and the limited learning opportunities of professional developers, I also analyze the extent to which the curriculum materials are educative and provide learning opportunities for professional developers.

As this study is an analysis of curricular materials, the main unit of analysis is each set of curriculum materials. A set of curriculum materials typically consists of the main textbook for teachers and possibly a facilitator's guide and physical objects called for in the activities described in the text. In my analysis, I explore the learning opportunities that each set of curriculum materials provide. I also analyze the learning opportunities in individual professional learning tasks (PLTs) within each curriculum and across all of the curriculum materials in my sample in order to make comparisons and describe trends.

This study was conducted in two stages (see Figures 3.1 and 3.2). The first stage was conducted in order to select the sample of professional development curriculum materials for analysis. A survey was conducted with 32 large scale professional development projects across the country in order to identify a sample of publicly available, professional development curriculum materials that were most commonly used with large numbers of middle school mathematics teachers.

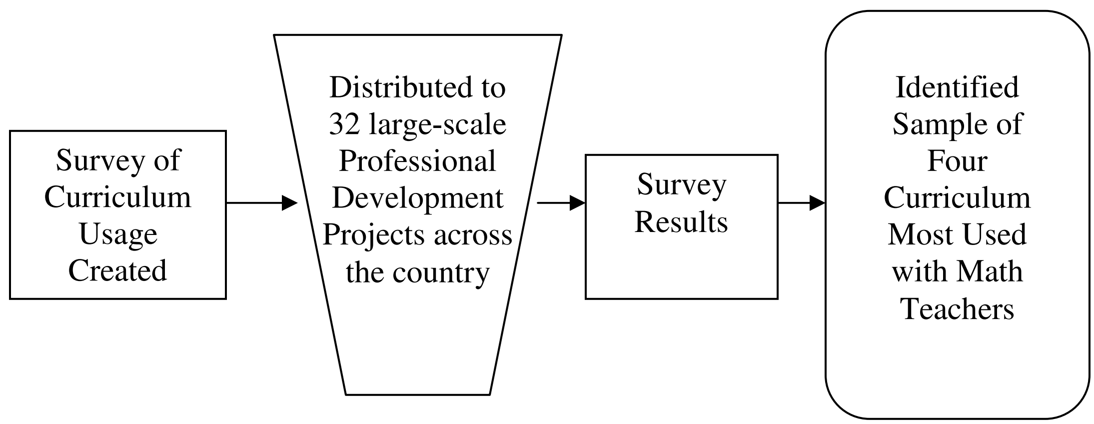


Figure 3.1. Dissertation study design. Stage 1: Sample selection process.

Once the sample of curriculum materials was identified, two types of analysis were conducted. In order to answer the first three research questions around teachers' opportunities to learn specific mathematical content and pedagogy central to middle school teaching, I designed an analytic framework to be used to analyze teachers' opportunities to learn in the sample of curriculum materials. In order to answer the fourth research question around the educative nature of the curriculum materials, I developed a second analytic framework to be used to identify the educative features present in the curricula. These two frameworks were used to identify the learning opportunities provided in the sample curriculum materials to mathematics teachers and professional developers.

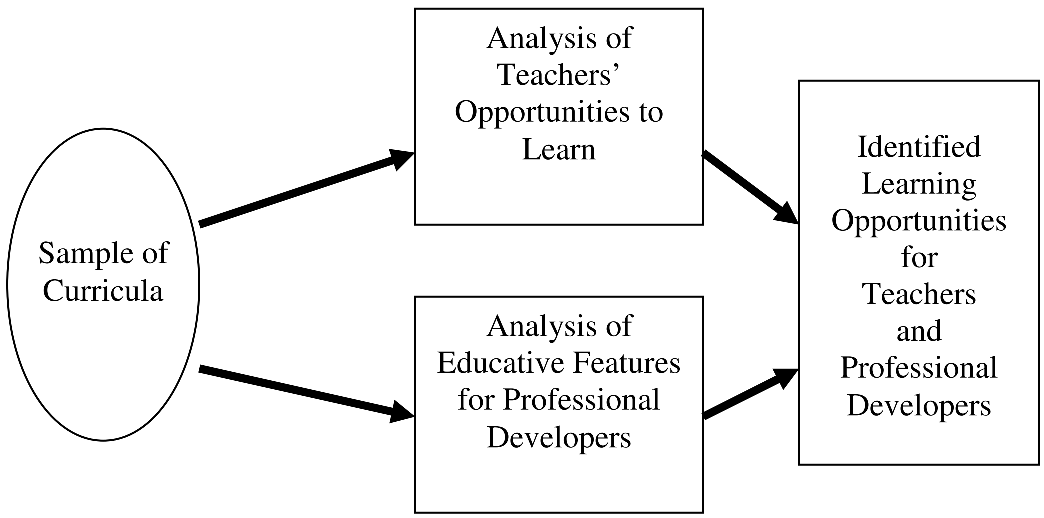


Figure 3.2. Dissertation study design. Stage 2: Curricular analyses.

Research Questions

This study is designed to learn the extent to which and the ways in which professional development curriculum materials provide opportunities for middle school teachers to learn about mathematical content and pedagogy. The first three research questions focus upon the opportunities provided to middle school mathematics teachers to learn about specific mathematical topics and pedagogy that are both central to the work of teaching mathematics and proven to be difficult for teachers. Namely, these research questions ask:

RQ1: To what extent and in what ways do the professional development curriculum materials provide opportunities for teachers to learn middle school mathematical content – specifically *proportionality, rational numbers, and linear equations and functions*?

RQ2: To what extent and in what ways do the professional development curriculum materials provide opportunities for teachers to learn about *using multiple representations* of *mathematical ideas*?

RQ3: To what extent and in what ways do the professional development curriculum materials provide opportunities for teachers to learn about *using cognitively demanding mathematical tasks in instruction*?

In previous chapters I have provided arguments for the appropriateness and importance of these research questions. In the later sections I will describe how I developed an analytic framework for analyzing teachers' opportunities to learn in the sample of professional development curricula that is specifically designed to answer these research questions.

As the actions of professional developers shape the learning opportunities provided by curriculum materials, this study also attends to facilitation. It does so by considering how the curriculum materials were designed to provide learning opportunities to professional developers about how to effectively facilitate professional development – that is by examining the educative features of the curricula. The fourth and final research question is dedicated to this pursuit by asking

RQ4: To what extend and in what ways do the professional development curriculum materials appear to be *educative for professional developers*?

My previously arguments about the importance of facilitation, the large range in experience of practicing professional developers and the limited learning opportunities available to them, provide a rationale for including this research question in the study. In order to answer it, I developed an analytic framework based on the research literature around educative curricula and effective facilitation of professional development that I used to identify and analyze educative features in the curricula.

Sample Selection Process

While there is a wide range of professional development curriculum materials in use in the United States, this study is focused on those that are used with a large number of middle school mathematics teachers. Thus, only those materials that fulfilled the following parameters were considered for inclusion in the sample:

- designed for use with middle grade (6-8) mathematics teachers
- publicly available so that it is probable that future groups of teachers will use the materials
- include artifacts from practice or research findings (materials that are more than a mere activities book)

An initial list of materials that were to be considered for inclusion was developed through examining materials listed both on an online database for professional development providers (Teacher Education Materials Project, 2010) and on commercial bookselling websites. This list was used to generate a survey that was disseminated to large professional development projects across the country and used to determine the most commonly used professional development curriculum materials (see Appendix A).

The survey was conducted with the purpose of identifying, based on empirical evidence, a sample of publicly available professional development curriculum materials that are being extensively used with large numbers of teachers across the United States. The survey lists seven professional development curricula designed for use with middle school mathematics teachers and asks participants about the number of teachers that each curriculum was used with, the extent to which projects used each curriculum, and the reasons that projects selected the curriculum materials for use. Aside from indicating the extent to which they used the listed curriculum materials, the participants were also asked to share information on any professional development curriculum materials they used with middle school mathematics teachers that were not listed on the survey.

The National Science Foundation funds many large scale professional development projects spread across the country. Using the online database of their funded professional development projects (National Science Foundation, 2010), I contacted by email and administered the survey to the principal investigators of 32 large-scale Math Science Partnership projects that provided professional development to middle school mathematics teachers. These professional development projects were conducted in a variety of contexts, urban to rural, across 21 states from coast to coast and Puerto Rico. The principal investigators were asked to either fill out the survey sent as an attachment or fill it out using an online format. There was a high participation rate on the survey of 84%, with 27 of the 32 projects sharing information and 5 not responding. The collected survey data illustrated the usage of professional development curriculum materials in projects working with 6203 teachers spread across the United States and Puerto Rico. The results of the survey of professional development curriculum usage are shown in Table 1.

Table 1

Survey Results on Curriculum Usage

Curriculum Name	Listed on Survey?	Percentage of Total Number of Teachers that have Used the Curriculum (N=6203)
Implementing Standards-Based Mathematics Instruction: A Casebook for Professional Development (Stein, Smith, Henningsen, & Silver, 2000)	Yes	45%
Improving Instruction in Rational Numbers and Proportionality: Using Cases to Transform Mathematics Teaching and Learning, Volume 1 (Smith, Silver, & Stein, 2005c)	Yes	18%
Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Strategies for Teachers (Lamon, 2005)	Yes	17%
Developing Mathematical Ideas, Number and Operations Part 2: Making Meaning for Operations (Schifter, Bastable, & Russell, 1999a)	No	14%
Mathematics Curriculum Topic Study: Bridging the Gap Between Standards and Practice (Keeley & Rose, 2006)	Yes	13%
Improving Instruction in Algebra: Using Cases to Transform Mathematics Teaching and Learning, Volume 2 (Smith, Silver, & Stein, 2005a)	Yes	6%
Windows on Teaching Math: Cases of Middle and Secondary Classrooms (Merseth, 2003)	Yes	6%
Mathematics Teaching Cases: Fractions, Decimals, Ratios, and Percents: Hard to Teach and Hard to Learn? (Barnett, Goldenstein, & Jackson, 1994)	Yes	4%

Table 1 shows that one curriculum, that was not listed on the survey but projects wrote about, was widely used by projects involving 14% of the total number of teachers. The survey results were used to select the final sample of professional development curriculum materials that were purchased and analyzed in this study.

Sample

Based on the survey results, a sample of curriculum materials commonly used with large numbers of teachers was selected for analysis. Three curricula listed on the survey were the most widely used in the professional development projects surveyed – each being used in projects involving over 15% of the total teachers involved in the surveyed projects. The fourth ranked curriculum was not listed on the survey as it is primarily categorized as a book for elementary mathematics teachers. However, several projects wrote that they used this curriculum. It would be interesting to learn what opportunities for learning this curriculum provides for middle school teachers since it is being used with them across the country. Therefore, I chose the four most commonly used sets of curriculum materials for the sample of professional development curriculum materials analyzed in this study. They are:

- *Implementing Standards-Based Mathematics Instruction: A Casebook for Professional Development* (Stein, et al., 2000) [ISBI]

The ISBI curriculum is focused on the use of cognitively demanding mathematical tasks in mathematics instruction. It provides an organized collection of narrative cases of mathematics instruction that is preceded by conceptual material that supports teachers to better understand and reflect on the use of cognitively demanding mathematical tasks. The six narrative cases depict instruction on a range of mathematical topics in different classroom contexts wherein students and teachers are using these challenging mathematical tasks.

- *Improving Instruction in Rational Numbers and Proportionality: Using Cases to Transform Mathematics Teaching and Learning, Volume 1* (Smith, et al., 2005c) [IIRP]

The IIRP curriculum uses narrative cases to depict the use of cognitively demanding mathematics tasks in classroom instruction on rational numbers and proportionality. It offers four detailed narrative cases around which teachers are provided opportunities to engage in a constellation of activities that focus their attention on the mathematical content and pedagogy addressed through the use of such tasks. The curriculum, in addition to offering these PLTs to teachers, dedicates sections of text to provide support to professional developers on how to facilitate teacher learning around these PLTs.

- *Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Strategies for Teachers* (Lamon, 2005) [TFRU]

The TFRU curriculum is designed to strengthen and deepen teachers' conceptual understanding of fractions and ratios specifically. The PLTs in the curriculum center teachers' exploration of these mathematical concepts in classroom instruction by providing many mathematical tasks that could be used with students and examples of how students from various grade levels have explored these concepts.

- *Developing Mathematical Ideas, Number and Operations Part 2: Making Meaning for Operations* (Schifter, et al., 1999a; Schifter, Bastable, & Russell, 1999b) [DMIMMO]

The DMIMMO curriculum uses brief narrative cases to illustrate the teaching and learning of operations with a focus on making meaning of these operations at K-6 grades. The teachers depicted in the cases use multiple representations and strategies to help students explore these concepts while maintaining high levels of cognitive demand. The DMIMMO curriculum consists of a casebook of PLTs for teachers and a facilitator's guide that provide professional developers with guidance on how to support teachers' learning from the activities in the casebook.

These four sets of curriculum materials were reported to have been used by 14 of the 27 responsive large scale professional development projects, often in conjunction with each other. 3759 teachers were engaged in the professional development of these 14 projects, thus 60% of all teachers in all of the projects surveyed were involved in the 14 projects using one or more of the curricula in the sample. The empirical data from the survey provides evidence that the four sets of curriculum materials in the study's sample are used with large numbers of teachers across the United States. Investigating the learning opportunities that these curriculum materials provide for teachers' and professional developers' learning will provide a glimpse into what can be learned in professional development for mathematics teachers across the country.

Development of Two Analytic Frameworks

In order to answer my specific research questions I developed two analytic frameworks to aid in my analysis. The first, the opportunity to learn analytic framework (OTL framework) was designed to be used to answer RQ1-3. Therefore, it directs attention towards and records information about how the curriculum materials and the PLTs within them present specific mathematical topics, the use of multiple representations of mathematical ideas, and the use of cognitively demanding mathematical tasks. The second analytic framework, the educative features framework (EF Framework) was designed to answer RQ4. It consists of a set of categories and specific features of support that could be designed into professional development curriculum materials to promote the learning of professional developers. The EF Framework is used to identify and record educative features present in the sampled curriculum materials.

In this section I provide detailed accounts of the development process used to develop the two analytic frameworks of this study. The purpose of this section is to explain the design of each framework and why they have specific characteristics. It is in the Data Analysis section that I provide detailed descriptions of how each framework was used in the analysis of the curriculum materials. The development of the OTL framework is presented first, with the development of the EF framework following.

Development of the opportunity to learn analytic framework. In order to identify and analyze teachers' opportunities to learn in a research-based and reproducible way, I developed the OTL framework to use in the analysis of teachers' opportunities to learn in professional development curriculum materials (Appendix B). Recognizing the power of considering the curriculum as a whole and also analyzing its separate professional learning tasks, the framework allows for both a horizontal and vertical analysis of curriculum materials. The approach of using both horizontal and vertical analyses was effectively used by Charalambous, Delaney, Hsu, and Mesa (2009) to compare very different elementary school curricula from Taiwan, Cyprus, and Ireland. I chose to use this approach in my study and adapted the horizontal and vertical analyses to help answer my study's particular research questions.

The horizontal analysis considers the structure of the curriculum materials and other surface level features, such as the number of pages. It is used to gather background information, and learn about the structure of the curriculum and the sequencing and coverage of topics. The most important information gained from the horizontal analysis is about the sequencing and coverage of topics that affect the opportunities to learn in the curriculum (Travers & Westbury, 1989; Valverde, et al., 2002). In my study, the horizontal analysis identifies PLTs within the curriculum materials and describes the mathematical topics they address, the artifacts of practice used within them, and the types of mathematical tasks and representations presented for teachers' consideration. This initial analysis sketches out the terrain for potential teacher learning that the curriculum presents. Additionally, the background information on the curriculum materials informs my analysis by providing contextual information, such as the year of publication, which can be used when comparing across curriculum materials. The structure of the curriculum materials is another important feature to consider as the look and structure of curriculum materials has been shown to affect how teachers and professional developers participate with them in practice (Remillard, 2005).

The OTL framework is also used to perform vertical analyses of individual PLTs to determine opportunities to learn provided to teachers by each PLT that was identified during the horizontal analysis. As discussed previously, each PLT may have two components: the mathematical task, and the link to practice component. The OTL framework considers features of each component that may provide teachers with opportunities to learn.

The mathematical task is analyzed based on its level of cognitive demand (Stein & Smith, 1998), use of representations (Goldin, 2002; Lemke, 2003), mathematical structure (Vermont Mathematics Partnership, 2008), and type of response required.

Studies have shown that cognitively demanding tasks are under-utilized in U.S. classrooms (Stigler & Hiebert, 1999) and that they are difficult for teachers to facilitate (Henningsen & Stein, 1997; Silver & Stein, 1996). Therefore, RQ3 is focused on teachers' opportunities to learn to use cognitively demanding tasks in professional development. In order to determine the level of cognitive demand of mathematical tasks in my framework, I have used the task analysis guide that was developed in the QUASAR project to categorize the level of cognitive demand of mathematical tasks with which students in middle grade mathematics classrooms might engage (Stein & Smith, 1998). The guide defines four levels of cognitive demand for mathematical tasks: memorization, procedures without connections, procedures with connections, and doing mathematics. The OTL framework lists the main characteristics of these four levels from the task analysis guide and provides a space for recording the determination made in regards to each mathematical task.

Learning to effectively use multiple representations of mathematical ideas is addressed in RQ2, thus, the framework also considers the representations that the mathematical task requires teachers to use as they engage with the mathematical ideas presented. The types of representations considered are (1) verbal descriptions, such as "three out of four squares", (2) symbols, such as ¾, (3) physical objects, such as unix cubes, plastic toys, or cups, (4) graphs, such as distance-time graphs, (5) tables, such as ratio tables, and (6) visual diagrams, such as partitioned and shaded shapes (Elliot & Kenney, 1996; Goldin, 2002; Kalchman & Koedinger, 2005; Leinhardt, Zaslavski, & Stein, 1990; Lemke, 2003; Pyke, 2003; Rider, 2007). These forms of representation are part of the language of mathematics and are used to create meaning in mathematics teaching and learning (O'Halloran, 2003; Sealy, 2009). Teachers who gain the capacity to use multiple representations and teach their students to do so are increasing students' representational competency, which is positively linked to student achievement (Brenner, et al., 1999; Brenner, et al., 1997).

The mathematical structure and context of proportional mathematical tasks affects the reasoning that students, and possibly teachers, use when engaging with them (Cramer, et al., 1993; Karplus, Pulos, & Stage, 1983; Vermont Mathematics Partnership, 2008). For example, different strategies may be employed when solving mathematical tasks wherein the ratio relationship is part to whole as compared to when it is part to part (Lamon, 1993b). Therefore, I have used portions of a framework developed to investigate proportional mathematical tasks (Vermont Mathematics Partnership, 2008) in my analytic framework to identify the structure of the mathematical tasks being analyzed.

The final feature of the mathematical task that is analyzed is the type of required response. Mathematical tasks can ask teachers to provide an explanation, a justification, or only an answer. While just providing an answer would engage teachers' content knowledge, providing an explanation or justification would also engage teachers' pedagogical content knowledge and require them to use representations of mathematical ideas with their peers. Thus, the first two types of responses would offer more opportunities for teacher learning as they could: engage more domains of teachers' knowledge, encourage the use of multiple representations of ideas, and spark collective discussion and reflection about what a good mathematical explanation entails.

The link to practice component of the PLTs is analyzed in terms of the ways it opens up mathematics instruction for individual or collective inquiry and reflection –

activities that promote teacher learning (Ball & Cohen, 1996, 1999; Wilson & Berne, 1999). The link to practice component may include an artifact of practice to ground teachers' inquiry. The type of artifact used may influence which aspects of instruction are available for collective inquiry. Thus, the analytic framework considers if the professional learning task uses artifacts of practice that have been shown to support teacher learning – narrative cases (Barnett, 1991; Derry, Wilsman, & Hackbarth, 2007; Harrington & Garrison, 1992; Merseth, 1996; Silver, et al., 2005), student work samples (Garet, et al., 2001; Little, Gearhart, Curry, & Kafka, 2003; Westheimer, 2008), and research study results (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Fennema, et al., 1996).

The link to practice component of the PLT may address certain pedagogical issues, present certain representations of mathematical ideas, and require teachers to participate in certain activities. In order to evaluate teachers' opportunities to reflect upon pedagogical issues, the analytic framework attends to which issues are explicitly addressed in the curriculum. A checklist of pedagogical issues that arise often in teacher education literature and in professional development sessions based on my experience is included in the framework, along with a space for indicating issues that arise but are not listed. Since several instructional practices by teachers have been linked to the maintenance or decline of the cognitive demand of mathematical tasks (Henningsen & Stein, 1997), the analytic framework also considers which of these practices are presented in the PLT for teacher reflection. The analytic framework also considers the representations of mathematical ideas that are presented to teachers in the link to practice component of the PLT. For example, teachers may be presented with student generated diagrams that they use to analyze student thinking.

In addition to attending to what pedagogical issues and representations of mathematical ideas are presented to teachers in the curriculum materials, the analytic framework is used to consider what teachers are being asked to do with the pedagogical issues and mathematical representations presented. Teachers may be asked to: discuss aspects of instructional practices; make conjectures about students' understanding and support their conjectures with evidence from artifacts of practice; reflect upon the pedagogical issues addressed and how those issues arise in their own classroom; or design a lesson that would build upon the students' understandings that they have analyzed. The OTL framework gathers information on what teachers are being asked to *do* which allows for consideration of those activities, in light of what research has found effective in supporting teacher learning.

Finally, the analytic framework looks across the entire PLT to consider how it provides opportunities for teachers to learn about multiplicative structures – a mathematical topic that undergirds proportionality. Sowder and colleagues (1998) have found that teachers need support in developing the conceptual understanding needed to teach multiplicative structures that are central to proportionality. They offer several recommendations for learning opportunities that teachers should be provided to gain this skill. I have included in my analytic framework a final check of whether their suggested recommendations are provided to teachers as they engage with PLTs.

During the vertical analysis of individual PLTs, the OTL framework is used to analyze a number of features. It analyzes features of both the mathematical task and link to practice components of PLTs as summarized in Table 2.

Table 2

Summary of the vertical analysis portion of the OTL framework

Analysis of the Mathematical Task	Level of cognitive demand (doing mathematics, procedures with connections, procedures without connections, or memorization)
	Use of representations required (verbal descriptions, symbols, graphs, tables, diagrams, or physical objects)
	Mathematical structure: - Mathematical topic (e.g. ratio, fraction, linear equation…etc) - Task type (e.g. comparison, missing value, qualitative…etc) - Numbers used (all integers, all rationals, or mixture) - Multiplicative relationships between and within measure spaces (all integer 1:2 & 3:6, some non-integer 2:3 & 10:15, or all non-integer 2:3 & 5:7.5) - Ratio relationship (part:whole, part:part, or n/a)
	Type of response required (answer only, explanation, or justification)
	Connections between representations required (e.g. verbal with symbolic…etc)
Analysis of Link to Practice Component	Artifact of practice used (narrative case, student work samples, research study results, or other)
	Pedagogical issues emphasized (e.g. unitizing, student struggle, use of multiple student solutions, student motivation…etc)
	Instructional factors associated with maintenance or decline of cognitive demand (e.g. sustained press for justification, students not held accountable for high level products/processes…etc)
	Use of representations (verbal descriptions, symbols, graphs, tables, diagrams, or physical objects)
	Type of response required (e.g. participation in discussion, written reflection, conducting a lesson…etc)
Opportunities to Develop Multiplicative Reasoning	Task requires teachers to reason explicitly in terms of quantities and quantitative relationships
	Task appropriately uses and contrasts additive and multiplicative reasoning
	Task exposes teachers to situations that allow them to see proportional reasoning as a complex process that evolves over a long period of time
	Task allows for connections to be made among forms of rational numbers and with concepts of ratio and proportion

The OTL framework was designed to provide a view of the learning opportunities across a curriculum and within each individual PLT of the curriculum. This framework was specifically designed to highlight teachers learning opportunities around specific mathematical topics (RQ1), the use of representations of mathematical ideas (RQ2), and the use of cognitively demanding tasks in instruction (RQ3). In the data analysis section I describe how the OTL framework was used to analyze the sample professional development curriculum materials and provide illustrative examples.

Development of the educative features framework. Educative professional development curriculum materials are designed to support the learning of professional developers as they use the curriculum to facilitate professional development with teachers. In order to investigate the educative features of the professional development curriculum, I followed the schema of Davis and Krajcik (2005) to first consider the challenges of the professional developers' work with teachers. With those challenges in mind, I considered what features should be designed into curriculum materials to lend support in meeting them. Besides challenges, I also considered recommendations from research about what general features should be included in any curriculum that was designed to be educative. Combining the generally recommended educative features and features that would provide support specific to the work of professional developers, I created the educative features framework (EF Framework) to aid in my examination of educative features within the sample of professional development curriculum materials (Appendix C).

Through a review of research literature on providing professional development and my experiences as a mathematics teacher educator, I identified several challenges to the work of professional developers. For example, professional developers need to conceptualize and support the ongoing and long-term learning of both individual teachers and groups of teachers in professional learning communities. The fostering and support of professional learning communities over time has been found to be challenging for professional developers whose previous experiences have generally been in providing short-term support in workshops or courses (Ball & Cohen, 1996, 1999; Mewborn, 2003; Stein, et al., 1999; Wilson & Berne, 1999). Thus, one of the categories of support that is included in the EF Framework is *supporting professional developers in building a professional learning community with teachers*. The research literature on the challenges in the work of professional development facilitation guided the conceptualization of what learning opportunities the curriculum materials should provide to professional developers.

General guidelines for the design of educative curriculum materials were compiled by considering recommendations from research on educative curriculum materials (Ball & Cohen, 1996; Beyer, Delgado, Davis, & Krajcik, 2009; Davis & Krajcik, 2005; Remillard, 2000, 2005; Schneider & Krajcik, 2002). These general guidelines, such as making visible the authors' pedagogical judgments (Davis & Krajcik, 2005) and providing rationales for unfamiliar suggestions or activities (Remillard, 2000), were used, in conjunction with identified challenges in the work of facilitating professional development, to create general categories of support in the analytic framework. Within each category, several features of support specific to the work of facilitating professional development were then explicated. These specific forms of support were created because they addressed particular challenges in the work of professional developers. For example, in addressing the challenge of anticipating and working with teachers' ideas, two different items of support that the framework considers are if the curriculum materials (a) *explain why supporting teacher learning means attending to teachers' knowledge, beliefs, and habits of practice* and (b) *provide guidance on when to expect certain common teacher ideas to emerge*. These items of support exemplify the two forms of support that are provided in educative curriculum materials: *rationales* and *implementation guidance*. Rationales are items of support that provide explicit justifications for instructional decisions and the design of activities by explaining why they are appropriate (Beyer, et al., 2009). Items of support that help professional developers to "know how to use the instructional approaches and activities in productive ways by making explicit their salient features" and adapt them successfully are implementation guidance (p. 13).

The full version of the EF framework (see Appendix C) provides information on the type of support that each item provides and offers examples of what such an item might look like in curriculum materials. A brief summary of the 6 categories and 20 items of support listed in the EF framework is presented in Table 3.

Table 3

Summary of EF Framework

#	Category	Items
1	Supporting professional developers in engaging teachers with *specific mathematical topics*	A. Explain the value of revisiting and deepening teachers' understanding of mathematical topics that they teach
		B. Explain the conceptual underpinnings of and the connections between the mathematical topics under discussion
		C. Provide guidance on which strategies to use to focus attention on the conceptual aspects of the mathematical topic(s)
		D. Provide guidance on which representations might support discussion of the mathematical topic(s)

2	Supporting professional developers in engaging teachers with *specific pedagogy*	A. Explain why the pedagogical moves being illustrated have been focused upon and provide evidence that they are effective in promoting student learning
		B. Provide guidance on how to model the "reform oriented" pedagogical moves with teachers that they are encouraged to use with their students
		C. Provide guidance on strategies to use to focus attention on specific aspects of classroom pedagogy
3	Supporting professional developers in engaging teachers with professional learning tasks *centered on the work of mathematics teaching*	A. Explain why learning activities centered on the work of teaching and situated in classroom practice are desirable for promoting teachers learning
		B. Provide guidance on how to collaborate with teachers to reflect upon and engage in collective inquiry into mathematics instruction in relation to the example from the PLT, their own practice, and more generally
4	Supporting professional developers in *anticipating and working with teachers' ideas* about mathematics teaching and learning	A. Explain why supporting teacher learning means attending to teachers' knowledge, beliefs, and habits of practice
		B. Provide guidance on when to expect certain common teacher ideas to emerge
5	Supporting professional developers in *building a professional learning community* with teachers	A. Explain why collegial support and learning within a community are important for teacher learning both individually and collectively
		B. Provide guidance on how to facilitate and create a common discourse for teachers to engage in collective inquiry about mathematics teaching and learning
		C. Provide guidance on ways to create and support a professional learning community of teachers

6	Supporting professional developers to provide *long-term, ongoing, and coherent professional development programs in various contexts*	A. Explain the benefits of an ongoing and long-term professional development program
		B. Explain the sequencing of PLTs over the curriculum and the overall learning goals for teachers that the sequence addresses
		C. Explain how the curriculum is aligned to state and national standards and current theories on teacher learning as a long-term process
		D. Explain the design and the teacher learning goals (both the learning of mathematical content and pedagogy) of individual PLTs
		E. Provide guidance on how and when to modify PLTs or the sequence of PLTs in order to fit the local culture or context of PD
		F. Provide guidance on various strategies to continuously engage teachers and when to employ them

The EF Framework is a set of categories and specific features of support that could be designed into professional development curriculum materials to promote the learning of professional developers. This framework was developed based on literature on the work of professional developers and on the design of educative curriculum. The EF Framework was used in my analysis to investigate the presence of educative features that provide learning opportunities to professional developers in the sample of professional development curriculum materials under investigation.

Data Analysis

In this section I will first describe the analysis of teachers' learning opportunities that serves to answer RQ1-3. I will describe how the OTL framework I developed was used to analyze the sample curriculum materials. I will then describe the analysis of the professional developers' learning opportunities that answers RQ4 about the educative features present in the sample professional development curriculum materials. A description of how the developed EF Framework was used in my analysis will be provided.

Analysis of teachers' opportunities to learn. To examine the opportunities for teacher learning, four phases of analysis were conducted. In phase 1, the OTL framework (Appendix B) was used to perform a horizontal analysis on each of the sets of curriculum materials. These horizontal analyses collect background information about the curriculum and identify the quantity, location, topic and sequence of PLTs within the curriculum materials. As I have defined PLTs as activities organized around artifacts of practice (Silver, 2009), I used the artifacts of practice to identify individual PLTs. For example, in the horizontal analysis of the IIRP curriculum (see Appendix D) the Randy Harris PLT consists of the activities organized around the case of Randy Harris and the case itself (Smith, et al., 2005c, pp. 8-25). In another curriculum that does not use narrative cases the PLT would center on a different artifact of practice. For example, the Tree Growth PLT in the TFRU curriculum consists of a mathematical task, a set of students' solutions, and instructions to assess the correctness of the solutions and rank them in terms of the sophistication of students reasoning (Lamon, 2005, pp. 29-30). Phase 1 provided contextual information about the curriculum, a vision of the sequence of PLTs, and identified the location of each PLT for further analysis in the next phase.

In phase 2, the OTL framework was used to conduct vertical analyses on each of the identified PLTs within each of the curriculum materials. The vertical analysis identifies, describes, and categorizes the opportunities to learn that are available to teachers engaged with these PLTs. An example of such vertical analysis on the Randy Harris PLT is provided in Appendix E. The vertical analysis begins by analyzing the mathematical task(s) in the PLT. The mathematical task in the Randy Harris PLT is categorized at the doing mathematics level of cognitive demand as it explores concepts, requires complex and nonalgorithmic thinking, and requires considerable cognitive effort. It requires the use of three of the six representations: verbal descriptions, symbols (decimals, fractions, and percents), and a diagram. Its mathematical structure is such that the task addresses the topics of fractions, decimals, and percents, and is non-proportional. The task requires an explanation as a response that connects symbols to the diagram presented in the task.

The Randy Harris PLT is centered on the artifact of practice of a narrative case that illustrates how the cognitively demanding mathematical task discussed is used by a 7th grade teacher, Randy Harris, with his class. Using the OTL framework, I indicated which of the 15 listed pedagogical issues arose in the PLT. Those issues emphasized in the PLT were: unitizing, student struggle, use of multiple student-generated representations, use of multiple student solutions, and explanations. Since instruction was depicted in the case, I could also identify, using the OTL framework, the classroom factors associated with cognitive demand that were illustrated for teacher reflection, such as a sustained press for justification. Teachers using the Randy Harris PLT were asked to do a large number of things around the case. They are asked to: participate in group discussion; analyze mathematical and pedagogical ideas raised in the case, such as what counts as making a connection between representations; write their reflections about the case; create and teach a lesson that uses one of the pedagogical moves observed in the case; and teach a lesson that uses the same mathematical task in the PLT. I have shared the characteristics that were identified to be present within the Randy Harris PLT. For each characteristic of a PLT identified to be present using the OTL framework, the user also writes a description of how and where in the PLT that characteristic presents itself. These detailed descriptions can be found in Appendix E.

After using the analytic framework in the first two phases to identify and analyze individual PLTs, the analysis in phase 3 looked across each set of professional development curriculum materials. The sequence of opportunities for learning within the identified professional learning tasks was analyzed to create a description of the overall learning opportunities provided to teachers who use a particular professional development curriculum. For example, I analyzed the progression of opportunities with which teachers have to engage, and make connections between various representations of mathematical ideas across each curriculum. This analysis brings together data from phases 1 and 2 to create a coherent description of specific types of learning opportunities within each set of curriculum materials.

