# The effect of trade credit on operational policies and on the relationship between banks, suppliers, and manufacturers

by

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## ABSTRACT

**The effect of trade credit on operational policies and on the relationship between banks, suppliers, and manufacturers**

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Companies in a broad range of industries and economies rely heavily on external sources to finance their operations. But, external financing could be expensive and/or difficult to obtain due to imperfections in real capital markets.

I focus on cash-constrained manufacturers that rely on external sources to finance their operations. More specifically, I focus on trade credit, the most important source of short-term financing, and analyze its effect on operational policies and on the relationship between banks, suppliers, and manufacturers. I discuss factors that affect supply reliability, fixed cost to work with a supplier, and the trade credit amount suppliers make available to manufacturers. Then, I determine how supply risk, financing constraints, and the dual role served by suppliers affect the financing and operational decisions of manufacturers. I also contrast differences in supply reliability, fixed cost to work with a supplier, and financing constraints between manufacturers in developed and developing economies to address how the economic

environment manufacturers operate in affects their financing and operational decisions. Afterwards, I analyze the effect of trade credit on manufacturers' financing and operational decisions when there is information asymmetry between banks and manufacturers about the credibility of manufacturers.

The analysis suggests that the optimal number of suppliers may increase as the availability of either internal financing or trade credit diminishes and as the cost to work with a supplier or the wholesale price increases. Surprisingly, as the standard deviation of a supplier yield's increases, the optimal number of suppliers could either increase or decrease. Ceteris paribus, manufacturers in developing economies will have more suppliers than comparable manufacturers in developed economies. But if, in developing economies, the cost to work with a supplier is very high or the manufacturer is close to bankruptcy, then this manufacturer may actually have fewer suppliers than its counterpart in developed economies. Cash-constrained manufacturers should only use trade-credit to finance their operations when borrowing from suppliers is cheap or when they are trying to signal their credibility to banks. Also, the availability of trade credit may increase or decrease operational costs and it may increase the total amount to borrow.

# CHAPTER I

# Introduction

## 1.1 Motivation.

Companies in a broad range of industries and economies rely heavily on external sources to finance their operations. But, external financing could be expensive and/or difficult to obtain due to asymmetric information between lenders and borrowers, high cost of capital of lenders, high cost of lenders to monitor the credibility of potential borrowers, credit rationing by lenders, and expensive loan terms sometimes extended by lenders with power advantage over manufacturers.

Banks often have lower costs of capital and easier access to capital than suppliers. Therefore, banks can offer loans at cheaper rates than suppliers if they can assess how likely manufacturers are to repay. Unfortunately, asymmetric information that often exists between banks and manufacturers causes banks to offer unfavorable loan terms or even deny loans to manufacturers. However, suppliers, being in the same industry as manufacturers, can sometimes better estimate the distribution of the demand for manufacturers' goods than banks and the competition among suppliers to sell goods to manufacturers might make suppliers lend to manufacturers with greater ease than banks. This is why, even when suppliers offer credit terms that come at high costs, the relatively easy access to supplier financing enables manufacturers to signal their credit quality to banks, which facilitates access to bank loans.

## 1.2 Contributions.

I focus on cash-constrained manufacturers that rely on external sources to finance their operations. More specifically, I focus on trade credit[1], the most important source of short-term financing, and analyze its effect on operational policies and on the relationship between banks, suppliers, and manufacturers. I discuss factors that affect supply reliability, fixed cost to work with a supplier, and the trade credit amount[2] suppliers make available to manufacturers. Then, I determine how supply risk, financing constraints, and the dual role served by suppliers affect the financing and operational decisions of manufacturers. I also contrast differences in supply reliability, fixed cost to work with a supplier, and financing constraints between manufacturers in developed and developing economies to address how the economic environment manufacturers operate in affects their financing and operational decisions. Afterwards, I analyze the effect of trade credit on manufacturers' financing and operational decisions when there are information asymmetry between banks and manufacturers about the credibility of manufacturers.

## 1.3 Main results.

The analysis suggests that the optimal number of suppliers may increase as the availability of either internal financing or trade credit diminishes and as the cost to

[1]Trade credit is the delay in payments from the buyer to the supplier of goods. One can think of trade credit as a loan extended by the supplier to the buyer. The buyer effectively obtains the loan from the supplier by not paying for the purchase initially, but it has to repay the loan later. The typical trade credit contracts in the United States are "net 30" and "2/10 net 30" (see Ng et al. (1999)). According to the former, the buyer does not have to pay for the purchase for 30 days, thus obtaining, effectively, a 30-day interest-free loan. According to the latter contract, the buyer receives a 2% discount if it pays for the purchase within 10 days, and it has up to 30 days to pay for the goods. Trade credit contracts vary by industry and country. It is not uncommon to see trade credit terms that have maturity longer and shorter than 30 days, and higher and lower implied interest rates.

[2]To better compare trade credit with other financing sources, trade credit is modeled as a cash-advance from suppliers to manufacturers instead of a delay in payments of goods.

work with a supplier or the wholesale price increases. Surprisingly, as the standard deviation of a supplier yield's increases, the optimal number of suppliers could either increase or decrease. Ceteris paribus, manufacturers in developing economies will have more suppliers than comparable manufacturers in developed economies. But if, in developing economies, the cost to work with a supplier is very high or the manufacturer is close to bankruptcy, then this manufacturer may actually have fewer suppliers than its counterpart in developed economies. Cash-constrained manufacturers should only use trade-credit to finance their operations when borrowing from suppliers is cheap or when they are trying to signal their credibility to banks. Also, the availability of trade credit may increase or decrease operational costs and it may increase the total amount to borrow.

## CHAPTER II

# Risk, Financing and the Optimal Number of Suppliers

### 2.1 Introduction.

Should manufacturers in developed economies work with more or fewer suppliers than manufacturers in developing economies? More generally, how does the number of suppliers for a manufacturer depend on the manufacturer's economic environment? To answer these questions I identify several economic and business factors that might affect the number of suppliers: supply risk, fixed costs to work with suppliers, and access to financing (particularly trade credit financing).

Supply risk has been recognized as one of the main reasons for manufacturers to diversify their supply base both in empirical research (see Wu and Choi (2005)) and in theory (see Minner (2003), Agrawal and Nahmias (1997), Tang (2006), Babich et al. (2007), Dada et al. (2007), Yang et al. (2009), Yang et al. (2008)). The severity of exposure to supply risks depends on where suppliers are located and a variety of other economic and business factors. For example, manufacturers operating in developing economies may have to cope with greater uncertainty about supplier reliability due to underdeveloped production, transportation, information, and financial infrastructures, as well as insufficient legal protection and political risk[1]. The ram-

[1] For example, Ukrainian government seized assets of a number of firms in a reprivatization campaign following the "Orange Revolution" (see Paskhaver and Verkhovodova (2007))

ifications of supply risk and, hence, the need for diversification also depend on the extent to which the owners of a firm are liable for the disruption consequences. If the owners could walk away from their financial and other obligations to their financial partners and customers, then the owners would be less concerned about supply risk and rely less on diversification. Although the connection between supply risk and diversification has been studied extensively, the effect of liability on this connection, which we study in this chapter, has received scant attention in the literature.

Fixed costs to work with suppliers create incentives for manufacturers to lean out their supply base. These costs can take various forms. For example, some manufacturers (e.g. in automotive and aerospace industries) must certify potential suppliers using rigorous and costly process before any part can be procured. Furthermore, just the initial certification may not be sufficient. To guarantee that suppliers perform according to the manufacturer's expectations, suppliers must be continuously monitored and the quality of their parts must be checked. Costs to monitor and contract enforcement is significantly higher in economies with less developed legal systems and information infrastructure.

In addition to raising capital in well-functioning and mature financial markets, the majority of companies in the U.S.A. (and other developed countries) rely on alternative sources, such as trade credit and private investments, to finance their strategic, tactical, and operational decisions. In fact, trade credit financing is the single largest source of external short-term corporate financing in the United States (see Rajan and Zingales (1995) and Petersen and Rajan (1997)). According to Rajan and Zingales (1995), accounts payable and creditors constituted 15%, while debt in current liabilities was only 7.4% of the total book value for an average non-financial firm in 1991.[2] Several studies (see Nilsen (2002) and Atanasova and Wilson (2003)) find that the reliance on trade credit financing increases when other sources of financing are restricted (for example, during times of monetary crunch in the economy). This is why, in developing economies, whose financial markets are still in their infancy, access to trade credit has profound consequences for a manufacturer's growth potential, competitive abilities, and survival. Several recent empirical studies emphasize the importance of trade credit as a financing source in developing economies. Fisman and Love (2003) observe that, in countries with weaker financial institutions, the industries with higher dependence on trade credit financing exhibit higher rates of growth, relative to other industries. Fisman (2001) discovers a correlation between the availability of trade credit financing and a manufacturer's operational performance. Using a sample of African firms, Fisman (2001) finds that firms with access to trade credit financing are less likely to experience stock-outs, and are more likely to have higher production-capacity utilization. To the best of my knowledge, the interactions among financing constraints, trade credit loans and the number of suppliers has not been studied before, a gap that this chapter aims to fill.

In this chapter, I analyze how financing constraints, the dual role played by suppliers as the providers of parts and the financiers of the manufacturer, supplier risk, limited liability of the manufacturer, and fixed costs to work with suppliers affect the manufacturer's choices of order quantities and the number of suppliers. Using a one-period model with homogeneous suppliers (and later a model with non-homogeneous suppliers), I consider the joint procurement and financing decisions of

[2] Given the prevalence of trade credit, examples of companies that rely heavily on this form of financing are easy to find. Consider, for instance, TenderCare International, Inc., which sells disposable baby diapers, natural formula wipes, and related products in the United States. According to this company's annual report, it had \$1.172 million in accounts payable, and \$0.007 million in short-term debt out of total \$1.218 million in total current liabilities in 2005. In the same year, the company's cost of goods sold was \$2.185 million. Thus, with $Days\ Payables\ Outstanding = Account\ Payables/COGS * 365 \approx 6\ months$, TenderCare International depends greatly on its suppliers for financing.

a manufacturer with access to limited internal financing and loans from suppliers, facing either an uncertain demand or an uncertain supply (random yield). In contrast to the traditional operational models where the decision maker is fully liable for losses, I model the decisions of the owners of the manufacturer, who may have limited liability (e.g. they could be limited-liability partners or shareholders; I will refer to the owners of the manufacturer as the shareholders in the sequel).

The contracts between suppliers and the manufacturer are assumed to be given exogenously. This is why the contract terms, in particular, the terms of the loans extended by the suppliers to the manufacturer do not change as I perform comparative statics analysis on the solution. There are several reasons for the contract terms to be fixed. In practice, contracts are renegotiated on a periodic basis and are not sensitive to short-term fluctuations in the manufacturer's condition. Furthermore, although I am analyzing a single product line of the manufacturer, the manufacturer might have other product lines and might be selling in multiple markets. Thus, the contract terms must take into account the manufacturer's overall condition and not just the condition of one of the manufacturer's subdivisions. I focus on manufacturers that do not have access to stock markets for additional financing (as is the case in some developing economies) or consider equity financing costs to be too high (see Pagano et al. (1998) for a discussion about the costs of going public and tradeoffs between equity vs. bank financing). The lack of equity investor scrutiny exacerbates the non-transparency of the manufacturer's operations to outside investors. This asymmetric information violates the perfect capital market assumption of Modigliani and Miller (1958) and prevents the lenders from reacting to changing conditions of the manufacturer. A number of empirical studies find that loan terms are fairly insensitive to individual firm conditions. For example, using data from a sample of small firms, Petersen and Rajan (1994) observe that, once a decision to extend a loan is made, the loan terms are determined based on industry practices, economy-wide factors, internal policies and conditions of the lender, and do not depend on the conditions of the borrower. Ng et al. (1999) find that, while there are significant differences between industries, the trade-credit contract terms are standardized within industries, with "net 30" and "2/10 net 30" being the most popular contracts.

Based on my analysis, I offer several testable hypotheses about the relationship between economic conditions and size of the manufacturer's supply base. Some of these hypotheses have already been confirmed in prior empirical studies; others still need to be empirically verified. Specifically, as one would expect, the analysis suggests that the alternative financing sources (internal financing and trade credit) are substitutable. That is, ceteris paribus, the manufacturer uses more suppliers if internal financing is not available. Surprisingly, I also find that, because of limited liability of the shareholders, the optimal production quantity could be increasing in fixed costs. Furthermore, the optimal number of suppliers could be increasing in fixed costs to work with suppliers as well, because working with more suppliers could relax the manufacturer's financing constraints. In addition, I observe that the limits on loans and the wholesale price affect the optimal number of suppliers in a non-monotone way. Interestingly, I find that the value of the shareholders and the optimal number of suppliers of the manufacturer could be increasing or decreasing in the standard deviation of the supplier random yield. This is a consequence of the tradeoffs between the expected profit and the value of the option to default that shareholders hold. I study the effects of limited liability and find that, when suppliers are perfectly reliable, the greater the loss for which the manufacturer is responsible, the smaller the order quantity the manufacturer will place and the fewer the number of suppliers with whom the manufacturer will work. However, when suppliers are not reliable, it may happen that the greater the loss the manufacturer is responsible for, the more suppliers the manufacturer may order from in order to take advantage of supply-risk diversification benefits.

Finally, I address the question whether manufacturers operating in developing economies must contract with more suppliers than manufacturers operating in developed economies. The answer is "no" if the fixed cost of an extra supplier is high. However, in this case, my model predicts that financing constraints will force manufacturers in developing economies to suboptimal levels of production and cause higher stock-out rates. This conclusion is consistent with the results of earlier empirical studies.

The remainder of the chapter is organized as follows. In the next section I discuss the related literature. The model, which contains both financing and operational decisions, is discussed in Section 2.3. In Section 2.4 I find the optimal financing decisions, given the optimal operational decisions. Then, I derive a series of analytical results on the optimal operational decisions and the role of financing constraints in the supplier selection. These analytical results guide a numerical study, the results of which are presented in Section 2.5. Section 2.6 considers a model with heterogeneous suppliers. Conclusions, model limitations, and future research directions are discussed in Section 2.7. Finally, the Appendix contains technical lemmas and proofs.

## 2.2 Literature review.

If capital markets were perfect, as Modigliani and Miller (1958) proved in their seminal paper, managers could consider financing decisions independently from the manufacturer's other decisions. However, imperfections of real capital markets, such as transaction costs, taxes, information asymmetry, and bankruptcy costs imply that managers could generate value for shareholders by jointly considering financing and non-financing decisions (see Harris and Raviv (1991) for a survey of research on the choice of capital structure and the effects of market imperfections). Several recent papers: Lederer and Singhal (1994), Li et al. (2005), Babich and Sobel (2004), Buzacott and Zhang (2004), Xu and Birge (2004), and Boyabatli and Toktay (2007) consider the value of combining financing, operational, and technology decisions. Similar to those papers, I will explicitly model the manufacturer's ordering and financing decisions and investigate how financing terms (in my case, trade credit terms) affect ordering decisions. For example, Buzacott and Zhang (2004) study how asset-based bank loans affect the ability of the manufacturer to grow (using a dynamic deterministic model) and how asset-based financing terms can be optimally set by banks (using a one-period Stackelberg game with a stochastic demand and several borrowers). Similar to the model in Buzacott and Zhang (2004), I consider the limited liability of the borrower. Unlike their model, however, I focus on the manufacturer's choice of the number of suppliers, driven by the tradeoff between the fixed costs of adding a supplier and the benefits of relaxing financing constraints or diversifying supply risk. The external financing in my model is provided by the suppliers (through trade credit), who perform a dual function by supplying components and offering financing to the manufacturer.

A number of researchers studied the effects of trade credit on inventory policies. For example, Gupta and Wang (2009) prove that, even in the presence of trade credit, the order-up-to inventory policy remains optimal for a discrete-time, joint inventory-financing model, and suggest an algorithm for computing the optimal stock level for a continuous-time model. The effects of delayed payments on the Economic Order Quantity model were investigated by Haley and Higgins (1973), Chapman et al. (1984), and Rachamandugu (1989). In this chapter, in addition to determining optimal order quantities (as is done in the cited articles), I will compute the optimal number of suppliers that a manufacturer should have.

If the supplier yields are random, then the manufacturer may benefit by ordering from several suppliers. The benefits of diversification[3] as a remedy for supplier random yields were considered, for example, by Anupindi and Akella (1993), Tomlin (2006), Tomlin (2007), Dada et al. (2007), and Federgruen and Yang (2007). Babich et al. (2007), and Babich (2006) quantify diversification benefits when suppliers are competing. However, the majority of the research in the random yield literature assumes that the supplier set is exogenously given (see Yano and Lee (1995) for an extensive review of random yield research). Among exceptions is the work by Agrawal and Nahmias (1997), who study tradeoffs between diversification benefits and supplier set-up costs using a one-period model with independent, multiplicative, normally distributed supplier yields. The authors consider cases of both identical and non-identical suppliers, provide conditions that the optimal order quantities must satisfy, and suggest numerical procedures for determining the optimal number of suppliers. Agrawal and Nahmias (1997) find that the optimal order quantity is likely to be increasing and the optimal profit is decreasing in the volatility of the supplier yield. They also find that the profit is increasing and the optimal order quantity is likely to be decreasing in the yield's mean.

I extend the analysis of the identical-supplier case of Agrawal and Nahmias (1997) by adding financing decisions and financing constraints, and by allowing for the lim-

[3] By diversification I mean holding a portfolio of contracts, instead of just one contract. In this chapter specifically, diversification means placing orders with several suppliers.

ited liability of the decision makers. These additional assumptions produce results different from those in Agrawal and Nahmias (1997). For example, while the optimal value of the objective function is increasing in the random yield's mean both in our model and in Agrawal and Nahmias (1997), I observe that with the limited liability assumption, the decision makers in my model may benefit from an increase in the volatility of supplier yields. While diversification benefits provide a powerful incentive for the manufacturer to order from several suppliers, financing constraints in the model may either hinder or encourage diversification. Unlike Agrawal and Nahmias (1997), I do not study in depth the supplier selection problem with non-identical suppliers, because I focus on financing decisions, financing constraints, limited liability, and their effects on the size of the supply base, rather than on interactions with individual suppliers. However, I do extend the analytical results of their model with heterogeneous suppliers to our setting.

A number of earlier studies observed effects of limited liability similar to our findings. Gollier et al. (1997) consider the investment problem of a risk-averse firm with limited liability. They show that the optimal exposure to risk is always larger under limited liability compared to full liability. Brander and Lewis (1986) show that limited liability may commit a leveraged firm to a more aggressive output stance. Because firms will have incentives to use financial structure to influence the output market, this demonstrates a new determinant of the debt-equity ratio. Faure-Grimaud (2000) shows that asymmetric information between banks and firms plays a crucial role in financing decisions and output market strategies. In his model, debt causes firms to compete less aggressively: the usual (positive) limited liability effect on quantities is offset by a negative one due to (endogenous) financing costs.

## 2.3 Model and assumptions.

Consider a one-period model with a manufacturer that procures a component from outside suppliers in order to meet random customer demand, denoted by $D$. I am modeling only one of possibly several businesses that the manufacturer may have. There exists a number of suppliers, and the managers of the manufacturer must decide with how many suppliers to contract (the number of suppliers contracted is denoted by $N$) and how much to order from each (the quantity ordered from supplier $i$ is denoted by $y_i$). Similar to Agrawal and Nahmias (1997), I assume that suppliers are identical to simplify the analysis. Therefore, each supplier receives the same order quantity: $y_i = y$. The total order placed with the suppliers is $z = Ny$.

**Demand.**

The customer demand $D$ is a random variable with probability density function (p.d.f.) $g$ and cumulative distribution function (c.d.f.) $G$. Define $\overline{G}(x) = 1 - G(x)$. I will also consider models with deterministic demand $D$.

**Assumption 2.1.** *Let demand $D$ be defined over a domain $[x_l, x_u]$ where $0 \leq x_l < x_u < \infty$. Let $G$ be strictly increasing and twice continuously differentiable. Furthermore, assume $\frac{g(x)}{\overline{G}^2(x)}$ is increasing.*

The requirement of a finite domain imposed by Assumption 2.1 is needed to rule out pathological cases where the manufacturer may find it desirable to order arbitrarily large quantities. The condition that $\frac{g(x)}{\overline{G}^2(x)}$ is increasing is a weaker condition than requiring that the demand distribution has an increasing failure rate (IFR) and, therefore, is satisfied by all IFR distributions (and their truncated versions), including the normal, Weibull and Gamma distributions. This assumption will be used to establish the unimodality of the objective function later in the chapter.

**Random yield.**

The yield of a supplier is random and independent across suppliers. Similar to Agrawal and Nahmias (1997), it will be convenient to assume that the yields are stochastically proportional and normally distributed. Therefore, if an order for $y$ units is placed with a supplier at the beginning of the planning horizon, a quantity $Xy$ will be delivered by the sales time, where $X \sim \mathcal{N}(\mu, \sigma)$. If the total order quantity $z = Ny$ is placed with $N$ suppliers, then a quantity $Q(N, z) = \overline{X}z$ will be delivered, where $\overline{X} \sim \mathcal{N}\left(\mu, \frac{\sigma}{\sqrt{N}}\right)$. To guarantee that the probability of supplier yield falling outside of the range [0,1] is negligible, I assume that $0 < \mu \pm 3\sigma < 1$.

**Operational costs and revenues.**

Each supplier charges the manufacturer $w$ per unit of the component when the order is placed. The timing of payments is not of great importance when suppliers are perfectly reliable. When suppliers are unreliable, the manufacturer prefers to pay after the delivery and only for items that are actually delivered. Suppliers prefer up-front payments for the orders that have been placed. Depending on the market power of the manufacturer and the suppliers, some combination of per-ordered and per-delivered payment contracts will be adopted.[4] Even if the manufacturer enjoys full market power, there are circumstances when only contracts with per-ordered payments are possible. For example, the cost of verifying the delivery size (e.g. via quality control) could be prohibitively high, leaving the manufacturer no alternative but to accept the entire order and rely on its customers to identify defective products. Furthermore, even if the inspection costs are low, in practice the manufacturer accepts the whole order if, e.g., 98% of the products in the samples are good, because the inspection process is not error free — error rates between 20%-30% are

[4]As Babich et al. (2007) demonstrate, in equilibrium the suppliers and the manufacturer could be indifferent between per-ordered and per-delivered payments.

not unusual (see Chapter 14 and Chapter 15 in Montgomery (2005)). I analyzed a general model with both per-ordered and per-delivered payments; however, the insights obtained were the same as those for the model with only per-ordered payments. Therefore, for the sake of exposition, I only present the simpler model with per-ordered payments.

In addition to variable costs, the manufacturer incurs a fixed cost of $C$ for working with a supplier. The fixed cost in this chapter represents an amalgam of various costs that a manufacturer has to incur by working with a supplier, including supplier-selection costs, contract-monitoring costs, legal fees, quality control expenses, etc. These costs encompass both physical and financial transaction costs. Thus, the cost of operations is $NC + wz$. Unmet demand becomes lost sales, and leftover inventory has no value. Therefore, the revenue of the manufacturer is $p \min[D, Q(N, z)]$, and the manufacturer's operational profit is $p \min[D, Q(N, z)] - wz - NC$.

**Financing.**

The manufacturer has two financing options for its operational decisions: internal capital and trade credit (recall that in Section 2.1 I argued that trade credit is the single most important form of external short-term financing). Assume that at the beginning of the planning horizon the internal capital (which is internally generated cash available to the manufacturer, e.g., retained earnings) is $I$. The manufacturer can invest at the rate of $r_I$.

The manufacturer may also use trade-credit contracts offered by the suppliers. As discussed in the introduction, a trade-credit contract allows the buyer to delay payments for the goods received, which is equivalent to the supplier offering a loan to the buyer.[5] For example, suppose that a manufacturer places an order for $y$ units

[5] See Chapter 30 (p. 812 - 840) of Brealey et al. (2006) for a description of trade-credit contract terms.

with a supplier. The supplier offers the manufacturer the choice either to make an immediate payment of $wy$ or to postpone paying for any part of the order until the end of the planning horizon. At the end of the planning horizon the manufacturer must pay a higher per unit amount $w(1 + r_S)$. Effectively, the manufacturer can take out a loan, $S$, up to the monetary value of the purchase, $wy$ (i.e. $S \leq wy$), with the supplier. The interest rate on this loan is $r_S$. In general, the supplier may offer a trade credit on only a part of the order (i.e. $S \leq \alpha y$, where $0 \leq \alpha \leq w$) and may even offer some amount, $\beta$, regardless of the order size, just for receiving an order from the manufacturer (i.e. $S \leq \alpha y + \beta$, where $0 \leq \alpha \leq w$ ). In most applications, $\beta = 0$. However, I derive results with a more general assumption: $\beta \geq 0$. The terms offered on the supplier loans can be better or worse than those of internal financing, depending on competition among suppliers, transaction costs, information asymmetry and other factors.[6] The absolute size of the loan from a supplier depends on the supplier's size (in financial terms), monetary supply in the economy, the ability of the supplier to access capital, and the credit-risk management activities of the supplier (lenders usually limit the size of a loan that can be offered to any single borrower). Thus, an additional constraint on the supplier loan is $S \leq \widehat{S}$, where $\widehat{S}$ is the upper limit on the loan, regardless of the manufacturer's order size. Putting together two constraints on the loan from a supplier, we obtain $S \leq \overline{S}(y) = \min(\widehat{S}, \alpha y + \beta)$.

If the fixed cost to work with a supplier, $C$, is greater than the absolute limit on the supplier loan, $\widehat{S}$, then the manufacturer cannot use additional suppliers to relax financing constraints. To make the problem more interesting, I assume that $C < \widehat{S}$, which, in practice, is a reasonable property. It is also natural to assume that the

[6] For discussions of factors affecting trade-credit terms, see Petersen and Rajan (1997), and Biais and Gollier (1997).

amount of money, $\beta$, the buyer receives from the supplier just by placing an order is less than the fixed cost to work with a supplier, $C$. This assumption is likely to be true in practice, because in most applications $\beta = 0$. In general, this assumption prevents the buyer from having infinite wealth at time 0 just by placing orders with an infinite number of suppliers (each supplier increasing the manufacturer's wealth by $\beta - C > 0$). To summarize, we make the following assumptions on the parameters of the supplier loans:

**Assumption 2.2.** $0 \leq \alpha < w$ *and* $0 \leq \beta < C < \widehat{S}$.

Furthermore, I assume (to avoid trivial solutions) that the rate of return on both financing sources, internal capital and trade credit, is small enough that the manufacturer is able to recover the wholesale price plus the interest by selling the product. That is:

**Assumption 2.3.** $p > (1 + \max\{r_S, r_I\})w$.

In this one-period model, I assume that the loan terms are fixed. One could think, for example, that the manufacturer makes decisions after having observed the loan rates and the loan limits offered by the lenders. As discussed in the introduction, in perfect capital markets, the loan terms would reflect the default probability of the borrower and the loans would be fairly priced. However, real markets are not perfect. For example, due to information asymmetry, suppliers may not be able to adjust their rates in response to the changing business risk of the manufacturer. Using data from a sample of small firms, Petersen and Rajan (1994) observe that, once a decision to extend a loan is made, the loan terms are determined based on industry practices, economy-wide factors, and internal policies and conditions of the lender, and are fairly insensitive to the conditions of the borrower. One may wonder if the loan limits are more sensitive to the changes in the borrower's state. As Berger and Udell (1995) discuss, although credit line agreements usually include clauses that allow banks to revoke credit in case of significant changes in the conditions of the borrower, these clauses can only be invoked based on verifiable events. Even when events are verifiable, Avery and Berger (1991) show that banks are reluctant to invoke these clauses. Likewise, suppliers may decide to extend favorable loan terms even to their risky customers, in order to benefit in the long run by avoiding the costs of their customers' defaults.

**Objective.**

The objective of the manufacturer's managers is to maximize the value for the manufacturer's shareholders, who have limited liability because of bankruptcy protection. We are considering a single-period model and, therefore, the value of the business is the value of its cash position at the end of the planning horizon. If the manufacturer loses money on this business, part of the losses can be absorbed by the other businesses that the manufacturer has. From the shareholder's perspective, they are liable for the losses from this business up to an amount $l < 0$. That is, if the cash position of this business is $x$, the shareholders receive $\max(l, x)$. Special cases of limited liability are $l = -\infty$, in which case the shareholders are liable for all losses (as is traditionally assumed in the operations literature) and $l = 0$, in which case the shareholders are not liable for losses at all (as would be the case if the manufacturer had only one business). In the subsequent sections I will highlight the results that are driven by the limited liability assumption.

**Timing of events and cash flows.**

At the beginning of the planning horizon, the manufacturer decides on the optimal number of suppliers, the total order quantity, and the financing sources and amounts.

It receives loans from the suppliers, $NS$, and pays operational costs, $NC + wz$.

The suppliers deliver product by the end of the planning horizon. Random demand is realized and the manufacturer collects revenue, $p \min[D, Q(N, z)]$, repays loans, $N(1 + r_S)S$, or declares bankruptcy.

**Mathematical formulation.**

With the assumptions listed above, the manufacturer's objective function is as follows:

$$E\left[\max\left\{l, p\min[D, Q(N,z)] - (1 + r_I)(wz + NC) - (r_S - r_I)NS + (1 + r_I)I\right\}\right]. \tag{2.1}$$

The first term inside the max operator, $l$, is a non-positive number and it represents the amount of losses the shareholders are liable for. The second term inside the max operator is the manufacturer's cash position at the end of the planning horizon. (Recall that if the manufacturer incurs a loss, in which case the cash position is negative, the shareholders are liable only up to an amount $l$.) The cash position itself consists of four terms. The first term, $p \min[D, Q(N, z)]$, is the revenue from sales, assumed to be collected at the end of the planning horizon. The second term captures the operational costs incurred by the manufacturer at the beginning of the planning horizon, $wz + NC$, inflated by the manufacturer's internal rate of return, $r_I$. The third term, $(r_S - r_I)NS$, is the interest paid on supplier loans. The fourth term, $(1 + r_I)I$, is the manufacturer's internal capital, $I$, inflated by internal rate of return, $r_I$.

Using $\max\{l, x\} = l + (x - l)^+$ and noting that $l$ is a constant, we can simplify the objective (2.1), obtaining the following optimization problem for the shareholders

$$\max_{S\geq 0, z\geq 0, N\in\mathbb{N}} E\left\{p\min[D, Q(N,z)] - (1+r_I)(wz+NC) - (r_S - r_I)NS + (1+r_I)I - l\right\}^+ \tag{2.2a}$$

subject to:

$$wz + NC \leq NS + I, \tag{2.2b}$$

$$S \leq \overline{S}(z/N). \tag{2.2c}$$

### 2.4 Model analysis.

I will first investigate the optimal financing decisions when the operational decisions are already fixed. Subsequently, I will investigate the optimal operational decisions.

#### 2.4.1 Financing decisions.

I will describe the optimal financing decision (i.e., the amount of supplier loan, $S$), provided that the operational decisions (i.e., order quantity, $z$, and number of suppliers, $N$) are fixed. In order for the operational decisions to be financially feasible, we need

$$wz + NC \leq N\overline{S}(z/N) + I. \tag{2.3}$$

Using the expression for the limit on the supplier loan $\overline{S}(y) = \min(\widehat{S}, \alpha y + \beta)$, one can write an expression for financing feasibility of the operational decisions given by (2.3) as follows

$$\begin{cases} (w-\alpha)z + (C-\beta)N \leq I, \\ wz - (\widehat{S} - C)N \leq I. \end{cases} \tag{2.4}$$

An inspection of problem (2.2) yields the following crucial observation: depending on whether the supplier loan is more ($r_S > r_I$) or less ($r_S < r_I$) expensive than the internal rate of return, the manufacturer will either borrow the smallest feasible or the largest feasible amount from suppliers. The following proposition presents the optimal supplier loan amounts:

**Proposition 2.1.** *Suppose that the operational decisions — number of suppliers, $N$, and order quantity, $z$ — are fixed and financially feasible (as in inequality (2.3)). Then the optimal loan amounts are*

***Case I:*** $\boldsymbol{r_I < r_S}$.

$$NS^* = (wz + NC - I)^+. \tag{2.5}$$

***Case II:*** $\boldsymbol{r_I > r_S}$.

$$NS^* = N\overline{S}(z/N). \tag{2.6}$$

Now, using optimal financing decisions (2.5), (2.6) and the expression for financing feasibility of operations decisions (2.4), we can rewrite optimization problem (2.2) as follows:

**Case I:** $\boldsymbol{r_I < r_S}$.

$$\begin{aligned}\max_{z\geq 0, N\in\mathbb{N}} E\Big\{&p\min\left[D, Q(N,z)\right] - (1+r_I)(wz+NC) \\ &- (r_S - r_I)(wz + NC - I)^+ + (1+r_I)I - l\Big\}^+\end{aligned} \tag{2.7a}$$

subject to:

$$(w-\alpha)z + (C-\beta)N \leq I, \tag{2.7b}$$

$$wz - (\widehat{S} - C)N \leq I. \tag{2.7c}$$

For **Case I**, the term $(wz+NC-I)^+$ in the objective function is zero if only internal financing is used, and it is positive if both internal and supplier financing are used. Therefore, the equation $(wz+NC)-I=0$ defines a *threshold of dual financing* for **Case I**.

**Case II: $\boldsymbol{r_I > r_S}$.**

$$\begin{aligned}\max_{z\geq 0, N\in\mathbb{N}} E\Big\{&p\min\left[D, Q(N,z)\right] - (1+r_I)(wz+NC)\\&+(r_I-r_S)\min\left(N\widehat{S}, \alpha z + N\beta\right) + (1+r_I)I - l\Big\}^+\end{aligned} \tag{2.8a}$$

subject to:

$$(w-\alpha)z + (C-\beta)N \leq I, \tag{2.8b}$$

$$wz - (\widehat{S}-C)N \leq I. \tag{2.8c}$$

For **Case II**, instead of a threshold of dual financing, we are concerned with the *threshold of exceeding supplier loan limit*; that is, whether the manufacturer will order so much from each supplier that it will reach the limit on the supplier loan $\widehat{S}$.

Instead of proceeding with the analysis of problems (2.7) (when $r_I < r_S$) and (2.8) (when $r_I > r_S$) separately, observe that they possess the same mathematical structure. Therefore, we can formulate a generalized problem, which highlights the salient model features and streamlines the analysis. That is, instead of repeating the same analysis twice, we can perform it only once with the generalized model and then apply the results derived to each special case. Define a generalized objective function $f(N,z)$ as follows:

$$f(N,z) = E[\Pi_L(N,z)]^+ \cdot 1_{\{T(N,z)\leq 0\}} + E[\Pi_R(N,z)]^+ \cdot 1_{\{T(N,z)>0\}} \tag{2.9}$$

where

$$\Pi_L(N,z) = \begin{cases} p\min\left[D, Q(N,z)\right] - (1+r_I)(wz+NC) + (1+r_I)I - l & \text{if } r_I < r_S, \\ p\min\left[D, Q(N,z)\right] - (1+r_I)(wz+NC) + \\ \qquad (r_I - r_S)(\alpha z + N\beta) + (1+r_I)I - l & \text{if } r_I > r_S; \end{cases} \tag{2.10}$$

$$\Pi_R(N,z) = \begin{cases} p\min\left[D, Q(N,z)\right] - (1+r_S)(wz+NC) + (1+r_S)I - l & \text{if } r_I < r_S, \\ p\min\left[D, Q(N,z)\right] - (1+r_I)(wz+NC) + \\ \qquad (r_I - r_S)N\widehat{S} + (1+r_I)I - l & \text{if } r_I > r_S; \end{cases} \tag{2.11}$$

$$T(N,z) = \begin{cases} wz + NC - I & \text{if } r_I < r_S, \\ \alpha z + N\beta - N\widehat{S} & \text{if } r_I > r_S. \end{cases} \tag{2.12}$$

Thus, the objective function consists of two components, defined using function $\Pi_L$ and $\Pi_R$, with function $T$ defining the threshold $\widehat{z}$ seperating the domains of $\Pi_L$ and $\Pi_R$. For a given $N$,

$$T(N, \widehat{z}) = 0. \tag{2.13}$$

Threshold $\widehat{z}$ is what is earlier called the *threshold of dual financing* and the *threshold of exceeding supplier loan limit*. Throughout the remainder of the chapter, I denote

$$f_L(N,z) = E[\Pi_L(N,z)]^+ \text{ and } f_R(N,z) = E[\Pi_R(N,z)]^+. \tag{2.14}$$

Furthermore, observe that $f_L(N,z)$ and $f_R(N,z)$ have the same structure; both can be written as $E\left\{p\min\left[D, Q(N,z)\right] - a_k z - b_k(N) - l\right\}^+$ for $k = L, R$, with $a_k$ and $b_k(N)$ given by:

$$a_L = \begin{cases} (1+r_I)w & \text{if } r_I < r_S, \\ (1+r_I)w - (r_I - r_S)\alpha & \text{if } r_I > r_S. \end{cases} \tag{2.15}$$

$$b_L(N) = \begin{cases} (1+r_I)NC - (1+r_I)I & \text{if } r_I < r_S, \\ (1+r_I)NC - (r_I - r_S)N\beta - (1+r_I)I & \text{if } r_I > r_S. \end{cases} \tag{2.16}$$

$$a_R = \begin{cases} (1+r_S)w & \text{if } r_I < r_S, \\ (1+r_I)w & \text{if } r_I > r_S. \end{cases} \tag{2.17}$$

$$b_R(N) = \begin{cases} (1+r_S)NC - (1+r_S)I & \text{if } r_I < r_S, \\ (1+r_I)NC - (r_I - r_S)N\widehat{S} - (1+r_I)I & \text{if } r_I > r_S. \end{cases} \tag{2.18}$$

There is a good reason to introduce new notation. With the original parameters, the subsequent expressions would have been difficult to parse. New parameters have simple and practical interpretations: $a_k, k = L, R$ is the manufacturer's unit variable procurement cost, and $b_k(N), k = L, R$ is the manufacturer's overhead cost.

In Section 2.4.1, we have explored the optimal financing decisions. Section 2.4.2 and Section 2.4.3 are dedicated to analyzing operational decisions.

### 2.4.2 Operational decisions under stochastic demand and deterministic yield.

I investigate the effect of financing constraints on the optimal operational decisions for the manufacturer when the demand, $D$, is stochastic and the supplier yield, $X$, is deterministic and perfect, i.e., $Q(N, z) = z$. That is, for Section 2.4.2, I make the following assumption:

**Assumption 2.4.** *The yields of the suppliers are deterministic and perfect, $X = 1$.*

In Section 2.4.3, we will turn our attention to the effect of diversification for the case with random yield and deterministic demand.

#### 2.4.2.1 Manufacturer's objective function.

I begin with the analysis of the objective function of our problem, by assuming only in Section 2.4.2.1, that problem constraints are not binding. Recall the definitions (2.9) and (2.14) of the manufacturer's objective function. Unfortunately, functions $f_k(N,z), k = L, R$ are not necessarily concave in $z$, for a fixed $N$. Hence, the function $f(N,z)$ is not concave in $z$ either. Nevertheless, under Assumptions (2.1) through (2.4), the function $f(N,z)$ is well-behaved, as stated in the following proposition:

**Proposition 2.2.** *Suppose $N$ is fixed. Then, $f(N,z)$ is unimodal in $z$.*

The key observation that yields the result of Proposition 2.2 is the following: the function $f(N,z)$ is given by $f_L(N,z)$ to the left of $\widehat{z}$ (i.e. for $z < \widehat{z}$) and by $f_R(N,z)$ to the right of $\widehat{z}$ (i.e. for $z > \widehat{z}$), and the two unimodal functions $f_L(N,z)$ and $f_R(N,z)$ intersect at $z = \widehat{z}$. Given this observation, the function $f(N,z)$ will be non-unimodal only if $f_L(N,z)$ is decreasing and $f_R(N,z)$ is increasing in $z$ at $z = \widehat{z}$. As it turns out, this cannot happen. Hence, the function $f(N,z)$ is unimodal. Furthermore, this observation leaves us with only three possibilities regarding the behavior of functions $f_L(N,z)$ and $f_R(N,z)$ at $\widehat{z}$. Either both $f_L(N,z)$ and $f_R(N,z)$ are decreasing in $z$ at $z = \widehat{z}$ (in which case, the optimal $z$ is given by the maximizer of $f_L(N,z)$), or both $f_L(N,z)$ and $f_R(N,z)$ are increasing in $z$ at $z = \widehat{z}$ (in which case the optimal $z$ is given by the maximizer of $f_R(N,z)$), or $f_L(N,z)$ is increasing and $f_R(N,z)$ is decreasing in $z$ at $z = \widehat{z}$ (in which case, the optimal $z$ is given by $\widehat{z}$). This observation is formalized in Lemma 2.4 (see appendix 2.8.2), which forms the basis for an algorithm (provided in appendix 2.8.2) to determine the optimal order quantity, $z^* \stackrel{\text{def}}{=} \arg\max_z\{f(N,z)\}$, for a given $N$.

One can show that $f_k(N, z), k = L, R$ is supermodular in order quantity, $z$, and overhead cost, $b_k$, for fixed $N$. Therefore,

**Proposition 2.3.** *Suppose that the number of suppliers, $N$, is fixed. Then, for $k = L, R$, $z_k^* \overset{def}{=} \arg\max_z\{f_k(N, z)\}$ is non-decreasing in the overhead cost, $b_k$.*

Because the overhead cost, $b_k$, is increasing in the fixed cost to work with a supplier, $C$, it follows that, if the optimal order quantity, $z^* = z_k^*$ for $k = L$ or $k = R$, then the optimal order quantity may be increasing in $C$. This is an effect of the limited liability assumption. In the model with full liability (i.e. $l = -\infty$) the optimal order quantity does not depend on the fixed cost, provided that the fixed costs are small enough for the manufacturer to be in business.

One would expect that the optimal order quantity of a limited-liability manufacturer will be higher than the optimal order quantity of a full-liability manufacturer. After all, limited liability curbs the manufacturer's overage costs, thereby inducing the manufacturer to stock larger quantities. This is, indeed, the case as stated in the following proposition:

**Proposition 2.4.** *Suppose that the number of suppliers, $N$, is fixed. Then, the more negative the liability level, $l$, the smaller the optimal order quantity, $z^*$.*

So far we have explored the choice of order quantity, $z$, assuming that the number of suppliers, $N$, is fixed and ignoring optimization constraints. Next, let us consider the choice of the number of suppliers, $N$, still ignoring optimization constraints.

**Proposition 2.5.** *Suppose that $z$ is fixed. Then:*

*(i) If $r_I < r_S$, then the objective function, $f(N, z)$ is decreasing in the number of suppliers, $N$, and the manufacturer will choose the optimal number of suppliers, $N^* = 1$.*

*(ii) If $r_I > r_S$ and the fixed cost to work with a supplier, $C > \frac{(r_I - r_S)}{(1+r_I)}\widehat{S}$, then the objective function, $f(N, z)$, is decreasing in the number of suppliers, $N$, and the manufacturer will choose the optimal number of suppliers, $N^* = 1$.*

*(iii) If $r_I > r_S$ and the fixed cost to work with a supplier, $C \leq \frac{(r_I - r_S)}{(1+r_I)}\beta$, then the objective function, $f(N, z)$ is increasing in the number of suppliers, $N$, and the manufacturer will choose to work with the largest possible number of suppliers (as long as the financing constraints are ignored).*

These results are intuitive. Ignoring the financing constraints, part $(i)$ of the proposition says that, if the internal capital is the cheaper source of financing, then the manufacturer will order from only one supplier, as there is no reason for the manufacturer to work with multiple suppliers and incur the fixed cost of $C$ for each one (recall that, in Section 2.4.2, there is no supply risk). On the other hand, as shown in part $(iii)$, when the suppliers are the cheaper source of financing and the cost to work with an extra supplier is much smaller than the guaranteed supplier loan amount, then the manufacturer may choose to work with multiple suppliers, because it can reinvest money borrowed from the suppliers in its other businesses. However, as shown in part $(ii)$, when the fixed cost to work with a supplier ($C$) is sufficiently close to the maximum loan available from a supplier ($\widehat{S}$), then the additional loan from a supplier will not be worth the additional fixed cost, and the manufacturer will again choose to work with only one supplier. Finally, note that Proposition 2.5 does not describe the behavior of the objective function when $r_I > r_S$, and $\frac{(r_I - r_S)}{(1+r_I)}\beta < C \leq \frac{(r_I - r_S)}{(1+r_I)}\widehat{S}$. In this case, depending on which part ($f_L$ or $f_R$) of the objective function we are considering, the objective function can be either increasing or decreasing in the number of suppliers $N$.

To further study the optimal choice of the number of suppliers, $N^*$, we have to consider the effects of the financing constraints.

#### 2.4.2.2 Manufacturer's problem with financing constraints.

When there is no limit on the total capital available to the manufacturer, Section 2.4.2.1 describes the optimal operational decisions of the manufacturer. Unfortunately, the manufacturer does not have access to unlimited capital. In Section 2.4.2.2, I consider the effect of financing constraints on the manufacturer's operational decisions.

Finding the value of optimal $N$ for a problem with financing constraints is not a trivial task. Even when the financing constraints are not binding, as $N$ changes, $f_L(N, z)$ and $f_R(N, z)$ may increase or decrease, and the unconstrained optimal order quantity, $z^*$, may switch between $z_L^*$, $z_R^*$ and $\widehat{z}$. This complicated relationship between $N$ and $z^*$, together with the discrete nature of $N$, makes analytical derivation of the optimal number of suppliers, $N^*$, unlikely. Propositions in Section 2.4.2.1 provide structural properties of the objective function, $f(N, z)$, and bounds on the optimal $N$ and $z$. In addition, the following propositions describe further bounds on the optimal values of $N$ and $z$ due to financing constraints.

First, I will derive a bound on the optimal number of suppliers, $N^*$, in the presence of financing constraints.

According to Assumption 2.2, the loan available from each supplier is less than the cost of ordering from that supplier by at least $C - \beta$, and the difference must be made up by the internally generated capital. Therefore, the amount of internal capital imposes a limit on the number of suppliers the manufacturer can work with (formally, this is seen from the first inequality in (2.4)).

**Proposition 2.6.** *The optimal number of suppliers, $N^*$, is limited by $N^* \leq \frac{I}{C-\beta}$.*

Next, I will present bounds on the optimal order quantity. The number of suppliers, $N$, restricts the feasible choices for the order quantity, $z$, because the number of suppliers affect the amount of loans available to the manufacturer. The following proposition (which follows from system (2.4) by fixing $N$) formalizes this relationship.

**Proposition 2.7.** *Suppose that the number of suppliers, $N$, is fixed and satisfies $N \leq \frac{I}{C-\beta}$. Then:*

*(i) If $N \leq \frac{\alpha I}{w(\widehat{S}-\beta)-\alpha(\widehat{S}-C)}$, then the optimal order quantity satisfies*

$$z \leq z_{max}(N) \stackrel{def}{=} \frac{I+(\widehat{S}-C)N}{w}. \tag{2.19}$$

*(ii) If $\frac{\alpha I}{w(\widehat{S}-\beta)-\alpha(\widehat{S}-C)} \leq N \leq \frac{I}{C-\beta}$, then the optimal order quantity satisfies*

$$z \leq z_{max}(N) \stackrel{def}{=} \frac{I-(C-\beta)N}{w-\alpha}. \tag{2.20}$$

In Section 2.4.2, we have addressed the manufacturer's operational decisions when the demand is stochastic and the yield is deterministic. Next I focus on the case in which the demand is deterministic and the yield is stochastic.

### 2.4.3 Operational decisions under deterministic demand and stochastic yield.

The manufacturer may decide to use several suppliers not only to acquire access to a larger capital pool (as we discussed in section 2.4.2), but also to diversify risk, if suppliers are not perfectly reliable. The tradeoffs between diversification benefits and set-up costs, without financing constraints and with full manufacturer's liability, have been studied by Agrawal and Nahmias (1997). Section 2.4.3 extends their analysis by considering a model with limited liability and financing constraints. This is a difficult model to analyze. Therefore, for Section 2.4.3, I make the following assumption:

**Assumption 2.5.** *The demand, $D$, is deterministic.*

Besides simplifying the analysis, this assumption may be useful for a problem where the demand uncertainty is much smaller than the supply uncertainty, for example, when the manufacturer has long-term contracts with the customers.

#### 2.4.3.1 Manufacturer's objective function.

Recall that the manufacturer's objective function is given by expressions (2.9) and (2.14). I will only consider operational decisions: number of suppliers, $N$, and order quantity, $z$, which satisfy

**Assumption 2.6.** $pD > a_k z + b_k(N), k = L, R.$

Assumption 2.6 can be made without loss of generality because, if it is violated, the manufacturer is guaranteed to have negative profit. The following lemma offers a convenient expression for $f_k, k = L, R$ (see equations (2.14) for definitions of $f_k, k = L, R$):

**Proposition 2.8.** *Suppose that the number of suppliers, $N$, is fixed. Let $\overline{X}$ be a random variable with c.d.f. $\Phi$ and p.d.f. $\phi$ and let $f_k, k = L, R$ be defined by equations (2.14). Define*

$$\Gamma(m) = \int_m^\infty x\phi(x)dx, \tag{2.21}$$

$$\gamma(m) = \Gamma'(m) = -m\phi(m) \tag{2.22}$$

*Then,*

$$\begin{aligned} f_k(z) =&(pD - a_k z - b_k - l) \int_{\frac{D}{z}}^\infty \phi(x)\,dx \\ &+ \int_{\frac{a_k z + b_k + l}{pz}}^{\frac{D}{z}} [pxz - a_k z - b_k - l]\phi(x)\,dx \end{aligned} \tag{2.23}$$

$$
\begin{aligned}
f_k'(z) &= p \int_{\frac{a_k z + b_k + l}{pz}}^{\frac{D}{z}} x\phi(x)\, dx - a_k \Pr\left[\overline{X} \geq \tfrac{a_k z + b_k + l}{pz}\right] \\
&= p\left[\Gamma\left(\tfrac{a_k z + b_k + l}{pz}\right) - \Gamma\left(\tfrac{D}{z}\right)\right] - a_k \Pr\left[\overline{X} \geq \tfrac{a_k z + b_k + l}{pz}\right]
\end{aligned} \tag{2.24}
$$

*and, if $\frac{\gamma(\cdot)}{\Gamma(\cdot)}$ is decreasing, $f_k, k = L, R$ are unimodal in $z$.*

Using this lemma, one can prove that the objective function of the model with the random supplier yield is unimodal in the total order quantity.

**Proposition 2.9.** *If the mean of supplier yields is normally distributed, that is $\overline{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{N}}\right)$, and Assumptions 2.2, 2.3, and 2.6 hold, then the manufacturer's objective function, $f(z)$, defined by 2.9, is unimodal in the order quantity, $z$.*

Thus, the optimization problem in Section 2.4.3 has the same structure as the optimization problem with certain yield and random demand in Section 2.4.2. If the number of suppliers, $N$, is fixed, we can find the optimal order quantity, $z^*$, using the analog of Lemma 2.4 in appendix 2.8.2. Unlike the model in Section 2.4.2, each component ($f_k(N,z), k = L, R$) of the objective function, is submodular in the order quantity, $z$, and the overhead cost, $b_k$. Therefore,

**Proposition 2.10.** *Suppose that the number of suppliers, $N$, is fixed. For $k = L, R$, if $b_k + l > 0$, then $z_k^* \stackrel{def}{=} \arg\max_z\{f_k(N,z)\}$ is non-increasing in the overhead cost, $b_k$, and as the liability level $l$ becomes more negative, $z_k^*$ increases (non-strictly). If $b_k + l < 0$, then $z_k^* \stackrel{def}{=} \arg\max_z\{f_k(N,z)\}$ is non-decreasing in the overhead cost, $b_k$, and as the liability level $l$ becomes more negative, $z_k^*$ decreases (non-strictly).*

Finding the optimal number of suppliers, $N^*$, for the model in Section 2.4.3 is as difficult as finding the optimal number of suppliers for the model in Section 2.4.2. To obtain further managerial insights into the manufacturer's optimal decisions, in particular, the optimal number of suppliers, we conduct a numerical study, which is discussed in the next section.

## 2.5 Numerical study.

Propositions in Section 2.4 provide structural properties of the objective function, $f(N, z)$, and bounds on the optimal $N$ and $z$. Such analytical result allow us to devise an efficient search algorithm to find the optimal solution, thus facilitating a numerical study. As I discuss in this section, the numerical study provides several valuable insights into the choice of optimal operational decisions under financing constraints.

### 2.5.1 Stochastic demand, deterministic yield.

First, I focus on the case in which the demand is random, but the yield is perfect and deterministic. The numerical study uses the following default values of model parameters: the rate of internal financing and the rate of supplier loans are $(r_I, r_S) = (0.2, 0.1)$[7], the per unit revenue is $p = 3$, the wholesale price is $w = 0.5$, the fixed cost to work with a supplier is $C = 20$, the parameters of the supplier loans are $\alpha = 0.48$ and $\beta = 4$, the internal capital is $I = 75$, the limit on the supplier loan is $\widehat{S} = 75$, the demand is normal with mean $\mu_D = 500$ and variance $\sigma_D^2 = 500$, and the liability level is $l = 0$.

**Effects of internal capital.**

Consider the effects of the internal capital first, depicted in Figure 2.1. As the internal capital, $I$, decreases, financing constraint forces the manufacturer to order smaller quantities (as shown in the right panel of Figure 2.1), which, in turn, causes a decrease in the manufacturer's revenues. Rather than suffer from a further decline in

[7]the graphs look either identical or similar when $(r_I, r_S) = (0.1, 0.2)$

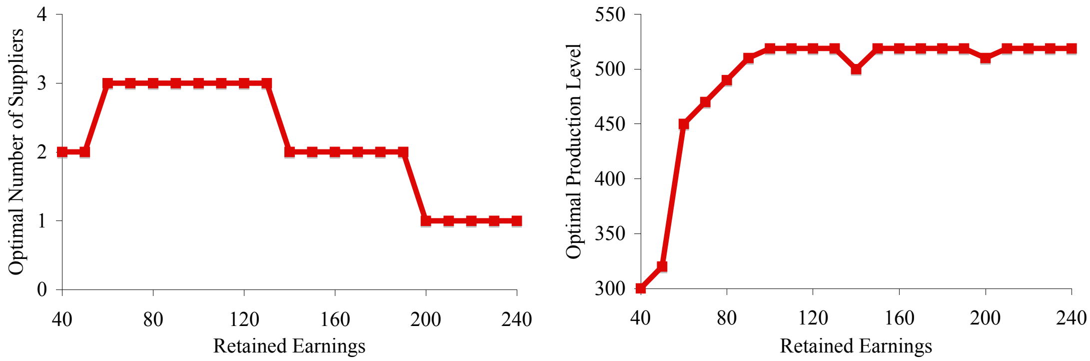


Figure 2.1: The effect of the internal capital.

revenues, the manufacturer may prefer to incur the fixed cost to work with an extra supplier who will provide the manufacturer with additional financing. Therefore, we observe in the left panel of Figure 2.1 that, for high $I$, the optimal number of suppliers, $N$, and the optimal order quantity, $z$, may increase as $I$ decreases. However, once the internal capital ($I$) becomes too small, the manufacturer will have to reduce the number of suppliers again. To see why, recall that the limit on the trade credit available from a supplier, $\overline{S}(z/N) = \min\{\widehat{S}, \alpha z/N + \beta\}$, is less than the cost of ordering from that supplier, $wz/N + C$. Therefore, for each supplier the manufacturer works with, the difference between the trade credit and the cost of ordering must be covered through the use of internal financing. If $I$ is too small, the manufacturer cannot afford the fixed cost to work with an additional supplier, which forces the manufacturer to reduce the number of suppliers. Similar behavior is observed in the supplier loan-limit study.

**Effects of fixed cost.**

An increase in the supplier fixed cost, $C$, may cause an increase in the optimal number of suppliers, $N$. One example of such behavior is depicted in Figure 2.4 by curves marked 'Developing,' which correspond to the default parameter set for these numerical examples. This surprising result is observed for moderate $C$, when the manufacturer is borrowing the absolute maximum amount, $\widehat{S}$, from each supplier. The following is the intuitive explanation for this phenomenon. As the fixed cost to work with a supplier, $C$, increases, because of the financing constraints (inequalities (2.4)), the manufacturer has to reduce the order quantity (and, hence, future revenues) in order to pay for the increased cost, $C$. If the manufacturer is borrowing the absolute maximum amount, $\widehat{S}$, (that is the second inequality in (2.4) is binding), the manufacturer can relax its financing constraints (and increase revenues) by adding suppliers. How many suppliers the manufacturer will add depends on the extra supplier cost, $C$, which appears in the objective function, and also on the value of $N$ for when the manufacturer runs out of internal capital to finance additional fixed costs to work with suppliers (i.e., the first inequality in constraint (2.4) becomes binding).

