Labor Market Frictions and Dynamic Labor Demand

by

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To my wife, Annette, and my parents

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CHAPTER I

Introduction

Simply acknowledging that the U.S. labor market does not behave like a frictionless, spot market introduces a number of complications into the analysis of labor demand. Economists' understanding of these questions is still growing. On the one hand, while there are undoubtedly frictions in the hiring and firing process, it has often been hard to identify concrete examples that could be rigorously evaluated. Recently, though, empirical work has identified clear, observable costs of adjusting employment and estimated the quantitative effects of these costs on firms' labor demand decisions. On the other hand, an exhaustive theoretical analysis of these frictions has been somewhat delayed because of the difficulty of the models, even if the intuition behind the models' behavior is sometimes apparent. Therefore, the development of robust theoretical results has also received more attention recently.

This dissertation contributes to both of these avenues of research. The first chapter explores the consequences of an employment adjustment cost that prevailed within the domestic motor vehicle industry for much of the last 20 years. The cost takes the form of a provision within the union labor agreement known as the JOBS Bank. This provision required domestic motor vehicle manufacturers to pay full salary to a worker for each week spent on layoff beyond an allotment specified in the contract. Because the JOBS Bank provision is literally written into the labor agreement, it is a readily observable and measurable cost of adjustment. This is one reason it is unique within the literature. In many cases, authors must specify stylized forms of adjustment costs – for instance, the cost of hiring workers grows linearly in the number of new hires – and estimate these, since no direct measures of the costs exist. Therefore, the concreteness and transparency of the JOBS Bank make it valuable from the standpoint of research. This chapter is able to take the cost of adjustment as given – the analysis does not rely on any assumptions about its form. It then estimates the effect of the JOBS Bank on employment dynamics at vehicle assembly plants.

The second and third chapters provide a theoretical analysis of labor demand models with frictions. In each chapter, the cost of adjustment is more stylized than the JOBS Bank. This sacrifices an element of realism but it has two payoffs. First, the costs may be more broadly applicable in that each of them likely summarizes, at least approximately, several frictions that are present in many sectors. This differs from the JOBS Bank, which is precisely measurable but specific to one industry. Second, the stylized costs of chapters two and three are relatively tractable, enabling a more complete theoretical analysis of their implications. In contrast, much of the analysis of the model of the JOBS Bank must be done by computational methods.

In the second chapter, firms post vacancies in order to match with unemployed workers. The employment adjustment friction takes the form of a cost that firms must pay to advertise each of their vacancies. The chapter also allows for a non-linear production technology and incorporates both idiosyncratic and aggregate innovations to productivity. The result is a rich model of the firm's labor demand problem. One of the key contributions of the paper is to show that the model is also, despite its many "moving parts," quite tractable. It thus provides a framework in which to consider the implications of employment frictions for the behavior of individual firms and for the aggregate labor market. Specifically, the paper provides a new result on the determination of the wage rate in environments with frictions; develops comparative statics for optimal dynamic labor demand; and provides a way to analytically aggregate over the decisions of heterogeneous firms in order to characterize the causes of aggregate unemployment fluctuations.

Reflecting the rich array of frictions that operate in the U.S. labor market, the third chapter confronts the firm with yet another cost of adjustment. This friction is a seemingly simple fixed cost of adjusting employment: it is paid whenever any (net) change is made to the stock of workers. Despite its apparent simplicity, the difficulty in analyzing fixed costs is well known: the presence of a discrete cost implies that employment changes discretely, meaning that it is necessary to determine the conditions under which the firm optimally "does nothing". The paper provides a new means by which to solve the firm's problem analytically, whereas the previous literature had generally resorted to numerical methods. The optimal forward-looking policy rule of the model may be shown to nest the solution of the corresponding static, or myopic, model. Moreover, for reasonable calibrations of the model, the myopic policy rule actually approximates forward-looking labor demand quite well. Since the myopic rule is readily understandable, this result provides a critical window onto the mechanics of a problem that has been considered quite difficult in the literature.

CHAPTER II

Layoff Costs and Dynamic Labor Demand: Evidence from the U.S. Auto Industry's Job Security Agreement

2.1 Introduction

In the fall of 1990, America's largest corporation, General Motors, accepted a new provision in its contract with the United Automobile Workers (UAW) that represented a significant departure from recent agreements. GM agreed to substantially restrict its ability to reduce its own workforce and, as a result, would guarantee full pay to workers for whom there was, in fact, no work to do. It was no coincidence that UAW members voted 80-20 to ratify the agreement; it was the most lop-sided vote in generations. The other two major domestic automobile producers, Ford Motor and Chrysler, agreed to the same provision one month later.

This paper studies some of the economic consequences of this provision, known as the Job Opportunity Bank Security, or JOBS Bank. Stated more precisely, the JOBS Bank required domestic automotive manufacturers to pay full salary to a worker for each week that worker spends on layoff beyond an allotment specified in the union agreement. This structure has an immediate implication for the producer's problem: if there is an allotment of weeks on layoff, a week "spent" now is one that cannot be deployed in the future if automobile demand deteriorates further within the current union contract. It is this trade-off between layoffs now or (potentially) later that forms one of the key problems posed by JOBS, and it is the focus of this paper.

I first present a stylized model of the manufacturer's problem that illustrates that JOBS generates an option value of production: a plant wants to produce, and safeguard its allotment of layoffs, in the event that vehicle demand declines further over the life of the contract. Within the model, this behavior is manifest in two ways. First, the plant does fewer layoffs, all else equal, as the number of past layoffs approaches the contractual allotment. Second, it implements more layoffs as uncertainty is resolved and the contract draws to a close.

I initially use the model to motivate a linear regression that relates weeks on layoff per month to the two critical JOBS-related variables: number of past layoffs (as a share of the allotment) and the fraction of the contract completed. For this, I draw on a comprehensive plant-level database that links layoffs at each U.S. assembly plant to the sales and inventories of products made there. I initially find evidence that is consistent with the key qualitative implications of the model.

However, further investigation casts doubt on a causal interpretation of the reduced form. When I re-estimate the regression model on data prior to 1990 – a period in which the JOBS Bank effect ought to be zero – I still find non-zero effects. The reduced form thus fails a key falsification test. The paper identifies potential sources of specification error that appear to contribute to a spurious JOBS Bank effect in both sample periods. The analysis of the reduced form concludes that the JOBS effect may in fact be almost zero.

The paper attempts to understand this result in the context of the structural model. It first applies a simulation-based estimator, indirect inference, to uncover the structural parameters and finds that the results also imply virtually no JOBS Bank effect. With the parameters in hand, it is possible to conduct a series of simulations that helps identify the mechanisms of the model behind this result. The analysis highlights, among other results, that a robust effect of the JOBS Bank requires counterfactually highly persistent innovations to product demand. To see why, consider a plant that receives an adverse shock and expects that its product demand will still likely be subdued in the future when another negative disturbance may arrive. This plant has an incentive to defer layoffs after the first shock, and instead "wait and see" if it will need its allotment of layoff weeks later in the contract when demand may be lower still. On the other hand, if shocks dissipate quickly, the length of the labor contract and the size of the allotment are such that the threat of JOBS binding is small.

The paper follows in a long line of work on employment frictions. For the purposes of research, the JOBS program has a distinct advantage in that it is literally written down as part of the UAW's agreements with the Detroit Three, or the major domestic automobile producers: GM, Ford Motor, and Chrysler. This allows me to de-couple the measurement of the cost from the study of its employment effects. This approach differs from macroeconomic studies that rely on a particular model to map from the data on employment outcomes into the size and shape of the employment adjustment cost. Since I may take the latter as given, this paper's analysis of its effects is not sensitive to the choice of any one parametric model used to estimate the cost.

This study is particularly closely related to work that has analyzed the effects of employment frictions introduced by state and federal laws. Research in this area identified observable frictions and attempted to estimate their causal effects on labor demand. Examples of frictions include the layoff taxes charged under states' unemployment insurance systems and wrongful dismissal statutes.[1] Even in this context, though, the tractable structure of the JOBS program and the availability of plant-level data make JOBS stand out. In other applications, the adjustment cost, though real, is sometimes hard to incorporate into a formal model[2] or the data are not available at the level of the decision-maker.[3] This paper benefits from a combination of a clear cost structure as well as a rich plant-level dataset on layoffs, sales, and inventories. These features allow me to relate outcomes more precisely to observable changes in incentives at the microeconomic level from month to month.

This paper now proceeds in six parts. Section 2 summarizes the essential details of the JOBS program. Section 3 begins with a simple example to illustrate the mechanism by which JOBS affects plant's labor demand decisions. It then presents a more complete model of the plant's problem and characterizes the optimal policy functions for layoffs, inventory, and sales. Section 4 uses the model to motivate a linear regression that relates weeks of layoff to accumulated layoffs and the share of the contract completed, the two JOBS-related variables. It estimates the regression equation on a rich, plant-level database that links layoffs at the plant level to sales and inventories of vehicles produced there. Various robustness tests are also conducted in this section, such as the use of the pre-treatment period as noted above. Section 5 estimates the structural parameters of the model by indirect inference and uses the estimates to conduct counterfactual simulations which shed light on the mechanisms of the model that induce the small JOBS Bank observed in the reduced form. Section 7 concludes and speculates on how one might still resurrect an JOBS Bank effect employment. Details regarding the data and proofs associated with the model are provided in two appendices in Section 8.

[1]On wrongful discharge laws, see Autor (2003); Autor, Donohue, and Schwab (2004, 2006); and, for a rebuttal, Bester, Conley, Hansen, and Vogelsang (2008). On unemployment insurance, see Anderson (1993) and Anderson and Meyer (2000).

[2]For instance, the channel through which wrongful discharge laws affect the labor demand decision depends, in part, on the likelihood of a employee-initiated lawsuit. This may be a difficult feature of the problem to formally model.

[3]To take one example, the study of layoff taxes under states' unemployment insurance systems generally requires access to establishment-specific layoff histories and tax rates. But these data are often hard to obtain.

2.2 The Detroit Three's Benefits for Laid-off Workers

Domestic motor vehicle manufacturers have offered some form of supplemental unemployment income for the last 50 years. The automobile manufacturers and the UAW first agreed to supplemental insurance in 1955, and it has been included in all contracts since. The Supplemental Unemployment Benefit (SUB) pays approximately 40 percent of a worker's weekly (pre-tax) salary when that worker is laid off for a full week.[4] The firm also continues to provide non-cash benefits, such as health care, to the laid-off worker.[5]

In 1984, firms agreed to a more expansive benefit for displaced workers. The JOBS program at General Motors paid full salary to any worker displaced for "structural" reasons, such as advances in automation (i.e., the introduction of new robots into a plant) (see Block, 2006). Ford agreed to adopt the virtually identical Protected Employee Program that year, and Chrysler followed suit in the subsequent year. For simplicity, I will generally refer to the job security programs at all Detroit Three firms as the "JOBS Bank", or simply JOBS. Workers laid off for "volume-related" reasons, i.e., a slump in sales that pushes down labor demand *given* the current level of automation, were not able to eligible to receive full salary under JOBS at this time.

The subject of this paper is the expansion of the JOBS program in the fall of 1990 to cover workers laid off due to a decline in automobile demand. This portion of the program remained in place through 2008, although it was modified in an important way in the fall 2007 negotiations and finally suspended in January 2009 in the midst of the financial crisis.[6] It did not entitle workers to full pay immediately upon layoff, as did the program introduced in 1984. Instead, the firms agreed to pay workers who were laid off due to "volume related declines attributable to market related conditions" (UAW, 1990) full salary for every week spent on layoff beyond a contractually set maximum. The upper bound was 36 weeks in the 1990 agreement, extended to 42 weeks in 1999 (when the UAW and the Detroit Three adopted a four-, rather than a three-, year contract), and

[4]The Detroit Three also pay a "short week" benefit to workers on layoff for less than a full week. For those days on which a worker is on layoff, he receives slighty more than 80 percent of his straight-time pay. Since short weeks are rare relative to full-week layoffs (Ford Motor, 2007), I abstract from short-week downtime in the subsequent analysis.

[5]The UAW agreement stipulates that a laid-off worker receives 95 percent of his or her after-tax earnings. Ford Motor (2007) estimates this to be about 72 percent of pre-tax earnings for the typical assembler. This is, in turn, split between state-administered unemployment insurance and the SUB. The firm partially reimburses the state through a tax paid to support the UI fund. Some authors treat this tax rate as a component of the marginal cost of a layoff. I will argue otherwise (see Section 4), so here, I do not fold this tax burden in with the SUB.

[6]The modification to the program in 2007 relaxed the restrictions on inter-plant transfers of JOBS Bank workers. Prior to that year, a JOBS Bank member was permitted to refuse any number of jobs outside of a 50 mile radius of his home plant and retain full salary. Because of this restriction, and data limitations, I abstract from the issue of inter-plant transfers in this paper. A list of the "placement zones" of the Ford Motor Corporation, which was reported in Appendix N of its 1990 agreement with the UAW, is available upon request. In the 2007 agreement, this restriction was relaxed. A worker was instead given three opportunities to accept a new position anywhere in the United States before he was removed from the JOBS Bank.

extended once more to 48 weeks in the 2003 (four-year) agreement. For each week on layoff below the maximum, the plant pays the SUB. Once that maximum was breached, the worker received full (straight-time) salary (and non-cash benefits) for every subsequent week on layoff.

As a result, if the worker transitions from SUB to the JOBS program, labor costs jump up sharply. Figure 1 illustrates this relation between SUB and JOBS program participation for a very simple case where the worker is on indefinite, or long-term, layoff. The numbers are drawn from the estimates discussed in Section 5. In short, even after the SUB is expanded to reflect non-cash benefits, it remains roughly 50 percent below compensation under the JOBS program.