In phase 4, those descriptions were used to compare and contrast teachers' learning opportunities across the sample of curriculum materials. In research and in life, comparison is a useful strategy for noticing trends and interesting characteristics (Philips, 2000; Strauss & Corbin, 1998). This final phase of analysis enables an exploration of trends in the provision of learning opportunities to teachers across curricula.

Analysis of educative features. The EF Framework is an analytic tool to be used to determine the presence of educative features in professional development curriculum materials that provide support specific to the work of facilitating teacher learning in professional development. Educative features are not common in curriculum materials since the materials are primarily designed to speak to teachers and address teacher learning. If they are present, then they will be found in sections of the curriculum materials that are explicitly directed towards professional developers, such as facilitator guides. In order for a section of text to be educative it needs to have a high degree of explicitness to signal to professional developers that there is something for them to learn about facilitating teacher learning. Therefore, I first identified the sections of the sample curriculum materials that were written to directly address professional developers, such as facilitator notes, and then used the EF Framework to analyze these sections of the sample curriculum materials.

Since a paragraph can be defined as a distinct section of text dealing with a particular point (Oxford University Press, 2010), it was reasonable to parse the text to be analyzed by paragraph. The paragraphs in each section of the curriculum materials under analysis were numbered and the EF Framework was used as a coding scheme on each paragraph to determine which, if any, categories of support were addressed and specifically which educative features were present. If an educative feature was present within a paragraph, then this was recorded as a sighting and the location recorded. It was possible for a single paragraph to contain no educative features or several features under different categories. For example, a paragraph might be discussing the mathematical topics being addressed in the curriculum and suggesting which representations might support discussions of those topics. This paragraph addresses Category 1 of *supporting professional developers in engaging teachers with specific mathematical topics* and the specific educative feature present is 1D where the authors *provide guidance on which* *representations might support discussion of the mathematical topic(s)*. My analyses, using the EF Framework, identified the instances where the curriculum materials provided learning opportunities to professional developers and specified what could be learned in each instance.

After each of the sample curriculum materials had been analyzed with the EF Framework, I looked across the results to consider trends in the data about the extent to which the curriculum materials were educative. I investigated which categories and items of support for the work of facilitating teacher learning were well addressed and which were over-looked. I also considered the prevalence of each type of support provided in the curriculum materials: rationale or implementation guidance. The results of my analysis are shared in Chapter 4.

Validity and Reliability

The validity of a research study has been described as "the trustworthiness of inferences drawn from data" (Eisenhart & Howe, 1992, p. 644). I strive to accomplish such trustworthiness in several ways. First, I maintain the *descriptive validity* of the study (Maxwell, 2002) by providing factually accurate accounts of the contents of the curricula under analysis for the reader. Second, I accomplish *interpretive validity* (Maxwell, 2002) by providing thick and rich descriptions of the study design, data collection, and data analysis, and by clearly showing the connection between the data and my findings (Eisenhart, 2006; Erickson, 1986; Freeman, deMarrais, Preissle, Roulston, & St.Pierre, 2007).

In the previous sections I have provided rich, detailed descriptions of the study design, data collection and data analysis used. Further, the method of using curricular analyses to explore the opportunities to learn that different sets of professional development curriculum materials provide has been used with success in many previous studies (McDonnell, 1995; Travers & Westbury, 1989; Valverde, et al., 2002). Conducting a curricular analysis to examine the opportunities to learn that curriculum materials provide has been proven to be a useful method for investigating the learning occurring in an instructional space. Thus, through using an established method and providing a thorough description of my specific study design and methodology, I have demonstrated that I have used appropriate and adequate methods to draw valid inferences from the data (Freeman, et al., 2007). In the next section, I clearly state the limitations of the study that clarify the generalizability of the inferences drawn from the data that are described in Chapters 4 and 5. In these final chapters, in order to maintain the trustworthiness of my inferences, I am careful to clearly illustrate the connections between the data and inferences and conclusions made (Erickson, 1986; Freeman, et al., 2007).

The reliability of the analytic frameworks developed in this study was important as these frameworks were designed to be used both in my curricular analysis and by other researchers in future studies. I designed the frameworks with the idea that researchers could use them to analyze professional development curriculum materials that were not included in this study's sample. Thus, the analytic frameworks were carefully designed to be explicit in meaning.

To ensure that the frameworks could reliably be used by others, inter-rater reliability checks were performed on both frameworks. In each check, two graduate students in education and I used either the OTL or EF Framework on a sub-sample of curriculum materials representing either two professional learning tasks or several pages of text specifically directed towards professional developers. The purpose of the checks was to ensure that the frameworks were explicit enough that they could be easily used by others and that such use would yield reliable results.

Raters were provided instructions introducing them to the framework being used, the sub-sample of curriculum materials to be analyzed, and the framework itself. After the two raters and I completed our analyses of the sub-sample of curriculum materials, the results were compared. In the test of the OTL framework agreement was achieved if 2/3 or 3/3 of the raters made the same decision. For example, the OTL framework lists 15 possible mathematical topics and asks the user to indicate the ones that were addressed in the PLT being analyzed. If two or more of us agreed that a particular topic, such as ratio, was addressed, then our agreement on that item would receive a score of 1. If we did not agree, then it would receive a score of 0. Each item within the OTL framework was assessed to see the extent of agreement reached. Using this scoring system, an initial inter-rater reliability check of the OTL framework achieved a level of rater agreement of 82% (agreement on 151 out of 181 items).

In the test of the EF framework the scoring for agreement differed slightly. I considered each paragraph of text in the sub-sample of curriculum materials to determine if there was unanimous agreement on how each paragraph was coded with the educative features listed in the EF framework. If there was unanimous agreement on the coding for a paragraph, then it would receive a score of 1. After modifying several versions of the EF Framework based on feedback from colleagues, a rate of agreement of 86% (agreement on 872 out of 1020 paragraphs) was achieved between raters doing an inter-rater reliability check.

Debrief sessions were held with raters of both frameworks to resolve disagreement. In these sessions, raters only identified a few sections of the frameworks as ambiguous. Based on their feedback, I modified the instructions for these sections to clarify how they should be used in the analysis of curriculum materials. The updated OTL and EF Frameworks were used in my analysis of the curricula in my sample. The results from the inter-rater reliability checks provide evidence to suggest that the OTL and EF Frameworks can reliably serve as a means for the wider research community to identify and categorize opportunities for learning for both teachers and professional developers.

Limitations

The sample of professional development curriculum materials used in the study are drawn from a national sample, used with large numbers of teachers, and, being publicly available, have the potential to be used with greater numbers of teachers in the future. Though these curriculum materials are widely used, I do not claim that these materials are representative of all professional development curriculum materials in the United States since many professional development programs use home-grown activities and materials (Banilower, et al., 2006). While the results of the study offer insights into the opportunities for learning that the sampled curriculum materials provide for teachers and professional developers, I do not make claims about the opportunities for learning provided generally by all professional development curriculum materials across the country. However, since large numbers of middle school teachers across the country either use or have the potential to use the sampled professional development curriculum materials, the findings, though particular, still have a national influence.

One limitation of the OTL framework is that the task analysis guide it uses for assessing the cognitive demand of mathematical tasks was developed by the same research project, the QUASAR project (Silver & Stein, 1996), from which two of the curricula in my sample emerged (Smith, et al., 2005c; Stein, et al., 2000). This introduces some circularity, but the task analysis guide is a widely used tool for assessing cognitive demand and the curricula became part of the study based on empirical data from the survey and not by design.

Professional development curriculum materials are just one component of professional development. While I can analyze the opportunities for learning that are available to teachers in the materials as they are written, these opportunities will be shaped by the professional developer(s) and the teachers as the PLTs within the materials are set up and enacted within professional development sessions. Thus, in my curriculum analysis I have included an analysis of the educative features of the materials that serve to both support the professional developer(s) in conducting professional development and convey the developers' intentions of how the curriculum should be enacted. However, no amount of analysis with static materials can predict the myriad ways that the opportunities for teacher learning can be shaped and molded during complex interactions between facilitator(s), teachers, curriculum materials, and environments. What can be said with confidence is that curriculum materials "exert probabilistic influences on the educational opportunities that take place" (Valverde, et al., 2002, p. 2) in professional development. Therefore, while my study does identify certain opportunities for teacher learning that professional development curriculum materials provide, it does not claim that teachers using said materials are guaranteed to learn these ideas. Teacher learning, like all human learning, is a complex process that depends on many variables (Putnam & Borko, 1997).

This study is limited to informing our knowledge of the opportunities for teacher learning that the sample professional development curriculum materials themselves can offer. While limited in scope, this study is an important step. Future research studies that build upon it can examine: the use of these same curriculum materials to explore how the identified opportunities for learning are shaped during enactment of the curriculum with professional developers; what it is that teachers learn from the professional development experience; and how teachers are then able to apply their learning to their instructional practices in mathematics classrooms to affect student learning.

Chapter IV - Identified Learning Opportunities in the Four Curricula: Results

Overview

In this chapter I present the results of my analyses of the learning opportunities provided to mathematics teachers and professional developers by the sample professional development curriculum materials. The purpose of this chapter is to present the results themselves. My interpretation of said results and my judgments about their implications for teacher learning and the support of professional developers will be presented in Chapter 5. The results of the analysis of the opportunities to learn provided to middle school mathematics teachers will be presented first. The results of the educative analysis will follow.

Opportunities for Teacher Learning in the Four Curricula

My analyses of the opportunities to learn provided to teachers in the four curricula in the sample were designed to answer the first three research questions. RQ1-3 ask to what extent and in what ways do the professional development curriculum materials provide opportunities for teachers to learn: *middle school mathematical content – specifically proportionality, rational numbers, and linear equations and functions; about using multiple representations of mathematical ideas;* and *about the use of cognitively demanding mathematical tasks in instruction*. In the following sections, I share the results of my analyses designed to answer these three questions.

Though I have used a simple diagram to represent mathematics instruction

(Figure 2.1), it is actually dynamic, complex, and challenging (Lampert, 2001). As mathematics instruction itself is the content to be learned in professional development, this presents a challenge for curriculum designers. One professional development curriculum can never adequately address all aspects of mathematics instruction, so choices need to be made about the focus. Designers of professional development curriculum materials need to decide which mathematical topics to focus upon, which common student understandings and misunderstanding should be presented, and which pedagogical issues to highlight. These decisions affect the learning opportunities that the curriculum can offer. In the following sections I will present results on the opportunities to learn provided by each of the four curricula in the sample (summarized in Appendices F, G, H, and I) and general trends in the results across the entire sample of curriculum materials.

Teachers' opportunities to learn in the Implementing Standards-Based Mathematics Instruction (ISBI) curriculum. The *Implementing Standards-Based Mathematics Instruction* (ISBI) curriculum (Stein, et al., 2000) grew out of the authors' work on the QUASAR project. This five year project investigated mathematics teaching and learning in urban middle school classrooms with a focus on the use of instructional tasks (Silver & Stein, 1996). From their analysis of hundreds of mathematics lessons from 1990 to 1995, QUASAR researchers had two main findings that are featured in the ISBI curriculum. One is that mathematical tasks have different levels of cognitive demand and require different degrees and kinds of student thinking (Stein, et al., 2000, p. 3). The other is that the cognitive demand of mathematical tasks can change as they are implemented in classrooms as illustrated by the Mathematical Tasks Framework (MTF) in Figure 1.2. The MTF is intended to be used by teachers throughout the activities in the book and beyond as a lens for reflection on mathematics teaching and learning and a "shared language for discussing instruction with their colleagues" (p. 4). Drawing on the research base of the QUASAR project, the ISBI curriculum focuses on supporting teachers to learn about the use of cognitively demanding tasks in mathematics instruction.

The ISBI curriculum is arranged in two parts. In the first part teachers are provided information about the concepts, frameworks, and research findings that are referenced throughout the book, such as cognitive demand, the MTF, and instructional factors found to support or inhibit high level mathematical reasoning. It is in this part that the first PLT, a task-sorting activity, is presented. The task-sorting activity provides teachers with an opportunity to learn how to recognize and determine the level of cognitive demand in mathematical tasks. In the second part of the curriculum, six narrative cases are presented that portray how cognitively demanding mathematical tasks can be used in classroom instruction. Teachers are encouraged to use the concepts, frameworks, and research findings from the first part to inform their discussions of the mathematics instruction depicted in the narrative cases. The six narrative cases each have a similar format. Initially a mathematical task or tasks is presented for teachers to solve and to discuss its possible solutions. Then, a classroom episode is described in which the mathematical task or one similar is used by a teacher or teachers with students. At the end of each case a set of discussion questions is presented to be used for reflection and/or group discussion. These questions direct teachers' attention towards characteristics, such as the mathematical goals of the lesson, the level of student engagement with the mathematical tasks, and instructional factors that influences the level of cognitive demand during the lesson. The six narrative cases present examples of how cognitively demanding mathematical tasks addressing a range of mathematical topics can be used with students in a range of classroom contexts by teachers with varying success at maintaining the level of cognitive demand.

The mathematical tasks presented in the task-sorting PLT and in the six narrative case PLTs address the mathematical topics of proportionality (which includes ratio, rate, similarity, scale and slope), rational numbers (which include fractions, decimals, and percents), and linear functions (which include linear equations and patterns), but do not specifically focus on these topics. As can be seen in Figure 4.1, a range of mathematical topics are addressed, including topics such as the multiplication of binomials.

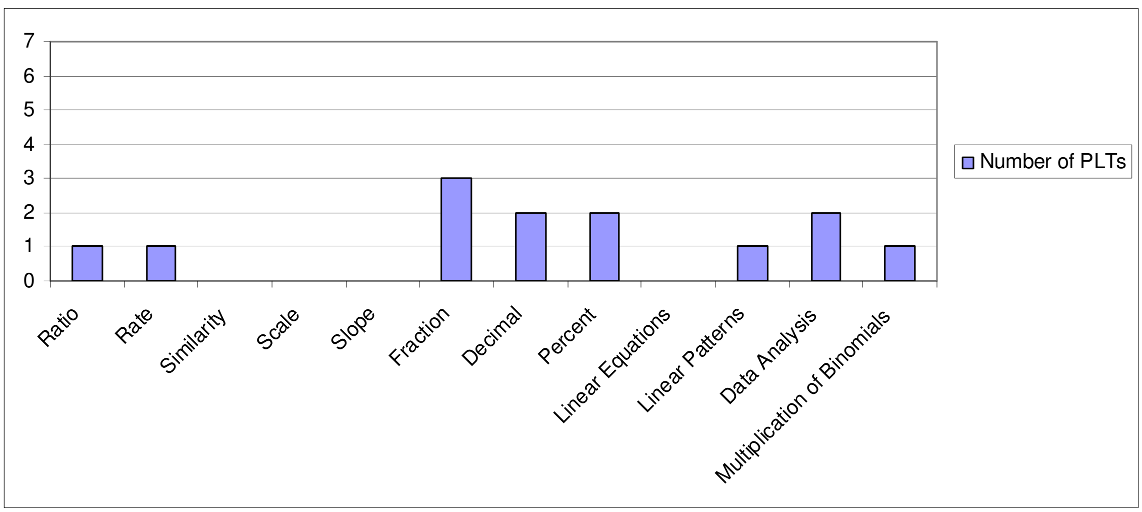


Figure 4.1. Mathematical topics presented in the ISBI curriculum.

Rational numbers (fractions, decimals, and percents) were addressed in 3 PLTs – the task-sorting PLT and two narrative cases. Proportionality (ratio and rate) and linear patterns were only addressed in the task sorting activity. Though the focus was not solely on those mathematical topics specified in RQ1, the other topics addressed, such as data analysis, are middle school mathematical content. Through solving these tasks and discussing the different ways they could be solved, teachers were provided an opportunity to revisit a range of middle school mathematical content and reflect upon the mathematical concepts to be learned.

One topic that people often have superficial understandings of is the conceptual connections between the forms of rational numbers: fractions, decimals and percents (Sowder, Armstrong, et al., 1998). The mathematical task presented in the PLT named *The Case of Ron Castleman* (Stein, et al., 2000, p. 47) provides an opportunity for teachers to link these different forms to a visual diagram – an area model. The task presents a non-conventional 4 x 10 grid and asks that 6 square tiles be shaded. The six tiles could be shaded in any way the solver selects. The solver is then instructed to use their shaded diagram to explain how to determine the percent, decimal part, and fraction part of the area that is shaded. This mathematical task is designed to push the solver beyond simple procedural thinking. The unconventional grid of 40 squares instead of the standard 100 requires the solver to think about what percentage of the whole grid the shaded portion represents. Further, by requiring the solver to use the diagram in their explanations it directs solver to reason visually rather than using numerical procedures. Additionally, the fact that the solver can shade the grid in any way they wish presents the opportunity for him or her to decide upon reasonable ways to unitize quantities on the grid. This decision shapes the way the task is solved. As can be seen in Figure 4.2, there are several ways that a teacher or students could shade the grid.

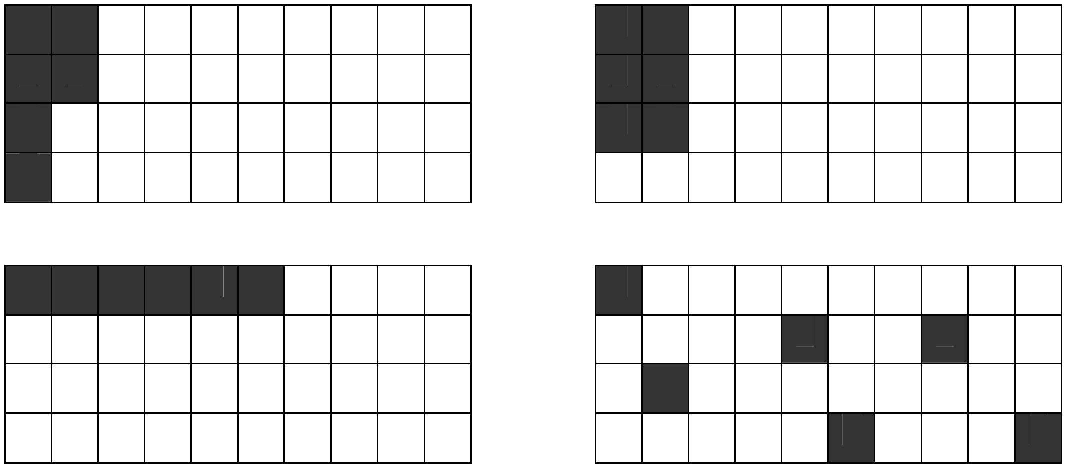


Figure 4.2. Examples of ways the grid in the mathematical task of the case of Ron Castleman can be shaded.

If a person decided to make the first shading, then it is likely that they were reasoning about the columns of the grid as 10%, 0.10, or 1/10th of the total area. They could then determine visually that the shaded area is 15%, 0.15, and 3/20^{ths} of the area.

I have chosen to highlight this mathematical task because it exemplifies the type of task presented in the ISBI curriculum in that it pushes teachers to move beyond the use of procedures and to delve into the concepts underlying the mathematical content addressed. After doing so, teachers then have an opportunity to see how these concepts play out in a classroom as teachers and students engage with the mathematical task(s). Through seeing how the task is taken up in a classroom setting, teachers are provided with the opportunity to see how students may understand or misunderstand mathematical concepts and how the teacher either facilitated or inhibited their learning from the task.

With its focus on cognitive demand, the ISBI curriculum provides teachers with opportunities to engage with mathematical tasks at every level of cognitive demand. In the task sorting activity opportunities are provided to teachers to learn how to differentiate mathematical tasks based on their level of cognitive demand. The opportunity is provided to sort mathematical tasks into the four levels of cognitive demand: memorization, procedures without connections, procedures with connections, and doing mathematics. In the narrative cases, opportunities were provided to observe how teachers and students use cognitively demanding tasks (4 at the "doing mathematics" level and 2 at the "procedures with connections" level) in the classroom. Pedagogical issues around such use arise in the PLTs using these cases. For example, issues around student explanations, student struggle, cognitive demand, and the use of multiple student solutions emerge in 43% or higher (3 or more out of 7) of the PLTs in the ISBI curriculum. Further, classroom-based instructional factors found to be associated with the maintenance or decline of the cognitive demand of mathematical tasks (Henningsen & Stein, 1997) also were found to be addressed, as seen in Figure 4.3.

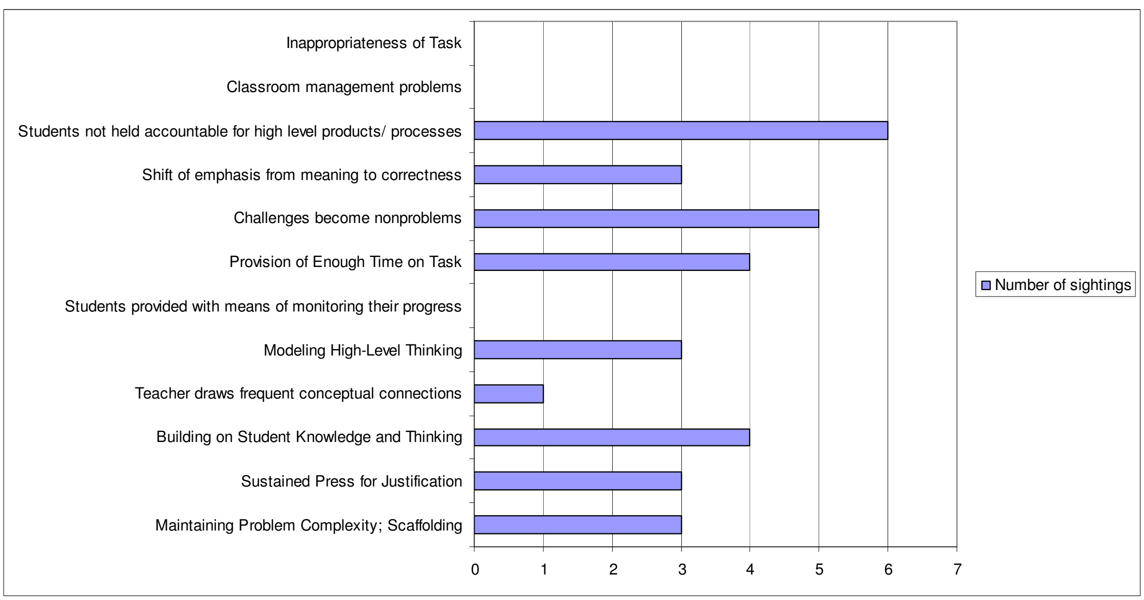


Figure 4.3. Instructional factors related to cognitive demand addressed in the ISBI curriculum.

These instructional factors, coupled with the pedagogical issues raised, provide opportunities for teachers to learn about the use of cognitively demanding mathematical tasks in instruction. For example, a sustained press for justification is associated with maintaining the level of cognitive demand as students engage with a mathematical task. This involves asking students to explain their thinking and justify their solutions. Students often struggle with providing explanations and if they are not held accountable for doing this, then the level of student thinking required by the task, the cognitive demand, can decline.

Continuing to use the case of Ron Castleman as an example, in his second period class Mr. Castleman succumbed to his desire to ease students' struggle and suggested that students start with what he thought was easier than initially finding the percent: finding the fractional part that was shaded instead. What resulted is that students simply counted that there were 6 squares out of 40 shaded and used numerical procedures to reduce the fraction, used division to find the decimal, and moved the decimal point to obtain the percent. The use of such procedures instead of reasoning using the diagram denied students an opportunity to understand the reasoning behind the conversions between the visual diagram, fractions, decimals, and percents. In Mr. Castleman's 6th period class, however he sustained his press for students to reason and justify their solutions using the diagram. Though they still struggled with finding the percent shaded, Mr. Castleman directed them back to the diagram and asked them questions that would allow them to build off of the way they had shaded the grid. Additionally when two students simply shared their numerical solution with the class and began to return to their seats, Mr. Castleman insisted that they provide a rationale for their solution at the overhead projector. Rachel, one of the students, "explained that since there were 40 squares in the diagram and the whole diagram needed to represent 100%, each small square would have to equal 2 ½ %" (p. 54). This statement allowed for a class discussion about the meaning of 100%. A discussion that was much needed as another student Derrick stated "are you saying that 100% = 1? I thought that 100% = 100!" (p. 54). This is a common misconception that 100% must refer to 100 objects. The class discussion around Rachel's explanation, the visual diagram, and Derrick's question allowed for the class to debunk this misconception and clarify the important idea that 100% can refer to 1 object divided into 100 parts. The case of Ron Castleman demonstrates how a sustained press for explanation can support student learning and the surfacing of students' understandings and misunderstandings about key mathematical ideas. It also illustrates how failure to press for explanation and to hold students accountable can reduce what can be learned.

The teachers depicted in the PLTs of the ISBI curriculum both succeed at pressing students for explanations and fail to do so. The consequences of both these actions on the learning that occurs around the mathematical task are illustrated in the cases for teachers to reflect upon. This is just one way that the narrative cases provide opportunities for teachers to learn about using cognitively demanding tasks in mathematics instruction and the instructional factors associated with such use.

Many representations are used in the PLTs of the ISBI curriculum. As seen in Table 4, every major form of representation is used in at least one of the PLTs. As mathematical language is multisemiotic and uses multiple representations in concert to make meaning (Lemke, 2003; Sealy, 2009), it is important to consider how the representations are being *used*. In my analyses of the cases that described how teachers used representation in their instruction, I noted which representations were being used with others.

Table 4

The extent to which representations of mathematical ideas are presented in the PLTs of the ISBI curriculum

Representation	Number of PLTs in which the representation is present	Percentage of PLTs in which the representation is present
Verbal Descriptions	7	100%
Symbols	4	57%
Graphs	1	14%
Tables	1	14%
Visual Diagram	5	71%
Physical Object	3	43%

I did so to track both the opportunities that were presented for teachers to make connections between representations of mathematical ideas and opportunities to learn how teachers use multiple representations in instruction to promote student learning. Figures 4.4 and 4.5 illustrate the connections made between the multiple representational forms made in the ISBI curriculum across its seven PLTs.

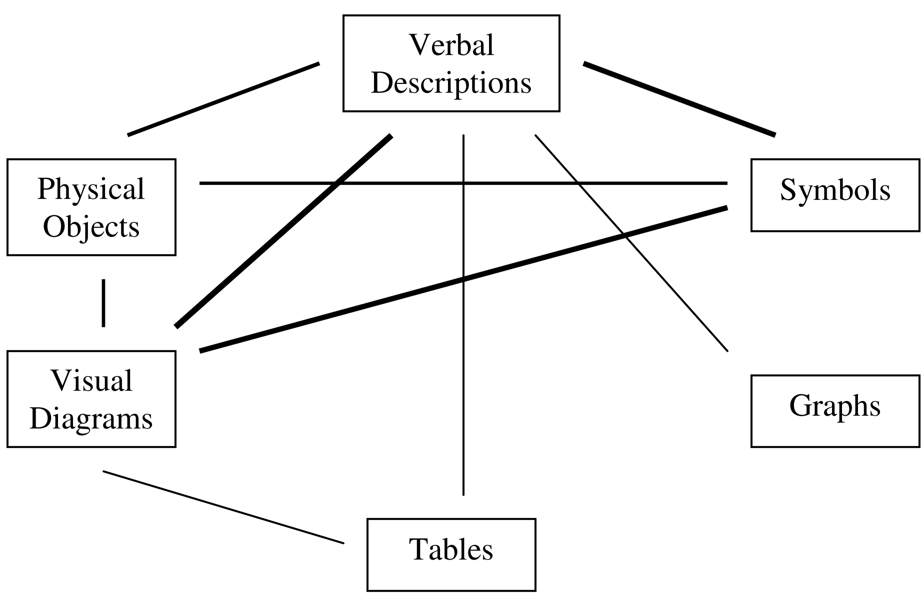


Figure 4.4. Strength of connections between representations in the ISBI curriculum.

As can be seen, several connections are made as multiple representations of mathematical ideas are presented in the cases. As illustrated by the weight of the lines in Figure 4.4, in the ISBI curriculum, connections between verbal descriptions, visual diagrams, and symbols are made more frequently and more strongly than other connections between representational forms. In the Case of Monique Butler (pp. 96-104), for example, algebra tiles and students' sketches were used in an algebra lesson around the multiplication of monomials and binomials. This case provides an example of how symbols, physical objects, and visual diagrams can be used together to help students learn how to perform these multiplications and understand why they work.

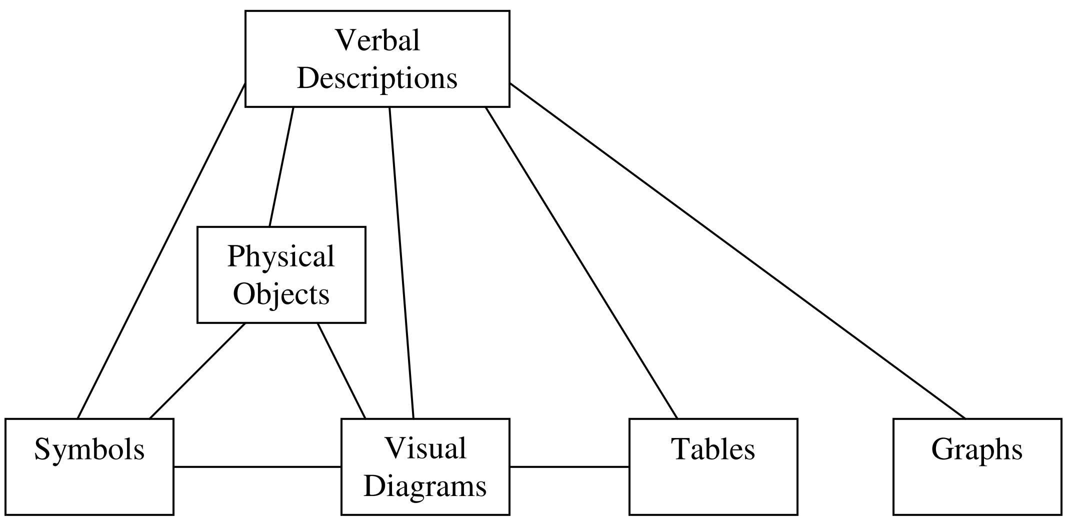


Figure 4.5. Degree of connectivity between representations in the ISBI curriculum.

Since graphs and tables are only used once, connections between these forms of representations and others are under-developed or missing, as illustrated in Figure 4.5. Specifically, the connection between symbols, graphs, and tables is not made in the ISBI curriculum. This is an important missing connection between representations as it is widely used in algebra when exploring functions.

Overall the ISBI curriculum provides opportunities for teachers to learn how to identify and about using cognitively demanding mathematical tasks. The task sorting activity provides opportunities for teachers to learn to differentiate mathematical tasks based on their cognitive demand. The MTF framework provides teachers with a lens to view instruction using cognitively demanding mathematical tasks that pays attention to the ways teachers' actions can support or inhibit the cognitive demand of tasks as they are used with students and hence affect students learning opportunities. With this lens in hand, teachers , through the six narrative cases, are provided opportunities to reflect on images of practice that illustrate how cognitively demanding tasks have be used in classroom instruction around a range of mathematical topics. By examining the mathematical tasks in the cases and the ways that students and teachers interacted with them, teachers are provided opportunities to learn to create multiple representations themselves and analyze the representations created by students. In particular, teachers are provided opportunities to see how the teacher and students in the cases used symbols and diagrams in their discussions of mathematical ideas.

Teachers' opportunities to learn in the Improving Instruction in Rational Numbers and Proportionality (IIRP) curriculum. The *Improving Instruction in Rational Numbers and Proportionality* (IIRP) curriculum (Smith, et al., 2005c) features PLTs that use narrative cases to portray how teachers and students use cognitively demanding mathematical tasks in middle school classrooms. These cases were developed by the Cases of Mathematics Instruction to Enhance Teaching (COMET) Project as it used the extensive data of the QUASAR project to create professional development materials for teachers. Being based on the same research base, the IIRP and ISBI curricula have a similar focus on the use of cognitively demanding mathematics tasks. However, the IIRP curriculum has a specific mathematical focus. It was published 5 years after the ISBI curriculum as part of a series that focused on specific mathematical topics: specifically, *rational numbers and proportionality* (Smith, et al., 2005c), algebra (Smith, et al., 2005a), and geometry and measurement (Smith, Silver, & Stein, 2005b). The IIRP curriculum provides teachers with opportunities to learn about using cognitively demanding mathematical tasks about rational numbers and proportionality specifically.

The IIRP curriculum presents teachers with four PLTs. These PLTs each center on a narrative case but include other components. They begin with an opening activity around a mathematical task. Following the opening activity, prompts are given about what to attend to when reading the narrative case. This is followed by the narrative case itself. Each of these detailed narrative cases illustrates the classroom factors that influence the cognitive demand of mathematical tasks during enactment and other pedagogical issues that impact instruction. The PLTs end with a set of prompts for teachers to engage in analysis of the case, to connect the case to their own classroom practices, and to their classroom curriculum materials. These prompts also orient teachers to the practices of collective inquiry, such as providing evidence from the cases for the claims they make about the mathematics instruction being discussed with their peers. Although there are only four PLTs in the IIRP curriculum, as I have chosen to define them, each one has these many components and asks teachers to engage in a number of varied activities.

The mathematical focus of the IIRP curriculum is squarely on the two mathematical topics of rational numbers and proportionality as shown in Figure 4.6.