**Effects of wholesale price.**

The number of suppliers may also be non-monotone in the wholesale price, $w$, as depicted in Figure 2.2 (in this numerical example, $C = 5$). As expected, when the

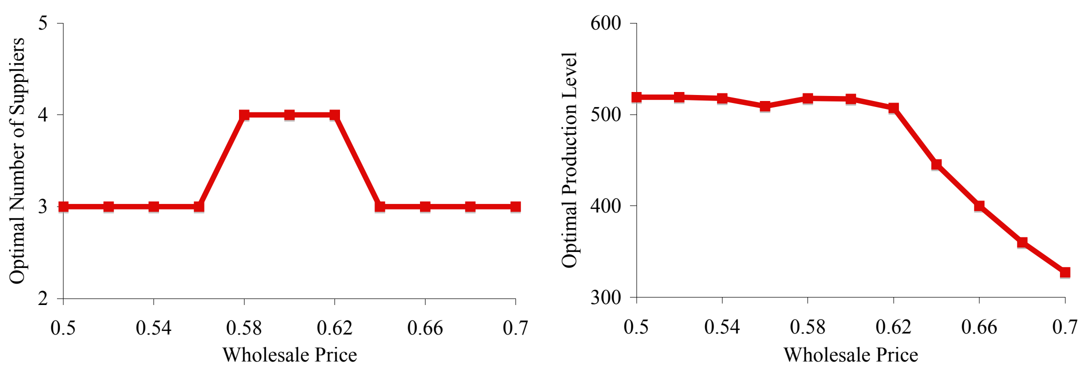


Figure 2.2: The effect of the wholesale price.

wholesale price, $w$, is large, the business is barely profitable, and the manufacturer works with few suppliers. As $w$ becomes smaller, the manufacturer would like to order a larger quantity (because the profit margin on each unit sold to the customer is larger), and in an effort to order a larger quantity, the manufacturer may find it preferable to work with additional suppliers so that it can raise the necessary cash. As we have already explained in our discussion on the effects of the fixed cost to work with a supplier, the manufacturer can relax its financing constraints by working with more suppliers, if the second inequality in (2.4) is binding. Once the wholesale price, $w$, is sufficiently small, the manufacturer reduces the number of suppliers again to save on the fixed cost to work with a supplier. Although this reduces the cash available for purchases, which, in turn, drives the order quantity down, the manufacturer still prefers saving the fixed cost, because the extent of the reduction in order quantity is dampened by the small wholesale price.

**Effects of the standard deviation of the demand.**

The left panel of Figure 2.3 demonstrates that, depending on the value of unit revenue, $p$, the optimal number of suppliers could be either increasing or decreasing in the standard deviation of the demand. The key to understanding this behavior is the right panel of Figure 2.3 and what we know about the relationship between the standard deviation of the demand and the optimal order quantity for the newsvendor problem (although, because of the limited liability, we do not have a newsvendor problem, the behavior of the optimal order quantity for our problem is similar). For the normally distributed demand, the optimal order quantity for the

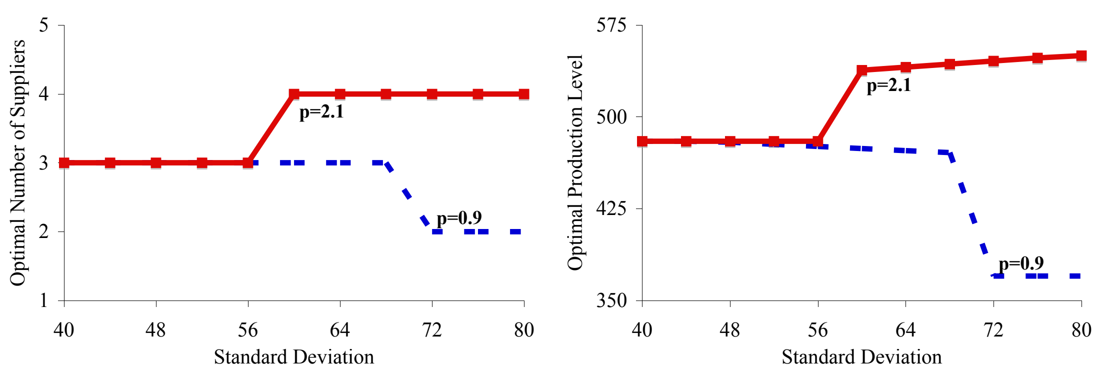


Figure 2.3: The effect of the demand standard deviation.

newsvendor problem is $z^{news} = \mu_D + \sigma_D \mathcal{N}^{-1}(1 - a/p)$, where $\mathcal{N}$ is the c.d.f. of a standard normal random variable, $a$ is the variable cost, $\mu_D$ is the demand's mean, and $\sigma_D$ is the demand's standard deviation. From this expression, if $a/p < 1/2$, then $z^{news}$ increases in $\sigma_D$, and if $a/p > 1/2$, then $z^{news}$ decreases in $\sigma_D$. The unconstrained order quantity for our problem (derived from the first order condition (2.36)) also depends on the value of ratio $a/p$. If $a/p$ is small (e.g. when $p = 2.1$, for the case $r_I > r_S$, $a_L/p = [(1 + r_I)w - (r_I - r_S)\alpha]/p = 0.263$ and $a_R/p = (1 + r_I)w/p = 0.286$), the optimal order quantity increases and, to finance this increase (to relax financing constraints), the manufacturer may increase the number of suppliers. Conversely, if $a/p$ is large (e.g. when $p = 0.9$, for the case $r_I > r_S$, $a_L/p = [(1 + r_I)w - (r_I - r_S)\alpha]/p = 0.613$ and $a_R/p = (1 + r_I)w/p = 0.667$), the unconstrained optimal order quantity decreases and the manufacturer can reduce the number of suppliers because financing constraints are no longer binding.

**Developing vs. developed economies.**

Finally, let us contrast the effect of financing constraints on manufacturers operating in developing and developed economies and that have access to only regional suppliers. Manufacturers in developed economies enjoy access to much more capital compared to manufacturers in developing economies.[8] In this numerical example the loan limits for the developing economy are set to $I = 75$, $\widehat{S} = 75$. Let loan limits for the developed country be $I = 75$, $\widehat{S} = 300$. Therefore, as Figure 2.4 illustrates, we would expect that for any given level of the fixed cost, $C$, manufacturers in developing economies will tend to have a greater number of suppliers (ceteris paribus).

Does this mean that we should expect to observe this disparity empirically? Not

[8]In addition to the capital availability, manufacturers operating in developed and developing economies may also face different costs of capital. Our numerical experiments showed that that the effects of capital costs are predictable: for instance, higher internal financing rate encourages the manufacturer to work with more suppliers.

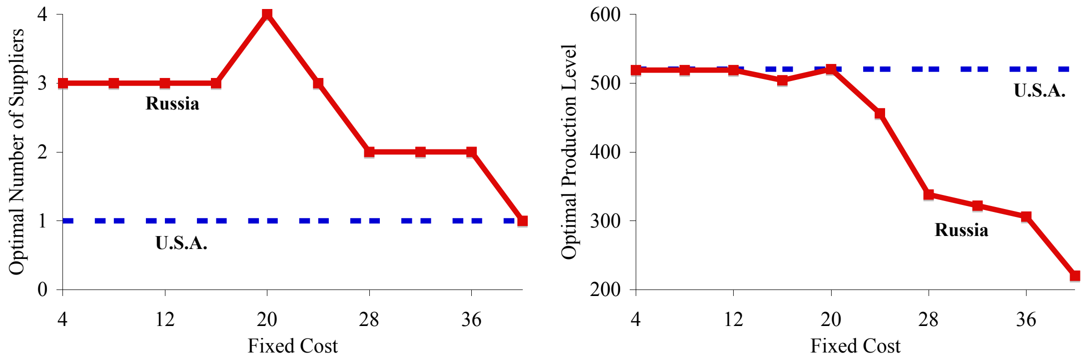


Figure 2.4: Developing vs. developed economies.

necessarily. Because manufacturers in developing economies will also tend to have greater fixed costs, $C$ (due to the operational inefficiencies), they may optimally keep the number of suppliers low. However, as Figure 2.4 illustrates, if $C$ is very large, the model predicts that these manufacturers will place smaller orders, have lower inventory (and hence, experience a higher frequency of stock-outs), an effect that has been empirically observed by Fisman (2001).

Here, I focused on the availability of external financing and fixed costs as the main differentiators between developed and developing economies. One could make further comparisons by focusing on other differentiators, e.g., supplier yield characteristics (suppliers in developed economies may be more reliable).

**Interactions between parameter values.**

Figure 2.5 shows the optimal number of suppliers as functions of the limit on supplier loans and the fixed cost to work with a supplier, the limit on supplier loan and the wholesale price, and the internal capital of the manufacturer and the demand mean. Lighter shades correspond to a higher number of suppliers. The black shade corresponds to $N^* = 1$; the white shade corresponds to $N^* = 3$. From Figure 2.5 we observe that the higher number of suppliers corresponds to low fixed cost and low supplier loan limit (left panel), or low wholesale price and low supplier loan limit

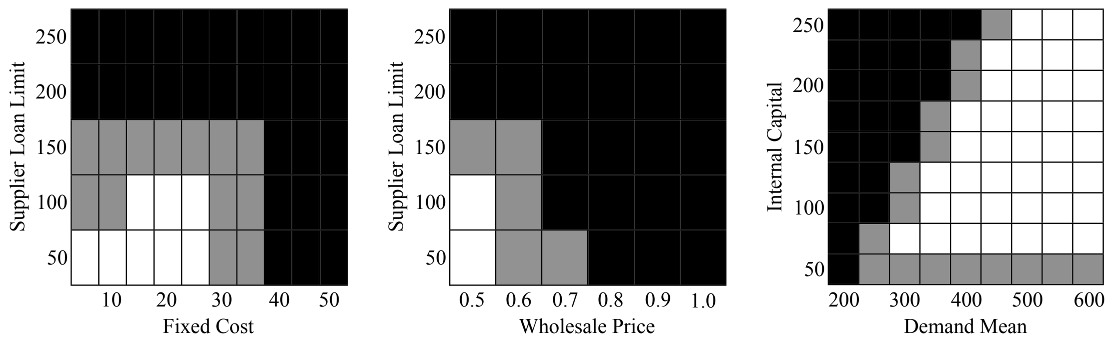


Figure 2.5: The optimal number of suppliers. Interactions between parameter values.

(center panel). The picture in the right panel of Figure 2.5 shows that limited internal capital may prevent the manufacturer from increasing the number of suppliers even as the demand for products (and hence demand for financing) increases.

### 2.5.2 Deterministic demand, stochastic yield.

Next, I discuss the numerical results when the yield is random. In this numerical study, I observed that the effects of the financing constraints, the fixed cost and the wholesale price are the same under this model as in the model with deterministic yield. Therefore, in what follows, I only focus on the effects of the random yield, which is characterized by its mean, $\mu$, and its standard deviation, $\sigma$, on the shareholders' value, the optimal number of suppliers, and the optimal order level. The presentation will focus on the case where the internal rate is lower than suppliers' interest rate ($r_I < r_S$). The results for the other case ($r_I > r_S$) are similar.

The numerical study uses the following default values of model parameters: unit revenue, $p = 1.75$, wholesale price, $w = 0.5$, interest rate on supplier loans, $r_S = 0.2$, rate on internal capital, $r_I = 0.1$, fixed cost to work with a supplier, $C = 5$, $\beta = 4.9$, $\alpha = 0.48$, absolute limit for supplier loans, $\widehat{S} = 225$, internal capital, $I = 20$, and demand, $D = 500$.

According to the left panel of Figure 2.6, the shareholders' value is increasing

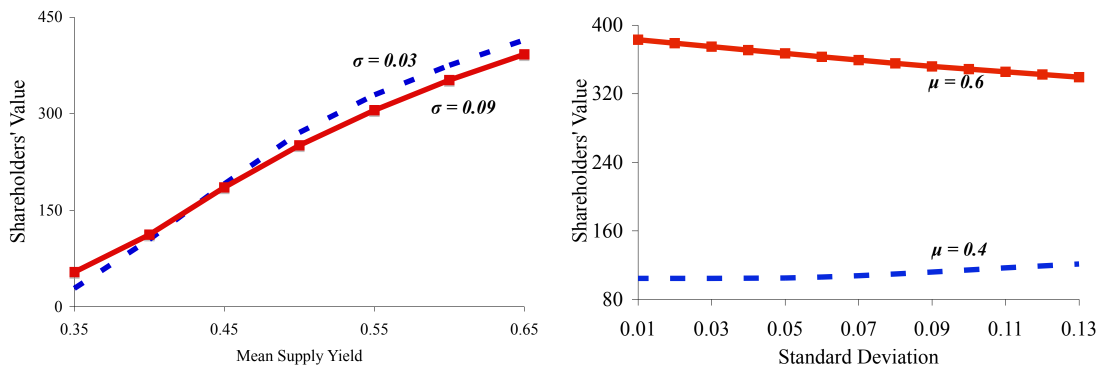


Figure 2.6: Shareholder value as a function of yield parameters.

as the expected yield, $\mu$, increases. This behavior is to be anticipated, because the manufacturer benefits if the average reliability of its suppliers increases.

Surprisingly, as shown in the right panel of Figure 2.6, the increase in the standard deviation, $\sigma$, of the supplier yield may result in either an increase or a decrease in the shareholders' value. If the expected yield is high (low) the shareholders' value decreases (increases) in the standard deviation of the yield. To understand this phenomenon, consider how functions $f_L$ and $f_R$ depend on mean yield $\overline{X} = \frac{\sum_{k=1}^{N} X_k}{N}$. Recall that $f_k(\overline{X}) = E\left[\Pi_k(\overline{X})\right]^+, k = L, R$ (see equation (2.14)). We can rewrite this expression as

$$f_k(\overline{X}) = E\left[\Pi_k(\overline{X})\right] + E\left[-\Pi_k(\overline{X})\right]^+ \quad k = L, R \tag{2.25}$$

The first term in (2.25) represents the manufacturer's expected profit. Because $\Pi_k$ is a concave function, as the standard deviation of yield increases, this term decreases. The second term in (2.25) represents the value of the option to default that shareholders hold (because they have limited liability). Function $[-\Pi_k(\cdot)]^+$ is convex and, therefore, this term increases as the standard deviation of the yield increases. Thus, the change in the shareholders' value, as the volatility of the yield, $\sigma$, increases, comes from the decrease in the expected profit and the increase in the value of the option to default. When the expected yield is small (e.g. when $\mu = 0.4$), the manufacturer is close to bankruptcy and, therefore, the value of the option provides the largest contribution to the shareholders' value. This means that the convex part of the objective function dominates, and the decision maker behaves as a risk-seeking agent and responds to the increasing volatility of the supplier yield by decreasing the number of suppliers, as shown in the left panel of Figure 2.7. When the expected yield is high (e.g. when $\mu = 0.6$), the expected profit provides the largest contribution to the shareholders' value. That is, the concave part of the objective function dominates, and the decision maker behaves as a risk averse agent and responds to the increasing volatility in supplier yield by increasing the number of suppliers.

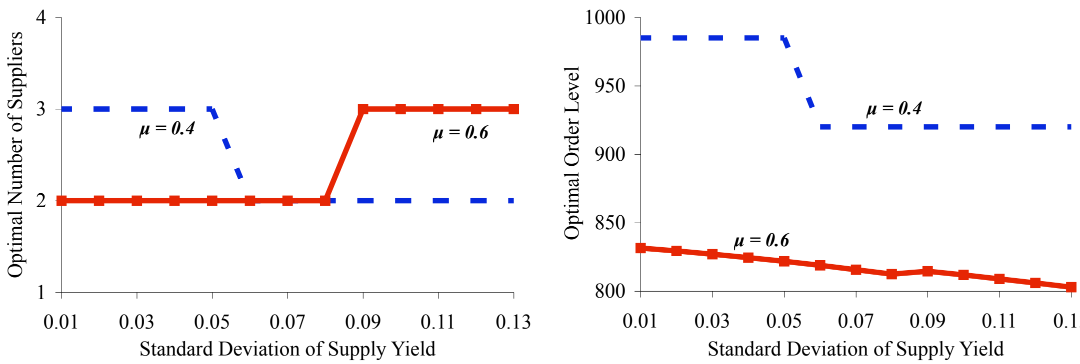


Figure 2.7: Effect of a supplier's yield uncertainty.

The left panels of Figures 2.7 and 2.8 confirm our intuition about diversification and the risk-averse behavior of the decision maker. I use the standard deviation of the order delivered as a proxy of risk and observe that, for the curves corresponding to $\mu = 0.6$, an increase in the optimal number of suppliers coincides with a decrease in risk. Thus, the decision maker is acting as a risk-averse agent. Similarly, for the curves corresponding to $\mu = 0.4$, a decrease in the number of suppliers corresponds to an increase in risk, indicating that the decision maker is acting as a risk-seeking

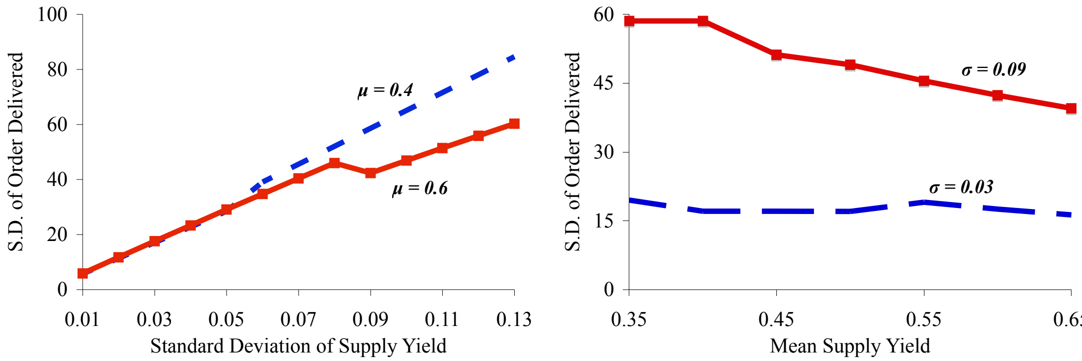


Figure 2.8: The standard deviation of the order delivered.

agent. I defer the discussion of the right panel of Figure 2.8 until our discussion of Figure 2.9.

Next, consider the right panel of Figure 2.7, which illustrates the effect of the standard deviation, $\sigma$, of supplier's yield on the optimal order quantity, $z^*$. I will focus on the curve corresponding to the expected yield, $\mu = 0.4$. Observe that the optimal order quantity decreases in $\sigma$. To understand this behavior, note that the profit of the manufacturer can be written as

$$\Pi = B - A\overline{X}z - p\max(D - \overline{X}z, 0), \tag{2.26}$$

for some constants, $A$ and $B$. This is a payoff on a portfolio consisting of a safe bank account ($B$), a short position in the underlying asset ($\overline{X}z$), and a short position in a put option ($\max(D - \overline{X}z, 0)$) on the underlying asset.[9] The distribution of the underlying asset is $\mathcal{N}\left(\mu z, \frac{\sigma}{\sqrt{N}} z\right)$. From the option theory, the value of the put option increases in the underlying asset's variance (in this case $\frac{\sigma^2}{N} z^2$). Therefore, assuming that the optimal number of suppliers, $N^*$, is constant, an increase in the standard deviation of the yield, $\sigma$, leads to an increase in the put option value ($E[\max(D - \overline{X}z, 0)]$) and a decrease in the expected profit ($E[\Pi]$). Shareholders can hedge against the effects of increasing $\sigma$ by reducing the optimal order quantity,

[9]See Hull (2000) for definitions and discussion of option contracts.

$z^*$. Finally, observe that for $\mu = 0.6$ the optimal order level increases when the number of suppliers changes from 2 to 3. The explanation for this behavior is akin to the results in Proposition 2.10. As the number of suppliers increases, the overhead cost increases and the maximizers of each of the two parts of the objective function increase.

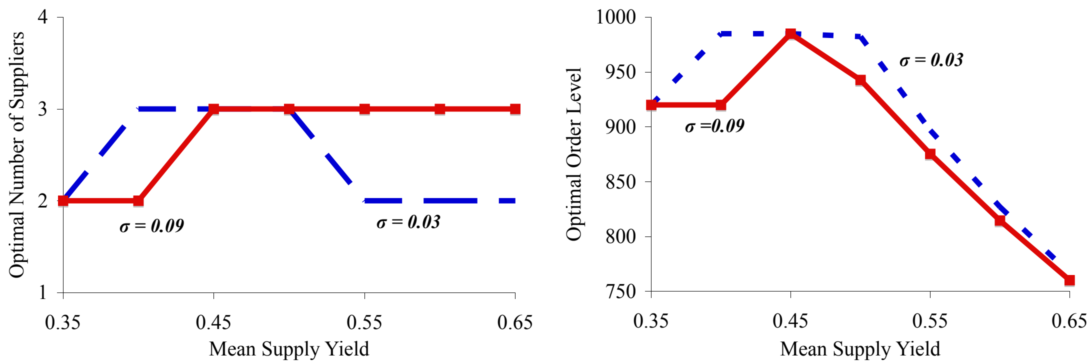


Figure 2.9: Effect of a supplier's expected yield.

Curves in the left panel of Figure 2.9 are formed due to the (now familiar) tradeoff between the risk-seeking and risk-averse behavior of the shareholders, and also due to a tradeoff between the benefits of diversification and the costs to work with suppliers. Comparing the left panel of Figure 2.9 and the right panel of Figure 2.8, observe that an increase in the number of suppliers corresponds to a decrease in risk, and a decrease in the number of suppliers corresponds to an increase in risk. When the expected yield, $\mu$, is small, the convex part of the shareholders' objective dominates and the shareholders behave as risk-seeking agents. As the expected supplier yield increases, the risk-seeking behavior is replaced by the risk-averse one and the number of suppliers increases. As the expected yield continues to grow, that is, as the suppliers become more reliable, the need for diversification becomes less pressing and the manufacturer can start saving on fixed costs by reducing the number of suppliers. Curves in the right panel of Figure 2.9 follow from the observation that, as the suppliers become more reliable, the manufacturer does not have to order as much to compensate for possible losses. Figure 2.10 illustrates the effect of limited

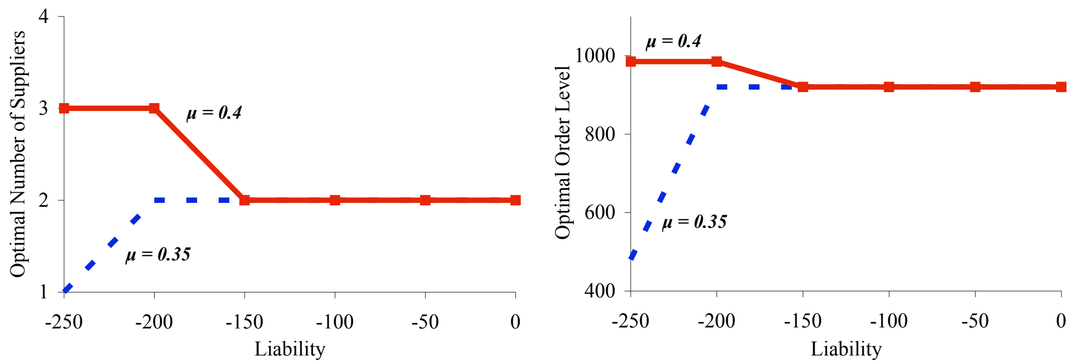


Figure 2.10: Effect of limited liability.

liability. In this numerical study the standard deviation of the supplier yields is $\sigma = 0.1$. First, while conducting numerical experiments, I observed that the limited liability manifests itself only when the manufacturer is fairly close to bankruptcy and, hence, the option to default on part of its obligations is valuable. Therefore, Figure 2.10 contains curves for two cases which bring the manufacturer close to bankruptcy: mean supplier yields $\mu = 0.35$ and $\mu = 0.4$. If supply were certain, the more negative liability level, $l$, becomes, the less valuable the business becomes, the smaller the order that will be placed, the less financing will be needed, and the fewer suppliers the manufacturer will work with. However, when supply is uncertain, the manufacturer benefits from diversification by working with more suppliers. Whether the number of suppliers increases or decreases as $l$ decreases depends on which of the two forces (financing required vs. diversification) dominates. For the graph corresponding to $\mu = 0.4$ in Figure 2.10 the diversification force prevailed.

Figure 2.11 highlights the interaction between limited liability and financing constraints due to supplier loan limits (left panel), and diversification choices for different values of yield parameters $\mu$ and $\sigma$. In this figure, I plot the optimal number of sup-

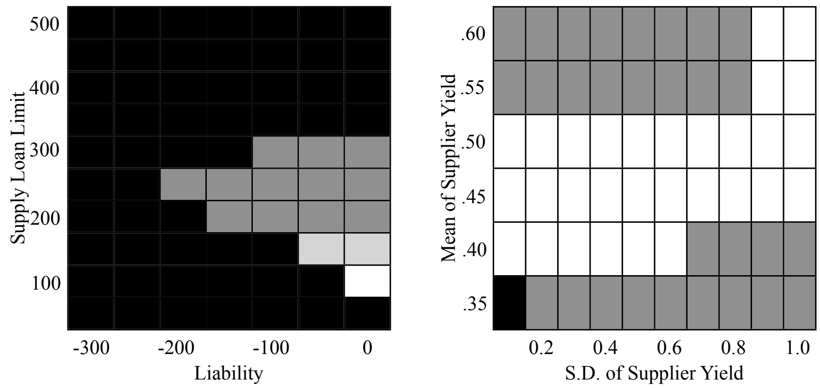


Figure 2.11: The optimal number of suppliers as a function of limited liability and supplier loan limit (left panel), and mean and standard deviation of the supplier yield (right panel).

pliers. The lighter shades correspond to the greater number of suppliers. The black color corresponds to $N^* = 1$.

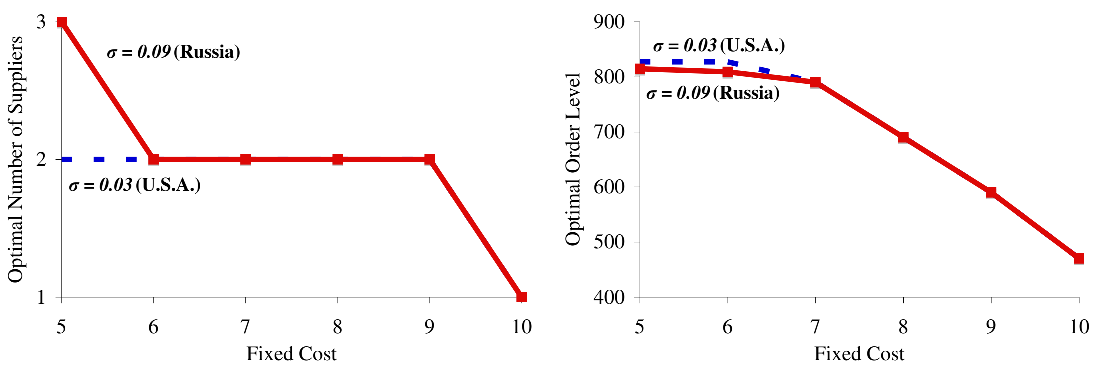


Figure 2.12: Developing vs. developed economies. Random yield. Mean value of the supplier yield is $\mu = 0.6$.

Finally, Figure 2.12[10] illustrates the effect of random yield the decisions of manufacturers in developing and developed economies and that have access to only regional suppliers. The figure shows that a greater volatility of supplier yields will encourage manufacturers in developing economies to have a greater number of suppliers if the fixed cost to work with the suppliers is not too high.

[10]In this example, $\mu = 0.6$.

## 2.6 Heterogeneous suppliers.

In this section I will discuss the implications of relaxing the assumption of a homogeneous supplier base. In general, suppliers could differ in a number of attributes: $(\mu_i, \sigma_i)$, distribution parameters of supplier $i$ yield; $w_i$, wholesale price charged by suppler $i$; $C_i$, fixed cost to work with supplier $i$; $\widehat{S}_i$, absolute limit on supplier $i$ loan amount; $(\alpha_i, \beta_i)$, parameters of the supplier $i$ trade credit loan; $r_i$, interest rate on supplier $i$ loan. The multi-attribute problem of selecting a subset of suppliers for the manufacturer to work with can only be solved numerically, except for special cases.

To begin, assume that the manufacturer can only work with a single supplier and, hence, the supplier selection problem becomes: which supplier should win the manufacturer's business. For each of the suppliers one needs to solve problem (2.2) with an additional constraint, $N = 1$, and then select the solution which offers the highest value of the objective function. It is not difficult to see that, everything else being equal, the manufacturer favors the supplier with the lowest $r_i$, $C_i$, or $w_i$, and the highest $\widehat{S}_i$, $\beta_i$, $\alpha_i$, or $\mu_i$. The effect of $\sigma_i$ is not immediately obvious. As discussed in Section 2.4.3, the presence of the option to default may encourage the manufacturer to take more risk, by working with suppliers whose yield distribution has a higher standard deviation.

A general model of (2.2), where the manufacturer can work with any number of suppliers, has the following mathematical form:

$$\begin{aligned}\max_{\{\vec{S},\vec{y}\}} E\Big\{&p\min[D, Q(\vec{y})] - (1+r_I)\sum_i \left(w_i y_i + C_i \mathbf{1}_{\{y_i>0\}}\right) \\ &- \sum_i (r_i - r_I)S_i + (1+r_I)I - l\Big\}^+\end{aligned} \tag{2.27a}$$

subject to:

$$\sum_i \left(w_i y_i + C_i \mathbf{1}_{\{y_i>0\}}\right) \leq \sum_i S_i + I, \tag{2.27b}$$

$$0 \leq S_i \leq \overline{S}_i(y_i), \tag{2.27c}$$

$$\overline{S}_i(y) = \min\left(\widehat{S}_i, \beta_i + \alpha_i y\right). \tag{2.27d}$$

where $Q(\vec{y})$ is the quantity received by the manufacturer from the suppliers (possibly a random variable). Similar to the analysis in Section 2.4.1, we can determine the optimal financing decisions, given a particular choice of operational decisions. Specifically, suppose that $\vec{y}$ is given. Consider only the suppliers that received positive orders ($y_i > 0$) from the manufacturer and sort them according to the value of the rates on their loans, $r_i$, in increasing order. Let $k = \min\{j : r_j \geq r_I\}$ be the index of the first supplier whose rate exceeds the rate on internal capital for the manufacturer. Then, for $1 \leq i < k$, the optimal loan amount is

$$S_i^* = \overline{S}_i(y_i). \tag{2.28}$$

For $i \geq k$, the optimal loan amount is

$$S_i^* = \min\left\{\overline{S}_i(y_i), \left[\sum_j (w_j y_j + C_j) - I - \sum_{j=1}^{i-1} S_j^*\right]^+\right\}. \tag{2.29}$$

Substituting the optimal loan amounts, $\vec{S}^*$, into problem (2.27) we derive an optimization problem with operational decisions, $\vec{y}$, only. Similar to the analysis in Section 2.4.1, the domain of the objective function of this optimization problem can be divided into regions such that, within each region, $k$, the objective function is written as $f_k(\vec{y}) = E\left\{p \min\left[D, Q(\vec{y})\right] - \sum_i (a_i y_i + b_i) - l\right\}^+$, where $b_i$ may depend on $\mathbf{1}_{\{y_i>0\}}$. The constraints for this optimization problem are

$$\sum_i w_i y_i - \sum_i \left(\widehat{S}_i - C_i\right) \mathbf{1}_{\{y_i>0\}} \leq I, \tag{2.30a}$$

$$\sum_i (w_i - \alpha_i)\, y_i + \sum_i (C_i - \beta_i)\, \mathbf{1}_{\{y_i > 0\}} \le I. \tag{2.30b}$$

While in the model with homogeneous suppliers we had only two parts (one per financing source) of the objective function, here there are as many parts as there are suppliers plus one (corresponding to financing from internal capital). This makes the problem with external financing too complex to analyze.

Let's focus on the effect of supplier random yield. To simplify, let's assume that the interest rate on internal financing is the lowest (i.e. $r_I \le r_i$, for all $i$), the manufacturer has more than sufficient internally generated capital to run the manufacturer (i.e. $I > \sum_i (w_i y_i + C_i)$ for all reasonable values of $\vec{y}$, implying, in particular, that no loans from the suppliers are needed and limited liability is never used), and demand, $D$, is deterministic. With these assumptions, we obtain a model similar to the non-identical suppliers model in Agrawal and Nahmias (1997). The essential difference between their model and ours lies in the assumption about payment from the manufacturer to the suppliers. I assumed that the manufacturer pays for the items orders, while they assumed (effectively) that the manufacturer pays only for the items delivered. Still, the similarity between models allows us to replicate the results in Agrawal and Nahmias (1997). Specifically, we can show that the objective function can be represented as follows:

$$E\left\{ p \min[D, Q(\vec{y})] - (1 + r_I) \sum_i \left( w_i y_i + C_i \mathbf{1}_{\{y_i > 0\}} \right) + (1 + r_I) I \right\} =$$
$$p \left[ D + (\overline{\mu} - D)\Phi(D) - \overline{\sigma}^2 \phi(D) \right] - (1 + r_I) \sum_i \left( w_i y_i + C_i \mathbf{1}_{\{y_i > 0\}} \right) + (1 + r_I) I, \tag{2.31}$$

where $\phi$ and $\Phi$ are p.d.f. and c.d.f. of the normal random variable $Q(\vec{y})$, whose mean is $\overline{\mu} = \sum_i \mu_i y_i$, and variance $\overline{\sigma}^2 = \sum_i \sigma_i^2 y_i^2$. Using Theorem 3 in Agrawal and Nahmias (1997), the objective function is concave. The first order condition for the order quantity with supplier $i$ is:

$$p\mu_i\Phi(D) - py_i\sigma_i^2\phi(D) = (1 + r_I)w_i. \tag{2.32}$$

Suppose that $\mu_i = \mu$ and $w_i = w$ for all $i$. Then, conditions (2.32) imply that the optimal order quantities to suppliers $i$ and $j$ are inversely proportional to the variances of supplier yields:

$$\frac{y_i^*}{y_j^*} = \frac{\sigma_j^2}{\sigma_i^2}. \tag{2.33}$$

Alternatively, one may argue that the supplier market is in equilibrium where the prices $w_i$ are proportional to the expected fraction the order suppliers deliver, that is, $w_i = A\mu_i$. In this case, the orders to suppliers will be:

$$\frac{y_i^*}{y_j^*} = \frac{\sigma_j^2/\mu_j}{\sigma_i^2/\mu_i}. \tag{2.34}$$

Agrawal and Nahmias (1997) solve a general non-identical supplier model only numerically. Because of the financing constraints, piecewise-defined objective function, and limited liability, our model is even more complex than that in Agrawal and Nahmias (1997). Therefore, we also cannot solve a general model analytically.

## 2.7 Conclusions, limitations, and extensions.

Numerous empirical studies report that trade credit (supplier financing) is the number one source of short-term financing in developed economies, accounting for as much as twice the amount of short-term bank loans. The role of trade credit is even more prominent in developing economies, where access to traditional sources of financing is severely limited. Other differences in business environments between developed and developing economies are the costs to work with suppliers and supplier reliability.

I study how trade credit financing, internal financing, cost to work with a supplier, and the supplier yield affect the optimal number of suppliers and the optimal order size. For this study I use a stylized model where the salient problem features are joint operational and financing decisions and financing constraints.

The theoretical and numerical analysis generated several testable hypotheses. Some of these hypotheses have already been confirmed in prior empirical studies. Others can be verified in future empirical work. I derived both anticipated and surprising results. These results can be explained by considering tradeoffs between the main elements of the model: financing constraints and their effect on feasible order quantity, cost to work with the suppliers and its effect on the objective function, order quantities and their effects on revenues, decomposition of the objective function into a concave profit term (which encourages diversification) and a convex option to default term (which encourages reducing the number of suppliers). For example, as the availability of either internal financing or supplier loans diminishes, the optimal number of suppliers may increase. To understand this, consider that the manufacturer, by paying the extra cost to work with additional suppliers, benefits by relaxing financing constraints and increasing order quantity, to earn higher expected revenues.

More surprising are the observations that an increase in the cost to work with a supplier or the wholesale price may result in an increase in the optimal number of suppliers.

Also surprising is that, as the standard deviation of a supplier yield's increases, the optimal number of suppliers could either increase or decrease. The intuitive explanation for this behavior is the tradeoff between the concave part of the objective function (which induces risk-averse preferences on the manufacturer) and the convex option to default (which encourages risk-seeking actions by the manufacturer).

The initial motivation for this research was the question: "Should one expect to observe empirically that manufacturers in developing economies work with more suppliers?" The answer to this question is "it depends." For example, everything being equal, the analysis suggests that manufacturers in developing economies will have more suppliers than comparable manufacturers in developed economies. But if, in developing economies, the cost to work with a supplier is very high or the manufacturer is close to bankruptcy, then that manufacturer may actually have fewer suppliers than its counterpart in developed economies. In this case, the analysis suggests that manufacturers in developing economies will place lower order quantities and will have higher stock-out probabilities, which matches perfectly the observations of the earlier empirical studies.

Thus, to answer the question: "Should one expect to observe empirically that manufacturers in developing economies work with more suppliers?" one needs a sophisticated empirical analysis, which carefully accounts for the factors that we considered. One of the main contributions of this chapter is to provide a set of testable hypotheses for future empirical studies.

To focus on the essential problem features, I have assumed away many other practical concerns. For example, in comparative statics analysis, financing terms (interest rates and loan limits) do not change as the manufacturer's financial conditions change. This presents an opportunity for the manufacturer to take advantage of its lenders. In practice, as long as the markets are not perfect Modigliani and Miller (1958) as defined by, this type of "mispricing" is possible.[11] I selected the simplest functional form (i.e., no changes in financing terms) to represent this phenomenon.

[11] This mispricing need not constitute an arbitrage, because market imperfections preclude market participants from creating an arbitrage portfolio.

Other assumptions are possible. However, as long as the functional form of mapping between manufacturer's financial state and financing terms is exogenously given, some form of mispricing will be present and the essential predictions of my model will not be altered, but the analysis will be more complex. For instance, I considered an extension to the model presented, where financing terms depend on the number of suppliers. For this more general model, I derived numerically the same insights as the ones presented in this study.

I assumed that the manufacturer uses only trade credit as the source of external financing. The analysis and the results are easily extended to the model where, in addition, the manufacturer can borrow from a bank. However, this adds unnecessary details to the presentation. For instance, instead of two parts in the objective function I would have to consider three parts.

I presented a model where the manufacturer pays up-front for the entire order placed with the supplier. As discussed in Section 2.3, there are many real-life systems where this is a good assumption. However, payments for items actually delivered are also common. I considered a more general model, where the manufacturer pays both for the order placed and for the parts received. The analysis of this more general model did not yield additional insights, and therefore, for the sake of exposition, I chose to use the simpler model.

To focus on the question about the number of suppliers, I assumed that suppliers were homogeneous. To address the question of supplier selection, a different model, emphasizing the differences among suppliers, is needed. While we have analyzed a model with heterogeneous suppliers, an in-depth research of the supplier selection problem, for example, extending analysis in Dada et al. (2007) and Federgruen and Yang (2007), is important and should be addressed in future studies.

Other generalizations, such as the dynamic relationship between lenders and borrowers and the role of asymmetric information, are also subjects for future research.

## 2.8 Appendix.

### 2.8.1 Proofs of Propositions 2.2 and 2.3.

These two proofs utilize Lemmas 2.1, 2.2 and 2.3, which are stated and proven following the propositions' proofs.

*Proof.* Proposition 2.2.

Recall that $f(N,z)$ is given by $f_L(N,z)$ for $z \le \widehat{z}$ and by $f_R(N,z)$ for $z > \widehat{z}$, where $f_L(N,z) = f_R(N,z)$ for $z = \widehat{z}$. Furthermore, observe from Lemma 2.2(i),(ii) that the function $f_k(N,z)$ is zero for $z$ values outside the range $\left[\frac{b_k+l}{p-a_k}, \frac{px_u-b_k-l}{a_k}\right]$, $k = L, R$. Therefore, we can divide the proof into two cases:

*Case 1:* $f(N,\widehat{z}) > 0$. In this case, $\widehat{z}$ must be in the ranges $\left[\frac{b_k+l}{p-a_k}, \frac{px_u-b_k-l}{a_k}\right]$, $k = L, R$.

*Case 2:* $f(N,\widehat{z}) = 0$. In this case, $\widehat{z}$ must be outside the ranges $\left[\frac{b_k+l}{p-a_k}, \frac{px_u-b_k-l}{a_k}\right]$, $k = L, R$.

In this proof, I deal with Case 1, which is more interesting. The second case is a degenerate case where either $f(N,z) = 0$ for all $z \le \widehat{z}$ or $f(N,z) = 0$ for all $z \ge \widehat{z}$, and the result could be proven similarly for that case.

Lemma 2.2 shows that $f_L(N,z)$ and $f_R(N,z)$ are unimodal. Observe that, by definition, $f(N,z) = f_L(N,z)$ for $z \le \widehat{z}$ and $f(N,z) = f_R(N,z)$ for $z > \widehat{z}$. Now, note that the function $f(N,z)$ will not be unimodal only if it is decreasing as $z$ approaches $\widehat{z}$ from below and starts increasing once $z$ exceeds $\widehat{z}$. Equivalently, the function $f(N,z)$ will not be unimodal only if $\frac{\partial f_L(N,\widehat{z})}{\partial z} < 0$ and $\frac{\partial f_R(N,\widehat{z})}{\partial z} > 0$. Now, I will prove by contradiction that this cannot happen. Suppose $\frac{\partial f_L(N,\widehat{z})}{\partial z} < 0$ and

$\frac{\partial f_R(N,\widehat{z})}{\partial z} > 0$. This, coupled with the fact that $f_L(N,\widehat{z}) = f_R(N,\widehat{z})$, implies that there must exist $z > \widehat{z}$ such that $f_L(N,z) < f_R(N,z)$. However, by Lemma 2.3, we must have $f_L(N,z) \geq f_R(N,z)$ for $z \geq \widehat{z}$, which yields a contradiction. Therefore, we can never have $\frac{\partial f_L(N,\widehat{z})}{\partial z} < 0$ and $\frac{\partial f_R(N,\widehat{z})}{\partial z} > 0$, and $f(N,z)$ is unimodal in $z$. ■

*Proof.* Proposition 2.3.

The result follows from the supermodularity of $f_k$ in $(z, b_k)$. To show supermodularity, consider $z$ such that $pz - a_k z - b_k - l \geq 0$. The derivative of $f_k$ with respect to $z$ is derived in Lemma 2.2 (see equation (2.38)). Taking a derivative of expression in (2.38) with respect to $b_k$, we find that

$$\frac{\partial^2 f_k}{\partial z_k \partial b_k} = \frac{a_k}{p}\, g\left(\frac{a_k z_k + b_k + l}{p}\right) \geq 0. \quad ■$$

**Lemma 2.1.** *Consider the model with stochastic demand and deterministic supplier yield from Section 2.1. We can write the function $f_k(N,z)$ as follows:*

$$f_k(N,z) = 1_{\{pz - a_k z - b_k - l \geq 0\}}\Bigg[-(a_k z + b_k + l)\overline{G}\left(\frac{a_k z + b_k + l}{p}\right) + pz\overline{G}(z) + p\int_{\frac{a_k z + b_k + l}{p}}^{z} xg(x)\,dx\Bigg], \tag{2.35}$$

*where $\overline{G}(x) = 1 - G(x)$.*

*Proof.* Lemma 2.1

For notational convenience, define $\lambda_k = a_k z + b_k + l$ and observe that $f_k(N,z) = E[p\min(D,Q(N,z)) - \lambda_k]^+$, $k = L, R$ (see equations (2.14), (2.15), (2.16), (2.17), and (2.18)). Recall that demand $D$ is stochastic with cdf $G$, and the quantity delivered, $Q(N,z)$, is deterministic and equal to $z$ by Assumption 2.4. Then:

$$\begin{aligned} f_k(N,z) &= -\lambda_k E\left[1_{\{p\min(D,z)-\lambda_k \geq 0\}}\right] + pE\left[\min(D,z)1_{\{p\min(D,z)-\lambda_k \geq 0\}}\right] \\ &= -\lambda_k 1_{\{pz-\lambda_k \geq 0\}} \Pr[pD - \lambda_k \geq 0] + p1_{\{pz-\lambda_k \geq 0\}} E\left[\min(D,z)1_{\{pD-\lambda_k \geq 0\}}\right] \end{aligned}$$

$$= -\lambda_k 1_{\{pz-\lambda_k \geq 0\}} \overline{G}\left(\frac{\lambda_k}{p}\right) + p 1_{\{pz-\lambda_k \geq 0\}} z E\left[1_{\{pD-\lambda_k \geq 0, D \geq z\}}\right]$$

$$+ p 1_{\{pz-\lambda_k \geq 0\}} E\left[D 1_{\{pD-\lambda_k \geq 0, D < z\}}\right]$$

$$= -\lambda_k 1_{\{pz-\lambda_k \geq 0\}} \overline{G}\left(\frac{\lambda_k}{p}\right) + p 1_{\{pz-\lambda_k \geq 0\}} z \overline{G}(z) + p 1_{\{pz-\lambda_k \geq 0\}} \int_{\frac{\lambda_k}{p}}^{z} x g(x)\, dx$$

$$= 1_{\{pz-\lambda_k \geq 0\}} \left[-\lambda_k \overline{G}\left(\frac{\lambda_k}{p}\right) + pz\overline{G}(z) + p \int_{\frac{\lambda_k}{p}}^{z} x g(x)\, dx\right].$$

■

**Lemma 2.2.** *Consider the model with stochastic demand and deterministic supplier yield from Section 2.4.2. If $px_u - a_k x_u - b_k - l < 0$, $k = L, R$, then the function $f_k(N, z) = 0$ for all order quantities $z$ in the domain of demand, $[x_l, x_u]$. Otherwise:*

*(i) $f_k(N, z) = 0$ for $z < \frac{b_k + l}{p - a_k}$.*

*(ii) $f_k(N, z) = 0$ for $z > \frac{px_u - b_k - l}{a_k}$.*

*(iii) $f_k(N, z)$ is unimodal in $z$.*

*(iv) There exists a unique $z_k \in \left[\frac{b_k + l}{p - a_k}, \frac{px_u - b_k - l}{a_k}\right]$ that maximizes $f_k(N, z)$ and satisfies*

$$p\overline{G}(z_k) = a_k \overline{G}\left(\frac{a_k z_k + b_k + l}{p}\right) = a_k \Pr\left[pD > a_k z_k + b_k + l\right]. \qquad (2.36)$$

*Proof.* Lemma 2.2.

Recall that the random variable $D$ has a density function defined over the domain $[x_l, x_u]$. First, suppose that $px_u - a_k x_u - b_k - l < 0$. Then, for any order quantity $z \in [x_l, x_u]$, we have $pz - a_k z - b_k - l < 0$ (because, by Assumption 2.3, $p > a_k$ for $k = L, R$). Therefore, for any order quantity $z \in [x_l, x_u]$, we have $1_{\{pz - a_k z - b_k - l \geq 0\}} = 0$. It now follows that $f_k(z) = 0$ for any $z \in [x_l, x_u]$ (See (2.35) in Lemma 2.1.).

Now, I turn to the more interesting case where $px_u - a_k x_u - b_k - l \geq 0$, $k = L, R$.

**Proof of (i):** If $z < \frac{b_k + l}{p - a_k}$, then $1_{\{pz - a_k z - b_k - l \geq 0\}} = 0$, and the result follows from

Lemma 2.1.

**Proof of (ii):** The inequality $z > \frac{px_u - b_k - l}{a_k}$ is equivalent to $\frac{a_k z + b_k + l}{p} > x_u$. Hence, the first term in brackets in (2.35) is zero. Furthermore, if $z > \frac{px_u - b_k - l}{a_k}$, then one can verify that $z > x_u$ (using also the current assumption that $px_u - a_k x_u - b_k - l \geq 0$). Hence, the second and third terms in (2.35) are also zero. It now follows that $f_k(z) = 0$.

**Proof of (iii):** By parts (i) and (ii), $f_k(z) = 0$ for $z < \frac{b_k + l}{p - a_k}$ and $z > \frac{px_u - b_k - l}{a_k}$. Therefore, we will conclude the proof if we can show that $f_k(z)$ is unimodal for $z \in \left[\frac{b_k + l}{p - a_k}, \frac{px_u - b_k - l}{a_k}\right]$. In this range, by Lemma 2.1, we have:

$$f_k(z) = -(a_k z + b_k + l)\overline{G}\left(\frac{a_k z + b_k + l}{p}\right) + pz\overline{G}(z) + p\int_{\frac{a_k z + b_k + l}{p}}^{z} xg(x)\,dx. \quad (2.37)$$

It is not difficult to check the following claim is true:

*Claim (a):* $f_k'(z) > 0$ at $z = \frac{b_k + l}{p - a_k}$.

In addition, I will now prove the following claim:

*Claim (b):* $f_k''(z) < 0$ whenever $f_k'(z) = 0$.

The first derivative of $f_k$ is

$$f_k'(z) = -a_k\overline{G}\left(\frac{a_k z + b_k + l}{p}\right) + p\overline{G}(z). \quad (2.38)$$

The first order condition is

$$a_k\overline{G}\left(\frac{a_k z + b_k + l}{p}\right) = p\overline{G}(z). \quad (2.39)$$

The second derivative of $f_k$ is

$$f_k''(z) = \frac{a_k^2}{p}g\left(\frac{a_k z + b_k + l}{p}\right) - pg(z) = p\left[g\left(\frac{a_k z + b_k + l}{p}\right)\frac{a_k^2}{p^2} - g(z)\right]. \quad (2.40)$$

Let $z_0$ satisfy the first order condition (2.39). Then,

$$f_k''(z_0) = p\overline{G}^2(z_0)\left[\frac{g\left(\frac{a_k z_0 + b_k + l}{p}\right)}{\overline{G}^2\left(\frac{a_k z_0 + b_k + l}{p}\right)} - \frac{g(z_0)}{\overline{G}^2(z_0)}\right] \leq 0, \quad (2.41)$$

where the inequality follows from the facts that $z_0 > \frac{a_k z_0 + b_k + l}{p}$ and $\frac{g(x)}{\bar{G}^2(x)}$ is increasing. Hence, we have shown that Claim (b) holds. Now, claim (a) implies that the function $f_k(z)$ starts increasing from zero at $z = \frac{b_k + l}{p - a_k}$. Furthermore, the function goes back to zero at $z = \frac{p x_u - b_k - l}{a_k}$ and any stationary point of the function $f_k(\cdot)$ in the range $\left[\frac{b_k+l}{p-a_k}, \frac{p x_u - b_k - l}{a_k}\right]$ is a local maximum by claim (b). Therefore, we conclude that there exists only one stationary point of the function $f_k(\cdot)$ in the range $\left[\frac{b_k+l}{p-a_k}, \frac{p x_u - b_k - l}{a_k}\right]$, and this stationary point is a maximizer. Hence, the function $f_k(\cdot)$ is unimodal. (If there were two stationary points, both of them would have to be local maxima by claim (b), which would require the existence of a local minimum in between these two local maxima, which contradicts claim (b).)

**Proof of (iv):** This follows from parts (i) through (iii) of the lemma. ■

**Lemma 2.3.** *If $z \geq \widehat{z}$, then $f_L(N, z) \geq f_R(N, z)$.*

*Proof.* Lemma 2.3 I first prove the lemma for the case where $r_I < r_S$. From equations (2.10), (2.11) and (2.12), observe that, when $r_I < r_S$, we have $\Pi_L(N, z) - \Pi_R(N, z) = (r_S - r_I)T(N, z)$. Furthermore, $\widehat{z}$ is defined in (2.13) such that $T(N, z) \geq 0$ for any $z \geq \widehat{z}$. Therefore, we conclude that $\Pi_L(N, z) - \Pi_R(N, z) \geq 0$ for any $z \geq \widehat{z}$. Hence, $f_L(N, z) = E[\Pi_L(N, z)]^+ \geq E[\Pi_R(N, z)]^+ = f_R(N, z)$ for any $z \geq \widehat{z}$. In the other case where $r_I > r_S$, we have $\Pi_L(N, z) - \Pi_R(N, z) = (r_I - r_S)T(N, z)$ from equations (2.10), (2.11) and (2.12). The lemma follows similarly for this case. ■

### 2.8.2 Proof of Proposition 2.4.

This proof utilizes Lemmas 2.4 and 2.5, which are stated and proven after the proposition's proof.

*Proof.* Proposition 2.4.

Again, I focus on the more interesting case where $f(N, \widehat{z}) > 0$ (as opposed to the degenerate case where $f(N, \widehat{z}) = 0$.) In this proof, I write $z^*(l)$ to make explicit the dependence of the optimal order quantity, $z^*$, on the maximum liability, $l$. Similarly, I add $l$ to the list of arguments for functions $f(N, z)$ and $f_k(N, z), k = L, R$. Define $z_k^*(l) = \arg\max\{f_k(N, z, l)\}$. Suppose $l_1 > l_2$. Our goal is to prove $z^*(l_1) \geq z^*(l_2)$. I will prove the result by considering four different cases, each one corresponding to one of the cases in the statement of Lemma 2.4.

*Case 1:* $\widehat{z} \leq 0$. In this case, from Lemma 2.4(i), it follows that $z^*(l_1) = z_R^*(l_1)$ and $z^*(l_2) = z_R^*(l_2)$. Now, the result follows because $z_R^*(l_1) \geq z_R^*(l_2)$ by Lemma 2.5.

*Case 2:* $\widehat{z} > 0$ and $\frac{\partial f_L(N, \widehat{z}, l_1)}{\partial z} < 0$. In this case, from Lemma2.4(ii), we know that $z^*(l_1) = z_L^*(l_1)$. Furthermore, because $f_k(N, z, l)$ is supermodular in $(z, l)$ (as shown in Lemma 2.5), it must be that $\frac{\partial f_L(N, \widehat{z}, l_2)}{\partial z} < 0$. Therefore, from Lemma2.4(ii), we know that $z^*(l_2) = z_L^*(l_2)$. The result now follows since $z_L^*(l_1) \geq z_L^*(l_2)$ by Lemma 2.5.

*Case 3:* $\widehat{z} > 0$, $\frac{\partial f_L(N, \widehat{z}, l_1)}{\partial z} > 0$ and $\frac{\partial f_R(N, \widehat{z}, l_1)}{\partial z} > 0$. In this case, we have $z^*(l_1) = z_R^*(l_1)$ by Lemma 2.4(iii). Furthermore, note that $z^*(l_1) \geq \widehat{z}$ (since $\frac{\partial f_R(N, \widehat{z}, l_1)}{\partial z} > 0$ and $f_R(N, z)$ is unimodal), which will be used in the rest of the proof. I will consider a number of subcases depending on the signs of $\frac{\partial f_L(N, \widehat{z}, l_2)}{\partial z}$ and $\frac{\partial f_R(N, \widehat{z}, l_2)}{\partial z}$.

*Case 3(a):* $\frac{\partial f_L(N, \widehat{z}, l_2)}{\partial z} < 0$. In this case, from Lemma 2.4(ii), we know that $z^*(l_2) = z_L^*(l_2)$. Furthermore, note that $z^*(l_2) \leq \widehat{z}$ (since $\frac{\partial f_L(N, \widehat{z}, l_2)}{\partial z} < 0$ and $f_L(N, z, l)$ is unimodal). The result now follows since $z^*(l_2) \leq \widehat{z} \leq z^*(l_1)$.

*Case 3(b):* $\frac{\partial f_L(N, \widehat{z}, l_2)}{\partial z} > 0$ and $\frac{\partial f_R(N, \widehat{z}, l_2)}{\partial z} > 0$. In this case, from Lemma 2.4(iii), we know that $z^*(l_2) = z_R^*(l_2)$. Since $z^*(l_1) = z_R^*(l_1)$, the result follows from Lemma 2.5.

*Case 3(c):* $\frac{\partial f_L(N, \widehat{z}, l_2)}{\partial z} > 0$ and $\frac{\partial f_R(N, \widehat{z}, l_2)}{\partial z} < 0$. In this case, from Lemma 2.4(iv), we know that $z^*(l_2) = \widehat{z}$. The result follows since $z^*(l_2) = \widehat{z} \leq z^*(l_1)$.

*Case 4:* $\widehat{z} > 0$, $\frac{\partial f_L(N,\widehat{z},l_1)}{\partial z} > 0$ and $\frac{\partial f_R(N,\widehat{z},l_1)}{\partial z} < 0$. In this case, from Lemma 2.4(iv), we know that $z^*(l_1) = \widehat{z}$. Furthermore, because $f_k(N,z,l)$ is supermodular in $(z,l)$, it must be that $\frac{\partial f_R(N,\widehat{z},l_2)}{\partial z} < 0$. Again, I consider a number of subcases depending on the signs of $\frac{\partial f_L(N,\widehat{z},l_2)}{\partial z}$ and $\frac{\partial f_R(N,\widehat{z},l_2)}{\partial z}$.

*Case 4(a):* $\frac{\partial f_L(N,\widehat{z},l_2)}{\partial z} < 0$. By Lemma 2.4(ii), we have $z^*(l_2) \leq \widehat{z}$. The desired result follows because $z^*(l_1) = \widehat{z}$.

*Case 4(b):* $\frac{\partial f_L(N,\widehat{z},l_2)}{\partial z} > 0$. Given that we also have $\frac{\partial f_R(N,\widehat{z},l_2)}{\partial z} < 0$, it follows from Lemma 2.4(iv) that $z^*(l_2) = \widehat{z}$. The result now follows because $z^*(l_1) = \widehat{z}$ as well. ■

**Lemma 2.4.** *Consider the model with stochastic demand and deterministic supplier yield from Section 2.4.2. Suppose $N$ is fixed. Then, the optimal order quantity*

$$z^* \in \left\{\arg\max_z\{f_L(N,z)\}, \arg\max_z\{f_R(N,z)\}, \widehat{z}\right\}. \tag{2.42}$$

*Furthermore, to find the optimal order quantity, $z^*$, one could use the following properties:*

*(i) If $\widehat{z} \leq 0$, then $z^* = \arg\max_z\{f_R(N,z)\}$.*

*(ii) If $\widehat{z} > 0$ and $\frac{\partial f_L(N,\widehat{z})}{\partial z} < 0$, then $z^* = \arg\max_z\{f_L(N,z)\}$.*

*(iii) If $\widehat{z} > 0$, $\frac{\partial f_L(N,\widehat{z})}{\partial z} > 0$ and $\frac{\partial f_R(N,\widehat{z})}{\partial z} > 0$, then $z^* = \arg\max_z\{f_R(N,z)\}$.*

*(iv) If $\widehat{z} > 0$, $\frac{\partial f_L(N,\widehat{z})}{\partial z} > 0$ and $\frac{\partial f_R(N,\widehat{z})}{\partial z} < 0$, then $z^* = \widehat{z}$.*

*Proof.* Lemma 2.4.