Contractually, workers who received full salary under JOBS were entitled to the benefit only for the duration of the extant contract. But in practice, no workers who received full salary under the JOBS program had their unemployment compensation reduced when a contract concluded. Therefore, workers have in fact remained under JOBS until recalled to production work (or until they received a buy-out offer from the firm).

It is quite a different matter for workers who accumulated many weeks on layoff over the life of an agreement but who never reached the contractual maximum. For instance, if a worker accumulated 10 weeks of layoff between, say, the 1990 and the 1993 negotiations, those weeks would not count toward his allotted 36 in the *next* contract. Rather, he would have to accumulate a full 36 weeks on layoff over the 1993-1996 agreement in order to become JOBS eligible. This is why, among human resources personnel, it is said that a worker on layoff who did not receive full salary under the JOBS program had his "clock" reset when a new agreement is signed. This is illustrated in Figure 2. For the purposes of the new contract, this worker's accumulated weeks on layoff are re-set to zero.

This feature of the agreement suggests that a firm might have an incentive to defer layoffs so as not to exhaust its allotment before the contract expires. Consider an assembly plant that is subject to uncertainty about the future evolution of demand. It has a specified number of weeks on layoff that it can "spend" over the life of any contract before the JOBS program becomes active. Intuitively, if it knows that, in some (bad) states of the world, it will have to shut down in the future, it does not want to have to pay full salary to laid off workers while it does so. Thus, it may keep the factory going now (even at a loss) in order to preserve its option to shut down in the future in a relatively more profitable way. This is the basic idea that will be investigated in the remainder of the paper.

2.3 Model

This section first introduces a simple model that helps make precise how JOBS influences labor demand. The main actor in the model is the individual automobile assembly plant. To communicate the intuition as clearly as possible, I abstract from several features of the plant's problem that are taken up subsequently in Section 3.2.

2.3.1 A Simple Example

Analytics

The key assumptions of the model are as follows. The plant faces a iso-elastic demand curve of the form $x = zP^{-\frac{1}{\varphi}}$, where x is sales, z is a demand shifter, and P is the product price.[7] For simplicity, I assume z is i.i.d. with probability distribution function G. Next, output is a binary variable: $y = 0$ or $y = Y = AN$, where N is the plant's labor force and A is output per worker. It follows that revenue is 0 if $y = 0$ and $zY^{\frac{\varphi-1}{\varphi}}$ if $y = Y$. Let W be the wage. If this were the only cost of production, no plant would actually breach the allotment since any layoff would sacrifice revenue but not reduce the cost of operation. This is a counterfactual implication – there were workers in the JOBS Bank. To remedy this, I introduce intermediate inputs. Specifically, I maintain the assumption throughout the paper that the plant must also spend C per worker to equip its employees with the materials needed for production. In this case, even if JOBS nearly binds, the plant may still shut down in order to avoid these other variable costs. The total cost of operation per worker is then $\bar{W} = W + C$. The profit in a period in which the plant does operate is then given by

$$\pi^Y(z) = zY^{\frac{\varphi-1}{\varphi}} - \bar{W}N,$$

where the superscript on π refers to the level of production.

If the plant does not produce, it pays a layoff benefit outlined in a labor agreement. Suppose the agreement lasts for two periods, and the plant is allowed one period of layoff. The second layoff within an agreement triggers the JOBS program, and workers are paid W for that second period, whereas they are paid a benefit, $B < W$, for the first period on layoff. Let $\tau = 1, 2$ denote the period of the contract. To keep track of layoffs over the agreement, let L_{-1} denote, as of the start

[7]It is not necessary to assume that the plant is a price setter. Perhaps a simpler way to proceed is to assume that the plant operates in a perfectly competitive market and faces shocks to the market price. Since I will retain this monopolistic demand structure throughout, I introduce it here for contintuity.

of the current period, the number of periods of layoff since the beginning of the contract. If $\tau = 1$, then $L_{-1} = 0$ by construction: the JOBS "clock" is re-set when a new contract is initiated.[8] If $\tau = 2$, then $L_{-1} = 0$ if no layoff was conducted in period one and 1 otherwise. Profit in the event the plant does not produce is then given by

$$\pi^0 (L_{-1}) = 0 - WN\mathbf{1}[L_{-1} = 1] - BN\mathbf{1}[L_{-1} = 0]$$

where $\mathbf{1}[\cdot]$ is an indicator function that equals one if the term inside the bracket is true.

The plant's decision problem is to devise a layoff policy rule. The model is solved backward, and the solution is depicted in Figure 3. Suppose a plant laid off its workers in period one, in which case it pays the full wage, W, if it shuts down in the terminal period of the contract. The plant compares π^Y and π^0 in period two and produces if (and only if)

$$z \geq \left(\bar{W} - W\right) N^{1/\varphi} \equiv Z(L_{-1}, \tau) = Z(1, 2).$$

If $L_{-1} = 0$, on the other hand, the plant produces if

$$z \geq \left(\bar{W} - B\right) N^{1/\varphi} \equiv Z(0, 2).$$

That $Z(0,2) > Z(1,2)$ follows from $W > B$. This states that, if the JOBS Bank binds, a layoff saves the plant relatively little in labor costs. Consequently, it continues to produce even as z falls below $Z(0,2)$.

What is of more interest is the first-period problem. The figure indicates that $Z(0,1) < Z(0,2)$, which means the plant produces over a lower range of zs in period one than in period two, given $L_{-1} = 0$ in both periods.[9] The gap between the two thresholds depends on the sum of two terms:

$$(W - B) N \int^{Z(1,2)} g(z')\, dz' + \int_{Z(1,2)}^{Z(0,2)} \left[\pi^0(0) - \pi^Y(z')\right] g(z')\, dz', \tag{2.1}$$

where g is the density function, $g = G'(z)$, and z' refers to the value of z next period. To understand this, suppose a plant uses up its allotment in period one. If z falls below $Z(1,2)$ in the next period,

[8]Consistent with the JOBS agreement, accumulated layoffs are reset to zero at the end of the contract if the plant conducts only one period of layoff throughout the agreement. For simplicity, I also reset accumulated layoffs to zero if the workers are in the JOBS Bank at the end of the contract. This is consistent with my treatment of the problem in the next subsection. It does deviate from the literal text of the JOBS agreement, where workers remain in the JOBS Bank *across* contracts. I discuss this further in section 3.2.3.

[9]Recall that, as of the first period, the number of layoffs since the beginning of the agreement is zero by construction, $L_{-1} = 0$.

it will be optimal to lay off despite the fact that JOBS will bind. But the plant will lose $W - B$ relative to what it would have lost if it had preserved its allotted layoff until period two. If instead z' falls in the range $[Z(1,2), \; Z(0,2)]$, the plant, which has exhausted its supply of layoffs, will produce and earn $\pi(z')$. But if it had preserved the option to shut down in the final period and pay only B, it would earn $\pi^0(0) - \pi^Y(z')$ more. These two considerations generate an option value of production in period one. In this example, this option value represents the *contract time effect*, as it drives a wedge between the optimal policies across two different periods of the contract that share the same L_{-1}.

Equation (2.1) also provides the simplest context in which to explore the effect of uncertainty on the option value. Uncertainty enters only through the density function, g, since the triggers, $Z(0,2)$ and $Z(1,2)$ are pinned down by the terminal period problem. An increase in uncertainty works through g to potentially raise the option value. Figure 4 illustrates the case where a mean-preserving spread raises the amount of mass above each point to the left of the trigger, $Z(0,2)$, and so increases the value of each term in the expression in (2.1). In this case, an increase in uncertainty raises the expected future cost of exhausting the allotment in period one by raising the likelihood of conducting a layoff in period two.[10] The two-period model is obviously a special case, but the essential point will survive in the generalizations of this model presented below.

One drawback of the two-period model is that, if the plant implements a layoff in period one, the JOBS program binds in a "hard" fashion in period two: the plant *will* pay W if it does another round of layoffs. But the JOBS program can influence the plant's labor demand decision even if it does not bind now or next period; as long as it binds in expectation in the future, it can affect the contemporaneous choice. To illustrate this point, it is necessary to extend the model.

Consider now a three-period problem where the plant is allowed two layoffs instead of one. Figure 5 illustrates the solution to the problem. There are three observations to make. First, as long as the plant arrives in period two with zero accumulated layoffs (i.e., $L_{-1} = 0$), the JOBS program is guaranteed *not* to bind within this particular agreement. This is why the layoff decision rule is identical across the three scenarios, $(L_{-1}, \tau) = (0,2)$, $(L_{-1}, \tau) = (0,3)$, and $(L_{-1}, \tau) = (1,3)$. In each case, the plant is not threatened by the prospect of JOBS, and so the thresholds for production

[10]Strictly speaking, the effect of a mean-preserving spread depends on the location of the trigger $Z(0,2)$ along the support of the z. Figure 4 actually illustrates this: a "fanning out" of the distribution must reduce mass within a certain range, so the trigger has to lie away from this subset. To be more precise, it is helpful to specialize g to be the lognormal probability density function. Then it is straightforward to show that a mean-preserving spread – raising the variance of z from, say, σ_l^2 to σ_h^2 – will increase the option value if

$$\bar{Z}(0,2) < \mu - \sqrt{\frac{\log(\sigma_h/\sigma_l)}{\sigma_l^{-2} - \sigma_h^{-2}}}.$$

are the same, i.e., $Z(0,2) = Z(0,3) = Z(1,3)$. In contrast, $Z(2,3)$ lies below these thresholds because, if $L_{-1} = 2$, another layoff will trigger the JOBS Bank.

Second, if the plant implements a layoff in period one, then it is more cautious about how it uses its last allotted layoff in period two. This caution is represented by the fact that $Z(0,2) > Z(1,2)$, which says that the plant will operate over a lower range of zs if it has one accumulated layoff rather than zero. The gap between these thresholds depends on

$$[W - B] N \int^{Z(2,3)} g(z')\, dz' + \int_{Z(2,3)}^{Z(1,3)} \left[\pi^0(1) - \pi^Y(z')\right] g(z')\, dz'$$

which resembles expression (2.1). The term is always positive since the plant prefers to shut down in the final period if it has one accumulated layoff and z falls below the threshold, $Z(1,3)$. Intuitively, this is the added expected profit the plant will earn in the final period if it preserves its layoff and is able to shut down relatively more profitably in the future. This option value represents the *effect of accumulated layoffs* on the layoff decision, as it drives a wedge between the optimal policies associated with two different values of L_{-1} (within the same contract period).

Third, there is a contract time effect that operates in the first period of the agreement. Note that, irrespective of the plant's action, JOBS will *not* bind in period one or two. Yet the impact of the JOBS agreement on the first-period decision can be seen in the gap between the thresholds, $Z(0,1)$ and $Z(0,2)$. That $Z(0,1) < Z(0,2)$ indicates the plant will produce over a lower range of zs in period one than in period two.

The wedge between these two thresholds depends on the sum of two components that make up the option value of production in period one. The first is given by

$$\int_{Z(1,2)}^{Z(0,2)} \left[\pi^0(0) - \pi^Y(z')\right] g(z').$$

To see the intuition behind this, suppose the plant foregoes a layoff in the initial period. It is able to deploy that layoff in the second period when z is moderately low, i.e., in the range $[Z(1,2),\ Z(0,2)]$. If the plant, on the other hand, uses a layoff in period one, it is constrained to operate in the next period if z falls in this range because it does not want to exhaust its allotment before the final period. Since $\pi^0(0) > \pi^Y(z')$ for $z \in [Z(1,2),\ Z(0,2)]$, this gives the plant a motive to conserve a layoff in period one.

To motivate the second component, suppose z falls below $Z(1,2)$ in period two, in which case the plant will have to execute another layoff. This is the source of two additional concerns to the plant. First, if z also falls below the threshold, $Z(2,3)$, in the third period, the JOBS Bank will bind, and the plant will pay W rather than B. This concern is represented in the top line of the expression given below. Second, if z does not fall that far in the final period but instead lies in the range $[Z(2,3),\ Z(1,3)]$, the plant that has used up its allotment has to operate. But if it had conserved a layoff, it would be able to shut down relatively more profitably and pay just B. This concern is captured in the bottom line of the expression. Together, these two represent the second component of the option value,

$$(W-B)\,N\int^{Z(1,2)}\int^{Z(2,3)} g(z'')\,dz''g(z')\,dz'$$
$$+\int^{Z(1,2)}\int_{Z(2,3)}^{Z(1,3)}\left[\pi^{0}(1)-\pi^{Y}(z')\right]g(z'')\,dz''g(z')\,dz'.$$

Quantitative Application

While this model is very simple, it still may be instructive to get a first look at the quantitative effect of the JOBS program. Even such a simple model helps reveal some of the mechanisms that contribute to a robust JOBS Bank effect. Many of the insights obtained from this analysis will carry over to the more realistic model presented in the next section.