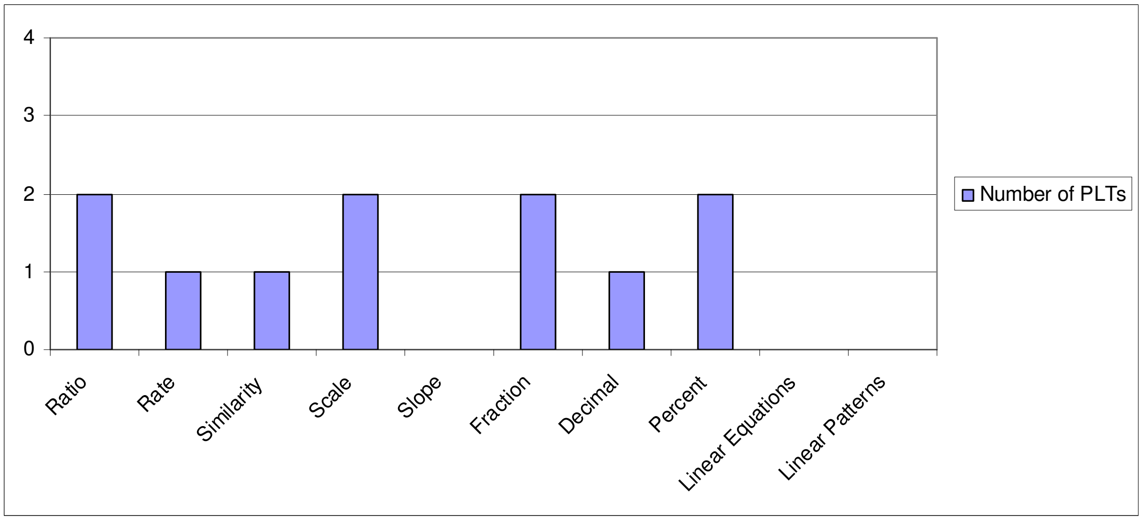


Figure 4.6. Mathematical topics presented in the IIRP curriculum.

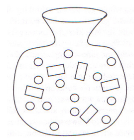
The IIRP curriculum provides opportunities for teachers to revisit the mathematical concepts underlying the topics of proportionality and rational numbers. However, the topic of linear equations and functions is not addressed.

The IIRP curriculum does provide many opportunities for teachers to develop their multiplicative reasoning as suggested by Sowder and colleagues (1998). The activities in 3 of the 4 PLTs around the mathematical tasks and narratives about their use in classroom instruction provided opportunities for teachers to "reason explicitly in terms of quantities and quantitative relationships" (p. 131), to reframe proportionality as a complex reasoning process that evolves over time (p. 137), and to make "connections among the forms of rational numbers and connections with the concepts of ratio and proportion" (p. 145).

The PLT around the case of Marie Hanson, as seen in Figure 4.7, is a good example of a PLT that provides opportunities for teachers to develop their multiplicative reasoning.

OPENING ACTIVITY

The candy jar contains Jolly Ranchers (the rectangles) and Jawbreakers (the circles). Please use this candy jar to solve Problems 1-3 and respond to the "Consider" question before reading the case.



SOLVE

1. Suppose that you have a larger candy jar with the same ratio of Jolly Ranchers to Jawbreakers, as shown in the candy jar. If the jar contains 100 Jolly Ranchers, how many Jawbreakers are there in the jar?
2. Suppose that you have an even larger candy jar with the same ratio of Jolly Ranchers to Jawbreakers as shown in the candy jar. If the jar contains 720 candies, how many of each kind of candy are there in the jar?
3. Suppose that you are making treats to hand out to trick-or-treaters on Halloween. Each treat is a small bag that contains 5 Jolly Ranchers and 13 Jawbreakers. If you have 50 Jolly Ranchers and 125 Jawbreakers how many complete small bags could you make?

CONSIDER

After you have solved Problems 1-3, try solving each problem using a different approach. What is the relationship between the two different approaches you used to solve each problem?

Figure 4.7. The mathematical task in the Case of Marie Hanson (Smith, et al., 2005c, pp. 26-27).

The mathematical task given in the opening activity provides opportunities for teachers to reason about ratios and fractions. While teachers may solve Problem 1 only using ratios, Problem 2 presents a scenario where using either ratios or fractions would be appropriate. In this problem, the total number of candies is presented and so a part-whole relationship could be used. Thus, teachers could either find an equivalent ratio to 5 Jolly Ranchers: 13 Jaw Breakers : 18 Candies or they could use fractions and determine that the number of Jolly Ranchers in a jar would be 5/18$^{\text{ths}}$ and the number of Jaw Breakers would be 13/18$^{\text{th}}$ of the total 720 candies. Recalling that ratios and fractions can be thought of as two overlapping sets (Clark, et al., 2003), problem 1 is in the realm of ratio only and problem 2 lies in the intersection of the two. This mathematical task provides an opportunity for teachers to make connections between ratios and fractions and their use in proportional tasks.

The narrative cases in the IIRP curriculum provide opportunities for teachers to reflect upon students' thinking about rational numbers and proportionality. For example, in the narrative case around the previous mathematical task, Ms. Hanson's students used both additive and multiplicative reasoning in their solutions. This student work provides an opportunity for teachers to contrast additive and multiplicative reasoning and to consider ways to help their own students shift to using multiplicative reasoning on proportional tasks.

Mathematical processes that underlie proportional reasoning are also made available for teachers to reflect upon. The important role of unitizing in solving problems involving rational numbers and ratios is highlighted in three of the four cases. Teachers using these cases are provided opportunities to reflect upon how one' perception of the whole can shape the solution strategy taken to solve mathematical tasks involving rational numbers or ratios.

The mathematical tasks used in the IIRP curriculum are cognitively demanding. Three of the four mathematical tasks are categorized as having the highest level of cognitive demand, "doing mathematics", and the fourth is at the "procedures with connections" level. The narrative cases provide examples of how such challenging tasks can be used in instruction. In the cases, many instructional factors and pedagogical issues related to using such cognitively demanding tasks are presented for teacher reflection. Issues around student struggle and the provision of student explanations arise in two or more of the cases. Figure 4.8 illustrates the extent to which the instructional factors associated with the maintenance or decline the cognitive demand of the mathematical task are provided in the curriculum.

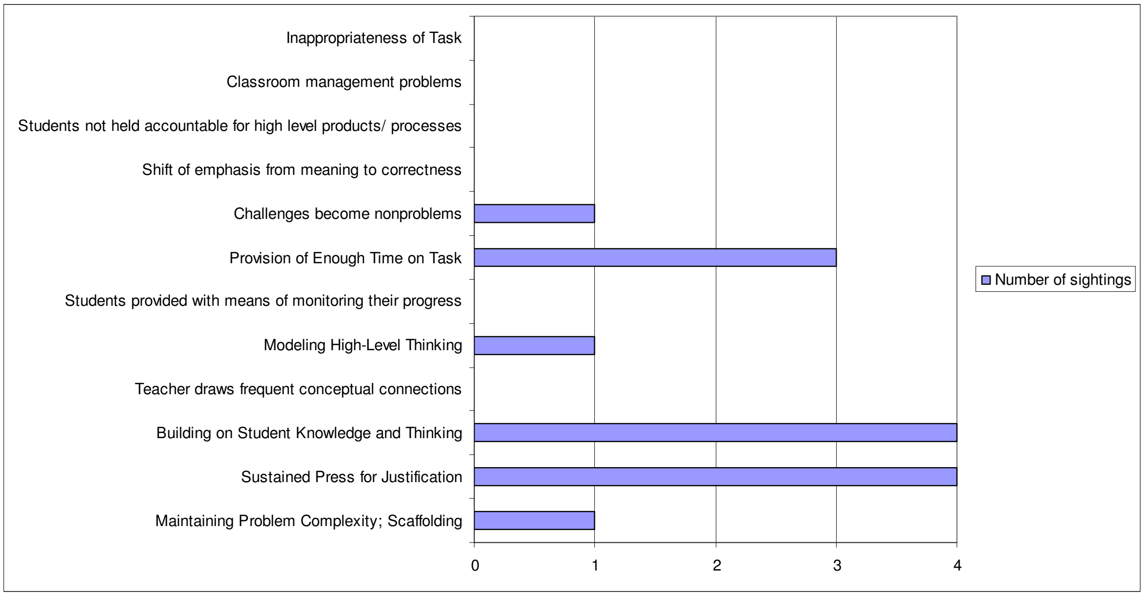


Figure 4.8. Instructional factors related to cognitive demand addressed in the IIRP curriculum.

As can be seen, the instructional moves that maintain the cognitive demand of tasks as they are enacted in classrooms are predominantly featured. In all the cases teachers both sustained a press for students to justify their thinking and built upon their thinking and prior knowledge. For example, Marie Hanson selected the task in Figure 4.6 because it built on her students' prior work with ratios. During her teaching, she consistently asked students to explain their solution strategies and to explain the ideas presented by others. At one point (pp. 33-36), she chose to start a class discussion by having a student, Jordan, share his incorrect strategy for Problem 1 that used additive instead of multiplicative reasoning and asking if students had any questions. This led to three other students explaining their different solutions that used multiplicative reasoning and the chance for the class to contrast additive and multiplicative reasoning and conclude that multiplication was the appropriate way to have quantities grow at the same rate and find equivalent ratios. Thus, the case illustrates how Ms. Hanson was able to successfully build on students' explanations of their thinking, both correct and incorrect, to help the entire class learn an important mathematical concept. In this PLT and others, the IIRP curriculum primarily provides examples of how teachers can successfully use cognitively demanding mathematical tasks with their students.

The IIRP curriculum provides teachers with opportunities to consider how diagrams, symbols, tables, and physical objects can be used in mathematical discussions. As seen in Table 5, there is heavy use of diagrams in both the opening activities and narrative cases of the PLTs in the IIRP curriculum.

Table 5

The extent to which representations of mathematical ideas are presented in the PLTs of the IIRP curriculum

Representation	Number of PLTs in which the representation is present	Percentage of PLTs in which the representation is present
Verbal Descriptions	4	100%
Symbols	3	75%
Graphs	0	0%
Tables	2	50%
Visual Diagram	4	100%
Physical Object	1	25%

Diagrams are involved in each of the four opening activities. In all four of the narrative cases the teacher in the case explicitly mentions the power of using diagrams to help students solve problems and build their understandings of mathematical ideas (p. 10; p. 29; p. 47; p. 68). After verbal descriptions and diagrams, symbols are the other prevalent form of representation of mathematical ideas used. The connections made between the multiple representations provided in the curriculum are illustrated in Figures 4.9 and 4.10.

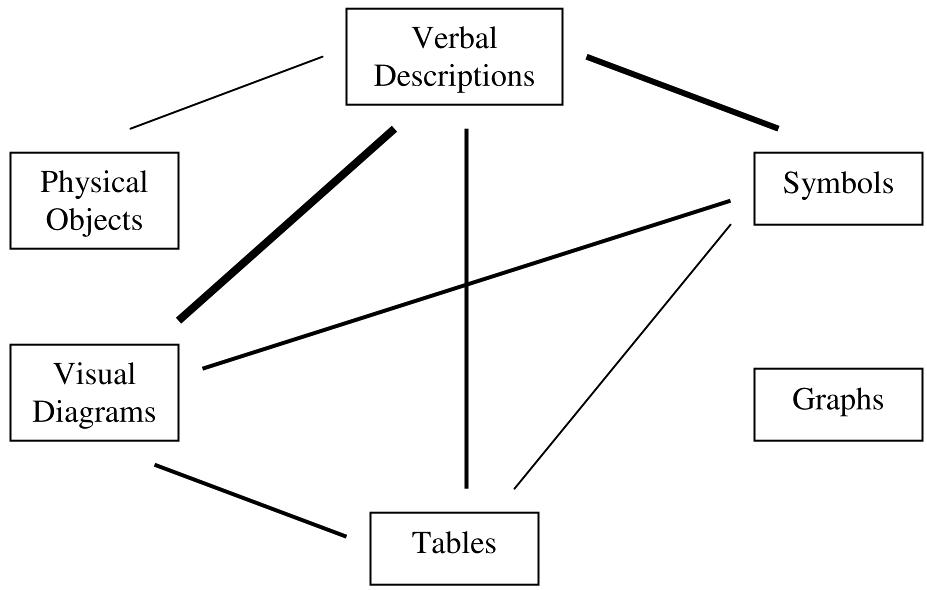


Figure 4.9. Strength of connections between representations in the IIRP curriculum.

Strong connections between visual diagrams and symbols are made during discussions described in the narrative cases of the IIRP curriculum. Connections between diagrams and tables in discussions are also frequently made, though to a lesser degree. Graphs are not used in the IIRP curriculum and physical objects are only used once.

This lack of use has consequences for the degree of connectivity between representational forms (Figure 4.10). While connections are made between verbal descriptions, tables, symbols, and diagrams, physical objects are only linked to their verbal description. As graphs are not used, teachers are not provided any opportunity to learn how to connect graphs to the other representational forms in the IIRP curriculum.

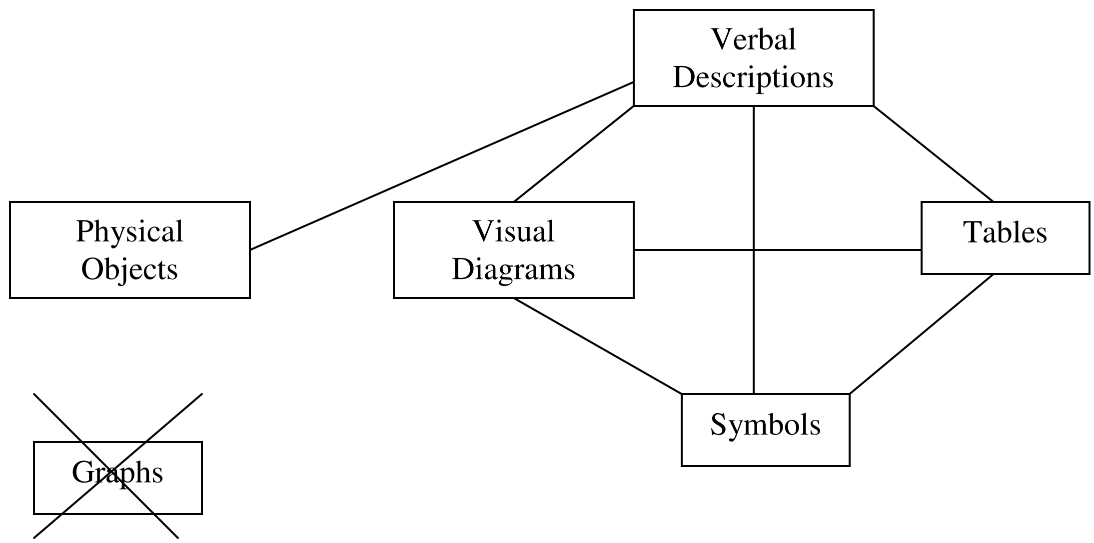


Figure 4.10. Degree of connectivity between representations in the IIRP curriculum.

The IIRP curriculum also focuses on using cognitively demanding mathematical tasks, but focuses, in particular, on tasks addressing the topics of rational numbers and proportionality. Linear equations and functions are not a topic addressed in this curriculum. Through the cases, teachers are provided opportunities to see and reflect upon how diagrams, symbols, and tables, in particular, can be used in classroom discussions of mathematical ideas. The four detailed narrative cases of the curriculum illustrate how teachers and students can interact with cognitively demanding tasks in ways that maintain the high level of thinking initially required by the task. The PLTs in the curriculum provide teachers with opportunities to learn from images of practice that depict the successful use of multiple representations in solving and using cognitively demanding tasks around rational numbers and proportionality in classroom instruction.

Teachers' opportunities to learn in the Teaching Fractions and Ratios for Understanding (TFRU) curriculum. Susan Lamon, the author of the *Teaching Fractions and Ratios for Understanding* (TFRU) curriculum (2005), has done extensive research on how students at different grade levels learn and think about fractions and ratios (1993a, 1993b, 1996, 2006). In the TFRU curriculum she shares her knowledge about these mathematical topics, how students think about them, and how they can be taught to build on students' understandings. She *unpacks* these mathematical topics for teachers (Ball & Bass, 2003) and explores their conceptual underpinnings. The TFRU curriculum is designed to push teachers beyond simple procedures and provide opportunities for them to refine and develop their conceptual knowledge of these mathematical topics while also deepening their knowledge of how students understand these ideas.

The TFRU curriculum (Lamon, 2005) has many activities for teachers. It provides 71 PLTs that include: sets of mathematical tasks for teachers to solve and explore mathematical ideas; samples of student work for teachers to analyze and investigate students' thinking about mathematics; prompts for teachers' reflection on mathematical concepts and pedagogical issues; and instructions for activities that teachers can perform in their classrooms to investigate student thinking about specific mathematical concepts. Over its pages it defines and explains mathematical content knowledge around ratios and fractions and provides many examples of mathematical tasks and student work on those tasks that illustrate those ideas. It provides teachers with 245 mathematical tasks to solve and reason through and 101 examples of students' work from grades 3 through 6 to analyze and reflect upon. The PLTs provide teachers with many opportunities to explore mathematical ideas and how students think about and develop their understandings of these ideas over time.

As the title suggests, the focus of the TFRU curriculum is on ratios and fractions. As shown in Figure 4.11, the curriculum provides opportunities for mathematics teachers to engage with mathematical topics related to proportionality and rational numbers.

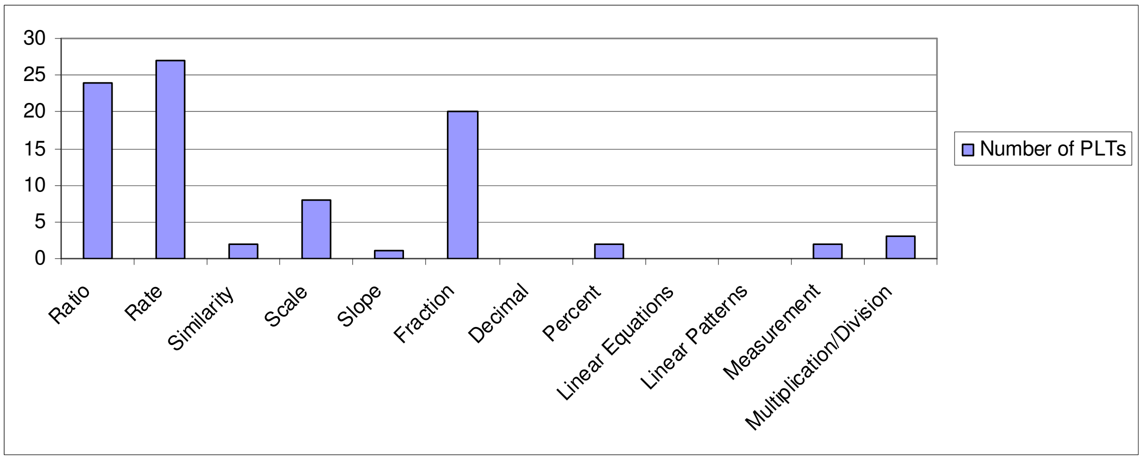
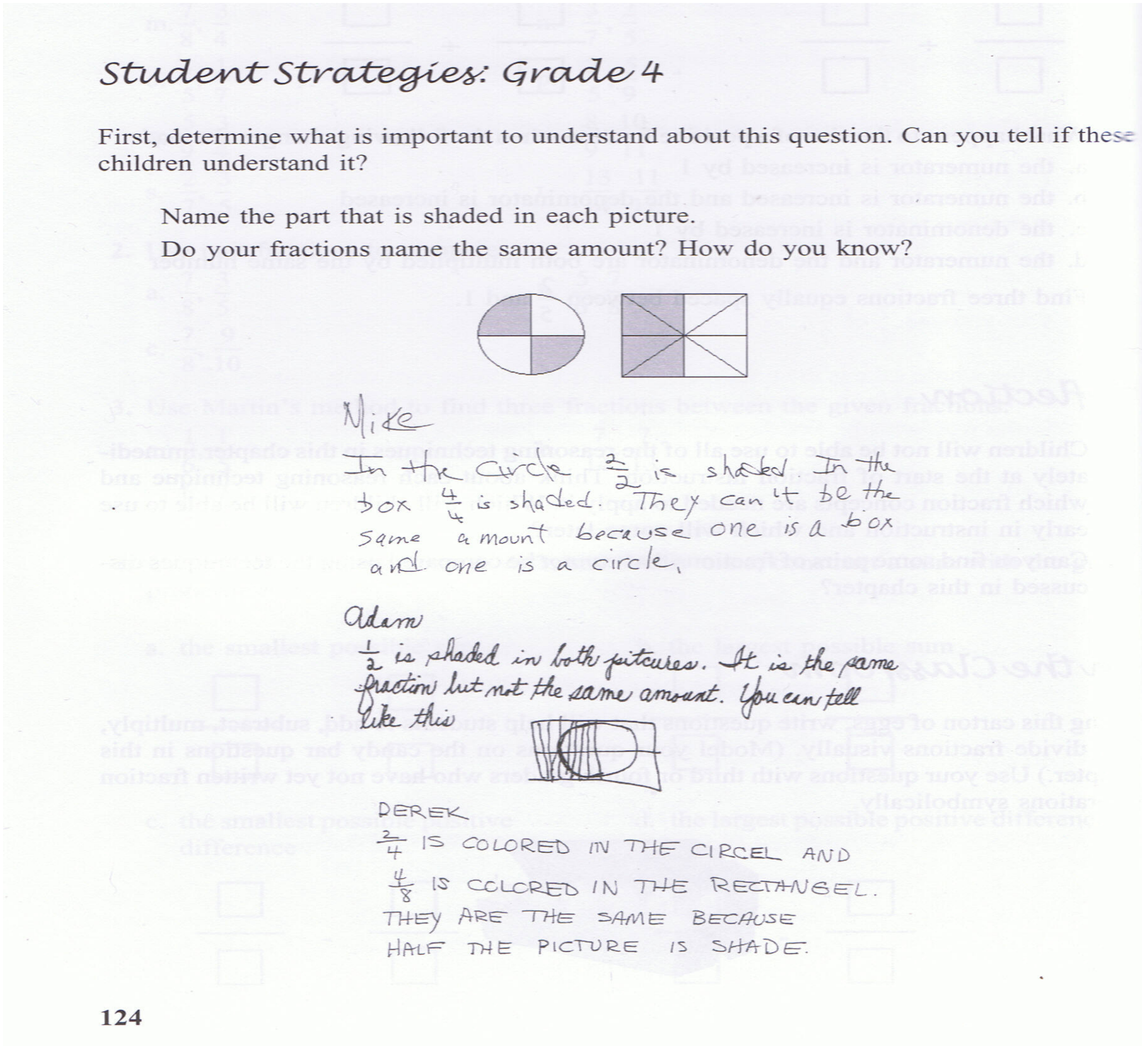


Figure 4.11. Mathematical topics presented in the TFRU curriculum.

However, the curriculum does not cover all forms of rational numbers as it focuses on fractions and does not address decimals. Proportionality (ratio, rate,…, slope) is well addressed, but linear equations and functions are not present. In addition to the mathematical topics in Figure 4.11, the TFRU curriculum provides opportunities for teachers to reflect on the role of unitizing (in 14 PLTs), the meaning of quantities (in 10 PLTs), and the differences between additive and multiplicative reasoning (in 7 PLTs). For example, the mathematical task and student work samples shown in Figure 4.12 provide teachers with an opportunity to explore part-whole comparisons with unitizing and to analyze student work to determine the extent to which students understand these mathematical ideas.



Student Strategies: Grade 4

First, determine what is important to understand about this question. Can you tell if these children understand it?

Name the part that is shaded in each picture.
Do your fractions name the same amount? How do you know?

Mike
In the Circle $\frac{2}{2}$ is shaded In the box $\frac{4}{4}$ is shaded. They can't be the same amount because one is a box and one is a circle.

Adam
$\frac{1}{2}$ is shaded in both pictures. It is the same fraction but not the same amount. You can tell like this

DEREK
$\frac{2}{4}$ IS COLORED IN THE CIRCEL AND $\frac{4}{8}$ IS COLORED IN THE RECTANGEL. THEY ARE THE SAME BECAUSE HALF THE PICTURE IS SHADE.

124

Figure 4.12. Example of a PLT addressing part-whole comparisons in the TFRU curriculum (Lamon, 2005, p. 124).

This PLT provides teachers with opportunities to reason about part-whole comparisons, the role of unitizing in defining fractions, and equivalence. In particular, Adam's diagram clearly illustrates that the mathematical task has two different wholes with different areas under consideration.

Over its 71 PLTs, the TFRU curriculum provides many opportunities for teachers to develop their multiplicative reasoning by meeting all four of the recommendations of

Sowder and colleagues (1998). The extent to which these four recommendations are addressed in the TFRU curriculum is illustrated in Figure 4.13.

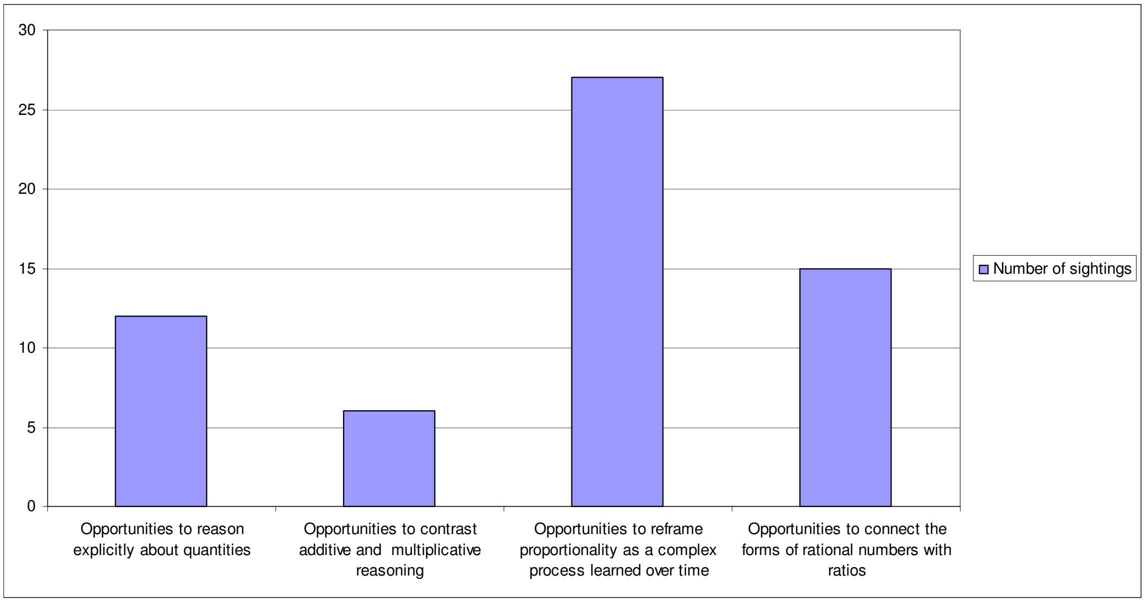


Figure 4.13. Opportunities to develop multiplicative reasoning provided in the TFRU curriculum.

The recommendation from Sowder and colleagues (1998) most prevalent is the provision of opportunities to reframe proportionality as a complex process that evolves over time. The main way that the TFRU curriculum provides these opportunities to teachers is through the provision of student work that illustrates how students across several grades reason and solve the same mathematical task. For example, in the PLT presented in Figure 4.14, student work from students in grades 3 through 6 is provided that illustrates how students' thinking and solution strategies evolve though the four grade levels.

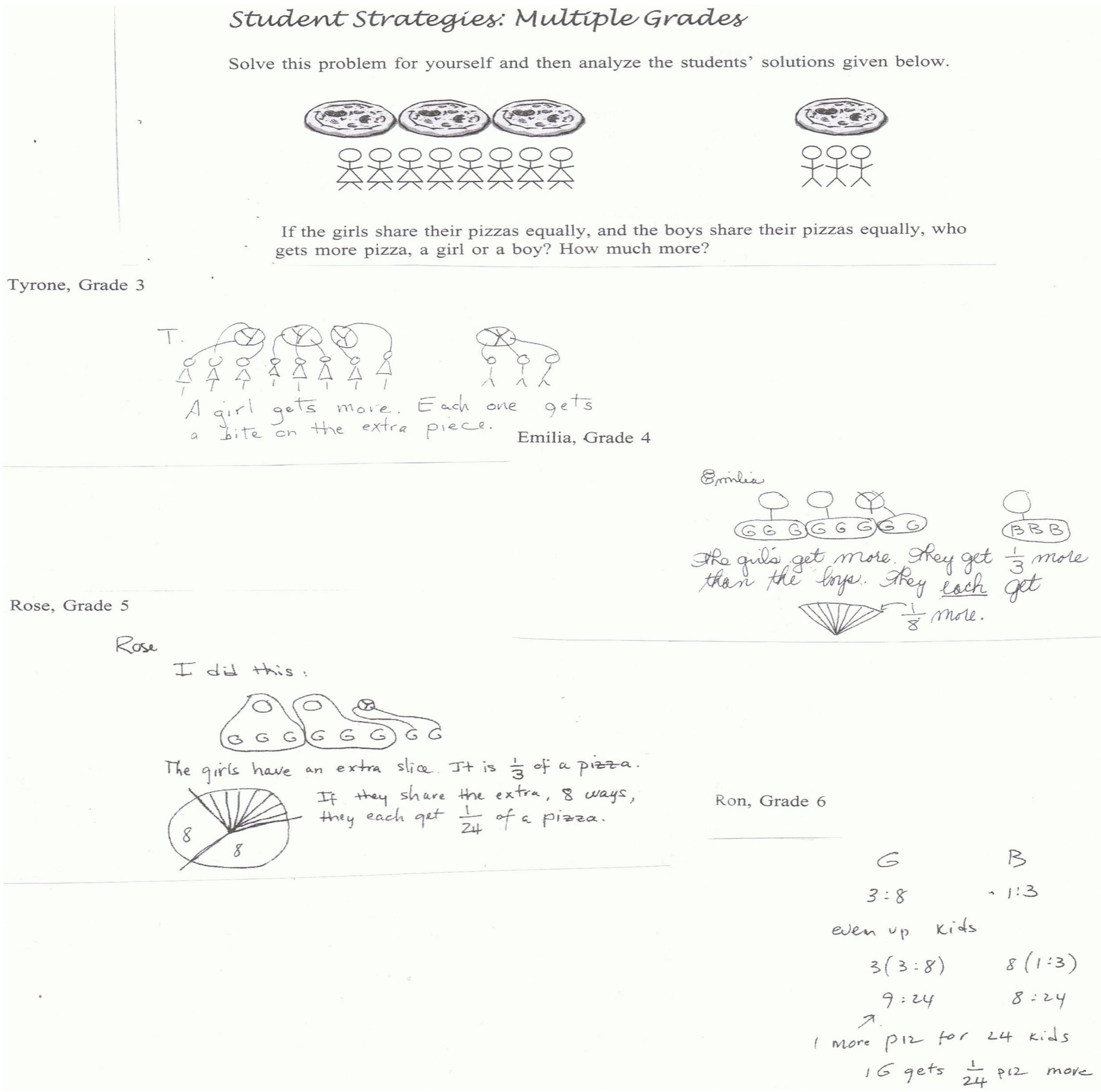


Figure 4.14. Example of a PLT using student work from various grade levels in the TFRU curriculum (Lamon, 2005, pp. 88-89).

As can be seen, students' strategies for comparing ratios shifted from visual to symbolic through the grades and their ability to answer the question of how much more pizza increased over the grades with their facility using partitioning strategies. This PLT provides teachers with an opportunity to analyze how students' understandings of proportionality and rational numbers evolve over time.

Of the 245 mathematical tasks presented in the TFRU curriculum, the majority are cognitively demanding for the students at the grade level with which they were used. The mathematical tasks were analyzed based on the grade level of the students indicated. From this analysis, 128 were categorized as “doing mathematics”, 115 as “procedures with connections” and 2 as “procedures without connections” (Stein & Smith, 1998). However, it should be noted that while many of these tasks may be cognitively demanding for the elementary students whose work is showcased, such tasks would not be as challenging for middle school students. For example, the task illustrated in Figure 4.14 could be categorized using the OTL framework as “doing mathematics” for a 3rd grader as that student has not yet learned an algorithm for solving such a task and the task would require considerable cognitive effort for a young child. For an 8th grader, however, such a task would be categorized as a “procedures without connections” task because an 8th grader would be familiar with algorithms to use for such tasks and the task does not require an explanation of thinking to be made. The mathematical tasks and student work provided in the TFRU curriculum are mainly cognitively demanding but such categorization pertains to the range of elementary and middle school grades specified.

While the TFRU curriculum (Lamon, 2005) provides many opportunities for teachers to solve cognitively demanding mathematical tasks and see how students solve such tasks, it does not provide examples of how such tasks can be used in classroom instruction. Rather, it instructs teachers to go into classrooms and use such tasks with students and interview them about their thinking as they solve the task (p. 14; p. 28; p. 38; p. 48; p. 76; p. 98; p. 111). Teachers using the curriculum are provided opportunities through these activities to learn from their own practice about how to use cognitively demanding tasks, but not opportunities to learn from the classroom practices of others. As a result of teacher instruction not being portrayed in the curriculum, teachers are not provided formal opportunities to reflect upon the classroom factors related to the use of cognitively demanding mathematical tasks in instruction.

Many representations are used in the TFRU curriculum. As can be seen from Table 6, every form of representation was used within the 71 PLTs of the curriculum.

Table 6

The extent to which representations of mathematical ideas are presented in the PLTs of the TFRU curriculum

Representation	Number of PLTs in which the representation is present	Percentage of PLTs in which the representation is present
Verbal Descriptions	70	99%
Symbols	21	30%
Graphs	2	3%
Tables	10	14%
Visual Diagram	47	66%
Physical Object	3	4%

As can be seen in Figures 4.15 and 4.16, in the TFRU curriculum connections are made between all six of the representational forms.