Once again, I focus on the more interesting case where $f(N,\widehat{z}) > 0$ (as opposed to the degenerate case where $f(N,\widehat{z}) = 0$.) For the purposes of this proof, define $z_k^* = \arg\max_z f_k(N,z)$ for $k = L, R$. We first prove properties (i) through (iv):

**Proof of (i):** If $\widehat{z} < 0$, then $f(N,z) = f_R(N,z)$ for all $z \geq 0$ and $z^* = z_R^*$, which concludes the proof.

**Proof of (ii):** If $\widehat{z} > 0$ and $\frac{\partial f_L(N,\widehat{z})}{\partial z} < 0$, then it must be that $\frac{\partial f_R(N,\widehat{z})}{\partial z} < 0$ as well. (Otherwise, we would obtain a contradiction to the unimodality of $f(N,z)$, which was proven in Proposition 2.2.) Now, because $f(N,z) = f_R(N,z)$ for $z \geq \widehat{z}$, it follows that $f(N,z)$ must be decreasing in $z$ for $z \geq \widehat{z}$ (since $\frac{\partial f_R(N,\widehat{z})}{\partial z} < 0$ and $f_R(N,z)$ is unimodal.) Therefore, it must be that $z_R^* \leq \widehat{z}$. In addition, we know that $f(N,z) = f_L(N,z)$ for $z \leq \widehat{z}$, and, furthermore, $z_L^* < \widehat{z}$ (since $\frac{\partial f_L(N,\widehat{z})}{\partial z} < 0$ and $f_L(N,z)$ is unimodal.) Therefore, we have $z^* = z_L^*$, which concludes the proof.

**Proof of (iii):** Since $f(N,z) = f_L(N,z)$ for $z \leq \widehat{z}$, it follows that $f(N,z)$ must be increasing in $z$ for $z \leq \widehat{z}$ (since $\frac{\partial f_L(N,\widehat{z})}{\partial z} > 0$ and $f_L(N,z)$ is unimodal.) Therefore, it must be that $z_L^* \geq \widehat{z}$. In addition, we know that $f(N,z) = f_R(N,z)$ for $z \geq \widehat{z}$, and, furthermore, $z_R^* > \widehat{z}$ (since $\frac{\partial f_R(N,\widehat{z})}{\partial z} > 0$ and $f_R(N,z)$ is unimodal.) Therefore, we have $z^* = z_R^*$, which concludes the proof.

**Proof of (iv):** Since $f(N,z) = f_L(N,z)$ for $z \leq \widehat{z}$, it follows that $f(N,z)$ must be increasing in $z$ for $z \leq \widehat{z}$ (since $\frac{\partial f_L(N,\widehat{z})}{\partial z} > 0$ and $f_L(N,z)$ is unimodal.) Furthermore, because $f(N,z) = f_R(N,z)$ for $z \geq \widehat{z}$, it follows that $f(N,z)$ must be decreasing in $z$ for $z \geq \widehat{z}$ (since $\frac{\partial f_R(N,\widehat{z})}{\partial z} < 0$ and $f_R(N,z)$ is unimodal.) Hence, we have $z^* = \widehat{z}$, which concludes the proof.

The statement that $z^* \in \{\arg\max_z\{f_L(N,z)\}, \arg\max_z\{f_R(N,z)\}, \widehat{z}\}$ now follows as a corollary to properties (i) through (iv). ■

**Algorithm for computing the optimal operational decisions — $N^*$ and $z^*$:**

(i) For each $N \leq \frac{I}{C-\beta}$.

  (a) Compute $\widehat{z}$: $T(N,\widehat{z}) = 0$.

  (b) Compute $z_{max}(N)$.

(c) If $\widehat{z} < 0$, then find $z^*(N)$ that maximizes $f_R(N,z)$ by searching over all $z \in [0, z_{max}(N)]$.

(d) If $\widehat{z} \geq 0$, then

i. If $\frac{\partial f_L(N,\widehat{z})}{\partial z} \leq 0$, then find $z^*(N)$ that maximizes $f_L(N,z)$ by searching over all $z \in [0, \widehat{z}]$.

ii. If $\frac{\partial f_L(N,\widehat{z})}{\partial z} > 0$ and $\frac{\partial f_R(N,\widehat{z})}{\partial z} > 0$, then find $z^*(N)$ that maximizes $f_R(N,z)$ by searching over all $z \in [\widehat{z}, z_{max}(N)]$.

iii. If $\frac{\partial f_L(N,\widehat{z})}{\partial z} > 0$ and $\frac{\partial f_R(N,\widehat{z})}{\partial z} \leq 0$, then $z^*(N) = \widehat{z}$.

(ii) Pick $N$ for which $f(N, z^*(N))$ is the largest.

**Lemma 2.5.** *Suppose that Assumptions (2.1), (2.2), (2.3), (2.4) hold. For $k = L, R$, let $z_k^* = \arg\max_z\{f_k(N,z)\}$. Then, $z_k^*$ is increasing in $l$.*

*Proof.* Lemma 2.5.

The result follows from the supermodularity of $f_k$ in $(z,l)$. Consider $z$ such that $pz - a_k z - b_k - l \geq 0$. The derivative of $f_k$ with respect to $z$ is given by equation (2.38). Taking the derivative of the expression in (2.38) with respect to $l$, we find that

$$\frac{\partial^2 f_k}{\partial z \partial l} = \frac{a_k}{p}\, g\left(\frac{a_k z + b_k + l}{p}\right) \geq 0. \qquad \blacksquare$$

### 2.8.3 Proofs of Propositions 2.5, 2.6, 2.7.

In this section, I provide the proofs of Propositions 2.5 through 2.7 followed by a lemma that is useful for these proofs.

*Proof.* Proposition 2.5.

**Proof of (i):** In order to prove the result, I will show that for any two integers

$N_1$ and $N_2$ such that $N_1 < N_2$, we have $f(N_1, z) - f(N_2, z) \geq 0$ when $r_I < r_S$. The desired result will then follow. Consider the following four cases:

*Case 1:* $T(N_1, z) \leq 0$ and $T(N_2, z) \leq 0$. In this case, by (2.9), $f(N_1, z) - f(N_2, z) = f_L(N_1, z) - f_L(N_2, z)$. The result now follows from Lemma 2.6(i).

*Case 2:* $T(N_1, z) > 0$ and $T(N_2, z) > 0$. In this case, $f(N_1, z) - f(N_2, z) = f_R(N_1, z) - f_R(N_2, z)$. The result now follows from Lemma 2.6(i).

*Case 3:* $T(N_1, z) \leq 0$ and $T(N_2, z) > 0$. In this case, $f(N_1, z) - f(N_2, z) = f_L(N_1, z) - f_R(N_2, z)$. Since $T(N_2, z) > 0$, we have $z \geq \widehat{z}$ at $N_2$, and, therefore, $f_L(N_2, z) \geq f_R(N_2, z)$ by Lemma 2.3. Furthermore, note that $f_L(N_1, z) \geq f_L(N_2, z)$ by Lemma 2.6(i). Hence, $f_L(N_1, z) \geq f_R(N_2, z)$, which yields the desired result.

*Case 4:* $T(N_1, z) > 0$ and $T(N_2, z) \leq 0$. This case cannot occur, since $T(N, z)$ is increasing in $N$.

**Proof of (ii):** The proof is similar to that of (i) and uses Lemma 2.6(ii) where the proof of (i) uses Lemma 2.6(i).

**Proof of (iii):** In order to prove the result, I will show that for any two integers $N_1$ and $N_2$ such that $N_1 < N_2$ , we have $f(N_1, z) - f(N_2, z) \leq 0$ when $r_I > r_S$. It will then follow that $N^* > 1$. Consider the following four cases:

*Case 1:* $T(N_1, z) \leq 0$ and $T(N_2, z) \leq 0$. In this case, $f(N_1, z) - f(N_2, z) = f_L(N_1, z) - f_L(N_2, z)$. The result now follows from Lemma 2.6(iii), since $f_L(N, z)$ is increasing $N$.

*Case 2:* $T(N_1, z) > 0$ and $T(N_2, z) > 0$. In this case, $f(N_1, z) - f(N_2, z) = f_R(N_1, z) - f_R(N_2, z)$. The result now follows from Lemma 2.6(iii), since $f_R(N, z)$ is increasing $N$.

*Case 3:* $T(N_1, z) \leq 0$ and $T(N_2, z) > 0$. Note from (12) that when $r_I > r_S$, $T(N, z)$ is decreasing in $N$ (since $\beta < \widehat{S}$ by Assumption 2.2). Therefore, this case cannot occur, since $T(N, z)$ is decreasing in $N$.

*Case 4:* $T(N_1, z) > 0$ and $T(N_2, z) \leq 0$. In this case, $f(N_1, z) - f(N_2, z) = f_R(N_1, z) - f_L(N_2, z)$. Since $T(N_1, z) > 0$, we have $z > \widehat{z}$ at $N_1$ and, therefore, $f_L(N_1, z) \geq f_R(N_1, z)$ by Lemma 2.3. Furthermore, note that $f_L(N_2, z) \geq f_L(N_1, z)$ by Lemma 2.6(iii). Combining these last two observations, we obtain $f_L(N_2, z) \geq f_R(N_1, z)$, which yields the desired result. ■

*Proof.* Proposition 2.6.

From the first of the two constraints stated in (2.4), it follows that we must have $(C - \beta)N \leq I$, which yields the desired result. ■

*Proof.* Proposition 2.7.

**Proof of (i):** When $N \leq \frac{\alpha I}{w(\widehat{S}-\beta)-\alpha(\widehat{S}-C)}$, it is not difficult to check any $z \geq 0$ that satisfies the second of the two constraints stated in (2.4) will satisfy the first one as well. Therefore, when $N \leq \frac{\alpha I}{w(\widehat{S}-\beta)-\alpha(\widehat{S}-C)}$, $z$ is bounded by the second constraint in (2.4), which yields the desired result.

**Proof of (ii):** When $\frac{\alpha I}{w(\widehat{S}-\beta)-\alpha(\widehat{S}-C)} \leq N \leq \frac{I}{C-\beta}$, it is not difficult to check that any $z \geq 0$ that satisfies the first of the two constraints stated in (2.4) will satisfy the second one as well. Therefore, in this case, $z$ is bounded by the first constraint in (2.4), which yields the desired result. ■

**Lemma 2.6.** *Consider the model with stochastic demand and deterministic supplier yield from Section 2.4.2. At a fixed z:*

*(i) If $r_I < r_S$, then both $f_L(N, z)$ and $f_R(N, z)$ are decreasing in $N$.*

*(ii) If $r_I > r_S$ and $C > \frac{(r_I - r_S)}{(1+r_I)}\widehat{S}$, then both $f_L(N, z)$ and $f_R(N, z)$ are decreasing in $N$.*

*(iii) If $r_I > r_S$ and $C < \frac{r_I - r_S}{1+r_I}\beta$, then both $f_L(N,z)$ and $f_R(N,z)$ are increasing in $N$.*

*Proof.* Lemma 2.6.

**Proof of (i):** Note from (2.10) that $\Pi_L(N,z)$ is decreasing in $N$ when $r_I < r_S$. (Note that $Q(N,z) = z$ in the model of Section 2.4.2.) Since $f_L(N,z)$ is defined in (2.14) to be a monotonic transformation of $\Pi_L(N,z)$, we observe that $f_L(N,z)$ is decreasing in $N$ when $r_I < r_S$. Similarly for $f_R(N,z)$.

**Proof of (ii):** Note that $C > \frac{(r_I - r_S)}{(1+r_I)}\widehat{S}$ is equivalent to $(1+r_I)C > (r_I - r_S)\widehat{S}$. Now, when $r_I > r_S$ and $(1+r_I)C > (r_I - r_S)\widehat{S}$, we note from (2.10) and (2.11) that both $\Pi_L(N,z)$ and $\Pi_R(N,z)$ are decreasing in $N$. The result follows, because $f_L(N,z)$ and $f_R(N,z)$ are monotonic transformations of $\Pi_L(N,z)$ and $\Pi_R(N,z)$, respectively.

**Proof of (iii):** Note that $C < \frac{r_I - r_S}{1+r_I}\beta$ is equivalent to $(r_I - r_S)\beta > (1+r_I)C$. Now, when $r_I > r_S$ and $(r_I - r_S)\beta > (1+r_I)C$, observe from (10) and (11) that both $\Pi_L(N,z)$ and $\Pi_R(N,z)$ are increasing $N$. The result follows because $f_L(N,z)$ and $f_R(N,z)$ are monotonic transformations of $\Pi_L(n,z)$ and $\Pi_R(n,z)$, respectively. ■

### 2.8.4 Proofs of Lemma 2.8 and Propositions 2.9 and 2.10.

*Proof.* Lemma 2.8.

First, I prove that $f_k(z)$, defined by (2.14), can be written as in (2.23). For notational convenience, define $\lambda_k = a_k z + b_k + l$. Observe that $f_k(N,z) = E[p\min(D, Q(N,z)) - \lambda_k]^+$, $k = L, R$. Recall that demand $D$ is deterministic and the quantity delivered, $Q(N,z)$, is given by $\overline{X}z$. Then:

$$
\begin{aligned}
f_k(N,z) &= -\lambda_k E\left[1_{\{p\min(D,\overline{X}z)-\lambda_k \geq 0\}}\right] + pE\left[\min(D,\overline{X}z)1_{\{p\min(D,\overline{X}z)-\lambda_k \geq 0\}}\right] \\
&= -\lambda_k 1_{\{pD-\lambda_k \geq 0\}} \Pr[p\overline{X}z - \lambda_k \geq 0] \\
&\quad + p1_{\{pD-\lambda_k \geq 0\}} E\left[\min(D,\overline{X}z)1_{\{p\overline{X}z-\lambda_k \geq 0\}}\right].
\end{aligned}
$$

Note that $pD - \lambda_k \geq 0$ by Assumption 2.6. Hence, $1_{\{pD-\lambda_k \geq 0\}} = 1$ and we have:

$$\begin{aligned} f_k(N,z) &= -\lambda_k \Pr[p\overline{X}z - \lambda_k \geq 0] + pE\left[\min(D, \overline{X}z)1_{\{p\overline{X}z-\lambda_k\geq 0\}}\right] \\ &= -\lambda_k \Pr[p\overline{X}z - \lambda_k \geq 0] + pE\left[\overline{X}z1_{\{p\overline{X}z-\lambda_k\geq 0, \overline{X}z\leq D\}}\right] \\ &+ pE\left[D1_{\{p\overline{X}z-\lambda_k\geq 0, \overline{X}z\geq D\}}\right]. \end{aligned}$$

Furthermore, by Assumption 2.6, we have $\frac{D}{z} \geq \frac{\lambda_k}{pz}$. Using this observation, we can write:

$$\begin{aligned} f_k(N,z) &= -\lambda_k \Pr[p\overline{X}z - \lambda_k \geq 0] + pE\left[\overline{X}z1_{\{p\overline{X}z-\lambda_k\geq 0, \overline{X}z\leq D\}}\right] + pD\Pr[p\overline{X}z \geq D] \\ &= -\lambda_k \Pr\left[\overline{X} \geq \frac{\lambda_k}{pz}\right] + pE\left[\overline{X}z1_{\{\frac{\lambda_k}{pz}\leq \overline{X}\leq \frac{D}{z}\}}\right] + pD\Pr\left[\overline{X} \geq \frac{D}{pz}\right]. \end{aligned}$$

The expression above can now be re-written as the expression presented in (2.23). The derivative presented in (2.24) can be verified by taking the derivative of (2.23).

Next, I prove the unimodality of $f_k(z)$ when $\frac{\gamma(\cdot)}{\Gamma(\cdot)}$ is decreasing. We start by writing (2.24) as follows:

$$f'_k(z) = -a_k\overline{\Phi}\left(\frac{a_kz+b_k+l}{pz}\right) - p\Gamma\left(\frac{D}{z}\right) + p\Gamma\left(\frac{a_kz+b_k+l}{pz}\right). \tag{2.43}$$

By taking the derivative of (2.43), we obtain:

$$f''_k(z) = -\phi\left(\frac{a_kz+b_k}{pz}\right)\frac{a_k(b_k+l)}{pz^2} + p\frac{D}{z^2}\gamma\left(\frac{D}{z}\right) - \frac{b_k+l}{z^2}\gamma\left(\frac{a_kz+b_k+l}{pz}\right). \tag{2.44}$$

In what follows, we let $K = \frac{a_kz+b_k+l}{pz}$ for convenience. In order to prove unimodality, it will be sufficient to prove that $f''_k(z) < 0$ whenever $f'_k(z) = 0$. To that end, we start by restating (2.44) as follows:

$$f''_k(z) = -\phi(K)\frac{a_k(b_k+l)}{pz^2} + p\frac{D}{z^2}\gamma\left(\frac{D}{z}\right) - \frac{b_k+l}{z^2}\frac{\gamma(K)}{\Gamma(K)}\Gamma(K).$$

We then substitute for $\Gamma(K)$ from (2.43) in the above equation to obtain:

$$f''_k(z)|_{f'_k(z)=0} = -\phi(K)\frac{a_k(b_k+l)}{pz^2} + p\frac{D}{z^2}\gamma\left(\frac{D}{z}\right) - \frac{b_k+l}{z^2}\frac{\gamma(K)}{\Gamma(K)}\frac{a_k\overline{\Phi}(K)+p\Gamma\left(\frac{D}{z}\right)}{p}.$$

Rearranging the terms, we can write:

$$\begin{aligned}f_k''(z)|_{f_k'(z)=0} =& \frac{p}{z^2}\Gamma\left(\frac{D}{z}\right)\left[D\frac{\gamma\left(\frac{D}{z}\right)}{\Gamma\left(\frac{D}{z}\right)} - \frac{b_k+l}{p}\frac{\gamma(K)}{\Gamma(K)}\right] \\ & - \phi(K)\frac{a_k(b_k+l)}{pz^2} - \frac{\gamma(K)\overline{\Phi}(K)}{\Gamma(K)}\frac{a_k(b_k+l)}{pz^2}.\end{aligned}$$

Substituting $\gamma(K) = -K\phi(K)$ in the above equation yields:

$$\begin{aligned}f_k''(z)|_{f_k'(z)=0} =& \frac{p}{z^2}\Gamma\left(\frac{D}{z}\right)\left[D\frac{\gamma\left(\frac{D}{z}\right)}{\Gamma\left(\frac{D}{z}\right)} - \frac{b_k+l}{p}\frac{\gamma(K)}{\Gamma(K)}\right] \\ & - \phi(K)\frac{a_k(b_k+l)}{pz^2}\left(1 - \frac{K\overline{\Phi}(K)}{\Gamma(K)}\right).\end{aligned} \tag{2.45}$$

Observe that, by Assumption 2.6, we have $pD > b_k + l$ and $\frac{D}{z} \geq K$. Also, note that $\gamma(\cdot) < 0$ (by definition). Now, invoke our assumption that $\frac{\gamma(\cdot)}{\Gamma(\cdot)}$ is decreasing to observe that the first term in (2.45) is negative. In addition, observe that $\Gamma(K) = K\overline{\Phi}(K) + \int_K^\infty \overline{\Phi}(x)dx$ (by integration by parts), so $1 - \frac{K\overline{\Phi}(K)}{\Gamma(K)} > 0$; hence, the second term in (2.45) is negative. Thus, we conclude that $f_k''(z)|_{f_k'(z)=0} < 0$, which concludes the proof of the lemma. ■

*Proof.* Proposition 2.9.

First, I prove that $f_k, k = L, R$ are unimodal in $z$. To that end, I need to prove that, for $\overline{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{N}}\right)$, $\frac{\gamma(\cdot)}{\Gamma(\cdot)}$ is decreasing, where $\gamma$ and $\Gamma$ are defined by (2.22) and (2.21). I can then apply Lemma 2.8. $\phi$ is the p.d.f. of a normal random variable with mean $\mu$ and standard deviation $\frac{\sigma}{\sqrt{N}}$. Define $t(m) = \frac{(m-\mu)\sqrt{N}}{\sigma}$ and let $\phi_S$ and $\Phi_S$ denote the p.d.f. and c.d.f. of the standard normal distribution, respectively. The following observations are standard:

$$\gamma(m) = -m\phi_S[t(m)]; \qquad \Gamma(m) = \mu\overline{\Phi}_S[t(m)] + \frac{\sigma}{\sqrt{N}}\phi_S[t(m)]. \tag{2.46}$$

From (2.46), it follows that

$$\frac{\Gamma(m)}{\gamma(m)} = -\frac{\mu\overline{\Phi}_S[t(m)]}{m\phi_S[t(m)]} - \frac{\sigma}{\sqrt{N}m} \tag{2.47}$$

Note that $\frac{\overline{\Phi}_S(\cdot)}{m\phi_S(\cdot)}$ is decreasing, because standard normal distribution has increasing failure rate. Using this observation, we note from (2.47) that $\frac{\Gamma(m)}{\gamma(m)}$ is increasing in $m$. Hence, $\frac{\gamma(m)}{\Gamma(m)}$ is decreasing in $m$.

Now the proof that the objective function is unimodal is similar to the proof of Proposition 2.2. ■

*Proof.* Proposition 2.10.

Take the derivative of the expression (2.24) with respect to $b$

$$\begin{aligned} \frac{\partial}{\partial b} h'(z) &= \frac{\partial}{\partial b}\left[p \int_{\frac{az+b+l}{pz}}^{\infty} x\phi(x)\, dx - a \int_{\frac{az+b+l}{pz}}^{\infty} \phi(x)\, dx\right] \qquad (2.48) \\ &= -\frac{1}{pz}\phi\left(\tfrac{az+b+l}{pz}\right)\left[p\,\tfrac{az+b+l}{pz} - a\right] \leq 0. \end{aligned}$$

■

# CHAPTER III

# Inventory of Cash-Constrained Firms and the Option to Acquire Future Financing

## 3.1 Introduction.

Companies in a broad range of industries and economies rely heavily on external sources to finance their operations. But, external financing could be expensive and/or difficult to obtain due to asymmetric information between lenders and borrowers, high cost of capital of lenders, high cost of lenders to monitor the credibility of potential borrowers, credit rationing by lenders, and expensive loan terms sometimes extended by lenders with power advantage over manufacturers.

Banks often have lower costs of capital and easier access to capital than suppliers. Therefore, banks can offer loans at cheaper rates than suppliers if they can assess how likely manufacturers are to repay. Unfortunately, asymmetric information that often exists between banks and manufacturers causes banks to offer unfavorable loan terms or even deny loans to manufacturers. However, the competition among suppliers to sell goods to manufacturers might make suppliers lend to manufacturers with greater ease than banks. This is why, even when suppliers offer credit terms that come at high costs, the relatively easy access to supplier financing enables manufacturers to signal their credit quality to banks, which facilitates access to bank loans.

In this chapter, I focus on cash-constrained manufacturers that rely on bank loans and trade credit[1] to finance their costs of production in expectation of the demand for final goods. The emphasis of my analysis is to answer the following questions: when should manufacturers use trade credit to finance their operations?; how does information asymmetry between banks and manufacturers affect the operational and financial decisions of manufacturers?; how does the availability of trade credit affect the operational and financial decisions of manufacturers?; what are the benefits and costs of the trade credit signal?

I consider a stochastic dynamic programming model to represent operations of a cash-constrained manufacturer. Then, I analyze and compare results of two problems: a problem where a bank's belief about the manufacturer's credibility does not depend on the loan repayment amount of the manufacturer; and a problem where a bank's belief about the manufacturer's credibility depends on the loan repayment amount of the manufacturer.

I show that financing considerations affect the optimal order-up-to level. Specifically, when the benefit to borrow from suppliers is greater than the benefit to use internal capital, manufacturers might borrow a high amount from suppliers and over-order to make use of cheap supplier loans and capitalize on high demands for finished goods; when the line of credit offered by banks is low, manufacturers might borrow a high amount from suppliers and over-order to signal their repayment capabilities to banks and capitalize on high demands for finished goods; when the line of credit offered by banks is high, manufacturers might borrow a high amount from banks and over-order to make use of cheap bank loans and capitalize on high demands for finished goods; when the line of credit offered by lenders is low and the opportunity cost of capital of manufacturers is high relative to other sources of financing, manu-

[1] See Chapters I and II to obtain a more detailed explanation of trade credit financing.

facturers might under-order to allocate their precious capital to projects with higher expected returns.

My analysis suggests that, if a bank extends a small line of credit to a manufacturer, then the manufacturer is better-off when the bank's belief about the manufacturer's credibility depends on the loan repayment amount of the manufacturer and, if a bank extends a big line of credit to a manufacturer, then the manufacturer is better-off when the bank's belief about the manufacturer's credibility does not depend on the loan repayment amount of the manufacturer. Also, I show that, if borrowing from the bank is very cheap compared to other sources of financing, then the amount to borrow is greater in the signaling model than in the no-signaling model.

### 3.2 Literature review.

Many implications of trade credit financing have been analyzed in the operations literature. For example, Haley and Higgins (1973), and Rachamandugu (1989) investigate trade credit's impact on the Economic Order Quantity model. Recently, Gupta and Wang (2009) show the order-up-to-inventory remains optimal for a discrete time combined inventory-financing model when trade credit is used and they prescribe an algorithm for computing the optimal stock level for a continuous time model.

Trade credit financing has also been analyzed in the finance literature. For example, Petersen and Rajan (1997) observe that suppliers appear to have an advantage over traditional financial institutions in lending to growing firms, especially if those firms' credit quality is suspect. According to Petersen and Rajan (1997), there are three main reasons why suppliers appear to have an advantage over traditional financial institutions in lending to growing firms: the suppliers are more capable than banks to repossess the goods from manufacturers in case of insolvency and sell them; the buyers are possible source of future business for suppliers; and there is a low cost of obtaining information from product market transactions, and perhaps from other suppliers. Fisman (2001) discovers a correlation between the availability of trade credit financing and a firm's operational performance. Using a sample of African firms, Fisman (2001) finds that firms with access to trade credit financing are less likely to experience stock-outs, and are more likely to have higher production-capacity utilization.

Various studies have been conducted on the relationship between lenders and borrowers when lenders are facing asymmetric information about the riskiness of the borrowers' projects. For example, Povel and Raith (2004) study financial contracting when an entrepreneur's investment and its resulting revenue are unobservable to the lender. Jaffee and Russell (1976) analyze the dynamics of a loan market in which borrowers have more information about the likelihood of default than the lenders. Sometimes borrowers can even consider taking actions to alleviate the asymmetric information that exists between them and lenders in order to obtain favorable loans from lenders. For example, Stiglitz and Weiss (1981) present a model of credit rationing in which among observationally identical borrowers some receive loans and others do not receive loans even if the potential borrowers are willing to accept higher interest rates than the market interest rates or put up more collateral than is demanded by the lenders.

In addition to studies on the asymmetric information between lenders and borrowers, various studies have been conducted on the use of trade credit by manufacturers to signal their repayment capabilities to banks. For example, Cook (1999), using data from a survey of 352 firms in Russia in 1995, empirically concludes that trade credit can incorporate the private information held by suppliers about their customers in the lending relation. The author argues that trade credit financing sends a positive signal to financial institutions which, without that signal, might be reluctant to lend to buyers. Cook's empirical findings are consistent with Alphonse et al. (2006) who use US small business data (NSSBF 1998) to show that trade credit can signal firm's quality and thus facilitates access to bank debt. Biais and Gollier (1997) consider a static model in which a supplier has private information about the manufacturers while the bank does not. In Biais and Gollier (1997), manufacturers' revenues do not depend on the operational decisions made by manufacturers and the decision to finance projects is solely dictated by exogenous signals received from banks and suppliers. Furthermore, in Biais and Gollier (1997), credit rationing occurs when the bank cannot always assess the quality of a firm with enough precision. As a result, many firms with positive net present value projects are denied credit from the bank. However, suppliers might find it profitable to extend credit to some of these firms.

I focus on the use of trade credit financing as a way for manufacturers to finance their productions and, at the same time, signal a high repayment capability to banks. Unlike in Biais and Gollier (1997) I do not consider consider a static setting with multiple decision-makers. Instead, I consider a multi-period model, which contains one bank, one supplier and one manufacturer where the manufacturer is the only decision-maker. The main motivation for the multi-period model is to address, in a dynamic setting, the effect of the trade credit on the bank loan terms. Unfortunately, tracing the results of a multi-pertiod model required me to sacrifice decisions made by the suppliers and the banks. Nonetheless, my multi-period optimization model contributes to the operations and finance literatures because I consider the joint effect of the loan terms, the manufacturer's operational policies and the manufacturer's financial decisions on each other. However, instead of following others who have considered the impact of operational policies on the loan interest rates set by the bank, I am analyzing the impact of the operational policies on the loan limits.

### 3.3 Model and assumptions.

I solve a discrete-time, dynamic, finite horizon, stochastic model with a manufacturer, a bank, and a supplier, where the manufacturer is the only decision-maker. The objective of the manufacturer is to maximize the expected net present value of the dividends received by its shareholders.

I assume that only short-term loans are available and the manufacturer can borrow from the bank, the supplier, and the shareholders. Manufacturers can also use retained earnings to finance operations. The supplier can produce unlimited number of goods and always delivers the number of goods requested by the manufacturer. Throughout the analysis, the interest rate on bank loans is $r^B$, the interest rate on supplier loans is $r^S$, the discount factor of the cash flows provided to the manufacturer's shareholders is $\alpha$. Also, demands, $\{D_t\}$, for any period $t \in \{1, T\}$ are i.i.d. with a p.d.f. $\phi$ and an invertible c.d.f. $\Phi$, the unit wholesale price is $\tau$, the manufacturer pays $m$ for production, the maximum credit offered by the supplier per unit of product bought by the manufacturer is $\lambda$, and the retail price is $p$. The trade credit amount cannot be greater than the wholesale price, $\lambda \leq \tau$. Thus, the total cost of a good, for the manufacturer, is $w$ where $w = \tau + m$. To ensure the manufacturer is able to make profit, $w < p$.

At the beginning of each period, $t$, the manufacturer observes its starting inventory level, $x_t$, the retained earnings level, $m_t$, and the bank loan limit, $\overline{B}_t$, before making decisions on the order quantity, $Q_t$, dividend amount, $\nu_t$, bank loan amount, $B_t$, and trade credit amount, $S_t$. If the retained earnings level is negative the manufacturer obtains capital from its shareholders and pays a default penalty fee, $\Psi(m_t)$, before borrowing from the bank, borrowing from the supplier, paying loan interests on the loans obtained, providing dividends to its shareholders and incurring the manufacturing cost of the goods ordered. The production quantity, bank loan, and supplier loan are nonnegative:

$$0 \leq Q_t, \qquad 0 \leq B_t, \qquad 0 \leq S_t. \tag{3.1}$$

To prevent the expenditures in period $t$ from surpassing the available financing, I require

$$\nu_t + \Psi(m_t) + wQ_t + r^B B_t + r^S S_t \leq m_t + B_t + S_t. \tag{3.2}$$

Because $\nu_t$ is the only variable that can be negative in the model, constraint (3.2) enables us to determine, at the beginning of any period $t$, if the manufacturer gives dividend to the shareholders or requests money from them to cover the expenses.

At the end of the period, the supplier delivers the products, demand, $D_t$, is realized and revenue, $R(x_t + Q_t, D_t)$, is generated. I assume the bank is repaid before the supplier. Therefore, at the end of the period, the manufacturer repays the principal on the bank loan. Then, the manufacturer pays the principal on the supplier loan.

My model is similar to the model analyzed in Li et al. (2005). As was done in their paper, it is convenient to introduce the order-up-to level,

$$y_t = x_t + Q_t, \tag{3.3}$$

and the target-capital-level,

$$M_t = m_t - \nu_t - \Psi(m_t) - w(y_t - x_t) - r^B B_t - r^S S_t. \tag{3.4}$$

The target-capital-level indicates the amount of capital at the manufacturer after paying out all costs.

The loan limit of the supplier is a function of the order quantity of the manufacturer, $Q_t$, and the maximum amount of capital the supplier is willing and/or capable to lend to the manufacturer, $\overline{S}$. Specifically, the loan limit of the supplier is $\min\left[\lambda Q_t, \overline{S}\right]$[2].

I consider two problems: a problem where the bank's belief about the manufacturer's credibility does not depend on the loan repayment amount of the manufacturer (no-signaling); and a problem where the bank's belief about the manufacturer's credibility depends on the loan repayment amount of the manufacturer (signaling). For the no-signaling problem, the bank loan limit at each period is $\overline{B}$. For the signaling problem, the bank loan limit, $\overline{B}_t$, depends on the most recent line of credit of the manufacturer, $B_{t-1} + S_{t-1}$, and the maximum amount the manufacturer was able to repay after realization of demand, $M_{t-1} + B_{t-1} + S_{t-1} + R(x_{t-1} + Q_{t-1}, D_{t-1})$. Specifically, the bank loan limit at $t = 1$ is $\overline{B}$ and, for $t = 2, 3, ..., T$, the bank loan limit is $\min[B_{t-1} + S_{t-1}, M_{t-1} + B_{t-1} + S_{t-1} + R(x_{t-1} + Q_{t-1}, D_{t-1})]$.

The interest rates on bank loans, $r^B$, and supplier loans, $r^S$ are exogenously given. Because the bank is the preferred lender, I assume that the rate, $r^B$, is less than or equal to the rate on trade credit terms, $r^S$, and the opportunity cost of capital of the shareholders, $(1 - \alpha)$. That is, $r^B < \min[r^S, (1 - \alpha)]$.

I consider the situation where the planning periods occur frequently and where time $T$ is relatively short. This is why $\alpha$, $\tau$, $m$, $p$, $\lambda$, $\overline{S}$, $r^B$, and $r^S$ are constant from

[2]See Chapter II to obtain a more detailed explanation for the choice of supplier loan limit used.

period 1 to period $T$[3].

The next period's starting inventory is given by

$$x_{t+1} = y_t - D_t. \tag{3.5}$$

The next period's starting capital is given by

$$m_{t+1} = M_t + R(y_t, D_t). \tag{3.6}$$

From equations (3.3), (3.4), (3.5) and (3.6), the decision variables in period $t$ can be specified as $y_t$, $M_t$, $B_t$ and $S_t$ instead of $Q_t$, $\nu_t$, $B_t$ and $S_t$. Therefore, for $t = 1, 2, ..., T$, let the history of states and actions be given by

$$\begin{aligned} \Pi_t \stackrel{\text{def}}{=} (&x_1, m_1, y_1, M_1, B_1, S_1, D_1 ..., x_{t-1}, m_{t-1}, y_{t-1}, M_{t-1}, B_{t-1}, S_{t-1}, D_{t-1}, \\ &x_t, m_t, y_t, M_t, B_t, S_t, D_t) \end{aligned} \tag{3.7}$$

and let

$$C = \sum_{t=1}^{T} \alpha^{t-1} \nu_t. \tag{3.8}$$

A non-anticipative policy is a rule, for each $t$, on how the manufacturer chooses $y_t$, $M_t$, $B_t$, $S_t$ as a function of $\Pi_t$. An optimal policy maximizes $E\Big[C | x_1 = x, m_1 = m, \overline{B}_1 = \overline{B}\Big]$ for each $\big(x, m, \overline{B}\big) \in \mathbb{R}^3$. Our goal is to characterize such optimal policy.

## 3.4 Analysis.

Although this problem appears to have three state variables ($x_t$, $m_t$ and $\overline{B}_t$), it can be reduced, similar to Li et al. (2005), to a problem with two state variables ($x_t$ and $\overline{B}_t$) and its solution can be specified almost explicitly.

Observe, from equation (3.4), that

$$\nu_t = m_t - \Psi(m_t) - w(y_t - x_t) - r^B B_t - r^S S_t - M_t. \tag{3.9}$$

[3] See Chapter II to obtain a more detailed argument for the stationary of $\lambda$, $\overline{S}$, $r^B$, and $r^S$.

If we substitute $\nu_t$ in equation (3.8) by its expression in equation (3.9), we derive:

$$C = \sum_{t=1}^{T} \alpha^{t-1} \left[ m_t - \Psi(m_t) - w(y_t - x_t) - r^B B_t - r^S S_t - M_t \right] \tag{3.10}$$

$$\begin{aligned} =& m_1 + wx_1 - \Psi(m_1) - wy_1 - r^B B_1 - r^S S_1 - M_1 \\ &+ \sum_{t=2}^{T} \alpha^{t-1} \left[ m_t - \Psi(m_t) - w(y_t - x_t) - r^B B_t - r^S S_t - M_t \right]. \end{aligned} \tag{3.11}$$

Recall, from equation (3.5) and equation (3.6), that $x_{t+1} = y_t - D_t$ and $m_{t+1} = M_t + R(y_t, D_t)$. If, for $t = 2, 3, ..., T$, we substitute $x_t$ and $m_t$ in equation (3.11) by their equivalent expressions in equation (3.5) and equation (3.6), then we derive:

$$\begin{aligned} C =& m_1 + wx_1 - \Psi(m_1) - \sum_{t=1}^{T-1} \alpha^t w D_t + \alpha R(y_1, D_1) - (1-\alpha) w y_1 \\ &- \alpha \Psi\Big( M_1 + R(y_1, D_1) \Big) - (1-\alpha) M_1 - r^B B_1 - r^S S_1 \\ &+ \sum_{t=2}^{T} \alpha^{t-1} \Big\{ \alpha R(y_t, D_t) - (1-\alpha) w y_t - \alpha \Psi\Big( M_t + R(y_t, D_t) \Big) \\ &- (1-\alpha) M_t - r^B B_t - r^S S_t \Big\}. \end{aligned} \tag{3.12}$$

For $(B, S, M, y) \in \mathbb{R}^4$, let

$$\begin{aligned} K(B, S, M, y) =& \alpha R(y, D) - (1-\alpha) w y \\ &- \alpha \Psi\Big( M + R(y, D) \Big) - (1-\alpha) M - r^B B - r^S S. \end{aligned} \tag{3.13}$$

Because a policy maximizes $E(C|\Pi_1)$ if and only if it maximizes

$$E(C|\Pi_1) - \left[ m_1 + wx_1 - \Psi(m_1) - \sum_{t=1}^{T-1} \alpha^t w E(D_t) \right], \tag{3.14}$$

I utilize equation (3.12) and equation (3.13) to optimize the following criterion:

$$E\left[ \sum_{t=1}^{T} \alpha^{t-1} K(y_t, M_t, B_t, S_t) \right], \tag{3.15}$$

subject to:

$$x_t \leq y_t, \tag{3.16}$$

$$0 \leq B_t \leq \overline{B}_t, \tag{3.17}$$

$$0 \leq S_t \leq \min\left[\lambda\left(y_t - x_t\right), \overline{S}\right], \tag{3.18}$$

$$0 \leq M_t + B_t + S_t. \tag{3.19}$$

The maximization of the expected value in equation (3.8) is equivalent to maximizing the expected value in (3.15). However, a dynamic program for the former problem has a state space consisting of three state variables, whereas the latter problem has two variables. This is why, with

$$\begin{aligned} K(B,S,M,y) =& E\Big[\alpha R\left(y,D\right) - (1-\alpha)wy - \alpha\Psi\Big(R\left(y,D\right) + M\Big) \\ & - (1-\alpha)M - r^B B - r^S S\Big], \end{aligned} \tag{3.20}$$

our general problem can be transformed into the following DP recursions.

**DP Recursion for the no-signaling problem**.

$$g_t\left(x\right) = \max_{B,S,M,y} J_t(B,S,M,y) \tag{3.21}$$

where

$$J_t(B,S,M,y) = K(B,S,M,y) + \alpha E\left[g_{t+1}\left(y-D\right)\right], \tag{3.22}$$

$$g_{T+1}\left(.\right) = 0. \tag{3.23}$$

**DP Recursion for the signaling problem**.

$$f_t\left(\overline{B},x\right) = \max_{B,S,M,y} J_t(B,S,M,y) \tag{3.24}$$

where

$$\begin{aligned} J_t(B,S,M,y) =& K(B,S,M,y) \\ & + \alpha E\left[f_{t+1}\Big(B+S+\min\left[M+R\left(y,D\right),0\right], y-D\Big)\right], \end{aligned} \tag{3.25}$$

$$f_{T+1}\left(.,.\right) = 0. \tag{3.26}$$

Optimization domain:

$$x \leq y, \tag{3.27}$$

$$0 \leq B \leq \overline{B}, \tag{3.28}$$

$$0 \leq S \leq \min\left[\lambda\left(y - x\right), \overline{S}\right], \tag{3.29}$$

$$0 \leq M + B + S. \tag{3.30}$$

In the rest of the analysis, I introduce a new variable, $L$, to represent the total loan amount requested by the manufacturer, $B + S$.

**Proposition 3.1.** *For a given target-capital level, $M$, and order-up-to level, $y$, the manufacturer, at optimality, will only consider borrowing from the supplier if the amount the manufacturer desires to borrow from the bank exceeds the bank loan limit.*

*Proof.* I show that it is always suboptimal to borrow from the supplier if bank loans are still available.

For any $t = 1, 2, ..., T$, if trade credit does not affect the bank's belief about the manufacturer's credibility, then

$$\begin{aligned} J_t(B, S, M, y) =& E\Big[\alpha R\left(y, D\right) - (1-\alpha)wy - \alpha\Psi\Big(R\left(y, D\right) + M\Big) \\ & - (1-\alpha)M - r^B B - r^S S\Big] + \alpha E\left[g_{t+1}\Big(y - D\Big)\right]. \end{aligned} \tag{3.31}$$

Because the interest rate on bank loans is lower than the interest rate on supplier loans it is suboptimal to borrow from the supplier if bank loans are still available.

For any $t = 1, 2, ..., T$, if trade credit affects the bank's belief about the manufacturer's credibility, then

$$\begin{aligned} J_t(B, S, M, y) =& E\Big[\alpha R\left(y, D\right) - (1-\alpha)wy - \alpha\Psi\Big(R\left(y, D\right) + M\Big) \\ & - (1-\alpha)M - r^B B - r^S S\Big] \\ & + \alpha E\left[f_{t+1}\Big(B + S + \min\left[M + R\left(y, D\right), 0\right], y - D\Big)\right]. \end{aligned} \tag{3.32}$$

Observe that, $f_{t+1}$ depends on the total loan amount, $B+S$, and not on the source of the loan. Therefore, because the interest rate on bank loans is lower than the interest rate on supplier loans, it is suboptimal to borrow from the supplier if bank loans are still available. ■

According to Proposition 3.1, the bank, lending at a lower rate than the supplier, is always the preferred lending source. This is why, if we define

$$\begin{aligned} K(L,M,y) =& E\Big[\alpha R(y,D) - (1-\alpha)wy - \alpha\Psi\Big(R(y,D)+M\Big) \\ & - (1-\alpha)M - r^B \min\left(L,\overline{B}\right) - r^S\left(L-\overline{B}\right)^+\Big], \end{aligned} \tag{3.33}$$

then our problems can be transformed into the following DP recursions.

**DP Recursion for the no-signaling problem**.

$$g_t(x) = \max_{L,M,y} J_t(L,M,y) \tag{3.34}$$

where

$$J_t(L,M,y) = K(L,M,y) + \alpha E\left[g_{t+1}(y-D)\right], \tag{3.35}$$

$$g_{T+1}(.) = 0. \tag{3.36}$$

**DP Recursion for the signaling problem**.

$$f_t\left(\overline{B},x\right) = \max_{L,M,y} J_t(L,M,y) \tag{3.37}$$

where

$$J_t(L,M,y) = K(L,M,y) + \alpha E\left[f_{t+1}\Big(L+\min\left[M+R(y,D),0\right], y-D\Big)\right], \tag{3.38}$$

$$f_{T+1}(.,.) = 0. \tag{3.39}$$

Optimization domain

$$x \le y, \tag{3.40}$$

$$0 \leq L \leq \overline{B} + \min\left[\lambda\left(y - x\right), \overline{S}\right], \tag{3.41}$$

$$0 \leq M + L. \tag{3.42}$$

**Lemma 3.1.** *At any period $t$, the shareholders' optimal value functions, $g_t$ and $f_t$, are non-increasing in the starting inventory, $x$. Furthermore, at any period $t$, the shareholders' optimal value function, $f_t$, is non-decreasing in the bank loan limit, $\overline{B}$.*

*Proof.* Recall, from (3.34), (3.37), (3.40), (3.41) and (3.42), that, for any $t$, $g_t$ and $f_t$ maximize $J_t(L, M, y)$ subject to $x \leq y$, $0 \leq L \leq \overline{B}+\min[\lambda\left(y - x\right), \overline{S}]$ and $0 \leq M+L$. But, the feasible region shrinks in $x$ while $J_t$ does not depend on $x$. Therefore, for any period $t$, the shareholders' optimal value functions, $g_t$ and $f_t$, are non-increasing in the starting inventory, $x$.

For any $t$, $f_t$ maximizes $J_t(L, M, y)$ subject to $x \leq y$, $0 \leq L \leq \overline{B}+\min[\lambda\left(y - x\right), \overline{S}]$ and $0 \leq M + L$. But, the feasible region expands in $\overline{B}$ while $J_t$ is non-decreasing in $\overline{B}$. Therefore, for any period $t$, the shareholders' optimal value function, $f_t$, is non-decreasing in the bank loan limit, $\overline{B}$. ■

**Lemma 3.2.** *If the revenue function, $R$, is concave and the penalty function, $\Psi$, is convex and decreasing, the immediate reward function, $K$, is concave.*

*Proof.* Let $\Omega = \Big\{L, M, y\Big\}$ be a convex set $\subset E_3$. Because expectation over the demand, $D$, preserves concavity, $K$ is concave on the convex set $\Omega$ if, for any $D$, $\alpha R\left(y, D\right) - (1-\alpha)wy - \alpha\Psi\Big(R\left(y, D\right) + M\Big) - (1-\alpha)M - r^B \min\left(L, \overline{B}\right) - r^S\left(L - \overline{B}\right)^+$ is concave. We can easily see that $-(1-\alpha)wy - (1-\alpha)M - r^B \min\left(L, \overline{B}\right) - r^S\left(L - \overline{B}\right)^+$ is concave on the convex set $\Omega$. Therefore, if $\alpha R\left(y, D\right) - \alpha\Psi\Big(R\left(y, D\right) + M\Big)$ is concave on the convex set $\Omega$, then $K$ is concave on the convex set $\Omega$.

Observe that, for any concave revenue function, $R$, $\alpha R\left(y, D\right)$ is concave because multiplying a positive number to a concave function results in a concave function. Furthermore, $-\alpha\Psi\Big(R\left(y,D\right)+M\Big)$ is concave if $\Psi$ is convex and decreasing. ■

In the rest of the analysis I assume that the revenue function is $R\left(y,D\right)=p\min(y,D)$ and the penalty function is $\Psi(m)=\psi(m)-\psi(m)^{+}$ where $\psi>0$. By Lemma 3.2, the immediate reward function $K$ is concave with these revenue and penalty functions and (3.33) becomes:

$$\begin{aligned}K(L,M,y)=&E\Big[\alpha p\min(y,D)-(1-\alpha)wy-(1-\alpha)M+\alpha\psi\Big(p\min(y,D)+M\Big)\\&-\alpha\psi\Big(p\min(y,D)+M\Big)^{+}-r^{B}\min\left(L,\overline{B}\right)-r^{S}\left(L-\overline{B}\right)^{+}\Big].\end{aligned}\tag{3.43}$$

and our problems can be transformed into the following DP recursions.

**DP Recursion for the no-signaling problem**.

$$g_t\left(x\right)=\max_{L,M,y}J_t(L,M,y)\tag{3.44}$$

where

$$J_t(L,M,y)=K(L,M,y)+\alpha E\left[g_{t+1}\left(y-D\right)\right],\tag{3.45}$$

$$g_{T+1}\left(.\right)=0.\tag{3.46}$$

**DP Recursion for the signaling problem**.

$$f_t\left(\overline{B},x\right)=\max_{L,M,y}J_t(L,M,y)\tag{3.47}$$

where

$$\begin{aligned}J_t(L,M,y)=&K(L,M,y)\\&+\alpha E\left[f_{t+1}\Big(L+\min\left[M+p\min(y,D),0\right],y-D\Big)\right],\end{aligned}\tag{3.48}$$

$$f_{T+1}\left(.,.\right)=0.\tag{3.49}$$

Optimization domain

$$x \leq y, \tag{3.50}$$

$$0 \leq L \leq \overline{B} + \min\left[\lambda\left(y-x\right), \overline{S}\right], \tag{3.51}$$

$$0 \leq M + L. \tag{3.52}$$

**Proposition 3.2.** *At any period $t$, it is suboptimal to have a positive target-capital-level before receiving the loans. That is, it is optimal to have $M \leq 0$.*

*Proof.* I show that it is always suboptimal to have $M > 0$ by showing that, if $M > 0$, then we will improve the objective value in the DP recursion optimizations by reducing $M$.

For any $t = 1, 2, ..., T$ and $M > 0$, if trade credit does not affect the bank's belief about the manufacturer's credibility, then

$$\begin{aligned} J_t(L, M, y) = & E\Big[\alpha p \min(y, D) - (1-\alpha)wy - (1-\alpha)M \\ & - r^B \min\left(L, \overline{B}\right) - r^S \left(L - \overline{B}\right)^+\Big] \\ & + \alpha E\left[g_{t+1}\left(y - D\right)\right]. \end{aligned} \tag{3.53}$$

and, if trade credit affects the bank's belief about the manufacturer's credibility, then

$$\begin{aligned} J_t(L, M, y) = & E\Big[\alpha p \min(y, D) - (1-\alpha)wy - (1-\alpha)M \\ & - r^B \min\left(L, \overline{B}\right) - r^S \left(L - \overline{B}\right)^+\Big] + \alpha E\Big[f_{t+1}\Big(L, y - D\Big)\Big]. \end{aligned} \tag{3.54}$$

Terms with $g_{t+1}$ and $f_{t+1}$ are independent of $M$. Because $1 - \alpha > 0$ and, for a fixed $L$ and $y$, $M$ is independent of the state variables, then, for any $t$, $J_t$ is decreasing in $M$. Therefore, any $M > 0$, is suboptimal. ■

By definition, $M$ indicates the target-capital level after paying out all costs and before receiving loans. Therefore, Proposition 3.2 implies that it is suboptimal to maintain a positive target-capital-level after paying out all costs and before receiving loans.

**Proposition 3.3.** *For the no-signaling problem, it is optimal to maintain a zero target-capital-level after receiving the total loan requested and before the realization of the demand. That is, for any $t = 1, 2, ..., T$, $L + M = 0$ is optimal in the no-signaling problem.*

*Proof.* For any $t = 1, 2, ..., T$, observe, from (3.45), that $\alpha E\left[g_{t+1}\left(y - D\right)\right]$ is independent of $L$ and, from (3.43), that $K(L, M, y)$ decreases in $L$. Therefore, (3.51) and (3.52) lead to $L = \max(0, -M)$ for any $t = 1, 2, ..., T$ in the no-signaling problem. Recall, from Proposition 3.2, that, at optimality, $M \leq 0$. Therefore, at optimality, we get $L = -M$ in the no-signaling problem. ■

According to Proposition 3.3, it is suboptimal, in the no-signaling problem, to borrow from the bank and/or the supplier and, afterwards, not passing the excess capital to the shareholders. This is why, in the no-signaling problem, it is optimal to maintain a zero target-capital-level after receiving the total loan requested and before the realization of the demand. Furthermore, Proposition 3.3 reduces the no-signaling problem into a problem with two decision variables. This is why, with

$$\begin{aligned} G(L,y) =& E\Big[\alpha p \min(y, D) - (1-\alpha) w y - \alpha\psi\Big(L - p\min(y,D)\Big)^+ \\ & + (1-\alpha)L - r^B \min\left(L, \overline{B}\right) - r^S\left(L - \overline{B}\right)^+\Big], \end{aligned} \tag{3.55}$$

and

$$\begin{aligned} F(L,M,y) =& E\Big[\alpha p \min(y, D) - (1-\alpha) w y - (1-\alpha)M + \alpha\psi\Big(M + p\min(y,D)\Big) \\ & - \alpha\psi\Big(M + p\min(y,D)\Big)^+ - r^B \min\left(L, \overline{B}\right) - r^S\left(L - \overline{B}\right)^+\Big], \end{aligned} \tag{3.56}$$

our problems can be transformed into the following DP recursions.

**DP Recursion for the no-signaling problem**.

$$g_t(x) = \max_{L,y} J_t(L, y) \tag{3.57}$$

where, $g_{T+1}(.) = 0$, and, for any $t = 1, 2, ..., T$,

$$J_t(L, y) = G(L, y) + \alpha E\left[g_{t+1}(y - D)\right]. \tag{3.58}$$

Optimization domain

$$x \leq y, \tag{3.59}$$

$$0 \leq L \leq \overline{B} + \min\left[\lambda(y - x), \overline{S}\right]. \tag{3.60}$$

**DP Recursion for the signaling problem**.

$$f_t\left(\overline{B}, x\right) = \max_{L,M,y} J_t(L, M, y) \tag{3.61}$$

where, $f_{T=1}(.,.) = 0$, and, for any $t = 1, 2, ..., T$,

$$\begin{aligned} J_t(L, M, y) =& F(L, M, y) \\ &+ \alpha E\left[f_{t+1}\Big(L + \min\left[M + p\min(y, D), 0\right], y - D\Big)\right]. \end{aligned} \tag{3.62}$$

Optimization domain

$$x \leq y, \tag{3.63}$$

$$0 \leq L \leq \overline{B} + \min\left[\lambda(y - x), \overline{S}\right], \tag{3.64}$$

$$M \leq 0, \tag{3.65}$$

$$0 \leq M + L. \tag{3.66}$$

Recall that we are interested in analyzing and comparing results of two problems: a two-period problem when the loan repayments do not affect the bank's belief about the manufacturer's creditworthiness and a two-period problem when the loan repayments affect the bank's belief about the manufacturer's creditworthiness. Before analyzing each problem separately, it is convenient to analyze the structure of the optimal solution of the one-period problem.

### 3.4.1 One-period problem.

We need to analyze:

$$\max_{L,y} J(L,y) \tag{3.67}$$

where

$$\begin{aligned} J(L,y) = & E\Big[\alpha p \min(y,D) - (1-\alpha)wy + (1-\alpha)L \\ & - \alpha\psi\Big(L - p\min(y,D)\Big)^+ - r^B \min\left(L,\overline{B}\right) - r^S\left(L-\overline{B}\right)^+\Big]. \end{aligned} \tag{3.68}$$

Optimization domain

$$x \leq y, \tag{3.69}$$

$$0 \leq L \leq \overline{B} + \min\left[\lambda\left(y-x\right), \overline{S}\right]. \tag{3.70}$$

Observe that, if $L > py$, then (3.68) is equivalent to

$$\begin{aligned} J(L,y) = & E\Big[\alpha p \min(y,D) - (1-\alpha)wy + (1-\alpha)L \\ & - \alpha\psi\Big(L - p\min(y,D)\Big) - r^B \min\left(L,\overline{B}\right) - r^S\left(L-\overline{B}\right)^+\Big], \end{aligned} \tag{3.71}$$

and, if $L \leq py$, then (3.68) is equivalent to

$$\begin{aligned} J(L,y) = & E\Big[\alpha p \min(y,D) - (1-\alpha)wy + (1-\alpha)L - \alpha\psi\left(L-pD\right)^+ \\ & - r^B \min\left(L,\overline{B}\right) - r^S\left(L-\overline{B}\right)^+\Big]. \end{aligned} \tag{3.72}$$

Throughout the analysis, I will use (3.68), or (3.71) and (3.72) for the expressions of $J(L,y)$.

**Lemma 3.3.** *If* $(1-\alpha) \leq \alpha\psi + r^B$*, then, at optimality,* $L \leq py$*.*

*Proof.* I show that, if $(1-\alpha) \leq \alpha\psi + r^B$, then, it is suboptimal to have $L > py$ by showing that, if $L > py$, then we will improve the objective value by reducing $L$.

Recall that $r^S > r^B$. Therefore, $(1-\alpha) \leq \alpha\psi + r^B$ implies that $(1-\alpha) \leq \alpha\psi + r^S$. This is why, if $(1-\alpha) \leq \alpha\psi + r^B$, then, from (3.71), $J(L, y)$ is decreasing in $L$ for any $y$ and $L > py$. Hence, $L > py$ is suboptimal when $(1-\alpha) \leq \alpha\psi + r^B$. ■

**Lemma 3.4.** *If* $\alpha\psi + r^B \leq (1-\alpha)$*, then, at optimality,* $L \geq \overline{B}$*.*

*Proof.* I show that, if $\alpha\psi + r^B \leq (1-\alpha)$, then, it is suboptimal to have $L < \overline{B}$ by showing that, if $L < \overline{B}$, then we will improve the objective value by increasing $L$.