The model is set at a weekly frequency since plants generally decide schedules on a weekly basis. The calibration largely relies on the estimates to be presented in Section 5. In particular, the allotment of layoffs and the length of the contract are now parameterized to be consistent with the agreements signed by the UAW and Detroit Three in the 1990s. I set the allotment, denoted by $\bar{L}$, to 36 weeks and the contract duration, denoted by T, to 156 weeks, or three years.[11] It would be tedious to derive the analytical solution here, so the model is solved numerically by backward induction. To obtain the numerical solution, however, we do have to specify the stochastic process for z. Above, z was assumed to be i.i.d. to ease the algebra. Here, I assume it follows a Gaussian geometric autoregressive process,

$$\log z=\mu+\rho\log z_{-1}+\varepsilon,\ \varepsilon\sim N\left(0,\sigma^{2}\right). \tag{2.2}$$

This law of motion will be used throughout this paper. I set the persistence parameter, ρ, to be 0.6 on a monthly basis.[12] The standard deviation of $\log z$ is defined as $s=\sigma/\sqrt{1-\rho^{2}}$ and set equal to

[11] The labor contracts signed in the 1990s were three-year agreements. Beginning in 1999, though, the contract lengthened to four years, and the allotment, $\bar{L}$, rose proportionally (to 48 weeks per agreement). Thus, the number of weeks allotted per month of the contract remained at one.

[12] Although a weekly model is preferable (see Ramey and Vine (2004) and Copeland and Hall (2009)), data

0.268. (One might just as well work with σ directly; I work with s since this is the parameter that is estimated in Section 5.) This calibration is consistent with that presented later in the paper and so a full discussion is deferred.[13] Given this, this section concentrates on the comparative statics of the model with regard to ρ and s.

The effect of L_{-1} on the (unconditional) weekly probability of a layoff is shown in Figure 6, given alternative values of τ. The probability of a layoff is calculated as

$$\Phi\left(\frac{\log Z\left(L_{-1},\tau\right)-\left(\mu/\left(1-\rho\right)\right)}{s}\right) \tag{2.3}$$

where $\mu/\left(1-\rho\right)=\mathrm{E}\left[\log z\right]$ is the mean of $\log z$. The figure illustrates that the probability of layoff this period declines as past weeks of layoff, L_{-1}, accumulate. This is consistent with the policy function derived in the context of the three-period model. The numerical analysis goes one step further and suggests that the *rate* of decline in the layoff probability increases in L_{-1}. While we did not predict any nonlinearity in our analysis above, it does appear sensible: as the limit is neared, the plant exercises greater restraint in its use of layoffs in order to safeguard the few weeks that remain in its allotment.

Under this calibration, the quantitative effect of L_{-1} is unnoticeable until the plant reaches roughly half of its allotment. At that point, the probability of layoff begins to fall as L_{-1} rises further. For a plant midway through the agreement, the probability declines from approximately 4 percent to 3.75 and then 3.1 percent as L_{-1} reaches 25 and 30 weeks. The implication of this is that the identification in the data of a strong JOBS Bank effect likely requires that a sufficient number of plants exhaust nearly 3/4 or so of their allotment. This point will re-emerge in the analysis of Section 5.

The effect of τ on the weekly probability of layoff is shown in the top panel of Figure 7. It confirms that the passage of time within the contract relieves downward pressure on layoffs. In addition, it indicates that the rate of increase declines as the agreement comes to a close. Intuitively, the marginal effect of the passage of time diminishes as it becomes clear that the plant will not breach the JOBS-imposed allotment. Another qualitative feature of the optimal policy that emerges from the figure is the interaction between τ and L_{-1}. The increase in the probability of layoff over the

limitations mean that the model introduced in the next section must be set at a monthly frequency. For the sake of consistency, though, I will report parameter values throughout *on a monthly basis* wherever possible. For instance, in the model of this section, I use $\rho = 0.88$. At a monthly frequency, this translates to $0.88^{1/4} = 0.6$.

[13] The calibration borrows from the discussion of Section 5. The values of the other parameters are set as follows: $A = 1.545$; $N = 2540$; $\bar{W} = 1$ (normalization); $C = 0.62$; $W = 0.38$; $B = 0.152$; and $\varphi = 3.5$.

contract is greater for a plant whose L_{-1} is initially relatively high. This reflects the fact that a high L_{-1} suppresses the probability of layoff, and so the passage of time relieves pressure on this plant to a greater degree than at facilities where L_{-1} is low.

Figure 7 demonstrates, however, that the quantitative effect of contract time is quite small relative to that exerted by L_{-1}. Indeed, the increase in the probability of layoff over the course of the entire agreement is less than one-tenth of one percentage point. In the discussion of the two-period model, I argued that the strength of the contract time effect is likely tied to the degree of uncertainty around the evolution of future vehicle demand. It is straightforward to illustrate this here. The bottom panel of Figure 7 shows how the probability of layoff varies over the contract when s is increased by roughly one fifth, to 0.326. This raises the probability of layoff for any τ, of course, which is why the scale of vertical axis differs from that in the top panel. But it also raises the importance of contract time: the weekly probability of layoff increases between 2.4 and 6.4 percent over the course of the first half of the agreement (from a level of 0.083 to 0.085 if $L_{-1} = 4$ and from 0.078 to 0.083 if $L_{-1} = 14$). This impact still appears modest, but represents literally an order of magnitude increase over the effect illustrated in the top panel, where s was smaller.

The quantitative model also allows one to investigate the impact of variations in ρ. The two-period model did not speak to this, but in the presence of the JOBS Bank, persistence is likely to matter because it determines the likelihood that a plant, which receives an adverse shock today, will face subdued household demand for the remainder of the contract. If so, the plant recognizes that any future negative shocks will occur when demand for its product is likely to still be relatively depressed. As a result, the plant defers layoffs now until it recovers, lest it exhaust its allotment and have to face future deteriorations in demand without any "cushion" in terms of allotted layoffs.

To summarize the effects of both s and ρ, I simulate the model and calculate the JOBS Bank effect for different choices of these parameters.[14] The results are given in Table 1. The first two columns report the probability of layoff with and without JOBS. These columns summarize the impact of the JOBS Bank on the layoff decision, aggregating across the effects of L_{-1} and τ. The results indicate that, unless ρ is sufficiently near one, JOBS may induce very little difference in the aggregate layoff rate. Given $\rho = 0.6$, for instance, even the 20 percent increase in s mentioned above does not, despite its strengthening of the contract time effect, generate a statistically significant reduction in the relative layoff rate under JOBS. As ρ rises, however, the relative probability of layoff declines appreciably. For $\rho = 0.9$, the weekly layoff probability under JOBS is almost 18

[14]Specifically, I run 100 simulations, each of which calculates the paths of layoffs for 50 plants over five 36-month contracts. The size of the panel and the structure of the contract mirrors the calibration used in the simulation experiments conducted later.

percent smaller (0.0334 versus 0.0406), given $s = 0.268$. If the variance of the innovation also rises, the gap between the layoff probabilities widens to 26 percent. This suggests that the interaction between s and ρ is powerful, at sufficiently high levels of ρ.

The next two columns provide a measure of the *individual* contributions of L_{-1} and τ. The first column reports the effect on the probability of layoff of an increasing L_{-1} in increments of one-standard deviation. (These calculations use equation (2.3).) The nonlinearity of the thresholds comes through starkly. For instance, given $\rho = 0.8$ and $s = 0.326$, the effect of raising L_{-1} from its mean by one-standard deviation is to reduce the probability of layoff by 1.1 percentage points. But raising it by another standard deviation generates a 3.1 percentage point reduction.

The second column reports the effect of raising τ in increments of 9 months, which represents the passage of one quarter of the contract. Again, the nonlinearity of the response is striking. In particular, the table indicates that, for ρ sufficiently high, the threshold function is convex, rather than concave, in τ. What is behind this? Earlier, it seemed as if the marginal importance of one month declined in contract time. The intuition was that, if the uncertainty regarding whether JOBS will bind is nearly resolved midway through the agreement, then the passage of one more month beyond that point does not affect plant behavior greatly. That conclusion is overturned when ρ is high because the uncertainty persists for much longer into the contract. The reason is that, if an adverse shock dissipates very slowly, the plant remains vulnerable, even halfway into the agreement, to a sustained period of slow sales that might exhaust the allotment. Consequently, the probability of layoff remains suppressed well into the second half of the agreement, and then rises rapidly as uncertainty is resolved.

2.3.2 A More General Model

I now describe a more general problem for the individual assembly plant. Unlike the model studied above, the model of this section is set at a monthly frequency, and the plant selects the number of weeks per month to operate. This decision was made because data on which estimation of the model relies is not available at a higher frequency. Also, unlike in Section 3.1, the model here allows the plant to store unsold units of the good it produces, i.e., the plant holds inventory. This is a necessary feature to add in any model of the motor vehicle industry since inventory fluctuations contribute to movements in layoffs. I first state the plant's problem and then describe the optimal policy. This analysis guides the development of the regression model in Section 4. A fuller discussion of the rationale behind some of the simplifying assumptions of the model is deferred until the final subsection.

The plant's problem

The plant seeks to maximize the expected present discounted value of profits, which is given by

$$\sum_{s=0}^{\infty} \beta^s \left[p_{t+s} x_{t+s} - \frac{\lambda}{2} \left(I_{t+s-1} - \alpha x_{t+s} \right)^2 - \omega_{t+s} \right],$$

where p_t is the price of a vehicle; x_t is sales; I_{t-1} is inventory brought into period t; and ω_t is the cost of production. The maximization is carried out subject to, among other constraints to be introduced, the law of motion for inventory,

$$I = I_{-1} + y(\ell) - x \tag{2.4}$$

where ℓ is the number of weeks of layoff per month and production, $y(\ell)$, takes a very simple form,

$$y(\ell) = AN(4 - \ell) \tag{2.5}$$

with N the number of workers available to the plant and A output per worker per week. The production function indicates that a layoff involves all workers at the plant: either all work or none at all. For vehicle assembly plants, this is a reasonable, though, stylized treatment of weekly layoffs. I do abstract, however, from other margins of output adjustment.

The three components of the objective function merit discussion. I handle each of these in turn.

Vehicle demand. A large literature studies the household's discrete choice problem over many differentiated vehicle products (or models) (see, among others, Berry, Levinsohn, and Pakes (1995) and Esteban and Shum (2007)). This literature often, however, must abstract from many of the details of the labor demand problem. In contrast, other research focuses solely on the factor demand problem, abstract from the product market, and treat sales as given (see Ramey and Vine, 2006). This paper attempts to take a middle ground. My focus here is labor demand, so the product market is kept intentionally simple. On the other hand, I find that a model in which sales are explicitly determined has important implications for the econometric analysis, even if that model must omit a number of features of the plant's actual decision problem. This is a point to which I return below.

I assume a representative household that seeks to maximize,

$$\sum_{s=0}^{\infty} \beta^s \left[u\left(C_{t+s}\right) + \theta\left(\mathcal{X}_{t+s}\right)\right]$$

where C is a nondurable (non-vehicle) aggregate and $\mathcal{X}$ is an aggregator of vehicle sales. I assume $\mathcal{X}$ is given by a constant elasticity of substitution aggregate,

$$\mathcal{X} = \left[\sum_j z_j x_j^{\frac{\varphi-1}{\varphi}}\right]^{\frac{\varphi}{\varphi-1}},$$

where z_j represents a preference for product j and x_j is the quantity purchased of product j. Household optimization implies a demand curve that faces each plant of the form,

$$\frac{p_j}{P} = z_j \left(\frac{x_j}{\mathcal{X}}\right)^{-\frac{1}{\varphi}},$$

where P is the Dixit-Stiglitz aggregate price index. If $\theta\left(\mathcal{X}\right) = \log \mathcal{X}$, it is straightforward to show that $P = \frac{1}{\lambda \mathcal{X}}$, where λ is the marginal utility of wealth. One may now write the assembly plant's revenue as $p_j x_j = \lambda^{-1} z_j \left(\frac{x_j}{\mathcal{X}}\right)^{\frac{\varphi-1}{\varphi}}$. Importantly, if the model is set in steady state, aggregate objects, such as λ and $\mathcal{X}$, are fixed. In that case, it makes sense to collect these terms as $\Omega = \left[\lambda \mathcal{X}^{\frac{\varphi-1}{\varphi}}\right]^{-1}$. Any $\Omega \neq 1$ can be captured by the choice of the mean of z. Thus, it is possible to normalize $\Omega = 1$ and write revenue as

$$px = zx^{\frac{\varphi-1}{\varphi}}, \tag{2.6}$$

where I have now dropped the subscript since each plant's problem, by symmetry, is identical.

Inventory. The penalty function associated with inventory imparts to the plant an incentive hold a sufficient number of units at the start of the period in order to satisfy a measure of sales. The form of the function dates back at least to Holt, Modigliani, and Simon (1955) and has often been used to study inventory behavior in produce-to-stock industries (see the discussion in Ramey and West, 1999). The penalty function was inspired by the initial, seminal work on inventory management in the presence of a fixed cost-to-order and stock-out penalty (see, for instance, Arrow, Harris, and Marschak (1951)). That work demonstrated that producers should build up "safety stocks" as orders, or sales, increase, which is captured by the presence of the target, αx. If $\alpha > 1$, in particular, the producer has an incentive to generally carry an excess of inventory over sales. (In the context of original *S,s* literature on inventory management, an $\alpha > 1$ would reflect a combination of large costs to order and stock-out penalties.) In this paper, it will be particularly important that α may exceed one, as the size of inventory stocks in the automotive industry generally equals two to three months of vehicle sales.[15] The incentive to hold inventory does have its limits, however. The penalty function here also exacts a price on producers that carry over a level of inventory far in excess of sales, as judged by the distance between I_{-1} and αx.

Cost of production. First, I outline the cost of production at an active plant; then the cost to the employer of a week of layoff without JOBS; and, finally, the cost of a week of layoff under JOBS.

The total cost of production at an active plant involves labor- and non-labor costs. Let w denote weekly straight-time pay (per worker) and let h represent non-pecuniary compensation accrued over a week (i.e., health care benefits). Then define $W = w + h$ as the wage bill of a single worker and WN as the wage bill of the plant as a whole. I assume non-labor costs are also proportional to the size of the workforce, as noted above. The idea here is the plant spend must spend C dollars per worker to supply that worker with the materials needed to produce a vehicle. It follows that the weekly production cost of an active plant is $(W + C)\,N \equiv \bar{W}N$.