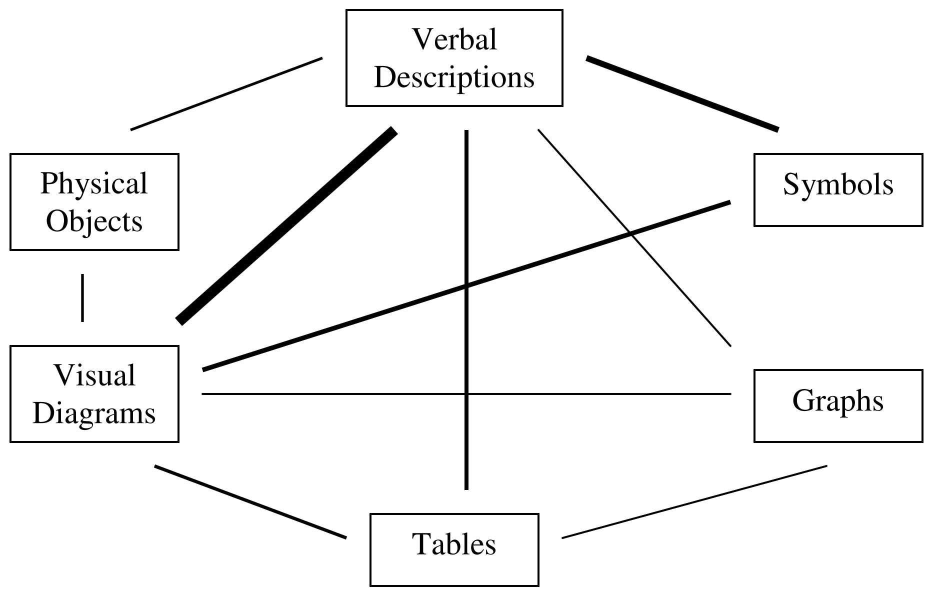


Figure 4.15. Strength of connections between representations in the TFRU curriculum.

Figure 4.15 illustrates that strong connections are made between verbal descriptions, diagrams, and symbols (in the form of fractional and ratio representations) throughout the curriculum. While connections are made with other representations, these three representations are widely used in concert by students whose work is presented for analysis to explore and communicate mathematical ideas.

Figure 4.16 illustrates that with all six forms of representations being used in the TFRU curriculum, many connections between representational forms are made and some are missing. Graphs are used twice in the curriculum and connected to tables, diagrams, and verbal descriptions. However, the connection between symbols, graphs, and tables, one important for algebra, is still missing.

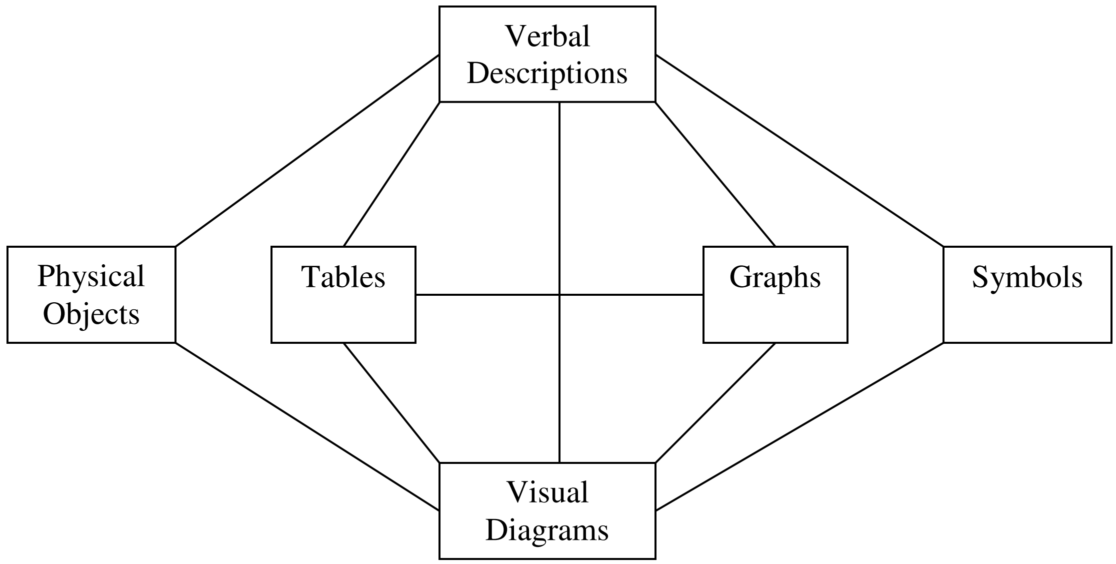


Figure 4.16. Degree of connectivity between representations in the TFRU curriculum.

Overall, through the many mathematical tasks that require the use of multiple representations, the TFRU curriculum provides opportunities for teachers to use multiple representations of mathematical ideas themselves in solving mathematical tasks. Through analysis of the student work samples, teachers also are provided opportunities to learn how students use multiple representations to solve tasks. In the two PLTs I have shared as examples in Figures 4.12 and 4.14, students used visual diagrams, verbal descriptions, and symbols to represent mathematical ideas and their solution processes. Thus, teachers are provided opportunities to both use multiple representations of mathematical ideas themselves and to analyze how students used such representations.

The TFRU curriculum provides opportunities for teachers to learn about rational numbers and proportionality by focusing on providing teachers with many opportunities to deepen their understanding of fractions and ratios conceptually and of how students come to learn these concepts. Through the PLTs that incorporate student work across many grade levels, teachers are provided opportunities to develop their multiplicative reasoning by learning how students learning and ability to communicate their understandings of fraction and ratio evolves over time and across grade levels. In the TFRU curriculum, fractions and ratios are represented using all six representational forms. Aside from creating multiple representations themselves, through numerous opportunities to analyze and collect student work, teachers are also provided opportunities to reflect upon the diverse student-generated representations that can be created. In solving mathematical tasks and analyzing student work, teachers are provided opportunities to use cognitively demanding tasks and see how they were solved by students. However, since classroom instruction was never depicted in the curriculum, teachers did not have opportunities to reflect on how such tasks can be used during instruction by another teacher.

Teachers' opportunities to learn in the Developing Mathematical Ideas: Making Meaning for Operations (DMIMMO) curriculum. The *Developing Mathematical Ideas: Making Meaning for Operations* (DMIMMO) curriculum (Schifter, et al., 1999a, 1999b) grew out of the work of staff and teachers involved in the Teaching for Big Ideas Project. In this four-year research and professional development project participating teachers wrote short narratives about the mathematical thinking of their students as part of their professional development. Later the teachers and staff of this project collaborated to organize these narratives into cases that provide opportunities for elementary and middle school teachers to reflect on mathematics teaching and learning (Education Development Center, 2005). The DMIMMO curriculum is aimed at supporting teachers to explore the teaching of number and operations to elementary and middle school students. While teachers may be comfortable and knowledgeable about using the operations, research suggests they may not be as knowledgeable about what these operations mean and how this meaning can be explained (Ball, 1990). The DMIMMO curriculum is designed to provide opportunities for teachers to revisit the operations and deepen their understanding of what they mean, how students understand them, and how they can be taught to students at different grade levels.

Due to the focus on operations, the DMIMMO curriculum is oft described as a curriculum for professional development with elementary school teachers rather than for middle school mathematics teachers (Teacher Education Materials Project, 2010). However, the survey data indicates that it is extensively used with middle school mathematics teachers. Given this use, it is important to determine what opportunities for learning it can provide to teachers at these middle grades.

The DMIMMO curriculum has 28 PLTs. Each is in the form of a brief narrative case of classroom instruction around a mathematical task. At the end of each case, teachers are asked to respond to reflection prompts about the cases. These prompts direct teachers' attention to mathematical concepts, students' thinking, and teacher's instruction for reflection and discussion.

As indicated by the title of the curriculum, the mathematical focus is on the operations of addition/subtraction and multiplication/division. As can be seen from Figure 4.17, the only other mathematical topics addressed are ratio and fractions.

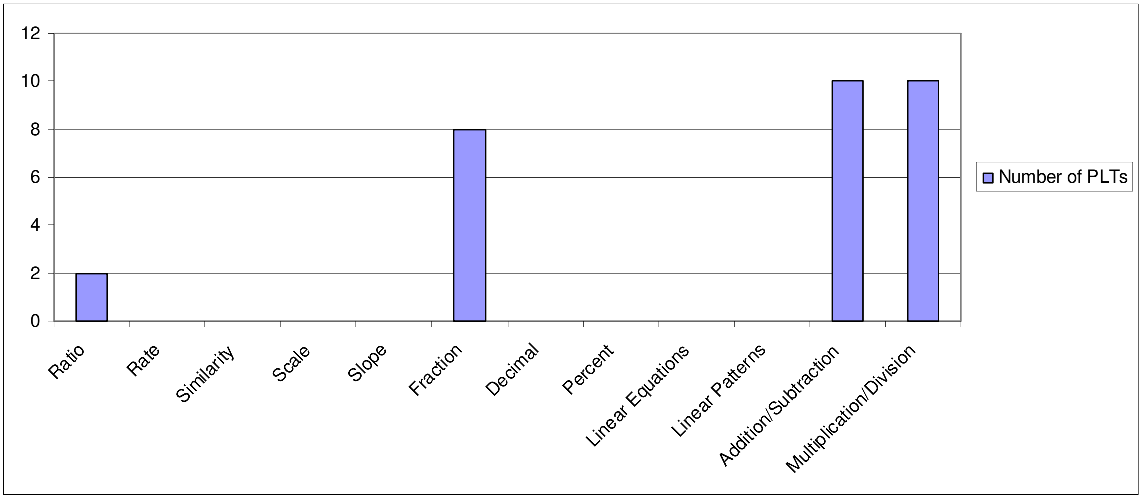


Figure 4.17. Mathematical topics presented in the DMIMMO curriculum.

Ratios and fractions are addressed to the extent that they appear in mathematical tasks related to using the operators with rational numbers and in proportional situations. For example, ratios and fractions are addressed in the mathematical task in case 27 of the curriculum: “You are giving a birthday party. From Ben and Jerry’s ice cream factory, you order 6 pints of ice cream. If you serve ¾ of a pint of ice cream to each guest, how many guests can be served?” (Schifter, et al., 1999a, p. 120). In the DMIMMO curriculum, decimals and percents, the other forms of rational numbers, are not addressed. Not surprisingly, given the focus on elementary mathematics, linear equations and functions are also not addressed.

Given the focus on operations, there are few opportunities provided for teachers to develop their multiplicative reasoning. In only two PLTs is additive and multiplicative reasoning contrasted and in only one are connections made between ratios and fraction. None of the other recommendations of Sowder and colleagues (1998) on developing teachers’ multiplicative reasoning are addressed.

The mathematical tasks presented in the DMIMMO curriculum are cognitively demanding for the elementary school students they are depicted being used with. All of the mathematical tasks were categorized at either the "doing mathematics" or "procedures with connections level". However, these mathematical tasks would, by and large, not be cognitively demanding for more advanced middle school students as they address the operations that by the middle grades would be procedures with which students should be fluent.

Since the DMIMMO curriculum is designed for K-6 use, it provides examples of how elementary and middle school teachers use cognitively demanding tasks with their students. Thus, though the mathematical tasks would not all be cognitively demanding for middle school students, the curriculum still provides examples of how a teacher could use challenging tasks with students – lower grade students though they be. The cases in the curriculum depict how teachers can make instructional decisions that maintain the level of cognitive demand of the mathematical tasks they use with students (see Figure 4.18).

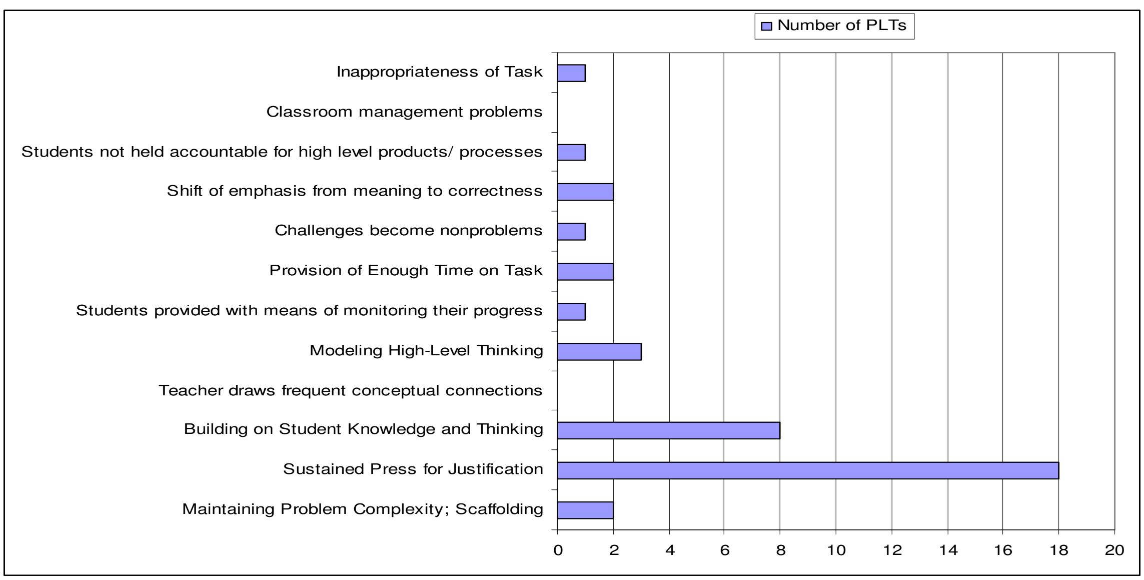


Figure 4.18. Instructional factors related to cognitive demand addressed in the DMIMMO curriculum.

The DMIMMO curriculum provides opportunities for teachers to see examples of teachers who ask students for explanations, use such explanations to gain insight into students' thinking, and then use such insight to build on students' understandings. For example in case 20, the teacher, Maryann, pressed students to explain their thinking as they solved a problem about sharing brownies among people at a party (p. 89; p. 90; p. 93). From this she was able to determine that her students were able to use strategies to find the fraction of brownie that each person would get but were not yet able to relate fractions with differing denominators. Her students arrived at the correct solution to the problem using partitioning and counting. Their solutions ranged in representation including 5/8, 2/4 and 1/8, ¼ + ¼ + 1/8 and ½ and 1/8. In reflecting on her students thinking, MaryAnn states "up ahead for my students is the work of sorting out the ideas behind adding fractions with unlike denominators. I'll be watching how they think this through, building on the ideas they already have in place" (p. 93). This narrative case provides an example to teachers of how a teacher can use information about students' understandings gained from pressing for explanations to help them determine both what students have learned and what they need to learn next to build on their current understandings.

The DMIMMO curriculum, with its mathematical focus, primarily uses verbal descriptions, visual diagrams, and symbols (number sentences) to describe operations on values in its PLTs. As can be seen in Table 7, some physical objects were used to represent the values being added or multiplied and one table was employed by a student to build a number sentence of multiplication.

Table 7

The extent to which representations of mathematical ideas are presented in the PLTs of the DMIMMO curriculum

Representation	Number of PLTs in which the representation is present	Percentage of PLTs in which the representation is present
Verbal Descriptions	28	100%
Symbols	21	75%
Graphs	0	0%
Tables	1	4%
Visual Diagram	19	68%
Physical Object	8	29%

The ways that these representations are used in combination and the connections made possible are depicted in Figure 4.19 and 4.20.

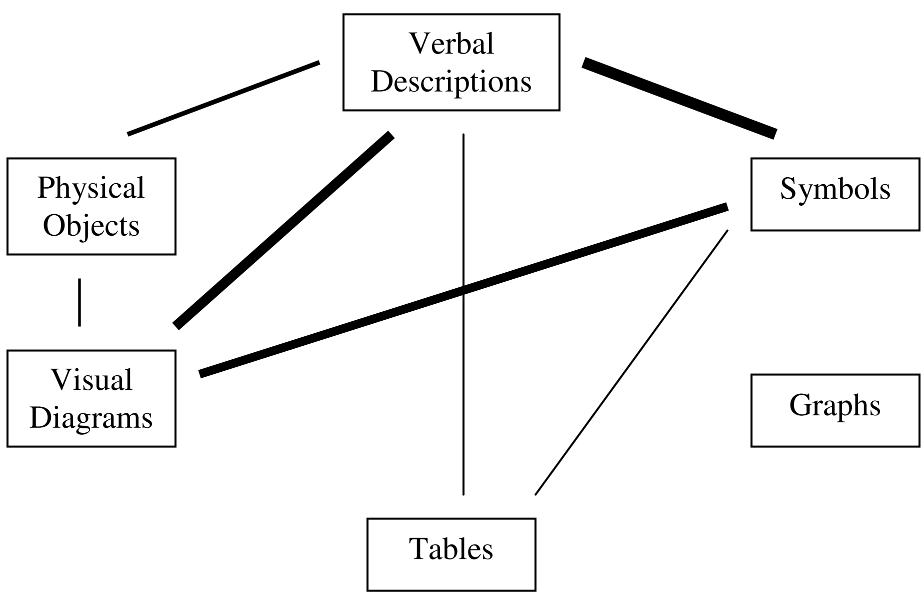


Figure 4.19. Strength of connections between representations in the DMIMMO curriculum.

The representations used most frequently in combination were symbols, diagrams, and verbal descriptions. The students and teachers in the narrative cases of the PLTs used diagrams in their discussions to create symbolic representations of operations being performed (number sentences) in the mathematical tasks.

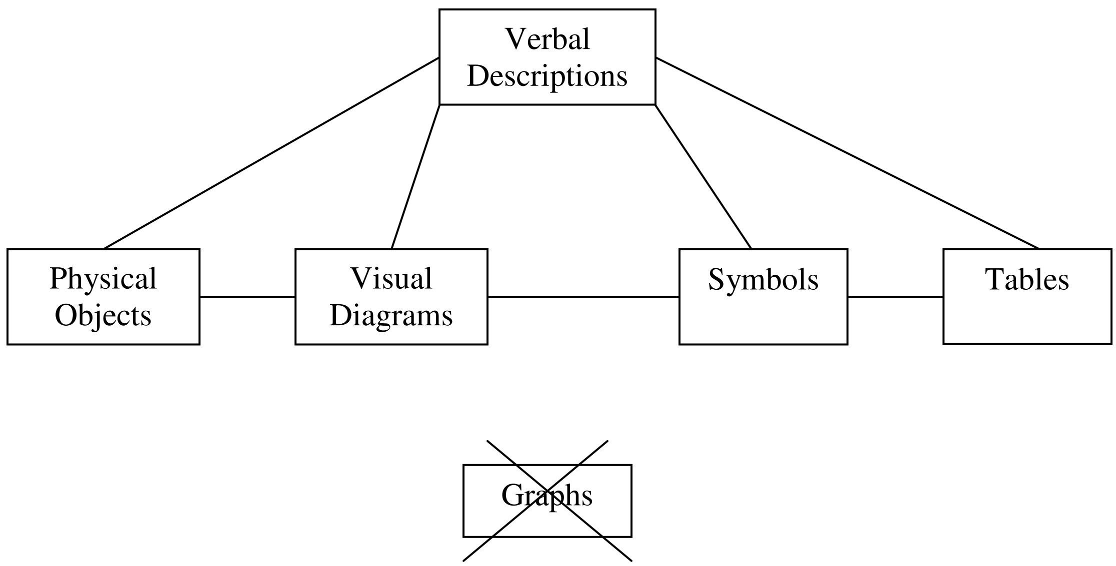


Figure 4.20. Degree of connectivity between representations in the DMIMMO curriculum.

As can be seen in Figure 4.20, connections are made between verbal descriptions, physical objects and visual diagrams, but not between physical objects and symbols. The students and teachers depicted in the PLTs make connections between visual diagrams of physical objects being added or multiplied to the symbols (number sentences) describing these operations, but not to the actual objects themselves. Graphs are not present in this curriculum nor connected to any other representational form. Teachers using this curriculum would be provided with opportunities to use and see some of the representations used in concert, but not all of the representations of mathematical ideas available.

The DMIMMO curriculum has a firm focus on the development of meaning for the operations of addition, subtraction, multiplication, and division. The mathematical tasks depicted in the narrative cases, while not cognitively demanding for middle school students, are challenging for the elementary students with which they are used. Thus, teachers are provided with opportunities to reflect on how the teachers in the cases use cognitively demanding tasks in their classroom instruction and are more or less successful at maintaining the level of cognitive demand of the tasks and students' opportunities for learning. Though graphs are not presented and tables used only once, the curriculum does provide teachers with many opportunities to learn about using multiple representations, mainly symbols and diagrams, in mathematical discussions to explore and communicate mathematical ideas.

Teachers' opportunities to learn across the four curricula. The four professional development curriculum materials in the sample are widely used with a large number of teachers across the United States of America. These curricula have different foci but all were chosen for use in professional development programs because they provide opportunities for middle grade mathematics teachers to learn mathematical content and pedagogy. In this section I will share what can be learned about teachers' opportunities to learn generally across the sample of curriculum materials. Such analysis is warranted as many projects use a combination of these curricula over time and it would be important to learn what teachers can learn from such a combination and what is still overlooked.

Over the four curricula, there are 110 identified professional learning tasks. These PLTs expose teachers to different aspects of mathematics teaching and learning and provide different learning opportunities. Summaries of the identified learning opportunities provided in the ISBI, IIRP, TFRU, and DMIMMO curricula are presented in Appendices F, G, H, and I respectively. Data on the different types of PLTs presented across the four curricula are presented in Table 8.

Table 8

The Different Types of Professional Learning Tasks available across the Four Curricula

Type of Professional Learning Task (PLT)	Number of PLTs
Sets of mathematical tasks	15
Samples of student work on mathematical tasks	23
Narrative cases of classroom instruction around mathematical tasks	38
Future classroom activities where teachers will engage with students around mathematical tasks	17
General reflection on mathematical and pedagogical ideas	17

Mathematical tasks are key components of all of the types of PLTs with the exception of the 17 PLTs that involve general reflection. In the 93 PLTs that feature mathematical tasks, 284 individual mathematical tasks are provided. The provision of so many mathematical tasks provides opportunities for teachers to individually revisit and grapple with the mathematical content they teach. Through the narrative cases and student work samples, they also are provided opportunities to compare and reflect upon how other learners understand and use these ideas. The extent to which different mathematical ideas are presented for teachers to engage with across the four curricula is presented in Figure 4.21.

The topics of ratio and fraction are most addressed across the sample. Proportionality encompasses ratio, rates, similarity, scale, and slope. This mathematical topic of middle school mathematics content is addressed in 72 PLTs – that is, in 65% of the PLTs across the four curricula.

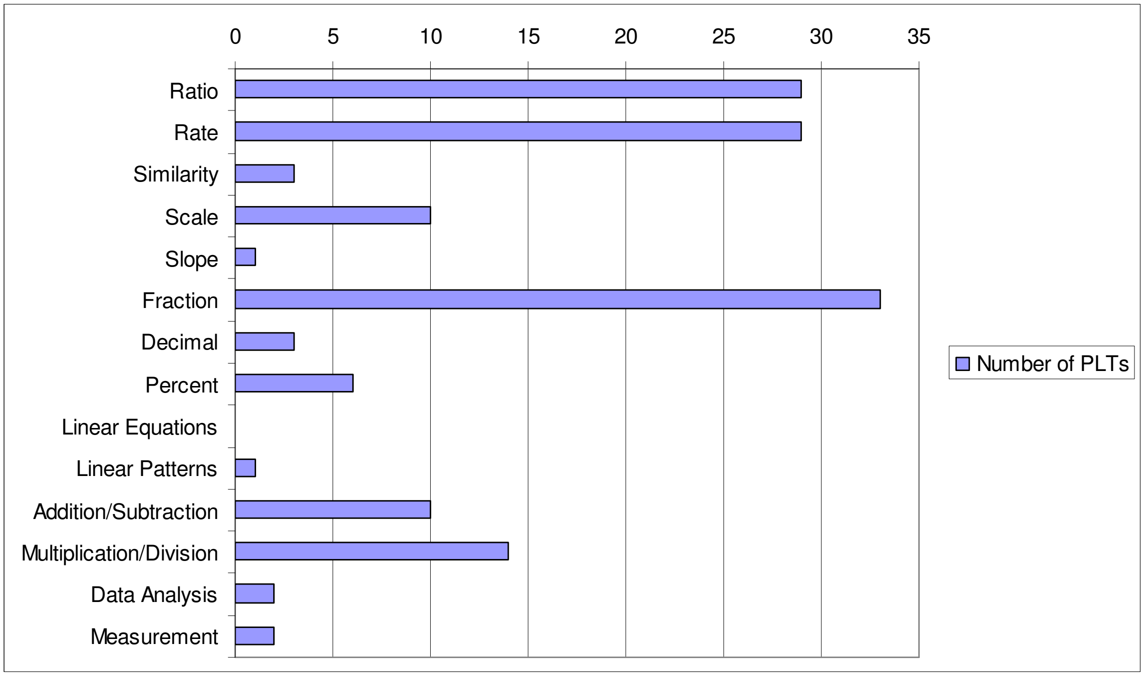


Figure 4.21. Mathematical topics presented across the sample curricula.

The second most frequently used topic, fractions, can be categorized, along with decimals and percents, under rational numbers. Rational numbers are addressed in 38% (42 out of 110) of the PLTs across the four curricula. The third topic of interest in RQ1, linear equations, is only addressed once across the four curricula. Teachers are provided with many opportunities to learn more about the operations which are foundational for proportionality and proportionality itself, yet they are provided with scarce opportunities to learn what proportionality forms a foundation for: linear equations and functions.

Many representations are used in each of the four curricula and opportunities to learn about how multiple representations can be used in classroom instruction are provided. When looking across the entire sample, it is apparent, as seen in Figures 4.22 and 4.23, which representations are prevalent and which connections between representations are made more often than others.

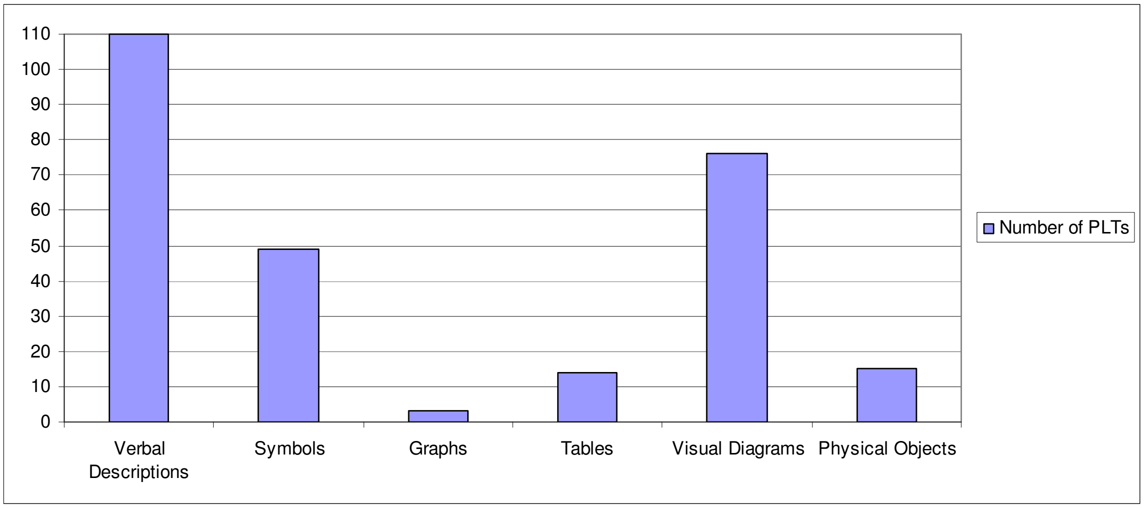


Figure 4.22. Representations of mathematical ideas presented across the four curricula.

Verbal descriptions are present in every PLT as we use words as our primary mode of communication. Aside from verbal descriptions, visual diagrams and symbols are the prevalent representational forms used across the four curricula. Graphs are only used in 3% (3 out of 110) of the PLTs. This scarce use has implications for the connections made between representations, illustrated in Figure 4.23.

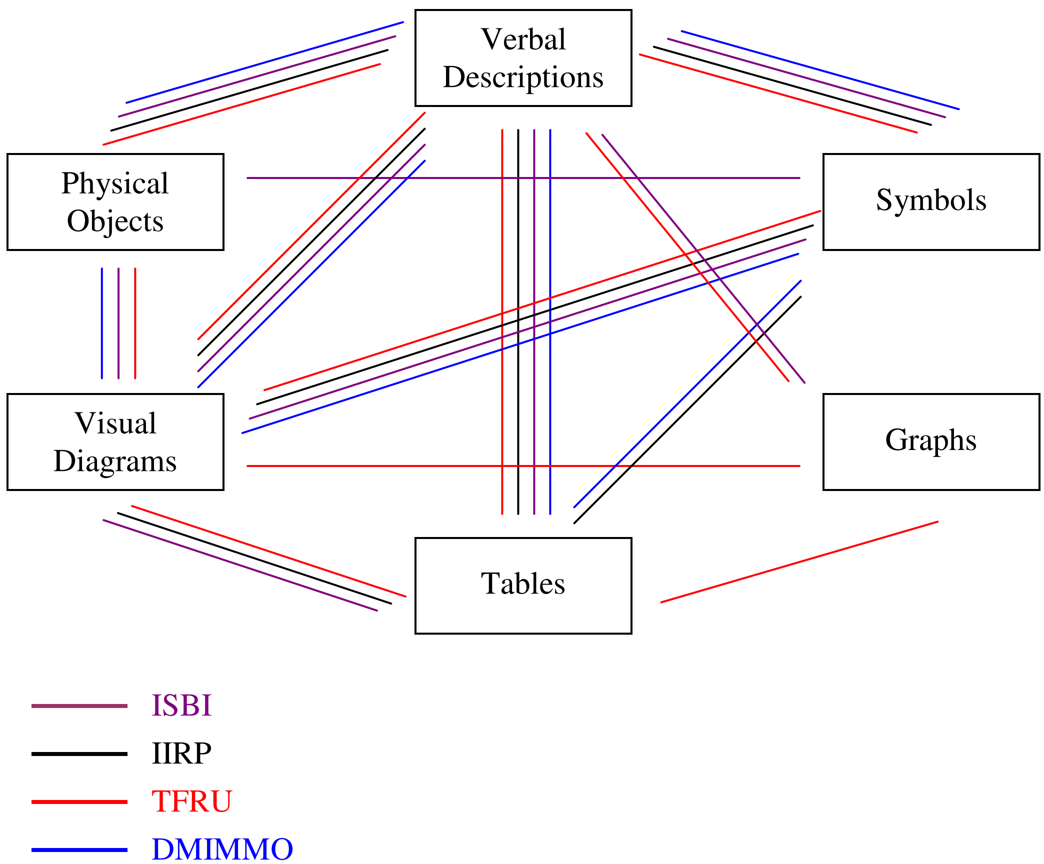


Figure 4.23. Connections made between representations in the four curricula.

As can be seen in Figure 4.23, the four curricula provide opportunities to learn to connect multiple representations of mathematical ideas. Within the PLTs, when solving mathematical tasks, teachers are provided with opportunities to use combinations of representations themselves. When reflecting on the narrative cases, teachers are provided with opportunities to analyze how students and other teachers use multiple representations to explore mathematical concepts. However, across the sample, due to the scarcity of graphs, teachers are provided with limited opportunities to connect graphs to visual diagrams and tables and no opportunity to connect graphical representations of mathematical ideas to symbolic ones.

The vast majority of mathematical tasks presented in the four curricula, 99% (280 out of 284), were cognitively demanding for the students for which their use was designated – whether elementary or middle school students. Teachers are provided extensive opportunities to use such cognitively demanding tasks themselves. The PLTs which used narrative cases, 38 PLTs across the four curricula, also provide teachers with examples of how such cognitively demanding tasks can be used by a teacher in his/her classroom instruction. These PLTs containing narrative cases showcase the classroom based factors that are associated with the maintenance or decline of the cognitive demand of mathematical task. Figure 4.24 illustrates the extent to which these factors are addressed in the 38 PLTs containing narrative cases provided across the four curricula.

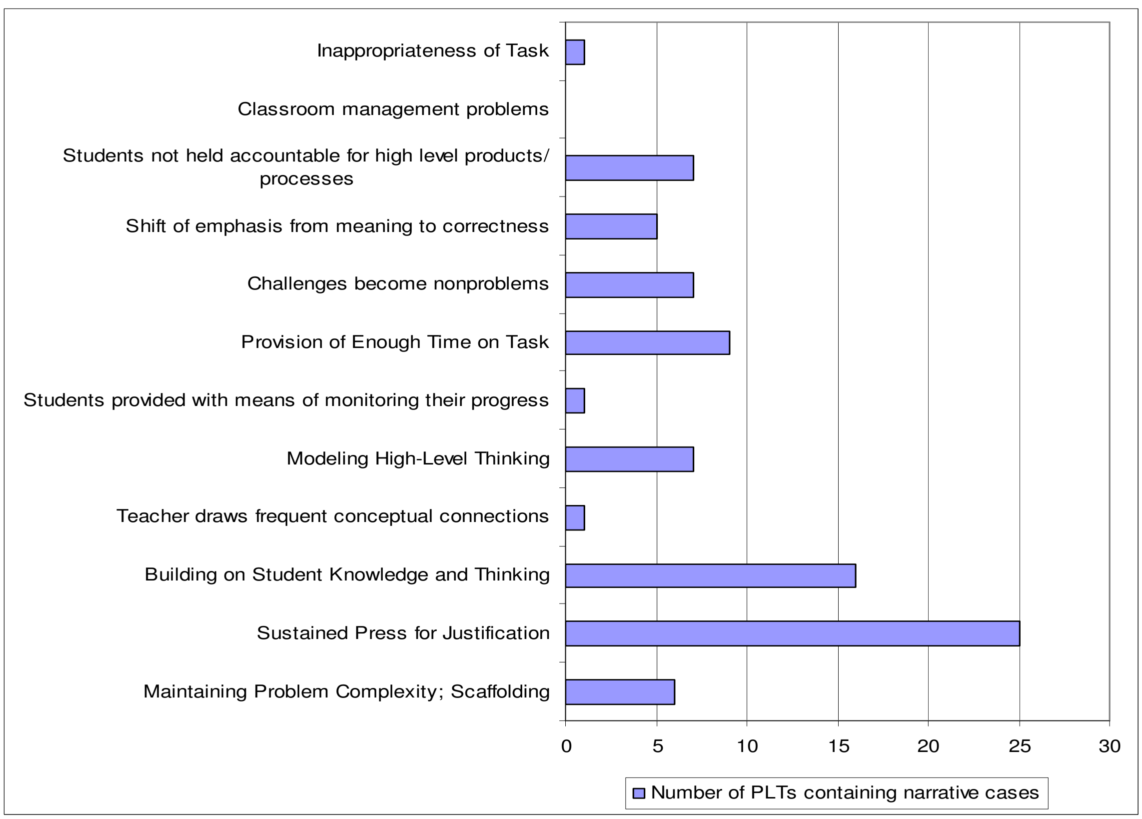


Figure 4.24. Instructional factors related to cognitive demand addressed across the four curricula.