Observe, from (3.68), that, if $L < \overline{B}$, then

$$J(L, y) = E\Big[\alpha p \min(y, D) - (1-\alpha)wy + (1-\alpha)L - \alpha\psi\Big(L - p\min(y, D)\Big)^+ - r^B L\Big]. \tag{3.73}$$

Furthermore, observe that, if $\alpha\psi + r^B \leq (1-\alpha)$, then $(1-\alpha)L - \alpha\psi\Big(L - p\min(y, D)\Big)^+ - r^B L$ is increasing in $L$. This is why, if $\alpha\psi + r^B \leq (1-\alpha)$, then it is suboptimal to have $L < \overline{B}$. ■

**Lemma 3.5.** *If* $\alpha\psi + r^S \leq (1-\alpha)$*, then, at optimality,* $L = \overline{B} + \min\left[\lambda\left(y - x\right), \overline{S}\right]$*.*

*Proof.* I show that, if $\alpha\psi + r^S \leq (1-\alpha)$, then, it is suboptimal to have $L < \overline{B} + \min\left[\lambda\left(y - x\right), \overline{S}\right]$ by showing that, if $L < \overline{B} + \min\left[\lambda\left(y - x\right), \overline{S}\right]$, then it is optimal to increase $L$ as much as possible.

Recall that $r^S > r^B$. Therefore, $\alpha\psi + r^S \leq (1-\alpha)$ implies that $\alpha\psi + r^B \leq (1-\alpha)$. Lemma 3.4 states that, if $\alpha\psi + r^B \leq (1-\alpha)$, then, at optimality, $L \geq \overline{B}$. Therefore, if $\alpha\psi + r^S \leq (1-\alpha)$, then, at optimality, $L \geq \overline{B}$.

Observe, from (3.68), that, if $L \geq \overline{B}$, then

$$\begin{aligned} J(L,y) =& E\Big[\alpha p \min(y,D) - (1-\alpha)wy + (1-\alpha)L \\ & - \alpha\psi\Big(L - p\min(y,D)\Big)^{+} - r^{B}\overline{B} - r^{S}(L-\overline{B})\Big]. \end{aligned} \tag{3.74}$$

Furthermore, observe that, if $\alpha\psi + r^S \leq (1-\alpha)$, then $(1-\alpha)L - \alpha\psi\Big(L - p\min(y,D)\Big)^{+} - r^S L$ is increasing in $L$. Recall, from (3.70), that $L \leq \overline{B} + \min\left[\lambda\left(y-x\right), \overline{S}\right]$. This is why, if $\alpha\psi + r^S \leq (1-\alpha)$, then it is optimal to have $L = \overline{B} + \min\left[\lambda\left(y-x\right), \overline{S}\right]$. ■

For various relationships between the financial parameters considered in our problem, Lemma 3.3, Lemma 3.4 and Lemma 3.5 shrink the search region for the optimal solution of our optimization problem. This is why, if $(1-\alpha) \leq \alpha\psi + r^B$, then our one-period problem can be transformed into the following:

$$\max_{L,y} J(L,y) \tag{3.75}$$

where

$$\begin{aligned} J(L,y) =& E\Big[\alpha p \min(y,D) - (1-\alpha)wy + (1-\alpha)L - \alpha\psi\left(L - pD\right)^{+} \\ & - r^{B}\min\left(L,\overline{B}\right) - r^{S}\left(L-\overline{B}\right)^{+}\Big]. \end{aligned} \tag{3.76}$$

Optimization domain

$$x \leq y, \tag{3.77}$$

$$0 \leq L \leq \overline{B} + \min\left[\lambda\left(y-x\right), \overline{S}\right], \tag{3.78}$$

$$L \leq py. \tag{3.79}$$

If $\alpha\psi + r^B \leq (1-\alpha) \leq \alpha\psi + r^S$, then the one-period problem can be transformed into the following:

$$\max_{L,y} J(L,y) \tag{3.80}$$

where

$$\begin{aligned} J(L,y) =& E\Big[\alpha p \min(y,D) - (1-\alpha)wy + (1-\alpha)L - \alpha\psi\Big(L - p\min(y,D)\Big)^{+} \\ & - r^{B}\overline{B} - r^{S}(L-\overline{B})\Big]. \end{aligned} \tag{3.81}$$

Optimization domain

$$x \leq y, \tag{3.82}$$

$$\overline{B} \leq L \leq \overline{B} + \min\left[\lambda\left(y-x\right), \overline{S}\right], \tag{3.83}$$

$$L \leq \max(\overline{B}, py). \tag{3.84}$$

If $\alpha\psi + r^{S} \leq (1-\alpha)$, then the one-problem can be transformed into the following:

$$\max_{y} J(y) \tag{3.85}$$

where

$$\begin{aligned} J(y) =& E\Big[\alpha p \min(y,D) - (1-\alpha)wy + (1-\alpha)\min\left[\overline{B} + \lambda\left(y-x\right), \overline{B} + \overline{S}\right] \\ & - \alpha\psi\Big(\min\left[\overline{B} + \lambda\left(y-x\right), \overline{B} + \overline{S}\right] - p\min(y,D)\Big)^{+} \\ & - r^{B}\overline{B} - r^{S}\min\left[\lambda\left(y-x\right), \overline{S}\right]\Big]. \end{aligned} \tag{3.86}$$

Optimization domain: $x \leq y$.

Before continuing the analysis of our problem, it is convenient to visualize the feasible regions of our problem.

With respect to $y$, the slope of the boundary where $L = py$ is more positive than the slope of the boundary where $L = \overline{B} + \lambda(y-x)$. Therefore, although, a priori, we do not know where the optimal solution falls, we know that, for any $x \geq \frac{\overline{B}}{p}$, there is no $y \geq x$ for which $py \leq \overline{B} + \lambda(y-x)$.

Figure 3.1 presents the feasible regions when $(1-\alpha) \leq \alpha\psi + r^{B}$: Figure 3.1 $(a)$ and Figure 3.1 $(b)$ represent the feasible region where $x \leq \frac{\overline{B}}{p}$; Figure 3.1 $(c)$ represents the feasible region where $x \geq \frac{\overline{B}}{p}$.

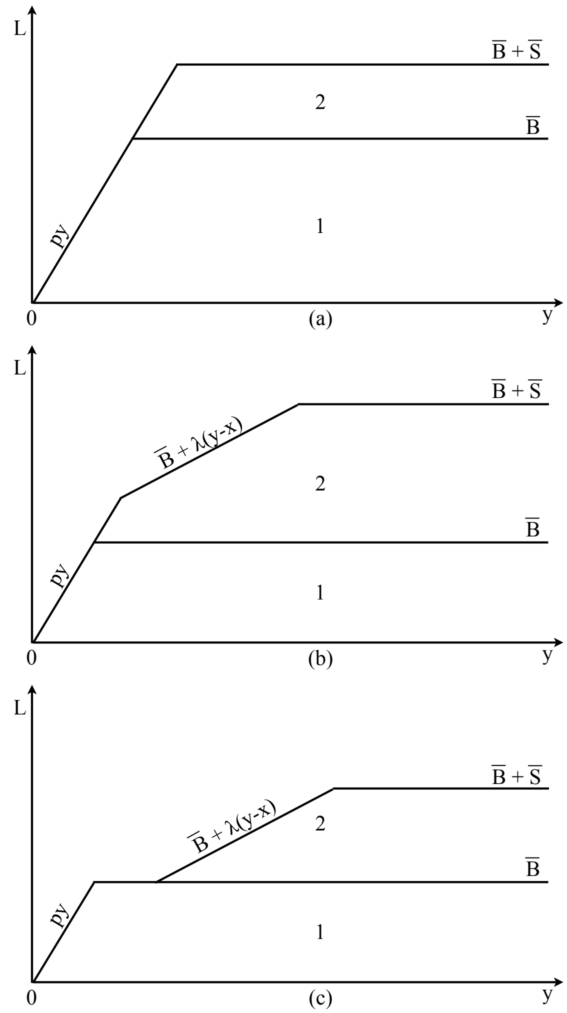


Figure 3.1: Feasible regions for the one-period problem when $(1-\alpha) \leq \alpha\psi + r^B$.

Figure 3.2 presents the feasible regions when $\alpha\psi + r^B \leq (1-\alpha) \leq \alpha\psi + r^S$: Figure 3.2 $(a)$ and Figure 3.2 $(b)$ represent the feasible region where $x \leq \frac{\overline{B}}{p}$; Figure 3.2 $(c)$ represents the feasible region where $x \geq \frac{\overline{B}}{p}$. Observe that, when $\alpha\psi + r^B \leq (1-\alpha)$, it is always cheaper to borrow from the bank than the shareholders even if borrowing from the bank results in a default penalty. Therefore, the optimal amount to borrow is always greater than $\overline{B}$ and, for any value of $\overline{B}$ and $x$, the optimal solution is in region 1, region 2 or at one of the boundaries that are shown in Figure 3.2.

Figure 3.3 presents the feasible regions when $(1-\alpha) \leq \alpha\psi + r^B$: Figure 3.3 $(a)$

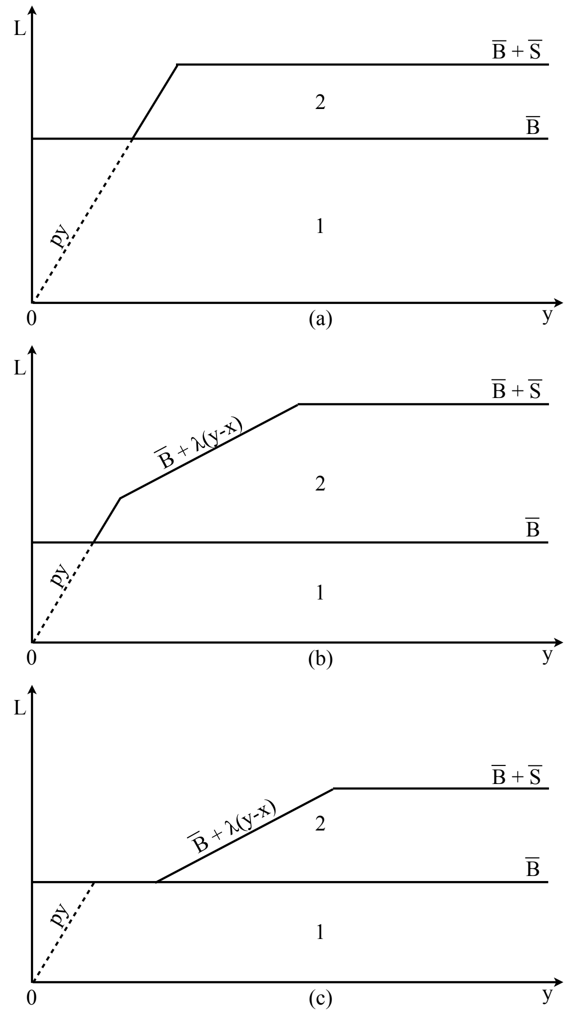


Figure 3.2: Feasible regions for the one-period problem when $\alpha\psi+r^B \leq (1-\alpha) \leq \alpha\psi+r^S$.

and Figure 3.3 ($b$) represent the feasible region where $x \leq 0$; Figure 3.3 ($c$) represents the feasible region where $x \geq 0$. Observe that, when $\alpha\psi + r^S \leq (1-\alpha)$, it is always cheaper to borrow from the bank and/or the supplier than the shareholders even if borrowing from the bank and/or the supplier results in a default penalty. Therefore, for any order-up-to level $y$, the optimal amount to borrow is always greater than $\min\left[\overline{B} + \lambda(y-x), \overline{B} + \overline{S}\right]$.

**Proposition 3.4.** $(1-\alpha) \leq \alpha\psi + r^B$. *Define* $L_b^0 \stackrel{def}{=} p\Phi^{-1}\left[\frac{(1-\alpha)-r^B}{\alpha\psi}\right]$ *and* $y^0 \stackrel{def}{=} \Phi^{-1}\left[\frac{\alpha p-(1-\alpha)w}{\alpha p}\right]$. *If the manufacturer borrows only from the bank and no constraints*

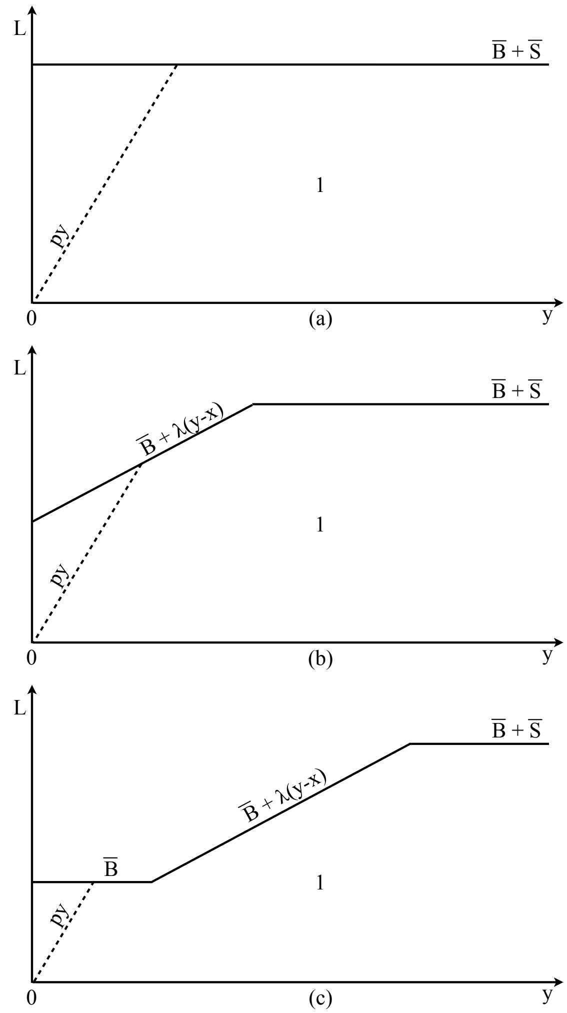


Figure 3.3: Feasible regions for the one-period problem when $\alpha\psi + r^S \leq (1-\alpha)$.

*are active, then the optimal amount to borrow and optimal order-up-to level are* $L_b^0$ *and* $y^0$ *respectively. That is, when* $x \leq y^0$*,* $0 \leq L_b^0 \leq \overline{B}$ *and* $L_b^0 \leq py^0$*, then the optimal amount to borrow is* $L_b^0$ *and optimal order-up-to level is* $y^0$*.*

*Proof.* If $(1-\alpha) \leq \alpha\psi + r^B$ and the manufacturer borrows only from the bank, then our problem becomes

$$\max_{L,y} J(L,y) \tag{3.87}$$

where

$$J(L,y) = E\Big[\alpha p \min(y,D) - (1-\alpha)wy - \alpha\psi\,(L-pD)^+ + \big[(1-\alpha) - r^B\big]\,L\Big]. \tag{3.88}$$

Optimization domain

$$x \le y, \tag{3.89}$$

$$0 \le L \le \overline{B}, \tag{3.90}$$

$$L \le py. \tag{3.91}$$

If we define $L_b^0 \overset{\text{def}}{=} p\Phi^{-1}\left[\frac{(1-\alpha)-r^B}{\alpha\psi}\right]$ and $y^0 \overset{\text{def}}{=} \Phi^{-1}\left[\frac{\alpha p-(1-\alpha)w}{\alpha p}\right]$, then, if $L = L_b^0$ and $y = y^0$, $\frac{\partial J(L,y)}{\partial L} = [(1-\alpha)-r^B] - \alpha\psi\Phi\left(\frac{L_b^0}{p}\right) = 0$ and $\frac{\partial J(L,y)}{\partial y} = \alpha p\,[1-\Phi(y^0)] - (1-\alpha)w = 0$. Because $J$ is a concave function, if $(1-\alpha) \le \alpha\psi + r^B$, then it is optimal to borrow $L_b^0$ and order-upto $y^0$ when $x \le y^0$, $0 \le L_b^0 \le \overline{B}$ and $L_b^0 \le py^0$. ∎

**Proposition 3.5.** *If $(1-\alpha) \le \alpha\psi + r^S$, the manufacturer is unable to borrow the desired amount from the bank and we define $L_{bs}^0 \overset{def}{=} p\Phi^{-1}\left[\frac{(1-\alpha)-r^S}{\alpha\psi}\right]$, then; when $x \le y^0$, and $\overline{B} \le L_{bs}^0 \le \min[\overline{B} + \lambda(y^0 - x), \overline{B} + \overline{S}, py^0]$; it is optimal to borrow $L_{bs}^0$ and order-up to $y^0$.*

*Proof.* If $(1-\alpha) \le \alpha\psi + r^S$ and the manufacturer borrows more than $\overline{B}$, then our problem becomes

$$\max_{L,y} J(L,y) \tag{3.92}$$

where

$$\begin{aligned} J(L,y) = E\Big[&\alpha p \min(y,D) - (1-\alpha)wy - \alpha\psi\Big(L - pD\Big)^+ \\ &+ (1-\alpha)L - r^B\overline{B} - r^S\left(L - \overline{B}\right)\Big], \end{aligned} \tag{3.93}$$

Optimization domain

$$x \leq y, \tag{3.94}$$

$$\overline{B} \leq L \leq \overline{B} + \min\left[\lambda\left(y - x\right), \overline{S}\right], \tag{3.95}$$

$$L \leq py. \tag{3.96}$$

Therefore, if we define $L^0_{bs} \stackrel{\text{def}}{=} p\Phi^{-1}\left[\frac{(1-\alpha)-r^S}{\alpha\psi}\right]$, then, if $L = L^0_{bs}$ and $y = y^0$, $\frac{\partial J(L,y)}{\partial L} = [(1-\alpha) - r^S] - \alpha\psi\Phi\left(\frac{L^0_{bs}}{p}\right) = 0$ and $\frac{\partial J(L,y)}{\partial y} = \alpha p\left[1 - \Phi(y^0)\right] - (1-\alpha)w = 0$. Because $J$ is a concave function, if $(1-\alpha) \leq \alpha\psi + r^S$, then it is optimal to borrow $L^0_{bs}$ and order-up to $y^0$ when the manufacturer is unable to borrow the desired amount from the bank, $x \leq y^0$, and $\overline{B} \leq L^0_{bs} \leq \min[\overline{B} + \lambda(y^0 - x), \overline{B} + \overline{S}, py^0]$. ■

**Proposition 3.6.** *It is suboptimal to borrow from the supplier when* $(1-\alpha) < r^S$.

*Proof.* Recall that, at optimality, $L > \overline{B}$ implies that the manufacturer borrows $L - \overline{B}$ from the supplier. Therefore, I will show that if $L > \overline{B}$ and $(1-\alpha) < r^S$, then it is optimal to decrease $L$.

When $L > \overline{B}$,

$$\begin{aligned} J(L,y) = &E\Big[\alpha p\min(y,D) - (1-\alpha)wy - \alpha\psi\Big(L - p\min(y,D)\Big)^+ \\ &+ (1-\alpha)L - r^B\overline{B} - r^S\left(L - \overline{B}\right)\Big], \end{aligned} \tag{3.97}$$

Optimization domain

$$x \leq y, \tag{3.98}$$

$$\overline{B} \leq L \leq \overline{B} + \min\left[\lambda\left(y - x\right), \overline{S}\right]. \tag{3.99}$$

Observe that, if $(1-\alpha) < r^S$, then, for any $L \geq \overline{B}$, $\frac{\partial J(L,y)}{\partial L} = -r^S < 0$. This is why, it is suboptimal to borrow from the supplier when $(1-\alpha) < r^S$. ■

Proposition 3.4 and Proposition 3.5 give the optimal unconstrained order-up-to level. Proposition 3.4 gives the optimal unconstrained amount to borrow from the bank. Proposition 3.5 gives the optimal unconstrained amount to borrow from the bank and supplier when the manufacturer is unable to borrow as much as desired from the bank. Proposition 3.6 gives conditions when it is never optimal to borrow from the supplier. I will show that joint consideration of financing and operational decisions in our problem leads to an optimal order-up-to level that is greater than or equal to the optimal order-up-to level in the traditional news-vendor problem. But, beforehand, I analyze properties and the structure of the constrained problem.

**Lemma 3.6.** *If the optimal order-up-to level is on the boundarywhere $y = x$, then, if $(1-\alpha) \leq \alpha\psi + r^B$, it must be optimal to borrow from the bank the minimum between $L_b^0$, $px$ and $\overline{B}$. Furthermore, if the optimal order-up-to level is on the boundarywhere $y = x$, then, if $\alpha\psi + r^B \leq (1-\alpha)$, it must be optimal to borrow $\overline{B}$ from the bank.*

*Proof.* First, I consider when $(1-\alpha) \leq \alpha\psi + r^B$. Then, I consider when $\alpha\psi + r^B \leq (1-\alpha)$.

**Case:** $(1-\alpha) \leq \alpha\psi + r^B$.

If the order-up-to level is on the boundarywhere $y = x$ then our problem becomes

$$J(L) = E\Big[\alpha p \min(x, D) - (1-\alpha)wx + [(1-\alpha) - r^B]L - \alpha\psi\Big(L - pD\Big)^+\Big]. \quad (3.100)$$

Optimization domain

$$0 \leq L \leq \overline{B}, \quad (3.101)$$

$$L \leq px. \quad (3.102)$$

Observe that $J$ is a concave function and

$$\frac{\partial J(L)}{\partial L} = [(1-\alpha) - r^B] - \alpha\psi\Phi\left(\frac{L}{p}\right). \quad (3.103)$$

Therefore, if $L = L_b^0$, then $\frac{\partial J(L)}{\partial L} = 0$. This is why, if $y = x$, then it must be that $L = \min(L_b^0, px, \overline{B})$.

**Case:** $\alpha\psi + r^B \leq (1-\alpha)$.

If the order-up-to level is on the boundarywhere $y = x$ then our problem becomes

$$J(L) = E\Big[\alpha p \min(x, D) - (1-\alpha)wx + [(1-\alpha) - r^B]L - \alpha\psi\Big(L - p\min(x, D)\Big)\Big]. \tag{3.104}$$

Optimization domain: $0 \leq L \leq \overline{B}$.

Observe that

$$\frac{\partial J(L)}{\partial L} = (1-\alpha) - r^B - \alpha\psi > 0. \tag{3.105}$$

This is why, if $y = x$, then it must be that $L = \overline{B}$. ∎

**Lemma 3.7.** *If the optimal amount to borrow is on the boundary where $L = py$ and we define $y_b^p$ as the order-up-to level that satisfies*

$$0 = \alpha p\left[1 - \Phi(y)\right] - (1-\alpha)w + [(1-\alpha) - r^B]p - \alpha\psi p\Phi(y). \tag{3.106}$$

*and $y_{bs}^p$ as the order-up-to level that satisfies*

$$0 = \alpha p\left[1 - \Phi(y)\right] - (1-\alpha)w + [(1-\alpha) - r^S]p - \alpha\psi p\Phi(y). \tag{3.107}$$

*then, for $L \leq \overline{B}$, the value function increases in $y$ when $y$ is less than $y_b^p$ and decreases in $y$ when $y$ is greater than $y_b^p$, and, for $L > \overline{B}$, the value function increases in $y$ when $y$ is less than $y_{bs}^p$ and decreases in $y$ when $y$ is greater than $y_{bs}^p$.*

*Proof.* If the amount to borrow is on the boundary where $L = py$, then, if $L \leq \overline{B}$, our problem becomes

$$\begin{aligned} J(y) = &E\Big[\alpha p \min(y, D) - (1-\alpha)wy - \alpha\psi\Big(py - pD\Big)^+ \\ &+ [(1-\alpha) - r^B]py\Big], \end{aligned} \tag{3.108}$$

and, if $\overline{B} \leq L$, our problem becomes

$$
\begin{aligned}
J(y) =& E\Big[\alpha p \min(y, D) - (1-\alpha)wy - \alpha\psi\Big(py - pD\Big)^{+} \\
&+ [(1-\alpha) - r^{S}]py\Big].
\end{aligned}
\tag{3.109}
$$

Optimization domain

$$
x \leq y, \tag{3.110}
$$

$$
0 \leq py \leq \overline{B} + \min\left[\lambda\left(y - x\right), \overline{S}\right]. \tag{3.111}
$$

Observe that $J$ is a concave function. Furthermore, observe that, when $L \leq \overline{B}$, we get:

$$
\frac{\partial J(y)}{\partial y} = \alpha p\left[1 - \Phi\left(y\right)\right] - (1-\alpha)w + [(1-\alpha) - r^{B}]p - \alpha\psi p\Phi(y) \tag{3.112}
$$

and, when $\overline{B} \leq L$, we get:

$$
\frac{\partial J(y)}{\partial y} = \alpha p\left[1 - \Phi\left(y\right)\right] - (1-\alpha)w + [(1-\alpha) - r^{S}]p - \alpha\psi p\Phi(y). \tag{3.113}
$$

Therefore, at optimality, if $L = py$ and we define $y_b^p$ as the order-up-to level that satisfies

$$
0 = \alpha p\left[1 - \Phi\left(y\right)\right] - (1-\alpha)w + [(1-\alpha) - r^{B}]p - \alpha\psi p\Phi(y) \tag{3.114}
$$

and $y_{bs}^p$ as the order-up-to level that satisfies

$$
0 = \alpha p\left[1 - \Phi\left(y\right)\right] - (1-\alpha)w + [(1-\alpha) - r^{S}]p - \alpha\psi p\Phi(y) \tag{3.115}
$$

then, for $L \leq \overline{B}$, the value function increases in $y$ when $y$ is less than $y_b^p$ and decreases in $y$ when $y$ is greater than $y_b^p$, and, for $L > \overline{B}$, the value function increases in $y$ when $y$ is less than $y_{bs}^p$ and decreases in $y$ when $y$ is greater than $y_{bs}^p$. ■

**Lemma 3.8.** *If the optimal amount to borrow is on the boundary where $L = \overline{B}$ and $\overline{B} \leq py^0$, then the value function increases in $y$ when $y$ is less than $y^0$ and decreases in $y$ when $y$ is greater than $y^0$. Furthermore, if the optimal amount to borrow is on the boundary where $L = \overline{B} + \overline{S}$ and $\overline{B} + \overline{S} \leq py^0$, then the value function increases in $y$ when $y$ is less than $y^0$ and decreases in $y$ when $y$ is greater than $y^0$.*

*Proof.* If the amount to borrow is on the boundary where $L = \overline{B}$ and $\overline{B} \leq py$, then our problem becomes

$$J(y) =E\Big[\alpha p \min(y, D) - (1-\alpha)wy - \alpha\psi\Big(\overline{B} - pD\Big)^+ + [(1-\alpha) - r^B]\overline{B}\Big]. \quad (3.116)$$

If the amount to borrow is on the boundarywhere $L = \overline{B} + \overline{S}$ and $\overline{B} + \overline{S} \leq py$, then our problem becomes

$$\begin{aligned} J(y) =&E\Big[\alpha p \min(y, D) - (1-\alpha)wy - \alpha\psi\Big(\overline{B} + \overline{S} - pD\Big)^+ \\ &+ (1-\alpha)\left(\overline{B} + \overline{S}\right) - r^B\overline{B} - r^S\overline{S}\Big]. \end{aligned} \quad (3.117)$$

Optimization domain: $x \leq y$.

Observe that $J$ is a concave function and

$$\frac{\partial J(y)}{\partial y} =\alpha p\left[1 - \Phi(y)\right] - (1-\alpha)w. \quad (3.118)$$

Therefore, if $y = y^0$, then $\frac{\partial J(y)}{\partial y} = 0$. This is why, the value function increases in $y$ when $y$ is less than $y^0$ and decreases in $y$ when $y$ is greater than $y^0$. ■

**Lemma 3.9.** *If we define $y^0_{fin}$ as the order-up-to level that satisfies*

$$0 =\alpha p(1+\psi)\left[1 - \Phi\left(y\right)\right] - (1-\alpha)w \quad (3.119)$$

*then, if the optimal amount to borrow is on the boundary where $L = \overline{B}$ and $\overline{B} > py^0_{fin}$, the value function increases in $y$ when $y$ is less than $y^0_{fin}$ and decreases in $y$ when $y$ is greater than $y^0_{fin}$. Furthermore, if the optimal amount to borrow is on the boundary* *where $L = \overline{B} + \overline{S}$ and $\overline{B} + \overline{S} > py^0_{fin}$, then the value function increases in $y$ when $y$ is less than $y^0_{fin}$ and decreases in $y$ when $y$ is greater than $y^0_{fin}$.*

*Proof.* If the amount to borrow is on the boundary where $L = \overline{B}$ and $\overline{B} > py$, then our problem becomes

$$J(y) = E\Big[\alpha p \min(y, D) - (1-\alpha)wy - \alpha\psi\Big(\overline{B} - p\min(y, D)\Big) + [(1-\alpha) - r^B]\overline{B}\Big]. \tag{3.120}$$

If the amount to borrow is on the boundary where $L = \overline{B}+\overline{S}$ and $\overline{B}+\overline{S} > py$, then our problem becomes

$$\begin{aligned} J(y) =& E\Big[\alpha p \min(y, D) - (1-\alpha)wy - \alpha\psi\Big(\overline{B} + \overline{S} - p\min(y, D)\Big) \\ &+ (1-\alpha)\left(\overline{B}+\overline{S}\right) - r^B\overline{B} - r^S\overline{S}\Big]. \end{aligned} \tag{3.121}$$

Optimization domain: $x \leq y$.

Observe that $J$ is a concave function and

$$\frac{\partial J(y)}{\partial y} = \alpha p(1+\psi)\left[1 - \Phi\left(y\right)\right] - (1-\alpha)w. \tag{3.122}$$

Therefore, if $y = y^0_{fin}$, then $\frac{\partial J(y)}{\partial y} = 0$. This is why, the value function increases in $y$ when $y$ is less than $y^0_{fin}$ and decreases in $y$ when $y$ is greater than $y^0_{fin}$. ■

**Lemma 3.10.** *If we define $y^\lambda$ as the order-up-to level that satisfies*

$$0 = \alpha p\left[1 - \Phi\left(y\right)\right] - (1-\alpha)w + \lambda\left\{[(1-\alpha) - r^S] - \alpha\psi\Phi\left(\frac{\overline{B} + \lambda(y-x)}{p}\right)\right\}, \tag{3.123}$$

*and define $y^\lambda_{fin}$ as the order-up-to level that satisfies*

$$0 = \alpha p(1+\psi)\left[1 - \Phi\left(y\right)\right] - (1-\alpha)w + \lambda[(1-\alpha) - r^S - \alpha\psi], \tag{3.124}$$

*then, if the optimal amount to borrow is on the boundary where $L = \overline{B} + \lambda(y-x)$, then, for any $y \geq \frac{\overline{B}-\lambda x}{p-\lambda}$, the value function increases in $y$ when $y$ is less than $y^\lambda$*

*and decreases in $y$ when $y$ is greater than $y^\lambda$. Furthermore, if the optimal amount to borrow is on the boundary where $L = \overline{B} + \lambda(y - x)$, then, for any $y < \frac{\overline{B}-\lambda x}{p-\lambda}$, the value function increases in $y$ when $y$ is less than $y^\lambda_{fin}$ and decreases in $y$ when $y$ is greater than $y^\lambda_{fin}$.*

*Proof.* First, we consider when $y \geq \frac{\overline{B}-\lambda x}{p-\lambda}$. Then, we consider when $y < \frac{\overline{B}-\lambda x}{p-\lambda}$.

**Case:** $y \geq \frac{\overline{B}-\lambda x}{p-\lambda}$.

If $y \geq x$ and the amount to borrow is on the boundary where $L = \overline{B} + \lambda(y - x)$, then our problem becomes

$$\begin{aligned} J(y) =& E\Big[\alpha p \min(y, D) - (1-\alpha)wy - \alpha\psi\Big(\overline{B} + \lambda(y-x) - pD\Big)^+ \\ & + (1-\alpha)[\overline{B} + \lambda(y-x)] - r^B\overline{B} - r^S\lambda(y-x)\Big], \end{aligned} \tag{3.125}$$

Optimization domain: $x \leq y$.

Observe that $J$ is a concave function and

$$\frac{\partial J(y)}{\partial y} = \alpha p\left[1 - \Phi\left(y\right)\right] - (1-\alpha)w + \lambda\left\{[(1-\alpha) - r^S] - \alpha\psi\Phi\left(\frac{\overline{B} + \lambda(y-x)}{p}\right)\right\}. \tag{3.126}$$

Therefore, if $y = y^\lambda$, then $\frac{\partial J(y)}{\partial y} = 0$. This is why, the value function increases in $y$ when $y$ is less than $y^\lambda$ and decreases in $y$ when $y$ is greater than $y^\lambda$.

**Case:** $y < \frac{\overline{B}-\lambda x}{p-\lambda}$.

If $y \geq x$ and the amount to borrow is on the boundary where $L = \overline{B} + \lambda(y - x)$, then our problem becomes

$$\begin{aligned} J(y) =& E\Big[\alpha p \min(y, D) - (1-\alpha)wy - \alpha\psi\Big(\overline{B} + \lambda(y-x) - p\min(y, D)\Big) \\ & + (1-\alpha)[\overline{B} + \lambda(y-x)] - r^B\overline{B} - r^S\lambda(y-x)\Big], \end{aligned} \tag{3.127}$$

Optimization domain: $x \leq y$.

Observe that $K$ is a concave function and

$$\frac{\partial J(y)}{\partial y} = \alpha p(1+\psi)\left[1 - \Phi\left(y\right)\right] - (1-\alpha)w + \lambda[(1-\alpha) - r^S - \alpha\psi]. \tag{3.128}$$

Therefore, if $y = y^{\lambda}_{fin}$, then $\frac{\partial J(y)}{\partial y} = 0$. This is why, the value function increases in $y$ when $y$ is less than $y^{\lambda}_{fin}$ and decreases in $y$ when $y$ is greater than $y^{\lambda}_{fin}$. ■

**Proposition 3.7.** *The optimal order up-to level in our problem is always greater than or equal to the optimal order-up-to level in the traditional news-vendor problem.*

*Proof.* Observe that, in the traditional news-vendor problem with no financing considerations (i.e., $M = 0$, $\Psi = 0$, $L = 0$, $m > w(y - x)$), our optimization problem is the following:

$$f(x) = \max_y J(y) \tag{3.129}$$

where

$$J(y) = E\Big[\alpha p \min(y, D) - (1 - \alpha) w y\Big]. \tag{3.130}$$

Optimization domain: $x \leq y$.

This leads to $y^0$ as to the optimal order-up-to level for the traditional news-vendor problem with no financing considerations. I will show that, in our problem, the conditions that lead to the optimal order-up-to level to be on one of the boundaries also lead to the order-up-to level to be greater than or equal to $y^0$. Then, I argue that even if the optimal order-up-to level is on one or more than one of the boundaries, then the optimal order-up-to level will still be greater than $y^0$. Throughout the analysis, I refer to $y^*$ as the optimal order-up-to level and $L^*$ as the optimal amount to borrow.

**Case:** $y^* = y^p_b$.

Observe that $L^* = p y^p_b \leq \overline{B}$. This leads to $\frac{\partial J(L,y)}{\partial y} = \alpha p \left[1 - \Phi(y)\right] - (1 - \alpha) w + [(1 - \alpha) - r^B] p - \alpha \psi p \Phi(y)$ and, at $y = y^0$, $\frac{\partial J(L,y)}{\partial y} = \alpha \psi p \left[\Phi\left(\frac{L^0_b}{p}\right) - \Phi(y^0)\right]$. Recall, from Proposition 3.4, that a necessary condition for $y^* = y^p_b$ is for $L^0_b \geq p y^0$. Because

$\Phi$ is a non-decreasing function, $\alpha\psi p\left[\Phi\left(\frac{L_b^0}{p}\right)-\Phi\left(y^0\right)\right]\geq 0$ and, hence, at $y=y^0$, $\frac{\partial J(L,y)}{\partial y}\geq 0$. The concavity of $J$ and the definition of $y_b^p$ imply that, if $y^*=y_b^p$, then $y_b^p\geq y^0$.

**Case:** $y^*=y_{bs}^p$.

Observe that $L^*=py_{bs}^p\geq\overline{B}$. This leads to $\frac{\partial J(L,y)}{\partial y}=\alpha p\left[1-\Phi\left(y\right)\right]-(1-\alpha)w+[(1-\alpha)-r^S]p-\alpha\psi p\Phi(y)$. Because of Proposition 3.4 and Proposition 3.5, $L^*=py_{bs}^p\geq\overline{B}$ implies that $L_{bs}^0\geq py^0$. This is why, at $y=y^0$, $\frac{\partial J(L,y)}{\partial y}=\alpha\psi p\left[\Phi\left(\frac{L_{bs}^0}{p}\right)-\Phi(y^0)\right]$. Because $\Phi$ is a non-decreasing function, $\alpha\psi p\left[\Phi\left(\frac{L_{bs}^0}{p}\right)-\Phi\left(y^0\right)\right]\geq 0$ and, hence, at $y=y^0$, $\frac{\partial J(L,y)}{\partial y}\geq 0$. The concavity of $J$ and the definition of $y_{bs}^p$ imply that, if $y^*=y_{bs}^p$, then $y_{bs}^p\geq y^0$.

**Case:** $y^*=y_{fin}^0$.

Observe, from Lemma 3.9, that, if $y^*=y_{fin}^0$, then $L^*>py_{fin}^0$. This leads to $\frac{\partial J(L,y)}{\partial y}=\alpha p(1+\psi)\left[1-\Phi\left(y\right)\right]-(1-\alpha)w$ and, at $y=y^0$, $\frac{\partial J(L,y)}{\partial y}=\alpha p\psi\left[1-\Phi\left(y^0\right)\right]$. Recall that $\Phi(.)\in[0,1]$. Therefore, $\alpha p\psi\left[1-\Phi\left(y^0\right)\right]\geq 0$ and, hence, at $y=y^0$, $\frac{\partial J(L,y)}{\partial y}\geq 0$. The concavity of $J$ and the definition of $y_{fin}^0$ imply that if $y^*=y_{fin}^0$, then $y_{fin}^0\geq y^0$.

**Case:** $y^*=y^\lambda$.

Observe, from Lemma 3.10, that, if $y^*=y^\lambda$, then $L^*\geq\overline{B}$ and $L^*\leq py^\lambda$. This leads to $\frac{\partial J(L,y)}{\partial y}=\alpha p\left[1-\Phi\left(y\right)\right]-(1-\alpha)w+\lambda\left\{[(1-\alpha)-r^S]-\alpha\psi\Phi\left(\frac{\overline{B}+\lambda(y-x)}{p}\right)\right\}$ and, at $y=y^0$, $\frac{\partial J(L,y)}{\partial y}=\lambda\left\{[(1-\alpha)-r^S]-\alpha\psi\Phi\left(\frac{\overline{B}+\lambda(y^0-x)}{p}\right)\right\}=\alpha\psi\lambda\left[\Phi\left(\frac{L_{bs}^0}{p}\right)-\Phi\left(\frac{\overline{B}+\lambda(y^0-x)}{p}\right)\right]$. Recall, that $\Phi$ is a non-decreasing function and, from Proposition 3.5, that a necessary condition for $L^*=\overline{B}+\lambda(y^\lambda-x)$ is for $L_{bs}^0\geq\overline{B}+\lambda(y^0-x)$. Therefore, $\alpha\psi\lambda\left[\Phi\left(\frac{L_{bs}^0}{p}\right)-\Phi\left(\frac{\overline{B}+\lambda(y^0-x)}{p}\right)\right]\geq 0$ and, hence, at $y=y^0$, $\frac{\partial J(L,y)}{\partial y}\geq 0$. The concavity of $J$ and the definition of $y^\lambda$ imply that if $y^*=y^\lambda$, then $y^\lambda\geq y^0$.

**Case:** $y^*=y_{fin}^\lambda$.

Observe, from Lemma 3.10, that, if $y^*=y_{fin}^\lambda$, then $L^*\geq\overline{B}$ and $L^*>py_{fin}^\lambda$. This leads to $\frac{\partial J(L,y)}{\partial y} = \alpha p(1+\psi)\left[1-\Phi\left(y\right)\right] - (1-\alpha)w + \lambda[(1-\alpha) - r^S - \alpha\psi]$ and, at $y = y^0$, $\frac{\partial J(L,y)}{\partial y} = \alpha p\psi\left[1-\Phi\left(y^0\right)\right] + \lambda[(1-\alpha) - r^S - \alpha\psi]$. Observe, from Lemma 3.3, that $L^* \geq \overline{B}$ and $L^* > py^{\lambda}_{fin}$ implies $(1-\alpha) \leq \alpha\psi + r^S$. Furthermore, recall that $\Phi(.) \in [0,1]$. Therefore, at $y = y^0$, $\frac{\partial J(L,y)}{\partial y} \geq 0$. The concavity of $J$ and the definition of $y^{\lambda}_{fin}$ imply that if $y^* = y^{\lambda}_{fin}$, then $y^{\lambda}_{fin} \geq y^0$.

**Case:** $y^* = x$.

Recall, from (3.69), that $y \geq x$. Therefore, $y^* = x$ only if the desired order-up-to level is less than or equal to $x$. Recall, from the previous cases analyzed, that $y^p_b$, $y^p_{bs}$, $y^0_{fin}$, $y^{\lambda}$ and $y^{\lambda}_{fin}$ are greater than or equal to $y^0$. Therefore, $y^* = x$ implies that either $\max(y^p_b, x) = x \geq y^0$, $\max(y^p_{bs}, x) = x \geq y^0$, $\max(y^0_{fin}, x) = x \geq y^0$, $\max(y^{\lambda}, x) = x \geq y^0$, $\max(y^{\lambda}_{fin}, x) = x \geq y^0$ or $\max(y^0, x) = x \geq y^0$. This is why, if $y^* = x$, then $x \geq y^0$.

We have just shown that, for any optimal amount to borrow, the desired order-up-to level is always greater than or equal to $y^0$. Therefore, if the amount to borrow is constrained, then the optimal order-up-to level is greater than or equal to $y^0$. Furthermore, if the optimal order-up-to level is at the intersection of two constraints, then it must be that the constrained order-up-to level is greater than or equal to $y^0$ otherwise the optimal order-up-to level would have been on none of the boundaries.

■

According to Proposition 3.7, borrowing from outside lenders at a cheaper rate than from the shareholders leads to savings for the manufacturer that might be used to increase the order-up-to level in order to generate a higher revenue if demand ends up being high. Therefore, the order-up-to level in this problem is different than the order up-to level of a traditional news-vendor problem with no financing considerations.

### 3.4.2 Two-period problems.

Observe that the feasible region in Figure 3.1, Figure 3.2, and Figure 3.3 is not always convex. Therefore, although $G$, $F$, $g_2$ and $f_2$ are concave functions, $g_1$ and $f_1$ need not be concave functions. Therefore, the two-period problem is difficult to analyze. I perform numerical analysis to gather insights and compare, for different set of economic parameters, the solutions for the two-period Dynamic signaling problem with the solutions for the two-period Dynamic no-signaling problem. Before presenting numerical examples, I introduce Lemma 3.11 and Lemma 3.12 to help us explain the results obtained in the numerical examples.

**Lemma 3.11.** *For the no-signaling problem, denote $y_t^*$ as the optimal order-up-to level and $L_t^*$ as the optimal amount to borrow for any period, $t$. Suppose the starting inventory is $x$. If the optimal solution at each period is unconstrained, then $y_1^* \leq y_2^*$ and $L_1^* = L_2^*$ .*

*Proof.* Recall, from Lemma 3.1, that, for any period $t$, $g_t(.)$ is a non-increasing function. Because

$$J_2(L, y) = G(L, y), \tag{3.131}$$

$$J_1(L, y) = G(L, y) + \alpha E\left[g_2\left(y - D\right)\right], \tag{3.132}$$

$\alpha > 0$ and $E\left[g_2\left(y - D\right)\right]$ is non-increasing in $y$, it must be, for the same starting inventory, $x$, that $y_1^* \leq y_2^*$ when the solution is unconstrained. Furthermore, because $E\left[g_2\left(y - D\right)\right]$ is independent of $L$, it must be, for the same starting inventory, $x$, that $L_1^* = L_2^*$ when the solution is unconstrained. ■

**Lemma 3.12.** *Denote by $L_{tNoSig}^*$ the optimal amount to borrow in the no-signaling problem for any period, $t$, and by $L_{tSig}^*$ the optimal amount to borrow in the signaling* *problem for any period, $t$. Then, for the same bank-loan limit, $\overline{B}$, and starting inventory, $x$, $L^*_{1NoSig} \leq L^*_{1Sig}$.*

*Proof.* Recall, from (3.45) and (3.48), that, for the no-signaling problem,

$$J_1(L, M, y) = K(L, M, y) + \alpha E\left[g_2\left(y - D\right)\right], \tag{3.133}$$

and, for the signaling problem,

$$J_1(L, M, y) = K(L, M, y) + \alpha E\Big[f_2\Big(L + \min\left[p\min(y, D) + M, 0\right], y - D\Big)\Big]. \tag{3.134}$$

Moreover, recall, from Lemma 3.1, that, for any period $t$, $f_t(.,.)$ is non-increasing in its first argument. Because $\alpha > 0$ and $E\left[f_2\Big(L + \min\left[p\min(y, D) + M, 0\right], y - D\Big)\right]$ is non-decreasing in $L$ and $M$, it must be, for the same bank-loan limit, $\overline{B}$ and starting inventory, $x$, that, at optimality, $L + M$ in the no-signaling problem is less than or equal to $L + M$ in the signaling problem. Therefore, for the same bank-loan limit, $\overline{B}$, and starting inventory, $x$, $L^*_{1NoSig} \leq L^*_{1Sig}$. ■

### 3.5 Numerical analysis.

Throughout the analysis, demand, $D_t$, for any period $t = 1, 2, ...T$ is i.i.d. with p.m.f. $\phi_m$. Specifically, for any mean demand, $\mu$, and its distance to the upper (lower) bound of possible demands, $\delta$, demand, $D_t$, for any period $t = 1, 2, ...T$ is in the integer set $[\mu-\delta, \mu-\delta+1, \mu-\delta+2, ..., \mu, \mu+1, \mu+2, ...\mu+\delta]$. Define $D_{low} = \mu-\delta$, $D_{high} = \mu + \delta$ and $\phi_m$ in the following way:

If $D_{low} > 0$, then, for any demand, $d \in [D_{low}, \mu]$,

$$\phi_m(d) = \frac{d}{Range}, \tag{3.135}$$

and, for any demand, $d \in [\mu, D_{high}]$,

$$\phi_m(d) = \frac{(D_{high} - d) + D_{low}}{Range}, \tag{3.136}$$

where

$$Range = 2 \times \sum_{D_{low}}^{\mu-1} + \mu. \tag{3.137}$$

If $D_{low} = 0$, then, for any demand, $d \in [0, \mu]$,

$$\phi_m(d) = \frac{d+1}{Range}, \tag{3.138}$$

and, for any demand, $d \in [\mu, D_{high}]$,

$$\phi_m(d) = \frac{(D_{high} - d) + 1}{Range}, \tag{3.139}$$

where

$$Range = 2 \times \sum_{1}^{\mu} + \mu + 1. \tag{3.140}$$

This demand distribution is

$$\phi_m(D_{low}) \leq \phi_m(D_{low} + 1) \leq ... \leq \phi_m(\mu), \tag{3.141}$$

$$\phi_m(\mu) \geq \phi_m(\mu + 1) \geq ... \geq \phi_m(D_{high}), \tag{3.142}$$

$$\phi_m(\mu - i) = \phi_m(\mu + i), \;\; i = 1, 2, .., \delta. \tag{3.143}$$

Figure 3.4 is an example with $\mu = 5$ and $\delta = 4$ that illustrates the properties of the demand distribution chosen.

Throughout the analysis, the mean of the demand distribution is $\mu = 100$, the distance from the mean of the demand distribution to the upper (lower) bound of the possible demands is $\delta = 60$, the unit retail price is $p = 5$, the total cost of a good is $w = 3.5$, the unit wholesale price is $\tau = 3$, the discount factor used by the shareholders is $\alpha = 0.8$, the maximum amount of capital the supplier is willing and/or capable to lend to the manufacturer is $\overline{S} = 800$, the order-up-to level of the traditional newsvendor model is $y^0 = 133$, and the starting inventory level is $x = 0$.

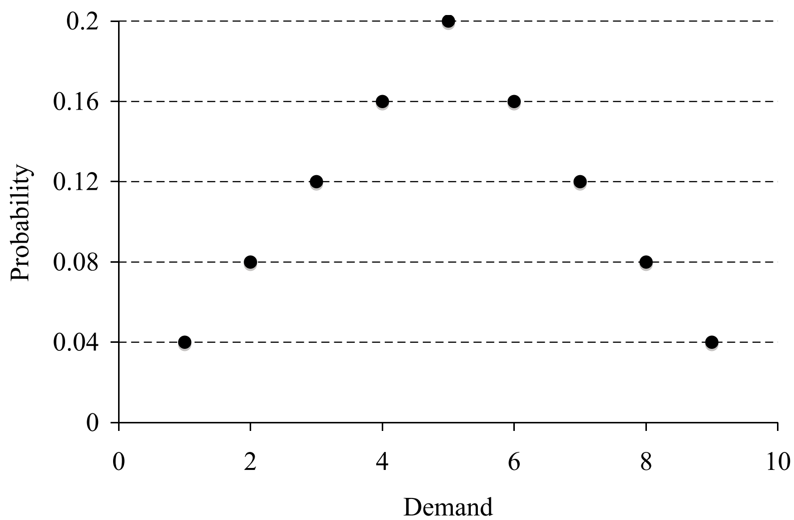


Figure 3.4: Example of triangular-uniform distribution with $\mu = 5$ and $\delta = 4$.

I will give the values for the interest rate on bank loans, $r^B$, the interest rate on the supplier loans, $r^S$, the default penalty for every unit of currency not repaid, $\psi$, the credit offered by the supplier for every unit of good lent to the manufacturer, $\lambda$, and the bank loan limit, $\overline{B}$, in the numerical examples that I present.

In all examples, I consider the situation where it is costly for the manufacturer to borrow and then default on its loan obligations (i.e., I consider when $(1-\alpha) < r^B + \alpha\psi$) as, otherwise, we would be focusing on the trivial case where the manufacturer borrows as much as possible from the bank and, hence, sends a non-credible signal about its expected repayment capability.

### 3.5.1 General results.

As expected (see Proposition 3.6 and Lemma 3.12), the total amount to borrow in the signaling problem is greater than or equal to the total amount to borrow in the no-signaling problem. In the no-signaling problem, it is suboptimal to borrow from the supplier when cheaper sources of financing are available. In both problems, the amount to borrow from the supplier decreases as the interest rate on the supplier loans increases. Furthermore, in both problems, the shareholders' value

- is increasing in the bank loan limit and the credit offered by the supplier for every unit of good lent to the manufacturer, and

- is decreasing in the interest rate on bank loans, the interest rate on the supplier loans, and the default penalty for every unit of currency not repaid.

When the bank loan limit in the current period is low, the shareholders' are better-off in the signaling problem compared to in the no-signaling problem because, in the signaling problem, the manufacturer can signal its credibility to the bank to obtain a higher line of credit in the next period. Surprisingly, when the bank loan limit in the current period is high, the shareholders are worse-off in the signaling problem compared to in the no-signaling problem; this is because, in the signaling problem,

- the benefit, at $t = 1$, to increase next period's bank loan limit is non-increasing in the current bank loan limit,

- a low realized demand at $t = 1$ reduces the next period's bank loan limit,

- the probability, at $t = 1$, that the next period's bank loan limit is higher than the current period's bank loan limit is non-increasing in the current bank loan limit.

In Section 3.5.2 to Section 3.5.4, I present results obtained with various sets of economic parameters to illustrate the results presented in Section 3.5.1. Then, I discuss additional findings. In Section 3.5.5, with different sets of parameters compared to the ones used in Section 3.5.1 to Section 3.5.4, I present additional results and argue that the properties of the results obtained in Section 3.5.1 to Section 3.5.4 hold for all sets of parameter values that satisfy $(1-\alpha) < r^B + \alpha\psi$.

### 3.5.2 $r^B$ is low and $\psi$ is low relative to the value of other parameters.

Table 3.1 to Table 3.3 contain results for the case where the manufacturer finds it very cheap to borrow from the bank and the default penalty for every unit of currency not repaid is low relative to the value of the other parameters. In Table 3.1 and Table 3.2 it is cheaper to borrow from the supplier instead of requesting money from the shareholders (i.e., $r^S < (1-\alpha)$). In Table 3.3, it is cheaper to request money from the shareholders instead of borrowing from the supplier (i.e., $(1-\alpha) < r^S$).

| | **Signaling** | | | | **No-signaling** | | | |
|---|---|---|---|---|---|---|---|---|
| **Bank Limit** | $L^*$ | $M^*$ | $y^*$ | $f$ | $L^*$ | $y^*$ | $g$ | **Row** |
| 100 | 380 | −380 | 140 | 621.250 | 365 | 133 | 602.725 | 1 |
| 200 | 470 | −470 | 135 | 633.606 | 465 | 133 | 621.810 | 2 |
| 300 | 565 | −565 | 133 | 641.765 | 555 | 132 | 635.178 | 3 |
| 400 | 580 | −580 | 132 | 646.873 | 555 | 133 | 645.020 | 4 |
| 500 | 580 | −580 | 132 | 651.873 | 555 | 133 | 654.082 | 5 |
| 600 | 600 | −600 | 132 | 656.843 | 600 | 133 | 662.512 | 6 |
| 700 | 675 | −675 | 135 | 658.950 | 675 | 135 | 665.780 | 7 |

Table 3.1: Borrowed amount $L$, target-capital level $M$, order-up-to level $y$, objective values $f$, $g$ in signaling and no-signaling problems as functions of the bank limit; $\mu = 100$, $\delta = 60$, $p = 5$, $w = 3.5$, $\tau = 3$, $y^0 = 133$, $r^B = 5\%$, $r^S = 10\%$, $\psi = 0.2$, $\lambda = 2$, $\overline{S} = 800$ and the row column is added to conveniently refer to specific rows.

As expected, the desired amount to borrow from the supplier is decreasing in the interest rate on the supplier loans (see Table 3.1 to Table 3.3). Surprisingly, the desired amount to borrow from the bank in the signaling problem is increasing in

| | Signaling | | | | No-signaling | | | |
|---|---|---|---|---|---|---|---|---|
| **Bank Limit** | $L^*$ | $M^*$ | $y^*$ | $f$ | $L^*$ | $y^*$ | $g$ | **Row** |
| 100 | 375 | $-375$ | 138 | 603.097 | 365 | 133 | 581.225 | 1 |
| 200 | 440 | $-440$ | 133 | 617.213 | 415 | 132 | 601.054 | 2 |
| 300 | 440 | $-440$ | 133 | 627.213 | 415 | 133 | 619.356 | 3 |
| 400 | 440 | $-440$ | 133 | 637.213 | 415 | 133 | 637.361 | 4 |
| 500 | 500 | $-500$ | 133 | 648.287 | 500 | 133 | 653.097 | 5 |
| 600 | 600 | $-600$ | 133 | 655.195 | 600 | 133 | 662.512 | 6 |
| 700 | 680 | $-675$ | 135 | 657.343 | 675 | 135 | 665.780 | 7 |

Table 3.2: Borrowed amount $L$, target-capital level $M$, order-up-to level $y$, objective values $f$, $g$ in signaling and no-signaling problems as functions of the bank limit; $\mu = 100$, $\delta = 60$, $p = 5$, $w = 3.5$, $\tau = 3$, $y^0 = 133$, $r^B = 5\%$, $r^S = 15\%$, $\psi = 0.2$, $\lambda = 2$, $\overline{S} = 800$ and the row column is added to conveniently refer to specific rows.

| | Signaling | | | | No-signaling | | | |
|---|---|---|---|---|---|---|---|---|
| **Bank Limit** | $L^*$ | $M^*$ | $y^*$ | $f$ | $L^*$ | $y^*$ | $g$ | **Row** |
| 100 | 125 | $-125$ | 133 | 565.477 | 100 | 133 | 563.727 | 1 |
| 200 | 225 | $-225$ | 133 | 592.328 | 200 | 133 | 590.727 | 2 |
| 300 | 325 | $-325$ | 133 | 615.826 | 300 | 133 | 616.067 | 3 |
| 400 | 400 | $-400$ | 133 | 634.820 | 400 | 133 | 637.312 | 4 |
| 500 | 500 | $-500$ | 133 | 647.922 | 500 | 133 | 653.097 | 5 |
| 600 | 600 | $-600$ | 133 | 654.830 | 600 | 133 | 662.512 | 6 |
| 700 | 695 | $-675$ | 135 | 657.050 | 675 | 135 | 665.780 | 7 |

Table 3.3: Borrowed amount $L$, target-capital level $M$, order-up-to level $y$, objective values $f$, $g$ in signaling and no-signaling problems as functions of the bank limit; $\mu = 100$, $\delta = 60$, $p = 5$, $w = 3.5$, $\tau = 3$, $y^0 = 133$, $r^B = 5\%$, $r^S = 25\%$, $\psi = 0.2$, $\lambda = 2$, $\overline{S} = 800$ and the row column is added to conveniently refer to specific rows.

the interest rate on the supplier loans (i.e., see row 7 in Table 3.1 to Table 3.3); this is because the benefit to over-borrow from the bank and bet on a high revenue to signal a high credibility increases as the interest rate on the supplier loans increases (see Table 3.1 to Table 3.3).

Unsurprisingly, the order-up-to level is higher than the order-up-to level in the traditional newsvendor problem when the manufacturer desires to borrow a high amount from the supplier and the benefit to relax the financing constraint $L \leq \overline{B} + \lambda(y - x)$ is bigger than the cost to produce additional goods, (i.e., see rows 1 and 2 in Table 3.1 and row 1 in Table 3.2). Surprisingly, as the amount to borrow from the supplier decreases, the order-up-to level may be lower than the order-up-to level in the traditional newsvendor problem (e.g., see rows 4, 5 and 6 in Table 3.1 and row 2 in Table 3.2). This is because, by under-ordering, the manufacturer reduces the probability and the consequence to not be able to borrow the desired amount in the next period if the demand for goods in the current period turns out to be low; a low demand leads to a high starting inventory in the next period and, hence, to a low supplier loan limit and, in the signaling problem, to a low bank limit in the next period.