The cost of a week of layoff to the producer is relatively straightforward in the absence of JOBS. Under the UAW agreement, each plant is still responsible for the non-wage component of compensation, h. In addition, the plant pays approximately a share, b, of the weekly salary under the SUB provision. (This share is roughly 40 percent, as noted above.) The total cost to the employer of a single worker's week of layoff is therefore $B \equiv bw + h$. It follows that the cost of operation over a month, in the absence of JOBS, is given by

$$\hat{\omega}\,(\ell) = BN\ell + \bar{W}N\,(4 - \ell)$$

This states that if the plant implements ℓ weeks of layoff, it pays its workforce BN for those weeks and $\bar{W}N$ for the $4 - \ell$ that it is active.

Under JOBS, the current cost of a week of layoff depends on cumulated layoffs. For each week on layoff in excess of the allotment, $\bar{L}$, the plant pays W to its laid-off workers. Denote the total weeks accumulated as of the start of the current period by $L_{-1} \in \left\{0, 1, ..., \bar{L}\right\}$. Then the cost of

[15] This is not the only way to generate realistic inventory behavior. First, some authors have argued that inventories of finished goods facilitate sales because they provide consumers the opportunity to browse a variety of products. This idea has led these authors to enter inventory directly into the demand function, which in turn generates a strong incentive to hold inventory. See Bils and Kahn (2002) and Copeland and Hall (2009) for applications. Second, Kahn (1987) builds on Carr and Karlin (1962) to provide a model where the firm is allowed only limited backlogging of unfilled orders and must select price and quantity before a demand shock is realized. In this case, inventory accumulation allows the firm to build up a buffer stock in order to meet demand in the event of a large positive shock.

operation is formally given by

$$\omega(\ell, L_{-1}) = \sum_{l=1}^{\ell} \begin{bmatrix} BN\mathbf{1}\left(L_{-1}+l \leq \bar{L}\right) \\ +WN\mathbf{1}\left(L_{-1}+l > \bar{L}\right) \end{bmatrix} + \bar{W}N(4-\ell), \tag{2.7}$$

where $\sum_{l=1}^{0}[\cdot] \equiv 0$. This departs from $\hat{\omega}(\ell)$ in that, under JOBS, the current cost of operation depends on the total number of weeks of layoff accumulated since the start of the contract.

The effect of contract time, denoted by $\tau \in \{1, 2, ..., T\}$, is not evident in (2.7), but it is there implicitly. The reason, as discussed further in the next section, is that the conclusion of one contract and beginning of the next implies that L_{-1} is reset to zero. In other words, L_{-1} implicitly depends on contract time, τ. In fact, the level of L_{-1} at the beginning of a contract is uniquely, though somewhat trivially, pinned down by $\tau = 1$, namely, $L_{-1} = 0$.

The optimal policy

The variables, $L_{-1}$and τ, which are introduced into the plant's problem by the JOBS Bank, affect only the current, and future expected, wage bills of the plant. The core of the problem is the optimal selection of weeks worked and inventory. Key features of this problem may be delineated in a simpler version of the model that omits JOBS. This section begins here and then incorporates the JOBS Bank.

The model without JOBS If the wage bill of the plant is given by $\hat{\omega}(\ell)$, then its problem is fully characterized by the Bellman equation,

$$V(I_{-1}, z) = \max_{I,\ell} \left\{ zx^{\frac{\varphi-1}{\varphi}} - \frac{\lambda}{2}(I_{-1} - \alpha x)^2 - \hat{\omega}(\ell) + \beta \int V(I, z')\, dG(z'|z) \right\}.$$

Since ℓ is discrete, we may rewrite this in two steps. First, for given ℓ, the value of the plant is given by

$$V^{\ell}(I_{-1}, z) = \max_{I} \left\{ z\left(I_{-1} + y(\ell) - I\right)^{\frac{\varphi-1}{\varphi}} - \frac{\lambda}{2}\left[I_{-1} - \alpha\left(I_{-1} + y(\ell) - I\right)\right]^2 + \beta \int V(I, z')\, dG(z'|z) \right\}. \tag{2.8}$$

Once I has been determined, the optimal number of weeks of layoff is selected:

$$V(I_{-1}, z) = \max_{\ell} \left\{ V^{\ell}(I_{-1}, z) - \hat{\omega}(\ell) \right\}.$$

The optimal policy is thus composed of two decision rules: one regulates the choice of inventory given ℓ, and the second determines when one value of ℓ is selected over the others. The choice of inventory satifies the first order condition, taken for a given ℓ:

$$\frac{\varphi-1}{\varphi} z x^{-\frac{1}{\varphi}} + \lambda\alpha\left[I_{-1} - \alpha x\right] = \beta \int V_I\left(I, z'\right) dG\left(z'|z\right), \quad \text{for all } \ell = 0, 1, \ldots, 4, \tag{2.9}$$

where $x = I_{-1} + y(\ell) - I$. This states that the plant adjusts inventory until the marginal value of sales this period is just offset by the expected future marginal value of foregone sales, i.e., inventory. The first order condition yields a sales policy rule, $x\left(I_{-1}, z; \ell\right)$, that is conditioned on ℓ and varies smoothly as a function of the state, $\left(I_{-1}, z\right)$.

The choice of weeks of layoff is determined by comparing adjacent ℓ-specific value functions. For instance, the plant sets $\ell =\underline{\ell}$ rather than $\ell = \overline{\ell}$, where $\underline{\ell}$¡ $\overline{\ell}$, only if $V^{\underline{\ell}}\left(I_{-1}, z\right) - \hat{\omega}\left(\underline{\ell}\right) > V^{\overline{\ell}}\left(I_{-1}, z\right) - \hat{\omega}\left(\overline{\ell}\right)$. Rearranging, this becomes

$$V^{\underline{\ell}}\left(I_{-1}, z\right) - V^{\overline{\ell}}\left(I_{-1}, z\right) > \hat{\omega}\left(\underline{\ell}\right) - \hat{\omega}\left(\overline{\ell}\right).$$

The left-hand side is the value of an increment of output to the plant. In the numerical solution of the model, this is found to be increasing in z: the value of additional output is, expectedly, higher when demand, z, is higher. It follows that, for any given pair $\left(\underline{\ell}, \overline{\ell}\right)$, with $\underline{\ell}< \overline{\ell}$, there exists a unique threshold, denoted by $Z_{\underline{\ell}\overline{\ell}}$, such that the plant is indifferent between $\underline{\ell}$ and $\overline{\ell}$ at $Z_{\underline{\ell}\overline{\ell}}$,

$$V^{\underline{\ell}}\left(I_{-1}, Z_{\underline{\ell}\overline{\ell}}\right) - V^{\overline{\ell}}\left(I_{-1}, Z_{\underline{\ell}\overline{\ell}}\right) = \hat{\omega}\left(\underline{\ell}\right) - \hat{\omega}\left(\overline{\ell}\right), \tag{2.10}$$

and prefers $\underline{\ell}$ for all $z > Z_{\underline{\ell}\overline{\ell}}$ and $\overline{\ell}$ for all z to the left of $Z_{\underline{\ell}\overline{\ell}}$. Moreover, by the implicit function theorem, the "value matching relation" (2.10) defines a continuous function $Z_{\underline{\ell}\overline{\ell}}\left(I_{-1}\right)$.[16] Generalizing this to any adjacent pair $\left(\underline{\ell}, \overline{\ell}\right)$, where $\underline{\ell} < \overline{\ell}$ and $\underline{\ell}, \overline{\ell} \in \{0, 1, 2, 3, 4\}$ suggests the following form

[16] If the ℓ-specific value functions are each concave, then the implicit function theorem also gives that the threshold, $Z_{\underline{\ell}\overline{\ell}}\left(I_{-1}\right)$, increases in I_{-1}. The concavity of each within-ℓ value function would appear to, in turn, follow directly from the concavity of the expected value function, $v(I, z) \equiv \int V\left(I, z'\right) dG\left(z'|z\right)$. One difficulty posed by the plant's problem, though, is that v is not necessarily concave. To see this, note that there are two effects of an increase in I on the marginal value of inventory. The first is that the value of an additional unit of inventory, for a given ℓ, declines in I. This result follows essentially from the concavity of current profit. The second effect is somewhat more subtle. If the "trigger", $Z_{\underline{\ell}\overline{\ell}}$, does in fact increase in I, this implies that an increment to inventory this period raises the probability that the plant will produce discretely less next period. All else equal, this reduces the expected supply of goods for sale, $I + y'$, in the future and *raises* the expected marginal value of beginning-of-period inventory. This effect potentially counteracts the first.

Numerically, it is not hard to check which effect dominates: just plot the function to be maximized for a given ℓ, namely, the expression in brackets in (2.8). If $v(I, z)$ is not concave, this will contort this function and generate multiple local maxima. The policy functions induced by the parameterizations considered in this paper were "well behaved".

for the optimal layoff policy, which is confirmed in the numerical solution of the model:

$$\ell = \begin{cases} 0 & if \quad z > Z_{01}(I_{-1}) \\ 1 & if \quad Z_{12}(I_{-1}) < z \le Z_{01}(I_{-1}) \\ \vdots & \vdots \quad \vdots \\ 4 & if \quad z \le Z_{34}(I_{-1}) \end{cases}, \tag{2.11}$$

with $Z_{01}(I_{-1}) > Z_{12}(I_{-1}) > \cdots Z_{34}(I_{-1})$.[17] Each Z is indexed by the pair of layoff weeks to which it applies, i.e., $Z_{34}(I_{-1})$ is the threshold such that the plant implements four weeks of layoff rather than three if and only if $z \le Z_{34}(I_{-1})$.

The model with the JOBS Bank The introduction of the JOBS Bank amends the wage bill, as in (2.7), and introduces two new state variables, namely, total weeks of layoff since the start of the contract, L_{-1}, and the month of the agreement, τ. It is helpful to begin with the laws of motion governing the evolution of L_{-1} and τ. Contract time evolves as

$$\tau = \mathbf{1}[\tau_{-1} = T] + (\tau_{-1} + 1) \times \mathbf{1}[\tau_{-1} < T]. \tag{2.12}$$

This states that if the last period was the terminal month of the contract, then τ is reset to one and otherwise accumulates as $\tau_{-1} + 1$. The accumulation equation for L_{-1} is given by

$$L = 0 \times \mathbf{1}[\tau_{-1} = T] + (L_{-1} + \ell) \times \mathbf{1}[\tau_{-1} < T]. \tag{2.13}$$

This states that, if last period was the terminal month of the contract, then L is reset to zero this period and the accumulation of layoffs begins anew. Otherwise, L is given by the sum of L_{-1} and the number of weeks of layoff implemented this period, ℓ.

The key observation is that, when L_{-1} and τ are reset, the plant faces fundamentally the same problem as it did at the beginning of the last agreement. Put another way, the state of the world is fully summarized by the quadruple, $(I_{-1}, z, L_{-1}, \tau)$; calendar time per se is irrelevant. It follows that the optimal policy is stationary, controlling for contract time.

An important implication of this is that beginning-of-contract value function uniquely characterizes the plant's problem. To see this, define the value of the plant in the first period of a contract

[17] This rank condition on the thresholds follows directly if the expected value function is concave. The preceding footnote, however, discusses why, in principle, the concavity of the expected value function cannot be taken for granted, though it holds for all parameterizations considered in this paper.

by $V_1(I, z', L) = V_1(I, z', 0) \equiv \upsilon(I, z')$ for a pair (I, z'). The value of the plant in the prior period is then given recursively by the Bellman equation,

$$\begin{aligned} V_T(I_{-1}, z, L_{-1}) &= (M\upsilon)(I_{-1}, z, L_{-1}) \\ &= \max_{\ell} \left\{ \max_{I} \left[\begin{array}{c} \pi(I, \ell;\ I_{-1}, z) \\ -\omega(\ell, 0) + \beta \int \upsilon(I, z')\, dG(z'|z) \end{array} \right] \right\}, \end{aligned}$$

where within-period profit is given by

$$\pi(I, \ell;\ I_{-1}, z) = z(I_{-1} + y(\ell) - I)^{\frac{\varphi-1}{\varphi}} - \frac{\lambda}{2}\left[I_{-1} - \alpha(I_{-1} + y(\ell) - I)\right]^2.$$

Here, I have introduced the operator, M, to make explicit the notion that the period T value function is obtained by application of the operator associated with right side of this equation to the function, υ. The formulation of the period $T-1$ value function is analogous. In the penultimate period of the agreement, the value of the plant is given by

$$V_{T-1}(I_{-2}, z_{-1}, L_{-2}) = (MV_T)(I_{-2}, z_{-1}, L_{-2}) = \left(M^2\upsilon\right)(I_{-2}, z_{-1}, L_{-2}),$$

where the term on the right-hand side is given by

$$\max_{\ell_{-1}} \left\{ \max_{I_{-1}} \left[\begin{array}{c} \pi(I_{-1}, \ell_{-1};\ I_{-2}, z_{-1}) - \omega(\ell_{-1}, L_{-2}) \\ +\beta \int \max_{\ell} \left\{ \max_{I} \left[\begin{array}{c} \pi(I, \ell;\ I_{-1}, z) - \omega(\ell, 0) \\ +\beta \int \upsilon(I, z')\, dG(z'|z) \end{array} \right] \right\} dG(z|z_{-1}) \end{array} \right] \right\}.$$

Inductively, it follows that the value function associated with each period of the contract is determined by υ. Since the optimal policy for a given $(I_{-1}, z, L_{-1}, \tau)$ is invariant with respect to calendar time, this means that the determination of υ uniquely characterizes the plant's problem. To uncover υ, note that, after repeated substitution, one finds

$$\upsilon = M^T \upsilon.$$

This states that the beginning-of-contract value function, υ, is the maximal fixed point of the mapping, M^T. The following proposition confirms that there is in fact just one fixed point of the map, thus giving the existence and uniqueness of υ.