The four curricula provide numerous examples of how teachers can sustain pressure for justification and build on students' knowledge and thinking. Examples are also provided of teachers engaging in practices that are associated with undermining the cognitive demand of mathematical tasks, such as not holding students accountable. Few examples of how to draw conceptual connections or provide students with the means for self-regulation are provided. Though classroom management is a big concern for novice teachers, it is not focused upon in any of the curricula in the sample. The curricula provide opportunities to reflect upon classroom factors that both maintain and undermine the cognitive demand of mathematical tasks.

Overall, the combination of PLTs across the four curricula provide opportunities for middle grade mathematics teachers to learn about proportionality and rational numbers, to learn to use multiple representations, and to learn about the use of cognitively demanding tasks in instruction. However, there are few opportunities to learn about linear equations and functions and to learn to use graphs with other representational forms. The implications of these findings will be discussed in Chapter 5.

Educative Features of the Four Curricula: Opportunities for Professional Developers' Learning

Individuals who conduct professional development with mathematics teachers in this country vary greatly in their backgrounds and level of preparation for the work of facilitating teacher learning (Banilower, et al., 2006; Zaslavsky & Leikin, 2004). While many may need additional preparation, there are few learning opportunities available to obtain it (Ball & Cohen, 1999). Therefore, it is especially important to have educative professional development curriculum materials available that are designed to support the learning of both professional developers and teachers in professional development settings.

There are 20 educative features or items of support for professional developers are listed on the EF Framework under 6 categories of support (Appendix C). My analyses identified instances in the curriculum where these features were present. In the following sections I will share the results of my analyses of the sample of four curricula to see the extent to which and the ways in which they appear to be educative for professional developers. I will begin by describing the results from my analysis of each of the curricula and then describe trends found when analysis was conducted across the four curricula.

Educative features in the Implementing Standards-Based Mathematics Instruction (ISBI) curriculum. The ISBI curriculum (Stein, et al., 2000) has guidance directed towards professional developers embedded throughout the book. Many sections of text, such as the "Learning from Cases" section (pp. 33-38), address both teachers and professional developers. There are some sections of text that are more relevant to professional developers but they were also written for this dual audience. Given the embedded nature of the guidance for professional developers, I analyzed more than half of the book looking for the presence of educative features – some 79 pages of text.

Over those 79 pages of text, I found instances of educative features 249 times. This means that there is an average of 3.15 sightings of educative features per page. These span 15 of the 20 specific items of support listed in the EF Framework. Though the educative features present span 75% (15 out of 20) of the items of support, some categories and items are present in greater frequency than others, as can be seen in Figure 4.25. Four categories of support are most prevalent. These top four categories are:

- ❖ Category 6: Supporting professional developers to provide long-term, ongoing, and coherent professional development programs in various contexts
- ❖ Category 5: Supporting professional developers in building a professional learning community with teachers
- ❖ Category 2: Supporting professional developers in engaging teachers with specific pedagogy
- ❖ Category 4: Supporting professional developers in anticipating and working with teachers' ideas about mathematics teaching and learning

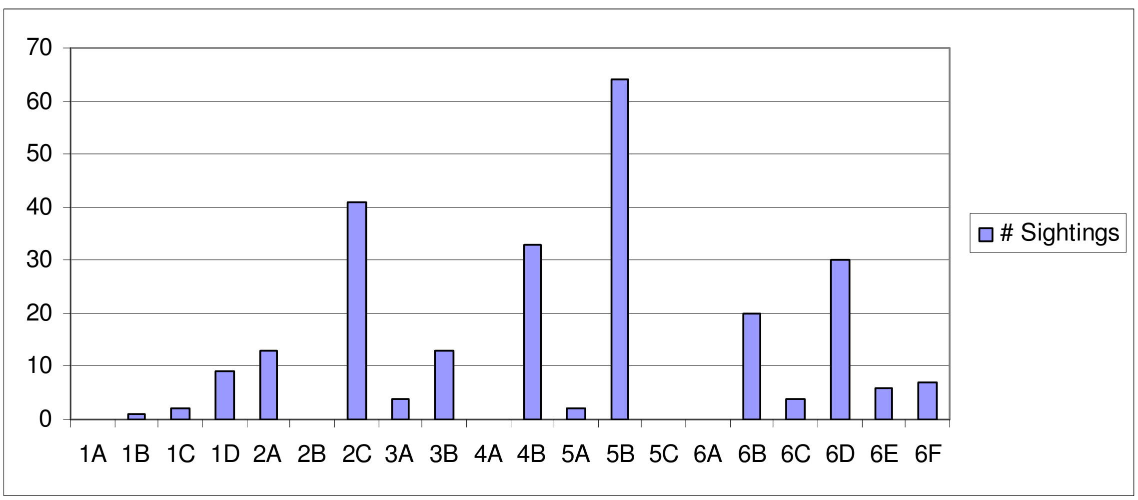


Figure 4.25. The frequency of educative features in the ISBI curriculum.

As can be seen from Figure 4.25, within these four categories there are four items of support that occurred most frequently. Namely:

- ➢ Item 5B: Provide guidance on how to facilitate and create a common discourse for teachers to engage in collective inquiry about mathematics teaching and learning

- Item 2C: Provide guidance on strategies to use to focus attention on specific aspects of classroom pedagogy.
- Item 4B: Provide guidance on when to expect certain common teacher ideas to emerge.
- Item 6D: Explain the design and the teacher learning goals (both the learning of mathematical content and pedagogy) of individual PLTs

Guidance on facilitating group discourse (item 5B) is provided in 64 paragraphs of text within the ISBI curriculum. The authors provide detailed guidance on how to facilitate discussions around the mathematical and pedagogical ideas addressed in the activities for teachers. For example the authors write:

> It is easy to get side-tracked with discussions about how a specific group of students would solve a particular task or to become overly concerned about achieving complete consensus on every task. (This has happened to us more than once!) The goal is not to achieve complete agreement but rather to provide teachers with a shared language for discussing tasks and their characteristics and to raise the level of discussion among teachers toward a deeper analysis of the relationship between the tasks they select or create and the level of cognitive engagement that will be required of students. It is important to remind participants to consider the purpose of the task-sorting activity more generally – to begin to consider how and why tasks differ and how these differences can impact opportunities for student learning. (p. 20)

In this example the authors provide guidance on how to facilitate discussion around the task-sorting activity. By sharing the goal of the discussion and suggestions for keeping participants focused on the purpose of the activity generally, the ISBI curriculum provides professional developers with a basis for making decisions on what to focus on during their facilitation of discussion around this PLT. In the curriculum, guidance provided by the authors is both specific to particular activities, such as the task-sorting activity, and general guidance that places the facilitation of discussion with in the larger context of teacher learning, such as the role of creating a shared language that can be used to facilitate collective inquiry.

Guidance on focusing teachers' attention on specific pedagogy (item 2C) is present in 41 paragraphs of text. One main contribution of the ISBI curriculum is that it draws attention to instructional factors that either inhibit or maintain the level of cognitive demand of mathematical tasks. Thus, most examples of this type of guidance explicate how the actions of the teacher in the narrative case relate to the maintenance of cognitive demand. For each case, the authors discuss factors that appear to be responsible for the task's decline or maintenance of cognitive demand and suggest that "these are the ideas toward which the discussion should be steered" (p. 58; 78; 93; 106; 119; 129). For example, the authors note in the Case of Fran Gorman and Kevin Cooper that:

> Students are not held accountable for high-level products or processes. Fran did not hold her students (or herself) accountable to two of the big ideas that she set out to engage with: the unit whole and the idea that multiplication does not always mean getting bigger. These ideas appeared to get lost in all of the step-by-step attention to details. (p. 79)

Not holding students accountable for high-level products or processes is one of the instructional factors linked to the decline of the level of cognitive demand of mathematical tasks during instruction and hence a lowering of students' opportunity to learn from the task. In the paragraphs in which guidance on focusing on specific pedagogy (item 2C) is present professional developers are provided with guidance on where to focus teachers' attention on such instructional factors.

It is useful for professional developers to have some idea of how teachers may respond the activities in a curriculum. Such anticipation of teachers' ideas helps in the planning process and in future facilitation. Guidance to professional developers on when to expect certain common teacher ideas to emerge (item 4B) is found in 33 paragraphs of the ISBI curriculum. It ranges from examples of common strategies that might be used to solve mathematical tasks in the cases (p. 63) to suggestions teachers might have about pedagogical decisions. For example, in a discussion of the case of Ron Castleman and his teaching of procedural algorithms before moving on conceptually based work the authors state:

> Participants may suggest that perhaps he should try the reverse order, beginning instead with the visually based work (indeed, this is the suggestion of the NCTM). By doing so, he will first be laying a conceptual foundation, without the interference of algorithms. After the students appear to have a good grasp of the meaning of percents, decimals, and fractions, they could be provided with the conventional conversion algorithms, with special attention to places where connections to underlying concepts can be made. (p. 60)

Here, the authors both cue the professional developer to a common teacher suggestion and provide an explanation for the basis of said suggestion.

Explanations of the design and learning goals of each PLT (item 6D) appears in 30 paragraphs throughout the ISBI curriculum to explain the design and learning goals of its 7 PLTs. The first PLT in the curriculum is a task-sorting activity which is introduced by the authors on page 18:

> One way that we have found to help teachers learn to differentiate levels of cognitive demand is through the use of a task-sorting activity. The long-term goal of this activity is to raise teachers' awareness of how mathematical tasks differ with respect to their levels of cognitive demand, thereby allowing them to better match tasks to goals for student learning. A task-sorting activity can also enhance teachers' ability to thoughtfully analyze the cases (which appear in Part II of this book), and ultimately, to become more analytic and reflective about the role of tasks in instruction.

The other six PLTs in the curriculum are narrative cases. Before going into specifics about each case, the authors first explain some learning goals for cases generally. They describe the mathematical topics being addressed as both "mainstays" of the middle grade mathematics curriculum and less frequently taught subjects. Aside from spanning a range of mathematical topics, the authors share that the cases also address a range of pedagogical issues beyond the featured topics of task implementation and cognitive demand. As an example, they share that the case of Fran Gorman and Kevin Cooper addresses how teachers collaborate and confer in team-planning. The authors provide a table of the issues embedded in the six cases of the book on p. 42. These examples illustrate the ways that explanations of PLT design appear in the curriculum and provide opportunities for professional developers to learn more about the activities they will use with mathematics teachers.

Though the last item discussed provides a rationale to professional developers about the design of PLTs, overall, the majority of support given in the ISBI curriculum 70% (175 out of 249) is in the form of implementation guidance. Guidance on how to facilitate common discourse, focus attention on specific pedagogy, and anticipate teachers' responses to the PLTs is provided to professional developers in the ISBI curriculum. This emphasis on implementation guidance is not surprising as it has also been the case in K-12 educative curriculum materials that have been researched (Beyer, et al., 2009).

Educative features in the Improving Instruction in Rational Numbers and Proportionality (IIRP) curriculum. Aside from an initial section about learning from cases that was written for both teachers and professional developers, the authors of the IIRP curriculum (Smith, et al., 2005c) wrote separate sections specifically for professional developers. These sections are about facilitating learning from each of the curriculum's cases and provided appendices presenting samples of teacher responses to the mathematical tasks within the cases. The initial section and these separate sections cover 65 pages of text.

In my analysis of these pages I had 340 sightings of educative features – some 5.23 sightings per page. The identified educative features cover 90% (18 out of 20) of the items listed in the EF Framework. Though a wide range of items were found to be present, certain categories and items are more heavily present as can be seen in Figure 4.26.

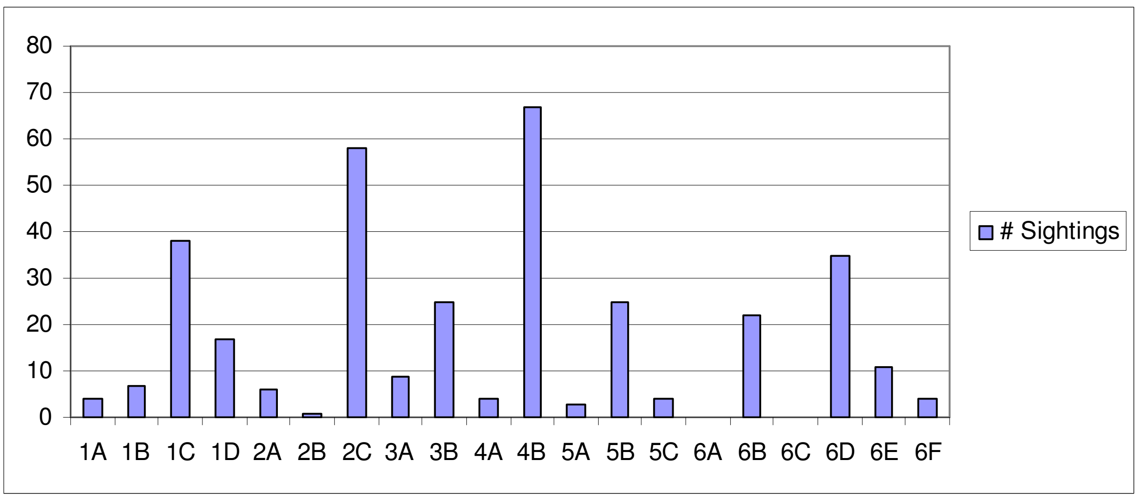


Figure 4.26. The frequency of educative features in the IIRP curriculum.

The top four categories of support present are:

- ❖ Category 6: Supporting professional developers to provide long-term, ongoing, and coherent professional development programs in various contexts
- ❖ Category 4: Supporting professional developers in anticipating and working with teachers' ideas about mathematics teaching and learning
- ❖ Category 1: Supporting professional developers in engaging teachers with specific mathematical topics

- ❖ Category 2: Supporting professional developers in engaging teachers with specific pedagogy

Within those categories, four items of support are prevalent:

- ➢ Item 4B: Provide guidance on when to expect certain common teacher ideas to emerge.
- ➢ Item 2C: Provide guidance on strategies to use to focus attention on specific aspects of classroom pedagogy.
- ➢ Item 1C: Provide guidance on which strategies to use to focus attention on the conceptual aspects of the mathematical topic(s)
- ➢ Item 6D: Explain the design and the teacher learning goals (both the learning of mathematical content and pedagogy) of individual PLTs

Since the authors provide appendices full of teachers' most common solutions to the mathematical tasks and reflection prompts in the PLTs, it makes sense that the provision of guidance on when to expect common teacher ideas (item 4B) is most prevalent in the IIRP curriculum. In addition to copious examples of teachers' work on the PLTs, the authors also provide guidance on where common misconceptions may occur. For example, in the section, "Facilitating Learning from The Case of Randy Harris", the authors warn professional developers to:

> Watch for a common misconception – treating the grids as if they were 10 x 10. If participants produce incorrect solutions that suggest this misconception (e.g. shading 72½ squares in Problem 1), try to help them focus on the total number of squares in the grid and the relative number of those squares that are shaded. (p. 86)

For each case the authors provide such guidance on when professional developers can expect common teacher ideas or misconceptions to emerge.

The second most frequent educative feature found in the IIRP curriculum is guidance on strategies to use to focus attention on specific aspects of classroom pedagogy (item2C). Such guidance is found in 58 paragraphs in the text. Cases depict episodes of classroom instruction and open up the pedagogy of the mathematics teacher depicted for analysis and reflection. In describing how cases can be used to support learning about teaching, the authors provide the following example:

> Analyzing instructional practice involves the interpretation of teacher thinking and action in the context of the overall lesson…For example, we frequently have used "The Case of Marcia Green" (Chapter 4) to help teachers learn to analyze teacher questioning. After teachers identify "good questions" asked by Marcia, they are encouraged to discern what makes the questions they have identified effective. Invariably, this leads to teachers' recognition of the importance of what else is going on at the point in the lesson at which the "good question" was asked. (p. 76)

By focusing attention on the case teacher's questions and asking teachers to analyze what makes a question effective, the authors suggest professional developers can help teachers reflect on the importance of considering classroom context when asking questions.

For each individual case, the authors provide professional developers with guidance and specific strategies that can be used to focus attention on certain aspects of mathematics pedagogy. One common strategy that they suggest is for participants to read and reflect on the case before meeting as a group. In addition, for each case they suggest that teachers focus on specific features of practice, such as the use of questioning.

Aside from guidance on ways to focus attention on aspects of pedagogy, the IIRP curriculum also provides guidance on ways to focus attention on conceptual aspects of mathematical topics (item 1C). This educative feature appears in 38 paragraphs throughout the curriculum. The authors provide general guidance, such as:

> The role of the facilitator during the Opening Activity is to elicit a variety of solution strategies to the problems and, to the extent possible, help teachers to identify how those strategies are both similar to and different from one another. We have found it useful to have teachers work on the tasks first individually, then in small groups, and finally to participate in large group discussion in which various solution strategies are made public. (p. 79)

In the quotation above the authors share general strategies for having teachers work with mathematical tasks in the opening activity. The strategy of comparing various solution strategies can encourage teachers to discuss the conceptual understandings that such strategies are build upon.

Guidance specific to the mathematical topics explored within individual cases is also provided in the IIRP curriculum. For example, strategies are suggested for helping teachers who are having difficulty to focus on the concepts being explored.

> If some participants are having difficulty getting started, suggest that they draw a picture or diagram to represent what is given or known, and then ask questions that will support their work. For example, on Problem 1, you might suggest that they draw a rectangle that has an area of 300, and then ask them questions that will help them focus on the relationship between the length and width (e.g., What do you know about the length and width of the rectangle you are trying to find? What does it mean for the ratio to be 4 to 3? What would some rectangles that have this relationship look like? How can you find the one you are looking for?) (p. 107)

In this example, guidance is provided both on ways to make mathematical concepts of similar figures and scale factor more visible to teachers and on the types of questions that professional developers can use to support teachers' learning about these concepts.

The fourth most common item of support is explanations of the design and learning goals of individual PLTs (item 6D). These are present in 35 paragraphs in the IIRP curriculum. The following excerpt provides an example of both the authors' general learning goal for its activities around cases and specific learning goal for the case of Marie Hanson.

> Our goal is for teachers, through reading and discussing cases, to connect the events depicted in the cases to an increasingly elaborated knowledge base of mathematics, teaching, and learning, and to their own practice. For example, in "The Case of Marie Hanson" (Chapter 3), teachers are exposed to Marie's thought processes as she monitors student work and decides which students to call on to present their solution strategies to the whole class and in what order. Marie decides to first call on a student who has solved the problem using an incorrect additive approach in order to air that misconception and then to move on to a student whose response was correct and accessible to other students in the class. In the case discussion, the facilitator can help teachers to view Marie's thinking and decisions as an instance of using student responses in a pedagogically productive way, a skill that applies in situations other than the specific situation in which Marie Hanson found herself. (pp. 76-77)

Though the provision of explanations of PLTs' design and learning goals is a rationale, implementation guidance is the predominant form of support provided in the IIRP curriculum. Implementation guidance on which common teacher ideas to anticipate and on how to focus attention on specific pedagogy and mathematical concepts is provided to professional developers. This form of support is provided 74% of the time in 250 out of 340 educative features.

Educative features in the Teaching Fractions and Ratios for Understanding (TFRU) curriculum. The TFRU curriculum (Lamon, 2005) had very few sections of text that address its facilitation and speaks to professional developers. In fact, only 4 pages in its preface address the facilitation of activities in the book. The vast majority of the curriculum, the other 232 pages, is devoted to presenting mathematical and pedagogical content and activities for teachers. Since there are only four pages of text addressing facilitation, few sightings of educative features were made. In fact, over the four pages educative features were found only 5 times for an average of 1.25 sightings per page.

As can be seen in Figure 4.27, the educative features present in the TFRU curriculum represent 3 categories and four items of support listed in the EF Framework.

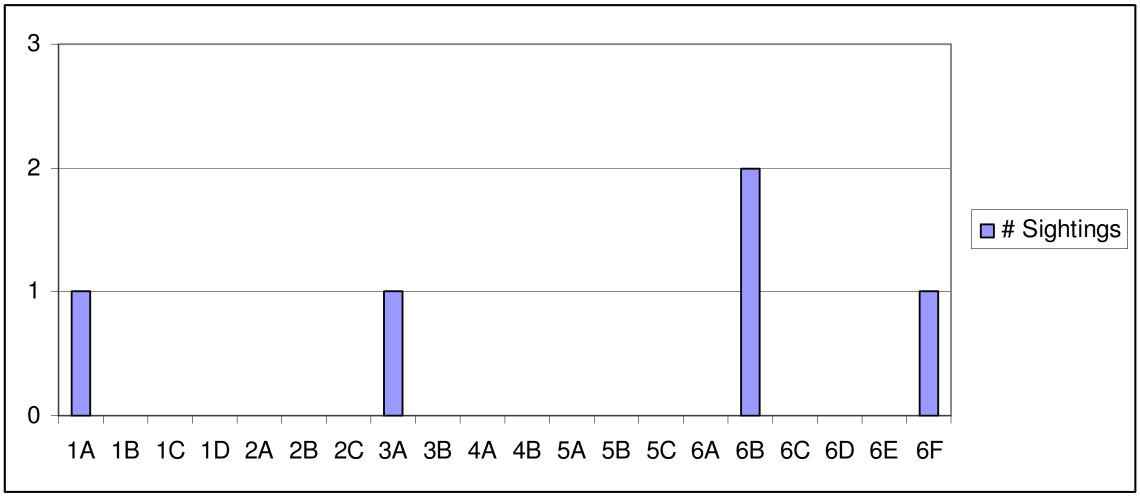


Figure 4.27. The frequency of educative features in the TFRU curriculum.

The categories are:

- ❖ Category 6: Supporting professional developers to provide long-term, ongoing, and coherent professional development programs in various contexts
- ❖ Category 1: Supporting professional developers in engaging teachers with specific mathematical topics
- ❖ Category 3: Supporting professional developers in engaging teachers with professional learning tasks centered on the work of mathematics teaching

The four items of support found in the curriculum are:

- ➢ Item 6B: Explain the sequencing of PLTs over the curriculum and the overall learning goals for teachers that the sequence addresses
- ➢ Item 1A: Explain the value of revisiting and deepening teachers' understanding of mathematical topics that they teach

- Item 6F: Provide guidance on various strategies to continuously engage teachers and when to employ them
- Item 3A: Explain why learning activities centered on the work of teaching and situated in classroom practice are desirable for promoting teachers learning

Lamon explains in the preface that the learning goal of the book is to push teachers beyond their current understandings of rational numbers and proportionality. In explaining the overall learning goals of the sequence of PLTs (item 6B), she states that the reader "will be challenged to refine and explain [his or her] thinking and to make sense – without falling back on the fraction rules and procedures [he or she has] relied upon throughout [his or her] life" (p. xiv). The learning goal is for teachers to strengthen their conceptual understandings.

In an earlier paragraph she explains the need for deepening teachers' understandings about rational number (item 1A). She states:

> Unfortunately, until recently, we have had little understanding of how proportional reasoning develops…Without a research base to inform decision-making about the important conceptual components of proportional reasoning, textbook approaches have unintentionally encouraged simplistic, mechanical treatment of ratio and proportions, highlighting the algebraic representation of proportion and the manipulation of symbols…For most people, this mantra is a proxy for reasoning about quantities and their relationships. (p. xiii)

With the goal of the book to push teachers beyond this procedural view of rational numbers and proportionality, Lamon explains why using mathematical tasks situated in classroom practice and designed for children (item 3A) with teachers can accomplish this aim. She states,

> This book is not a textbook as much as it is a resource book. One of its underlying assumptions is that facilitating teacher understanding using the same questions and activities that can be used with children, is one way to help teachers to build

> the comfort and confidence they need to talk to children about complex mathematics.

The TFRU curriculum provides guidance on some strategies to use through this process of revisiting the complex mathematics involved in middle grade mathematics instruction (item 6F). It suggests that teachers using the book "include time for persona reasoning and reflection as well as time to discuss the materials with others" (p. xiv). Such discussions will leave teachers "better prepared to orchestrate discussion" (p. xiv) in their own classrooms.

Though they are very few instances of education features in the TFRU curriculum, 80% (4 out of 5) of them are rationales about the goals and underlying assumptions of the author. The curriculum is designed with the goal of developing teachers' conceptual understanding of the mathematical concepts they teach. These explanations about the curricular design provide some indication of the ways in which the book could be used by professional developers, though little guidance on how to do so.

Educative features in the Developing Mathematical Ideas: Making Meaning for Operations (DMIMMO) curriculum. Like the IIRP curriculum, the DMIMMO curriculum (Schifter, et al., 1999a, 1999b) has a few sections addressing both teachers and professional developers and several sections devoted to providing guidance to professional developers on the facilitation of its activities for teachers. The introduction section of its casebook addresses both teachers and professional developers as it outlines the learning goals of the curriculum. The DMIMMO curriculum not only provides separate sections of text specifically for professional developers, but also provides these sections in a separate book – the facilitator's guide. The facilitator's guide has three sections. The first provides general tips for facilitation and specific guidance on facilitating each session in the casebook with teachers. The other two sections provide narratives of the experiences of a professional developer and two teachers with the curriculum. The sections of text from the casebook and facilitator's guide that explicitly addressed professional developers - the introduction section of the casebook and the first section of the guide - were analyzed.

These sections of text covered 81 pages and within those pages I found 169 sightings of educative features. The DMIMMO curriculum has an average of 2.09 sightings per page of educative features. These features represent 13 of the 20 listed items of support in the EF Framework, as can be seen in Figure 4.28.

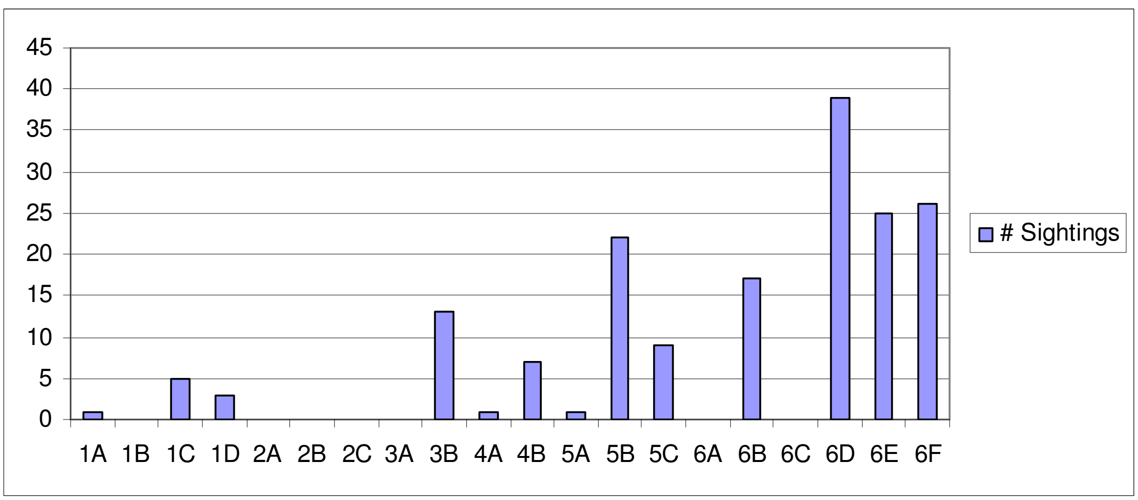


Figure 4.28. The frequency of educative features in the DMIMMO curriculum.

The categories of support most emphasized within this curriculum are:

- Category 6: Supporting professional developers to provide long-term, ongoing, and coherent professional development programs in various contexts
- Category 5: Supporting professional developers in building a professional learning community with teachers

- Category 3: Supporting professional developers in engaging teachers with professional learning tasks centered on the work of mathematics teaching
- Category 1: Supporting professional developers in engaging teachers with specific mathematical topics

The items of support that were most prevalent in the guidance to facilitators are:

- Item 6D: Explain the design and the teacher learning goals (both the learning of mathematical content and pedagogy) of individual PLTs
- Item 6F: Provide guidance on various strategies to continuously engage teachers and when to employ them
- Item 6E: Provide guidance on how and when to modify PLTs or the sequence of PLTs in order to fit the local culture or context of PD
- Item 5B: Provide guidance on how to facilitate and create a common discourse for teachers to engage in collective inquiry about mathematics teaching and learning

The authors of the DMIMMO curriculum provide professional developers with clear explanations of the learning goals envisioned for the activities in each session. This item of support, 6D, is found in 39 paragraphs across the facilitator's guide. It is present, for example, in a section where the authors describe the purposes of an activity involving the collection and analysis of student work.

> At the beginning and end of the Making Meaning for Operations seminar, participants bring in samples of their students' work. This activity serves a dual purpose. First, it provides a forum for participants to discuss the mathematical understandings of their students with other teachers. As the teachers work in small groups, examining the student work and sharing their assessments, the discussion fosters a kind of collegial interaction that is integral to a successful DMI seminar and is, at the same time, likely to be new for many teachers. This assignment also serves as a tool for reflection, offering participants the opportunity to track their changing views from the beginning to the end of the seminar. (p. 22)

Guidance on strategies to use to continuously engage teachers (item 6F) are present in 26 paragraphs in the curriculum. An example from the curriculum is provided below:

> There are a few strategies particular to facilitating case discussions: The whole group can begin with one of the focus questions on the handout you distributed for that session. As the discussion of children's mathematics unfolds, you should ask that particular line numbers in the casebook be cited. This technique draws everyone into the specifics of the case, focuses conversation on the mathematical ideas of the children, and helps avoid generalized critiques of classrooms and teaching strategies. (p. 8)

This example illustrates how the DMIMMO curriculum provides professional developers with strategies, such as using focus questions, for facilitating discussion based on evidence and promoting collective inquiry they can use in their facilitation of this curriculum and others.

The provision of guidance on how to modify the curriculum's activities to fit local contexts (item 6E) is present in 25 paragraphs of text. One example of the way guidance is provided about adapting the curriculum is in regards to the format and timing. As the authors state:

> The facilitator's guides lay out a plan for covering both DMI seminars on Number and Operations in 16 sessions that meet for 3 hours each. However, facilitators have conducted DMI seminars in other configurations…While the agendas spell out the order and a suggested duration for the activities in each session, you can adapt them to fit your situation. (p. 19)

Further the authors provide comments from facilitators who have used different formats about their experiences using the curriculum in each configuration (pp. 10 – 11).

Building a professional learning community with teachers is challenging. One aspect of such a community is having a common discourse. The provision of guidance on how to facilitate and create a common discourse for teachers to use in collective inquiry (item 5B) is found in 22 paragraphs of text. The example I will share is one from a section for professional developers entitled “facilitating group discussion”.

> To initiate whole-group discussion, you might choose among several strategies:
> - Bring out an issue that caused confusion in small groups: “It seems that lots of people are struggling with the idea that…and so I thought we could come together to see if we can sort it out.”
> - Begin with an idea that some groups fund stimulating: “Many of you were discussing [issue X]. Now I think it would be worth our while to discuss where you agree and where your interpretations and perspectives differ.”
> - Highlight a point made in one small group that is important for everyone to consider “[Participant’s name] said something that left me thinking…”
> - Draw the group’s attention to an issue that has been ignored in small groups: “As I went from group to group, I heard lots of interesting and important ideas, but I didn’t hear anyone talking about [issue Y].”
>
> (p. 8)

In this excerpt the authors provide professional developers with concrete strategies that they can use to initiate discussion, engage teachers, and move towards a common discourse.

Though the authors provided explanations about their learning goals and other rationales, the major form of support provided to professional developers is, once again, implementation guidance. This form of support is provided 65% of the time in 110 out of 169 educative features. Professional developers using the facilitator’s guide of the DMIMMO curriculum are provided with guidance on how to facilitate discussions, keep teachers engaged in their learning, and modify the PLTs to fit their local context.

Educative features across the four curricula. The four curricula differ significantly in the extent to which they appear to be educative, as illustrated in Figure 4.29. They range from the TFRU curriculum which provides a scarce 5 educative features to the IIRP curriculum which offers a wide offering of 340 educative features to support professional developers.