Recall that it is cheaper to borrow from the bank instead of borrowing from the shareholders (i.e., $r^B < (1 - \alpha)$). This is why, when the benefit to borrow from the bank is high and the manufacturer is able to borrow a high amount from the bank, the order-up-to level might be greater than the order-up-to level in the traditional newsvendor model (see row 7 in Table 3.1 to Table 3.3).

### 3.5.3 $r^B$ is high and $\psi$ is low relative to the value of other parameters.

Table 3.4 to Table 3.6 contain results for the case where the manufacturer finds it a little expensive to borrow from the bank and the default penalty for every unit of currency not repaid is low relative to the value of other parameters.

| | **Signaling** | | | | **No-signaling** | | | |
|---|---|---|---|---|---|---|---|---|
| **Bank Limit** | $L^*$ | $M^*$ | $y^*$ | $f$ | $L^*$ | $y^*$ | $g$ | **Row** |
| 100 | 370 | −370 | 135 | 584.003 | 365 | 133 | 572.225 | 1 |
| 200 | 440 | −440 | 133 | 591.164 | 415 | 132 | 583.054 | 2 |
| 300 | 440 | −440 | 133 | 596.164 | 415 | 133 | 592.356 | 3 |
| 400 | 440 | −440 | 133 | 601.164 | 425 | 133 | 601.361 | 4 |
| 500 | 500 | −500 | 133 | 605.874 | 500 | 133 | 608.097 | 5 |
| 600 | 555 | −555 | 133 | 606.622 | 555 | 133 | 609.082 | 6 |

Table 3.4: Borrowed amount $L$, target-capital level $M$, order-up-to level $y$, objective values $f$, $g$ in signaling and no-signaling problems as functions of the bank limit; $\mu = 100$, $\delta = 60$, $p = 5$, $w = 3.5$, $\tau = 3$, $y^0 = 133$, $r^B = 10\%$, $r^S = 15\%$, $\psi = 0.2$, $\lambda = 2$, $\overline{S} = 800$ and the row column is added to conveniently refer to specific rows.

The results in Section 3.5.3 have the same properties as the results in Section 3.5.2. The main difference is that the benefit of the manufacturer to signal its credibility is smaller in Section 3.5.3 than in Section 3.5.2 because borrowing from the bank in Section 3.5.3 is more expensive than in Section 3.5.2.

### 3.5.4 Effect of $\lambda$ on the decisions of the manufacturer.

I compare the decisions of the manufacturer for different pairs of $\lambda$ and $\overline{B}$. I use the same parameter values as in Section 3.5.2 but the properties of my results apply to all sets of parameters. The results obtained are found in Table 3.7.

As expected, the desired amount to borrow from the supplier is non-decreasing

| | Signaling | | | | No-signaling | | | |
|---|---|---|---|---|---|---|---|---|
| **Bank Limit** | $L^*$ | $M^*$ | $y^*$ | $f$ | $L^*$ | $y^*$ | $g$ | **Row** |
| 100 | 125 | $-125$ | 133 | 555.477 | 100 | 133 | 554.727 | 1 |
| 200 | 225 | $-225$ | 133 | 573.342 | 200 | 133 | 572.727 | 2 |
| 300 | 300 | $-300$ | 133 | 588.663 | 300 | 133 | 589.067 | 3 |
| 400 | 400 | $-400$ | 133 | 599.850 | 400 | 133 | 601.312 | 4 |
| 500 | 500 | $-500$ | 133 | 605.509 | 500 | 133 | 608.097 | 5 |
| 600 | 555 | $-555$ | 133 | 606.257 | 555 | 133 | 609.082 | 6 |

Table 3.5: Borrowed amount $L$, target-capital level $M$, order-up-to level $y$, objective values $f$, $g$ in signaling and no-signaling problems as functions of the bank limit; $\mu = 100$, $\delta = 60$, $p = 5$, $w = 3.5$, $\tau = 3$, $y^0 = 133$, $r^B = 10\%$, $r^S = 25\%$, $\psi = 0.2$, $\lambda = 2$, $\overline{S} = 800$ and the row column is added to conveniently refer to specific rows.

| | Signaling | | | | No-signaling | | | |
|---|---|---|---|---|---|---|---|---|
| **Bank Limit** | $L^*$ | $M^*$ | $y^*$ | $f$ | $L^*$ | $y^*$ | $g$ | **Row** |
| 100 | 100 | $-100$ | 133 | 545.727 | 100 | 133 | 545.727 | 1 |
| 200 | 200 | $-200$ | 133 | 554.727 | 200 | 133 | 554.727 | 2 |
| 300 | 300 | $-300$ | 133 | 561.894 | 300 | 133 | 561.894 | 3 |
| 400 | 400 | $-400$ | 133 | 564.880 | 400 | 133 | 564.880 | 4 |
| 500 | 415 | $-415$ | 133 | 564.923 | 415 | 133 | 564.923 | 5 |

Table 3.6: Borrowed amount $L$, target-capital level $M$, order-up-to level $y$, objective values $f$, $g$ in signaling and no-signaling problems as functions of the bank limit; $\mu = 100$, $\delta = 60$, $p = 5$, $w = 3.5$, $\tau = 3$, $y^0 = 133$, $r^B = 15\%$, $r^S = 25\%$, $\psi = 0.2$, $\lambda = 2$, $\overline{S} = 800$ and the row column is added to conveniently refer to specific rows.

in $\lambda$. Furthermore, because the amount to borrow is low and the unconstrained target-capital position is very negative, the constraint $L + M \geq 0$ makes it optimal to maintain a zero target-capital position before the realization of the demand.

| | | Signaling | | | | No-signaling | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Bank Limit** | $\lambda$ | $L^*$ | $M^*$ | $y^*$ | $f$ | $L^*$ | $y^*$ | $g$ | **Row** |
| 50 | 1 | 185 | −185 | 135 | 587.161 | 185 | 135 | 571.580 | 1 |
| | 2 | 330 | −330 | 140 | 613.621 | 320 | 135 | 591.419 | 2 |
| | 3 | 460 | −460 | 137 | 625.721 | 445 | 132 | 605.181 | 3 |
| 100 | 1 | 235 | −235 | 135 | 599.587 | 235 | 135 | 584.932 | 4 |
| | 2 | 380 | −380 | 140 | 621.250 | 365 | 133 | 602.725 | 5 |
| | 3 | 505 | −505 | 135 | 630.617 | 495 | 132 | 613.370 | 6 |
| 150 | 1 | 285 | −285 | 135 | 610.801 | 285 | 135 | 597.664 | 7 |
| | 2 | 420 | −420 | 135 | 627.879 | 415 | 133 | 612.922 | 8 |
| | 3 | 555 | −555 | 135 | 634.432 | 540 | 130 | 620.141 | 9 |
| 200 | 1 | 335 | −335 | 135 | 620.626 | 335 | 135 | 609.500 | 10 |
| | 2 | 470 | −470 | 135 | 633.606 | 465 | 133 | 621.810 | 11 |
| | 3 | 585 | −585 | 132 | 637.284 | 555 | 131 | 625.780 | 12 |

Table 3.7: Borrowed amount $L$, target-capital level $M$, order-up-to level $y$, objective values $f$, $g$ in signaling and no-signaling problems as functions of the bank limit; credit limit $\lambda$, $\mu = 100$, $\delta = 60$, $p = 5$, $w = 3.5$, $\tau = 3$, $y^0 = 133$, $r^B = 5\%$, $r^S = 10\%$, $\psi = 0.2$, $\overline{S} = 800$ and the row column is added to conveniently refer to rows; the row column is added to conveniently refer to specific rows.

Surprisingly, the order-up-to level is non-monotone in $\lambda$. Recall that the desired amount to borrow from the supplier decreases as the bank loan limit increases. This is why, for a given order-up-to level, the benefit to relax the financing constraint decreases as the bank loan limit increases and it increases as $\lambda$ increases. Because it is expensive to produce goods, the manufacturer sets a high order-up-to level only when the expected benefit to obtain additional financing is high relative to the additional production costs (e.g., see rows 1 to 11 in the Signaling columns of Table 3.7 and see rows 1, 2, 4, 7, and 10 in the No-signaling columns of Table 3.7). Similarly, the manufacturer never sets a high order-up-to level when the expected benefit to obtain additional financing is low relative to the additional production costs (e.g., see rows 5, 8, and 11 in the No-signaling columns of Table 3.7). A similar discussion as the one in Section 3.5.2 provides the reason for order-up-to levels that are lower than the order-up-to level in the traditional newsvendor problem (e.g., see row 12 in the Signaling columns of Table 3.7 and rows 3, 6, 9 and 12 in the No-signaling columns of Table 3.7).

### 3.5.5 Discussion.

I considered various sets of examples that satisfy $(1-\alpha) < r^B + \alpha\psi$. But, I did not present all the examples analyzed because the properties of the results obtained in Section 3.5.1 to Section 3.5.4 hold for all set of economic parameters that satisfy $(1-\alpha) < r^B + \alpha\psi$. The main difference between the results of the examples studied is the change in the magnitude of the behaviors observed (see Table 3.8 to Table 3.13 in Section 3.7 for additional results)

## 3.6 Conclusions, limitations and extensions.

I find that the benefit to signal a high repayment capability to banks is high when it is inexpensive to borrow from banks compared to using other financing sources. But, when the benefit to signal their repayment capability is high, manufacturers might borrow from suppliers instead of requesting capital from shareholders even if borrowing from suppliers is very expensive.

Although the decisions of banks and suppliers were not considered, I study the effect of manufacturers' decisions on the interest rates at which manufacturers borrow and the results and insights gathered help us grasp the effect of banks using loan repayments of manufacturers as an indication of the credibility of manufacturers.

My problem is a more general problem than the traditional newsvendor problem. I consider the possibility of both internal and external financing and account for the relationship between operational and financing decisions. I find, when manufacturers relax financing constraints and/or want to signal their credibility to banks, that manufacturers might order up to a level greater than the order-up-to level in the traditional news-vendor problem. However, when the line of credit offered by lenders is low and the opportunity cost of capital of manufacturers is high relative to other sources of financing, manufacturers might order up to a level less than the order-up-to level in the traditional news-vendor problem in order to allocate their precious capital to projects with higher expected returns.

My analysis contributes to the finance literature because I explicitly model operational decisions in a dynamic framework, and, simultaneously focus on the use of trade credit to update the bank loan limit instead of using trade credit to adjust the interest rate on the bank loan terms. My analysis contributes to the operations literature because I consider the effect of trade credit on the bank loan limit and, simultaneously focus on the effect of indirect financial costs on operational decisions. Future studies should simultaneously include the effect of manufacturers' decisions on the interest rates at which manufacturers borrow, incorporate multiple decision-makers to the framework used in this chapter, and study the choice of the loan terms offered by the lenders. Also, although it would be challenging to study a problem in which the credit signal is a function of loan repayments observed over multiple periods, adding such feature to my model would be an interesting problem to analyze.

## 3.7 Appendix.

| | **Signaling** | | | | **No-signaling** | | | |
|---|---|---|---|---|---|---|---|---|
| **Bank Limit** | $L^*$ | $M^*$ | $y^*$ | $f$ | $L^*$ | $y^*$ | $g$ | **Row** |
| 100 | 370 | −370 | 135 | 615.953 | 365 | 133 | 600.771 | 1 |
| 200 | 470 | −470 | 135 | 625.777 | 460 | 131 | 616.054 | 2 |
| 300 | 490 | −490 | 133 | 631.056 | 465 | 133 | 626.311 | 3 |
| 400 | 490 | −490 | 133 | 636.056 | 465 | 133 | 635.430 | 4 |
| 500 | 500 | −500 | 133 | 641.019 | 500 | 133 | 643.782 | 5 |
| 600 | 555 | −555 | 133 | 642.140 | 555 | 133 | 645.140 | 6 |

Table 3.8: Borrowed amount $L$, target-capital level $M$, order-up-to level $y$, objective values $f$, $g$ in signaling and no-signaling problems as functions of the bank limit; $\mu = 100$, $\delta = 60$, $p = 5$, $w = 3.5$, $\tau = 3$, $y^0 = 133$, $r^B = 5\%$, $r^S = 10\%$, $\psi = 0.3$, $\lambda = 2$, $\overline{S} = 800$ and the row column is added to conveniently refer to specific rows.

| | **Signaling** | | | | **No-signaling** | | | |
|---|---|---|---|---|---|---|---|---|
| **Bank Limit** | $L^*$ | $M^*$ | $y^*$ | $f$ | $L^*$ | $y^*$ | $g$ | **Row** |
| 100 | 370 | −370 | 135 | 600.333 | 355 | 132 | 579.408 | 1 |
| 200 | 380 | −380 | 133 | 610.893 | 355 | 133 | 598.260 | 2 |
| 300 | 300 | −300 | 133 | 620.893 | 355 | 133 | 616.330 | 3 |
| 400 | 425 | −425 | 133 | 632.034 | 400 | 133 | 633.604 | 4 |
| 500 | 500 | −500 | 133 | 640.099 | 500 | 133 | 643.782 | 5 |
| 600 | 555 | −555 | 133 | 641.220 | 555 | 133 | 645.260 | 6 |

Table 3.9: Borrowed amount $L$, target-capital level $M$, order-up-to level $y$, objective values $f$, $g$ in signaling and no-signaling problems as functions of the bank limit; $\mu = 100$, $\delta = 60$, $p = 5$, $w = 3.5$, $\tau = 3$, $y^0 = 133$, $r^B = 5\%$, $r^S = 15\%$, $\psi = 0.3$, $\lambda = 2$, $\overline{S} = 800$ and the row column is added to conveniently refer to specific rows.

| | Signaling | | | | No-signaling | | | |
|---|---|---|---|---|---|---|---|---|
| **Bank Limit** | $L^*$ | $M^*$ | $y^*$ | $f$ | $L^*$ | $y^*$ | $g$ | **Row** |
| 100 | 125 | −125 | 133 | 565.477 | 100 | 133 | 563.727 | 1 |
| 200 | 225 | −225 | 133 | 592.275 | 200 | 133 | 590.727 | 2 |
| 300 | 310 | −310 | 133 | 614.656 | 300 | 133 | 615.237 | 3 |
| 400 | 400 | −400 | 133 | 631.412 | 400 | 133 | 633.604 | 4 |
| 500 | 500 | −500 | 133 | 639.901 | 500 | 133 | 643.782 | 5 |
| 600 | 555 | −555 | 133 | 641.022 | 555 | 133 | 645.260 | 6 |

Table 3.10: Borrowed amount $L$, target-capital level $M$, order-up-to level $y$, objective values $f$, $g$ in signaling and no-signaling problems as functions of the bank limit; $\mu = 100$, $\delta = 60$, $p = 5$, $w = 3.5$, $\tau = 3$, $y^0 = 133$, $r^B = 5\%$, $r^S = 25\%$, $\psi = 0.3$, $\lambda = 2$, $\overline{S} = 800$ and the row column is added to conveniently refer to specific rows.

| | Signaling | | | | No-signaling | | | |
|---|---|---|---|---|---|---|---|---|
| **Bank Limit** | $L^*$ | $M^*$ | $y^*$ | $f$ | $L^*$ | $y^*$ | $g$ | **Row** |
| 100 | 370 | −370 | 135 | 581.252 | 355 | 132 | 570.408 | 1 |
| 200 | 380 | −380 | 133 | 586.508 | 355 | 133 | 580.260 | 2 |
| 300 | 380 | −380 | 133 | 591.508 | 355 | 133 | 589.330 | 3 |
| 400 | 400 | −400 | 133 | 596.640 | 400 | 133 | 597.604 | 4 |
| 500 | 465 | −465 | 133 | 598.198 | 465 | 133 | 599.430 | 5 |

Table 3.11: Borrowed amount $L$, target-capital level $M$, order-up-to level $y$, objective values $f$, $g$ in signaling and no-signaling problems as functions of the bank limit; $\mu = 100$, $\delta = 60$, $p = 5$, $w = 3.5$, $\tau = 3$, $y^0 = 133$, $r^B = 10\%$, $r^S = 15\%$, $\psi = 0.3$, $\lambda = 2$, $\overline{S} = 800$ and the row column is added to conveniently refer to specific rows.

| | Signaling | | | | No-signaling | | | |
|---|---|---|---|---|---|---|---|---|
| **Bank Limit** | $L^*$ | $M^*$ | $y^*$ | $f$ | $L^*$ | $y^*$ | $g$ | **Row** |
| 100 | 125 | $-125$ | 133 | 565.477 | 100 | 133 | 554.727 | 1 |
| 200 | 225 | $-225$ | 133 | 573.290 | 200 | 133 | 572.727 | 2 |
| 300 | 300 | $-300$ | 133 | 587.862 | 300 | 133 | 586.237 | 3 |
| 400 | 400 | $-400$ | 133 | 596.441 | 400 | 133 | 597.604 | 4 |
| 500 | 465 | $-465$ | 133 | 598.00 | 465 | 133 | 599.430 | 5 |

Table 3.12: Borrowed amount $L$, target-capital level $M$, order-up-to level $y$, objective values $f$, $g$ in signaling and no-signaling problems as functions of the bank limit; $\mu = 100$, $\delta = 60$, $p = 5$, $w = 3.5$, $\tau = 3$, $y^0 = 133$, $r^B = 10\%$, $r^S = 25\%$, $\psi = 0.3$, $\lambda = 2$, $\overline{S} = 800$ and the row column is added to conveniently refer to specific rows.

| | Signaling | | | | No-signaling | | | |
|---|---|---|---|---|---|---|---|---|
| **Bank Limit** | $L^*$ | $M^*$ | $y^*$ | $f$ | $L^*$ | $y^*$ | $g$ | **Row** |
| 100 | 100 | $-100$ | 133 | 545.727 | 100 | 133 | 545.727 | 1 |
| 200 | 200 | $-200$ | 133 | 554.727 | 200 | 133 | 554.727 | 2 |
| 300 | 300 | $-300$ | 133 | 561.092 | 300 | 133 | 561.237 | 3 |
| 400 | 355 | $-355$ | 133 | 562.113 | 355 | 133 | 562.330 | 4 |

Table 3.13: Borrowed amount $L$, target-capital level $M$, order-up-to level $y$, objective values $f$, $g$ in signaling and no-signaling problems as functions of the bank limit; $\mu = 100$, $\delta = 60$, $p = 5$, $w = 3.5$, $\tau = 3$, $y^0 = 133$, $r^B = 15\%$, $r^S = 25\%$, $\psi = 0.3$, $\lambda = 2$, $\overline{S} = 800$ and the row column is added to conveniently refer to specific rows.

| | Signaling | | | | No-signaling | | | |
|---|---|---|---|---|---|---|---|---|
| **Bank Limit** | $L^*$ | $M^*$ | $y^*$ | $f$ | $L^*$ | $y^*$ | $g$ | **Row** |
| 100 | 100 | $-100$ | 133 | 545.727 | 100 | 133 | 545.727 | 1 |
| 200 | 200 | $-200$ | 133 | 554.727 | 200 | 132 | 554.727 | 2 |
| 300 | 300 | $-300$ | 133 | 562.695 | 300 | 133 | 562.897 | 3 |
| 400 | 400 | $-400$ | 133 | 568.288 | 400 | 133 | 569.019 | 4 |
| 500 | 500 | $-500$ | 133 | 571.118 | 500 | 133 | 572.412 | 5 |
| 600 | 555 | $-555$ | 133 | 571.492 | 555 | 133 | 572.905 | 6 |
| 700 | 555 | $-555$ | 133 | 571.492 | 555 | 133 | 572.905 | 7 |

Table 3.14: Borrowed amount $L$, target-capital level $M$, order-up-to level $y$, objective values $f$, $g$ in signaling and no-signaling problems as functions of the bank limit; $\mu = 100$, $\delta = 60$, $p = 5$, $w = 3.5$, $\tau = 3$, $y^0 = 133$, $r^B = 15\%$, $r^S = 25\%$, $\psi = 0.1$, $\lambda = 2$, $\overline{S} = 800$ and the row column is added to conveniently refer to specific rows.

## CHAPTER IV

# Signaling Manufacturers' Private Information Through Operational Decisions and Trade Credit

### 4.1 Introduction.

Companies in a broad range of industries and economies rely heavily on external sources to finance their operations. But, external financing could be expensive and/or difficult to obtain due to asymmetric information between lenders and borrowers, high cost of capital of lenders, high cost of lenders to monitor the credibility of potential borrowers, credit rationing by lenders, and expensive loan terms sometimes extended by lenders with power advantage over manufacturers.

Banks often have lower costs of capital and easier access to capital than suppliers. Therefore, banks can offer loans at cheaper rates than suppliers if they can assess how likely manufacturers are to repay. Unfortunately, asymmetric information that often exists between banks and manufacturers causes banks to offer unfavorable loan terms or even deny loans to manufacturers. However, suppliers, being in the same industry as manufacturers, can sometimes better estimate the distribution of the demand for manufacturers' goods than banks. This is why, even when suppliers offer credit terms that come at high costs, the relatively easy access to supplier financing enables manufacturers to signal their credit quality to banks, which facilitates access to bank loans.

In this chapter, I focus on cash-constrained manufacturers that rely on bank loans and trade credit[1] to finance their costs of production in expectation of the demand for final goods. The emphasis of my analysis is to answer the following questions: when should manufacturers use trade credit to finance their operations?; how does information asymmetry between banks and manufacturers affect the operational and financial decisions of manufacturers?; how does the availability of trade credit affect the operational and financial decisions of manufacturers?; who benefits from the availability of trade credit (manufacturers, end-customers, overall economy)?

To understand the effect of asymmetric information and trade credit financing on the supply chain, I analyze and compare results of three problems: a problem with symmetric information between banks and manufacturers; a problem with asymmetric information between banks and manufacturers in which trade credit is unavailable; a problem with asymmetric information between banks and manufacturers in which trade credit is available. Then, I determine the winners and losers when trade credit is available to manufacturers. Finally, I analyze the effect of the economic environment and the cost structure of each type of manufacturer on the benefit to have trade credit financing available in the supply-chain.

My analysis suggests that manufacturers can often signal their credit quality to banks through their ordering decisions. As long as their is a risk of default on the loans extended by the suppliers, the use of trade credit financing allows banks to update their own beliefs about manufacturers credibility because suppliers offer terms that signal manufacturers risk to banks. Also, I find that high-risk manufacturers suffer from trade credit because they obtain expensive loans which prevents them from ordering high quantities. However, low-risk manufacturers benefit from trade

[1]See Chapters I and II to obtain a more detailed explanation of trade credit financing.

credit because they obtain cheap loans and do not need to over-order and/or over-borrow to signal their credibility to banks.

### 4.2 Literature review.

Many implications of trade credit financing have been analyzed in the operations literature. For example, Haley and Higgins (1973), and Rachamandugu (1989) investigate trade credit's impact on the Economic Order Quantity model. Recently, Gupta and Wang (2009) show the order-up-to-inventory remains optimal for a discrete time combined inventory-financing model when trade credit is used. They prescribe an algorithm for computing the optimal stock level for a continuous time model.

Trade credit financing has also been analyzed in the finance literature. For example, Petersen and Rajan (1997) observe that suppliers appear to have an advantage over traditional financial institutions in lending to growing firms, especially if those firms' credit quality is suspect. According to Petersen and Rajan (1997), there are three main reasons why suppliers appear to have an advantage over traditional financial institutions in lending to growing firms: the suppliers are more capable than banks to repossess the goods from manufacturers in case of insolvency and sell them; the buyers are a possible source of future business for suppliers; and there is a low cost of obtaining information from product market transactions, and perhaps from other suppliers. Fisman (2001) discovers a correlation between the availability of trade credit financing and a firm's operational performance. Using a sample of African firms, Fisman (2001) finds that manufacturers with access to trade credit financing are less likely to experience stock-outs, and are more likely to have higher production-capacity utilization.

The previous studies provide interesting insights and arrive at important conclusions. However, when there is asymmetric information between banks and manufacturers, the previous studies do not simultaneously capture the effect of trade credit financing on the operational decisions of manufacturers and the ability of manufacturers to receive cheap loans from banks.

This study is not the first one to simultaneously analyze bank financing and trade credit financing when there is asymmetric information between lenders and borrowers. Cook (1999), using data from a survey of 352 firms in Russia in 1995, empirically concludes that trade credit can incorporate the private information held by suppliers about their customers in the lending relation. The author argues that trade credit financing sends a positive signal to financial institutions which, without that signal, might be reluctant to lend to buyers. Cook's empirical findings are consistent with Alphonse et al. (2006) who use US small business data (NSSBF 1998) to show that trade credit can signal firm's quality and thus facilitates access to bank debt. Biais and Gollier (1997) consider a static model in which a supplier has private information about the manufacturers while a bank does not. In Biais and Gollier (1997), manufacturers' revenues do not depend on the operational decisions made by manufacturers and the decision to finance projects is solely dictated by exogenous signals received from banks and suppliers. Furthermore, in Biais and Gollier (1997), credit rationing occurs when the bank cannot assess the quality of a firm with enough precision. As a result, many firms with positive net present value projects are denied credit from the bank. However, suppliers might find it profitable to extend credit to some of these firms.

Similar to Biais and Gollier (1997), I consider a model with banks, suppliers and manufacturers where suppliers have information advantages over banks. However, in my model, manufacturers determine their level of investment through their operational decisions, the banks' belief about manufacturers' type depends on exogenous signals but also on the financial and operational decisions made by manufacturers, the revenue generated by manufacturers depends on the investment decisions, and the interest rates offered by lenders determines which project the manufacturer will choose to implement.

This chapter not only captures the effect of trade credit and asymmetric information on the return of manufacturers' investments but it also captures the effect of asymmetric information along with trade credit on operational decisions of manufacturers which is a major contribution of this chapter. Contributions to the operations literature are made by considering the effect of asymmetric information and loan terms on manufacturers' operational policies. I also contribute to the finance literature because I consider and analyze the impact of operational policies on trade credit terms and bank loan terms when there is asymmetric information between banks and manufacturers.

## 4.3 Model and assumptions.

I solve a Bayesian game among three risk-neutral players: a cash-constrained manufacturer, a cash-constrained supplier, and a bank. There are two types of manufacturers in the economy ($\theta = 1, 2$) and the cost structure of each type is the same and known by every player. The only difference between each type of manufacturer is that, demand, $D_\theta$, for each type, $\theta = 1, 2$, of manufacturer is a random variable whose values are $D_\theta = H$ with probability $\pi_\theta^H$ and $D_\theta = L$ with probability $\pi_\theta^L$ where $\theta \in \{1, 2\}$, $E(D_1) \geq E(D_2)$, and $H > L$.

It is a common knowledge that a fraction $\alpha$ of the manufacturer population is of the type 1 and a fraction $1 - \alpha$ of the manufacturer population is of the type

2. The manufacturer and the supplier know the manufacturer's type but the bank is unable to determine, a priori, the type of the manufacturer that is requesting a loan. Therefore, the bank relies on trade credit terms and/or the manufacturer's operational decisions to determine the type of the manufacturer that is requesting a loan.

### 4.3.1 The problem of the manufacturer.

The objective of the manufacturer is to maximize its value at the end of the planning horizon.

At the beginning of the planning horizon, the manufacturer has no capital and its decisions are: the production quantity, $y$; trade credit amount, $L_{sm}$; and bank loan amount, $L_{bm}$. The manufacturer can order an unlimited number of goods but cannot sell to the supplier: $0 \leq y$. We assume that goods last one period, the supplier always delivers the number of goods requested by the manufacturer, and unsold goods have no scrap value. The manufacturer cannot lend money to the bank or the supplier but it can borrow as much as it desires from the bank: $0 \leq L_{bm}$; and the amount it can borrow from the supplier is proportionally bounded by the order quantity: $0 \leq L_{sm} \leq \lambda y$.

The manufacturer uses the loan received to cover its total procurement cost, $\tau y$, and its cost to manufacturer goods, $my$. The total manufacturing cost is $wy$ where $w = \tau + m$. Because the manufacturer has no initial capital, it borrows more than the total production cost: $L_{bm} + L_{sm} \geq wy$.

At the end of the planning horizon, the manufacturer receives revenue, $p \min(D, y)$, where $p$ is exogenously given. Then, if possible, the manufacturer repays $R_{sm}L_{sm}$ and $R_{bm}L_{bm}$ where $R_{sm}$ is the gross interest on the trade credit and $R_{bm}$ is the gross interest on the bank loans to the manufacturer. The manufacturer produces goods immediately after receiving loans and the manufacturer can lend money to outside investors at the gross interest rate of $R_m$.

The problem of any manufacturer of type $\theta = 1, 2$ is summarized below:

$$M_\theta = \max_{y, L_{bm}, L_{sm}} E_\theta \left[ R_m(L_{bm} + L_{sm} - wy) + p \min(D, y) - R_{bm}L_{bm} - R_{sm}L_{sm} \right]^+ \tag{4.1}$$

subject to:

$$0 \leq y, \tag{4.2}$$

$$0 \leq L_{sm} \leq \lambda y, \tag{4.3}$$

$$0 \leq L_{bm}, \tag{4.4}$$

$$wy \leq L_{bm} + L_{sm}. \tag{4.5}$$

### 4.3.2 The problem of the supplier.

The objective of the supplier is to maximize its value at the end of the planning horizon.

At the beginning of the planning horizon, the supplier has no capital and borrows $L_{bs}$ from the bank to cover its production cost, $\kappa y$, and the trade credit to provide to the manufacturer, $L_{sm}$. The supplier sells goods to the manufacturer immediately after receiving loans. There is no lead-time between the time the supplier requests the loan from the bank and the time the supplier sells the goods to the manufacturer. Bank loan to the supplier, $L_{bs}$, and revenue from selling goods to the manufacturer, $\tau y$, must cover the supplier's production cost, $\kappa y$, and the trade credit loan to the manufacturer, $L_{sm}$: $L_{sm} + \kappa y \leq L_{bs} + \tau y$. The payment for the goods, $\tau y$, is always greater or equal to the production cost of the goods, $\kappa y$: $\kappa \leq \tau$. Furthermore, the trade credit amount cannot be greater than the wholesale cost: $\lambda \leq \tau$. The supplier controls the wholesale price of the good, $\tau$, and the trade credit limit, $\lambda$[2].

I assume that the bank gets repaid before the supplier. Therefore, at the end of the planning horizon, the supplier waits for the bank to collect a loan repayment from the manufacturer. Then, the supplier collects a loan repayment from the manufacturer.

For any demand, $D$, the capital position of the manufacturer after making a loan repayment to the bank is given by $[(L_{bm} + L_{sm} - wy)R_m + p\min(D, y) - R_{bm}L_{bm}]^+$. Therefore, for any demand, $D$, the supplier receives from the manufacturer the minimum of $R_{sm}L_{sm}$ and the capital position of the manufacturer, $[(L_{bm}+L_{sm}-wy)R_m + p\min(D, y) - R_{bm}L_{bm}]^+$.

The supplier can lend money to outside investors at the gross interest rate of $R_s$. The problem of the supplier is summarized below:

$$S_\theta = \max_{\tau,\lambda,L_{bs},R_{sm}} E_\theta\Big\{ \min\left[R_{sm}L_{sm}, \left[R_m\left(L_{bm} + L_{sm} - wy\right) + p\min(D, y) - R_{bm}L_{bm}\right]^+\right] \\ + R_s\left(L_{bs} + \tau y - L_{sm} - \kappa y\right) - R_{bs}L_{bs}\Big\}^+ \tag{4.6}$$

subject to:

$$\kappa y \leq \tau y, \tag{4.7}$$

$$L_{sm} + \kappa y \leq L_{bs} + \tau y, \tag{4.8}$$

$$L_{sm} \leq \lambda y \leq \tau y. \tag{4.9}$$

The supplier, being in the same industry as the manufacturer, is assumed to have perfect information about the manufacturer's type. Therefore, the expectation in the supplier's problem is based on the manufacturer's type.

[2]See Chapter II to obtain a more detailed explanation for the choice of supplier loan limit used.

### 4.3.3 The problem of the bank.

The objective of the bank is to maximize its value at the end of the planning horizon.

At the beginning of the planning horizon, the bank borrows externally at a cost of capital $R$ (gross interest rate) to satisfy the loan requests of the supplier and the bank, $L_{bm} + L_{sm}$.

At the end of the planning horizon, the bank collects loan repayments from the supplier and the manufacturer. For any demand, $D$, the capital position of the manufacturer before making a loan repayment to the bank is $R_m(L_{bm} + L_{sm} - wy) + p\min(D, y)$ and the capital position of the supplier after receiving a loan repayment from the manufacturer and before making a loan repayment to the bank is $R_s(L_{bs} + \tau y - L_{sm} - \kappa y) + \min\{R_{sm}L_{sm}, [R_m(L_{bm} + L_{sm} - wy) + p\min(D, y) - R_{bm}L_{bm}]^+\}$. Therefore, for any demand, $D$, the bank receives from the manufacturer the minimum of $R_{bm}L_{bm}$ and the capital position of the manufacturer, $R_m(L_{bm} + L_{sm} - wy) + p\min(D, y)$ and it receives from the supplier the minimum of $R_{bs}L_{bs}$ and the capital position of the supplier, $R_s(L_{bs} + \tau y - L_{sm} - \kappa y) + \min\{R_{sm}L_{sm}, R_m[(L_{bm} + L_{sm} - wy) + p\min(D, y) - R_{bm}L_{bm}]^+\}$. Then, the bank repays to its outside investors, $R(L_{bs} + L_{bm})$ from the loan repayments received and/or from other assets. To ensure the bank does not always lose money, the gross interest rate on the trade credit, $R_{sm}$, and the gross interest rate on the bank loan, $R_{bm}$, are both greater than or equal to the cost of capital of the bank, $R$. Furthermore, the bank only lends if, in expectation, it does not lose money on the loan extended.

The problem of the bank is summarized below:

$$B = \max_{R_{bs}, R_{bm}} \{E[B_s(D)] + E[B_m(D)]\} \tag{4.10}$$

subject to:

$$0 \leq E\left[B_s\left(D\right)\right], \tag{4.11}$$

$$0 \leq E\left[B_m\left(D\right)\right], \tag{4.12}$$

$$R \leq R_{bs}, \tag{4.13}$$

$$R \leq R_{bm}. \tag{4.14}$$

where,

$$B_m(D) = \min\Big[R_m\left(L_{bm} + L_{sm} - wy\right) + p\min(D, y), R_{bm}L_{bm}\Big] - RL_{bm}, \tag{4.15}$$

$$\begin{aligned} B_s(D) = \min\Big\{ & \min\left[\left(R_m\left(L_{bm} + L_{sm} - wy\right) + p\min(D, y) - R_{bm}L_{bm}\right)^+, R_{sm}L_{sm}\right], \\ & + R_s\left(L_{bs} + \tau y - L_{sm} - \kappa y\right), R_{bs}L_{bs}\Big\} - RL_{bs}. \end{aligned} \tag{4.16}$$

The expectations in the bank's problem are based on the bank's belief about the manufacturer's type, which depends on the actions taken by the manufacturer and the actions taken by the supplier. I make the following assumption.

I assume that the bank always observe the quantity, $y$, ordered by the manufacturer, the trade credit amount, $L_{sm}$, obtained by the manufacturer and the terms, $R_{sm}$ and $\lambda$, offered to the manufacturer. This is why the expectations in the bank's problem are based on the bank's belief about the manufacturer's type, which depends on the actions taken by the manufacturer and the actions taken by the supplier.

The bank will not lend more than is needed to finance the manufacturer's production (i. e., $L_{bm} \leq wy - L_{sm}$). But, recall that the total amount borrowed by the manufacturer should be high enough for the manufacturer to finance its production cost (i. e., $L_{bm} + L_{sm} \geq wy$). Therefore, at optimality,

$$L_{bm} = wy - L_{sm}, \tag{4.17}$$

Let $\mu_\theta$ be the probability the bank believes the manufacturer is of type $\theta$. By Bayes rule,

$$\mu_1\left(y, L_{sm}, R_{sm}, \lambda\right) = \frac{Pr[y, L_{sm}, R_{sm}, \lambda|\theta = 1]\alpha}{Pr[y, L_{sm}, R_{sm}, \lambda|\theta = 1]\alpha + Pr[y, L_{sm}, R_{sm}, \lambda|\theta = 2](1-\alpha)} \tag{4.18}$$

and

$$\mu_2\left(y, L_{sm}, R_{sm}, \lambda\right) = \frac{Pr[y, L_{sm}, R_{sm}, \lambda|\theta = 2]\alpha}{Pr[y, L_{sm}, R_{sm}, \lambda|\theta = 1]\alpha + Pr[y, L_{sm}, R_{sm}, \lambda|\theta = 2](1-\alpha)} \tag{4.19}$$

where $Pr[y, L_{sm}, R_{sm}, \lambda|\theta = 1]$ is the probability that given the manufacturer of type $\theta = 1$, the supplier and the manufacturer would, at optimality, simultaneously choose actions $y$, $L_{sm}$, $R_{sm}$, $\lambda$. Similarly, $Pr[y, L_{sm}, R_{sm}, \lambda|\theta = 2]$ is the probability that given the manufacturer of type $\theta = 2$, the supplier and the manufacturer would, at optimality, simultaneously choose actions $y$, $L_{sm}$, $R_{sm}$, $\lambda$.

From the bank's point of view, the probability, for any Bayesian belief of the bank regarding a manufacturer's type, that the high demand occurs is $\widetilde{\pi}^H$ and the probability the low demand occurs is $\widetilde{\pi}^L$ where: $\widetilde{\pi}^H = \mu_1\left(y, L_{sm}, R_{sm}, \lambda\right)\pi_1^H + \mu_2\left(y, L_{sm}, R_{sm}, \lambda\right)\pi_2^H$, and $\widetilde{\pi}^L = \mu_1\left(y, L_{sm}, R_{sm}, \lambda\right)\pi_1^L + \mu_2\left(y, L_{sm}, R_{sm}, \lambda\right)\pi_2^L$.

When the bank believes, for given $y$, $L_{sm}$, $R_{sm}$ and $\lambda$, the manufacturer is a type-1 manufacturer: $\mu_1\left(y, L_{sm}, R_{sm}, \lambda\right) = 1$ and $\mu_2\left(y, L_{sm}, R_{sm}, \lambda\right) = 0$. This leads to $\widetilde{\pi}^H = 1\pi_1^H + 0\pi_2^H = \pi_1^H$ and $\widetilde{\pi}^L = 1\pi_1^L + 0\pi_2^L = \pi_1^L$.

When the bank believes, for given $y$, $L_{sm}$, $R_{sm}$ and $\lambda$, the manufacturer is a type-2 manufacturer: $\mu_1\left(y, L_{sm}, R_{sm}, \lambda\right) = 0$ and $\mu_2\left(y, L_{sm}, R_{sm}, \lambda\right) = 1$. This leads to $\widetilde{\pi}^H = 0\pi_1^H + 1\pi_2^H = \pi_2^H$ and $\widetilde{\pi}^L = 0\pi_1^L + 1\pi_2^L = \pi_2^L$.

When the bank, for given $y$, $L_{sm}$, $R_{sm}$ and $\lambda$, is unsure of the manufacturer's type: $\mu_1\left(y, L_{sm}, R_{sm}, \lambda\right) = \frac{1\times\alpha}{1\times\alpha+1\times(1-\alpha)} = \alpha$ and $\mu_2\left(y, L_{sm}, R_{sm}, \lambda\right) = \frac{1\times(1-\alpha)}{1\times\alpha+1\times(1-\alpha)} =$

$(1-\alpha)$. This leads to $\widetilde{\pi}^H = \alpha\pi_1^H + (1-\alpha)\pi_2^H$ and $\widetilde{\pi}^L = \alpha\pi_1^L + (1-\alpha)\pi_2^L$.

In the rest of the analysis, I use $\pi_P^H = \alpha\pi_1^H + (1-\alpha)\pi_2^H$ to refer to the probability the demand will be $H$ when the bank pools type-1 and type-2 manufacturers together because it cannot determine the manufacturer's type. Similarly, I use $\pi_P^L = \alpha\pi_1^L + (1-\alpha)\pi_2^L$ to refer to the probability the demand will be $L$ when the bank pools type-1 and type-2 manufacturers together because it cannot determine the manufacturer's type.

### 4.4 Analysis.

Throughout the analysis, I consider the case in which the supplier competes with other suppliers for the business with manufacturers and the bank competes with other banks for the business with manufacturers and suppliers. Thus, the supplier earns zero profit on the goods sold to the manufacturer, $\tau = \kappa$, has no positive cash flow at the beginning of the planning horizon, $L_{sm} + \kappa y = L_{bs} + \tau y$, and makes zero expected profit on the trade credit extended to the manufacturer, $E_\theta \Big\{ \min \left[ R_{sm}L_{sm}, [p\min(D,y) - R_{bm}L_{bm}]^+ \right] - R_{bs}L_{bs} \Big\}^+ = 0$.

Similarly, the bank makes zero expected profit on the loan extended to the supplier, $E\left[B_s\left(D\right)\right] = 0$, and on the loan extended to the manufacturer, $E\left[B_m\left(D\right)\right] = 0$. The fierce competition between suppliers and banks for the business with manufacturers makes all the profit realized by the supply chain to go to the manufacturer. One way to model this is to think that the decisions of the supplier and the decisions of the bank are dictated by the manufacturer, as long as the bank and the supplier make zero expected profit.

Recall that the bank is not always able to determine the manufacturer's type. However, the supplier knows the manufacturer's type and always offers break-even interest rates designed for the manufacturer's type. This is why, I refer to the objective value of a type-$\theta$ manufacturer as

$$K_{\theta\rho} = E_{\theta}\left[p \min\left(D, y\right) - R_{bm}L_{bm} - R_{sm}L_{sm}\right]^{+} \tag{4.20}$$

where $\rho = 1$ when the bank offers to the manufacturer an interest rate designed for type-1 manufacturers (type-1 rate), $\rho = 2$ when the bank offers to the manufacturer an interest rate designed for type-2 manufacturers (type-2 rate), $\rho = P$ when the bank pools type-1 and type-2 manufacturers together and offers to the manufacturer a pooling interest rate (pooling rate), and the expectation depends on the manufacturers type.

To avoid analyzing a situation where the manufacturer can make money by borrowing as much as possible from the bank to lend to outside investors, the interest rate at which the manufacturer can lend to investors, $R_m$, is less than or equal to the opportunity cost of capital of the bank, $R$: $R_m \leq R$. Similarly, $R_s \leq R$. Also, I have assumed $R_m$ and $R_s$ to be small relative to $R$. For simplicity, $R_m = 1$ and $R_s = 1$, but the structure of the results would not change if $1 < R_m < R$ and $1 < R_s < R$.

To understand the effect of asymmetric information and trade credit on manufacturers' operational and financial policies I first determine the menu of interest rates the bank and the supplier simultaneously offer to the manufacturer based on the decisions of the manufacturer. Second, I determine the symmetric information equilibria. Third, I determine the asymmetric information equilibria when trade credit is unavailable to the manufacturer. Then, I determine the asymmetric information equilibria when trade credit is available to the manufacturer. Afterwards, I determine who benefits and who suffers when there is asymmetric between banks and manufacturers and trade credit is available to manufacturers. Finally, I analyze the effect of the economic environment and the cost structure of each type of manufacturer on the benefit of having trade credit financing available in the supply-chain.

Observe in (4.3) that, for any $\lambda$ and $L_{sm} \in [0, \lambda y]$, the supplier can adjust its exposure to lending to the manufacturer by the choice of $R_{sm}$. Therefore, to simplify the exposition of the results I assume the manufacturer can borrow from the supplier as much as the wholesale price of the goods: $\lambda = \tau$. In other words, $L_{sm} \leq \tau y$.

### 4.4.1 Menus of interest rates offered by the lenders.

**Lemma 4.1.** *Suppose the system is in equilibrium where the bank and the supplier are breaking even and $L_{bm} + L_{sm} = wy$:*

$$E_\theta\{\min[R_{sm}L_{sm}, (p\min(D, y) - R_{bm}L_{bm})^+] - R_{bs}L_{bs}\}^+ = 0, \tag{4.21}$$

$$E\{\min[p\min(D, y), R_{bm}L_{bm}] - RL_{bm}\} = 0, \tag{4.22}$$

$$E\{\min[(p\min(D, y) - R_{bm}L_{bm})^+, R_{sm}L_{sm}, R_{bs}L_{bs}] - RL_{bs}\} = 0. \tag{4.23}$$

*Then, the equilibrium quantities satisfy the following relationships:*

$$R_{sm}L_{sm} = R_{bs}L_{bs}, \tag{4.24}$$

$$R_{bm} = \begin{cases} R \quad \textit{when } RL_{bm} \leq p\min(L, y), \\ \frac{RL_{bm} - \pi_\rho^L p\min(L,y)}{\pi_\rho^H L_{bm}} \quad \textit{when} \begin{cases} p\min(L, y) < RL_{bm}, \quad \textit{and} \\ L_{sm} = 0, \quad \textit{and} \\ RL_{bm} \leq \pi_\rho^H p\min(H, y) + \pi_\rho^L p\min(L, y), \end{cases} \\ \frac{RL_{bm} - \pi_\theta^L p\min(L,y)}{\pi_\theta^H L_{bm}} \quad \textit{when} \begin{cases} p\min(L, y) < RL_{bm}, \quad \textit{and} \\ L_{sm} > 0, \quad \textit{and} \\ Rwy \leq \pi_\theta^H p\min(H, y) + \pi_\theta^L p\min(L, y), \end{cases} \\ \textit{No solution otherwise,} \end{cases} \tag{4.25}$$

$$R_{sm} = \begin{cases} R \quad when\ Rwy \leq p\min(L,y), \\ \frac{RL_{sm}-\pi_\theta^L[p\min(L,y)-RL_{bm}]}{\pi_\theta^H L_{sm}} \quad when \begin{cases} RL_{bm} \leq p\min(L,y) \leq Rwy, \quad and \\ Rwy \leq \pi_\theta^H p\min(H,y) + \pi_\theta^L p\min(L,y), \end{cases} \\ \frac{R}{\pi_\theta^H} \quad when \begin{cases} p\min(L,y) < RL_{bm}, \quad and \\ Rwy \leq \pi_\theta^H p\min(H,y) + \pi_\theta^L p\min(L,y), \end{cases} \\ No\ solution\ otherwise. \end{cases} \tag{4.26}$$

*Proof.* Let us split the proof into multiple parts.

**Proof of equation (4.24).**

The break-even condition for the supplier,

$$0 = E_\theta \left\{ \min\left[ R_{sm}L_{sm}, (p\min(D,y) - R_{bm}L_{bm})^+ \right] - R_{bs}L_{sm} \right\}^+ \tag{4.27}$$

can be written as

$$\begin{aligned} 0 =& \pi_\theta^H \left\{ \min\left[ R_{sm}L_{sm}, (p\min(H,y) - R_{bm}L_{bm})^+ \right] - R_{bs}L_{bs} \right\}^+ \\ &+ \pi_\theta^L \left\{ \min\left[ R_{sm}L_{sm}, (p\min(L,y) - R_{bm}L_{bm})^+ \right] - R_{bs}L_{bs} \right\}^+ . \end{aligned} \tag{4.28}$$

As discussed at the beginning of Section 4.4, $\tau = \kappa$ and $L_{sm} + \kappa y = L_{bs} + \tau y$. Therefore, $L_{bs} = L_{sm}$. Replacing $L_{bs}$ by $L_{sm}$ in equation (4.28) we obtain:

$$\begin{aligned} 0 =& \pi_\theta^H \left\{ \min\left[ R_{sm}L_{sm}, (p\min(H,y) - R_{bm}L_{bm})^+ \right] - R_{bs}L_{sm} \right\}^+ \\ &+ \pi_\theta^L \left\{ \min\left[ R_{sm}L_{sm}, (p\min(L,y) - R_{bm}L_{bm})^+ \right] - R_{bs}L_{sm} \right\}^+ . \end{aligned} \tag{4.29}$$

Observe that $R_{sm}$ depends on the choice of $L_{sm}$, $L_{bm}$, $R_{bs}$ and $R_{bm}$. Therefore, to prove equation (4.24), I consider different ranges of values of $L_{sm}$, $L_{bm}$, $R_{bs}$ and $R_{bm}$ and determine the corresponding value of $R_{sm}$.

If $L_{sm} = 0$, then equation (4.29) is automatically satisfied.

Suppose $L_{sm} > 0$, I study the choice of $R_{sm}$ that ensures the equation (4.29) is satisfied.

There are three possible ranges of values for $R_{sm}L_{sm}$ and we do not know, a priori, which one holds in equilibrium:

(i) $R_{sm}L_{sm} \leq [p\min(L,y) - R_{bm}L_{bm}]^+$,

(ii) $[p\min(L,y) - R_{bm}L_{bm}]^+ \leq R_{sm}L_{sm} \leq [p\min(H,y) - R_{bm}L_{bm}]^+$,

(iii) $[p\min(H,y) - R_{bm}L_{bm}]^+ \leq R_{sm}L_{sm}$.

The last condition: $[p\min(H,y) - R_{bm}L_{bm}]^+ \leq R_{sm}L_{sm}$ cannot be true in equilibrium, because the manufacturer is guaranteed to earn negative profit. Therefore, I do not need to analyze this condition in equilibrium.

With condition (i), then equation (4.29) becomes

$$\begin{aligned} 0 =& \pi_\theta^H \left(R_{sm}L_{sm} - R_{bs}L_{sm}\right)^+ + \pi_\theta^L \left(R_{sm}L_{sm} - R_{bs}L_{sm}\right)^+ \\ =& \left(\pi_\theta^H + \pi_\theta^L\right)\left(R_{sm}L_{sm} - R_{bs}L_{sm}\right)^+ \\ =& \left(R_{sm}L_{sm} - R_{bs}L_{sm}\right)^+ . \end{aligned} \tag{4.30}$$

Observe that it must be true that $R_{sm}L_{sm} \geq R_{bs}L_{sm}$, for, otherwise, the supplier is guaranteed to lose money on the trade credit loan. Hence, from equation (4.30) I conclude that $R_{sm} = R_{bs}$.

With condition (ii), equation (4.29) becomes

$$0 = \pi_\theta^H \left(R_{sm}L_{sm} - R_{bs}L_{sm}\right)^+ + \pi_\theta^L \left\{[p\min(L,y) - R_{bm}L_{bm}]^+ - R_{bs}L_{sm}\right\}^+ . \tag{4.31}$$

As we already observed, $R_{sm}L_{sm} \geq R_{bs}L_{sm}$. Therefore, equation (4.31) is equivalent to

$$0 = \pi_\theta^H \left(R_{sm}L_{sm} - R_{bs}L_{sm}\right) + \pi_\theta^L \left\{[p\min(L,y) - R_{bm}L_{bm}]^+ - R_{bs}L_{sm}\right\}^+ . \tag{4.32}$$

In equation (4.32), if $[p\min(L,y)-R_{bm}L_{bm}]^{+}-R_{bs}L_{sm}\leq 0$ then, as before, I conclude that $R_{sm}=R_{bs}$.

The only non-trivial case is $[p\min(L,y)-R_{bm}L_{bm}]^{+}-R_{bs}L_{sm}>0$, which can happen if $p\min(L,y)-R_{bm}L_{bm}>0$. Thus, if $p\min(L,y)-R_{bm}L_{bm}>0$ and $[p\min(L,y)-R_{bm}L_{bm}]^{+}-R_{bs}L_{sm}>0$ then equation (4.32) is equivalent to

$$0 =\pi_\theta^H\left\{R_{sm}L_{sm}-R_{bs}L_{sm}\right\}+\pi_\theta^L\left\{p\min(L,y)-R_{bm}L_{bm}-R_{bs}L_{sm}\right\}. \tag{4.33}$$

Because both terms of the summation on the right hand side of equation (4.33) are non-negative, each of them must be equal to 0. In particular, $R_{sm}=R_{bs}$.

**Proof of equation** (4.25) **and equation** (4.26).

Let us start by determining the bank's equilibrium values for $R_{bm}L_{bm}$.

The break-even condition for the bank on the loan given to the manufacturer,

$$0 =E\{\min[p\min(D,y),R_{bm}L_{bm}]-RL_{bm}\} \tag{4.34}$$

can be written as

$$0 =\pi_\rho^H\min[p\min(H,y),R_{bm}L_{bm}]+\pi_\rho^L\min[p\min(L,y),R_{bm}L_{bm}]-RL_{bm}. \tag{4.35}$$

Observe that $R_{bm}$ depends on the choice of $L_{bm}$. Therefore, to prove equation (4.25), I consider different ranges of values of $L_{bm}$ and determine the corresponding value of $R_{bm}$.

If $L_{bm}=0$, then equation (4.35) is automatically satisfied.

Suppose $L_{bm}>0$, I study the choice of $R_{bm}$ that ensures the equation (4.35) is satisfied.

Equation (4.35) has the following form $f\left(R_{bm}L_{bm}\right)=RL_{bm}$ where

$$f(x) =\pi_\rho^H\min[p\min(H,y),x]+\pi_\rho^L\min[p\min(L,y),x]. \tag{4.36}$$

$f(x)$ is piecewise linear with three segments. The segments are $x < p\min(L,y)$, $x \in [p\min(L,y), p\min(H,y)]$, $x > p\min(H,y)$. The values on these segments are, correspondingly, $f(x) \in [0, p\min(L,y)]$, $f(x) \in [p\min(L,y), \pi_\rho^H p\min(H,y) + \pi_\rho^L p\min(L,y)]$ and $f(x) = \pi_\rho^H p\min(H,y) + \pi_\rho^L p\min(L,y)$.

Therefore, there are three possible ranges of values for $RL_{bm}$ I need to analyze and we do not know, a priori, which one holds in equilibrium:

(i) $RL_{bm} \in [0, p\min(L,y)]$,

(ii) $RL_{bm} \in \left[p\min(L,y), \pi_\rho^H p\min(H,y) + \pi_\rho^L p\min(L,y)\right]$,

(iii) $RL_{bm} \in \left[\pi_\rho^H p\min(H,y) + \pi_\rho^L p\min(L,y), \infty\right)$.

The last condition: $RL_{bm} \in \left[\pi_\rho^H p\min(H,y) + \pi_\rho^L \eta\min(L,y), \infty\right)$ cannot be true in equilibrium, because the bank is guaranteed to earn negative profit. Therefore, I do not need to analyze this condition in equilibrium.

With condition (i), equation (4.35) becomes

$$0 = R_{bm}L_{bm} - RL_{bm}. \tag{4.37}$$

We conclude that:

$$R_{bm} = R. \tag{4.38}$$

With condition (ii), equation (4.35) becomes

$$0 = \pi_\rho^H R_{bm}L_{bm} + \pi_\rho^L p\min(L,y) - RL_{bm}. \tag{4.39}$$

We conclude that:

$$R_{bm} = \frac{RL_{bm} - \pi_\rho^L p\min(L,y)}{\pi_\rho^H L_{bm}}. \tag{4.40}$$

Let us determine the supplier equilibrium values for $R_{bm}L_{bm}$.

The break-even condition for the bank on the loan given to the supplier is given by the following equation

$$0 = E\{\min[(p\min(D,y) - R_{bm}L_{bm})^+, R_{sm}L_{sm}, R_{bs}L_{bs}] - RL_{sm}\}. \tag{4.41}$$

Recall, from equation (4.24), that $R_{bs}L_{bs} = R_{sm}L_{sm}$. Therefore, because the supplier knows the manufacturer's type (i. e., $\pi_\rho^D = \pi_\theta^D$ for $\theta = 1$ or 2 and $D = L$ or $H$), equation (4.41) can be written as

$$\begin{aligned} 0 =& E\{\min[(p\min(D,y) - R_{bm}L_{bm})^+, R_{sm}L_{sm}]\} - RL_{sm} \\ =& \pi_\theta^H \min\{[p\min(H,y) - R_{bm}L_{bm}]^+, R_{sm}L_{sm}\} \\ &+ \pi_\theta^L \min\{[p\min(L,y) - R_{bm}L_{bm}]^+, R_{sm}L_{sm}\} - RL_{sm}. \end{aligned} \tag{4.42}$$

Observe that $R_{sm}$ depends on the choice of $R_{bm}$, $L_{bm}$ and $L_{sm}$. Therefore, to prove equation (4.26), I consider different ranges of values of $R_{bm}$, $L_{bm}$ and $L_{sm}$ and determine the corresponding value of $R_{sm}$.

If $L_{sm} = 0$, then equation (4.42) is automatically satisfied.

Suppose $L_{sm} > 0$, I study the choice of $R_{bm}$ that ensures the equation (4.42) is satisfied.

Equation (4.42) has the form $f(R_{sm}L_{sm}) = RL_{sm}$ where

$$f(x) = \pi_\theta^H \min\{[p\min(H,y) - R_{bm}L_{bm}]^+, x\} + \pi_\theta^L \min\{[p\min(L,y) - R_{bm}L_{bm}]^+, x\} \tag{4.43}$$

and equation (4.25) has the supplier equilibrium values for $R_{bm}L_{bm}$ which depend on the value of $RL_{bm}$.