Proposition *The operator, M^T, has a unique fixed point in the space of bounded and continuous functions.*

The proposition serves two purposes. First, it reveals that the solution of a relatively novel dynamic program actually boils down to the solution of a straightforward fixed-point problem. Second, the proposition provides guidance on the numerical solution for the policy rules. The proof indicates that, since the contraction is performed on υ, a natural way to proceed is to solve back through the labor agreement, given an initial guess for υ. That is, I first conjecture the value of the plant in the initial period of a contract, when $L_{-1} = 0$ and $\tau = 1$. Then I solve backward through the prior contract until one arrives at the implied value of the plant in the first period of that contract, $\hat{\upsilon}$. If υ and $\hat{\upsilon}$ are within a specified tolerance, then I stop. Otherwise, I iterate until I locate the fixed point.[18]

Numerical solution of the model indicates that the form of the optimal sales and layoff policies identified above generalize to the model with JOBS. Each variable is now a function of the JOBS-related state variables, (L_{-1}, τ), in addition to I_{-1} and z. In regards to the thresholds in particular, recall that in the simple model of Section 3.1 found that each trigger function was decreasing in L_{-1} and increasing in τ. These predictions will apply with full force to the model augmented with inventory, as will become clear in the course of the simulation analysis below.

This completes the description of the model, which will serve as the framework for the quantitative work to follow.

Discussion of the model's assumptions

To keep the model tractable and focused, the prior section made a number of assumptions regarding market structure, the production function, and the demand curve, among others. This section attempts to briefly discuss the rationale for those assumptions.

Competitive structure. Each plant makes a single, differentiated product. The model imagines, in other words, a universe of monopolistically competitive assembly plants. Therefore, I abstract from strategic interaction among producers. Given the presence of just a few major automotive firms, this is not the most natural assumption. I make it principally for tractability. In this, the paper follows several others which focused on factor demand and/or inventory dynamics and sought to simplify along other dimensions.[19] Whether market structure is critical to the response

[18] I am grateful to Rudi Bachmann for a discussion that helped clarify the mechanics of this algorithm.

[19] See Ramey and Vine (2006) and Copeland and Hall (2009). In Blanchard's widely cited 1983 paper, he assumed perfectly competitive manufacturers.

of optimal labor demand to the JOBS program or other frictions is a topic that must be left for future work.

Aggregate steady state. The model unfolds within an aggregate steady state. This is an apparently strong restriction. However, there are two points to bear in mind. First, this does not in any way restrict the data analysis in section 4, which controls for aggregate variation. Second, it is important to stress that the *necessary* ingredient in the model is idiosyncratic risk in the sense that it is this variation that "spreads" plants over different pairs of (L_{-1}, τ) and ultimately provides identification of the contract effects. While this does not imply idiosyncratic risk is sufficient (i.e., the dynamics of the model are not literally invariant to the steady state assumption), it does help justify why the baseline investigation focuses only on idiosyncratic variation.[20]

Retail-manufacturer relationship. I assume the manufacturer stores unsold output for future sale, i.e., the producer manages an inventory stock. In this, I follow a long literature that abstracts from the relationship between manufacturers and automobile dealer and instead assumes that the producer manages its own inventory stock.[21] This assumes that the identity of the firm that manages the inventory is not critical to the relation between inventory and production. The intuition for this is as follows. Because inventory-related costs are quadratic in inventory, the build-up of unsold units raises the marginal cost of inventory accumulation and motivates the producer to reduce production and price in order to clear out the backlog of unsold vehicles. In a model with a retailer and the manufacturer, the build-up of inventory at the former would lead to fewer orders and place pressure on the manufacturer to reduce output and price. That is, the relationship between inventory accumulation and vehicle prices and production would be qualitatively similar in these two models.[22]

Production technology. I concentrate on the decision regarding the number of weeks of layoff per month: this is the sole margin of output adjustment in the model. In reality, of course, a plant has a number of other margins along which it may adjust (see Bresnahan and Ramey, 1994). Here, I argue why I keep the model focused solely on weekly layoffs.

A typical plant operates for five days per week and produces vehicles in two shifts, each of

[20]Cooper, Haltiwanger, and Willis (2005) make a similar argument in their analysis of plant-level employment and hours data. They remove aggregate time effects from the panel. Accordingly, they argue, the model that is estimated on the regression-adjusted data does not include aggregate risk; identification is achieved from idiosyncratic variation.

[21]See Blanchard (1983), Blanchard and Melino (1986), Ramey and Vine (2006), and Copeland and Hall (2008).

[22]This does assume that the retailer faces an upward-sloped marginal cost schedule for inventory. How might one motivate a quadratic storage penalty at the retail level? One clear source of storage costs is floorplan interest, or the gross carrying cost of a new vehicle. In the absence of a binding capacity constraint on storage, however, this appears to be essentially linear in inventory. On the other hand, if the size of the retail lot is fixed in the short run, the storage cost would be linear only up to the lot size and vertical thereafter. This correspondence is convex but not particularly convenient. A quadratic cost of holding inventory can be thought of as a means of "smoothing out" this kink.

which work eight hours per day and make s vehicles per hour. To adjust production, a plant may in principle work along any one of these margins. For instance, to reduce monthly production, it may choose not to operate for a certain number of weeks per month, in which case it lays off all of its workers for that time. To raise output, it can operate overtime, which may involve up to two extra hours per weekday and an eight-hour workday on Saturday. In addition, it may adjust the number of shifts and/or the hourly speed of assembly, s. If it reduces the number of shifts from two to one, for instance, half of the assembly plant workers go on indefinite layoff and production will be cut roughly in half. If the hourly line speed is reduced, that generally involves reductions in the size of each shift's workforce, as fewer workers are needed to produced fewer vehicles per hour.

I abstract from line speed and shift adjustments, which implies that the plant has a fixed pool of N potential workers. Ramey and Vine (2004) note that line speed adjustments are rare, which suggests that their omission is not too consequential. In addition, over my sample, shift adjustments are also relatively rare. I calculate that there is a 4.67 percent chance that a plant reduces a shift within a given year, and 2.75 percent chance that it adds a shift. While it is true that many entrants into the JOBS program were workers whose shifts were removed, it remains the case that the decision which faces most plants most of the time is whether to conduct a temporary (weekly) layoff. Therefore, I also fix the number of shifts.

One might argue, however, that it is the absence of shift adjustments that points to the influence of the JOBS Bank: the cost to remove a shift is more substantial under JOBS, which means that the cost to reverse the addition of a new shift is also higher. Does the omission of shift changes lead one to understate the JOBS-related effect on employment? In response, it is important to note that, if JOBS placed such strong upward pressure on shifts, it would have likely also placed strong upward pressure on weeks worked, for a given number of shifts. After all, a strong JOBS effect on the choice of shifts implies that vehicle demand conditions were so adverse that plants would have liked to reduce their workforce but the cost of indefinite layoffs under JOBS was too high. Those same vehicle demand conditions would have given the plant a motive to arrange weekly layoffs with an eye toward their implications for the JOBS Bank allotment, $\bar{L}$. Thus, while this paper's approach may underestimate the effect of the JOBS Bank on Detroit Three employment, it should reveal whether there was, in fact, any effect at all.[23]

The production function in (4.2) also excludes other factors of production, such as capital and materials. This is not out of line with earlier research that analyzed labor dynamics in the

[23] The other margin on which plants adjust, as mentioned above, is overtime. I abstract from overtime because my focus is on the extensive margin, and, in particular, on output reductions. It seems unlikely that the model's conclusions will be overturned by the inclusion of an overtime margin.

automotive industry. Given my focus, these simplifications are not crucial. For instance, layoffs in this model are short-lived; I do not investigate the decision to replace workers permanently with new machines. In light of that, the interaction between the JOBS program and the investment decision does not bear significantly on the issue at hand.[24] Therefore, I simply assume the plant must purchase a single non-labor input whose cost is proportional to the size of the workforce. (This is the portion, CN, of the cost of operation in Section 3.1.) This is intended to capture the notion that each worker requires a certain number of inputs, such as materials and capital services, to do his or task. I return to this point when I discuss the cost of production in more detail in Section 5.

JOBS. The JOBS program in the model is simplified slightly from its real-world analogue. In reality, workers who are in the JOBS Bank at the conclusion of a labor agreement remain in JOBS until recalled to work. These are often workers on indefinite layoff, many of whom had their shift removed. But in the baseline model, I abstract from the shift decision. This means that the model does not generate the sort of long-term layoff spells that are consistent with shift reductions. In that case, little is lost if one assumes that, regardless of the number of accumulated layoffs at the conclusion of an agreement, the plant is granted a fresh allotment at the beginning of the next, and the JOBS "clock" is reset.[25]

In addition, I assume the cost structure under the JOBS program is treated as given by the plant when it solves its problem. In other words, I do not model the determination of the union contract. This means each plant assumes it cannot affect the terms of future agreements via any action of its own in the current period. It also assumes that the JOBS program is fixed over the life of the current agreement. These assumptions simplify matters slightly, but I judge them to be quite reasonable over my sample period. The JOBS program was eventually suspended, but not until two of the Detroit Three were on the verge of bankruptcy. Since the program appeared to hold under less catastrophic circumstances, I simply assume it fixed in the model.

Lastly, the JOBS Bank literally allots $\bar{L}$ weeks *per worker*, whereas in the model, the allotment is granted to a plant. For my purposes, this distinction is not critical, though. As I mentioned, I focus on the implications of the JOBS Bank for the choice of weeks of layoff per month. These actions are,

[24]The reader may be concerned that, while the model distinguishes among reasons for a layoff, the data do not. But fortunately, within the auto industry, there are agreed-upon standards regarding the reason for a layoff, which allows me to map ℓ in the model to layoff events in the data. I will discuss this further in section 4.

[25]In other words, if a worker is in the JOBS Bank at the end of an agreement, it is unlikely that he will remain *continually on* layoff for a significant amount of time under the new contract. Since the contract language grants a fresh allotment of layoffs as soon as the worker is recalled, resetting the "clock" to zero at the start of the new agreement is a reasonable simplification.

in general, applied to an entire plant's workforce.[26] Adjustments such as shift reductions and line speed slowdowns, on the other hand, result in indefinite layoffs that affect a subset of the workers at a plant. To study the implications of JOBS for these margins of adjustment, I would need to track individual workers in order cumulate their weeks on layoff. The data I have do not allow me to do this. This is another reason I restrict this study to the decision regarding weeks of layoff per month.

2.3.3 Roadmap

In the remainder of the paper, I put the structural model to work in two ways. First, I use it to help develop a linear reduced-form specification of the layoff policy rule that is, conceptually, very similar to a linear probability regression model. Section 4 formulates the regression specification, introduces a rich panel dataset on the domestic motor vehicle industry, and tests for effects of the JOBS Bank-related variables, L_{-1} and τ. In Section 5, I assess the reduced-form estimates in the context of the structural model. Specifically, this section estimates the structural model and evaluates its ability to induce the reduced-form results. I then use the estimated model to conduct a set of counterfactual simulations that help shed light on the features of the plant's environment that contribute to the strength of the JOBS Bank effects.

2.4 Reduced-form Evidence

2.4.1 Motivation

The rule governing weeks of layoff per month is a function of an unobservable, plausibly serially correlated variable, z, and two endogenous regressors, I_{-1} and L_{-1} (in addition to the exogenous τ). This endogeneity is the principal challenge to consistent estimation of the effect of the JOBS-related variables on the choice of ℓ. In this subsection, I discuss two ways to approach this estimation problem in a single-equation context and argue that each is either ill-advised or computationally infeasible. I conclude that the return to nonlinear single-equation methods is not obviously that high. As a result, I pursue a simpler strategy with regard to the estimation of the reduced form, and complement this with a structural estimation exercise in the next section.

Any single-equation estimation strategy must begin with a Taylor series approximation to the threshold functions, $\{Z_{01}, ..., Z_{34}\}$. Once this is made, it initially appears possible to estimate the

[26] Occasionally, a plant will alternate shifts, that is, work the day shift one week and the night shift the next. But both shifts spend the same number of weeks on layoff over the whole calendar month, which is the unit of analysis here.

layoff policy by maximum likelihood. The difficulty associated with this approach is that z is unobserved and assumed to be potentially serially correlated.[27] As a result, if the econometrician simply omits z_{-1}, the coefficients on the endogenous regressors, I_{-1} and L_{-1}, will be inconsistently estimated. In panels with only a few periods, it would be feasible to treat this problem by integrating the sequence of unobservable realizations of z out of the likelihood (in a procedure analogous to integrating the fixed effect out of a probit model). But this is a much more daunting task when there are over 150 months of data for the average plant. In that case, the likelihood would be a 150-dimensional integral since the autoregressive process of z indirectly links each realization with its entire history.