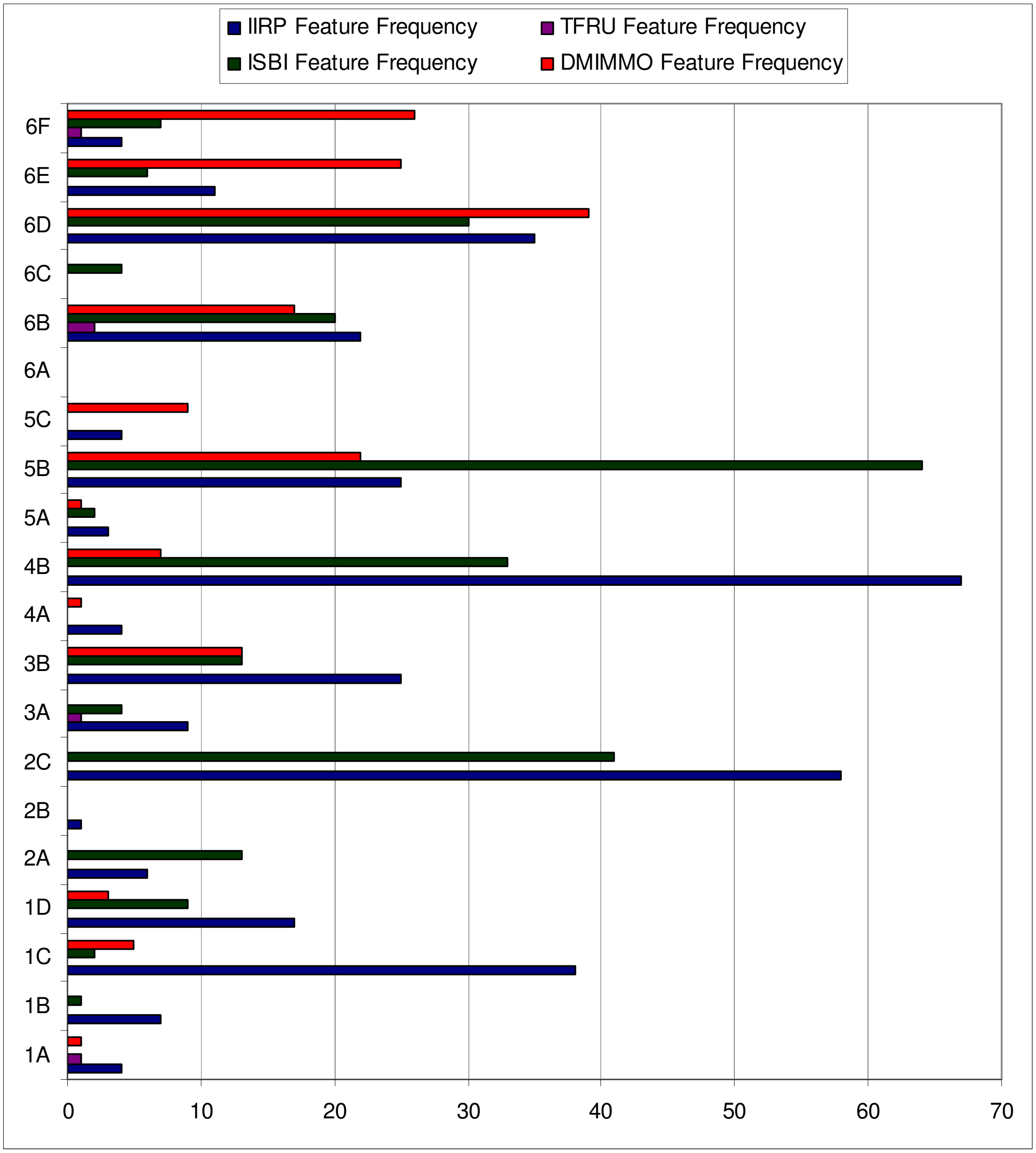


Figure 4.29. The frequency of educative features present in each of the four curricula.

From the survey data, however, it is apparent that professional developers often use the four sets of curriculum materials in conjunction with each other. Since this is the case it would make sense to explore the learning opportunities provided to professional developers using all four of the curricula at once.

As can be seen in Table 9, the curricula address different and overlapping educative features. Looking across the 229 pages of text directed towards professional developers in all four curricula, I analyzed the extent to which the text was educative for professional developers and considered the ways in which it supported their learning about teacher education.

Table 9

Results of the Educative Analysis across the Four Curricula

		ISBI	**IIRP**	**TFRU**	**DMIMMO**	***Across the Sample***	
1. Specific Mathematical Topics	1A	0	4	1	1	*6*	*88*
	1B	1	7	0	0	*8*	
	1C	2	38	0	5	*45*	
	1D	9	17	0	3	*29*	
2. Specific pedagogy	2A	13	6	0	0	*19*	*119*
	2B	0	1	0	0	*1*	
	2C	41	58	0	0	*99*	
3. Focus on practice	3A	4	9	1	0	*14*	*65*
	3B	13	25	0	13	*51*	
4. Anticipating teachers	4A	0	4	0	1	*5*	*112*
	4B	33	67	0	7	*107*	
5. Building learning communities	5A	2	3	0	1	*6*	*130*
	5B	64	25	0	22	*111*	
	5C	0	4	0	9	*13*	
6. Providing ongoing, coherent PD in different contexts	6A	0	0	0	0	*0*	*249*
	6B	20	22	2	17	*61*	
	6C	4	0	0	0	*4*	
	6D	30	35	0	39	*104*	
	6E	6	11	0	25	*42*	
	6F	7	4	1	26	*38*	
Total Sightings		**249**	**340**	**5**	**169**	***763***	
Number of Pages		79	65	4	81	*229*	

As can be seen from Table 9 and Figure 4.30, the sample of curriculum materials provides certain categories of support more than others. Category 6, which include explanations about the design and learning goals of teacher learning activities, is understandably most common. While Category 3, which supports professional developers in engaging teachers with PLTs centered on the work of mathematics teaching, is the least present across the four curricula.

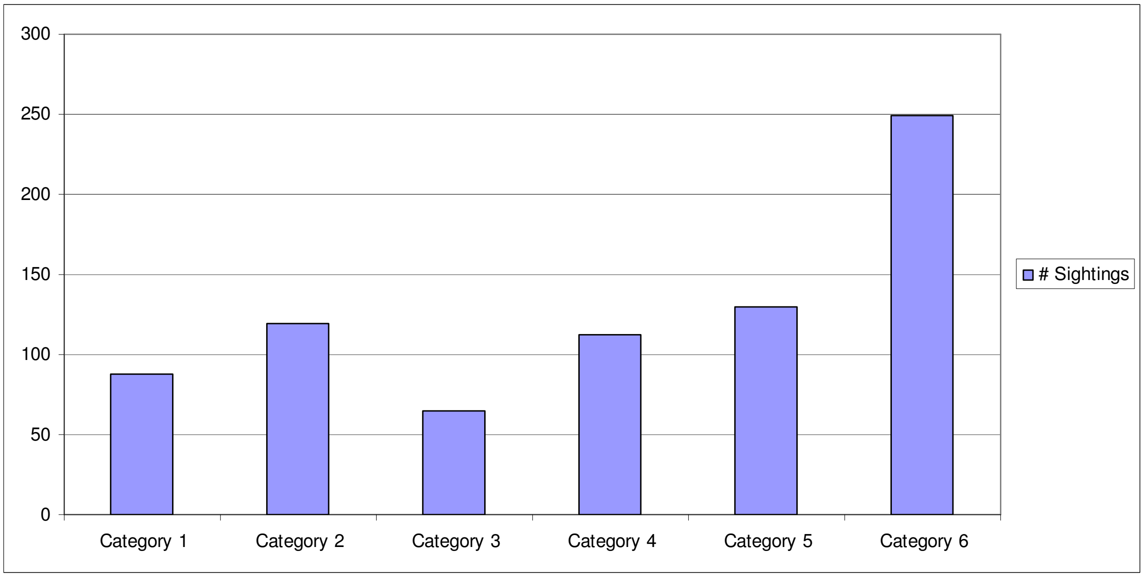


Figure 4.30. Distribution of categories of support across the four curricula.

Though the educative features presented in the individual curricula differs, looking across all four curricula at the overall distribution of the education features provides valuable information on which features are emphasized and which are not addressed. As can be seen in Table 9 and Figure 4.31, the items of support that are featured heavily in individual curricula remain the most frequently provided across the sample. They are:

- Item 5B: Provide guidance on how to facilitate and create a common discourse for teachers to engage in collective inquiry about mathematics teaching and learning
- Item 4B: Provide guidance on when to expect certain common teacher ideas to emerge.
- Item 6D: Explain the design and the teacher learning goals (both the learning of mathematical content and pedagogy) of individual PLTs

➢ Item 2C: Provide guidance on strategies to use to focus attention on specific aspects of classroom pedagogy.

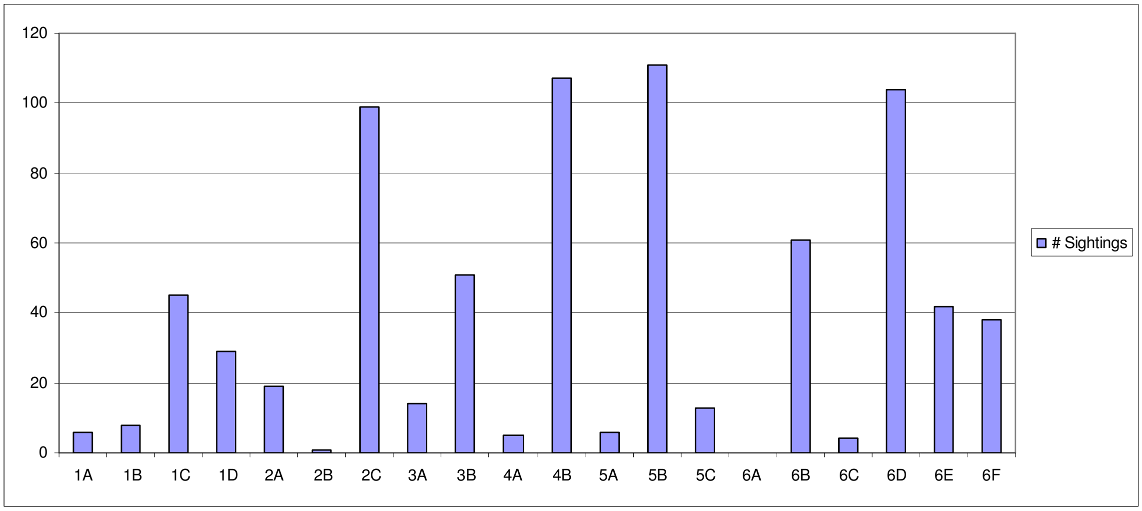


Figure 4.31. The frequency of educative features across the four sets of curriculum materials.

Professional developers using all four of the curricula would be provided with ample (99 or more) opportunities to learn: how to facilitate discourse (5B); when to expect common teacher ideas to emerge (4B); why the activities were designed in a particular way and what teachers are expected to learn from them (6D); and how to focus teachers' attention on specific aspects of pedagogy laid open for investigation in PLTs (2C).

However, several educative features are not well addressed across the sample of curriculum materials. Professional developers using the four curricula would have no opportunity to learn why ongoing and long-term professional development if beneficial (6A) and few (5 or less) opportunities to learn how to model "reform oriented" pedagogical moves with teachers (2B), how the curriculum materials are aligned to state or national standards (6C), or why attending to teachers' knowledge, beliefs, and habits of practice is important in supporting their learning (4A).

Overall, the four sets of professional development curriculum materials appear to have many educative features. Over the 229 pages of text directed towards professional developers, 763 educative features were identified for an average of 3.33 educative features per page of text. The majority of these features, 70% (536 out of 763), are in the form of implementation guidance that provide direction to professional developers on how to do many of the practices involved in the work of supporting teacher learning. These findings will be discussed in Chapter 5.

Chapter V - Discussion and Conclusions

In order to improve the quality of mathematics instruction offered to children and youth and increase student engagement and achievement in mathematics, attention to teachers' capacity to do the work of mathematics teaching is paramount. For middle school mathematics teachers, professional development is the main site to learn about mathematics teaching and learning. This is due to the fact that their college preparation usually does not focus upon the mathematical topics central to middle school mathematical content (Conference Board of the Mathematical Sciences, 2001). With professional development being the main site for the learning of middle school mathematics teachers, it is important that we, as a research community, develop an understanding of teacher learning in this setting. In order to do so we must attend to both what teachers have an opportunity to learn and how their learning is facilitated. However, in the literature on professional development little attention has been given to the content of professional development and what teachers have an opportunity to learn (Garet, et al., 2001). This study addresses this gap in the research and explores the opportunities to learn provided to middle school mathematics teachers in a sample of four, widely used, sets of professional development curriculum materials.

The four curricula analyzed in this study are used with large numbers of middle school mathematics teachers. In my survey of professional development projects across the United States and Puerto Rico I found that each of these materials was used in projects involving at least 14% of the 6203 middle school mathematics teachers participating in the surveyed projects. Aside from being used with large numbers of teachers in the population surveyed, these curricula, being publicly available, can be and probably are being used with many other mathematics teachers across the country. The results of this study, described in the previous chapter, shed light on what large numbers of mathematics teachers can learn about mathematics teaching and learning from using these curricula and also what professional developers can learn about supporting teacher learning in professional development from such use.

In this final chapter, I discuss the results of my study and relate the findings to the research literature base. I describe the potential contributions and implications that the study offers theoretically, methodologically, and practically. I recommend several research studies that could be conducted in the future to build upon this study. Finally, I offer concluding remarks about what I consider to be the most important contributions of the study and how the study addresses the research problem that inspired its design.

Opportunities for Teacher Learning in the Sample of Professional Development Curriculum Materials

My purpose in identifying the opportunities to learn provided to middle grade mathematics teachers in the sample of professional development curricula is to fill a gap in the educational community's knowledge about the content to be learned in mathematics professional development in the United States. Through my analysis I have started to fill that gap by identifying the learning opportunities provided to teachers in the

PLTs of each of the four, widely used curricula in my sample. My goal in doing so is not to compare these curricula, but rather to describe the relative strengths of each curriculum and how each one can be used as a resource to support teacher learning in professional development. It is my hope that my findings will be used to inform the selection and sequencing of these curricula so that teachers can best benefit from the learning opportunities that each provide.

The findings of this study shed light on the ways these professional development curriculum materials provide opportunities for teachers to learn about mathematical content and pedagogy. In the following sections I will discuss the learning opportunities provided for teachers to learn about specific mathematical content, the use of multiple representations, and the use of cognitively demanding tasks across the four curricula. Additionally, I will discuss how the design of PLTs seems to influence the learning opportunities they provide and how the identified learning opportunities align with what teachers are called upon to learn to do.

Teachers' opportunities to learn about middle grade mathematics – proportionality, rational numbers, and linear equations and functions (RQ1). A national survey of teachers found that one of the core features of effective professional development was a focus on content knowledge (Garet, et al., 2001). All four of the curricula maintain a clear focus on mathematics. Across the sample, 85% (93 out of 110) of the PLTs feature mathematical tasks and the other 15% ask teachers to reflect on mathematical ideas. Teachers using the four curricula have the opportunity to solve 284 mathematical tasks themselves, analyze student work samples on at least 23 of them, and reflect on instructional activities around at least 55 of them. These activities provide extensive opportunities for teachers to revisit and reflect on mathematical content. Such opportunities are needed because, while teachers have strong procedural knowledge of mathematics, they often lack conceptual understanding of the ideas that underpin those procedures (Mewborn, 2003).

Opportunities are provided in the professional development curriculum materials for teachers to focus on the concepts underpinning common procedures. The DMIMMO curriculum pushes teachers to make meaning of operations and understand conceptually how and why the operations work. The TFRU and IIRP curricula push on teachers to refine their thinking about proportionality and rational numbers and move beyond procedures to be able to understand and explain the mathematical ideas at play. For example, they are asked to solve proportional problems without using any established procedures or algorithms. The ISBI curriculum addresses a number of mathematical topics but also focuses on the development of conceptual understanding over procedural fluency. For example, in the case of Ron Castleman, it does so by requiring teachers to use visual reasoning rather than common procedures to explain how to determine the percent, decimal, and fraction part of the grid’s area that was shaded (Figure 4.2).

Across the four curricula, the mathematics being addressed is presumably not new for teachers. However, the opportunity to develop deeper conceptual understandings of these mathematical ideas may be a new experience for teachers. In this study I am particularly interested in the extent to which these types of experiences were provided to learn more about three core mathematical topics for middle grade mathematics: proportionality, rational numbers, and linear equations and functions.

Proportionality is a key and overarching concept in middle grade mathematics (Lanius & Williams, 2003; Lesh, et al., 1988; Shield & Dole, 2008) and, often, a source of difficulty for teachers – who experience difficulties in using proportional reasoning (Cramer, et al., 1993; Hillen, 1996), discussing it conceptually (Thompson & Thompson, 1994, 1996), and supporting students to deal with proportional situations (Sowder & Philipp, 1995). Given teachers' struggles with this topic, it is appropriate that proportionality is the mathematical topic most focused upon in the sampled professional development curriculum materials developed for middle grade mathematics teachers. It is addressed in all four curricula and in 65% (72 out of 110) of all the PLTs across them.

Aside from being extensively addressed, attention was also paid to the reasoning that underlies proportionality: multiplicative reasoning. Sowder and colleagues (1998) made four recommendations for the professional development, based on research on the teaching and learning of multiplicative structures, to support the development of teachers' multiplicative reasoning. All of the curricula meet at least two of these recommendations and the ISBI and TFRU curricula meet all four. The four curricula provide opportunities for teachers to reflect on the multiplicative relationships that characterize proportional situations. Additionally opportunities are provided for teachers to make connections among the forms of rational numbers and ratios.

Rational numbers is a topic that is intertwined with proportionality (Clark, et al., 2003). It is addressed across the sample of curriculum materials to varied degrees. While rational numbers are the focus of the TFRU curriculum and in PLTs in the ISBI and IIRP curricula, they are minimally addressed in the DMIMMO curriculum. Overall, rational numbers were addressed in 38% (42 out of 110) of the PLTs across the four curricula –

with fractions being the main form of rational number considered. Such a focus on fractions is warranted as teachers struggle with operations involving fractions (Ma, 1999) and with the connection between fractions and ratios (Smith, 2002).

Another topic intertwined with and often built off of proportionality is linear equations and functions. While linear equations and functions are a key topic in middle grade mathematics (National Council of Teachers of Mathematics, 2006) and they pose a great source of difficulty for students (Kieran, 2006), they are basically overlooked in all four curricula. Only once did a linear pattern emerge and that was in one of many mathematical tasks involved in the task sorting activity of the ISBI curriculum. It is problematic that the curriculum materials did not provide opportunities for teachers to learn more about the mathematics teaching and learning of this important topic. I am not suggesting that any one curriculum should be expected to attend to every topic of middle school mathematics. It is a good and strategic choice to focus the attention of a professional development curriculum on a particular mathematical topic or set of ideas, as the TFRU and IIRP curriculum do. Such focus provides teachers with the repeated opportunities over time needed to learn something well. What I am suggesting is that it is problematic that, though linear functions are an important part of middle school mathematics content, commonly used curriculum materials do not address this topic and the curriculum materials that do provide opportunities to learn about it are not often chosen for use in professional development. For example, a book that directly addresses linear functions, the *Improving Instruction in Algebr*a curriculum (Smith, et al., 2005a) was used with only 6% of the teachers participating in the professional development projects surveyed. If curriculum materials that address linear functions are not also included for use, then teachers will not being given opportunities to revisit and strengthen their conceptual understanding of linear equations and functions.

Overall, the PLTs in all the four curricula provide opportunities for teachers to move beyond their procedural knowledge and deepen their understandings of the concepts underlying procedures in a range of mathematical topics. The four curricula are focused upon making meaning of operations, proportionality, rational numbers, and a few other topics, such as data analysis. In regards to the three mathematical topics of interest, the sample of professional development curriculum materials provided extensive opportunities to learn about proportionality, many opportunities to learn about rational numbers, especially fractions, and scarce opportunities to learn about linear equations and functions. Given the centrality of proportionality and rational numbers, and the fact that linear functions builds on a firm understanding of these two other topics, such a distribution of learning opportunities is understandable. However, there is a need to ensure that at some point in their professional development experience teachers are provided opportunities to learn more about linear functions to prepare them to help students build a firm conceptual foundation for high school and college algebra.

Teachers' opportunities to learn about using multiple representations of mathematical ideas (RQ2). Mathematical language is multisemiotic (O'Halloran, 2003). We use words, physical objects and gestures, visual diagrams, symbols, graphs, and tables in concert to make meaning of mathematical ideas and to communicate mathematically with others. For this reason, communication and representation are two of the five process standards that "highlight ways of acquiring and using content knowledge" in mathematics (National Council of Teachers of Mathematics, 2000, p. 29).

The use of multiple representations is core to communication in mathematics and in the words of Mary Lindquist, NCTM president 1992-1994, communication is "the essence of teaching, assessing, and learning in mathematics" (Elliot & Kenney, 1996, p. 2) and making connections between representations is part of communicating in mathematics (Herbel-Eisenmann, 2002; Kaput, 1989).

Developing students' ability to use multiple representations is crucial for their learning of mathematics. Brenner and colleagues (1999) have shown that students' ability to use multiple representations flexibly, their representational competence, is a key component to students' competent mathematical thinking and problem solving. Kieran (2007) has suggested that in order to help students successfully learn algebra it is critical to teach them how to use the interrelationships between representations of mathematical ideas to build and make meaning.

Though using multiple representations is central to communicating mathematically and to mathematics teaching and learning, such use presents significant challenges to teachers. Ball (1993b) has referred to representing mathematical ideas as one of the main dilemmas of mathematics teaching. Teachers need to consider which representations or combination of representations will best illustrate the characteristics of the mathematical idea to which students need to attend. For example, teachers need to decide which representations to use when teaching students about fractions. Textbooks primarily provide diagrams that illustrate the part-whole relationship, such as shaded segments of a square or circle. While such diagrams are useful, if they are not used in combination with other types of representations the result can be problematic. Carraher (1996) found that the exclusive use of these types of diagrams increased students' difficulties in understanding the connection between fractions and ratios. In the TFRU curriculum, Lamon (2005) explicitly rejects that the term fraction only refers to the interpretation of rational numbers as part-whole comparisons. She argues that teachers should provide students with opportunities to learn that rational numbers can be interpreted as part-whole comparisons, measures, operators, quotients, and ratios and rates. Further, she states that students should use multiple representations (symbols, words, pictures, and physical objects) to explore each of these interpretations (p. 24). The lack of use of multiple representations of rational numbers can hamper student learning. Professional development curriculum materials can help by preparing teachers to use multiple interpretations and representations of this mathematical topic.

Representation can be conceived of as both a verb and a noun (Pape & Tchoshanov, 2001). Representation can refer to the active process of representing a mathematical idea, such as the activity a learner engages in as they create diagrams or tables while solving mathematical tasks. Representation can also refer to an object that is the manifestation of a mathematical idea or a person's conception of that idea, such as a diagram. The professional development curriculum materials analyzed provide many opportunities for teachers to use both conceptions of multiple representations (verbal descriptions, symbols, graphs, tables, visual diagrams, and physical objects) of mathematical ideas. Across the four curricula, 89% (98 out of 110) of the PLTs use more than one form of representation. Across the PLTs of the four curricula, teachers are provided with opportunities to use all six forms of representation themselves when solving mathematical tasks. This provides opportunities for teacher to gain practical skills in creating and using multiple representations to reason mathematically. In the PLTs featuring student work, narrative cases, and future classroom activities, teachers are provided opportunities to reflect on how students and teachers may use representations to build understandings of and communicate mathematical ideas. From the narrative cases, they can observe how teachers could make connections between representational forms to highlight concepts that students are learning. In the example from the case of Ron Castleman that was shared, Mr. Castleman connected his students' diagrams of shaded 4 x 10 grids (Figure 4.2) to the percentage symbol to have them discuss the meaning of percent and what 100% represented in that problem context.

As can be seen in Figures 4.22 and 4.23, the forms of representation are used to different extents and the degree to which connections are made between also varies. As we primarily use words to communicate, verbal descriptions are present throughout the PLTs. Aside from the verbal, diagrams and symbols are the two representational forms most often used in each of the curricula. Teachers are provided extensive opportunities to see how teachers and students engage in the mathematical practice of using these different representations to model mathematics. Since this is one of the eight practices espoused in the new Common Core Expectations (2010) and Stylianou (2010) has found that middle school mathematics teachers have an underdeveloped view of using representations as a process and mathematical process, it is beneficial that teachers are provided such opportunities in the analyzed curricula to refine their conception of using multiple representations.

Graphs are rarely used across the curricula. Thus, limited opportunities are provided to teachers to learn to use graphs themselves or in combination with other forms of representation. This is problematic as graphical representations are particularly well suited to display dynamic processes (O'Halloran, 2003) and to infuse symbolic representations, such as equations, with meaning (Romberg, Fennema, & Carpenter, 1993). Even with linear functions not being a topic of focus in the four curricula, it was surprising to find such limited use of graphs even in the context of proportional situations well suited to such a representation. Graphs were used only a few times to explore speed using distance/time graphs.

Due to the limited use of graphs, in none of the four curricula are connections made between graphs and symbols. This missing connection is problematic because in algebra the use of graphs, symbols (equations), and tables in combination is commonplace. Making connections between these forms of representation is listed as a learning expectation in national and state documents (Michigan Department of Education, 2006; National Council of Teachers of Mathematics, 2000). Kieran (2006) has argued that the teaching and learning of functions as algebraic objects requires the combined use of their graphical, tabular and symbolic representations. Though functions are not a mathematical topic of focus for the curricula in the sample, it is still important to lay a foundation for using this triad of representations to explore linear functions in middle school and other types of functions in future grade levels. Hopefully, professional development curriculum materials that focus on linear equations and functions would present teachers with many opportunities to make connections between the combinations of representations used in exploring such mathematical ideas.

Teachers' opportunities to learn about using cognitively demanding tasks in instruction (RQ3). The vast majority of mathematical tasks provided in the four curricula were cognitively demanding for the students at the grade level with which they were used. However, many of these tasks were designed for use with elementary rather than middle school students. In both the TFRU and DMIMMO curricula, many tasks are presented that are to be used with elementary students. While these tasks would be cognitively demanding for elementary students being introduced to the mathematical ideas, they usually would not present a significant challenge to middle grade students who would have had several years experience using these ideas. Therefore, the degree to which middle school mathematics teachers were provided opportunities to engage with mathematical tasks that are cognitively demanding for their own students was not as high as the quantitative results would suggest. However, the provision of elementary level tasks and accompanying student work or narrative cases does provide teachers with useful information about the prior knowledge that their students may bring into their classrooms.

Of the PLTs that provide examples of tasks that would be cognitively demanding for middle school students, most focus on tasks used with 6^{th} or 7^{th} grade students. There are few examples of the use of cognitively demanding tasks with 8^{th} grade students. This, no doubt, is linked to the mathematical topics being focused upon – rational numbers and proportionality are focused upon more heavily in the grade level expectations of 6^{th} and 7^{th} grades (Common Core State Standards Initiative, 2010; National Council of Teachers of Mathematics, 2006). However, it would be useful for teachers to have opportunities to reflect upon images of mathematics instruction using cognitively demanding tasks at the 8th grade, such as tasks concerning solving linear equations.

There seems to be a connection between the use of cognitively demanding tasks in PLTs and opportunities provided for teachers to focus on concepts over procedures and to explore the use of multiple representations. The very characteristics of cognitively demanding tasks invite such opportunities. Mathematical tasks at the “procedures with connections” level focus attention on the use of procedures for the purpose of developing conceptual knowledge and are usually represented in multiple ways. Those at the “doing mathematics” level require students to explore and understand concepts and do not have an algorithmic or predictable way to solve them. Thus, the mere use of such cognitively demanding mathematical tasks can move teachers’ attention to concepts rather than procedures, and encourage teachers to both use and make connections between multiple representations.

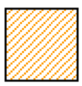
The opportunity to solve and analyze cognitively demanding mathematical tasks in PLTs has been shown to support teacher learning. The use and analysis of such mathematical tasks in professional development provides teachers with opportunities to develop their knowledge of mathematics and of pedagogy (Koeliner, et al., 2007) by offering the stimulus to rethink their selection process of mathematical tasks to use in instruction (Arbaugh & Brown, 2005). When such mathematical tasks are coupled with narrative cases in PLTs, the opportunities for teacher learning are increased. While the mathematical tasks expose teachers to *what* cognitively demanding tasks are like, the narrative cases portray *how* they can be used in mathematics teaching and learning. The cases illustrate the classroom factors that are associated with teachers either maintaining or undermining level of cognitive demand as they use such tasks in instruction (Henningsen & Stein, 1997). They also open for teacher reflection the pedagogical issues that emerge with the use of cognitively demanding tasks, such as student struggle and the management of multiple student solutions (Silver, et al., 2005). Due to the open-ended nature of cognitively demanding mathematical tasks, students will come up with many varied solutions. Teachers need to manage these multiple student solutions and facilitate productive mathematical discussions that help students benefit from the various strategies being discussed. Stein and colleagues (2008) have identified five practices that can support such discussions and can be shared with teachers to support their use of cognitively demanding mathematical tasks in instruction. They are: (1) anticipating student responses, (2) monitoring students' work on and engagement with tasks, (3) selecting particular students to present their mathematical responses, (4) purposefully sequencing the student responses that will be displayed, and (5) helping the class make mathematical connections between different students' responses and between student responses and key mathematical ideas (p.321).

The use of cognitively demanding mathematical tasks can be beneficial for both teachers and students. Teachers, through their analysis of such tasks and their preparation for using them during professional development, are presented with opportunities to develop their knowledge of mathematical content and pedagogy and, hence, their capacity to do the work of mathematics teaching. Students who use such tasks are provided with opportunities to deepen their conceptual understandings of mathematics and their ability to problem solve (Silver & Stein, 1996). However, the use of cognitively demanding mathematical tasks has been scarce in U.S. classrooms (Hiebert, et al., 2003), possibly due to the challenges they present to teachers, such as drawing on teachers' often under-developed knowledge of mathematical concepts (Mewborn, 2003), requiring the management of multiple student solutions (Silver, et al., 2005) and the facilitation of classroom discussions. The four curricula provide opportunities for teachers to develop the conceptual knowledge needed to use such tasks and to reflect upon the pedagogical issues and classroom factors that are associated with such use. The curricula, especially the ISBI and IIRP curricula, provide opportunities for teachers to become prepared to use these beneficial tasks in their mathematics instruction.

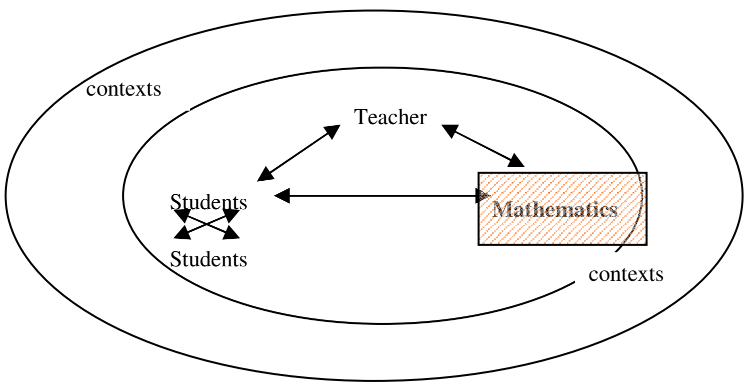
The influence of PLT design on learning opportunities provided for teachers. More than 20 years ago, Doyle (1988) analyzed the work in mathematics classrooms and asserted that tasks provide the context for the thinking and learning that occurs. In professional development spaces, tasks continue to provide that function. For that reason, I have focused upon analyzing the learning opportunities that professional learning tasks can provide for mathematics teachers. Through my analysis I have observed that the design of PLTs seems to have an effect on the type of context that they provide for thinking and the learning opportunities that they can provide to teachers. In this section I will share my thoughts on this matter.

Professional learning tasks can be designed in a variety of ways. Across the four curricula five types of PLTs were encountered: (1) sets of mathematical tasks, (2) samples of student work on mathematical tasks, (3) narrative cases of classroom instruction around mathematical tasks, (4) future classroom activities where teachers will engage with students around mathematical tasks, and (5) general reflection on mathematical and pedagogical issues. These PLTs use different means and artifacts of practice to focus teachers' attention of different aspects of mathematics teaching and learning and, hence, provided different learning opportunities to teachers.

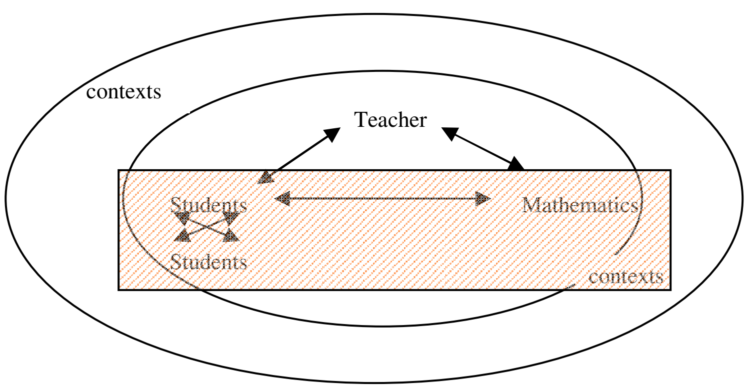
The content of teacher learning in professional development, as illustrated by Figure 2.2, is mathematics teaching and learning (Nipper & Sztajn, 2008) as represented by the instructional triangle (Cohen, et al., 2003). As can be seen in Figure 5.1, different types of PLTs focus attention on different areas of that triangle.



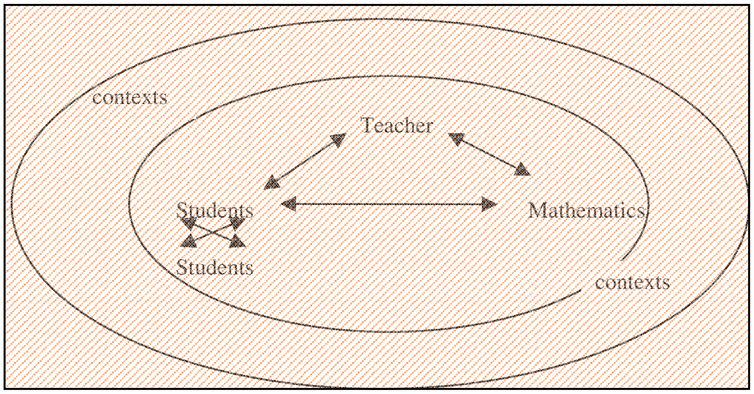
Shaded area indicates the area of mathematics instruction focused upon by PLT(s)



Focus of PLTs featuring sets of mathematical tasks



Focus of PLTs featuring samples of student work



Focus of PLTs featuring narrative cases of classroom instruction and/or future classroom activities where teachers will engage with students around mathematical tasks and/or general reflection on mathematical and pedagogical ideas.