Suppose $RL_{bm} \in [0, p\min(L,y)]$ holds. Then, from equation (4.25), $R_{bm}L_{bm} = RL_{bm}$ and the expressions in equation (4.43) have the following form $f\left(R_{sm}L_{sm}\right) = RL_{sm}$

where

$$f(x) =\pi_\theta^H \min\left[p\min(H,y) - RL_{bm}, x\right] + \pi_\theta^L \min\left[p\min(L,y) - RL_{bm}, x\right]. \tag{4.44}$$

$f(x)$ is piecewise linear with three segments. The segments are $x < p\min(L,y) - RL_{bm}$, $x \in [p\min(L,y) - RL_{bm}, p\min(H,y) - RL_{bm}]$, $x > p\min(H,y) - RL_{bm}$. The values on these segments are, correspondingly, $f(x) \in [0, p\min(L,y) - RL_{bm}]$, $f(x) \in [p\min(L,y) - RL_{bm}, \pi_\theta^H p\min(H,y) + \pi_\theta^L p\min(L,y) - RL_{bm}]$ and $f(x) = \pi_\theta^H p\min(H,y) + \pi_\theta^L p\min(L,y) - RL_{bm}$.

Let us solve for $x$ in $f(x) = RL_{sm}$.

If $RL_{sm} \in [0, p\min(L,y) - RL_{bm}]$, then

$$x =RL_{sm}. \tag{4.45}$$

Thus,

$$R_{sm} =R. \tag{4.46}$$

If $RL_{sm} \in \left[p\min(L,y) - RL_{bm}, \pi_\theta^H p\min(H,y) + \pi_\theta^L p\min(L,y) - RL_{bm}\right]$, then

$$\pi_\theta^H x + \pi_\theta^L[p\min(L,y) - RL_{bm}] =RL_{sm}. \tag{4.47}$$

Equivalently,

$$x =\frac{RL_{sm} - \pi_\theta^L[p\min(L,y) - RL_{bm}]}{\pi_\theta^H}. \tag{4.48}$$

Thus,

$$R_{sm} =\frac{RL_{sm} - \pi_\theta^L[p\min(L,y) - RL_{bm}]}{\pi_\theta^H L_{sm}}. \tag{4.49}$$

If $RL_{sm} \in \left[\pi_\theta^H p\min(H,y) + \pi_\theta^L p\min(L,y) - RL_{bm}, \infty\right)$, there is no solution to $f(x) = RL_{sm}$.

Suppose $RL_{bm} \in \left[p\min(L,y), \pi_\theta^H p\min(H,y) + \pi_\theta^L p\min(L,y)\right]$. From (4.25), $R_{bm} = \frac{RL_{bm} - \pi_\theta^L p\min(L,y)}{\pi_\theta^H L_{bm}}$. Note that,

$$p\min(L,y) - \frac{RL_{bm} - \pi_\theta^L p\min(L,y)}{\pi_\theta^H} = \frac{p\min(L,y) - RL_{bm}}{\pi_\theta^H} < 0 \tag{4.50}$$

and

$$p\min(H,y) - \frac{RL_{bm} - \pi_\theta^L p\min(L,y)}{\pi_\theta^H} = \frac{\pi_\theta^H p\min(H,y) + \pi_\theta^L p\min(L,y) - RL_{bm}}{\pi_\theta^H} \geq 0. \tag{4.51}$$

Therefore, expression (4.43) becomes

$$f(x) = \pi_\theta^H \min\left[\frac{\pi_\theta^H p\min(H,y) + \pi_\theta^L p\min(L,y) - RL_{bm}}{\pi_\theta^H}, x\right]. \tag{4.52}$$

$f(x)$ is piecewise linear, with two segments, for $x \leq \frac{\pi_\theta^H p\min(H,y) + \pi_\theta^L p\min(L,y) - RL_{bm}}{\pi_\theta^H}$ and $x > \frac{\pi_\theta^H p\min(H,y) + \pi_\theta^L p\min(L,y) - RL_{bm}}{\pi_\theta^H}$. The values on these segments are, correspondingly, $f(x) \in [0, \pi_\theta^H p\min(H,y) + \pi_\theta^L p\min(L,y) - RL_{bm}]$ and $f(x) = \pi_\theta^H p\min(H,y) + \pi_\theta^L p\min(L,y) - RL_{bm}$.

Let us solve for $x$ in $f(x) = RL_{sm}$.

If $RL_{sm} \in \left[0, \frac{\pi_\theta^H p\min(H,y) + \pi_\theta^L p\min(L,y) - RL_{bm}}{\pi_\theta^H}\right]$, then

$$\pi_\theta^H x = RL_{sm}. \tag{4.53}$$

Equivalently,

$$x = \frac{RL_{sm}}{\pi_\theta^H}. \tag{4.54}$$

Thus,

$$R_{sm} = \frac{R}{\pi_\theta^H}. \tag{4.55}$$

If $RL_{sm} \in \left[\frac{\pi_\theta^H p\min(H,y) + \pi_\theta^L p\min(L,y) - RL_{bm}}{\pi_\theta^H}, \infty\right)$, there is no solution to $f(x) = RL_{sm}$. Observe from condition (i), equation (4.49), condition (ii) and equation that, when $RL_{sm} \in \left[p\min(L,y) - RL_{bm}, \pi_\theta^H p\min(H,y) + \pi_\theta^L p\min(L,y) - RL_{bm}\right]$, the bank always knows the manufacturer's type. Recall from equation (4.17) that $L_{bm} + L_{sm} = wy$. Therefore, this leads to the bank knowing the manufacturer's type when $Rwy \in \left[p\min(L,y), \pi_\theta^H p\min(H,y) + \pi_\theta^L p\min(L,y)\right]$ and concludes the proof for equation (4.26). Furthermore, because $L_{sm} > 0$, and the bank knows the manufacturer's type when $Rwy \in \left[p\min(L,y), \pi_\theta^H p\min(H,y) + \pi_\theta^L p\min(L,y)\right]$, equation (4.40) becomes:

$$R_{bm} = \frac{RL_{bm} - \pi_\theta^L p\min(L,y)}{\pi_\theta^H L_{bm}}. \tag{4.56}$$

when $L_{sm} > 0$ and $RL_{bm} \in \left[p\min(L,y), \pi_\theta^H p\min(H,y) + \pi_\theta^L p\min(L,y)\right]$. ■

Lemma 4.1 states that, in equilibrium, the supplier promises to pay the bank the same amount the manufacturer promises to pay the supplier: $R_{bs}L_{bs} = R_{sm}L_{sm}$. Lemma 4.1 also gives the break-even interest rates the bank and the supplier offer to a manufacturer of type $\theta$ when the manufacturer places an order equal to $y$ from the supplier, borrows $L_{sm}$ from the supplier and borrows $L_{bm}$ from the bank. Recall the optimization problem of a manufacturer of the type $\theta$ presented in equation (4.1) to equation (4.5). Lemma 4.1 provides the menu of interest rates offered, in equilibrium, by the bank and the supplier. Substituting the interest rates in the objective value, $K_{\theta\rho}$, with their break-even expressions in Lemma 4.1, we obtain the following expressions for the objective value of a type-$\theta$ manufacturer:

$$K_{\theta\rho} = \begin{cases} E_\theta\left[p\min\left(D,y\right) - Rwy\right]^+ & \text{when} \quad Rwy \le p\min(L,y), \\ E_\theta\left[\frac{\pi_\rho^H p\min(D,y)+\pi_\rho^L p\min(L,y)-Rwy}{\pi_\rho^H}\right]^+ & \text{when} \begin{cases} p\min(L,y) < Rwy, \quad \text{and} \\ L_{sm} = 0, \end{cases} \\ E_\theta\left[\frac{\pi_\theta^H p\min(D,y)+\pi_\theta^L p\min(L,y)-Rwy}{\pi_\theta^H}\right]^+ & \text{when} \begin{cases} p\min(L,y) < Rwy, \quad \text{and} \\ L_{sm} > 0. \end{cases} \end{cases} \tag{4.57}$$

To avoid analyzing the situation where the objective function of all manufacturers is 0, I make the following assumption.

**Assumption 4.1.** *The unit revenue is greater than the unit financing cost of a manufacturer:* $p > Rw$.

**Lemma 4.2.** *In equilibrium, the order quantity, $y$, of any manufacturer is always greater than or equal to lowest demand value, $L$:* $y \ge L$.

*Proof.* I consider when $0 \le y < L$ and show that it is suboptimal for the manufacturer to order a quantity less than $L$.

Because the two possible demand values are never less than $L$, when $0 \le y < L$, the expressions for the objective function, $K_{\theta\rho}$, in equation (4.57) become

$$K_{\theta\rho} = E_\theta\left(py - Rwy\right)^+. \tag{4.58}$$

The bank's belief about a manufacturer's type depends on decisions made by all manufacturers. But, observe that, when $0 \le y < L$, all manufacturers receive the same interest rate regardless of the bank's belief about a manufacturer's type. This is why, when $0 \le y < L$, each type of manufacturer optimizes its objective value without the need to anticipate the decisions of other manufacturers and the corresponding interest rate that it will receive from the lenders.

Recall, from Assumption 4.1, that $p > Rw$. Therefore, the expression for $y \in [0, y]$ in (4.58) along with Assumption 4.1 lead to $y \geq L$ as the optimal action of manufacturers. ■

According to Lemma 4.2 it is suboptimal for manufacturers to order a quantity that is less than the smallest demand that can be observed. Thus, combining Assumption 4.1 and Lemma 4.2 with the expressions for the interest rates received by the manufacturer we get:

$$R_{bm} = \begin{cases} R \text{ when } RL_{bm} \leq pL, \\ \frac{RL_{bm} - \pi_\rho^L pL}{\pi_\rho^H L_{bm}} \text{ when } \begin{cases} pL < RL_{bm}, \text{ and} \\ L_{sm} = 0, \text{ and} \\ RL_{bm} \leq \pi_\rho^H p \min(H, y) + \pi_\rho^L pL, \end{cases} \\ \frac{RL_{bm} - \pi_\theta^L pL}{\pi_\theta^H L_{bm}} \text{ when } \begin{cases} pL < RL_{bm}, \text{ and} \\ L_{sm} > 0, \text{ and} \\ Rwy \leq \pi_\theta^H p \min(H, y) + \pi_\theta^L pL, \end{cases} \\ \text{No solution otherwise.} \end{cases} \tag{4.59}$$

$$R_{sm} = \begin{cases} R \quad \text{when } Rwy \leq pL, \\ \frac{RL_{sm} - \pi_\theta^L [pL - RL_{bm}]}{\pi_\theta^H L_{sm}} \quad \text{when} \begin{cases} RL_{bm} \leq pL \leq Rwy, \quad \text{and} \\ Rwy \leq \pi_\theta^H p \min(H, y) + \pi_\theta^L pL, \end{cases} \\ \frac{R}{\pi_\theta^H} \quad \text{when} \begin{cases} pL < RL_{bm}, \quad \text{and} \\ Rwy \leq \pi_\theta^H p \min(H, y) + \pi_\theta^L pL, \end{cases} \\ \text{No solution otherwise.} \end{cases} \tag{4.60}$$

Similarly, combining Assumption 4.1 and Lemma 4.2 with the expressions for the objective value of any manufacturer, $K_{\theta\rho}$, in equation (4.57) we get:

$$K_{\theta\rho} = \begin{cases} E_\theta \left[ p \min\left(D, y\right) - Rwy \right]^+ & \text{when} \quad L \leq y \leq \frac{pL}{Rw}, \\ E_\theta \left[ \frac{\pi_\rho^H p \min(D,y) + \pi_\rho^L pL - Rwy}{\pi_\rho^H} \right]^+ & \text{when} \quad \frac{pL}{Rw} < y \quad \text{and} \quad L_{sm} = 0, \\ E_\theta \left[ \frac{\pi_\theta^H p \min(D,y) + \pi_\theta^L pL - Rwy}{\pi_\theta^H} \right]^+ & \text{when} \quad \frac{pL}{Rw} < y \quad \text{and} \quad L_{sm} > 0. \end{cases} \tag{4.61}$$

Before investigating for various equilibria, let us introduce the notation that we use throughout the analysis:

- $y_{\theta\rho}^B$ is, for any $\rho = \{1, 2, P\}$, the best-response order quantity of a type-$\theta$ manufacturer.

- $R_{bm\theta\rho}^B$ is, for any $\rho = \{1, 2, P\}$, the interest rate that a manufacturer of the type-$\theta$ receives from the bank when it orders its best-response order quantity.

- $R_{sm\theta\rho}^B$ is, for any $\rho = \{1, 2, P\}$, the interest rate that a manufacturer of the type-$\theta$ receives from the supplier when it orders its best-response order quantity.

- $K_{\theta\rho}^B$ is, for any $\rho = \{1, 2, P\}$, the objective value of a type-$\theta$ manufacturer that orders $y_{\theta\rho}^B$.

### 4.4.2 Symmetric information equilibria between the bank and the manufacturer.

When there is symmetric information between the bank and the manufacturer, the bank offers to the manufacturer a menu of interest rates based on the manufacturer's type (i.e., $\rho = \theta$), the manufacturer's order quantity, the loan amount the manufacturer desires from the bank, and the loan amount the manufacturer desires from the supplier. Also, because the bank knows the manufacturer's type, the manufacturer does not benefit from signaling its type to the bank nor can the manufacturer pretend to be of the other type. Therefore, in equilibrium, the optimal operational decision of the manufacturer is to order its best-response order quantity to the menu of interest rates offered by the bank.

**Proposition 4.1.** *In a symmetric information equilibrium, the order quantity, $y$, of a manufacturer of the type $\theta$ is:*

$$y^B_{\theta\theta} = \begin{cases} H & \text{when} \quad Rw \leq \pi^H_\theta p, \\ L & \text{when} \quad \pi^H_\theta p \leq Rw. \end{cases} \tag{4.62}$$

*and the objective value of the manufacturer is:*

$$K^B_{\theta\theta} = \begin{cases} \pi^H_\theta pH + \pi^L_\theta pL - RwH & \text{if} \quad y = H, \\ (p - Rw)L & \text{if} \quad y = L. \end{cases} \tag{4.63}$$

*Proof.* Recall that the realized demand is either $L$ or $H$ where $L \leq H$. Therefore, because $L < \frac{pL}{Rw}$, $L < H$, and $H$ can be greater than or equal to $\frac{pL}{Rw}$, we need to analyze two cases: $H \leq \frac{pL}{Rw}$ and $\frac{pL}{Rw} < H$.

If $H \leq \frac{pL}{Rw}$, the objective value in equation (4.61) becomes:

$$K_{\theta\theta} = \begin{cases} E_\theta \left[p \min(D, y) - Rwy\right]^+ & \text{when} \quad L \leq y \leq H, \\ E_\theta \left(pD - Rwy\right)^+ & \text{when} \quad H \leq y \leq \frac{pL}{Rw}, \\ E_\theta \left(\frac{\pi_\theta^H pD + \pi_\theta^L pL - Rwy}{\pi_\theta^H}\right)^+ & \text{when} \quad \frac{pL}{Rw} < y. \end{cases} \tag{4.64}$$

where $K_{\theta\theta}$ is continuous in $y$.

Observe that, when $y \geq H$, $K_{\theta\theta}$ is non-increasing in $y$.

When $y \in [L, H]$,

$$K_{\theta\theta} = E_\theta \left[p \min(D, y) - Rwy\right]^+ \tag{4.65}$$

$$= \pi_\theta^H \left(py - Rwy\right)^+ + \pi_\theta^L \left(pL - Rwy\right)^+ \tag{4.66}$$

$$= \pi_\theta^H \left(py - Rwy\right) + \pi_\theta^L \left(pL - Rwy\right) \tag{4.67}$$

$$= \pi_\theta^H py + \pi_\theta^L pL - Rwy. \tag{4.68}$$

For all $y \in [L, H]$, this leads to $K_{\theta\theta}$ increasing in $y$ when $Rw \leq \pi_\theta^H p$ and decreasing in $y$ when $\pi_\theta^H p \leq Rw$. Therefore, if $H \leq \frac{pL}{Rw}$, $y_{\theta\theta}^B = H$ when $Rw \leq \pi_\theta^H p$ and $y_{\theta\theta}^B = L$ when $\pi_\theta^H p \leq Rw$.

If $\frac{pL}{Rw} < H$, the objective value in equation (4.61) becomes:

$$K_{\theta\theta} = \begin{cases} E_\theta \left[p \min(D, y) - Rwy\right]^+ & \text{when} \quad L \leq y \leq \frac{pL}{Rw}, \\ E_\theta \left(\frac{\pi_\theta^H p \min(D,y) + \pi_\theta^L pL - Rwy}{\pi_\theta^H}\right)^+ & \text{when} \quad \frac{pL}{Rw} < y \leq H, \\ E_\theta \left(\frac{\pi_\theta^H pD + \pi_\theta^L pL - Rwy}{\pi_\theta^H}\right)^+ & \text{when} \quad H \leq y. \end{cases} \tag{4.69}$$

where $K_{\theta\theta}$ is continuous in $y$.

Observe that, when $y \geq H$, $K_{\theta\theta}$ is non-increasing in $y$.

When $y \in \left[L, \frac{pL}{Rw}\right]$,

$$K_{\theta\theta} = E_\theta \left[p \min(D, y) - Rwy\right]^+ \tag{4.70}$$

$$=\pi_\theta^H (py - Rwy)^+ + \pi_\theta^L (pL - Rwy)^+ \tag{4.71}$$

$$=\pi_\theta^H (py - Rwy) + \pi_\theta^L (pL - Rwy) \tag{4.72}$$

$$=\pi_\theta^H py + \pi_\theta^L pL - Rwy. \tag{4.73}$$

and, when $y \in \left(\frac{pL}{Rw}, H\right]$,

$$K_{\theta\theta} = E_\theta \left[\frac{\pi_\theta^H p \min(D, y) + \pi_\theta^L pL - Rwy}{\pi_\theta^H}\right]^+ \tag{4.74}$$

$$=\pi_\theta^H \left(\frac{\pi_\theta^H py + \pi_\theta^L pL - Rwy}{\pi_\theta^H}\right)^+ + \pi_\theta^L \left(\frac{pL - Rwy}{\pi_\theta^H}\right)^+ \tag{4.75}$$

$$=\pi_\theta^H \left(\frac{\pi_\theta^H py + \pi_\theta^L pL - Rwy}{\pi_\theta^H}\right)^+ \tag{4.76}$$

$$= \left(\pi_\theta^H py + \pi_\theta^L pL - Rwy\right)^+. \tag{4.77}$$

For all $y \in [L, H]$, this leads to $K_{\theta\theta}$ increasing in $y$ when $Rw \leq \pi_\theta^H p$ and non-increasing in $y$ when $\pi_\theta^H p \leq Rw$. Therefore, if $\frac{pL}{Rw} < H$, $y_{\theta\theta}^B = H$ when $Rw \leq \pi_\theta^H p$ and $y_{\theta\theta}^B = L$ when $\pi_\theta^H p \leq Rw$.

Combining the results obtained for $H \leq \frac{pL}{Rw}$ and $\frac{pL}{Rw} < H$ gives us the optimal order quantities of all manufacturers. We can now compute the objective value that corresponds to each optimal order quantity.

The objective value of a type-$\theta$ manufacturer if it orders $H$ is

$$K_{\theta\theta}^B = \left(\pi_\theta^H pH + \pi_\theta^L pL - RwH\right)^+. \tag{4.78}$$

Observe that if $y = H$ then $Rw \leq \pi_\theta^H p$. Because $\pi_\theta^H pH \leq \pi_\theta^H pH + \pi_\theta^L pL$, we get:

$$K_{\theta\theta}^B = \pi_\theta^H pH + \pi_\theta^L pL - RwH. \tag{4.79}$$

If a type-$\theta$ manufacturer orders $L$ the objective value is

$$K_{\theta\theta}^B = (p - Rw)L. \tag{4.80}$$

■

Proposition 4.1 states the optimal order quantities with the corresponding objective value of the manufacturer when there is symmetric information between the bank and the manufacturer. Equation (4.17), equation (4.59), and equation (4.60) provide the optimal amounts borrowed by manufacturers and the break-even interest rates of the bank and supplier for any quantity ordered by the manufacturer. Therefore, combining Proposition 4.1, equation (4.17) with the expressions in equation (4.59), and equation (4.60) gives us the equilibria when there is symmetric information between the bank and the manufacturer.

#### 4.4.2.1 Equilibria.

The optimal decisions of the bank and each type of manufacturer are summarized in Table 4.1 and Table 4.2. In each table, column 1 and column 2 describe the conditions for the equilibria presented. I add column 3 to conveniently refer to specific rows. Column 4 and column 5 specify, for a type-1 manufacturer, the equilibrium order quantities and interest rates on loans from the bank to the manufacturer. Column 6 and column 7 specify, for a type-2 manufacturer, the equilibrium order quantities and interest rates on loans from the bank to the manufacturer. The expressions in equation (4.61) for the objective function, $K_{\theta\rho}$, of any type of manufacturer are independent of the borrowing source. Therefore, to simplify the exposition, I assume the manufacturer only borrows from the bank. Furthermore, I use $R^B_{bm\theta} \stackrel{\text{def}}{=} \frac{RwH - \pi^L_\theta pL}{\pi^H_\theta wH}$ to refer to the interest rate the bank offers to a manufacturer of the type $\theta$ when $pL \leq RwH$ and the manufacturer orders $H$.

#### 4.4.2.2 Discussion.

In equilibrium, the manufacturer orders $H$ or $L$. But, because type-1 manufacturers have a higher expected demand for final goods than type-2 manufacturers it

| $RwH \leq pL$ | | | Symmetric | | | |
|---|---|---|---|---|---|---|
| | | | Type 1 | | Type 2 | |
| **Subcase** | | | $y^*$ | $R^*_{bm}$ | $y^*$ | $R^*_{bm}$ |
| $\pi_1^H H \leq L$ | $RwH \leq \pi_2^H pH$ | 1 | $H$ | $R$ | $H$ | $R$ |
| | $\pi_2^H pH \leq RwH \leq \pi_1^H pH$ | 2 | $H$ | $R$ | $L$ | $R$ |
| | $\pi_1^H pH \leq RwH \leq pL$ | 3 | $L$ | $R$ | $L$ | $R$ |
| **Subcase** | | | | | | |
| $\pi_2^H H \leq L \leq \pi_1^H H$ | $RwH \leq \pi_2^H pH$ | 4 | $H$ | $R$ | $H$ | $R$ |
| | $\pi_2^H pH \leq RwH \leq pL$ | 5 | $H$ | $R$ | $L$ | $R$ |
| **Subcase** | | | | | | |
| $L \leq \pi_2^H H$ | $RwH \leq pL$ | 6 | $H$ | $R$ | $H$ | $R$ |

Table 4.1: Symmetric information equilibria between the bank and the manufacturer when $RwH \leq pL$.

is optimal for type-1 manufacturers to order $H$ and type-2 manufacturers order $L$ when $\pi_2^H p \leq Rw \leq \pi_1^H p$.

When, for a high order quantity, the smallest possible revenue that can be generated by the manufacturer is greater than or equal to its loan repayment obligations (i. e., $pL \geq RwH$), the manufacturer is always able to repay its loan repayment obligations. This leads to the bank and the supplier offering the lowest interest rate possible to the manufacturer because the loans given to either type of manufacturer are always riskless.

When, for a high order quantity, the smallest possible revenue that can be generated by the manufacturer is less than its loan repayment obligations (i. e., $pL \leq RwH$), there is a risk that the manufacturer will default on its loan repayment obligations. Due to this risk, the interest rate on the total loan amount received by the

| $pL < RwH$ | | | **Symmetric** | | | |
|---|---|---|---|---|---|---|
| | | | **Type 1** | | **Type 2** | |
| **Subcase** | | | $y^*$ | $R^*_{bm}$ | $y^*$ | $R^*_{bm}$ |
| $\pi_1^H H \leq L$ | $pL \leq RwH$ | 1 | $L$ | $R$ | $L$ | $R$ |
| **Subcase** | | | | | | |
| $\pi_2^H H \leq L \leq \pi_1^H H$ | $pL \leq RwH \leq \pi_1^H pH$ | 2 | $H$ | $R_1^B$ | $L$ | $R$ |
| | $\pi_1^H pH \leq RwH$ | 3 | $L$ | $R$ | $L$ | $R$ |
| **Subcase** | | | | | | |
| $L \leq \pi_2^H H$ | $pL \leq RwH \leq \pi_2^H pH$ | 4 | $H$ | $R_1^B$ | $H$ | $R_1^B$ |
| | $\pi_2^H pH \leq RwH \leq \pi_1^H pH$ | 5 | $H$ | $R_1^B$ | $L$ | $R$ |
| | $\pi_1^H pH \leq RwH$ | 6 | $L$ | $R$ | $L$ | $R$ |

Table 4.2: Symmetric information equilibria between the bank and the manufacturer when $pL < RwH$. Recall $R^B_{bm\theta} = \frac{RwH - \pi_\theta^L pL}{\pi_\theta^H wH}$.

manufacturer is higher than the opportunity cost of capital of the bank.

### 4.4.3 Asymmetric information equilibria between the bank and the manufacturer when trade credit is unavailable.

I classify the equilibria solutions into two types: separating equilibria and pooling equilibria.

When the bank offers a menu of type-1 and type-2 rates to the manufacturer, a separating equilibrium exists if

- a type-$\theta$ manufacturer chooses the menu of type-$\theta$ rates,
- the quantity ordered by a type-1 manufacturer is different from the quantity ordered by a type-2 manufacturer,
- the bank offers a type-$\theta$ rate to a type-$\theta$ manufacturer.

When the bank offers a menu of type-1 and type-2 rates to the manufacturer, a pooling equilibrium exists if

- each manufacturer orders the same quantity,
- each manufacturer receives the same rate.

**Lemma 4.3.** *For a given order quantity, $y$, $K_{\theta 1}(y) \geq K_{\theta P}(y) \geq K_{\theta 2}(y)$ and, for a given $\rho$ and order quantity, $y$, $K_{1\rho}(y) \geq K_{2\rho}(y)$.*

*Proof.* I first show that $K_{\theta\rho}(y)$ is non-decreasing in the value of $\pi_\rho^H$. Because $\pi_2^H \leq \pi_P^H \leq \pi_1^H$, this leads to $K_{\theta 1}(y) \geq K_{\theta P}(y) \geq K_{\theta 2}(y)$. Then I show that $K_{\theta\rho}(y)$ is non-decreasing in $\pi_\theta^H$. Because $\pi_2^H \leq \pi_1^H$, this leads to $K_{1\rho}(y) \geq K_{2\rho}(y)$.

If $y \in \left[L, \frac{pL}{Rw}\right]$, then $K_{\theta\rho}$ is independent of $\rho$. Therefore, $K_{\theta 1}(y) = K_{\theta P}(y) = K_{\theta 2}(y)$.

If $y \in \left(\frac{pL}{Rw}, \infty\right)$,

$$K_{\theta\rho} = E_\theta \left[\frac{\pi_\rho^H p \min(D, y) + \pi_\rho^L pL - Rwy}{\pi_\rho^H}\right]^+ \tag{4.81}$$

$$= \pi_\theta^H \left[\frac{\pi_\rho^H p \min(H, y) + \pi_\rho^L pL - Rwy}{\pi_\rho^H}\right]^+ + \pi_\theta^L \left(\frac{pL - Rwy}{\pi_\rho^H}\right)^+. \tag{4.82}$$

Recall that $y \in \left(\frac{pL}{Rw}, \infty\right)$. Therefore,

$$K_{\theta\rho} = \pi_\theta^H \left[\frac{\pi_\rho^H p \min(H, y) + \pi_\rho^L pL - Rwy}{\pi_\rho^H}\right]^+ \tag{4.83}$$

$$= \pi_\theta^H \left[(p \min(H, y) - pL) + \frac{pL - Rwy}{\pi_\rho^H}\right]^+. \tag{4.84}$$

Because $y \in \left(\frac{pL}{Rw}, \infty\right)$ and $\pi_\rho^H \geq 0$, this causes $K_{\theta\rho}$ to be non-decreasing in $\pi_\rho^H$. Recall that $\pi_2^H \leq \pi_P^H \leq \pi_1^H$. Therefore, $K_{\theta 1}(y) \geq K_{\theta P}(y) \geq K_{\theta 2}(y)$.

If $y \in \left[L, \frac{pL}{Rw}\right]$,

$$K_{\theta\rho} = E_\theta \left[p \min(D, y) - Rwy\right]^+, \tag{4.85}$$

$$=\pi_\theta^H \left[p\min\left(H,y\right)-Rwy\right]^+ + \pi_\theta^L \left(pL-Rwy\right), \tag{4.86}$$

$$=\pi_\theta^H \left\{\left[p\min\left(H,y\right)-Rwy\right]^+ - \left(pL-Rwy\right)\right\} + \left(pL-Rwy\right). \tag{4.87}$$

Recall that $y \geq L$ and $H > L$. Therefore, $p\min\left(H,y\right) \geq pL$. Because, $y \leq \frac{pL}{Rw}$, we can rewrite equation (4.87) in the following way:

$$K_{\theta\rho} =\pi_\theta^H \left[p\min\left(H,y\right)-pL\right] + \left(pL-Rwy\right). \tag{4.88}$$

Recall that $\pi_\theta^H \geq 0$ and $p\min\left(H,y\right) \geq pL$. This causes $K_{\theta\rho}$ to be non-decreasing in $\pi_\theta^H$. Because $\pi_1^H \geq \pi_2^H$, for any given $\rho$, this leads to $K_{1\rho}(y) \geq K_{2\rho}(y)$.

If $y \in \left(\frac{pL}{Rw}, \infty\right)$,

$$K_{\theta\rho} =\pi_\theta^H \left[\left(p\min\left(H,y\right)-pL\right) + \frac{pL-Rwy}{\pi_\rho^H}\right]^+. \tag{4.89}$$

Recall that $\pi_\theta^H \geq 0$. This causes $K_{\theta\rho}$ to be non-decreasing in $\pi_\theta^H$. Because $\pi_1^H \geq \pi_2^H$, for any given $\rho$, this leads to $K_{1\rho}(y) \geq K_{2\rho}(y)$. ■

Lemma 4.3 says that type-1 rates are preferred by all manufacturers. Lemma 4.3 also says that the objective value of a type-1 manufacturer is greater than or equal to the objective value of a type-2 manufacturer if both types of manufacturer order the same quantity and receive the same interest rate from the bank. This implies that, if both types receive the same rates, then a type-1 manufacturer is able to signal its type and receive type-1 rates by ordering order quantities that a type-2 manufacturer will find suboptimal to order.

For a given order quantity threshold imposed by the bank, a separating equilibrium exists if

- a type-2 manufacturer orders a quantity less than the threshold order quantity and, hence, receives a type-2 rate;

- a type-1 manufacturer orders a quantity greater than or equal to the threshold order quantity and, hence, receives a type-1 rate;
- no manufacturer in the economy prefers to choose an order quantity that will make it receive an interest rate that is intended for the other type of manufacturer.

When the bank anticipates not being able to determine the manufacturer's type by the manufacturer's order quantity, a pooling equilibrium exists if

- the bank offers the menu of pooling rates to the manufacturer,
- a type-2 manufacturer orders the same quantity as a type-1 manufacturer,
- a type-1 manufacturer prefers to receive a pooling rate instead of signaling its type to the bank to obtain a type-1 rate.

To prevent having more than one equilibrium for a given set of economic parameters, I make the following assumptions.

**Assumption 4.2.** *A type-2 manufacturer chooses an interest rate designed for its type instead of choosing other rates if the maximum value of its objective function when it chooses the set of rates designed for its type is equal the maximum value of its objective function when it chooses another set of rates,*

**Assumption 4.3.** *If, in equilibrium, a type-$\theta$ manufacturer chooses the rate designed for its type when the bank offers a menu of type-1 and type-2 rates to the manufacturer, the bank will never offer a single set of rates for which type-1 rates are offered for order quantities greater than or equal to a threshold order quantity and type-2 rates are offered for order quantities less than the threshold order quantity.*

Before summarizing the equilibria that I am investigating, let us introduce the notation that I use throughout the analysis:

- $y^T$ is, in a separating equilibrium, the smallest threshold order quantity that can be set by the bank to discourage a type-2 manufacturer from pretending to be a type-1 manufacturer.

- $y^T_{\theta 1}$ is the quantity $\left(y^T_{\theta 1} \geq y^T\right)$ that maximizes the objective value of a type-$\theta$ manufacturer when the bank expects to be able to determine a manufacturer's type by setting a threshold order quantity, $y^T$; $y^B_{11} \leq y^T = y^T_{11}$.

- $y^T_{\theta 2}$ is the quantity $\left(y^T_{\theta 2} < y^T\right)$ that maximizes the objective value of a type-$\theta$ manufacturer when the bank expects to be able to determine a manufacturer's type by setting a threshold order quantity, $y^T$; $y^B_{\theta 2} = y^T_{\theta 2} < y^T$.

- $L^T_{bm\theta\rho}$ is, for $\rho = 1$ and 2, the loan amount that a manufacturer of the type-$\theta$ requests from the bank when the bank sets a threshold order quantity and the manufacturer orders $y^T_{\theta\rho}$.

- $L^T_{sm\theta\rho}$ is, for $\rho = 1$ and 2, the loan amount that a manufacturer of the type-$\theta$ requests from the supplier when the bank sets a threshold order quantity and the manufacturer orders $y^T_{\theta\rho}$.

- $R^T_{bm\theta\rho}$ is, for $\rho = 1$ and 2, the interest rate that a manufacturer of the type-$\theta$ receives from the bank when the bank sets a threshold order quantity and the manufacturer orders $y^T_{\theta\rho}$ .

- $R^T_{sm\theta\rho}$ is, for $\rho = 1$ and 2, the interest rate that a manufacturer of the type-$\theta$ receives from the supplier when the bank sets a threshold order quantity and the manufacturer orders $y^T_{\theta\rho}$.

- $K_{\theta 1}^T$ is the objective value of a type-$\theta$ manufacturer when the bank sets a threshold order quantity and the manufacturer orders $y_{\theta 1}^T$; $K_{\theta 1}^T \leq K_{\theta 1}^B$.
- $K_{\theta 2}^T$ is the objective value of a type-$\theta$ manufacturer when the bank sets a threshold order quantity and the manufacturer orders $y_{\theta 2}^T$; $K_{\theta 2}^T = K_{\theta 2}^B$.

Below are the equilibria we investigate for.

**Separating equilibria when the bank offers a menu of type-1 and type-2 rates.**

$$\begin{cases} K_{22}^B \geq K_{21}^B, \\ K_{11}^B \geq K_{12}^B, \\ y_{22}^B \neq y_{11}^B. \end{cases} \tag{4.90}$$

**Pooling equilibria when the bank offers a menu of type-1 and type-2 rates.**

$$\begin{cases} K_{22}^B \geq K_{21}^B, \\ K_{11}^B \geq K_{12}^B, \\ y_{22}^B = y_{11}^B, \\ R_{bm22}^B = R_{bm11}^B. \end{cases} \tag{4.91}$$

**Separating equilibria when the bank offers a single set of rates.**

$$\begin{cases} K_{21}^B > K_{22}^B, \\ K_{22}^T > K_{21}^T, \\ K_{11}^T \geq K_{12}^T, \\ K_{11}^T \geq K_{1P}^T, \\ L \leq y_{22}^B = y_{22}^T \leq y_{11}^B < y^T = y_{11}^T. \end{cases} \tag{4.92}$$

**Pooling equilibria when the bank offers a single set of rates.**

$$\begin{cases} K_{21}^{B} > K_{22}^{B}, \\ K_{11}^{T} \geq K_{12}^{T}, \\ K_{1P}^{B} \geq K_{11}^{T}, \\ K_{2P} \geq K_{22}^{B}, \\ y_{2P} \ = \ y_{1P}^{B}. \end{cases} \tag{4.93}$$

To obtain the equilibria, I determine the best-response order quantities of each type of manufacturer to the set of interest rates designed for the other type. Then, I determine the best-response order quantities of type-1 manufacturers when all manufacturers are offered the set of pooling interest rates. Observe that pooling equilibria only exist when type-2 manufacturers do not suffer from pretending to be type-1 manufacturers and type-1 manufacturers prefer to use pooling rates instead of signaling their type. This is why, in pooling equilibria, the quantity ordered by all manufacturers in the economy is the best-response order quantity of type-1 manufacturers when all manufacturers are offered the set of pooling interest rates. Therefore, when all manufacturers are offered the set of pooling interest rates, I determine the order quantities of type-2 manufacturers based on the best-response quantities obtained for type-1 manufacturers. Afterwards, I provide conditions for

- equilibria when manufacturers of both types order their best-response order quantities and receive type-specific rates,
- equilibria when type-1 manufacturers signal their type with order quantities that are not their best-response order quantities to type-1 rates, and
- equilibria when all manufacturers order the same quantity and receive pooling

rates from banks.

Then, I present all the equilibria.

#### 4.4.3.1 Best-responses of manufacturers.

**Lemma 4.4.** *If a type-1 manufacturer receives the set of type-2 interest rates presented in equation* (4.59) *where* $L_{bm} = 0$ *and* $\rho = 2$*, then its best-response order quantity is the following:*

$$y_{12}^B = \begin{cases} H & when \quad L \leq \pi_2^H H \quad and \quad RwH \leq \pi_2^H pH, \\ \frac{pL}{Rw} & when \quad L \leq \pi_2^H H \quad and \quad \pi_2^H pH \leq RwH \leq \pi_1^H pH, \\ L & when \quad L \leq \pi_2^H H \quad and \quad \pi_1^H pH \leq RwH, \\ H & when \quad \pi_2^H H \leq L \leq \pi_1^H H \quad and \quad RwH \leq pL, \\ \frac{pL}{Rw} & when \quad \pi_2^H H \leq L \leq \pi_1^H H \quad and \quad pL \leq RwH \leq \pi_1^H pH, \\ L & when \quad \pi_2^H H \leq L \leq \pi_1^H H \quad and \quad \pi_1^H pH \leq RwH, \\ H & when \quad \pi_1^H H \leq L \quad and \quad RwH \leq \pi_1^H pH, \\ L & when \quad \pi_1^H H \leq L \quad and \quad \pi_1^H pH \leq RwH. \end{cases} \tag{4.94}$$

*and the objective value of the manufacturer is:*

$$K_{12}^B = \begin{cases} \pi_1^H pH + \pi_1^L pL - RwH & if \quad RwH \leq pL \quad and \quad y_{12}^B = H, \\ \pi_1^H \left( \frac{\pi_2^H pH + \pi_2^L pL - RwH}{\pi_2^H} \right) & if \quad pL < RwH \quad and \quad y_{12}^B = H, \\ \pi_1^H (p - Rw) \frac{pL}{Rw} & if \quad y_{12}^B = \frac{pL}{Rw}, \\ (p - Rw)L & if \quad y_{12}^B = L. \end{cases} \tag{4.95}$$

*Proof.* The expressions for the objective value of a type-1 manufacturer that receives pooling rates are obtained by setting $\rho$ to $P$ in equation (4.61). If $H \leq \frac{pL}{Rw}$, then for any $y \in [L, \infty)$, either $L \leq y < H$, $H \leq y \leq \frac{pL}{Rw}$ or $\frac{pL}{Rw} < y$. If $\frac{pL}{Rw} < H$, then for any $y \in [L, \infty)$, either $L \leq y \leq \frac{pL}{Rw}$, $\frac{pL}{Rw} < y \leq H$ or $H \leq y$. I consider these cases separately to study the best-response order quantities of type-1 manufacturers when they receive the set of type-2 rates. Then, I provide the corresponding objective value to the best-response order quantities of type-1 manufacturers.

If $H \leq \frac{pL}{Rw}$, then the objective value in equation (4.61) becomes:

$$K_{12} = \begin{cases} E_1\left[p\min\left(D, y\right) - Rwy\right]^+ & \text{when} \quad L \leq y \leq H, \\ E_1\left(pD - Rwy\right)^+ & \text{when} \quad H \leq y \leq \frac{pL}{Rw}, \\ E_1\left(\frac{\pi_2^H pD + \pi_2^L pL - Rwy}{\pi_2^H}\right)^+ & \text{when} \quad \frac{pL}{Rw} < y. \end{cases} \tag{4.96}$$

where $K_{12}$ is continuous in $y$.

Observe that, when $y \geq H$, $K_{12}$ is non-increasing in $y$.

When $y \in [L, H]$,

$$K_{12} = E_1\left[p\min\left(D, y\right) - Rwy\right]^+ \tag{4.97}$$

$$= \pi_1^H\left(py - Rwy\right)^+ + \pi_1^L\left(pL - Rwy\right)^+ \tag{4.98}$$

$$= \pi_1^H\left(py - Rwy\right) + \pi_1^L\left(pL - Rwy\right) \tag{4.99}$$

$$= \pi_1^H py + \pi_1^L pL - Rwy. \tag{4.100}$$

For all $y \in [L, H]$, this leads to $K_{12}$ increasing in $y$ when $Rw \leq \pi_1^H p$ and $K_{12}$ decreasing in $y$ when $\pi_1^H p \leq Rw$. Therefore, if $H \leq \frac{pL}{Rw}$, $y = H$ when $Rw \leq \pi_1^H p$ and $y = L$ when $\pi_1^H p \leq Rw$.

If $\frac{pL}{Rw} < H$, then the objective value in equation (4.61) becomes:

$$K_{12} = \begin{cases} E_1\left[p\min\left(D,y\right) - Rwy\right]^+ & \text{when} \quad L \le y \le \frac{pL}{Rw}, \\ E_1\left[\frac{\pi_2^H p\min(D,y)+\pi_2^L pL - Rwy}{\pi_2^H}\right]^+ & \text{when} \quad \frac{pL}{Rw} < y \le H, \\ E_1\left(\frac{\pi_2^H pD+\pi_2^L pL-Rwy}{\pi_2^H}\right)^+ & \text{when} \quad H \le y. \end{cases} \tag{4.101}$$

where $K_{12}$ is continuous in $y$.

Observe that, when $y \ge H$, $K_{12}$ is non-increasing in $y$.

When $y \in \left[L, \frac{pL}{Rw}\right]$,

$$K_{12} = E_1\left[p\min\left(D,y\right) - Rwy\right]^+ \tag{4.102}$$

$$= \pi_1^H py + \pi_1^L pL - Rwy. \tag{4.103}$$

$K_{12}$ is increasing in $y$ when $Rw \le \pi_1^H p$ and $K_{12}$ is decreasing in $y$ when $\pi_1^H p \le Rw$.

When $y \in \left(\frac{pL}{Rw}, H\right]$,

$$K_{12} = E_1\left[\frac{\pi_2^H p\min(D,y) + \pi_2^L pL - Rwy}{\pi_2^H}\right]^+ \tag{4.104}$$

$$= \pi_1^H\left(\frac{\pi_2^H py + \pi_2^L pL - Rwy}{\pi_2^H}\right)^+ + \pi_1^L\left(\frac{pL - Rwy}{\pi_2^H}\right)^+ \tag{4.105}$$

$$= \pi_1^H\left(\frac{\pi_2^H py + \pi_2^L pL - Rwy}{\pi_2^H}\right)^+. \tag{4.106}$$

$K_{12}$ is increasing in $y$ when $Rw \le \pi_2^H p$ and $K_{12}$ is decreasing in $y$ when $\pi_2^H p \le Rw$.

Recall that we are analyzing when $\frac{pL}{Rw} \le H$ and, by definition, $\pi_1^H$ is greater than or equal to $\pi_2^H$. Therefore, we have to consider $L \le \pi_2^H H$, $\pi_2^H H \le L \le \pi_1^H H$ and $\pi_1^H H \le L$ when considering conditions on $RwH$. By combining the results obtained for $y \in \left[L, \frac{pL}{Rw}\right]$, $y \in \left(\frac{pL}{Rw}, H\right]$ and $y \geq H$ we derive:

$$y_{12}^B = \begin{cases} H & \text{when} \quad L \leq \pi_2^H H \quad \text{and} \quad pL \leq RwH \leq \pi_2^H pH, \\ \frac{pL}{Rw} & \text{when} \quad L \leq \pi_2^H H \quad \text{and} \quad \pi_2^H pH \leq RwH \leq \pi_1^H pH, \\ L & \text{when} \quad L \leq \pi_2^H H \quad \text{and} \quad \pi_1^H pH \leq RwH, \\ \frac{pL}{Rw} & \text{when} \quad \pi_2^H H \leq L \leq \pi_1^H H \quad \text{and} \quad pL \leq RwH \leq \pi_1^H pH, \\ L & \text{when} \quad \pi_2^H H \leq L \leq \pi_1^H H \quad \text{and} \quad \pi_1^H pH \leq RwH, \\ L & \text{when} \quad \pi_1^H H \leq L \quad \text{and} \quad pL \leq RwH. \end{cases} \tag{4.107}$$

Combining the results obtained for $H \leq \frac{pL}{Rw}$ and $\frac{pL}{Rw} < H$ gives us the best-response order quantities of type-1 manufacturers when they receive the set of type-2 rates. We can now compute the objective value that corresponds to each best-response of a type-1 manufacturer that receives the set of type-2 rates.

When a type-1 manufacturer receives the set of type-2 rates, its objective value if $H \leq \frac{pL}{Rw}$ and it orders $H$ is:

$$K_{12}^B = \pi_1^H pH + \pi_1^L pL - RwH. \tag{4.108}$$

If $\frac{pL}{Rw} < H$ and it orders $H$, then the objective value is:

$$K_{12}^B = \pi_1^H \left( \frac{\pi_2^H pH + \pi_2^L pL - RwH}{\pi_2^H} \right)^+ . \tag{4.109}$$

Observe from equation (4.107) that, if $\frac{pL}{Rw} < H$ and $y_{21}^B = H$, then $Rw \leq \pi_2^H p$. Because $\pi_2^H pH \leq \pi_2^H pH + \pi_2^L pL$, we get:

$$K_{12}^B = \pi_1^H \left( \frac{\pi_2^H pH + \pi_2^L pL - RwH}{\pi_2^H} \right). \tag{4.110}$$

If a type-1 manufacturer orders $\frac{pL}{Rw}$, then the objective value is:

$$K_{12}^B = \pi_1^H \left( p\frac{pL}{Rw} - Rw\frac{pL}{Rw} \right) + \pi_1^L \left( pL - Rw\frac{pL}{Rw} \right) \tag{4.111}$$

$$=\pi_1^H (p - Rw) \frac{pL}{Rw}. \tag{4.112}$$

If it orders $L$, then the objective value is:

$$K_{12}^B = (p - Rw)L. \tag{4.113}$$

■

**Lemma 4.5.** *If a type-1 manufacturer receives the set of pooling interest rates given by equation (4.59) where $L_{bm} = 0$ and $\rho = P$, then its best-response order quantity is the following:*

$$y_{1P}^B = \begin{cases} H & when \quad L \le \pi_P^H H \quad and \quad RwH \le \pi_P^H pH, \\ \frac{pL}{Rw} & when \quad L \le \pi_P^H H \quad and \quad \pi_P^H pH \le RwH \le \pi_1^H pH, \\ L & when \quad L \le \pi_P^H H \quad and \quad \pi_1^H pH \le RwH, \\ H & when \quad \pi_P^H H \le L \le \pi_1^H H \quad and \quad RwH \le pL, \\ \frac{pL}{Rw} & when \quad \pi_P^H H \le L \le \pi_1^H H \quad and \quad pL \le RwH \le \pi_1^H pH, \\ L & when \quad \pi_P^H H \le L \le \pi_1^H H \quad and \quad \pi_1^H pH \le RwH, \\ H & when \quad \pi_1^H H \le L \quad and \quad RwH \le \pi_1^H pH, \\ L & when \quad \pi_1^H H \le L \quad and \quad \pi_1^H pH \le RwH. \end{cases} \tag{4.114}$$

*and the objective value of the manufacturer is:*

$$K_{1P}^B = \begin{cases} \pi_1^H pH + \pi_1^L pL - RwH & if \quad RwH \le pL \quad and \quad y_{1P}^B = H, \\ \pi_1^H \left( \frac{\pi_P^H pH + \pi^L pL - RwH}{\pi_P^H} \right)^+ & if \quad pL < RwH \quad and \quad y_{1P}^B = H, \\ \pi_1^H (p - Rw) \frac{pL}{Rw} & if \quad y_{1P}^B = \frac{pL}{Rw}, \\ (p - Rw)L & if \quad y_{1P}^B = L. \end{cases} \tag{4.115}$$

*Proof.* The expressions for the objective value of a type-1 manufacturer that receives pooling rates are obtained by setting $\rho$ to $P$ in equation (4.61). If $H \leq \frac{pL}{Rw}$, then for any $y \in [L, \infty)$, either $L \leq y \leq H$, $H \leq y \leq \frac{pL}{Rw}$ or $\frac{pL}{Rw} < y$. If $\frac{pL}{Rw} < H$, then for any $y \in [L, \infty)$, either $L \leq y \leq \frac{pL}{Rw}$, $\frac{pL}{Rw} < y \leq H$ or $H \leq y$. I consider these cases separately to study the best-response order quantities of type-1 manufacturers when they receive the set of pooling rates. Then, I provide the corresponding objective value to the best-response order quantities of type-1 manufacturers.

If $H \leq \frac{pL}{Rw}$, the objective value in equation (4.61) becomes:

$$K_{1P} = \begin{cases} E_1 \left[p \min\left(D, y\right) - Rwy\right]^+ & \text{when} \quad L \leq y \leq H, \\ E_1 \left(pD - Rwy\right)^+ & \text{when} \quad H \leq y \leq \frac{pL}{Rw}, \\ E_1 \left(\frac{\pi_P^H pD + \pi_P^L pL - Rwy}{\pi_P^H}\right)^+ & \text{when} \quad \frac{pL}{Rw} < y. \end{cases} \tag{4.116}$$

where $K_{1P}$ is continuous in $y$.

Observe that, when $y \geq H$, $K_{1P}$ is non-increasing in $y$.

When $y \in [L, H]$,

$$\begin{aligned} K_{1P} =& E_1 \left[p \min\left(D, y\right) - Rwy\right]^+ && (4.117) \\ =& \pi_1^H py + \pi_1^L pL - Rwy. && (4.118) \end{aligned}$$

For all $y \in [L, H]$, this leads to $K_{1P}$ increasing in $y$ when $Rw \leq \pi_1^H p$ and $K_{1P}$ decreasing in $y$ when $\pi_1^H p \leq Rw$. Therefore, if $H \leq \frac{pL}{Rw}$, $y = H$ when $Rw \leq \pi_1^H p$ and $y = L$ when $\pi_1^H p \leq Rw$.

If $\frac{pL}{Rw} < H$, the objective value in equation (4.61) becomes:

$$K_{1P} = \begin{cases} E_1 \left[p \min\left(D, y\right) - Rwy\right]^+ & \text{when} \quad L \leq y \leq \frac{pL}{Rw}, \\ E_1 \left[\frac{\pi_P^H p \min(D,y) + \pi_P^L pL - Rwy}{\pi_P^H}\right]^+ & \text{when} \quad \frac{pL}{Rw} < y \leq H, \\ E_1 \left(\frac{\pi_P^H pD + \pi_P^L pL - Rwy}{\pi_P^H}\right)^+ & \text{when} \quad H \leq y. \end{cases} \tag{4.119}$$

where $K_{1P}$ is continuous in $y$.

Observe that, when $y \geq H$, $K_{1P}$ is non-increasing in $y$.

When $y \in \left[L, \frac{pL}{Rw}\right]$,

$$K_{1P} = E_1 \left[p \min\left(D, y\right) - Rwy\right]^+ \tag{4.120}$$

$$= \pi_1^H py + \pi_1^L pL - Rwy. \tag{4.121}$$

$K_{1P}$ is increasing in $y$ when $Rw \leq \pi_1^H p$ and $K_{1P}$ is decreasing in $y$ when $\pi_1^H p \leq Rw$.

When $y \in \left(\frac{pL}{Rw}, H\right]$,

$$K_{1P} = E_1 \left[\frac{\pi_P^H p \min(D, y) + \pi_P^L pL - Rwy}{\pi_P^H}\right]^+ \tag{4.122}$$

$$= \pi_1^H \left(\frac{\pi_P^H py + \pi_P^L pL - Rwy}{\pi_P^H}\right)^+ + \pi_1^L \left(\frac{pL - Rwy}{\pi_P^H}\right)^+ \tag{4.123}$$

$$= \pi_1^H \left(\frac{\pi_P^H py + \pi_P^L pL - Rwy}{\pi_P^H}\right)^+. \tag{4.124}$$

$K_{1P}$ is increasing in $y$ when $Rw \leq \pi_P^H p$ and $K_{1P}$ is decreasing in $y$ when $\pi_P^H p \leq Rw$.

Recall that we are analyzing when $\frac{pL}{Rw} < H$ and, by definition, $\pi_1^H$ is greater than or equal to $\pi_P^H$. Therefore, when considering conditions on $RwH$ we have to consider when $L \leq \pi_P^H H$, $\pi_P^H H \leq L \leq \pi_1^H H$ and $\pi_1^H H \leq L$. By combining the results obtained for $y \in \left[L, \frac{pL}{Rw}\right]$, $y \in \left(\frac{pL}{Rw}, H\right]$ and $y \geq H$ we derive:

$$y_{1P}^B = \begin{cases} H & \text{when} \quad L \leq \pi_P^H H \quad \text{and} \quad pL \leq RwH \leq \pi_P^H pH, \\ \frac{pL}{Rw} & \text{when} \quad L \leq \pi_P^H H \quad \text{and} \quad \pi_P^H pH \leq RwH \leq \pi_1^H pH, \\ L & \text{when} \quad L \leq \pi_P^H H \quad \text{and} \quad \pi_1^H pH \leq RwH, \\ \frac{pL}{Rw} & \text{when} \quad \pi_P^H H \leq L \leq \pi_1^H H \quad \text{and} \quad pL \leq RwH \leq \pi_1^H pH, \\ L & \text{when} \quad \pi_P^H H \leq L \leq \pi_1^H H \quad \text{and} \quad \pi_1^H pH \leq RwH, \\ L & \text{when} \quad \pi_1^H H \leq L \quad \text{and} \quad pL \leq RwH. \end{cases} \tag{4.125}$$

Combining the results obtained for $H \leq \frac{pL}{Rw}$ and $\frac{pL}{Rw} < H$ gives us the best-response order quantities of type-1 manufacturers when they receive the set of pooling rates. We can now compute the objective value that corresponds to each best-response order quantities of a type-1 manufacturer that receives the set of pooling rates.

When a type-1 manufacturer receives the set of pooling rates, its objective value if $H \leq \frac{pL}{Rw}$ and it orders $H$ is:

$$K_{1P}^{B} = \pi_1^H pH + \pi_1^L pL - RwH. \tag{4.126}$$

If $\frac{pL}{Rw} < H$ and it orders $H$ the objective value is:

$$K_{1P}^{B} = \pi_1^H \left( \frac{\pi_P^H pH + \pi_P^L pL - RwH}{\pi_P^H} \right)^+. \tag{4.127}$$

Observe from equation (4.125) that, if $\frac{pL}{Rw} < H$ and $y_{2P}^B = H$, then $Rw \leq \pi_P^H p$. Because $\pi_P^H pH \leq \pi_P^H pH + \pi_P^L pL$, we get:

$$K_{1P}^{B} = \pi_1^H \left( \frac{\pi_P^H pH + \pi_P^L pL - RwH}{\pi_P^H} \right). \tag{4.128}$$

If a type-1 manufacturer orders $\frac{pL}{Rw}$ the objective value is:

$$K_{1P}^{B} = \pi_1^H (p - Rw) \frac{pL}{Rw}. \tag{4.129}$$

If it orders $L$ the objective value is:

$$K_{1P}^{B} = (p - Rw)L. \tag{4.130}$$

■

**Corollary 4.1.** *In equilibrium, if the type-1 and type-2 manufacturers are pooled* *together, then the optimal order quantity of type-2 manufacturers is the following:*

$$y_{2P} = \begin{cases} H & \text{when} \quad L \leq \pi_P^H H \quad \text{and} \quad RwH \leq \pi_P^H pH, \\ \frac{pL}{Rw} & \text{when} \quad L \leq \pi_P^H H \quad \text{and} \quad \pi_P^H pH \leq RwH \leq \pi_1^H pH, \\ L & \text{when} \quad L \leq \pi_P^H H \quad \text{and} \quad \pi_1^H pH \leq RwH, \\ H & \text{when} \quad \pi_P^H H \leq L \leq \pi_1^H H \quad \text{and} \quad RwH \leq pL, \\ \frac{pL}{Rw} & \text{when} \quad \pi_P^H H \leq L \leq \pi_1^H H \quad \text{and} \quad pL \leq RwH \leq \pi_1^H pH, \\ L & \text{when} \quad \pi_P^H H \leq L \leq \pi_1^H H \quad \text{and} \quad \pi_1^H pH \leq RwH, \\ H & \text{when} \quad \pi_1^H H \leq L \quad \text{and} \quad RwH \leq \pi_1^H pH, \\ L & \text{when} \quad \pi_1^H H \leq L \quad \text{and} \quad \pi_1^H pH \leq RwH. \end{cases} \tag{4.131}$$

*and the objective value of the manufacturer is:*

$$K_{2P} = \begin{cases} \pi_2^H pH + \pi_2^L pL - RwH & \text{if} \quad RwH \leq pL \quad \text{and} \quad y_{2P} = H, \\ \pi_2^H \left( \frac{\pi_P^H pH + \pi_P^L pL - RwH}{\pi_P^H} \right)^+ & \text{if} \quad pL < RwH \quad \text{and} \quad y_{2P} = H, \\ \pi_2^H (p - Rw) \frac{pL}{Rw} & \text{if} \quad y_{2P} = \frac{pL}{Rw}, \\ (p - Rw)L & \text{if} \quad y_{2P} = L. \end{cases} \tag{4.132}$$

*Proof.* In equilibrium, if the bank offers the set of pooling interest rates to manufacturers, $y_{2P} = y_{1p}^B$. Lemma 4.5 states the best-response order quantities and the corresponding objective value of type-1 manufacturers if the bank offers the set of pooling rates. Therefore, equaling $y_{2P}$ to the values obtained for $y_{1P}^B$ in Lemma 4.5, setting $\rho$ to $P$, and using the conditions in Lemma 4.5, gives us conditions for different values of $y_{2P}$ and $K_{2P}$. ∎

We have just described the best-response order quantities and their corresponding objective value for type-1 manufacturers when they receive the set of type-2 rates and the set of pooling rates. We have also described the order quantities of type-2 manufacturers and their corresponding objective value when type-2 manufacturers receive the set of pooling rates. Now, we will describe the best-response order quantities and their corresponding objective value for type-2 manufacturers when they receive the set of type-1 rates. Before analyzing the best-response order quantities and their corresponding value for type-2 manufacturers when type-2 manufacturers receive the set of type-1 rates, I denote $\pi^T := \frac{\pi_2^H\left(\pi_1^H H+\pi_1^L L\right)-\pi_1^H L}{\pi_2^H H-\pi_1^H L}$ and introduce the following lemma.