An alternative way to treat this sort of omitted variables problem is to estimate by simulation. In this case, maximum simulated likelihood (MSL) would appear particularly attractive since it naturally accommodates the discrete nature of the outcome variable.[28] In brief, while a 150-dimensional integral is exceptionally hard to evaluate by quadrature methods, it is not necessarily that hard to simulate.[29] The principal challenge of MSL in my case is computational. To take account of heterogeneity across plants, it will be important to include product (vehicle) effects and product-specific time trends. The presence of these controls will add roughly 280 additional parameters to be estimated. This is far too many parameters for MSL to reasonably accommodate.[30]

This discussion points me in the direction of a two-step strategy. I consider a simple, linear reduced-form equation that is, conceptually, similar to the linear probability model still often used in the discrete-choice literature. This affords a means to quickly and transparently recover the qualitative effects of the regressors. In Section 5, I use the linear regression as part of the basis for

[27] To see why this is so significant, it is helpful to inspect the likelihood function for ℓ_{jt}, which is the layoff decision of plant j in time t. The likelihood may be written as

$$\Pr(\ell_{jt}=0)^{1[\ell_{jt}=0]} \times \Pr(\ell_{jt}=1)^{1[\ell_{jt}=1]} \times ... \times \Pr(\ell_{jt}=4)^{1[\ell_{jt}=4]}$$

where, for instance,

$$\Pr(\ell_{jt}=0) = \Pr(z_{jt} > Z_{01}(I_{j,t-1}, L_{j,t-1}, \tau_t) \mid z_{j,t-1}, z_{j,t-2}, ...)$$

If z_{jt} follows a Gaussian geometric AR(1), as assumed above, this becomes,

$$\Pr(\ell_{jt}=0) = \Phi \Pr([\log Z_{01}(I_{j,t-1}, L_{j,t-1}, \tau_t) \; - \rho \log z_{j,t-1}] \; / \; \sigma)$$

Thus, $z_{j,t-1}$ appears as an unobserved regressor.

[28] For an introduction to MSL, see Stern (1997); Arias and Cox (1999); and Train (2009). The first two also include discussions of efficient simulators. Hajivassiliou (1999) touches on a number of practical concerns associated with implementation of MSL.

[29] To carry out MSL, again take a Taylor approximation to the threshold functions and provide an initial conjecture for the parameters associated with each of their arguments. Then it is possible to efficiently simulate a sequence of realizations of z that is consistent with the observed data on weeks of layoff and the (parameterized) optimal policy and to, therefore, "build up" the likelihood function by simulation. Maximization then takes place as it otherwise would (i.e., by application of gradient-based solver to improve on this initial guess), but here the objective to be maximized is the simulated log likelihood.

[30] To numerically evaluate the Jacobian of the likelihood, one would have to perturb each of these model-related effects; simulate the sequence of zs associated with the new parameters; and re-calculate the log likelihood function.

structural estimation, which takes the nonlinear structure of the model more seriously.

2.4.2 An Estimation Equation

I now develop a reduced-form approach to the estimation of the effects of the JOBS program, i.e., the effects of the variables, L_{-1} and τ on the choice of weeks of layoff. As seen in section 3, the weeks-of-layoff decision is conditioned on z, the demand disturbance; I_{-1}, the level of inventory available at the start of the current period; L_{-1}, the number of weeks on layoff implemented since the beginning of the current contract; and τ, the period of the current contract. Given these state variables, I initially specify the linear relation,

$$\ell = \gamma_0 + \gamma_z z + \gamma_I I_{-1} + \gamma_L L_{-1} + \gamma_\tau \tau$$

I make one modification here that is preserved throughout this discussion. Since z affects revenue multiplicatively (see (2.6)), it is sensible to restrict it to be non-negative. A particularly simple way to implement this is to assume that z follows a geometric AR(1) process given in equation (2.2). For this reason, I modify the linear projection to relate ℓ to $\log z$, rather than z:[31]

$$\ell = \gamma_0 + \gamma_z \log z + \gamma_I I_{-1} + \gamma_L L_{-1} + \gamma_\tau \tau$$

To turn into this an estimable equation, one must first address the endogeneity between ℓ and $\log z$. Given (2.2), this is relatively straightforward to treat. The law of motion of $\log z$ allows one to rewrite this specification as

$$\ell = \bar{\gamma}_0 + \rho\gamma_z \log z_{-1} + \gamma_I I_{-1} + \gamma_L L_{-1} + \gamma_\tau \tau + \xi$$

where $\bar{\gamma}_0 \equiv \gamma_0 + \gamma_z \mu$ and $\xi \equiv \gamma_z \varepsilon$ is the regression error. If $\log z_{-1}$ were observable, this would be a valid reduced-form specification.

The problem is that z_{-1} is in fact unobservable. Fortunately, the structural model suggests how one may proxy for z_{-1}. To see this, it is useful to inspect Figure 8. The right-hand panel plots the inverse of the layoff policy rule for a given (I_{-1}, L_{-1}, τ). Notice that the step-function shape of the policy accords with the form of (2.11). The figure indicates that for a given triple, (I_{-1}, L_{-1}, τ), it is possible to identify *the range* in which z must lie if the econometrician knows ℓ. This makes sense: if ℓ is high, that must mean that z is relatively low. The left side of the panel then shows

[31] A full log-linear specification is inadmissible since ℓ and L_{-1} may be zero.

that, given (I_{-1}, L_{-1}, τ) and ℓ, one may invert the sales policy rule and use sales, x, to pinpoint the value of the z *within this range.* Together, then, (I_{-1}, L_{-1}, τ), ℓ, and x completely characterize the position of the unobservable, z. This argument suggests that a useful proxy for $\log z_{-1}$ involves these five variables. For this reason, we amend the estimation equation as follows,

$$\ell = \bar{\gamma}_0 + \gamma_\ell \ell_{-1} + \gamma_x x_{-1} + \gamma_{I1} I_{-1} + \gamma_{I2} I_{-2} + \gamma_{L1} L_{-1} + \gamma_{L2} L_{-2} + \gamma_{\tau 1} \tau + \gamma_{\tau 2} \tau_{-1} + \xi.$$

Fortunately, it is possible to simplify this equation slightly. In my sample period, T is relatively long (at least 36 months), and so the number of contracts, five, is not that large. This has implications for how much of the information on the right side of this equation is actually needed *in practice* to proxy for z_{-1}. For instance, if L_{-1} and ℓ_{-1} are in the information set, then L_{-2} is almost always implicit since, for $\tau_{-1} < T$, $L_{-1} = \ell_{-1} + L_{-2}$. If the number of contracts is not that large, then this is the case that will hold for almost all of the observations in the sample period. Similarly, for $\tau_{-1} < T$, equation (2.12) states that $\tau = \tau_{-1} + 1$, which means that, given τ (and a constant), τ_{-1} is almost always implicit. These observations allow us to drop L_{-2} and τ_{-1} and suffer almost no loss of information:

$$\ell = \bar{\gamma}_0 + \gamma_\ell \ell_{-1} + \gamma_x x_{-1} + \gamma_{I1} I_{-1} + \gamma_{I2} I_{-2} + \gamma_L L_{-1} + \gamma_\tau \tau + \xi$$

To complete the empirical specification, a number of additional amendments to this baseline are needed. First, the theoretical model omits any sources of persistent heterogeneity. But it is arguable that estimation should allow different vehicles to have different average levels of demand. In the context of (2.2), it is helpful to think of this as if the intercept is the sum of a common component, μ, and model-specific components, μ_j. As a result, model-specific dummies ought to be included in the regression.[32] This acknowledges that a vehicle such as the Buick Lesabre, with average sales of almost 12,000 per month, is better modeled as if it possesses a higher μ than, for instance, the Buick Park Avenue, which is also a General Motors-produced full-size automobile but which averaged just over 4,000 sales per month.[33]

Next, it is important to add controls for fluctuations in aggregate vehicle demand. To do so in

[32] Perhaps it is simplest to define model by way of example. The Ford Taurus is a model, whereas the Taurus SE and SHO are known as trims. In this case, the SE is the base trim, which means it is the simplest available and the least expensive. The SHO essentially looks like the SE but offers a number of options (i.e., push-button start, more powerful engine) that are not available in the lower-price trims.

[33] Average sales are calculated over the period, October 1991 through June 2004. This is the interval over which Buick produced the Park Avenue in the United States. (Prior to 1991, Park Avenue was not regarded as a model; it was a trim level of the Buick Electra.)

a way that places the least structure on the data, I simply include time effects. Therefore, I include a dummy for every calendar quarter in the sample. In addition, I include 11 monthly dummies to pick up seasonal sources of variation in demand that are common across vehicles.

A third concern regarding the current form of the estimation equation is that it might take the AR(1) assumption for $\log z$ too seriously. Fortunately, this may be relaxed. If $\log z$ follows an AR(q), for instance, apply the logic of Figure 8 repeatedly to obtain,

$$\ell = \bar{\gamma}_0 + \sum_{s=1}^{q} \gamma_{\ell s}\ell_{-s} + \sum_{s=1}^{q} \gamma_{xs}x_{-s} + \sum_{s=1}^{q+1} \gamma_{Is}I_{-s} + \gamma_L L_{-1} + \gamma_\tau \tau + \eta' \mathbf{X} + \xi,$$

where, for reasons discussed above, it is sensible to drop the higher-order lags of L and τ. In the analysis below, alternative lag orders for z are considered. Here, $\mathbf{X}$ includes the model effects; quarterly time dummies; and seasonal effects discussed above.

Lastly, I make two additional changes to the estimation equation before I proceed to discuss the data and present the baseline results. As written, inventory and sales are in levels; instead, in all regressions to be reported in this paper, I instead specify both in logs. It is often thought within the industry that the inventory-sales *ratio* is a useful predictor of weeks on layoff. If $\log(I_{-1})$ and $\log(x_{-1})$ are entered, the regression allows (but does not assume) that the ratio of these objects affects weeks on layoff. In short, the log specification balances the implications of the theoretical model with independent information on the plant's decision rule available from industry analysts and management. Second, since contract length varies slightly over the sample, I normalize L_{-1} by $\bar{L}$ and τ and contract length, T. Intuitively, an L_{-1} of 18 weeks implies a potentially different option value if $\bar{L} = 36$ weeks as opposed to $\bar{L} = 48$. One straightforward way to adjust for this is to express L_{-1} relative to $\bar{L}$. A similar argument applies for τ.[34] Thus, the estimation equation is

$$(2.14)\quad \ell = \bar{\gamma}_0 + \sum_{s=1}^{q} \gamma_{\ell s}\ell_{-s} + \sum_{s=1}^{q} \gamma_{xs}\log(x_{-s}) + \sum_{s=1}^{q+1} \gamma_{Is}\log(I_{-s}) + \gamma_L\left(L_{-1}/\bar{L}\right) + \gamma_\tau(\tau/T) + \eta'\mathbf{X} + \xi$$

2.4.3 Plant-level Panel Data

To estimate (2.14), I link plant-level observations on weeks of layoff with data on sales and inventory for those vehicles produced at each assembly facility. My sample is October 1990 through September 2007. Data on weeks of layoff for most of this period (October 1990 through Decem-

[34]Section 3 did not consider variation in $\bar{L}$ and τ explicitly, but the effect of ρ on the strength of the JOBS Bank variables hints at the importance of contract length in particular. That section found that, as ρ increased for a given T, the JOBS Bank effect on the weekly layoff decision also rose. It stands to reason, and I have confirmed this in separate simulations, that a reduction in T, for a given ρ, has the same effect since it raises the half-life of a demand shock relative to the contract length.

ber 2006) have been assembled by Ramey and Vine (2004, 2008). Their source is an industry publication, *Automotive News*, which provides weekly reports as to whether a plant's workforce is on layoff and, if it is, the reason for downtime.[35] Four reasons are possible: holiday, model changeover, inventory adjustment, and supply shortage. A model changeover refers to plant-wide shutdown undertaken to prepare the facility to produce a new vehicle. Inventory adjustment is what I have referred to as a volume-related reason. Supply shortage includes strikes; parts shortages; or shutdowns due to acts of God (e.g., a tornado damages a plant's paint facility).

This level of detail with respect to the reason for layoff is important for my purposes because, as noted, only layoffs due to volume count against a plant's allotment, $\bar{L}$, under the JOBS' program. Therefore, I define ℓ as a layoff due to what *Automotive News* classifies as an inventory adjustment and L as the cumulation of these weeks on layoff over the life of a contract. There are five such contracts in my sample. Each starts in September or October and lasts for three or four years. The five contracts span the intervals 1990-1993; 1993-1996; 1996-1999; 1999-2003; and 2003-2007.[36]

Figures 9 and 10 summarize the time series and cross-sectional variation in the data. Figure 9 displays total weeks on layoff per month across all active U.S. assembly plants. The sharp increases in 1990-91 and in the fourth quarter of 2000 coincide with, or just slightly lead, two NBER-dated recessions. Weeks of layoff were also elevated in 2005 and 2006. GM posted an overall loss of $10 billion in 2005 while Ford Motor lost around $1.5 billion in North America and saw its market share hit an 80-year low. Figure 10 shows the distribution of total weeks on layoff within the labor agreement covered by my sample. Based on Figure 9, I aggregate the second and third contracts since these appear to span similar periods of robust growth in the vehicle market. I also aggregate the final two agreements since these span years of relatively lackluster growth. Relatively few plants bump up against the allotment but as argued above, it is not necessary that JOBS binds directly for it to influence plant behavior. The figure also reveals that majority of variation needed to identify the contract effects likely is provided by the first, fourth, and fifth agreements.

To take the model to data, I match layoff outcomes with observations on sales and inventory. These data are taken from Ward's electronic databank, which reproduces the published series in its

[35]To fill out the sample, I received data for 2007 directly from *Automotive News*.