Figure 5.1. Illustration of the areas of mathematics instruction highlighted by the types of PLTs used in the four curricula.

PLTs featuring mathematical tasks provide opportunities for teachers to explore their own understandings and misunderstandings of mathematics and the mathematical content that they teach. Revisiting and strengthening middle school mathematics teachers' knowledge of the mathematical content they teach is important (Mewborn, 2003). It is important both because it builds teachers' capacity to teach the content to others (Shulman, 1986) and there is evidence that many teachers do not have a strong conceptual understanding of the mathematics they teach, specifically the topics of *proportionality* (Cramer, et al., 1993; Hillen, 1996; Sowder & Philipp, 1995; Thompson & Thompson, 1994, 1996), *rational numbers* (Ma, 1999), and *linear functions* (Even, 1993). PLTs that feature mathematical tasks for teachers to explore provide much-needed opportunities for teachers to focus upon the mathematical ideas they teach (Koeliner, et al., 2007) and, with the support of peers and professional developers, to come to "know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility" (National Council of Teachers of Mathematics, 2000, p. 17).

Thus, the PLTs featuring mathematical tasks provide opportunities for teachers to develop their knowledge of the mathematical content they teach.

PLTs featuring student work samples provide opportunities for teachers to explore not only mathematical ideas, but also students' understandings and misunderstandings of those mathematical ideas. Such PLTs provide opportunities for teachers to develop knowledge and skills that increase their capacity to do the work of mathematics teaching. Teachers can develop knowledge of the concepts underpinning the mathematical content they teach. They can develop skills in analyzing student work to make deductions about student thinking that can inform instructional decisions. In their analysis of student work, teachers also have the opportunity to learn collectively (Westheimer, 2008) and to pay attention to specific items, such as students' use of multiple representations. Ryken (2009) suggests that reflection on students' use of representations can help educators rethink their teaching to more focus on multiple meanings and student thinking. Additionally, analyzing student work on mathematics tasks can provide teachers will the experience to better anticipate how their own students might approach similar mathematical tasks in their planning for instruction (Stein, et al., 2008). PLTs that focus on the interactions between students and mathematics allows for student thinking about mathematical ideas to be the center of teacher reflection.

PLTs featuring narrative cases provide opportunities for teachers to investigate the learning and teaching of mathematics in another person's classroom context. By providing a window into another teacher's mathematics instruction the case provides teachers with alternate images of practice than those they have personally experienced. Such images can include images of instructional practices that have been empirically shown to support student learning, such as using multiple representations (Brenner, et al., 1999) and cognitively demanding tasks (Silver & Stein, 1996). In addition to providing alternative images of practice, PLTs featuring narrative cases open up classroom mathematics instruction itself for collective inquiry. Teachers using such cases have opportunities to consider the mathematics being studied, students' engagement and understandings of said mathematics, and how the actions of the teacher shape and influence the mathematics being learned and how students engage with it. Teachers can reflect upon the mathematics, students' thinking, the teacher's pedagogy, and/or all three. Such PLTs "allow teachers to access, utilize, and develop knowledge of mathematics content, pedagogy, and student learning simultaneously" (Ponte, et al., 2009, p. 191). As such PLTs afford opportunities for such simultaneous learning they have been widely used (Ball & Cohen, 1999; Barnett, 1998; Derry, et al., 2007; Harrington & Garrison, 1992; Merseth, 1996; Silver, et al., 2007; Silver, et al., 2005).

The PLTs that prompt teachers to engage in future classroom activities with students provide opportunities for teachers to investigate the learning and teaching of those mathematical ideas in their own classroom context. These classroom activities include both conducting lessons using the mathematical tasks explored previously in the professional development curriculum materials and interviewing students around certain mathematical tasks. Both of these activities serve to provide teachers with opportunities learn in and from practice by connecting what they learned in professional development contexts to what they can learn from engaging in instructional practices in their own classroom contexts (Borko, 2004; Grossman, et al., 2008). Conducting lessons provides teachers an opportunity to try to apply what they learned in professional development about mathematics teaching and leaning to their own teaching. Conducting interviews with student around specific mathematical ideas provides teachers with opportunities to increase their knowledge of students' thinking (Jenkins, 2010).

Finally, PLTs featuring general reflection prompts provide opportunities for teachers to reflect upon issues of mathematics content and pedagogy generally and outside of the particular experiences they have had with the cases or in their classrooms. This provides teachers with opportunities to extend things they have learned about mathematics teaching and learning that was tied to specific contexts or experiences to more general contexts. Such PLTs provide opportunities for teachers to reflect on what the work of mathematics teaching entails generally and to reflect upon that work. Thus, teachers are provided opportunities to become knowledgeable both in and about teaching mathematics.

A question of alignment. In the quest to improve student achievement in mathematics nationally, attention has rightly fallen on the quality of mathematics instruction that students receive. Thus, the spotlight is on teachers. Teachers are being asked to transform their teaching practices (Stein, et al., 1999). They are being asked to move away from traditional instructional practices, such as lecturing and assigning large amounts of simple mathematical tasks to develop students' procedural fluency, and towards instructional practices found to better support student learning (National Council of Teachers of Mathematics, 2003), such as facilitating mathematical discussions and using cognitively demanding tasks to develop all strands of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive dispositions (National Research Council, 2001). In order for teachers to achieve such transformations in their mathematics teaching work they require support and professional development (Mewborn, 2003). We cannot expect teachers to change their instruction if opportunities are not provided for them to learn about and how to enact new instructional practices (Elmore, 2004).

Is there alignment between what teachers are being asked to do and what they are provided opportunities to learn to do? While this study does not answer this question generally, it does describe the degree to which the sampled professional development curriculum materials provide opportunities for teachers to learn the key mathematical content and pedagogy specified in RQ 1-3. Teachers are asked to help students develop their conceptual and procedural knowledge of proportional, rational numbers, and linear functions in the middle grades (National Council of Teachers of Mathematics, 2006). In the four curricula in my sample, teachers are provided opportunities to deepen their conceptual understandings of proportionality and rational numbers and develop their capacity to teach these mathematical topics. However, the curricula provide few opportunities for teachers to learn about linear functions and to develop their capacity to teach the topic which is often used to introduce algebra.

As using multiple representations supports meaning making in mathematics (Cuoco, 2001; Lemke, 2003; O'Halloran, 2003; Pape & Tchoshanov, 2001) and supports the participation and learning of students who are English-language learners (Brenner, et al., 1997; Chamot & O'Malley, 1994; Coggins, Kravin, Coates, & Carroll, 2007), teachers are encouraged to use them in their teaching. Such use is also recommended by the two process standards of communication and representation of mathematical ideas in the Standards document (National Council of Teachers of Mathematics, 2000). The four curricula provide many opportunities for teachers to use and make connections between multiple representations and ideas, especially connections between verbal descriptions, visual diagrams, and symbols. However, opportunities to learn to make connections between tables, graphs, and symbols, connections called for in the content expectations around linear functions (National Council of Teachers of Mathematics, 2000), are rarely provided.

One of the main consequences of the results from the TIMSS 1999 video study (Hiebert, et al., 2003; Stigler & Hiebert, 1999) is that there was a push for teachers to use more challenging mathematical tasks with students and use them without removing the challenging aspects for students. The four curricula, especially the ISBI and IIRP curricula, provided opportunities for teachers to learn about using cognitively demanding tasks themselves and about how such tasks can be used with students in instruction. The PLTs featuring narrative cases illustrate how teachers can use cognitively demanding tasks with students and either undermine students' learning opportunities by removing the challenging aspects of the task or maintain the learning opportunities by using the task as given and sustaining a press for explanation.

Overall, there seems to be a fair alignment between what teachers are being asked to do and what the sampled professional development curriculum materials provide them opportunities to learn to do. However, the lack of alignment around linear functions highlights the fact that the range of what teachers are being asked to do is larger than what these four curricula can address. Designers of long-term professional development programs for teachers should consider what combinations of professional development curriculum materials over time would provide the learning opportunities that would best align with the range of mathematical content and pedagogy needed for the work of teaching mathematics in the middle grades.

Educative Features of the Sample of Professional Development Curriculum Materials (RQ4)

In the United States a remarkable range of individuals facilitate teacher learning in professional development as professional developers. They vary in their level of training in mathematics and in teacher education, ranging from research mathematicians with little experience working with teachers to veteran science teacher educators new to working in the field of mathematics. They vary greatly in their preparation to work with mathematics teachers (Banilower, et al., 2006). Professional developers play an important role in teacher learning. Through the choices they make in how they set-up and enact the PLTs in professional development curriculum materials, they influence the learning opportunities that teachers have in professional development settings. They play an important role in the professional development of mathematics teachers, but due to their varied backgrounds probably need support in learning how to best fulfill this role. Unfortunately that support is difficult to find as there are few learning opportunities available to professional developers (Ball & Cohen, 1999).

Educative professional development curriculum materials can offer one avenue of support to the diverse individuals acting as professional developers. Educative professional development curriculum materials are designed to support the learning of both teachers and professional developers in professional development settings. They can provide opportunities for professional developers to learn how to support teacher learning around the PLTs in the curriculum and in learning activities generally. Though the learning that can be gained is probably not enough in itself to fully develop professional developers' capacity to facilitate the learning of mathematics teachers, it presents a useful learning resource for professional developers.

The learning needs of professional developers varies with their backgrounds. Professional developers' knowledge and backgrounds can vary along two continuums, as seen in Figure 5.2: that of mathematics and that of teacher education.

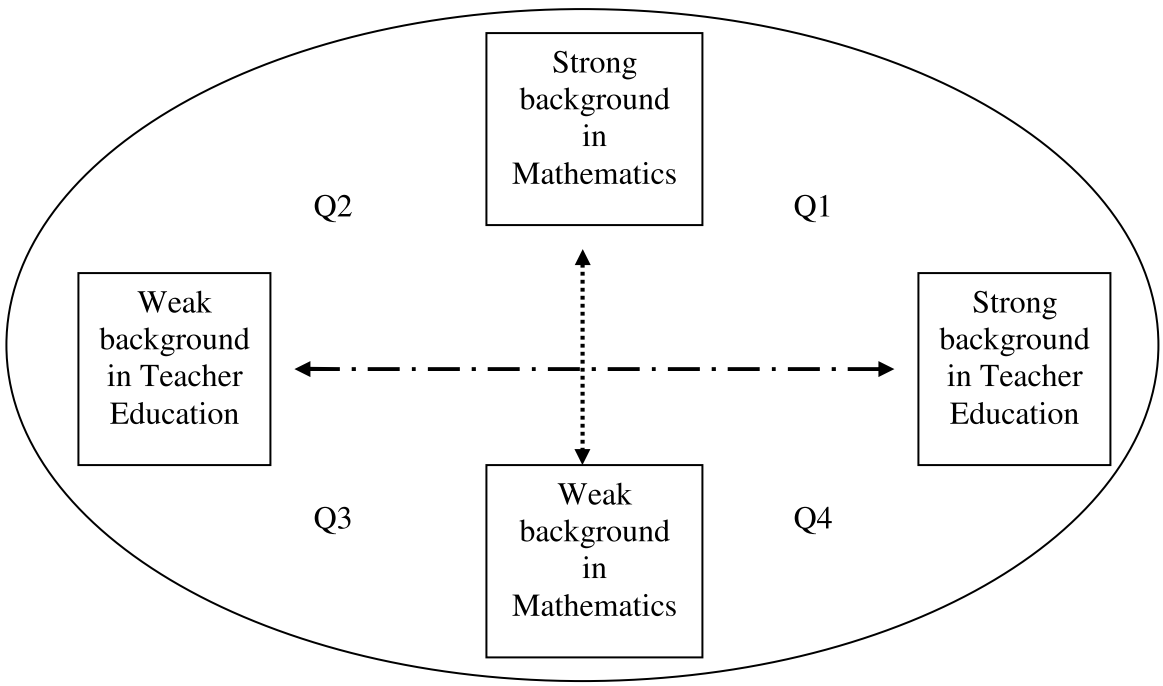


Figure 5.2. Continuums along which professional developers differ in their preparation for the work of supporting teacher learning.

Professional developers could be anywhere along either continuum. Their learning needs would vary depending on their location. For example, a research mathematician with a strong background in mathematics but little experience working with teachers would be in quadrant 2 of Figure 5.2 and would likely need to learn about facilitating teacher learning. He or she would likely need to learn about how to support teachers to engage in collective inquiry into mathematics instruction. In comparison, a veteran science teacher educator coming into mathematics professional development may already have knowledge of how to engage teachers in reflection about instruction but may need to learn about the mathematical content being addressed. Educative professional development curriculum materials face the challenge of meeting the varied learning needs of the diverse population of professional developers.

In designing the EF Framework I took into account the challenges of the work of facilitating teacher learning in professional development that would face an individual with a weak background in both mathematics and teacher education. Assuming such a low level of preparation for the work resulted in a framework that includes 20 different items of support that professional developers could potentially find helpful. Not every professional developer would need every item of support in the framework. Continuing the example, a research mathematician would probably not need to have explained to him or her the conceptual underpinnings of a mathematical topic (item 1B of Appendix C), but probably would need an explanation about why certain pedagogical moves are being focused upon (item 2A). Given the wide variety in the backgrounds of professional developers (Banilower, et al., 2006) considering a range of educative features in analysis seems prudent. Such an analysis would reveal the learning opportunities suited to a variety of professional developers.

The educative features most frequently present in the four curricula are: the provision of guidance on how to facilitate common discourse (item 5B); the provision of guidance on anticipating common teacher ideas (item 4B); and the explanations of the design and learning goals of individual PLTs in the curriculum (item 6D). These features relate to supporting collective inquiry and common discourse and to planning for facilitation. As they relate to activities core to facilitating professional development, it seems appropriate that these are the ones most commonly provided to professional developers. The facilitation of a common discourse (item 5B) supports the building of professional learning communities which research has shown can support teacher learning (Borko, 2004; Cochran-Smith & Lytle, 1999; Hammerness, et al., 2005). Professional developers have found building such learning communities that use common discourse to engage in collective inquiry of mathematics teaching and learning extremely useful for supporting teacher learning and extremely difficult to do (Stein, et al., 1999). In the four curricula, the guidance on how to do so provides professional developers with opportunities to learn this much needed skill. Planning for professional development is important and both guidance on anticipating teacher ideas (item 4B) and explanations of the design and goals of PLTs (item 6D) support professional developers' planning. Effective professional development has sessions that are coherent with teachers' other learning activities (Garet, et al., 2001). Information about the learning goals and design of PLTs supports professional developers in organizing coherent learning opportunities for teachers over time. In these activities with teachers professional developers will be called upon to lead productive discussions about mathematical content and pedagogy. Just as one of the five practices for orchestrating productive mathematical discussions with students is to anticipate their reactions to mathematical tasks (Stein, et al., 2008), professional developers can better support discussions with teachers by anticipating their reactions to PLTs. The information provided about common teacher ideas and reactions to PLTs (4B) supports professional developers in being able to anticipate teacher ideas.

Several educative features were infrequently present in the curricula. Explanations of the importance of long-term professional development (item 6A) were not present at all. While there may be several possible explanations, this may be due to the fact that there is not shared meaning for what “long term” professional development means. The provision of guidance on how to model reform oriented pedagogy (item 2B) was only sighted once, possibly also due to ambiguity over what counts as a “reform oriented” practice. Explanations of how the curriculum is aligned with state and national standards (item 6C) were only present a few times. This was probably done to ensure the longevity of the curricula. Standards documents seem to change faster than professional development curricula. In fact, as I am writing this the new Common Core State Standards (2010), which may overshadow the NCTM Standards (2000), are being finalized.

The four curricula in my sample vary greatly in the extent to which they can be described as educative. The average number of educative features per page varies with the IIRP curriculum having an average of 5.23, the ISBI curriculum having one of 3.15, the DMIMMO curriculum having one of 2.09, and the TFRU curriculum having an average of only 1.25. However, the ways that the curricula focus on certain features over others does seem to be fairly consistent across the sample. The variance in the degree to which the four curricula appear to be educative should not be taken as a ranking of their value. Rather, the variance in the extent and ways in which they are educative simply affects for whom they will be most useful.

Recall that professional developers have a variety of backgrounds and preparation for their work and will need differing levels and types of learning opportunities. The four curricula offer different learning opportunities to professional developers. The learning opportunities they do provide can be thought to correspond to professional developers' backgrounds and the quadrants in Figure 5.2. A professional developer in the first quadrant (Q1) with a strong background in both mathematics and teacher education would easily be able to use any of the four curricula with teachers. However, a professional developer in the second or third quadrants with a weak background in teacher education would benefit from the support offered by the educative features in the IIRP and DMIMMO curricula. The TFRU curriculum, though is has few educative features could be helpful for a professional developer if they were in the fourth quadrant. Such an individual already has a strong background in teacher education but, having a weak background in mathematics, would benefit from the wealth of opportunities the TFRU curriculum provides to learn specific mathematical content.

The professional development curriculum materials analyzed in this study showcase the different degrees to which curriculum materials can be categorized as *educative professional development curriculum materials*. Educative professional development curriculum materials provide varied opportunities for professional developers to learn about how to support teachers in mathematics professional development. With the challenges involved in effectively supporting teacher learning (Stein, et al., 1999) and the limited learning opportunities available to learn how to do so (Ball & Cohen, 1999), educative professional development curriculum materials can offer much needed support to professional developers.

Potential Contributions and Implications

In order to improve the quality and effectiveness of mathematics teaching and learning in the United States, teachers' learning in professional development is crucial (Mewborn, 2003; Sykes, 1996). Effectively supporting teacher learning in professional development settings is a serious and yet to be solved research problem (Borko, 2004). In part, it remains an unsolved problem as in order to effectively support teaching learning in professional development such learning needs to be better understood. Understanding teacher learning in any context includes attention to both the content and pedagogy involved (Ball & Cohen, 1999; Wilson & Berne, 1999). However, little attention has been paid in the research literature to the content of professional development (Garet, et al., 2001). This study addresses this gap in the research literature by mapping out the content to be learned in professional development programs that employ the four, commonly used, professional development curricula. By identifying one half of the puzzle, the content that can be learned, this study contributes towards our improved understanding of teacher learning in professional development contexts.

This study not only explored the content of teacher learning in professional development but also explored how curriculum materials can support professional developers in their work with teachers. My extension of the concept of educative curriculum materials from the K-12 space to the professional development space is an important contribution of this study. Educative curriculum materials have traditionally been conceptualized as school curriculum designed to support both teacher and student learning (Beyer, et al., 2009; Davis & Krajcik, 2005; Schneider & Krajcik, 2002). I have extended this concept to introduce the notion of educative professional development curriculum materials that are designed to support both professional developer and teacher learning and open up an interesting avenue for exploring how to support the learning of professional developers.

Developing professional developers who can support the learning of mathematics teachers is a long-term and continuous process. Individuals come into the work from a range of backgrounds (Banilower, et al., 2006) and need to continually adapt their knowledge, skills, and dispositions to the new realities of what teachers need to learn to do (Stein, et al., 1999). Ongoing opportunities to learn to do this dynamic work are sorely needed but scarcely found (Ball & Cohen, 1999). Further, the knowledge and skills needed for this work is understudied. Elliott and colleagues (2009) assert that "filling the knowledge gap in the research on leading PD [professional development] is an urgent issue if teacher learning is to be improved and adequately addressed" (p. 365). I suggest that educative professional development curriculum materials can provide an exciting avenue for professional developers to learn about doing the work of leading professional development. This study by exploring the learning opportunities provided to professional developers through educative professional development curriculum materials contributes to the small, but growing, research on learning to lead professional development.

The results of this study have practical implications for teacher education and the design of programs for professional development. For teacher educators and facilitators of professional development, the identification of the opportunities for teacher learning presented in the analyzed curricula can inform their selection of both curriculum materials and PLTs within materials. The four curricula have different foci and I have identified the different combinations of learning opportunities that they provide to teachers. In order to design effective professional development that is on-going with coherent learning opportunities (Garet, et al., 2001) instead of the fragmented courses previously offered (Ball & Cohen, 1999), professional developers can use this information to sequence their use of the curricula to best achieve their instructional goals, maintain coherence, and capitalize on the strengths of each curriculum. They can also consider how to supplement the learning these curricula offer in order to provide teachers with opportunities to learn the content that is not addressed in these curricula, such as the mathematical topic of linear functions or experiences connecting symbolic and graphical representations. The findings of this study about what learning opportunities were and were not provided can be used to inform teacher educators' and professional developers' curricular choices as they plan professional development programs that support teacher learning over time.

The results of this study's educative analysis contribute to our understanding of how to support the work of teacher educators and professional developers. I suggest that one avenue for the ongoing learning of professional developers can be educative professional development curriculum materials. While learning from these curricula will probably not be sufficient to learn all that a professional developer needs to know, it can be a useful support for them. Professional developers, especially novice professional developers, would be well served to identify and use curriculum materials that are particularly educative, such as the IIRP curriculum. This could be aided by information being shared within the educational community about the extent to which and the ways in which various professional development curriculum materials are educative. With such information professional developers could select materials that would best support their own learning needs. For example, professional developers may choose to use the ISBI curriculum to learn about how to differentiate and use cognitively demanding tasks or the TFRU curriculum to learn more about the mathematical concepts underlying ratio and fraction. The results of my study can contribute to the selection of curriculum by professional developers as it identifies the ways that each of the four curricula would be particularly helpful supporting the learning of professional developers from different backgrounds.

The results of this study have implications for the design of curriculum for professional development. It provides curriculum designers with information on the learning opportunities that four widely used curricula provide for mathematics teachers and professional developers. It, thereby, also identifies the content that these curricula do not address and that can be addressed by other curricula, such as linear functions. It supports the design of educative professional development curriculum materials by listing the educative features that can be designed into curriculum materials to support professional developers and explains the need to do so.

Designers of professional development curriculum materials can glean from the study's results that they should, as much as possible, include multiple representations and provide opportunities to make connections between them, address mathematical topics of early algebra, such as linear functions, select artifacts of practice for use in PLTs that direct attention to the desired aspects of mathematics teaching and learning, and write with both teachers and professional developers in mind. When writing with professional developers in mind, authors can clearly articulate what mathematical content and pedagogy they are choosing to focus upon and why, and can provide guidance to professional developers about how to support teachers to achieve the authors' learning goals.

The methods and analytic frameworks that were developed and used in this study constitute another potential contribution of the study. Researchers can replicate the study itself to explore the content of professional development curriculum materials designed for other groups of teachers, such as those who teach high school. The analytic frameworks could be used to analyze the learning opportunities for mathematics teachers and professional developers in other samples of professional development curriculum materials. For example, the OTL framework could be used to analyze and compare the learning opportunities provided to mathematics teachers by professional development curricula used in different countries. The EF Framework could be employed to further explore whether and how various professional development curricula are educative. It could be useful in future studies that seek to explore how educative professional development curriculum materials can support the learning of professional developers and impact the professional development experiences of teachers.

In this chapter I have shared my hypothesis about how different designs for PLTs affect teachers' learning opportunities. I have suggested that the type of artifact of practice chosen for use directs teachers' attention to different aspects of mathematics instruction and affords different learning opportunities. For example, mathematical tasks allow teachers to focus on the mathematics being taught (Zaslavsky & Leikin, 2004) while narrative cases allow teachers to also reflect upon how that mathematics can be taught and learned in various classroom contexts (Harrington & Garrison, 1992). This hypothesis can encourage further thought about how PLTs shape the context of teachers' thinking and the learning that can occur.

Building on the results of my study, several studies can be conducted in the future to explore teacher learning in professional development settings. Such studies could be organized around the Professional Learning Tasks Framework (Stein, et al., 2001) that illustrates how PLTs unfold during use in professional development (see Figure 5.3).

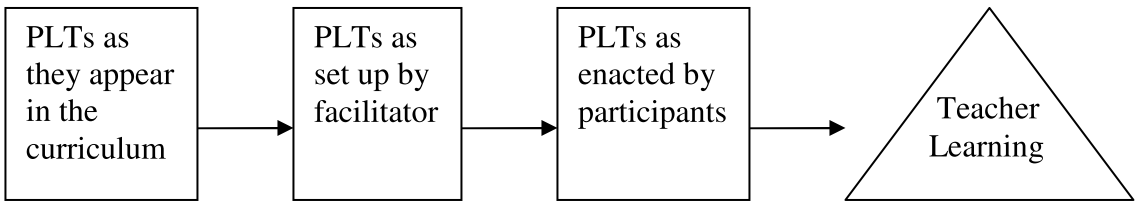


Figure 5.3. The professional learning tasks framework.

My study investigated the first square of the framework: the learning opportunities provided to teachers in the PLTs as they appear in the curriculum. Future research studies could build on my study to investigate the other elements of the framework.

Future Research

This study not only fills a gap in our knowledge about the content of teacher learning in professional development, but also provides a foundation upon which several future research studies can be built. My study addresses the first square in the PLT framework (Figure 5.3) and investigates PLTs as they appear in professional development curriculum materials. Researchers can use the methods and results of my study to explore teacher learning and the link between professional development curriculum materials and teacher learning in different ways and in different contexts.

Using the methodology of my study, a researcher could continue to investigate the first square of the PLT framework but also extend the analysis of teacher learning opportunities in professional development curriculum materials from a national to an international level. Given the range of student achievement in mathematics across countries and the important role that teachers have on student achievement, attention to teacher learning across countries is warranted. Researchers at 60 research institutions worldwide, in collaboration with the International Association for the Evaluation of Educational Achievement (IEA), have studied teacher learning in the TEDS-M project by conducting a comparative study of the teacher preparation of primary and secondary mathematics teachers (International Association for the Evaluation of Educational Achievement, 2008). While this study would shed light on how teachers are prepared in various countries, it will not address how they are supported in their teaching. Such information is important as it influences teachers' instruction. For example, in the TIMSS 1999 Video Study, 59% of teachers in the Netherlands reported that cooperative work with other teachers played a major role in their lesson content (Hiebert, et al., 2003). A comparative study of teacher education that focuses on professional development as a site for such collaborations would explore the ways teachers are supported in their work and allow us to better understand how such support influences their mathematics instruction. Such a study could be conducted using the methods and analytic frameworks developed in my study. This study would provide information on the opportunities to learn provided to practicing teachers across countries and give insight into the type of on-going support they receive through their careers. If such a study were carried out with countries that had scored higher than the United States on international comparison studies (Baldi, et al., 2007; Hiebert, et al., 2003), then one could also investigate if there is a significant relationship between teachers' opportunities for continued professional learning and student scores between countries. Thus, the study could build on both my dissertation study and existing international comparison studies to explore professional development curriculum materials.

Future research studies could continue the investigation of the first square of the PLT framework by modifying the methods and foci of my study. Researchers could take a more nuanced view and consider teachers' opportunities to learn specific classroom practices, such as how to manage student errors. They could also use an elaborated theory of teacher knowledge, such as mathematical knowledge for teaching, to delve into teachers' opportunities to learn "specialized content knowledge" (Ball, Thames, & Phelps, 2008) which is an essential part of developing teachers' capacity to successfully do the work of mathematics teaching.

Researchers could build upon the results of my study to investigate how the learning opportunities I have identified play out in the second two squares of the PLT framework which cover the implementation of the curriculum in professional development. By recording and investigatig professional development sessions in which participants engage with PLTs from the curricula analyzed in this study, researchers can explore how the pedagogy used by the professional developer(s) and the interactions among participants shape the previously identified learning opportunities. In their investigations researchers could focus on teacher leaning around the use of representations, the use of cognitively demanding mathematical tasks, or specific topics in middle grade mathematics.

Several research studies could be designed to address the PLT framework as a whole. Building on my study, they could explore how the learning opportunities provided in the curriculum and shaped through enactment in professional development sessions are translated into teacher learning as evidenced by teachers' classroom instructional practices. One study could explore the extent to which and the ways in which teachers take advantage in professional development of the learning opportunities in the sample curriculum materials and apply that learning in their classroom mathematics instruction. Another study could be conducted to test my hypotheses about the link between the design of PLTs and teacher learning about specific aspects of mathematics instruction. Since the four curricula provided different opportunities to learn about the instructional factors associated with using cognitively demanding mathematical tasks in instruction, a study could be designed to explore the impact of these differences. The study could investigate how the differences in learning opportunities presented in the curricula impacted teachers' learning and the classroom instructional practices they employed when using cognitively demanding tasks.

Conclusion

Professional development is an important site for middle school mathematics teachers to learn ideas central to improving their instructional practice. Since many teacher preparation programs do not focus on the mathematical topics addressed in the middle grades (Conference Board of the Mathematical Sciences, 2001; Hillen, 1996), professional development is the main site wherein teachers can explore these mathematical ideas, how students think about and learn them, and how they can effectively teach them. In professional development teachers can engage with professional learning tasks to explore mathematics instruction and learn about mathematical content and pedagogy. Given the importance of this space for teacher learning, it is problematic that little is known about what teachers are provided opportunities to learn in professional development.

This study addresses this research problem and builds up our understanding of teacher learning in professional development by exploring the opportunities to learn provided by widely used professional development curriculum materials to middle school mathematics teachers. It also addresses how curriculum materials might also provide opportunities to learn to professional developers in how to support teachers' learning. Through a curricular analysis of four curricula, I have identified the opportunities these widely used curricula provide for teachers to learn about specific mathematical topics, about the use of multiple representations, and the use of cognitively demanding mathematical tasks, and the opportunities they provide for professional developers to learn about supporting teacher learning.

To develop our understanding of teacher learning in professional development settings we need to better understand both *what* teachers learn and *how* they are taught (Wilson & Berne, 1999). I have charted out *what* large numbers of teachers using the four curricula have opportunities to learn in professional development. My dissertation study, therefore, has contributed to a better understanding of middle school mathematics teachers' learning in professional development and has set the stage for future studies that can research *how* these learning opportunities unfold in professional development. It has contributed to a better understanding of teacher learning in and through professional development.

Appendix A:
Survey of Mathematics Professional Development Curriculum Usage

Please state **your name**: ____

Please state the **name of your project**: ____

What is the approximate **total number** of mathematics teachers of grades 5-8 for whom your project has provided professional development? ____

To what extent did you use the following published materials with teachers of mathematics in grades 5-8?	Never used with teachers	Used less than half of the resource	Used more than half of the resource	Used all of the resource	If you used the materials**, what goal(s) did you believe they could help you achieve?** (e.g. promote teachers' learning about mathematical content, pedagogy…etc) Additionally, if your project has multiple cohorts of teachers, then please indicate the cohorts with which the curriculum was used.
Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Strategies for Teachers (Lamon, 2005)					
Improving Instruction in Rational Numbers and Proportionality: Using Cases to Transform Mathematics Teaching and Learning, Volume 1 (Smith, Silver, Stein, Boston, et al, 2005)					

To what extent did you use the following published materials with teachers of mathematics in grades 5-8?	Never used with teachers	Used less than half of the resource	Used more than half of the resource	Used all of the resource	If you used the materials**, what goal(s) did you believe they could help you achieve?** (e.g. promote teachers' learning about mathematical content, pedagogy…etc) Additionally, if your project has multiple cohorts of teachers, then please indicate the cohorts with which the curriculum was used.
Improving Instruction in Algebra: Using Cases to Transform Mathematics Teaching and Learning, Volume 2 (Smith, Silver, Stein, Henningsen, et al, 2005)					
Mathematics Teaching Cases: Fractions, Decimals, Ratios, and Percents: Hard to Teach and Hard to Learn? (Barnett, Goldenstein, & Jackson, 1999)					
Windows on Teaching Math: Cases of Middle and Secondary Classrooms (Merseth, 2003)					
Implementing Standards-Based Instruction: A Casebook for Professional Development (Stein, Smith, Henningsen, & Silver, 2000)					
Mathematics Curriculum Topic Study: Bridging the Gap Between Standards and Practice (Keeley & Rose, 2006)					

If your project uses publicly available, professional development curriculum materials that were <u>NOT</u> listed, then please write the name(s) of those materials below	Used less than half of the resource	Used more than half of the resource	Used all of the resource	**What goal(s) did you believe the materials could help you achieve?** (e.g. promote teachers' learning about mathematical content, pedagogy…etc) Additionally, if your project has multiple cohorts of teachers, then please indicate the cohorts with which the curriculum was used.

I would be happy to send you a **summary of the study's findings** at its conclusion in summer 2010. Please indicate if you would prefer NOT to receive a summary of the study's findings.

□ No, thank you. Please do not send me a summary of the study's findings.

If you have any **comments or suggestions** for my study, then please feel encouraged to share them below.

____
____
____
____
____

Thank you for taking the time to participate!

Please return to Jenny Sealy Badee, 1600 School of Education, University of Michigan, 610 E. University Ave, Ann Arbor, MI, 48109-1259

Appendix B:
Framework for the Analysis of Teachers' Opportunities to Learn in the Sampled Curricula (OTL Framework)

Horizontal Analysis of the Curriculum

Background information

Title	
Year of Publication	
Number of books / Accompanying materials	
Number of pages	
Profile of authors at the time of publication	
Publisher	
Grade level specifics	
Suggested timeframe for use	

Overall Structure & Sequence

Page #	Description	Math topic(s)	Link to practice – artifact(s) of practice used	Representations presented in mathematical task	Representations presented in link to practice component

Vertical Analysis of Individual PLTs

Section 1: Analysis of the Mathematical Task:

In this section only consider the mathematical task itself. What does the math task ask the solver to do? *In the section 2, you will continue to analyze the solutions and discussion associated with the task.*

Choose the level of cognitive demand of the task.

Level of Cognitive Demand	**Present**	**Description**
Memorization - reproducing or memorizing facts - procedure not useful - not ambiguous - no connection to underlying concepts		
Procedures Without Connections - algorithmic - little ambiguity - no connection to underlying concepts - focused on correct answer - no explanation of thinking required		
Procedures With Connections - focus on use of procedure to develop concepts - broad procedures linked to concepts - represented in multiple ways and connections made to develop meaning - some cognitive effort required		
Doing Mathematics - complex & nonalgorithmic thinking - explore concepts, processes or relationships - self regulation - access relevant knowledge and experiences - examine task constraints - considerable cognitive effort required		

Indicate the representations the solver MUST use.