**Lemma 4.6.** *If $L \leq \pi_2^H H$, then $\pi_2^H \leq \pi^T \leq \pi_1^H$.*

*Proof.* Recall that $\pi^T = \frac{\pi_2^H\left(\pi_1^H H+\pi_1^L L\right)-\pi_1^H L}{\pi_2^H H-\pi_1^H L}$. Therefore, I need to show that, if $L \leq \pi_2^H H$,

$$\pi_2^H \leq \frac{\pi_2^H\left(\pi_1^H H+\pi_1^L L\right)-\pi_1^H L}{\pi_2^H H-\pi_1^H L} \tag{4.133}$$

and

$$\frac{\pi_2^H\left(\pi_1^H H+\pi_1^L L\right)-\pi_1^H L}{\pi_2^H H-\pi_1^H L} \leq \pi_1^H. \tag{4.134}$$

Recall that $\pi_1^H \leq 1$. Therefore, if $L \leq \pi_2^H H$, $\pi_2^H H - \pi_1^H L \geq 0$. This is why, if $L \leq \pi_2^H H$, equation (4.133) and equation (4.134) are equivalent to

$$\pi_2^H(\pi_2^H H - \pi_1^H L) \leq \pi_2^H\left(\pi_1^H H+\pi_1^L L\right)-\pi_1^H L \tag{4.135}$$

and

$$\pi_2^H\left(\pi_1^H H+\pi_1^L L\right)-\pi_1^H L \leq \pi_1^H(\pi_2^H H - \pi_1^H L). \tag{4.136}$$

Recall that, for any $\theta$, $\pi_\theta^L = 1 - \pi_\theta^H$. Therefore, observe with a little algebra that equation (4.135) and equation (4.136) are satisfied if

$$(\pi_1^H - \pi_2^H)L \leq (\pi_1^H - \pi_2^H)\pi_2^H H \tag{4.137}$$

and

$$\pi_1^H(\pi_1^H - \pi_2^H) \leq \pi_1^H - \pi_2^H. \tag{4.138}$$

Recall that $\pi_2^H \leq \pi_1^H$. Therefore, $(\pi_1^H - \pi_2^H) \geq 0$. This leads to equation (4.137) and equation (4.138) being equivalent to

$$L \leq \pi_2^H H \tag{4.139}$$

and

$$\pi_1^H \leq 1. \tag{4.140}$$

$\pi_1^H$ is always less than or equal to 1. Therefore, equation (4.140) is always satisfied and equation (4.139) is satisfied if $L \leq \pi_2^H H$. ■

**Lemma 4.7.** *If a type-2 manufacturer receives the set of type-1 interest rates given by equation* (4.59) *where* $L_{bm} = 0$ *and* $\rho = 1$, *then its best-response order quantity is the following:*

$$y_{21}^B = \begin{cases} H & \textit{when} \quad L \leq \pi_2^H H \quad \textit{and} \quad RwH \leq \pi^T pH, \\ L & \textit{when} \quad L \leq \pi_2^H H \quad \textit{and} \quad \pi^T pH \leq RwH, \\ H & \textit{when} \quad \pi_2^H H \leq L \quad \textit{and} \quad RwH \leq \pi_2^H pH, \\ L & \textit{when} \quad \pi_2^H H \leq L \quad \textit{and} \quad \pi_2^H pH \leq RwH. \end{cases} \tag{4.141}$$

*and the objective value of the manufacturer is:*

$$K_{21}^B = \begin{cases} \pi_2^H pH + \pi_2^L pL - RwH & \textit{if} \quad RwH \leq pL \quad \textit{and} \quad y_{21}^B = H, \\ \pi_2^H \left( \frac{\pi_1^H pH + \pi_1^L pL - RwH}{\pi_1^H} \right)^+ & \textit{if} \quad pL < RwH \quad \textit{and} \quad y_{21}^B = H, \\ (p - Rw)L & \textit{if} \quad y_{21}^B = L. \end{cases} \tag{4.142}$$

*Proof.* The expressions for the objective value of a type-2 manufacturer that receives type-1 rates are obtained by setting $\rho$ to 1 in equation (4.61). If $H \leq \frac{pL}{Rw}$, then for any $y \in [L, \infty)$, either $L \leq y \leq H$, $H \leq y \leq \frac{pL}{Rw}$ or $\frac{pL}{Rw} < y$. If $\frac{pL}{Rw} < H$, then for any $y \in [L, \infty)$, either $L \leq y \leq \frac{pL}{Rw}$, $\frac{pL}{Rw} < y \leq H$ or $H \leq y$. I consider these cases separately to study the best-response order quantities of type-1 manufacturers when they receive the set of pooling rates. Then, I provide the corresponding objective value to the best-response order quantities of type-1 manufacturers.

If $H \leq \frac{pL}{Rw}$, then the objective value in equation (4.61) becomes:

$$K_{21} = \begin{cases} E_2 \left[ p \min(D, y) - Rwy \right]^+ & \text{when} \quad L \leq y \leq H, \\ E_2 (pD - Rwy)^+ & \text{when} \quad H \leq y \leq \frac{pL}{Rw}, \\ E_2 \left( \frac{\pi_1^H pD + \pi_1^L pL - Rwy}{\pi_1^H} \right)^+ & \text{when} \quad \frac{pL}{Rw} < y. \end{cases} \tag{4.143}$$

where $K_{21}$ is continuous in $y$.

Observe that, when $y \geq H$, $K_{21}$ is non-increasing in $y$.

When $y \in [L, H]$,

$$K_{21} = E_2 \left[ p \min(D, y) - Rwy \right]^+ \tag{4.144}$$

$$= \pi_2^H py + \pi_2^L pL - Rwy. \tag{4.145}$$

For all $y \in [L, H]$, this leads to $K_{21}$ increasing in $y$ when $Rw \leq \pi_2^H p$ and $K_{21}$ decreasing in $y$ when $\pi_2^H p \leq Rw$. Therefore, if $H \leq \frac{pL}{Rw}$, $y = H$ when $Rw \leq \pi_2^H p$

and $y = L$ when $\pi_2^H p \leq Rw$.

If $\frac{pL}{Rw} < H$, then the objective value in equation (4.61) becomes:

$$K_{21} = \begin{cases} E_2 \left[p \min\left(D, y\right) - Rwy\right]^+ & \text{when} \quad L \leq y \leq \frac{pL}{Rw}, \\ E_2 \left[\frac{\pi_1^H p \min(D,y) + \pi_1^L pL - Rwy}{\pi_1^H}\right]^+ & \text{when} \quad \frac{pL}{Rw} < y \leq H, \\ E_2 \left(\frac{\pi_1^H pD + \pi_1^L pL - Rwy}{\pi_1^H}\right)^+ & \text{when} \quad H \leq y. \end{cases} \tag{4.146}$$

where $K_{21}$ is continuous in $y$.

Observe that, when $y \geq H$, $K_{21}$ is non-increasing in $y$.

When $y \in \left[L, \frac{pL}{Rw}\right]$,

$$K_{21} = E_2 \left[p \min\left(D, y\right) - Rwy\right]^+ \tag{4.147}$$

$$= \pi_2^H py + \pi_2^L pL - Rwy. \tag{4.148}$$

$K_{21}$ is increasing in $y$ when $Rw \leq \pi_2^H p$ and $K_{21}$ is decreasing in $y$ when $\pi_2^H p \leq Rw$.

When $y \in \left(\frac{pL}{Rw}, H\right]$,

$$K_{21} = E_2 \left[\frac{\pi_1^H p \min(D, y) + \pi_1^L pL - Rwy}{\pi_1^H}\right]^+ \tag{4.149}$$

$$= \pi_2^H \left(\frac{\pi_1^H py + \pi_1^L pL - Rwy}{\pi_1^H}\right)^+ + \pi_2^L \left(\frac{pL - Rwy}{\pi_1^H}\right)^+ \tag{4.150}$$

$$= \pi_2^H \left(\frac{\pi_1^H py + \pi_1^L pL - Rwy}{\pi_1^H}\right)^+. \tag{4.151}$$

$K_{21}$ is increasing in $y$ when $Rw \leq \pi_1^H p$ and $K_{21}$ is decreasing in $y$ when $\pi_1^H p \leq Rw$.

Recall that we are analyzing when $\frac{pL}{Rw} < H$ and, by definition, $\pi_1^H$ is greater than or equal to $\pi_2^H$. Therefore, when considering conditions on $RwH$ we have to consider when $L \leq \pi_2^H H$, $\pi_2^H H \leq L \leq \pi_1^H H$ and $\pi_1^H H \leq L$. By combining the results obtained for $y \in \left[L, \frac{pL}{Rw}\right]$, $y \in \left(\frac{pL}{Rw}, H\right]$ and $y \geq H$ we derive:

$$y_{21}^B = \begin{cases} H & \text{when} \quad L \leq \pi_2^H H \quad \text{and} \quad pL \leq RwH \leq \pi_2^H pH, \\ H \text{ or } L & \text{when} \quad L \leq \pi_2^H H \quad \text{and} \quad \pi_2^H pH \leq RwH \leq \pi_1^H pH, \\ L & \text{when} \quad L \leq \pi_2^H H \quad \text{and} \quad \pi_1^H pH \leq RwH, \\ L & \text{when} \quad \pi_2^H H \leq L \quad \text{and} \quad pL \leq RwH. \end{cases} \tag{4.152}$$

A type-2 manufacturer orders $L$ instead of $H$ if its objective value when it orders $L$ is greater than or equal to its objective value when it orders $H$. Therefore, based on the expressions for the objective value of type-2 manufacturers, when $L \leq \pi_2^H H$ and $\pi_2^H p \leq Rw \leq \pi_1^H p$, a type-2 manufacturer orders $L$ instead of $H$ iff

$$\pi_2^H pL + \pi_2^L pL - RwL \geq \pi_2^H \left(\frac{\pi_1^H pH + \pi_1^L pL - RwH}{\pi_1^H}\right)^+ \tag{4.153}$$

and orders $H$ otherwise.

Because $\pi_1^H pH \leq \pi_1^H pH + \pi_1^L pL$,

$$\pi_2^H \left(\frac{\pi_1^H pH + \pi_1^L pL - RwH}{\pi_1^H}\right)^+ = \pi_2^H \left(\frac{\pi_1^H pH + \pi_1^L pL - RwH}{\pi_1^H}\right). \tag{4.154}$$

Hence, when $L \leq \pi_2^H H$ and $\pi_2^H p \leq Rw \leq \pi_1^H p$, a manufacturer orders $L$ instead of $H$ if

$$\pi_2^H pL + \pi_2^L pL - RwL \geq \pi_2^H \left(\frac{\pi_1^H pH + \pi_1^L pL - RwH}{\pi_1^H}\right). \tag{4.155}$$

Recall that, for any $\theta$, $\pi_\theta^L = 1 - \pi_\theta^H$. Therefore, observe with a little algebra that equation (4.155) is satisfied if

$$\left(\pi_2^H H - \pi_1^H L\right) Rw \geq \left[\pi_2^H \left(\pi_1^H H + \pi_1^L L\right) - \pi_1^H L\right] p \tag{4.156}$$

Recall that $\pi^T = \frac{\pi_2^H \left(\pi_1^H H + \pi_1^L L\right) - \pi_1^H L}{\pi_2^H H - \pi_1^H L}$ and $\pi_1^H \leq 1$. Because $\pi_1^H \leq 1$ and we are in the case when $L \leq \pi_2^H H$, $\pi_2^H H - \pi_1^H L$ is always greater than or equal to 0. Therefore, equation (4.156) is equivalent to

$$Rw \geq \pi^T p. \tag{4.157}$$

Recall from Lemma 4.6 that $\pi_2^H \leq \pi^T \leq \pi_1^H$. Therefore, equation (4.157) holds when $L \leq \pi_2^H H$ and $\pi^T p \leq Rw \leq \pi_1^H p$. This leads to

$$y_{21}^B = \begin{cases} H & \text{when} \quad L \leq \pi_2^H H \quad \text{and} \quad pL \leq RwH \leq \pi^T pH, \\ L & \text{when} \quad L \leq \pi_2^H H \quad \text{and} \quad \pi^T pH \leq RwH \leq \pi_1^H pH. \end{cases} \tag{4.158}$$

Combining the results obtained for $H \leq \frac{pL}{Rw}$ and $\frac{pL}{Rw} < H$ gives us the best-response order quantities of type-2 manufacturers to when they receive the set of type-1 rates designed for type-1 manufacturers. We can now compute the objective value that corresponds to each best-response of a type-2 manufacturer that receives the set of type-1 rates.

When a type-2 manufacturer receives the set of type-1 rates, its objective value if $H \leq \frac{pL}{Rw}$ and it orders $H$ is:

$$K_{21}^B = \pi_2^H pH + \pi_2^L pL - RwH. \tag{4.159}$$

If $\frac{pL}{Rw} < H$ and it orders $H$ the objective value is:

$$K_{21}^B = \pi_2^H \left( \frac{\pi_1^H pH + \pi_1^L pL - RwH}{\pi_1^H} \right)^+. \tag{4.160}$$

Observe from equation (4.152) that, if $\frac{pL}{Rw} < H$ and $y_{21}^B = H$, then $Rw \leq \pi_1^H p$. Because $\pi_1^H pH \leq \pi_1^H pH + \pi_1^L pL$, we get:

$$K_{21}^B = \pi_2^H \left( \frac{\pi_1^H pH + \pi_1^L pL - RwH}{\pi_1^H} \right). \tag{4.161}$$

If it orders $L$ the objective value is:

$$K_{21}^B = (p - Rw)L. \tag{4.162}$$

■

#### 4.4.3.2 Equilibria.

**Lemma 4.8.** *A type-1 manufacturer does not have to consider signaling its type with order quantities that are not its best-response order quantities to the set of type-1 rates:*

$$if \begin{cases} H \leq \frac{pL}{Rw}, \quad or \\ \frac{pL}{Rw} < H \quad and \quad L \leq \pi_2^H H \quad and \quad \pi^T pH \leq RwH, \quad or \\ \frac{pL}{Rw} < H \quad and \quad \pi_2^H H \leq L. \end{cases} \tag{4.163}$$

*Proof.* I show that, if (4.163) holds, then type-2 manufacturers do not benefit by pretending to be type-1 manufacturers and, hence, type-1 manufacturers do not have to consider signaling their type with order quantities that are not their best-response order quantities to the set of type-1 rates.

**Case:** $H \leq \frac{pL}{Rw}$.

Proposition 4.1 and Lemma 4.7 state that, if $H \leq \frac{pL}{Rw}$ and $Rw \leq \pi_2^H p$, then $K_{22}^B = K_{21}^B = \pi_2^H pH + \pi_2^L pL - RwH$. Also, Proposition 4.1 and Lemma 4.7 state that, if $H \leq \frac{pL}{Rw}$ and $\pi_2^H p \leq Rw$, then $K_{22}^B = K_{21}^B = (p - Rw)L$. Therefore, type-2 manufacturers do not benefit by pretending to be type-1 manufacturers.

**Case:** $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $\pi^T pH \leq RwH$.

Proposition 4.1 and Lemma 4.7 state that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $\pi^T pH \leq RwH$, then $K_{22}^B = K_{21}^B = (p - Rw)L$. Therefore, type-2 manufacturers do not benefit by pretending to be type-1 manufacturers.

**Case:** $\frac{pL}{Rw} < H$ and $\pi_2^H H \leq L$.

According to Proposition 4.1 and Lemma 4.7, if $\frac{pL}{Rw} < H$, $\pi_2^H H \leq L$ and $\pi_2^H pH \leq RwH$, then $K_{22}^B = K_{21}^B = (p - Rw)L$. Therefore, type-2 manufacturers do not benefit by pretending to be type-1 manufacturers. ∎

**Proposition 4.2.** *If $H \leq \frac{pL}{Rw}$ and $Rw \leq \pi_2^H p$, then there are pooling equilibria where all manufacturers in the economy order $H$, and the interest rate on the loan repayment of each manufacturer is the bank's cost of capital, $R$.*

*Proof.* I show that, if $H \leq \frac{pL}{Rw}$ and $Rw \leq \pi_2^H p$, there are pooling equilibria where all manufacturers in the economy order $H$, and the interest rate on the loan repayment of each manufacturer is the bank's cost of capital, $R$, because all the conditions in equation (4.91) are satisfied.

According to Lemma 4.8, if $H \leq \frac{pL}{Rw}$, a type-1 manufacturer does not have to consider signaling its type to the bank with order quantities that are not its best-response order quantities to type-1 rates because, if $H \leq \frac{pL}{Rw}$, type-2 manufacturers do not benefit by pretendingto be type-1 manufacturers. This leads to equilibria where the bank offers a menu of type-1 and type-2 rates to the manufacturer, the manufacturer chooses the set of rates designed for its type, and the manufacturer orders its best-response order quantity to the set of interest rates it receives from the bank.

According to Proposition 4.1, if $H \leq \frac{pL}{Rw}$ and $Rw \leq \pi_2^H p$, then $y_{11}^B = H$, $K_{11}^B = \pi_1^H pH + \pi_1^L pL - RwH$, $y_{22}^B = H$, and $K_{22}^B = \pi_2^H pH + \pi_2^L pL - RwH$. Recall from Lemma 4.4 and Lemma 4.7 that, if $H \leq \frac{pL}{Rw}$ and $Rw \leq \pi_2^H p$, then $y_{12}^B = H$, $K_{12}^B = \pi_1^H pH + \pi_1^L pL - RwH$, $y_{21} = H$, and $K_{21} = \pi_2^H pH + \pi_2^L pL - RwH$.

Observe that $K_{22}^B = K_{21}^B$ and $y_{22}^B = y_{11}^B$. Therefore, the only equilibria that can exist, if $H \leq \frac{pL}{Rw}$, $Rw \leq \pi_2^H p$, are pooling equilibria that satisfy equation (4.91). Observe from equation (4.59) that the interest rate on the loan repayment of each manufacturer is the bank's cost of capital, $R$, because when $H \leq \frac{pL}{Rw}$, there is no risk of default when manufacturers order $H$. Because $K_{11}^B = K_{12}^B$ and $R_{bm22}^B = R_{bm11}^B$, then all the conditions in equation (4.91) are satisfied. Therefore, there are pooling equilibria where all manufacturers in the economy order $H$, and the interest rate on the loan repayment of each manufacturer is the bank's cost of capital, $R$. ■

**Proposition 4.3.** *If $H \leq \frac{pL}{Rw}$ and $\pi_2^H p \leq Rw \leq \pi_1^H p$, then there are separating equilibria where type-1 manufacturers order $H$, type-2 manufacturers order $L$, and the interest rate on the loan repayment of each manufacturer is the bank's cost of capital, $R$.*

*Proof.* I show that, if $H \leq \frac{pL}{Rw}$ and $\pi_2^H p \leq Rw \leq \pi_1^H p$, then there are separating equilibria where type-1 manufacturers order $H$, type-2 manufacturers order $L$, and the interest rate on the loan repayment of each manufacturer is the bank's cost of capital, $R$, because all the conditions in equation (4.90) are satisfied.

According to Lemma 4.8, if $H \leq \frac{pL}{Rw}$, a type-1 manufacturer does not have to consider signaling its type to the bank with order quantities that are not its best-response order quantities to type-1 rates because, if $H \leq \frac{pL}{Rw}$, type-2 manufacturers do not benefit by pretendingto be type-1 manufacturers. This leads to equilibria where the bank offers a menu of type-1 and type-2 rates to the manufacturer, the manufacturer choses the set of rates designed for its type, and the best-response order quantity of a type-1 manufacturer is different than the best-response order quantity of a type-2 manufacturer.

According to Proposition 4.1, if $H \leq \frac{pL}{Rw}$ and $\pi_2^H p \leq Rw \leq \pi_1^H p$, then $y_{11}^B = H$, $K_{11}^B = \pi_1^H pH + \pi_1^L pL - RwH$, $y_{22}^B = L$ and $K_{22}^B = (p - Rw)L$. Recall from Lemma 4.4 and Lemma 4.7 that, if $H \leq \frac{pL}{Rw}$ and $\pi_2^H p \leq Rw \leq \pi_1^H p$, then $y_{1P}^B = H$, $K_{12}^B = \pi_1^H pH + \pi_1^L pL - RwH$, $y_{21}^B = L$, and $K_{21}^B = (p - Rw)L$.

Observe that $K_{22}^B = K_{21}^B$ and $y_{22}^B \neq y_{11}^B$. Therefore, the only equilibria that can exist, if $H \leq \frac{pL}{Rw}$, $\pi_2^H p \leq Rw \leq \pi_1^H p$, are separating equilibria that satisfy equation

(4.90). Because $K_{11}^B = K_{12}^B$, then all the conditions in equation (4.90) are satisfied.

Observe from equation (4.59) that the interest rate on the loan repayment of each manufacturer is the bank's cost of capital, $R$, because when $H \leq \frac{pL}{Rw}$, there is no risk of default when manufacturers order $H$. This leads to separating equilibria where type-1 manufacturers order $H$, type-2 manufacturers order $L$, and the interest rate on the loan repayment of each manufacturer is the bank's cost of capital, $R$. ■

**Proposition 4.4.** *If $\pi_1^H p \leq Rw$, then there are pooling equilibria where all manufacturers in the economy order $L$, and the interest rate on the loan repayment of each manufacturer is the bank's cost of capital, $R$.*

*Proof.* I show that, if $\pi_1^H p \leq Rw$, then there are pooling equilibria where all manufacturers in the economy order $H$, and the interest rate on the loan repayment of each manufacturer is the bank's cost of capital, $R$, because all the conditions in (4.91) are satisfied.

According to Lemma 4.8,

$$if \begin{cases} H \leq \frac{pL}{Rw}, \quad or \\ \frac{pL}{Rw} < H \quad and \quad L \leq \pi_2^H H \quad and \quad \pi^T pH \leq RwH, \quad or \\ \frac{pL}{Rw} < H \quad and \quad \pi_2^H H \leq L. \end{cases} \tag{4.164}$$

a type-1 manufacturers does not have to consider signaling its type to the bank with order quantities that are not its best-response order quantities to type-1 rates because, under the conditions in Lemma 4.8, type-2 manufacturers do not benefit by pretendingto be type-1 manufacturers. This leads to equilibria where the bank offers a menu of type-1 and type-2 rates to the manufacturer, the manufacturer chooses the set of rates designed for its type, and the manufacturer orders its best-response order quantity to the set of interest rates it receives from the bank.

According to Proposition 4.1 and Lemma 4.4, if $\pi_1^H p \leq Rw$, then $y_{11}^B = y_{22}^B = y_{12}^B = L$ and $K_{11}^B = K_{22}^B = K_{12}^B = (p - Rw)L$.

Recall from Lemma 4.7 that,

$$y_{21}^B = \begin{cases} L & \text{when} \quad L \leq \pi_2^H H \quad \text{and} \quad \pi^T pH \leq RwH, \\ L & \text{when} \quad \pi_2^H H \leq L \quad \text{and} \quad \pi_2^H pH \leq RwH. \end{cases} \tag{4.165}$$

Recall that $\pi_2^H \leq \pi_1^H$ and from Lemma 4.6 that if $L \leq \pi_2^H H$, then $\pi_2^H \leq \pi^T \leq \pi_1^H$. Therefore, if $\pi_1^H p \leq Rw$, then $y_{21}^B = L$ and $K_{21}^B = (p - Rw)L$.

Observe that $K_{22}^B = K_{21}^B$ and $y_{22}^B = y_{11}^B$. Therefore, the only equilibria that can exist, if $\pi_1^H p \leq Rw$, are pooling equilibria that satisfy equation (4.91). Observe from equation (4.59) that the interest rate on the loan repayment of each manufacturer is the bank's cost of capital, $R$, because when a manufacturer orders $L$, there is no risk of default. Because $K_{11}^B = K_{12}^B$ and $R_{bm22}^B = R_{bm11}^B$, then all the conditions in equation (4.91) are satisfied. Therefore, there are pooling equilibria where all manufacturers in the economy order $L$, and the interest rate on the loan repayment of each manufacturer is the bank's cost of capital, $R$. ∎

**Proposition 4.5.** *There are separating equilibria where a type-1 manufacturer orders $H$, a type-2 manufacturer orders $L$, the interest rate on the loan repayment of a type-1 manufacturer is $\frac{RwH - \pi_1^L pL}{\pi_1^H wH}$, and the interest rate on the loan repayment of a type-2 manufacturer is the bank's cost of capital, $R$,*

$$if \begin{cases} \frac{pL}{Rw} < H \;\; and \;\; L \leq \pi_2^H H \;\; and \;\; \pi^T pH \leq RwH \leq \pi_1^H pH, \;\; or \\ \frac{pL}{Rw} < H \;\; and \;\; \pi_2^H H \leq L \;\; and \;\; RwH \leq \pi_1^H pH. \end{cases} \tag{4.166}$$

*Proof.* I show that, if (4.166) holds, then there are separating equilibria where a type-1 manufacturer orders $H$, a type-2 manufacturer orders $L$, the interest rate on the loan repayment of a type-1 manufacturer is $\frac{RwH-\pi_1^L pL}{\pi_1^H wH}$, and the interest rate on the loan repayment of a type-2 manufacturer is the bank's cost of capital, $R$, because all the conditions in equation (4.90) are satisfied.

According to Lemma 4.8, if (4.166) holds, then a type-1 manufacturer does not have to consider signaling its type to the bank with order quantities that are not its best-response order quantities to type-1 rates because, if (4.166) holds, then type-2 manufacturers do not benefit by pretending to be type-1 manufacturers. This leads to equilibria where the bank offers a menu of type-1 and type-2 rates to the manufacturer, the manufacturer chooses the set of rates designed for its type, and the best-response order quantity of a type-1 manufacturer is different than the best-response order quantity of a type-2 manufacturer.

According to Proposition 4.1, if $\frac{pL}{Rw} < H$ and $\pi_2^H p \leq Rw \leq \pi_1^H p$, then $y_{11}^B = H$, $K_{11}^B = \pi_1^H pH + \pi_1^L pL - RwH$, $y_{22}^B = L$ and $K_{22}^B = (p - Rw)L$. Recall, from Lemma 4.4 and Lemma 4.7, that

$$y_{12}^B = \begin{cases} \frac{pL}{Rw} & \text{when} \quad L \leq \pi_2^H H \quad \text{and} \quad \pi_2^H pH \leq RwH \leq \pi_1^H pH, \\ \frac{pL}{Rw} & \text{when} \quad \pi_2^H H \leq L \quad \text{and} \quad pL \leq RwH \leq \pi_1^H pH, \end{cases} \tag{4.167}$$

and

$$y_{21}^B = \begin{cases} L & \text{when} \quad L \leq \pi_2^H H \quad \text{and} \quad \pi^T pH \leq RwH, \\ L & \text{when} \quad \pi_2^H H \leq L \quad \text{and} \quad \pi_2^H pH \leq RwH. \end{cases} \tag{4.168}$$

Recall that $\pi_2^H \leq \pi_1^H$ and from Lemma 4.6 that, if $L \leq \pi_2^H H$, then $\pi_2^H \leq \pi^T \leq \pi_1^H$. Therefore, if (4.166) holds, then $y_{12}^B = \frac{pL}{Rw}$, $K_{12}^B = \pi_1^H (p - Rw)\frac{pL}{Rw}$, $y_{21}^B = L$, and $K_{21}^B = (p - Rw)L$. Observe that $K_{22}^B = K_{21}^B$ and $y_{22}^B \neq y_{11}^B$. Therefore, the only equilibria that can exist if (4.166) holds are separating equilibria that satisfy the conditions in (4.90). Let us verify if the conditions in (4.90) are satisfied.

Substituting $K^B_{11}$, $K^B_{12}$, $K^B_{22}$, $K^B_{21}$, $y^B_{11}$ and $y^B_{22}$ in the conditions in (4.90) by their values gives

$$(p - Rw)L \geq (p - Rw)L \qquad \text{and} \tag{4.169}$$

$$\pi_1^H pH + \pi_1^L pL - RwH \geq \pi_1^H (p - Rw)\frac{pL}{Rw} \qquad \text{and} \tag{4.170}$$

$$L \neq H. \tag{4.171}$$

It is obvious that the first and third conditions in (4.169) are satisfied. Let us determine if and when the second condition in equation (4.169) is satisfied. Recall that, for any $\theta$, $\pi_\theta^L = 1 - \pi_\theta^H$. Therefore, observe with a little algebra that the second condition in equation (4.169) is satisfied if

$$(\pi_1^H p - Rw)H \geq (\pi_1^H p - Rw)\frac{pL}{Rw}. \tag{4.172}$$

Recall that $Rw \leq \pi_1^H p$. Therefore, $\pi_1^H p - Rw \geq 0$. This leads to the inequalities in (4.172) being equivalent to

$$H \geq \frac{pL}{Rw}. \tag{4.173}$$

Recall that $\frac{pL}{Rw} < H$. Therefore, equation (4.173) is satisfied.

Observe from equation (4.59) that the interest rate on the loan repayment of a type-1 manufacturer is $\frac{RwH - \pi_1^L pL}{\pi_1^H wH}$ and the interest rate on the loan repayment of a type-2 manufacturer is the bank's cost of capital, $R$. This leads to separating equilibria where type-1 manufacturers order $H$, type-2 manufacturers order $L$, the interest rate on the loan repayment of a type-1 manufacturer is $\frac{RwH - \pi_1^L pL}{\pi_1^H wH}$, and the interest rate on the loan repayment of a type-2 manufacturer is the bank's cost of capital, $R$. ■

We have presented in Proposition 4.2, Proposition 4.3, Proposition 4.4 and Proposition 4.5 all the equilibria when it is optimal, if presented with a menu of type-1 rates and type-2 rates, for type-2 manufacturers to choose the set of type-2 rates. To simplify the analysis of when it is optimal, if presented with a menu of type-1 rates and a menu of type-2 rates, for type-2 manufacturers to choose the set of type-1 rates it is convenient to denote $\pi^P := \frac{\pi_2^H\left(\pi_P^H H+\pi_P^L L\right)-\pi_P^H L}{\pi_2^H H-\pi_P^H L}$, $\overline{y}^T := \frac{\pi_2^H\left(\pi_1^H pH+\pi_1^L pL\right)-\pi_1^H (p-Rw)L}{\pi_2^H Rw}$, $\overline{R}^T := \frac{R[\pi_2^H pH-(p-Rw)L]}{\pi_2^H\left(\pi_1^H pH+\pi_1^L pL\right)-\pi_1^H (p-Rw)L}$ because these expressions appear more than once in the analysis. Similarly, I introduce Lemma 4.9, Lemma 4.10, Lemma 4.11, Lemma 4.12 and Lemma 4.13 to facilitate the exposition of the results.

**Lemma 4.9.** *If $L \leq \pi_2^H H$, then $\pi_2^H \leq \pi^P \leq \min\left(\pi_P^H, \pi^T\right)$.*

*Proof.* Recall that $\pi^P = \frac{\pi_2^H\left(\pi_P^H H+\pi_P^L L\right)-\pi_P^H L}{\pi_2^H H-\pi_P^H L}$ and $\pi^T = \frac{\pi_2^H\left(\pi_1^H H+\pi_1^L L\right)-\pi_1^H L}{\pi_2^H H-\pi_1^H L}$. Therefore, we need to show that, if $L \leq \pi_2^H H$,

$$\pi_2^H \leq \frac{\pi_2^H\left(\pi_P^H H+\pi_P^L L\right)-\pi_P^H L}{\pi_2^H H-\pi_P^H L}, \tag{4.174}$$

$$\frac{\pi_2^H\left(\pi_P^H H+\pi_P^L L\right)-\pi_P^H L}{\pi_2^H H-\pi_P^H L} \leq \pi_P^H, \tag{4.175}$$

and

$$\frac{\pi_2^H\left(\pi_P^H H+\pi_P^L L\right)-\pi_P^H L}{\pi_2^H H-\pi_P^H L} \leq \frac{\pi_2^H\left(\pi_1^H H+\pi_1^L L\right)-\pi_1^H L}{\pi_2^H H-\pi_1^H L}. \tag{4.176}$$

Recall that $\pi_P^H \leq \pi_1^H \leq 1$. Therefore, if $L \leq \pi_2^H H$, $\pi_2^H H - \pi_P^H L \geq 0$. This leads to $\pi_2^H H - \pi_1^H L \geq 0$. This is why, if $L \leq \pi_2^H H$, then equation (4.174), equation (4.175) and equation (4.176) are equivalent to

$$\pi_2^H(\pi_2^H H - \pi_P^H L) \leq \pi_2^H\left(\pi_P^H H+\pi_P^L L\right)-\pi_P^H L, \tag{4.177}$$

$$\pi_2^H\left(\pi_P^H H+\pi_P^L L\right)-\pi_P^H L \leq \pi_P^H(\pi_2^H H - \pi_P^H L), \tag{4.178}$$

and

$$(\pi_2^H H - \pi_1^H L)[\pi_2^H \left(\pi_P^H H + \pi_P^L L\right) - \pi_P^H L] \leq (\pi_2^H H - \pi_P^H L)[\pi_2^H \left(\pi_1^H H + \pi_1^L L\right) - \pi_1^H L] \tag{4.179}$$

respectively. Recall that, for any $\theta$, $\pi_\theta^L = 1 - \pi_\theta^H$. Therefore, observe with a little algebra that equation (4.177), equation (4.178) and equation (4.179) are satisfied if

$$(\pi_P^H - \pi_2^H)L \leq (\pi_P^H - \pi_2^H)H, \tag{4.180}$$

$$(\pi_P^H - \pi_2^H)\pi_P^H \leq (\pi_P^H - \pi_2^H), \tag{4.181}$$

and

$$(H - L)L \leq \pi_2^H H(H - L). \tag{4.182}$$

Recall that $L \leq H$ and $\pi_2^H \leq \pi_P^H$. Therefore, $(\pi_P^H - \pi_2^H) \geq 0$. This leads to equation (4.180), equation (4.181) and equation (4.182) being equivalent to

$$L \leq H, \tag{4.183}$$

$$\pi_P^H \leq 1, \tag{4.184}$$

and

$$L \leq \pi_2^H H. \tag{4.185}$$

$L$ is always less than or equal to $H$ and $\pi_P^H$ is always less than or equal to 1. Therefore, equation (4.183), equation (4.184) are always satisfied and equation (4.185) is satisfied if $L \leq \pi_2^H H$. ■

**Lemma 4.10.** *If $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $RwH \leq \pi^T pH$, then a type-1 manufacturer has to consider signaling its type with order quantities that are not its best-response order quantities to the set of type-1 rates.*

*Proof.* I show that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $RwH \leq \pi^T pH$, then $K_{21}^B > K_{21}^B$. From Lemma 4.6, if $L \leq \pi_2^H H$, then $\pi^T \geq \pi_2^H$. Therefore, I show that,

$$if \begin{cases} \frac{pL}{Rw} < H \quad and \quad L \leq \pi_2^H H \quad and \quad RwH \leq \pi_2^H pH, \quad or \\ \frac{pL}{Rw} < H \quad and \quad L \leq \pi_2^H H \quad and \quad \pi_2^H pH \leq RwH \leq \pi^T pH, \end{cases} \tag{4.186}$$

then $K_{21}^B > K_{21}^B$.

**Case:** $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $RwH \leq \pi_2^H pH$.

Proposition 4.1 and Lemma 4.7 state that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $RwH \leq \pi_2^H pH$, $y_{22}^B = y_{21}^B = H$, $K_{22}^B = \pi_2^H pH + \pi_2^L pL - RwH$ and $K_{21}^B = \pi_2^H \left(\frac{\pi_1^H pH + \pi_1^L pL - RwH}{\pi_1^H}\right)$. Therefore, $K_{21}^B > K_{22}^B$ if

$$\pi_2^H \left(\frac{\pi_1^H pH + \pi_1^L pL - RwH}{\pi_1^H}\right) > \pi_2^H pH + \pi_2^L pL - RwH. \tag{4.187}$$

Recall that, for any $\theta$, $\pi_\theta^L = 1 - \pi_\theta^H$. Therefore, observe with a little algebra that equation (4.187) is satisfied if

$$\pi_2^H (pL - RwH) > \pi_1^H (pL - RwH). \tag{4.188}$$

Recall that $\frac{pL}{Rw} < H$. Therefore, $pL - RwH < 0$ and equation (4.188) is satisfied if

$$\pi_2^H < \pi_1^H. \tag{4.189}$$

$\pi_2^H$ is always less than $\pi_1^H$. Therefore, equation (4.189) is always satisfied. Hence, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $RwH \leq \pi_2^H pH$, type-2 manufacturers would always prefer to order $H$ and receive a type-1 rate instead of ordering $H$ and receiving the type-2 rate.

**Case:** $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $\pi_2^H pH \leq RwH \leq \pi^T pH$.

Proposition 4.1 and Lemma 4.7 state that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $\pi_2^H pH \leq RwH \leq \pi^T pH$, $K_{22}^B = (p - Rw)L$ and $K_{21}^B = \pi_2^H \left(\frac{\pi_1^H pH + \pi_1^L pL - RwH}{\pi_1^H}\right)$. Observe that we have already compared $(p - Rw)L$ and $\pi_2^H \left(\frac{\pi_1^H pH + \pi_1^L pL - RwH}{\pi_1^H}\right)$ in equation (4.153) to equation (4.157). Furthermore, recall from Lemma 4.6 that, if $L \leq \pi_2^H H$, $\pi_2^H \leq \pi^T$. Therefore, from equation (4.158), we conclude that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $\pi_2^H pH \leq RwH \leq \pi^T pH$, $K_{22}^B > K_{21}^B$. Hence, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $\pi_2^H pH \leq RwH \leq \pi^T pH$, type-2 manufacturers would always prefer to order $H$ and receive type-1 rates instead of ordering $L$ and receiving type-2 rates. ■

**Lemma 4.11.** *If $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $RwH \leq \pi_2^H pH$, then a type-1 manufacturer is unable to signal its type to the bank with order quantities that are not its best-response order quantities to type-1 rates.*

*Proof.* According to Lemma 4.10, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $RwH \leq \pi_2^H pH$, a type-2 manufacturer always prefers to order $H$ and receive a type-1 rate instead of ordering $H$ and receive a type-2 rate. But, if the bank sets a threshold order quantity $y^T$ for which no type-2 manufacturer finds it optimal to order a quantity greater than or equal to $y^T$ and receive a type-1 rate, a type-1 manufacturers might be able to signal its type to the bank by ordering a quantity $y \geq y^T$.

I show that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $RwH \leq \pi_2^H pH$, then $K_{21}^B > K_{22}^B$ and there exist no $y^T$ for which $K_{22}^T > K_{21}^T$ and $K_{11}^T \geq K_{12}^T$ are simultaneously satisfied. This leads to a type-2 manufacturer to pretend to be a type-1 manufacturer and prevents a type-1 manufacturer from being able to signal its type with order quantities that are not its best-response order quantities to type-1 rates.

Recall, from Proposition 4.1 and Lemma 4.7, that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and

$RwH \leq \pi_2^H pH$, then

$$\begin{cases} y_{22}^B = H, \\ y_{21}^B = H, \\ K_{22}^B = \pi_2^H pH + \pi_2^L pL - RwH, \\ K_{21}^B = \pi_2^H \left( \frac{\pi_1^H pH + \pi_1^L pL - RwH}{\pi_1^H} \right). \end{cases} \tag{4.190}$$

Lemma 4.3 states that, for a given order quantity, $y$, $K_{\theta 1}(y) > K_{\theta 2}(y)$. Observe that $y_{22}^B = y_{21}^B = H$. This leads to $K_{21}^B > K_{22}^B$ which implies that, if presented with a menu of type-1 and type-2 rates, a type-2 manufacturer will always choose the set of type-1 rates.

Observe, from equation (4.61), that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, and $RwH \leq \pi_2^H pH$, then

$$\begin{cases} K_{21} = \pi_2^H \left[ \frac{\pi_1^H p \min(H,y) + \pi_1^L pL - Rwy}{\pi_1^H} \right]^+, \\ K_{11} = \left[ \pi_1^H p \min(H,y) + \pi_1^L pL - Rwy \right]^+, \\ K_{12} = \pi_1^H \left[ \frac{\pi_2^H p \min(H,y) + \pi_2^L pL - Rwy}{\pi_2^H} \right]^+. \end{cases} \tag{4.191}$$

Recall that $y_{11}^B = H$, this leads to us having to consider order quantity thresholds above $H$ when comparing $K_{22}^T$ with $K_{21}^T$ and when comparing $K_{11}^T$ with $K_{12}^T$ because no $y^T \leq H$ can ensure a separating equilibria. Therefore, a manufacturer has to order a quantity greater than $H$ in order to receive type-1 rates from the bank. This leads to

$$\begin{cases} K_{21} = \pi_2^H \left[ \frac{\pi_1^H pH + \pi_1^L pL - Rwy}{\pi_1^H} \right]^+, \\ K_{11} = \left[ \pi_1^H pH + \pi_1^L pL - Rwy \right]^+. \end{cases} \tag{4.192}$$

Recall, from Lemma 4.4, that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, and $RwH \leq \pi_2^H pH$, then $y_{12}^B = H$ and $K_{12}^B = \pi_1^H \left( \frac{\pi_2^H pH + \pi_2^L pL - RwH}{\pi_2^H} \right)$. Observe that $y_{12}^B = H < y^T$. This leads to $y_{12}^T = y_{12}^B = H$ and

$$K_{12}^T = \pi_1^H \left( \frac{\pi_2^H pH + \pi_2^L pL - RwH}{\pi_2^H} \right). \tag{4.193}$$

Recall, by the definition of $y^T$, that the bank is able to impose a threshold quantity, $y^T$, to the manufacturer if there exists at least one order threshold quantity, $y^T$, for which the following conditions are satisfied:

$$\begin{cases} K_{22}^T \geq K_{21}, \\ K_{11} \geq K_{12}^T. \end{cases} \tag{4.194}$$

$K_{22}^T > K_{21}$ if

$$\pi_2^H pH + \pi_2^L pL - RwH > \pi_2^H \left( \frac{\pi_1^H pH + \pi_1^L pL - Rwy}{\pi_1^H} \right)^+ \tag{4.195}$$

and $K_{11} \geq K_{12}^T$ if

$$\left(\pi_1^H pH + \pi_1^L pL - Rwy\right)^+ \geq \pi_1^H \left( \frac{\pi_2^H pH + \pi_2^L pL - RwH}{\pi_2^H} \right). \tag{4.196}$$

Recall that $\pi_1^H \left( \frac{\pi_2^H pH + \pi_2^L pL - RwH}{\pi_2^H} \right) > 0$. Therefore, equation (4.196) is never satisfied when $y \in \left[ \frac{\pi_1^H pH + \pi_1^L pL}{Rw}, \infty \right)$ because $y \in \left[ \frac{\pi_1^H pH + \pi_1^L pL}{Rw}, \infty \right)$ leads to $\pi_1^H pH + \pi_1^L pL - Rwy \leq 0$.

When $y \in \left[ 0, \frac{\pi_1^H pH + \pi_1^L pL}{Rw} \right)$, equation (4.195) becomes

$$\pi_2^H pH + \pi_2^L pL - RwH > \pi_2^H \left( \frac{\pi_1^H pH + \pi_1^L pL - Rwy}{\pi_1^H} \right) \tag{4.197}$$

and equation (4.196) becomes

$$\pi_1^H pH + \pi_1^L pL - Rwy \geq \pi_1^H \left( \frac{\pi_2^H pH + \pi_2^L pL - RwH}{\pi_2^H} \right). \tag{4.198}$$

Recall that, for any $\theta$, $\pi_\theta^L = 1 - \pi_\theta^H$. Therefore, observe with a little algebra that equation (4.197) and equation (4.198) are satisfied if

$$y > \frac{\pi_1^H RwH - (\pi_1^H - \pi_2^H) pL}{\pi_2^H Rw} \tag{4.199}$$

and

$$\frac{\pi_1^H RwH - (\pi_1^H - \pi_2^H)pL}{\pi_2^H Rw} \geq y. \tag{4.200}$$

Observe that there is no order quantity, $y$, for which equation (4.199) and equation (4.200) are simultaneously satisfied. Therefore, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $RwH \leq \pi_2^H pH$, the bank is unable to set a threshold order quantity for which type-2 manufacturers prefer to order a quantity below that threshold and for which type-1 manufacturers prefer to order a quantity equal or above that threshold order quantity.

■

**Lemma 4.12.** *If $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, $\pi_2^H p \leq Rw \leq \pi^T p$ and type-1 manufacturers in the economy signal their type to the bank with order quantities that are not their best-response order quantities to type-1 rates, then*

$$y_{11}^T = y^T = \overline{y}^T, \tag{4.201}$$

$$y_{22}^T = y_{22}^B = L, \tag{4.202}$$

$$K_{11}^T = \frac{\pi_1^H (p - Rw)L}{\pi_2^H}, \tag{4.203}$$

$$K_{22}^T = K_{22}^B = (p - Rw)L. \tag{4.204}$$

*Proof.* According to Lemma 4.10, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, $\pi_2^H p \leq Rw \leq \pi^T p$ a type-2 manufacturer benefits if it gets access to the set of type-1 rates and orders a quantity equal to the best response order quantity of type-1 manufacturers to the set of type-1 rates. Therefore, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $\pi_2^H p \leq Rw \leq \pi^T p$, a type-1 manufacturer has to consider signaling its type to the bank with order quantities that are not its best-response order quantities in order to avoid being pooled with type-2 manufacturers and getting more expensive rates than type-1 rates. .

I show that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $\pi_2^H p \leq Rw \leq \pi^T p$, then $K_{21}^B > K_{22}^B$ and there exist a $y^T$ for which $K_{22}^T > K_{21}^T$ and $K_{11}^T \geq K_{12}^T$ are simultaneously satisfied. Afterwards, I determine, for when the bank sets a threshold order quantity to determine the type of the manufacturer, the value of $y^T$, $y_{11}^T$, $y_{22}^T$, $K_{11}^T$, and $K_{22}^T$.

Recall, from Proposition 4.1 and Lemma 4.7, that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $\pi_2^H p \leq Rw \leq \pi^T p$, then

$$\begin{cases} y_{22}^B = L, \\ y_{21}^B = H, \\ K_{22}^B = (p - Rw)L, \\ K_{21}^B = \pi_2^H \left( \frac{\pi_1^H pH + \pi_1^L pL - RwH}{\pi_1^H} \right). \end{cases} \tag{4.205}$$

Observe that, $K_{21}^B > K_{22}^B$, if

$$\pi_2^H \left( \frac{\pi_1^H pH + \pi_1^L pL - RwH}{\pi_1^H} \right) > (p - Rw)L. \tag{4.206}$$

Observe with a little algebra that equation (4.206) is satisfied if

$$\left[\pi_2^H \left(\pi_1^H H + \pi_1^L L\right) - \pi_1^H L\right] p > \left(\pi_2^H H - \pi_1^H L\right) Rw. \tag{4.207}$$

Recall that $\pi^T = \frac{\pi_2^H \left(\pi_1^H H + \pi_1^L L\right) - \pi_1^H L}{\pi_2^H H - \pi_1^H L}$ and $\pi_1^H \leq 1$. Because $\pi_1^H \leq 1$ and we are in the case when $L \leq \pi_2^H H$, then $\pi_2^H H - \pi_1^H L$ is always greater than or equal to 0. Therefore, equation (4.207) is equivalent to

$$\pi^T p > Rw. \tag{4.208}$$

Recall that we are in the case when $Rw < \pi^T p$. Therefore, equation (4.208) is satisfied and, if presented with a menu of type-1 and type-2 rates, a type-2 manufacturer will always choose the set of type-1 rates.

Observe, from equation (4.61), that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, and $\pi_2^H p \leq Rw \leq \pi^T p$, then,

$$\begin{cases} K_{21} = \pi_2^H \left[\frac{\pi_1^H p \min(H,y) + \pi_1^L pL - Rwy}{\pi_1^H}\right]^+, \\ K_{11} = \left[\pi_1^H p \min(H,y) + \pi_1^L pL - Rwy\right]^+, \\ K_{12} = \pi_1^H \left[\frac{\pi_2^H p \min(H,y) + \pi_2^L pL - Rwy}{\pi_2^H}\right]^+. \end{cases} \tag{4.209}$$

Recall that $y_{11}^B = H$, this leads to us having to consider order quantity thresholds above $H$ when comparing $K_{22}^T$ with $K_{21}^T$ and when comparing $K_{11}^T$ with $K_{12}^T$ because no $y^T \leq H$ can ensure a separating equilibria. Therefore, a manufacturer has to order a quantity greater than $H$ in order to receive type-1 rates from the bank. This leads to

$$\begin{cases} K_{21} = \pi_2^H \left[\frac{\pi_1^H pH + \pi_1^L pL - Rwy}{\pi_1^H}\right]^+, \\ K_{11} = \left[\pi_1^H pH + \pi_1^L pL - Rwy\right]^+. \end{cases} \tag{4.210}$$

Recall from Lemma 4.4 that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, and $RwH \leq \pi_2^H pH$, then $y_{12}^B = \frac{pL}{Rw}$ and $K_{12}^B = \pi_1^H(p - -Rw)\frac{pL}{Rw}$. Observe that $y_{12}^B = \frac{pL}{Rw} < H < y^T$. This leads to $y_{12}^T = y_{12}^B = H$ and

$$K_{12}^T = \pi_1^H \left(\frac{\pi_2^H pH + \pi_2^L pL - RwH}{\pi_2^H}\right). \tag{4.211}$$

Recall, by the definition of $y^T$, that the bank is able to impose a threshold order quantity, $y^T$, to the manufacturer if there exists at least one order threshold quantity, $y^T$, for which the following conditions are satisfied:

$$\begin{cases} K_{22}^T \geq K_{21}, \\ K_{11} \geq K_{12}^T. \end{cases} \tag{4.212}$$

$K_{22}^T > K_{21}$ if

$$(p - Rw)L > \pi_2^H \left(\frac{\pi_1^H pH + \pi_1^L pL - Rwy}{\pi_1^H}\right)^+. \tag{4.213}$$

and $K_{11} \geq K_{12}^T$ if

$$\left(\pi_1^H pH + \pi_1^L pL - Rwy\right)^+ \geq \pi_1^H \left(\frac{\pi_2^H pH + \pi_2^L pL - RwH}{\pi_2^H}\right). \tag{4.214}$$

Recall that $\pi_1^H \left(\frac{\pi_2^H pH + \pi_2^L pL - RwH}{\pi_2^H}\right) > 0$. Therefore, equation (4.213) is never satisfied when $y \in \left[\frac{\pi_1^H pH + \pi_1^L pL}{Rw}, \infty\right)$ because $y \in \left[\frac{\pi_1^H pH + \pi_1^L pL}{Rw}, \infty\right)$ leads to $\pi_1^H pH + \pi_1^L pL - Rwy \leq 0$.

When $y \in \left[0, \frac{\pi_1^H pH + \pi_1^L pL}{Rw}\right)$, equation (4.213) becomes

$$(p - Rw)L > \pi_2^H \left(\frac{\pi_1^H pH + \pi_1^L pL - Rwy}{\pi_1^H}\right) \tag{4.215}$$

and equation (4.214) becomes

$$\left(\pi_1^H pH + \pi_1^L pL - Rwy\right) \geq \pi_1^H \left(\frac{\pi_2^H pH + \pi_2^L pL - RwH}{\pi_2^H}\right). \tag{4.216}$$

Recall that, for any $\theta$, $\pi_\theta^L = 1 - \pi_\theta^H$. Therefore, observe with a little algebra that equation (4.215) and equation (4.216) are satisfied if

$$y > \frac{\pi_2^H \left(\pi_1^H pH + \pi_1^L pL\right) - \pi_1^H (p - Rw)L}{\pi_2^H Rw} \tag{4.217}$$

and

$$\frac{\pi_1^H RwH - (\pi_1^H - \pi_2^H)pL}{\pi_2^H Rw} \geq y \tag{4.218}$$

equation (4.217) and equation (4.218) can be simultaneously satisfied if

$$\frac{\pi_2^H \left(\pi_1^H pH + \pi_1^L pL\right) - \pi_1^H (p - Rw)L}{\pi_2^H Rw} < \frac{\pi_1^H pH + \pi_1^L pL}{Rw}, \tag{4.219}$$

$$\frac{\pi_2^H \left(\pi_1^H pH + \pi_1^L pL\right) - \pi_1^H (p - Rw)L}{\pi_2^H Rw} < \frac{\pi_1^H RwH - (\pi_1^H - \pi_2^H)pL}{\pi_2^H Rw}, \tag{4.220}$$

and

$$H < \frac{\pi_2^H \left(\pi_1^H pH + \pi_1^L pL\right) - \pi_1^H (p - Rw)L}{\pi_2^H Rw}. \tag{4.221}$$

Observe with a little algebra that equation (4.219), equation (4.220) and equation (4.221) are satisfied if

$$-\pi_1^H(p - Rw)L < 0, \tag{4.222}$$

$$\pi_2^H\pi_1^H p(H - L) < \pi_1^H Rw(H - L), \tag{4.223}$$

and

$$Rw(\pi_2^H H - \pi_1^H L) < \pi_2^H \left(\pi_1^H H + \pi_1^L L\right) - \pi_1^H L. \tag{4.224}$$

Recall that $L < H$, $L \leq \pi_2^H H$, $0 \leq \pi_1^H \leq 1$ and, from Assumption 4.1, that $p > Rw$. Therefore, $p - Rw > 0$, $H - L > 0$ and $\pi_2^H H - \pi_1^H L$. This leads to equation (4.222), equation (4.223) and equation (4.224) being satisfied if

$$-L < 0, \tag{4.225}$$

$$\pi_2^H p < Rw, \tag{4.226}$$

and

$$Rw < \frac{\pi_2^H \left(\pi_1^H H + \pi_1^L L\right) - \pi_1^H L}{\pi_2^H H - \pi_1^H L}. \tag{4.227}$$

Recall that $L \geq 0$, $\pi^T = \frac{\pi_2^H\left(\pi_1^H H + \pi_1^L L\right) - \pi_1^H L}{\pi_2^H H - \pi_1^H L}$ and $\pi_2^H p \leq Rw \leq \pi^T p$. Therefore, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $\pi_2^H p \leq Rw \leq \pi^T p$, equation (4.225), equation (4.226) and equation (4.227) are satisfied.

The bank will set $y^T$ to be the smallest order quantity for which equation (4.215) is satisfied. Recall that $\overline{y}^T = \frac{\pi_2^H(\pi_1^H pH + \pi_1^L pL) - \pi_1^H(p - Rw)L}{\pi_2^H Rw}$. To have a tractable analysis I set $\overline{y}^T$ as the threshold order quantity and assume that this threshold order quantity discourages type-2 manufacturers from pretending to be type-1 manufacturers. We get:

$$y_{11}^T = y^T = \overline{y}^T, \tag{4.228}$$

$$y_{22}^T = y_{22}^B = L, \tag{4.229}$$

$$K_{11}^T = \frac{\pi_1^H (p - Rw)L}{\pi_2^H}, \tag{4.230}$$

$$K_{22}^T = K_{22}^B = (p - Rw)L. \tag{4.231}$$

■

**Lemma 4.13.** *If $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, $\pi^P p \leq Rw \leq \pi^T p$, then a type-1 manufacturer signals its type with order quantities that are not its best-response order quantities to type-1 rates. However, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, $\pi_2^H p \leq Rw \leq \pi^P p$, a type-1 manufacturer prefers receiving pooling rates instead of signaling its type with order quantities that are not its best-response order quantities to type-1 rates.*

*Proof.* Lemma 4.10 states that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $Rw \leq \pi^T p$, then a type-1 manufacturer has to consider signaling its type to the bank. Lemma 4.11 states that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $Rw \leq \pi_2^H p$, then a type-1 manufacturer cannot signal its type to the bank. However, Lemma 4.12 states that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $\pi_2^H p \leq Rw \leq \pi^T p$, then a type-1 manufacturer can signal its type to the bank.

I show that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $\pi_2^H p \leq Rw \leq \pi^T p$, then $K_{21}^B > K_{22}^B$ and $K_{11}^T > K_{12}^T$. Afterwards, I compare $K_{11}^T$ with $K_{1P}^B$ to provide conditions under which a type-1 manufacturers signals its type with order quantities that are not its best-response order quantity to the set of type-1 rates. Then, I compare $K_{11}^T$ with $K_{1P}^B$ to provide conditions under which a type-1 manufacturer prefers to be pooled with type-2 manufacturers instead of signaling its type to the bank with order quantities that are not its best-response order quantity to the set of type-1 rates.