[36]The JOBS program literally allots $\bar{L}$ weeks per worker, but, for my purposes, this distinction is not critical. As I mentioned in Section 3, I focus on the implications of the JOBS program for the choice of weeks of layoff per month. These actions are, in general, applied to an entire plant's workforce. (Occasionally, a plant will alternate shifts, that is, work the day shift one week and the night shift the next. But both shifts spend the same number of weeks on layoff over the whole calendar month, which is the unit of analysis here.) Adjustments such as shift reductions and line speed slowdowns, on the other hand, result in indefinite layoffs that affect a subset of the workers at a plant. To study the implications of JOBS for these margins of adjustment, I would like to be able to track individual workers in order cumulate their weeks on layoff. The data I have do not allow me to do this. This is one reason I restrict this study to the decision regarding weeks of layoff per month, for which the Ramey and Vine data are more appropriate.

Automotive Yearbook.[37] The data are monthly and available by model. To be clear, the inventory of a particular model is *not* attributed directly to any one assembly plant in the sense that the plant produced each vehicle in that stock. However, Ward's does report the models produced by each plant in each month. In this sense, it is possible to match models with plants. I calculate the empirical analogues of I and x for a plant as the sum of inventory (respectively, sales) across all models produced by that facility. Appendix A includes a more detailed discussion of how inventory and sales are matched to plants. I argue there that this is the best feasible strategy.

To provide a better sense of the panel, I now report some features of the plant-vehicle matched database. Figure 11 reveals that the number of vehicles under production in any one month in my sample varied from a low in the mid 50s to a high in the mid 80s.[38] Next, I consider how these vehicles are distributed across plants over time. First, I find that the typical plant produces roughly two vehicles per month. This indicates that there are *multi-product plants* – facilities make more than one vehicle at a time. Second, there is *product turnover* across time (within plant): the typical plant makes four vehicles over the whole time it is in my sample. Third, in a typical month, a little over 50 percent of the plants make a product, or a basket of products, that is unique to that plant. In a little less than 30 percent of the plants, at least one product is shared with another plant, and in 12 percent of the plants, at least one product is shared with two other plants. This indicates the degree of *product overlap* across plants. These three features of the data do conflict with the structural model set out in Section 3. That model assumed that one plant made one (differentiated) product. In Appendix A, I discuss this issue in more detail and argue that the regression model (2.14) remains a useful way to approach the data.

2.4.4 Results

Table 2 reports estimates of equation (2.14), where four lags of each variable based on standard lag selection criteria.[39] Consistent with the optimal policy rule (2.11) (see also Figure 8), inventory exerts upward pressure on weeks of layoff, and higher sales, a proxy for z, are associated with fewer weeks on layoff. In addition, the first two lags of ℓ, also proxies for z, are statistically significant.[40]

[37] I thank Dan Vine at the Federal Reserve Board for providing me access to the Ward's data at the Board.

[38] There is a sudden decline in the number of models in the fall of 2003. This occurs because eight plants were closed over the life of the 2003-2007 labor agreement and so the models made at these facilities were excluded from the analysis (see Appendix A).

[39] I inspect Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) lag selection statistics. The formula for the former is $-2 \times \log(\text{likelihood}) + 2\times$degrees of freedom, and, for BIC, $-2 \times \log(\text{likelihood}) + \log(n) \times$degrees of freedom, where n is the number of plant-month observations. The choice of four lags does not depend on whether n is interpreted as the total number of plant-month data points or as the number of clusters (plants).

[40] The sign of the coefficients on ℓ_{-1} and ℓ_{-2} are, of course, opposite those of the coefficients on the other proxy for z, namely lagged sales, since the former are negatively correlated with z and the latter, positively correlated.

When I turn to the JOBS-related variables, the signs are also in line with the theoretical model. First, a higher L_{-1} deters weeks of layoffs, so the coefficient on it is negative. The intuition for this result is that, as L_{-1} approaches the allotment $\bar{L}$, the option value of a week of layoff rises: the plant wants to preserve the option to shut down (relatively) more profitably in the future if demand deteriorates further. Second, the passage of contract time leads to an increase, all else equal, in weeks of layoff. As the contract draws to a close, uncertainty over vehicle demand throughout the life of the current agreement is resolved, and the option value of deferment falls.[41]

The specification reported in column one remains relatively conservative in one respect. While it includes model effects, it does not allow for any trends in vehicle taste. This might have important implications for the JOBS variables in particular: adverse long-run movements in demand, if omitted, would likely be correlated with L_{-1}. Since plants that face these trends will tend to implement more layoffs, this will bias up the coefficient on L_{-1}.[42]

Column two of Table 2 reports results that include vehicle-specific trends. The effect on L_{-1} is substantial and consistent with the concern regarding omitted variable bias. The effect on τ is also noticeable. This might simply be due to the fact that τ and L_{-1} are positively correlated, which implies a negative correlation between τ and the vehicle trends. As a result, the omission of the trends biases a positive coefficient on τ down in the direction of zero. The effect of the introduction of the vehicle trends on the other coefficients is not that substantial, although one difference is that now one cannot reject the null that the coefficients on $\log I_{-1}$ and $\log x_{-1}$ are equal in magnitude and of opposite sign.

One obstacle to a causal interpretation of Table 2 is the potential for correlation between the error and the endogenous regressors. This would occur if the error displays serial correlation, in which case ε_t would be correlated, via its own lag, with the endogenous regressors, such as $\log(I_{t-1})$ and L_{t-1}. The lag selection tests should minimize this risk since any appreciable correlation in the error due to, for instance, omitted lags of ℓ would show up in relatively high AIC and BIC scores. Nonetheless, it is still worthwhile to formally test for serial correlation in the error. Wooldridge (2002), section 7.8.5) recommends a simple procedure. First, calculate the series of residuals, $\hat{u}$, from the regression in column two. Then include $\hat{u}_{t-1}$ as a regressor and re-estimate the equation.

[41] The standard errors shown in Table 2 are robust to heteroskedasticity and serial correlation within plant. If the layoff policy rule were in fact linear, the law of motion of z would imply that the regression error is iid. Since the linear reduced-form model (2.14) is, however, an approximation, it is not obvious how seriously to push this implication. It does appear to be rejected by the data: I have found that corrections for heteroskedasticity and serial correlation appreciably affect the standard errors. Therefore, I simply report clustered errors in the table.

[42] To be concrete, an adverse trend in vehicle demand would be reflected in a "saw-toothed" pattern for L_{-1} in which the peak of L_{-1} at the conclusion of each contract exceeds that of the prior contract (and then resets to zero). This upward trend in L_{-1} is (negatively) correlated with the trend in the "taste" for that vehicle.

That $\hat{u}_{t-1}$ is a generated regressor turns out to have no effect on inference in this (special) case: under the null of zero serial correlation, the asymptotic distribution of the t-statistic associated with a generated regressor is the same as if the regressor were measured perfectly. I estimate the coefficient on $\hat{u}_{t-1}$ to be -0.074 with a standard error of 0.093.

Another obstacle to inference in the baseline regression model relates to the presence of spatial correlation in the data. Perhaps the most common robust covariance estimator for panel data, due to Arellano (1987), accounts for serial correlation within plant (it "clusters" by plant) but not for correlation across space (i.e., between, say, plant j in time t and plant k in time $t + \Delta$). The estimates in column two of Table 2 report Arellano standard errors. In the next two columns, I report spatially robust standard errors based on two estimators. The first is due to Driscoll and Kraay (1998). Foote (2007) shows that the Driscoll-Kraay estimator generalizes Arellano to allow the temporal mean of a plant's residuals to co-vary with that of other plants. An alternative multi-way cluster estimator is provided by Thompson (2006), who illustrates how to use separately clustered covariance estimators – clustered, that is, with respect to time and plant – to calculate standard errors robust to spatial correlation. To implement Thompson's formulas, the practitioner must specify some interval Δ of time such that, if observations on two plants are separated by more than Δ periods in time, the correlation between them is zero.[43]

Table 3 reports standard errors corrected for spatial correlation. On the whole, the correction does not substantively affect inference: although the standard errors increase, the JOBS variables remain significant at conventional levels. Adjustment for spatial correlation may have relatively modest effects because the regression already included several controls for common sources of variation, such as (calendar) time effects, seasonal dummies, and model-specific effects and trends.

2.4.5 Robustness

In this subsection, I investigate the robustness of the baseline results along three dimensions. First, the strength of the JOBS Bank effects depended heavily on the inclusion of vehicle-specific trends. This warrants a deeper look. In particular, if the baseline estimates were influenced by only a handful of vehicles, that would raise concern about the robustness of the results to minor variations in the composition of the sample. Second, in the simulation analysis of Section 3, it appeared that the effects of the JOBS Bank on the weekly layoff decision were nonlinear in L_{-1}

[43] These formulas are finite sample implementations of a general asymptotic covariance estimator for correlated vector processes. See Hayashi (section 6.5) for an introductory discussion and, in particular, for a concise statement of Gordin's conditions that make precise the degree of serial correlation permitted. See White (1984) for a proof of Gordin's central limit theorem.

and τ. This suggests that higher-order terms in L_{-1} and τ be included in equation (2.14) in order test the full set of the model's implications. Third, to interpret the reduced form estimates on τ and L_{-1} as the causal effect of the JOBS Bank, it stands to reason that the point estimates on τ and L_{-1} should vanish if we run regression (2.14) on data prior to the JOBS Bank's introduction in October 1990. If, on the other hand, the JOBS-related variables are significant predictors of weeks on layoff in the pre-1990 period, then they likely "stand in" for omitted variables that affect the layoff decision in both sample periods. We take up each of these tasks in turn.

Subsample stability

I initially conjectured that the inclusion of vehicle specific trends is sensible on the grounds that, in their absence, their effect will be attributed to L_{-1} and generate a spurious positive correlation between accumulated layoffs and contemporaneous downtime. The baseline results were consistent with this interpretation. Nonetheless, since the regression estimates depended so strongly on the introduction of vehicle-specific time trends, it is important to take a deeper look.

To this end, I re-run the regression 144 times, once for each vehicle trend. On each iteration, I drop a single trend and leave the remainder in. (All of the vehicle fixed effects are included in all regressions.) This allows me to identify the ten vehicles whose trends, if omitted, most substantially reduce the estimated effect of L_{-1} (i.e., their removal pushes the estimate nearer to zero). These models make up slightly less than 7 percent of the total number of vehicles.

Figure 12 reports the ten most influential models and their individual contributions to the estimated effect of L_{-1}. For instance, the removal of the trend specific to the Mercury Villager minivan reduces the estimate effect of L_{-1} by a little over 0.035 (from -0.394 to -0.358). What lies behind this is a strong adverse trend in Villager sales in the 1990s. Sales per worker at the Ohio plant that produced the minivan fell from nearly 3 to just over 1.5 in the second half of the 1990s. As a result, layoffs increased from 2 weeks for the whole of the 1993-1996 agreement to 15 weeks over the 1996-1999 accord and 28 weeks in the subsequent three years. Section 4 argued that the linear regression attributes, rightly, at least a portion of this increase to an adverse trend in minivan sales rather than any positive dependence between contemporaneous layoffs and L_{-1}.

The figure reports the impact on the baseline result if all ten of these vehicles are removed from the regression. The estimated effect of L_{-1} declines by nearly 40 percent, as it falls from -0.394 to -0.236. It does remain statistically significant, though. Thus, the magnitude of the effect of L_{-1} is moderately sensitive to variations in the composition of the sample, though the qualitative result remains intact.[44]

Nonlinear contract effects

Thus far, I have entered the JOBS-related variables linearly into the regression. Yet there is good reason to believe, in light of the analysis of Section 3, that these effects are in fact nonlinear. For instance, as L_{-1} approaches the allotment, the impact of an additional increase in L_{-1} rises. In other words, the marginal effect increases as the contract binds (in expectation) more severely. In contrast, the marginal effect of contract time appeared to diminish as time wore on (at least for ρ sufficiently small). Intuitively, the option value of deferment declines as uncertainty is resolved and the possibility that the JOBS Bank will bind recedes. Consequently, the passage of one more month means less nearer to the end of the contract than the beginning. Lastly, in the structural model, there is likely to be an interaction effect between L_{-1} and τ. In particular, the simulations of Section 3 indicated that the marginal effect of τ on the probability of layoff increases in L_{-1}. The reason is that the passage of time is more significant to a plant where L_{-1} is already quite high and, thus, where it is more likely that JOBS might bind. As a result, an increase in τ has a greater effect on that plant's probability of layoff than it does on a comparable plant where L_{-1} is relatively low.

These arguments motivate the inclusion of second order terms. The baseline equation (2.14) becomes

$$\begin{aligned}(2.15)\quad & \bar{\gamma}_0 + \sum_{s=1}^{q} \gamma_{\ell s} \ell_{-s} + \sum_{s=1}^{q} \gamma_{xs} \log(x_{-s}) + \sum_{s=1}^{q+1} \gamma_{Is} \log(I_{-s}) \\ & + \gamma_{L1}\left(L_{-1}/\bar{L}\right) + \gamma_{L2}\left(L_{-1}/\bar{L}\right)^2 + \gamma_{\tau 1}(\tau/T) + \gamma_{\tau 2}(\tau/T)^2 + \gamma_{L\tau}\left(L_{-1}/\bar{L}\right)(\tau/T) + \eta'\mathbf{X} + \xi,\end{aligned}$$

where $\mathbf{X}$ continues to include the model-specific trends discussed above. Table 4 reports the results.[45] At first, I include the quadratic terms and no interaction.To better understand the point estimates associated with L_{-1} and L_{-1}^2, Figure 13 graphs the total effect of each JOBS-related variable on weeks of layoff per month. The top panel reveals that the effect of L_{-1} does rise at a faster rate as L_{-1} increases. The point estimates on τ and τ^2 do not conform so neatly with the model's predictions, though. As seen in the bottom panel, the point estimates associated with contract time imply that the marginal effect tapers off as τ rises and then turns negative. The

[44] As noted above, the introduction of model trends strengthened the effect of both accumulated layoffs, L_{-1}, and contract time. It follows that the removal of these ten vehicles also weakened the contract time effect, but it, too, remained significant.