Use of Representations Those **required** to be used (**given or explicitly asked for in the task itself**)	**Present**	**Description**
Verbal		
Symbolic / Equation		
Graph		
Table		
Diagram/Model		
Physical Object		

Choose the appropriate categories that describe the mathematics in the task.

Mathematical Structure	**Present**	**Description**
Mathematical Topic: 1. ratio 2. rate (density) 3. rate (related to time) 4. rate (buy/consume) 5. similarity 6. scale 7. probability 8. percents 9. linear equations 10. linear patterns and relationships 11. slope 12. frequency distributions 13. fractions 14. decimals 15. other		
Task Type: 1. ratio 2. rate 3. comparison 4. missing value 5. scale factor 6. qualitative 7. non-proportional		
Numbers: 1. all integers 2. all rationals 3. both integers and rationals 4. other		
Multiplicative Relationships between and within measure spaces 1. all integers (e.g. 1:2 and 3:6) 2. some non-integers (e.g. 2:3 & 10:15) 3. all non-integers (e.g. 2:3 & 5:7.5)		

Ratio Relationships 1. Part:Whole 2. Part:Part		

Indicate what the mathematical task requires the solver to do:

Required Response	Present	Description
Type of response to be given: 1. Answer only 2. Explanation 3. Justification		
Requires that connections be made between: 1. Verbal 2. Symbolic/Equation 3. Graph 4. Table 5. Diagram/Model 6. Physical object		

Section2: Analysis of the Link to Practice Component

Now analyze the materials that constitute the rest of the PLT (artifacts of practice, reflection questions….etc)

Choose the main artifact of practice presented for use with teachers.

Artifact of Practice Used	Present	Description
Narrative Case (description of instruction)		
Student work samples		
Research study results		
Other		

Indicate the pedagogical issues that are emphasized in the PLT.

Pedagogical Issues	Present	Description
Additive versus Multiplicative reasoning		
Unitizing		
Equations – using formal symbols		
Ratio Referent		
Meaning of quantities		
Cross Product		
Student Struggle		
Cognitive Demand		
Use of multiple representations		
Use of multiple student solutions		

Language		
Explanations/Justifications		
Student Motivation		
Student thinking		
Other		

If instruction is described, then indicate which instructional factors were illustrated for teachers' consideration.

Instructional Factors associated with the Maintenance or Decline of Cognitive Demand	**Present**	**Description**
Maintaining problem complexity / Assisting through scaffolding (+)		
Sustained press for justification (+)		
Building on student knowledge and thinking (+)		
Teacher draws frequent conceptual connections (+)		
Modeling high-level thinking (+)		
Students provided with means of monitoring their progress (+)		
Provision of enough time on task		
Challenges become nonproblems (-)		
Shift of emphasis from meaning to correctness (-)		
Students not held accountable for high level products/processes (-)		
Classroom management problems (-)		
Inappropriateness of task (-)		

Indicate which representations of mathematical ideas were presented for teachers' consideration.

Use of representations Those provided in the artifacts examined or explicitly called for	**Present**	**Description**
Verbal		
Symbolic / Equation		
Graph		
Table		
Diagram/Model		
Physical object		

Indicate what teachers are asked to do.

Type of response required	Present	Description
Participation in group discussion - prompts ask for evidence to support conjectures offered - prompts orient participants to each other - prompts require teachers to relate learning to own practice		
Analysis - Analysis of student thinking and learning - Analysis of teacher pedagogical moves - Analysis of mathematical task		
Written reflection		
Creation of object (e.g. lesson plan)		
Make a lesson plan or conduct a lesson using a cognitively demanding task		
Other		

Section3: Additional Analyses

Consider the entire PLT and indicate if it presented any of the following opportunities to teachers.

Opportunities to develop multiplicative reasoning	Present	Description
Task requires teachers to reason explicitly in terms of quantities and quantitative relationships		
Task appropriately uses and contrasts additive and multiplicative reasoning		
Task exposes teachers to situations that allow them to see proportional reasoning as a complex process that evolves over a long period of time		
Task allows for connections to be made among forms of rational numbers and with concepts of ratio and proportion.		

Appendix C:
A Framework for the Analysis of the Educative Features in Professional Development Curriculum Materials
(EF Framework)

This framework is to be used to analyze professional development curriculum materials designed for use with teachers to see the extent to which and the ways in which the materials also provide opportunities for **professional developers** to learn to support teacher learning in professional development settings.

#	Category	Items	Type	Example(s)
1	Supporting professional developers in engaging teachers with **specific mathematical topics**	E. Explain the value of revisiting and deepening teachers' understanding of mathematical topics that they teach	Rationale	"Despite strong consensus that proportional reasoning is central to the development of solid mathematical proficiency in the middle grades, there is much evidence that this domain is exceptionally challenging for students. Moreover, some teachers have difficulty differentiating situations in which the comparison between quantities is multiplicative rather than additive, tend to use additive strategies when multiplicative approaches would be appropriate, and do not recognize ratios as a multiplicative comparison." (Smith, et al., 2005c, p. 77)
		F. Explain the conceptual underpinnings of and the connections between the mathematical topics under discussion	Rationale	"The mathematical model for proportional relationships is a linear function of the form y=kx, where k is called the constant of proportionality. Thus, y is a constant multiple of x. Equivalently, two quantities are proportional when they vary in such away that they maintain a constant ratio: y/x = k." (Lamon, 2005, p. 4)
		G. Provide guidance on which strategies to use to focus attention on the conceptual aspects of the mathematical topic(s)	Implementation guidance	[E.g. Have teachers solve proportional problems in at least 2 different ways and without the use of standard algorithms]

		H. Provide guidance on which representations might support discussion of the mathematical topic(s)	Implementation guidance	[E.g. Suggest that teachers employ ratio tables to demonstrate the multiplicative relationship between and within ratios. Also provide examples of how students may use ratio tables to reinforce their incorrect additive reasoning and how teachers can extend the tables to challenge such thinking.]
2	Supporting professional developers in engaging teachers with **specific pedagogy**	D. Explain why the pedagogical moves being illustrated have been focused upon and provide evidence that they are effective in promoting student learning	Rationale	[E.g. Research on how people learn has pointed out the importance on building on students' prior knowledge when introducing ideas. Consider how the teacher in the case makes use of or misses opportunities to use students' prior experiences and knowledge to support learning.]
		E. Provide guidance on how to model the "reform oriented" pedagogical moves with teachers that they are encouraged to use with their students.	Implementation guidance	[E.g. Suggest that professional developers build upon teacher's prior experience. Encourage professional developers to have teachers discuss the pedagogy described in a case in relation to an instance of their use of the pedagogy in their own classroom.]
		F. Provide guidance on strategies to use to focus attention on specific aspects of classroom pedagogy.	Implementation guidance	[E.g. Consider the ways in which the teacher uses and directs attention to graphs. How is the graph used to illustrate the mathematical ideas? How could the graph have been used more effectively?]

3	Supporting professional developers in engaging teachers with professional learning tasks **centered on the work of mathematics teaching**	C. Explain why learning activities centered on the work of teaching and situated in classroom practice are desirable for promoting teachers learning	Rationale	[E.g. Teachers, like anyone else, learn best when they can connect their learning to prior knowledge and experiences. Situating their learning in classroom practices allows for teachers to draw on their prior knowledge and supports connecting the ideas learned to the classroom context in which they are to be utilized.]
		D. Provide guidance on how to collaborate with teachers to reflect upon and engage in collective inquiry into mathematics instruction in relation to the example from the PLT, their own practice, and more generally	Implementation guidance	"If you have additional time, you may want to provide teachers with opportunities to consider their own practice and continue to explore the mathematical and pedagogical ideas on which the case is based…in Chapter 6 suggests three types of activities you might want to assign teachers for these purposes." (Smith, et al., 2005c, p. 87)
4	Supporting professional developers in **anticipating and working with teachers' ideas** about mathematics teaching and learning	C. Explain why supporting teacher learning means attending to teachers' knowledge, beliefs, and habits of practice	Rationale	[E.g. The goal of the current reform is to enact changes in teaching practices in US math classrooms. To reach this goal "the kind of learning that will be required has been described as transformative, that is, as requiring wholesale changes in deeply held beliefs, knowledge, and habits of practice" (Stein, et al., 1999, p. 238).]
		D. Provide guidance on when to expect certain common teacher ideas to emerge.	Implementation guidance	"Watch for a common misconception – attempting to increase the number of candies by adding a constant value to each number in the original ratio…If participants use this incorrect additive strategy, you may want to ask them to consider the relationship between the Jawbreakers and the Jolly Ranchers…This might help participants realize that adding a constant value to each term of the original ratio does not produce an equivalent ratio." (Smith, et al., 2005c, p. 93)

5	Supporting professional developers in building a **professional learning community** with teachers	D. Explain why collegial support and learning within a community are important for teacher learning both individually and collectively	Rationale	“These cases are written to stimulate collaborative reflection through discussion. Group interactions enable one to consider alternative perspectives and question the status quo.” (Goldenstein, Barnett-Clarke, & Jackson, 1994, p. xv)
		E. Provide guidance on how to facilitate and create a common discourse for teachers to engage in collective inquiry about mathematics teaching and learning	Implementation guidance	[E.g. The text could suggest the establishment of group norms for making conjectures, having discussions, and justifying claims based on evidence. For example, teachers could be required to provide evidence from the artifacts of practice being analyzed for their claims about what occurred.]
		F. Provide guidance on ways to create and support a professional learning community of teachers	Implementation guidance	[E.g. Facilitators should ensure that teachers experience a core set of shared activities around which future interactions can be built.]
6	Supporting professional developers to provide **long-term, ongoing, and coherent** professional development programs **in**	G. Explain the benefits of an ongoing and long-term professional development program	Rationale	[E.g. Teacher learning is a long term, nonlinear process that can be supported by an ongoing, coherent professional development program wherein teachers have opportunities to revisit ideas and use prior knowledge and experiences to actively build new understandings.]
		H. Explain the sequencing of PLTs over the curriculum and the overall learning goals for teachers that the sequence addresses	Rationale	[E.g. Several key instructional issues are repeatedly touched upon in the tasks and, thus, can be revisited and discussed in greater and greater depth. For example, the use of multiple representations appears frequently. Teachers can initially discuss the value of using multiple representations and later discuss ideal combinations of representations to convey particular mathematical ideas.]

	various contexts	I. Explain how the curriculum is aligned to state and national standards and current theories on teacher learning as a long-term process	Rationale	"The cases do contain illustrations of many ideas advocated by reform documents such as the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics* (1989) and *Professional Standards for Teaching Mathematics* (1991). For example, they highlight alternative assessment methods, the use of manipulatives, the relevance of language and writing, connections among mathematical ideas…" (Goldenstein, et al., 1994, p. xiv)
		J. Explain the design and the teacher learning goals (both the learning of mathematical content and pedagogy) of individual PLTs	Rationale	"The primary purpose of the Opening Activity is to engage teachers with the mathematical ideas that they will encounter when they read the case…The problems are intended to highlight important aspects of proportional reasoning…and allow teachers to develop strategies that make sense (e.g., using a ratio table, finding a unit rate, applying a scale factor)." (Smith, et al., 2005c, p. 93)
		K. Provide guidance on how and when to modify PLTs or the sequence of PLTs in order to fit the local culture or context of PD	Implementation guidance	"The cases can be used in a variety of ways. For example, facilitators may want to build a focused course around all four cases...In other situations, facilitators may want to select one or two specific cases to blend into an existing teacher education agenda…Whatever the situation, in the chapters that follow facilitators will find the support they need to make optimal use of the each of the cases." (Smith, et al., 2005c, p. 80)
		L. Provide guidance on various strategies to continuously engage teachers and when to employ them	Implementation Guidance	[E.g. To illustrate how the cases can support teachers' instructional improvement, the facilitator could collaborate with teachers to create a list of instructional practices that they feel are especially supportive of student learning and difficult to enact. The group could then analyze how the teacher(s) in the cases managed the use of these practices.]

Definitions:

Professional Developers – Individuals who facilitate professional development activities and support teachers' learning to improve their mathematics instruction

Professional Learning Tasks (PLTs) – The tasks and activities given in the curricula for teachers to do that involve working with a mathematical task and a representation of how the mathematical ideas in the task play out in instructional practice.

Appendix D:
Example of Horizontal Analysis using the OTL Framework

Horizontal Analysis
of the
Improving Instruction in Rational Numbers and Proportionality (IIRP) curriculum

Background information

H1	Title	Improving Instruction in Rational Numbers and Proportionality
H2	Year of Publication	2005
H3	Number of books / Accompanying materials	1
H4	Number of pages	136
H5	Profile of developers at the time of publication	- Margaret Smith is a professor at the University of Pittsburg. She has a PhD in mathematics education, experience as a math teacher, and a focus on teacher education. - Edward Silver is a professor at the University of Michigan. He has an EdD in mathematics education, experience as a math teacher, and focuses on a range of topics including using challenging tasks in instruction. - Mary Kay Stein is a professor at the University of Pittsburg. She has a PhD in educational psychology, and a focus on educational reform. - Melissa Boston is a doctoral student at the University of Pittsburg. She has experience as a math teacher and her research focuses upon the use of cases to facilitate teacher learning. - Marjorie Henningsen is a professor at the American University of Beirut. She has a PhD in mathematics education and a focus on professional development. - Amy Hillen is a doctoral student at the University of Pittsburg.
H6	Publisher	Teachers College Press
H7	Grade level specifics	Middle Grades 6-8

Overall Structure & Sequence

Page #	Description	Math topic(s)	Link to practice – artifact(s) of practice used	Representations presented in math task	Representations presented in link to practice component
3-7	Facilitator Notes about using cases				

Page #	Description	Math topic(s)	Link to practice – artifact(s) of practice used	Representations presented in math task	Representations presented in link to practice component
8-25	PLT The case of Randy Harris	Connecting fractions, decimals & percents	Narrative case Prompts to analyze case, connect to own practice and connect to curriculum and research	Diagram (non-conventional grid) Decimal Fraction Percent	Diagram (grids with various shadings) Decimal Fraction Percent
26-43	PLT The case of Marie Hanson	Ratio Proportional reasoning	Narrative case Prompts to analyze case, connect to own practice and connect to curriculum and research	Diagram Word problems	Diagrams of ratio relationship (given and unit rate) Ratio Table Symbolic
44-57	PLT The case of Marcia Green	Similar figures Scale factor	Narrative case Prompts to analyze case, connect to own practice and connect to curriculum and research	Diagram Manipulatives used to create enlarged diagram	Diagrams about change in side lengths Diagram about change in area Symbolic
58-71	PLT The case of Janice Patterson	Ratio Percents	Narrative case Prompts to analyze case, connect to own practice and connect to curriculum and research	Word problems Diagrams to consider and use in solving the problem	Table Diagrams Symbolic (teachers thinking)
75-108	Facilitator notes about facilitating learning from the cases				
109-129	Appendices A-D Samples of teacher responses to the cases				
130-136	References and about the authors				

Appendix E:
Example of Vertical Analysis on a Professional Learning Task

<u>Vertical Analysis of the Case of Randy Harris (IIRP)</u>

<u>Section 1: Analysis of the Mathematical Task:</u>

In this section only consider the mathematical task itself. What does the math task ask the solver to do? *In the section 2, you will continue to analyze the solutions and discussion associated with the task.*

Choose the level of cognitive demand of the task.

Level of Cognitive Demand	**Present**	**Description**
Memorization		
Procedures Without Connections		
Procedures With Connections		
Doing Mathematics	**X**	- complex and nonalgorithmic thinking - explore concepts, processes or relationships - self regulation - access relevant knowledge and experiences - examine task constraints - considerable cognitive effort required

Indicate the representations the solver MUST use.

Use of Representations Those **required** to be used (**given or explicitly asked for in the task itself**)	**Present**	**Description**
Verbal	**X**	Description of task and call for explanation of solution
Symbolic / Equation	**X**	Decimal, fractional, and percentage representations of a shaded area of a rectangular grid
Graph		
Table		
Diagram/Model	**X**	Unconventional grids (8x10, 8x12, 8x9) provided that need to be shaded and the shaded area quantified.
Physical Object		

Choose the appropriate categories that describe the mathematics in the task.

Mathematical Structure	Present	Description
Mathematical Topic: 1. ratio 2. rate (density) 3. rate (D=RT) 4. rate (buy/consume) 5. similarity 6. scale 7. probability 8. percents 9. linear equations 10. linear patterns and relationships 11. slope 12. frequency distributions 13. fractions 14. decimals 15. other	**8** **13** **14**	The problem called for user to shade in on the grids an area represented by a 1. decimal 2. fraction and 3. percent. Then the user is asked to also indicate the other ways in which that area can be represented to cover fractional, decimal and percentage representations.
Task Type: 1. ratio 2. rate 3. comparison 4. missing value 5. scale factor 6. qualitative 7. non-proportional	**7**	The problem is about translating between visual, decimal, fractional and percentage representations of quantity – portion of the area of a rectangle.
Numbers: 1. all integers 2. all rationals 3. both integers and rationals 4. other	**2**	All the portions being represented are rational numbers.
Multiplicative Relationships between and within measure spaces 1. all integers (e.g. 1:2 and 3:6) 2. some non-integers (e.g. 2:3 & 10:15) 3. all non-integers (e.g. 2:3 & 5:7.5)	**n/a**	
Ratio Relationships 1. Part:Whole 2. Part:Part	**n/a**	

Indicate what the mathematical task requires the solver to do:

Required Response	Present	Description
Type of response to be given: 1. Answer only 2. Explanation 3. Justification	**2**	Required to explain how each problem was solved.

<u>Requires</u> that connections be made between: 1. Verbal 2. Symbolic/Equation 3. Graph 4. Table 5. Diagram/Model 6. Physical Object	**2** **5**	- The task requires that connections be made between the visual, fractional, decimal and percentage representations of a portion of the area of a rectangle. - Additionally the final question asks users to consider how the diagram helped them "make sense of the equivalence among fractions, decimals and percents?"

Section2: Analysis of the Link to Practice Component

Now analyze the materials that constitute the rest of the PLT (artifacts of practice, reflection questions….etc)

Choose the main artifact of practice presented for use with teachers.

Artifact of Practice Used	Present	Description
Narrative Case (description of instruction)	**X**	Randy Harris is a 7^{th} grade teacher in a diverse school. In his introduction it describes his commitment to teaching math for conceptual understanding, the value he sees in making connections between mathematical ideas and the power of diagrams to make sense of mathematical situations. "He sees the use of diagrams as a central component in building understanding" p.10. [para. 1-4]
Student work samples		
Research study results		
Other		

Indicate the pedagogical issues that are emphasized in the PLT.

Pedagogical Issues	Present	Description
Additive versus Multiplicative reasoning		
Unitizing	**X**	- In Denise's explanation for Set A #4, she states that 45% = 0.45 and this confuses Ramon. She then explains the relationship using 10x10 grids. Mr. Harris notes that Ramon's confusion is

		around the unit whole, so he has Denise explain and visually show the difference between 0.45 and 0.45% [23-25]
Equations – using formal symbols		
Ratio Referent		
Meaning of quantities		
Cross Product		
Student Struggle	**X**	- Though he knew the problems in the task would be challenging, Mr. Harris did not start with an example as this could funnel their thinking towards a particular solution path [29]
Cognitive Demand		
Use of multiple representations	**X**	- Diagrams as central to building understanding [4] - Goal of the lesson is to help students move comfortable between representations [6] - Peter used a visual approach to solve Set A #3 of rearranging the shaded portion to resemble ¾ [21]
Use of multiple student solutions	**X**	- Making connections: Wrote the three solution strategies on the board (Natalie's computational, Deanna's computational, Mr.Harris's percentage per square computation) [19] - Making connections: After student discussion, Mr.Harris recapped the strategies used on #1: method 3 from previous day, column as 10%, and multiplying decimal by total number of squares in grid - Selection: Mr. Harris noticed Devon had an interesting solution strategy so when he asked for volunteers and Devon volunteered, he chose to have Devon come up and share his partitioning strategy: using

		columns 25% → 12.5% . [37] He then asked Darlene (who has shy and had not volunteered but had an interesting solution) to share: using rows [40]
Language		
Explanations/Justifications	**X**	- David's explanation of 1/5 as 20% based on memorized fact [12] - Natalie's explanation that she counted squares to get the fraction and divided using the calculator was not satisfactory to Randy Harris [14-15] → asking students what percentage the squares represented [17] - After working on the task as a class, the students were asked to solve the final problem individually and in as many ways as possible as a quiz. Mr. Harris saw this as a chance for individual accountability and for him to assess their understanding as individuals [41]
Student Motivation		
Student thinking		
Other		

If instruction is described, then indicate which instructional factors were illustrated for teachers' consideration.

Instructional Factors associated with Decline of Maintenance of Cognitive Demand	**Present**	**Description**
Maintaining problem complexity / Assisting through scaffolding (+)		
Sustained press for justification (+)	**X**	Asks Marilyn to use and explain method 3 to find 72.5% of the 10x8 grid
Building on student knowledge and thinking (+)	**X**	- The students have used 10x10 grids to relate fractions and decimals. Just started using various grids to explore percents [5] - Day 1: Work with grids already

		shaded → Day 2: Task (unconventional grid and need to determine number of squares to shade themselves) - Mr.Harris referred students to method 3 from the previous day to help them think about the task [33]
Teacher draws frequent conceptual connections (+)		
Modeling high-level thinking (+)		
Students provided with means of monitoring their progress (+)		
Provision of enough time on task		
Challenges become nonproblems (-)	**X**	Simplifies task by changing order and asking leading questions about method 3.
Shift of emphasis from meaning to correctness (-)		
Students not held accountable for high level products/processes (-)		
Classroom management problems (-)		
Inappropriateness of task (-)		

Indicate which representations of mathematical ideas were presented for teachers' consideration.

Use of representations Those provided in the artifacts examined or explicitly called for	**Present**	**Description**
Verbal	**X**	Students' verbal explanations of their mathematical thinking
Symbolic / Equation		
Graph		
Table		
Diagram/Model	**X**	The diagrams help the reader and those in the class to better understand students' explanations. - diagrams illustrated the various ways that the grids could be partitioned and thought about - they illustrated students' explanations
Physical Object		

Indicate what teachers are asked to do.

Type of response required	Present	Description
Participation in group discussion - prompts ask for evidence to support conjectures offered - prompts orient participants to each other - prompts require teachers to relate learning to own practice - other	X	Analyzing the Case - prompts ask for evidence to support conjectures offered about teacher moves that supported or inhibited students' learning of mathematics - prompts orient participants to each other
Analysis - Analysis of student thinking and learning - Analysis of teacher pedagogical moves - Analysis of mathematical task	X	Extending your Analysis - Compare mathematical difficulty of problem sets A & B - What counts as making a connection among representations and when does this occur? - Mr. Harris did not publically list all the strategies students offered. Compare those in the case to those he listed and analyze his selection - To what extent and how does Mr.Harris hold students accountable for their mathematical explanations? - To what extent and how does Mr.Harris hold students accountable for using the diagram? - How did the work on Set A support or inhibit student thinking on Set B?
Written reflection	X	- Think about Ramon's difficulty distinguishing 0.45 and 0.45%. Think about why this is difficult for students and where in their instruction it may have arisen - Mr.Harris values helping students move from 10x10 to nonconventional grids. Reflect upon this and offer your opinion - Reflect upon Mr.Harris's decision to ignore David's suggestion [33] - Do you think Mr. Harris met his lesson's goals? Use evidence to support your claim
Creation of object (e.g. lesson plan)	X	- Develop and analyze additional solution strategies that make use of

		the diagram - Consider the list of moves you saw Randy Harris do and pick one that has implications for your own teaching. Plan and teach a lesson that addresses the identified pedagogy
Make a lesson plan or conduct a lesson using a cognitively demanding task	X	- Teach a lesson using the task and record it. Reflect on the lesson, analyze your moves, and compare yourself to Randy Harris
Other		

Section3: Additional Analyses

Consider the entire PLT and indicate if it presented any of the following opportunities to teachers.

Opportunities to develop multiplicative reasoning	**Present**	**Description**
Task requires teachers to reason explicitly in terms of quantities and quantitative relationships	X	The use of the nonconventional grids and diagram in explanations requires teachers to reason explicitly about the shaded portion of area of the grid.
Task appropriately uses and contrasts additive and multiplicative reasoning	**n/a**	
Task exposes teachers to situations that allow them to see proportional reasoning as a complex process that evolves over a long period of time - explore the conceptual components of proportional reasoning - links to other topics in the middle school curriculum	**n/a**	
Task allows for connections to be made among forms of rational numbers and with concepts of ratio and proportion.	X	Allows students to see relationship between part and whole in various representational forms

Appendix F:
Summary of Teachers' Learning Opportunities provided in the 7 PLTs of the ISBI Curriculum

The Implementing Standards-Based Instruction (ISBI) curriculum *A shaded box represents a PLT in which the item was present.*							
The PLTs featured:							
Mathematical task(s)	■						
Student work samples							
Future classroom activity							
Narrative case		■	■	■	■	■	■
General reflection prompts							
Teachers were asked to:							
Solve math tasks		■	■	■	■	■	■
Discuss with peers		■	■	■	■	■	■
Analyze	■						
Reflect in writing							
Create an object (lesson plan)							
Teach a lesson							
Interview students							
Proportionality:							
Ratio	■						
Rate	■						
Similarity							
Scale							
Slope							
Rational Numbers:							
Fraction	■	■	■				
Decimal	■	■					
Percent	■	■					
Linear Functions:							
Linear patterns and relationships	■						
Linear equations							
Supports for developing multiplicative reasoning:							
Requires explicit reasoning about quantities and relationships	■		■				■
Contrasts additive and multiplicative reasoning							
Illustrates proportional reasoning as a complex process that evolves							

Makes connections among the forms of rational numbers and ratios	■	■					
Representations used:							
Symbols	■	■	■		■		
Graphs						■	
Tables							■
Visual Diagrams	■	■	■		■		■
Physical Objects	■		■		■		
Cognitive Demand of the math tasks:							
Memorization	■						
Procedures without connections	■						
Procedures with connections	■		■		■		
Doing mathematics	■	■		■		■	■
Classroom factors associated with cognitive demand illustrated:							
Maintaining problem complexity / Assisting through scaffolding (+)		■	■	■			
Sustained press for justification (+)		■	■	■			
Building on student knowledge and thinking (+)		■	■	■	■		
Teacher draws frequent conceptual connections (+)			■				
Modeling high-level thinking (+)			■	■		■	
Students provided with means of monitoring their progress (+)							
Provision of enough time on task (+ and -)		■		■	■		■
Challenges become nonproblems (-)		■	■	■	■	■	
Shift of emphasis from meaning to correctness (-)		■	■				■
Students not held accountable for high level products/processes (-)		■	■	■	■	■	■
Classroom management problems (-)							
Inappropriateness of task (-)							

Appendix G:
Summary of Teachers' Learning Opportunities provided in the 4 PLTs of the IIRP Curriculum

The Improving Instruction in Rational Numbers and Proportionality (IIRP) curriculum *A shaded box represents a PLT in which the item was present.*				
The PLTs featured:				
Mathematical task(s)				
Student work samples				
Future classroom activity				
Narrative case				
General reflection prompts				
Teachers were asked to:				
Solve math tasks				
Discuss with peers				
Analyze				
Reflect in writing				
Create an object (lesson plan)				
Teach a lesson				
Interview students				
Proportionality				
Ratio				
Rate				
Similarity				
Scale				
Slope				
Rational Numbers:				
Fraction				
Decimal				
Percent				
Linear Functions:				
Linear patterns and relationships				
Linear equations				
Supports for developing multiplicative reasoning:				
Requires explicit reasoning about quantities and relationships				
Contrasts additive and multiplicative reasoning				
Illustrates proportional reasoning as a complex process that evolves				

Makes connections among the forms of rational numbers and ratios				
Representations used:				
Symbols				
Graphs				
Tables				
Visual Diagrams				
Physical Objects				
Cognitive Demand of the math tasks:				
Memorization				
Procedures without connections				
Procedures with connections				
Doing mathematics				
Classroom factors associated with cognitive demand illustrated:				
Maintaining problem complexity / Assisting through scaffolding (+)				
Sustained press for justification (+)				
Building on student knowledge and thinking (+)				
Teacher draws frequent conceptual connections (+)				
Modeling high-level thinking (+)				
Students provided with means of monitoring their progress (+)				
Provision of enough time on task (+ and -)				
Challenges become nonproblems (-)				
Shift of emphasis from meaning to correctness (-)				
Students not held accountable for high level products/processes (-)				
Classroom management problems (-)				
Inappropriateness of task (-)				

Appendix H:
Summary of Teachers' Learning Opportunities provided in the 17 Chapters of the TFRU Curriculum

The Teaching Fractions and Ratios for Understanding (TFRU) curriculum *A shaded box represents a PLT in which the item was present.*																	
The PLTs featured:																	
Mathematical task(s)	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
Student work samples	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
Future classroom activity	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
Narrative case																	
General reflection prompts	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
Teachers were asked to:																	
Solve math tasks	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
Discuss with peers																	
Analyze	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
Reflect in writing	■	■	■	■	■	■	■	■	■	■	■	■	■	■		■	■
Create an object (lesson plan)			■					■		■						■	
Teach a lesson					■						■		■				
Interview students	■	■	■	■		■	■	■	■					■	■		■
Proportionality:																	
Ratio	■	■	■	■	■	■	■	■	■		■	■	■	■	■		
Rate	■	■	■	■		■	■	■	■				■		■	■	■
Similarity																■	■
Scale	■				■								■				■
Slope				■													

Rational Numbers:																	
Fraction		■		■		■	■	■		■	■	■	■	■	■		
Decimal																	
Percent													■				■
Linear Functions:																	
Linear patterns and relationships																	
Linear equations																	
Supports for developing multiplicative reasoning:																	
Requires explicit reasoning about quantities and relationships	■				■		■	■			■		■	■		■	■
Contrasts additive and multiplicative reasoning		■	■						■						■		
Illustrates proportional reasoning as a complex process that evolves	■		■	■	■	■	■	■	■	■		■	■		■	■	■
Makes connections among the forms of rational numbers and ratios		■				■	■			■	■	■		■	■		
Representations used:																	
Symbols		■				■	■	■	■	■	■	■	■	■			■
Graphs																■	
Tables	■			■				■	■				■			■	■
Visual Diagrams	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
Physical Objects					■								■				
Cognitive Demand of the math tasks: (Note: These tasks are given and assessed at elementary grade levels)																	
Memorization																	
Procedures without connections			■												■		
Procedures with connections	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
Doing mathematics	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■

Classroom factors associated with cognitive demand illustrated:																	
Maintaining problem complexity / Assisting through scaffolding (+)																	
Sustained press for justification (+)																	
Building on student knowledge and thinking (+)																	
Teacher draws frequent conceptual connections (+)																	
Modeling high-level thinking (+)																	
Students provided with means of monitoring their progress (+)																	
Provision of enough time on task (+ and -)																	
Challenges become nonproblems (-)																	
Shift of emphasis from meaning to correctness (-)																	
Students not held accountable for high level products/processes (-)																	
Classroom management problems (-)																	
Inappropriateness of task (-)																	

Appendix I:
Summary of Teachers' Learning Opportunities provided in
the 28 PLTs of the DMIMMO Curriculum

The Developing Mathematical Ideas: Making Meaning for Operations (DMIMMO) curriculum *A shaded box represents a PLT in which the item was present.*																												
The PLTs featured:																												
Mathematical task(s)																												
Student work samples																												
Future classroom activity																												
Narrative case	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
General reflection prompts																												
Teachers were asked to:																												
Solve math tasks	■	■	■	■	■	■	■	■	■	■	■		■	■	■	■	■	■	■	■	■	■		■	■		■	■
Discuss with peers	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
Analyze																												
Reflect in writing	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
Create an object (lesson plan)																												
Teach a lesson																												
Interview students																												
Proportionality:																												
Ratio						■									■													
Rate																												
Similarity																												
Scale																												
Slope																												

Rational Numbers:																												
Fraction																				■	■	■	■	■	■		■	■
Decimal																												
Percent																												
Linear Functions:																												
Linear patterns and relationships																												
Linear equations																												
Supports for developing multiplicative reasoning:																												
Requires explicit reasoning about quantities and relationships																												
Contrasts additive and multiplicative reasoning												■	■															
Illustrates proportional reasoning as a complex process that evolves																												
Makes connections among the forms of rational numbers and ratios																				■								
Representations used:																												
Symbols								■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
Graphs																												
Tables													■															
Visual Diagrams				■	■	■		■		■	■			■		■	■		■	■	■	■	■	■	■	■	■	■
Physical Objects	■		■	■	■	■	■		■																			
Cognitive Demand of the math tasks:																												
Memorization																												
Procedures without connections																												
Procedures with connections	■	■	■	■				■	■	■	■		■					■	■		■	■		■	■		■	■
Doing mathematics					■	■	■							■	■	■	■			■								

Classroom factors associated with cognitive demand illustrated:																												
Maintaining problem complexity / Assisting through scaffolding (+)															■										■			
Sustained press for justification (+)	■		■	■	■	■	■	■	■	■	■		■				■	■	■	■		■			■			■
Building on student knowledge and thinking (+)		■		■		■			■										■	■			■					
Teacher draws frequent conceptual connections (+)																												
Modeling high-level thinking (+)	■																			■								■
Students provided with means of monitoring their progress (+)		■																										
Provision of enough time on task (+ and -)																		■		■								
Challenges become nonproblems (-)															■													
Shift of emphasis from meaning to correctness (-)						■															■							
Students not held accountable for high level products/processes (-)												■																
Classroom management problems (-)																												
Inappropriateness of task (-)									■																			

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