Recall, from Proposition 4.1 and Lemma 4.7, that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $\pi_2^H p \leq Rw \leq \pi^T p$, then

$$\begin{cases} y_{22}^B = L, \\ y_{21}^B = H, \\ K_{22}^B = (p - Rw)L, \\ K_{21}^B = \pi_2^H \left( \frac{\pi_1^H pH + \pi_1^L pL - RwH}{\pi_1^H} \right). \end{cases} \tag{4.232}$$

Therefore, $K_{22}^B > K_{21}^B$ if

$$(p - Rw)L > \pi_2^H \left( \frac{\pi_1^H pH + \pi_1^L pL - RwH}{\pi_1^H} \right). \tag{4.233}$$

From the analysis performed in Lemma 4.7, I conclude that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $\pi_2^H p \leq Rw \leq \pi^T p$, then (4.235) is satisfied. This implies that, if presented with a menu of type-1 and type-2 rates, a type-2 manufacturer will always choose the set of type-1 rates.

Recall from Lemma 4.12 that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $\pi_2^H p \leq Rw \leq \pi^T p$ and $y^T = \overline{y}^T$, then the bank is able to separate manufacturers. This implies that a type-1 manufacturer prefers to order a quantity greater than or equal to $\overline{y}^T$ instead of ordering a quantity below the threshold order quantity (i. e., $K_{11}^T > K_{12}^T$).

Recall from (4.92) that a necessary condition for a type-1 manufacturer to signal its type with order quantities that are not its best-response order quantity to the set of type-1 rates is $K_{11}^T \geq K_{1P}^B$. Also, recall from (4.93) that a necessary condition for a type-1 manufacturer to prefer to be pooled with type-2 manufacturers instead of signaling its type to the bank with order quantities that are not its best-response order quantity to the set of type-1 rates. Let us compare $K_{11}^T$ with $K_{1P}^B$.

Lemma 4.12 states that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, $\pi_2^H p \leq Rw \leq \pi^T p$, then $K_{11}^T = \frac{\pi_1^H(p-Rw)L}{\pi_2^H}$. Lemma 4.5 states that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, $Rw \leq \pi_P^H p$, then $K_{1P}^B = \pi_1^H\left(\frac{\pi_P^H pH + \pi_P^L L - RwH}{\pi_P^H}\right)$ and, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, $\pi_P^H p \leq Rw \leq \pi_1^H p$, then $K_{1P}^B = \pi_1^H(p - Rw)\frac{pL}{Rw}$. Lemma 4.6 states that $\pi_2^H \leq \pi^T \leq \pi_1^H$. Therefore, because $\pi_2^H \leq \pi_P^H \leq \pi_1^H$, we need to compare $K_{11}^T$ with $K_{1P}^B$ when $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, $\pi_2^H p \leq Rw \leq \min(\pi_P^H p, \pi^T p)$ and when $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, $\min(\pi_P^H p, \pi^T p) \leq Rw \leq \pi^T p$.

**Case:** $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, $\pi_2^H p \leq Rw \leq \min(\pi_P^H p, \pi^T p)$.

Observe from Lemma 4.12 and Lemma 4.5 that $K_{11}^T \geq K_{1P}^B$ if

$$\frac{\pi_1^H(p-Rw)L}{\pi_2^H} \geq \pi_1^H\left(\frac{\pi_P^H pH + \pi_P^L L - RwH}{\pi_P^H}\right). \tag{4.234}$$

Observe with a little algebra that equation (4.235) is satisfied if

$$(\pi_2^H H - \pi_P^H L)Rw \geq \pi_2^H(\pi_P^H pH + \pi_P^L L) - \pi_P^H pL. \tag{4.235}$$

Recall that $\pi^P = \frac{\pi_2^H\left(\pi_P^H H + \pi_P^L L\right) - \pi_P^H L}{\pi_2^H H - \pi_P^H L}$ and $\pi_1^H \leq 1$. Because $\pi_P^H \leq 1$ and we are in the case when $L \leq \pi_2^H H$, $\pi_2^H H - \pi_P^H L$ is always greater than or equal to 0. Therefore, equation (4.235) is equivalent to

$$Rw \geq \pi^P p. \tag{4.236}$$

Lemma 4.9 states that $\pi^P \leq \min(\pi_P^H, \pi^T)$. Therefore, equation (4.236) is satisfied and type-1 manufacturers signal their type with order quantities that are not their best-response order quantities if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, $\pi^P p \leq Rw \leq \min(\pi_P^H p, \pi^T p)$. However, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, $\pi_2^H p \leq Rw \leq \pi^P p$, type-1 manufacturers prefer to be pooled with type-2 manufacturers instead of signaling their type to the bank with order quantities that are not their best-response order quantities to type-1 rates.

**Case:** $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, $\min(\pi_P^H p, \pi^T p) \leq Rw \leq \pi^T p$.

Observe from Lemma 4.12 and Lemma 4.5 that $K_{11}^T \geq K_{1P}^B$ if

$$\frac{\pi_1^H(p-Rw)L}{\pi_2^H} \geq \pi_1^H(p-Rw)\frac{pL}{Rw}. \tag{4.237}$$

Recall from Assumption 4.1 that $p > Rw$. Therefore, $p - Rw > 0$. Furthermore, recall that $\pi_1^H \geq 0$, $R \geq 0$, $w \geq 0$ and $L \geq 0$. Therefore, equation (4.237) is satisfied if

$$Rw \geq \pi_2^H p. \tag{4.238}$$

Because $\pi_2^H \leq \pi_P^H$, equation (4.238) is satisfied and type-1 manufacturers signal their type to the bank with order quantities that are not their best-response order quantities if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, $\min(\pi_P^H p, \pi^T p) \leq Rw \leq \pi^T p$. ■

**Proposition 4.6.** *If $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $Rw \leq \pi^P p$, then there are pooling equilibria where all manufacturers in the economy order $H$, and the interest rate on the loan repayment of each manufacturer is $\frac{RwH - \pi_P^L pL}{\pi_P^H wH}$.*

*Proof.* I show that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $Rw \leq \pi^P p$, then there are pooling equilibria where all manufacturers in the economy order $H$, and the interest rate on the loan repayment of each manufacturer is $\frac{RwH - \pi_P^L pL}{\pi_P^H wH}$ because all the conditions in equation (4.93) are satisfied.

Lemma 4.10 states that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $RwH \leq \pi^T pH$, type-2 manufacturers want to pretend to be type-1 manufacturers (i. e., $K_{21}^B > K_{22}^B$). Lemma 4.12 states that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $\pi_2^H p \leq Rw \leq \pi^T p$, type-1 manufacturers are able to signal their type to the bank with order quantities that are not their best-response order quantities to type-1 rates (i. e., $K_{11}^T > K_{22}^T$). Furthermore, Lemma 4.13, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, $\pi_2^H p \leq Rw \leq \pi^P p$, type-1 manufacturers prefer to be pooled with type-2 manufacturers instead of signaling their type to the bank with order quantities that are not their best-response order quantities to type-1 rates (i. e., $K^B_{1P} \geq K^T_{11}$).

Recall from Lemma 4.9 that, if $L \leq \pi^H_2 H$, $\pi^H_2 \leq \pi^P \leq \pi^T$. Therefore, according to equation (4.93), Lemma 4.10, Lemma 4.11 and Lemma 4.13, if $\frac{pL}{Rw} \leq H$, $L \leq \pi^H_2 H$ and $RwH \leq \pi^P pH$, then there are pooling equilibria if $K_{2P} \geq K^B_{22}$ or no equilibria if $K^B_{22} > K_{2P}$.

If $\frac{pL}{Rw} < H$, $L \leq \pi^H_2 H$ and $Rw \leq \pi^H_2 p$, Proposition 4.1 states that $K^B_{22} = \pi^H_2 pH + \pi^L_2 pL - RwH$ and $y^B_{22} = H$. If $\frac{pL}{Rw} < H$, $L \leq \pi^H_2 H$ and $\pi^H_2 p \leq Rw \leq \pi^P p$, Proposition 4.1 states that $K^B_{22} = (p - Rw)L$ and $y^B_{22} = L$. If $\frac{pL}{Rw} < H$, $L \leq \pi^H_2 H$ and $Rw \leq \pi^H_P p$, Corollary 4.1, states that $K_{2P} = \pi^H_2 \left(\frac{\pi^H_P pH + \pi^L_P pL - RwH}{\pi^H_P}\right)$ and $y_{2P} = H$. Recall that, if $L \leq \pi^H_2 H$, $\pi^H_2 \leq \pi^P \leq \pi^H_P$. Therefore, I compare $K^B_{22}$ with $K_{2P}$ under $\frac{pL}{Rw} < H$, $L \leq \pi^H_2 H$ and $Rw \leq \pi^H_2 p$ and under $\frac{pL}{Rw} < H$, $L \leq \pi^H_2 H$ and $\pi^H_2 p \leq Rw \leq \pi^P p$.

**Case:** $\frac{pL}{Rw} < H$, $L \leq \pi^H_2 H$ and $Rw \leq \pi^H_2 p$.

According to Lemma 4.3, for a given order quantity, $y$, $K_{\theta P} \geq K_{\theta 2}$. Because $y^B_{22} = y_{2P} = H$ and $\pi^H_P \geq \pi^H_2$, Lemma 4.3 leads to $K_{2P} \geq K^B_{22}$.

**Case:** $\frac{pL}{Rw} < H$, $L \leq \pi^H_2 H$ and $\pi^H_2 p \leq Rw \leq \pi^P p$.

$K_{2P} \geq K^B_{22}$ if

$$\pi^H_2 \left(\frac{\pi^H_P pH + \pi^L_P pL - RwH}{\pi^H_P}\right) =(p - Rw)L. \tag{4.239}$$

Observe with a little algebra that equation (4.239) is satisfied if

$$(\pi^H_2 H - \pi^H_P L)Rw \geq \pi^H_2 (\pi^H_P pH + \pi^L_P L) - \pi^H_P pL. \tag{4.240}$$

We have already shown in the analysis in Lemma 4.13 that, if $\frac{pL}{Rw} < H$, $L \leq \pi^H_2 H$, $\pi^P p \leq Rw \leq \pi^T p$, then equation (4.240) is satisfied.

Observe from equation (4.59) that the interest rate on the loan repayment of all manufacturers in the economy is $\frac{RwH - \pi_P^L pL}{\pi_P^H wH}$. Therefore, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $Rw \leq \pi^P p$, there are pooling equilibria where all manufacturers in the economy order $H$, and the interest rate on the loan repayment of each manufacturer is $\frac{RwH - \pi_P^L pL}{\pi_P^H wH}$. ■

**Proposition 4.7.** *If $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, $\pi^P p \leq Rw \leq \pi^T p$ and $y^T = \overline{y}^T$, then there are separating equilibria where type-1 manufacturers order $\overline{y}^T$, type-2 manufacturers order $L$, the interest rate on the loan repayment of type-1 manufacturer is $\overline{R}^T$, and the interest rate on the loan repayment of type-2 manufacturers is the bank's cost of capital, $R$.*

*Proof.* I show that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, $\pi^P p \leq Rw \leq \pi^T p$ and $y^T = \overline{y}^T$, then there are separating equilibria where type-1 manufacturers order $\overline{y}^T$, type-2 manufacturers order $L$, the interest rate on the loan repayment of type-1 manufacturer is $\overline{R}^T$, and the interest rate on the loan repayment of type-2 manufacturers is the bank's cost of capital, $R$ because all the conditions in equation (4.92) are satisfied.

Lemma 4.10 states that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $RwH \leq \pi^T pH$, then type-2 manufacturers want to pretend to be type-1 manufacturers (i. e., $K_{21}^B > K_{22}^B$). Lemma 4.13 states that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, $\pi^P p \leq Rw \leq \pi^T p$, then a type-1 manufacturer prefers to signal its type with order quantities that are not its best-response order quantities to type-1 rates instead of receiving pooling rates (i. e., $K_{11}^T \geq K_{12}^T$ and $K_{11}^T \geq K_{1P}^T$). But, recall that, when the bank sets a threshold order quantity to separate manufacturers by their type, a type-2 manufacturer always prefers to order below the threshold order quantity. Therefore, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, $\pi^P p \leq Rw \leq \pi^T p$, then $K_{22}^T > K_{21}^T$ and $L \leq y_{22}^B = y_{22}^T \leq y_{11}^B < y^T = y_{11}^T$.

Observe that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $RwH \leq \pi^T pH$, then all the conditions in equation (4.93) are satisfied. Furthermore, observe, from equation (4.59), that the interest rate on the loan repayment of a type-1 manufacturer is $\overline{R}_{bm}^T$ and the interest rate on the loan repayment of a type-2 manufacturer is the bank's cost of capital, $R$, because there is no risk of default when manufacturers order $L$. This leads to separating equilibria where type-1 manufacturers order $\overline{y}^T$, type-2 manufacturers order $L$, the interest rate on the loan repayment of type-1 manufacturer is $\overline{R}^T$, and the interest rate on the loan repayment of type-2 manufacturers is the bank's cost of capital, $R$. ■

### 4.4.4 Asymmetric information equilibria between the bank and the manufacturer when trade credit is available to the manufacturer.

Similar to 4.4.3, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$, $RwH \leq \pi^T pH$, then a type-1 manufacturer must consider signaling its type to the bank. In order for a type-1 manufacturer to signal its type and obtain type-1 rates from the bank,

- there must exist a threshold order quantity, $y^T$, set by the bank for which the bank expects order quantities greater than or equal to the threshold to come from a type-1 manufacturer and order quantities below the threshold to come from a type-2 manufacturer, or
- there must exist a threshold loan amount, $L_{sm}^T$, set by the bank for which the bank expects to determine the manufacturer's type when the manufacturer borrows any amount from the supplier above that threshold loan amount.

Observe, from equation (4.61) that, if

$$\begin{cases} y > \frac{pL}{Rw}, \\ L_{sm} > 0, \end{cases} \tag{4.241}$$

then the trade credit terms signal the manufacturer's type to the bank.

Recall from equation (4.17) that $L_{bm} + L_{sm} = wy$. Therefore equation (4.241) is equivalent to

$$\begin{cases} L_{sm} > \frac{pL}{R} - L_{bm}, \\ L_{sm} > 0. \end{cases} \tag{4.242}$$

Because the interest rate received by manufacturers only reveals their type when the manufacturers borrow from the supplier an amount greater than $\frac{pL}{R} - L_{bm}$, in equilibrium, $L^T_{sm}$ is the smallest amount above $\left(\frac{pL}{R} - L_{bm}\right)^+$. In order to be able to have a tractable analysis we make the following assumptions.

**Assumption 4.4.** *All manufacturers prefer to borrow from the bank.*

**Assumption 4.5.** *A type-1 manufacturer prefers to borrow from the bank and only borrows just enough from the supplier to signal its type to the bank.*

**Assumption 4.6.** *It is enough for a type-1 manufacturer to signal its type to the bank by borrowing an amount equal to* $\left(\frac{pL}{R} - L_{bm}\right)^+$ *from the supplier.*

If we denote $\left(\frac{pL}{R} - L_{bm}\right)^+$ by $\overline{L}^T_{sm}$, (4.242), Assumption 4.4, Assumption 4.5 and Assumption 4.6 lead to $L^T_{sm} = \overline{L}^T_{sm}$. Recall that the manufacturer cannot borrow from the supplier more than $\tau y$. Therefore, a type-1 manufacturer can only signal its type to the bank when $\overline{L}^T_{sm} \leq \tau y$.

Observe in equation (4.61) that, when a manufacturer borrows from the supplier any amount greater than $\overline{L}^T_{sm}$, its objective value is the same as its objective value in 4.4.2. Because type-1 rates are preferred by all manufacturers, a type-1 manufacturer uses trade credit and order its best-response quantities to receive type-1 rates whenever it needs to signal its type to the bank. Recall from Proposition 4.1 that, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $RwH \leq \pi^T pH$, then $y^B_{11} = H$ which leads to equation (4.241) being satisfied for any $L_{sm} > 0$. Furthermore, recall from the definition of

$\tau$ that $\tau > 0$ and, from Lemma 4.2, that, at optimlaity, $y \geq L > 0$. Therefore, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $RwH \leq \pi^T pH$, then $L_{sm}^T = 0 < \tau y$. This is why, if $\frac{pL}{Rw} < H$, $L \leq \pi_2^H H$ and $RwH \leq \pi^T pH$, then a type-1 manufacturer can always signal its type to the bank with trade credit and the optimal decisions of manufacturers, banks and suppliers in the economy in Section 4.4.4 are the same as the optimal decisions in Section 4.4.2.

#### 4.4.5 Compilation of all the equilibria.

Observe from (4.60) and (4.59) that, regardless of the bank's belief about the manufacturer's type, the interest on the manufacturer's loan is the opportunity cost of capital of the bank, $R$, if $H \leq \frac{pL}{Rw}$ and a manufacturer orders a quantity less than or equal to $H$. According to Proposition 4.1, (4.60) and (4.59), if $H \leq \frac{pL}{Rw}$, then manufacturers never order a quantity greater than $H$. Therefore, if $H \leq \frac{pL}{Rw}$, then the results obtained in Section 4.4.2, Section 4.4.3 and Section 4.4.4 are identical. This is why, I will not discuss the results of the case when $H \leq \frac{pL}{Rw}$.

Table 4.3 is a compilation of the equilibria for the case $\frac{pL}{Rw} < H$. Columns 1 and 2 provide the conditions for each equilibrium found. I add column 3 to conveniently refer to specific rows. Columns 4 to 8 specify the equilibrium threshold trade credit levels, loans from the supplier to the manufacturer, order quantities, interest rates on loans from the supplier to the manufacturer, and interest rates on loans from the bank to the manufacturer when there is asymmetric information between banks and manufacturers and trade credit is available. Columns 9 to 11 specify the equilibrium threshold order quantities, order quantities, and interest rates on loans from the bank to the manufacturer when there is asymmetric information between banks and manufacturers and trade credit is unavailable. Columns 12 and 13 specify the equilibrium order quantities and interest rates on loans from the bank to the manufacturer when there is asymmetric information between banks and manufacturers.

To simplify the exposition of the results, trade credit is never used when there is asymmetric information between the bank and the manufacturer and if a type-1 manufacturer, by ordering its best-response order quantity to type-1 rates, can signal its type to the bank without using trade credit; trade credit is never used when there is symmetric information between the bank and the manufacturer because, for any order quantity $y$, the total loan repayment per amount borrowed under symmetric information is independent of a manufacturer's choice of financing; I use "$-$" to refer to when, in equilibrium, no threshold trade credit level or threshold order quantity is set by the bank to determine the manufacturer's type; $R^{\theta}_{sm} \stackrel{\text{def}}{=} \frac{RL_{sm}-\pi^{L}_{\theta}(pL-RL_{bm})}{\pi^{H}_{\theta}L_{sm}}$ and $R^{\rho}_{bm} \stackrel{\text{def}}{=} \frac{RL_{bm}-\pi^{L}_{\rho}pL}{\pi^{H}_{\rho}L_{bm}}$.

### 4.4.6 Effect of asymmetric information.

Let us compare columns Symmetric Information with Asymmetric Information, No Trade Credit in Table 4.3.

When the probability of a high demand for type-2 manufacturers is *low relative to the spread between the two possible demand values* (subcase rows 6, 7, 13 and 14), equilibrium order quantities and interest rates on the loans from the bank to manufacturers are unaffected by asymmetric information between the bank and the manufacturer. The reason for this is that the expected revenue of type-2 manufacturers are too low for them to pretend to be type-1 manufacturers when type-1 manufacturers order a high quantity.

When the probability of a high demand for type-2 manufacturers is *high relative to the spread between the two possible demand values and the opportunity cost of capital*

| $pL < RwH$ | | | Asymmetric | | | | | | | | Sym. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | T. C. | | | | | No T. C. | | | T. C. | |
| | | | $L^T_{sm}$ | $L^*_{sm}$ | $y^*$ | $R^*_{sm}$ | $R^*_{bm}$ | $y^T$ | $y^*$ | $R^*_{bm}$ | $y^*$ | $R^*_{bm}$ |
| **Subcase** | | | **Type 1** | | | | | | | | | |
| $\frac{L}{H} \leq \pi_2^H$ | $Rw \leq \pi_2^H p$ | 1 | $\overline{L}^T_{sm}$ | $\overline{L}^T_{sm}$ | $H$ | $R^1_{sm}$ | $\frac{R}{\pi_1}$ | – | $H$ | $R^P_{bm}$ | $H$ | $R^1_{bm}$ |
| | $\pi_2^H p \leq Rw \leq \pi^P p$ | 2 | $\overline{L}^T_{sm}$ | $\overline{L}^T_{sm}$ | $H$ | $R^1_{sm}$ | $\frac{R}{\pi_1}$ | – | $H$ | $R^P_{bm}$ | $H$ | $R^1_{bm}$ |
| | $\pi^P p \leq Rw \leq \pi^T p$ | 3 | $\overline{L}^T_{sm}$ | $\overline{L}^T_{sm}$ | $H$ | $R^1_{sm}$ | $\frac{R}{\pi_1}$ | $\overline{y}^T$ | $\overline{y}^T$ | $\overline{R}^T$ | $H$ | $R^1_{bm}$ |
| | $\pi^T p \leq Rw \leq \pi_1^H p$ | 4 | – | 0 | $H$ | 0 | $R^1_{bm}$ | – | $H$ | $R^1_{bm}$ | $H$ | $R^1_{bm}$ |
| | $\pi_1^H p \leq Rw$ | 5 | – | 0 | $L$ | 0 | $R$ | – | $L$ | $R$ | $L$ | $R$ |
| $\pi_2^H \leq \frac{L}{H}$ | $Rw \leq \pi_1^H p$ | 6 | – | 0 | $H$ | 0 | $R^1_{bm}$ | – | $H$ | $R^1_{bm}$ | $H$ | $R^1_{bm}$ |
| | $\pi_1^H p \leq Rw \leq p$ | 7 | – | 0 | $L$ | 0 | $R$ | – | $L$ | $R$ | $L$ | $R$ |
| **Subcase** | | | **Type 2** | | | | | | | | | |
| $\frac{L}{H} \leq \pi_2^H$ | $Rw \leq \pi_2^H p$ | 8 | $\overline{L}^T_{sm}$ | 0 | $H$ | 0 | $R^2_{bm}$ | – | $H$ | $R^P_{bm}$ | $H$ | $R^2_{bm}$ |
| | $\pi_2^H p \leq Rw \leq \pi^P p$ | 9 | $\overline{L}^T_{sm}$ | 0 | $L$ | 0 | $R$ | – | $H$ | $R^P_{bm}$ | $L$ | $R$ |
| | $\pi^P p \leq Rw \leq \pi^T p$ | 10 | $\overline{L}^T_{sm}$ | 0 | $L$ | 0 | $R$ | $\overline{y}^T$ | $L$ | $R$ | $L$ | $R$ |
| | $\pi^T p \leq Rw \leq \pi_1^H p$ | 11 | – | 0 | $L$ | 0 | $R$ | – | $L$ | $R$ | $L$ | $R$ |
| | $\pi_1^H p \leq Rw$ | 12 | – | 0 | $L$ | 0 | $R$ | – | $R$ | $L$ | $L$ | $R$ |
| $\pi_2^H \leq \frac{L}{H}$ | $Rw \leq \pi_1^H p$ | 13 | – | 0 | $L$ | 0 | $R$ | – | $L$ | $R$ | $L$ | $R$ |
| | $\pi_1^H p \leq Rw \leq p$ | 14 | – | 0 | $L$ | 0 | $R$ | – | $L$ | $R$ | $L$ | $R$ |

Table 4.3: Equilibria when $pL < RwH$. Recall that $\pi^P = \frac{\pi_2^H\left(\pi_P^H H+\pi_P^L L\right)-\pi_P^H L}{\pi_2^H H-\pi_P^H L}$, $\pi^T = \frac{\pi_2^H\left(\pi_1^H H+\pi_1^L L\right)-\pi_1^H L}{\pi_2^H H-\pi_1^H L}$, $\overline{y}^T = \frac{\pi_2^H\left(\pi_1^H pH+\pi_1^L pL\right)-\pi_1^H(p-Rw)L}{\pi_2^H Rw}$, $\overline{L}^T_{sm} = \left(\frac{pL}{R}-L_{bm}\right)^+$, $\overline{R}^T = \frac{R[\pi_2^H pH-(p-Rw)L]}{\pi_2^H\left(\pi_1^H pH+\pi_1^L pL\right)-\pi_1^H(p-Rw)L}$, $R^\theta_{sm} = \frac{RL_{sm}-\pi^L_\theta(pL-RL_{bm})}{\pi^H_\theta L_{sm}}$ and $R^\rho_{bm} = \frac{RL_{bm}-\pi^L_\rho pL}{\pi^H_\rho L_{bm}}$.

*of the bank is not too high*, asymmetric information increases the order quantities placed by manufacturers (subcase rows 3 and 9), increases the interest rates received by type-1 manufacturers (subcase rows 1, 2, 3), and decreases the interest rates received by type-2 manufacturers (subcase rows 8 and 9). The reason for this is that the expected revenue of type-2 manufacturers is high enough for them to pretend to be type-1 manufacturers (subcase rows 1, 2, 3, 8, 9 and 10). If the opportunity cost of capital of the bank is low, type-2 manufacturers are capable of ordering high quantities and type-1 manufacturers find it suboptimal to signal their type by over-ordering (subcase rows 1, 2, 8 and 9). As a result, when the opportunity cost of capital, $R$, of the bank is low, type-2 manufacturers order the same quantity as type-1 manufacturers and the bank offers pooling rates to both types of manufacturers. As the opportunity cost of capital of the bank increases, type-2 manufacturers find it more difficult to pretend to be type-1 manufacturers (subcase rows 3, 4, 10 and 11). This allows type-1 manufacturers to signal their type either by over-ordering or ordering their best-response quantities to type-1 rates when it is too costly for type-2 manufacturers to mimic the best-response order quantity of type-1 manufacturers. As a result, type-2 manufacturers order a lower quantity than type-1 manufacturers and banks offer type-specific rates to all manufacturers for medium values of the opportunity cost of capital of the bank.

#### 4.4.7 Effect of the availability of trade credit.

Let us compare columns Asymmetric Information, No Trade Credit with Asymmetric Information, Trade Credit in Table 4.3.

When a manufacturer borrows from the supplier and there is a risk the manufacturer will default on its loan obligation with the supplier, the interest rate on the loan offered by the supplier reveals the manufacturer's type (subcase rows 1, 2 and 3). This is why type-1 manufacturers borrow just enough from the supplier when, without the trade credit signal, type-1 manufacturers are unable to credibly signal their type to banks through their ordering decisions (rows 1 and 2).

The objective value of all manufacturers under Symmetric Information are the same as under Asymmetric Information, Trade Credit. Therefore, trade credit *can eliminate asymmetric information* without adding additional costs to manufacturers.

#### 4.4.8 Winners and losers of the availability of trade credit when there is asymmetric information between the bank and the manufacturer.

I consider when $pL < RwH$, $L \leq \pi_2^H H$ and $Rw \leq \pi^T p$ because that is when the operational and financial decisions of manufacturers are affected by the availability of trade credit.

##### 4.4.8.1 Type-1 manufacturers.

**Expected profit of a type-1 manufacturer when trade credit is available**.

$$K_{11}^T = \pi_1^H pH + \pi_1^L pL - RwH. \tag{4.243}$$

**Expected profit of a type-1 manufacturer when trade credit is unavailable**.

**Case:** $Rw \leq \pi^P p$**.**

$$K_{1P}^B = \pi_1^H \left( \frac{\pi_P^H pH + \pi_P^L pL - RwH}{\pi_P^H} \right). \tag{4.244}$$

**Case:** $\pi^P p \leq Rw \leq \pi^T p$**.**

$$K_{11}^T = \frac{\pi_1^H (p - Rw)L}{\pi_2^H}. \tag{4.245}$$

**Expected benefit of having trade credit available for a type-1 manufacturer**.

**Case:** $Rw \leq \pi^P p$**.**

$$\text{Benefit}_1 = \pi_1^H pH + \pi_1^L pL - RwH - \pi_1^H \left( \frac{\pi_P^H pH + \pi_P^L pL - RwH}{\pi_P^H} \right) \tag{4.246}$$

$$= \frac{(\pi_1^L \pi_P^H - \pi_1^H \pi_P^L)pL - (\pi_P^H - \pi_1^H)RwH}{\pi_P^H} \tag{4.247}$$

$$= \frac{[(1-\pi_1^H)\pi_P^H - \pi_1^H(1-\pi_P^H)]pL - (\pi_P^H - \pi_1^H)RwH}{\pi_P^H} \tag{4.248}$$

$$= \frac{(\pi_P^H - \pi_1^H)(pL - RwH)}{\pi_P^H}. \tag{4.249}$$

We have a positive benefit because $\pi_P^H \leq \pi_1^H$ and $RwH \geq pL$.

**Case:** $\pi^P p \leq Rw \leq \pi^T p$**.**

$$\text{Benefit}_1 = \pi_1^H pH + \pi_1^L pL - RwH - \frac{\pi_1^H (p - Rw)L}{\pi_2^H} \tag{4.250}$$

$$= \frac{\pi_2^H(\pi_1^H pH + \pi_1^L pL) - \pi_1^H pL - Rw(\pi_2^H H - \pi_1^H L)}{\pi_2^H} \tag{4.251}$$

$$= \frac{(\pi^T p - Rw)(\pi_2^H H - \pi_1^H L)}{\pi_2^H}. \tag{4.252}$$

We have a positive benefit because $L \leq \pi_2^H H$, $\pi_1^H \leq 1$ and $Rw \leq \pi^T p$.

#### 4.4.8.2 Type-2 manufacturers.

**Expected profit of a type-2 manufacturer when trade credit is available**.

**Case:** $Rw \leq \pi_2^H p$**.**

$$K_{22}^B = \pi_2^H pH + \pi_2^L pL - RwH. \tag{4.253}$$

**Case:** $\pi_2^H p \leq Rw \leq \pi^T p$**.**

$$K_{22}^B = (p - Rw)L. \tag{4.254}$$

**Expected profit of a type-2 manufacturer when trade credit is unavailable**.

**Case:** $Rw \leq \pi^P p$**.**

$$K_{22}^P = \pi_2^H \left( \frac{\pi_P^H pH + \pi_P^L pL - RwH}{\pi_P^H} \right). \tag{4.255}$$

**Case:** $\pi^P p \leq Rw \leq \pi^T p$**.**

$$K_{22}^B = (p - Rw)L. \tag{4.256}$$

**Expected benefit of having trade credit available for a type-2 manufacturer**.

**Case:** $Rw \leq \pi_2^H p$**.**

$$\text{Benefit}_2 = \pi_2^H pH + \pi_2^L pL - RwH - \pi_2^H \left( \frac{\pi_P^H pH + \pi_P^L pL - RwH}{\pi_P^H} \right) \tag{4.257}$$

$$= \frac{(\pi_2^L \pi_P^H - \pi_2^H \pi_P^L) pL - (\pi_P^H - \pi_2^H) RwH}{\pi_P^H} \tag{4.258}$$

$$= \frac{[(1-\pi_2^H)\pi_P^H - \pi_2^H(1-\pi_P^H)]pL - (\pi_P^H - \pi_2^H) RwH}{\pi_P^H} \tag{4.259}$$

$$= \frac{(\pi_P^H - \pi_2^H)(pL - RwH)}{\pi_P^H}. \tag{4.260}$$

We have a negative benefit because $\pi_P^H > \pi_2^H$ and $RwH > pL$.

**Case:** $\pi_2^H p \leq Rw \leq \pi^P p$**.**

$$\text{Benefit}_2 = (p - Rw)L - \pi_2^H \left( \frac{\pi_P^H pH + \pi_P^L pL - RwH}{\pi_P^H} \right) \tag{4.261}$$

$$= \frac{\pi_P^H pL - \pi_2^H(\pi_P^H pH + \pi_P^L pL) + Rw(\pi_2^H H - \pi_P^H L)}{\pi_P^H} \tag{4.262}$$

$$= \frac{(\pi_P^H L - \pi_2^H H)(\pi^P p - wR)}{\pi_P^H}. \tag{4.263}$$

We have a negative benefit because $L \leq \pi_2^H H$, $\pi_P^H \leq 1$ and $Rw \leq \pi^P p$.

**Case:** $\pi^P p \leq Rw \leq \pi^T p$**.**

$$\text{Benefit}_2 = (p - Rw)L - (p - Rw)L \tag{4.264}$$

$$= 0. \tag{4.265}$$

Therefore, if $\pi^P p \leq Rw \leq \pi^T p$, then a type-2 manufacturer does not benefit from having trade credit available.

#### 4.4.8.3 Overall economy.

In expectation, the competition among banks and among suppliers make the bank and the supplier make zero profit and the system profit to be equal to the manufacturers' profits. Therefore, when there is asymmetric information between the bank and the manufacturer, the expected benefit of making trade credit available for the system is equal to $Pr[\theta = 1]E[\text{System Benefit}|\theta = 1] + Pr[\theta = 2]E[\text{System Benefit}|\theta = 2]$ where $Pr[\theta = 1] = \alpha$ and $Pr[\theta = 1] = 1 - \alpha$.

**Case:** $pL < Rw \leq \pi_2^H p$**.**

$$\begin{aligned}
\text{Expected System Benefit} =& \alpha\frac{(\pi_P^H - \pi_1^H)(pL - RwH)}{\pi_P^H} \\
&+ (1-\alpha)\frac{(\pi_P^H - \pi_2^H)(pL - RwH)}{\pi_P^H} \\
=& \frac{\pi_P^H\,(pL - RwH) - \pi_P^H\,(pL - RwH)}{\pi_P^H} \\
=& 0.
\end{aligned} \tag{4.266}$$

**Case:** $\pi_2^H p \leq Rw \leq \pi^P p$**.**

$$\begin{aligned}
\text{Expected System Benefit} =& \alpha\frac{(\pi_P^H - \pi_1^H)(pL - RwH)}{\pi_P^H} \\
&+ (1-\alpha)\left[(p - Rw)L - \pi_2^H\left(\frac{\pi_P^H pH + \pi_P^L pL - RwH}{\pi_P^H}\right)\right] \\
=& \frac{(1-\alpha)\pi_P^H RwH - (1-\alpha)\pi_P^H RwL}{\pi_P^H} \\
&+ \frac{(1-\alpha)\pi_2^H\pi_P^H pL - (1-\alpha)\pi_2^H\pi_P^H pH}{\pi_P^H} \\
=& (1-\alpha)(H-L)(Rw - \pi_2^H p).
\end{aligned} \tag{4.267}$$

We have a positive benefit because $0 < L < H$, $0 \leq \alpha \leq 1$ and we are analyzing when $Rw \geq \pi_2^H p$.

**Case:** $\pi^P p \leq Rw \leq \pi^T p$**.**

$$\text{Expected System Benefit} = \alpha\left[\frac{(\pi^T p - Rw)(\pi_2^H H - \pi_1^H L)}{\pi_2^H}\right]. \tag{4.268}$$

We have a positive benefit because $0 \leq \alpha \leq 1$, $\pi_2^H \geq 0$, and we are analyzing when $L \leq \pi_2^H H$ and $Rw \leq \pi^T p$.

#### 4.4.8.4 End-customers.

To determine the effect of trade credit to the end-customers we measure the expected number of stock-outs at the end of the planning horizon in the overall economy while assuming that a lost sale for a manufacturer does not result in an increase in the demand for the final goods of other manufacturers. For a manufacturer of type-$\theta$, the expected number of stock-outs at the end of the planning horizon when the manufacturer orders $y$ units is equal to $\pi_\theta^H(H-y)^+ + \pi_\theta^L(L-y)^+$. Therefore, at the end of the planning horizon, the expected number of stock-outs in the overall economy (ENSO) is equal to $Pr[\theta = 1]E[\text{Number of stock-outs}|\theta = 1] + Pr[\theta = 2]E[\text{Number of stock-outs}|\theta = 2]$.

**Case:** $pL < Rw \leq \pi_2^H p$**.**

When trade credit is unavailable,

$$\begin{aligned} ENSO =& \alpha\left[\pi_1^H(H-H)^+ + \pi_1^L(L-H)^+\right] \\ &+ (1-\alpha)\left[\pi_2^H(H-H)^+ + \pi_2^L(L-H)^+\right] \\ =& 0. \end{aligned} \tag{4.269}$$

When trade credit is available,

$$\begin{aligned} ENSO =& \alpha\left[\pi_1^H(H-H)^+ + \pi_1^L(L-H)^+\right] \\ &+ (1-\alpha)\left[\pi_2^H(H-H)^+ + \pi_2^L(L-H)^+\right] \\ =& 0. \end{aligned} \tag{4.270}$$

**Case:** $\pi_2^H p \leq Rw \leq \pi^P p$**.**

When trade credit is unavailable,

$$\begin{aligned} ENSO =& \alpha\left[\pi_1^H(H-H)^+ + \pi_1^L(L-H)^+\right] \\ &+ (1-\alpha)\left[\pi_2^H(H-L)^+ + \pi_2^L(L-H)^+\right] \\ =& 0. \end{aligned} \tag{4.271}$$

When trade credit is available,

$$\begin{aligned} ENSO =& \alpha\left[\pi_1^H(H-H)^+ + \pi_1^L(L-H)^+\right] \\ &+ (1-\alpha)\left[\pi_2^H(H-L)^+ + \pi_2^L(L-L)^+\right] \\ =&(1-\alpha)\pi_2^H(H-L). \end{aligned} \tag{4.272}$$

**Case:** $\pi^P p \le Rw \le \pi^T p$**.**

When trade credit is unavailable,

$$\begin{aligned} ENSO =& \alpha\left[\pi_1^H(H-\overline{y}^T)^+ + \pi_1^L(L-\overline{y}^T)^+\right] \\ &+ (1-\alpha)\left[\pi_2^H(H-L)^+ + \pi_2^L(L-L)^+\right] \\ =&(1-\alpha)\pi_2^H(H-L). \end{aligned} \tag{4.273}$$

When trade credit is available,

$$\begin{aligned} ENSO =& \alpha\left[\pi_1^H(H-H)^+ + \pi_1^L(L-H)^+\right] \\ &+ (1-\alpha)\left[\pi_2^H(H-L)^+ + \pi_2^L(L-L)^+\right] \\ =&(1-\alpha)\pi_2^H(H-L). \end{aligned} \tag{4.274}$$

#### 4.4.8.5 Discussion.

Unsurprisingly, type-1 manufacturers, the most credible manufacturers, prefer to have trade credit available while type-2 manufacturers prefer not to have trade credit available. Surprisingly, end-customers suffer from trade credit because, when type-1 manufacturers use trade credit to signal their type, fewer goods are ordered by manufacturers which increases the expected number of stock-outs in the economy.

The availability of trade credit discourages manufacturers from over-ordering but it does not affect the demand distribution of manufacturers. Therefore, the overall economy benefits from trade credit because trade credit decreases overall production costs.

### 4.4.9 Effect of the economic parameters on the expected benefit of having trade credit available.

I consider when $pL < RwH$, $L \leq \pi_2^H H$ and $Rw \leq \pi^T p$ because that is when the operational and financial decisions of manufacturers are affected by the availability of trade credit.

#### 4.4.9.1 Type-1 manufacturers.

**Case:** $Rw \leq \pi^P p$**.**

$$\frac{\partial \text{Benefit}_1}{\partial \pi_1^H} = \frac{-(1-\alpha)\pi_2^H(pL - RwH)}{(\pi_P^H)^2} \geq 0 \tag{4.275}$$

$$\frac{\partial \text{Benefit}_1}{\partial \pi_2^H} = \frac{(1-\alpha)\pi_1^H(pL - RwH)}{(\pi_P^H)^2} \leq 0 \tag{4.276}$$

$$\frac{\partial \text{Benefit}_1}{\partial H} = \frac{(\pi_1^H - \pi_P^H)Rw}{\pi_P^H} \geq 0 \tag{4.277}$$

$$\frac{\partial \text{Benefit}_1}{\partial L} = \frac{-(\pi_1^H - \pi_P^H)p}{\pi_P^H} \leq 0 \tag{4.278}$$

$$\frac{\partial \text{Benefit}_1}{\partial p} = \frac{-(\pi_1^H - \pi_P^H)L}{\pi_P^H} \leq 0 \tag{4.279}$$

$$\frac{\partial \text{Benefit}_1}{\partial R} = \frac{(\pi_1^H - \pi_P^H)wH}{\pi_P^H} \geq 0 \tag{4.280}$$

$$\frac{\partial \text{Benefit}_1}{\partial w} = \frac{(\pi_1^H - \pi_P^H)RH}{\pi_P^H} \geq 0 \tag{4.281}$$

$$\frac{\partial \text{Benefit}_1}{\partial \alpha} = \frac{\pi_1^H(\pi_1^H - \pi_2^H)(pL - RwH)}{(\pi_P^H)^2} \leq 0. \tag{4.282}$$

When trade credit is unavailable, the interest rate received by type-1 manufacturers become more expensive as the riskiness of type-2 manufacturers increases. Recall that the objective value of a manufacturer is non-increasing in the interest rate received. This is why, when trade credit is unavailable, the expected profit of type-1 manufacturers is non-decreasing in the expected profit of type-2 manufacturers. Recall that the benefit that type-1 manufacturers obtain when trade credit is available depends on the difference between the objective value when trade credit is available and the objective value when trade credit is unavailable. This explains why the benefit that type-1 manufacturers obtain when trade credit is available is non-decreasing in $\pi_2^H$, $p$, and non-increasing in $R$, and $w$.

As the proportion of type-1 manufacturers in the economy increases, the rates received by manufacturers when trade credit is unavailable decreases. This is why the benefit that type-1 manufacturers obtain when trade credit is available is non-increasing in $\alpha$. However, because type-2 manufacturers become less risky as the smallest demand value that can be generated increases, the benefit that type-1 manufacturers obtain when trade credit is available is non-increasing in $L$. Also, the decrease in interest rates received by type-1 manufacturers as $\pi_1^H$ increases leads to the benefit that type-1 manufacturers obtain when trade credit is available to be non-decreasing in $\pi_1^H$.

**Case: $\pi^P p \leq Rw \leq \pi^T p$.**

$$\frac{\partial \text{Benefit}_1}{\partial \pi_1^H} = \frac{(\pi_2^H H - L)p + (Rw - \pi_2^H p)L}{\pi_2^H} \geq 0 \tag{4.283}$$

$$\frac{\partial \text{Benefit}_1}{\partial \pi_2^H} = \frac{\pi_1^H (p - Rw)}{(\pi_2^H)^2} \geq 0 \tag{4.284}$$

$$\frac{\partial \text{Benefit}_1}{\partial H} = \pi_1^H p - Rw \geq 0 \tag{4.285}$$

$$\frac{\partial \text{Benefit}_1}{\partial L} = \frac{(\pi_2^H - \pi_2^H \pi_1^H - \pi_1^H)p + Rw\pi_1^H}{\pi_2^H} \leq 0 \tag{4.286}$$

$$\frac{\partial \text{Benefit}_1}{\partial p} = \frac{\pi^T (\pi_2^H H - \pi_1^H L)}{\pi_2^H} \geq 0 \tag{4.287}$$

$$\frac{\partial \text{Benefit}_1}{\partial R} = \frac{-w(\pi_2^H H - \pi_1^H L)}{\pi_2^H} \leq 0 \tag{4.288}$$

$$\frac{\partial \text{Benefit}_1}{\partial w} = \frac{-R(\pi_2^H H - \pi_1^H L)}{\pi_2^H} \leq 0 \tag{4.289}$$

$$\frac{\partial \text{Benefit}_1}{\partial \alpha} = 0. \tag{4.290}$$

When trade credit is unavailable, the higher the expected profit that can be generated by type-2 manufacturers, the higher the order quantities of type-1 manufacturers have to be in order to discourage type-2 manufacturers from pretending to be type-1 manufacturers. This is why, when trade credit is unavailable, the objective value of type-1 manufacturers is non-increasing in the expected revenue of type-2 manufacturers and non-decreasing in the expected costs of type-2 manufacturers. Hence, the reason the benefit that type-1 manufacturers obtain when trade credit is available is non-decreasing in $\pi_2^H$, $H$, $p$, and non-increasing in $R$, and $w$. However, type-2 manufacturers become less risky as $L$, the smallest demand value that can be generated, increases. This is why the benefit that type-1 manufacturers obtain when trade credit is available is non-increasing in $L$.

Observe that the benefit that type-1 manufacturers obtain when trade credit is available is independent of $\alpha$. Also, a similar discussion as the one provided for $\pi_1^H$ in the case when $Rw \leq \pi^P p$ gives that the benefit type-1 manufacturers obtain when trade credit is available is non-decreasing in $\pi_1^H$.

#### 4.4.9.2 Type-2 manufacturers.

**Case:** $Rw \leq \pi_2^H p$.

$$\frac{\partial \text{Benefit}_2}{\partial \pi_1^H} = \frac{\alpha \pi_2^H (pL - RwH)}{(\pi_P^H)^2} \leq 0 \tag{4.291}$$

$$\frac{\partial \text{Benefit}_2}{\partial \pi_2^H} = \frac{-\alpha \pi_1^H (pL - RwH)}{(\pi_P^H)^2} \geq 0 \tag{4.292}$$

$$\frac{\partial \text{Benefit}_2}{\partial H} = \frac{-(\pi_P^H - \pi_2^H) Rw}{\pi_P^H} \leq 0 \tag{4.293}$$

$$\frac{\partial \text{Benefit}_2}{\partial L} = \frac{(\pi^H - \pi_2^H) p}{\pi_P^H} \geq 0 \tag{4.294}$$

$$\frac{\partial \text{Benefit}_2}{\partial p} = \frac{(\pi_P^H - \pi_2^H) L}{\pi_P^H} \geq 0 \tag{4.295}$$

$$\frac{\partial \text{Benefit}_2}{\partial R} = \frac{-(\pi_P^H - \pi_2^H) wH}{\pi_P^H} \leq 0 \tag{4.296}$$

$$\frac{\partial \text{Benefit}_2}{\partial w} = \frac{-(\pi_P^H - \pi_2^H) RH}{\pi_P^H} \leq 0 \tag{4.297}$$

$$\frac{\partial \text{Benefit}_2}{\partial \alpha} = \frac{\pi_2^H(\pi_1^H - \pi_2^H)(pL - RwH)}{(\pi_P^H)^2} \leq 0. \tag{4.298}$$

A similar discussion as the one provided for type-1 manufacturers explains the sensitivity of the benefit of type-2 manufacturers to have trade credit available when there is asymmetric information between the bank and the manufacturers.

**Case:** $\pi_2^H p \leq Rw \leq \pi^P p$**.**

$$\frac{\partial \text{Benefit}_2}{\partial \pi_1^H} = \frac{\alpha\pi_2^H(pL - RwH)}{(\pi_P^H)^2} \leq 0 \tag{4.299}$$
$$\frac{\partial \text{Benefit}_2}{\partial \pi_2^H} = \frac{-\pi_2^H p(H-L) - \alpha\pi_1^H(pL - RwH)}{(\pi_P^H)^2} \leq 0 \tag{4.300}$$
$$\frac{\partial \text{Benefit}_2}{\partial H} = \frac{-\pi_2^H(p\pi_P^H - Rw)}{\pi_P^H} \leq 0 \tag{4.301}$$
$$\frac{\partial \text{Benefit}_2}{\partial L} = \frac{p(\pi_P^H + \pi_P^H\pi_2^H - \pi_2^H) - Rw\pi_P^H}{\pi_P^H} \geq 0 \tag{4.302}$$
$$\frac{\partial \text{Benefit}_2}{\partial p} = \frac{\pi_P^H(L - \pi_2^H H) - \pi_2^H L(1 - \pi_P^H)}{\pi_P^H} \leq 0 \tag{4.303}$$
$$\frac{\partial \text{Benefit}_2}{\partial R} = \frac{-w(\pi_P^H L - \pi_2^H H)}{\pi_P^H} \geq 0 \tag{4.304}$$
$$\frac{\partial \text{Benefit}_2}{\partial w} = \frac{-R(\pi_P^H L - \pi_2^H H)}{\pi_P^H} \geq 0 \tag{4.305}$$
$$\frac{\partial \text{Benefit}_2}{\partial \alpha} = \frac{\pi_2^H(\pi_1^H - \pi_2^H)(pL - RwH)}{(\pi_P^H)^2} \leq 0. \tag{4.306}$$

A similar discussion as the one provided for type-1 manufacturers explains the sensitivity of the benefit of type-2 manufacturers to have trade credit available when there is asymmetric information between the bank and the manufacturers.

**Case:** $\pi^P p \leq Rw \leq \pi^T p$**.**

The objective value of type-2 manufacturers is the same when trade credit is available and when trade credit is unavailable. Therefore, the availability of trade credit does not affect type-2 manufacturers.

#### 4.4.9.3 Overall economy.

**Case:** $Rw \leq \pi_2^H p$**.**

Recall that lenders are always breaking-even on the loans extended to manufacturers. Because the total number of goods produced in the economy is the same when trade credit is available and when trade credit is unavailable, the overall repayment risk in the economy does not change with trade credit. This is why the availability of trade credit does not affect the overall economy.

**Case:** $\pi_2^H p \leq Rw \leq \pi^P p$**.**

$$\frac{\partial \text{System}}{\partial \pi_1^H} = 0 \tag{4.307}$$

$$\frac{\partial \text{System}}{\partial \pi_2^H} = -(1-\alpha)p(H-L) \leq 0 \tag{4.308}$$

$$\frac{\partial \text{System}}{\partial H} = (1-\alpha)(Rw - \pi_2^H) \geq 0 \tag{4.309}$$

$$\frac{\partial \text{System}}{\partial L} = -(1-\alpha)(Rw - \pi_2^H) \leq 0 \tag{4.310}$$

$$\frac{\partial \text{System}}{\partial p} = -(1-\alpha)\pi_2^H(H-L) \leq 0 \tag{4.311}$$

$$\frac{\partial \text{System}}{\partial R} = (1-\alpha)w(H-L) \geq 0 \tag{4.312}$$

$$\frac{\partial \text{System}}{\partial w} = (1-\alpha)R(H-L) \geq 0 \tag{4.313}$$

$$\frac{\partial \text{System}}{\partial \alpha} = -(H-L)(Rw - \pi_2^H p) \leq 0. \tag{4.314}$$

A similar discussion as the one provided for type-1 and type-2 manufacturers explains the sensitivity of the benefit of type-2 manufacturers to have trade credit available when there is asymmetric information between the bank and the manufacturers.

**Case:** $\pi^P p \leq Rw \leq \pi^T p$**.**

$$\frac{\partial \text{System}}{\partial \pi_1^H} = \alpha \frac{(\pi_2^H H - L)p - (\pi_2^H p - Rw)L}{\pi_2^H} \geq 0 \tag{4.315}$$

$$\frac{\partial \text{System}}{\partial \pi_2^H} = \alpha \frac{\pi_1^H (p - Rw)}{(\pi_2^H)^2} \geq 0 \tag{4.316}$$

$$\frac{\partial \text{System}}{\partial H} = \alpha(\pi_1^H p - Rw) \geq 0 \tag{4.317}$$

$$\frac{\partial \text{System}}{\partial L} = \alpha \frac{(\pi_2^H - \pi_2^H \pi_1^H - \pi_1^H)p + Rw\pi_1^H}{\pi_2^H} \leq 0 \tag{4.318}$$

$$\frac{\partial \text{System}}{\partial p} = \alpha \frac{\pi^T (\pi_2^H H - \pi_1^H L)}{\pi_2^H} \geq 0 \tag{4.319}$$

$$\frac{\partial \text{System}}{\partial R} = \alpha \frac{-w(\pi_2^H H - \pi_1^H L)}{\pi_2^H} \leq 0 \tag{4.320}$$

$$\frac{\partial \text{System}}{\partial w} = \alpha \frac{-R(\pi_2^H H - \pi_1^H L)}{\pi_2^H} \leq 0 \tag{4.321}$$

$$\frac{\partial \text{System}}{\partial \alpha} = 0. \tag{4.322}$$

A similar discussion as the one provided for type-1 and type-2 manufacturers explains the sensitivity of the benefit of type-2 manufacturers to have trade credit available when there is asymmetric information between the bank and the manufacturers.

## 4.5 Conclusions, limitations and extensions.

I focus on the operational and financial decisions of cash-constrained manufacturers when there is asymmetric information between banks and manufacturers about the distribution of the demand for final goods of manufacturers. When there is asymmetric information between banks and manufacturers, a high-risk manufacturer never suffers if the bank confuses it for a low-risk manufacturer and, hence, offers cheap rates to this manufacturer. This can lead to high-risk manufacturers being able to pass for low-risk manufacturers and order high quantities when it is not too expensive for them to do so. Asymmetric information between banks and manufacturers can also lead to a decrease in the objective value of low-risk manufacturers because low-risk manufacturers might have to over-order in order to signal their type to banks.

Consequently, the default risk of manufacturers in the economy increases significantly with asymmetric information. This is why asymmetric information negatively affects the business of lenders and low-risk manufacturers. However, the end-customers and the high-risk manufacturers benefit from asymmetric information.

Similar to the results obtained from empirical studies, my analysis suggests that trade credit financing does not only reduce financial constraints but its use by manufacturers allows them to obtain loan terms from banks that more accurately reflect their credibility. The use of trade credit itself does not alleviate the asymmetric information that exists between banks and manufacturers: manufacturers can only send credit signals to banks with trade credit when there is a risk that manufacturers will default on loans obtained from suppliers. This is why, although I presented results for the case when banks have higher priority on loan repayments than suppliers, the insights gathered for the case when suppliers have higher priority on loan repayments than banks are similar.

This study not only captures the effect of trade credit and asymmetric information on the return of manufacturers' projects but it also captures the effect of asymmetric information along with trade credit on the operational decisions of manufacturers which is a major contribution. For example, in contrast to Biais and Gollier (1997), the break-even interest rates offered by lenders determine the operational investment of each type of manufacturer.

Although the results obtained are primarily driven by pure competition among banks and suppliers, the insights gathered from my analysis would have remained the same if I had considered a more complicated model in which manufacturers are more powerful than banks and suppliers. Furthermore, considering a model with more than two types of manufacturers would not have changed the insights gathered from this model because manufacturers can always signal their type by borrowing a risky amount from the supplier. Therefore, considering a model with more than two types of manufacturers would have just made the analysis more complicated without justifying the complexity in deriving the incremental results obtained.

My model, albeit simple, provides qualitative insights that can help foster future research on the effect asymmetric information between lenders and manufacturers and the effect of trade credit financing on the operational and financial decisions of manufacturers. For example, it would be interesting to study collusion possibilities between suppliers and manufacturers against other suppliers and banks. It would also be interesting to determine, when more than two demand possibilities can occur, if there are situations when manufacturers with higher expected number of end-customers receive higher interest rates for placing the same order quantity as manufacturers with lower expected number of end-customers.

## CHAPTER V

# Conclusion

The theoretical and numerical analysis generated several testable hypotheses. Some of these hypotheses have already been confirmed in prior empirical studies. Others can be verified in future empirical work. I derived both anticipated and surprising results.

Concerning how supply risk, financing constraints, and the dual role served by suppliers affect supplier selection, the analysis suggests that, as the availability of either internal financing or supplier loans diminishes, the optimal number of suppliers may increase. To understand this, consider that manufacturers, by paying the extra cost to work with additional suppliers, benefit by relaxing their financing constraints and increasing their order quantities, to earn higher expected revenues. More surprising are the observations that an increase in the cost to work with a supplier or the wholesale price may result in an increase in the optimal number of suppliers. Also surprising is that, as the standard deviation of a supplier yield's increases, the optimal number of suppliers could either increase or decrease.

A significant part of the research was to address if one expects to observe empirically that manufacturers in developing economies work with more suppliers, The answer to this question is "it depends." For example, everything being equal, my analysis suggests that manufacturers in developing economies will have more suppliers than comparable manufacturers in developed economies. But if, in a developing economy, the cost to work with a supplier is very high or the manufacturer is close to bankruptcy, then that manufacturer may actually have fewer suppliers than its counterpart in a developed economy. In this case, the analysis suggests that manufacturers in developing economies will place lower order quantities and will have higher stock-out probabilities, which matches perfectly the observations of the earlier empirical studies.

Thus, to answer the question: "Should one expect to observe empirically that manufacturers in developing economies work with more suppliers?" one needs a sophisticated empirical analysis, which carefully accounts for the factors that we considered. One of the main contributions of this research is to provide a set of testable hypotheses for future empirical studies.

Concerning when should manufacturers use trade-credit, the analysis indicates that cash-constrained manufacturers should only use trade-credit to finance their operations when borrowing from suppliers is cheap, and when manufacturers are trying to signal their credibility to banks. Although financing operations with trade credit can be more expensive than financing operations with other sources, manufacturers, especially when they receive a relatively small line of credit from banks and/or there is asymmetric information between banks and manufacturers, may benefit in the long-run by using trade credit to finance their operations. The analysis suggests that, similar to the results obtained from empirical studies, trade credit financing does not only reduce financing constraints but its use by manufacturers allows them to obtain loan terms from banks that more accurately reflect their credibility.

My analysis contributes to the finance literature because I explicitly model operational decisions in a dynamic framework, and, simultaneously focus on the use of trade credit to update the bank loan limit instead of using trade credit to adjust the interest rate on the bank loan terms. My analysis contributes to the operations literature because I consider the effect of trade credit on the bank loan limit and, simultaneously focus on the effect of indirect financing costs on operational decisions.

The models analyzed, albeit simple, provide qualitative results that ought to be considered in practice. Better understanding of the effect of trade credit on operational policies and on the relationship between banks, suppliers, and manufacturers can add great value to the shareholders of cash-constrained manufacturers.

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