[45] Since the coefficients on the non-JOBS variables do not materially differ from those presented in Table 2, they are suppressed in Table 4.

switch in the sign is hard to reconcile with the structural model. This might suggest that the quadratic form is too restrictive, or, equivalently, that the second-order approximation still fails to capture the nonlinearities embedded in the true threshold functions. But this same qualitative picture emerges even when third- and fourth-order terms are entered.

The introduction of the interaction term yields another puzzle. The final column of Table 4 shows that $\gamma_{L\tau}$ is estimated to be negative, which is in opposition to the prediction. The point estimate suggests that the passage of time is more significant to a plant with few past layoffs than a plant with many – even though the latter is virtually unencumbered by JOBS and has relatively little reason to modify its behavior as time passes. The negative sign of the estimated interaction effect, although statistically insignificantly different from zero, is worrisome insofar as it suggests that the reduced-form effects may not reflect the option-value concerns embedded in the structural model.

A falsification test

The final robustness test is to run the regression (2.14) on the pre-1990 data. This pre-dates the JOBS Bank and represents, therefore, a period over which the hypothesis regarding JOBS is known to be false. If the baseline estimates reported in Table 2 reflect the causal effect of JOBS, the coefficients on L_{-1} and τ should vanish over this period. It turns out that they do not.

I estimate equation (2.14) over the period October 1973 through September 1990. The start of the sample is determined by a combination of data availability and the union bargaining schedule. Data on weeks of layoff at the plant level are available from Bresnahan and Ramey (1994) beginning in January 1972. The UAW was in the middle of an agreement at that time. The next agreement began in the fall of 1973, which is when I start the sample. The period I selected includes six UAW-Detroit Three contracts.

As above, I construct a plant-level panel dataset that includes observations on weeks of layoff, inventories, and sales. Data on weeks of layoff at the plant level are available from Bresnahan and Ramey through December 1984 and beginning again in January 1990. I gathered data to fill in the years 1985-1989. For this, I also used *Automotive News*.[46] With regard to sales and inventory, Wards' electronic database includes data on sales beginning in January 1980, but its inventory stretches back only to January 1985. For cars, I was able to gather monthly sales and inventory data by model from *Automotive News* and *Wards Automotive Reports* going back to 1973,

[46]For randomly selected weeks, I cross checked *Automotive News*' reports with those in *Ward's Automotive Reports*. On weeks of layoff, the two almost always agreed. The only discrepancies I regularly found related to reports of overtime hours at Chrysler plants. I do not use overtime hours in any of the work presented here.

as required. For light trucks and SUVs, however, detail by model was not provided. Therefore, I include only car assembly plants in my sample from October 1973 through December 1984. It is not yet clear whether this assumption is harmless. On the one hand, car production made up at least three-quarters of total output, on average, at each Detroit Three firm over this sample period. At the same time, layoffs at light truck plants were sometimes much more common, particularly in the 1980-82 downturn.

To link data on plant-level layoffs to sales and inventory, I am able to use Wards' electronic database from January 1985 through September 1990. Prior to that, I use reports in Wards *Automotive Yearbooks* and in Automotive News' *Almanac* that show the vehicles scheduled to be made at each plant for each model year. The detail in these reports does degrade going back in time. If vehicles are relocated from one plant to another or suspended altogether, reports beginning in the late 1970s appear to regularly report this. Prior to that, these updates are not available. This will generate some additional noise in the inventory and sales data.

For the 1973-1990, I construct the JOBS-related (counterfactual) variables as follows. In the period over which JOBS was operative, the allotment of weeks on layoff over a contract was generally equal to one quarter of the total number of weeks in that contract. For instance, for three of the five contracts in the October 1990 – September 2007 period, the contract was 36 months long and the allotment was 36 weeks, or 9 months. I use this ratio to generate allotments in the pre-JOBS period. The construction of contract time, τ, remains straightforward.[47]

Once the data are constructed, I re-estimate equation (2.14). The results are provided in Table 5. The coefficients on lagged layoffs, inventory, and sales appear reasonable and broadly similar to those obtained over the post-1990 sample. Perhaps the only noticeable difference is the pattern of coefficients on inventory: here lags one and two, rather than only the first (as in Table 2), are significant.

The results with regard to the JOBS-related variables forcefully challenge the claim that the reduced form is able to uncover the causal effect of the JOBS Bank. The reason is that the JOBS-related variables are estimated to be statistically significant contributors to the layoff decision in the period *even before the JOBS Bank took effect.* The coefficient on L_{-1} is negative and significant

[47] One complication that arise in the construction of the JOBS variables is that some contracts in the pre-JOBS period were re-opened prior to completion. For instance, GM and Ford re-opened their 1979 agreement five and eight months, respectively, prior to the expiration of that agreement in September 1982. In this case, the re-negotiation appears to have been largely unforeseen, triggered by substantial losses at Ford in particular in the recession of that period. For this reason, I construct the data as if the plants anticipated the contract to extend through September 1982. In contrast, participants' expectations are less clear in the case of a brief, one-year interim agreement signed between Chrysler and the UAW in December 1982. That contract was re-opened nine months later, and two-year agreement negotiated. Therefore, I simply drop this short contract from the sample.

at the 2 percent level and quantitatively very similar to that found over the later sample period. The effect of contract time is also statistically significant at the 8 percent level, though noticeably smaller than in the JOBS period. To formally determine whether the difference across the two periods in the difference in the JOBS Bank effects is statistically significant, I merge the samples and interact L_{-1} and τ with a dummy variable that takes the value of one for each month after October 1990 and zero otherwise.[48] The regression equation isolates the "true" JOBS Bank effect. The results, also shown in Table 5, indicate that virtually no effect of L_{-1} but a significant effect of contract time, though the magnitude is diminished relative to the baseline estimate.[49]

2.4.6 A Reassessment of the Baseline Result

What might explain why both JOBS related variables predict of the layoff rate before *and after* the JOBS Bank was introduced? This section offers up some possibilities, though future work is needed to pursue these avenues more fully.

With respect to accumulated layoffs, it is possible that L_{-1} serves as a proxy for the disincentives for layoffs under experience-rated unemployment insurance. Such a system is funded, in part, by a tax on firms that increases in the number of layoffs that the firm has performed. This implies a qualitatively similar effect as JOBS on the weekly layoff decision: a sequence of past layoffs raises the tax burden and reduces the probability of further layoffs.

I doubt, however, that this is behind the pre-treatment effects. The simple reason is that the Detroit Three likely pay the maximal tax rate under the UI system, which means its tax burden does not increase with layoffs but functions more like a fixed cost of operation. To see this, consider the state of Michigan, where the Detroit Three operated 26 assembly plants over the course of the 1973-1990 period. In Michigan, a firm's tax bill equals the product of taxable payroll and the tax rate, the most important component of which is calculated as

$$\min\left\{0.063,\ \frac{\text{5 years of UI benefits charged to firm}}{\text{5 years of taxable payroll}}\right\},$$

[48]The regression on the merged sample allows the vehicle effects and vehicle-specific trends to vary across the sub-samples. Other than L_{-1} and τ, these are the only variables whose coefficients are permitted to vary across the two periods.

[49]If one allows L_{-1} to affect the weekly layoff decision in a nonlinear way, the post-1990 regression results do distinguish themselves at least slightly from the pre-1990 estimates. For instance, I re-estimated equation (2.15), where linear and quadratic terms in L_{-1} were included, on the pre-1990 sample. The quadratic term is more powerful in the post-1990 sample, which is consistent with the highly convex shape of the structural policy functions shown in Figure 6. However, the effects are estimated so imprecisely that the difference is not statistically significant. The same result obtains if one adopts a more flexible specification with regard to L_{-1}. For instance, I formed a piecewise linear spline in L_{-1}, which allows the effect of accumulated layoffs to differ when L_{-1} is relatively large. The effect of L_{-1}, conditional on $L_{-1} \geq 0.5$, appears to be stronger in the post-1990 sample, which is consistent with the results based on the quadratic specification. Again, however, the estimates are too imprecise to allow one to firmly reject the null that the pre- and post-1990 effects are identical.

that is, the rate increases in the level of UI benefits claimed by that firm's workers, but the upper limit is 6.3 percent. The key observation here is that assembly plants generally conduct plant-wide downtime – the entire plant's workforce is laid off for a week, for instance. As a result, it takes relatively few weeks of layoff per plant per year for the firm to accumulate total UI charges on the order of 5 to 10 percent of taxable payroll. In Michigan, it takes just one to two weeks of layoff per year per plant to reach the upper bound of 6.3 percent.[50]

A more likely explanation for the result that L_{-1} is significant is that accumulated weeks of layoff serves as a proxy for hard-to-measure costs of changing production. Specifically, if the cost of adjusting output increases in the number of recent down weeks, then accumulated layoffs would predict fewer current layoffs. What are possible sources of such a cost?

First, there are restrictions in the labor agreements, in addition to JOBS, on weekly downtime. For instance, the UAW agreement with Chrysler states that a plant may implement plant-wide layoffs for up two consecutive weeks, after which the plant must place low-seniority workers on indefinite layoff (and recall high-seniority workers) or negotiate for an exception with the local union chapter. General Motors' agreement with the union directs the firm to formally consult with the UAW regarding continued downtime if each active shift has consistently worked less than 32 hours per week. Again, one suspects that this is designed to give voice to the union's preference for indefinite layoffs over frequent plant-wide shutdowns. As a result, persistent deviations of accumulated layoffs from the guidelines articulated in the labor contract would likely strain relations between the UAW and management. Since the union would seek redress of some form, it follows that accumulated layoffs may signal increases in the shadow price of labor.[51]

Second, frequent shutdowns at final assemblers likely drive up the cost of production at their suppliers and may result in higher intermediate input prices for the assembly plants. To see why, suppose an engine plant supplies several final assembly facilities.[52] If orders from one assembler decline, the engine plant may not be able to reduce the workforce: as long as some orders are made,

[50]Taxable payroll per worker is the first \$9500 earned per year. Based on their wages, workers in the UAW generally receive the maximal UI benefit of \$362 per week. Thus, the annual number of weeks of layoff, ℓ, at which a firm will "max out" the tax rate satisfies

$$0.063 = \frac{5 \times \$362 \times \ell \times N}{5 \times \$9500 \times N} = \frac{\$362}{\$9500}\ell,$$

where N denotes the size of the workforce at a typical plant. This yields $\ell = 1.65$.

[51]Of course, if it were relatively inexpensive to remove a shift, the plant would just do this and short-circuit any increase in the shadow price. But the consensus in the literature (see Hall, 2000 and Ramey and Vine, 2004) is that there are substantial costs involved in the removal of a shift. This is why the plant will pay some price in terms of its relations with the union to run up accumulated layoffs, but there may be a limit to this approach.

[52]This example was motivated by a conversation with Dan Luria, though he should not be held liable for any errors that follow.

each worker is needed to execute a specialized task on his particular machine. As a result, a decline in orders may just reduce utilization and drive up unit costs. All else equal, this gives the assembly plant an incentive to smooth production after a sequence of regular shutdowns.

These arguments indicate that accumulated layoffs may be costly. However, they do *not* subscribe any particular meaning to the number of layoffs done since the beginning of a labor contract. Thus, they suggest a simple test: include a rolling sum of past layoffs, which is orthogonal to the contract structure, and see if the statistical significance of L_{-1} vanishes. The rolling sum is calculated as the number of weeks on layoff during the prior 36 week interval. Unlike L_{-1}, this variable does not reset to zero at the beginning of contracts. When the rolling sum is included in a "horserace" with L_{-1}, the estimated coefficient declines to -0.1 and is insignificantly different from zero. The coefficient on the rolling sum is -0.42, which is reminiscent of the estimated effect of L_{-1} in the baseline regression. This is at least consistent with the argument that L_{-1} functioned as a proxy for the cost of high-frequency adjustments to employment and output.

The estimated effect of contract time presents a greater challenge for interpretation. This is for two reasons. First, Table 5 suggests that there is a significant post-1990 effect of τ. Yet it is hard to imagine a JOBS Bank-related mechanism that would generate a contract time effect and yet no role for accumulated layoffs. Indeed, in simulations of the structural model reported below, these effects go hand in hand: perturbations of a parameter that accentuate one effect tend to make the other more significant, too.

Second, as for the pre-1990 effect, it is instructive to consult a paper by Bils (1990). He documents a downward trend in motor vehicle industry employment growth within each labor contract over the period from 1958 to 1984. Bils tried to replicate this result within a bargaining model in which a monopoly union sets the wage, and the firm chooses labor. The firm in this model has an incentive to reduce employment near the end of the contract in order to put downward pressure on the future contract wage. However, within the model, there is a second, countervailing force on employment growth. Specifically, if employment adjustments are costly to reverse, the union is aware that the current choice of labor is sensitive to the future path of wage growth. As a result, the union front-loads compensation, that is, it pre-commits to relatively slow future wage growth. This effect overwhelms the firm's incentive to reduce employment. As a result, Bils finds that for plausible parameter values, the model predicts that employment actually increases through the contract. The source of the apparent contract time effect, then, remains an open question.

2.5 Structural Estimation

This section has two objectives. It first investigates whether the structural model is able to induce the reduced-form estimates found in the prior section. In particular, it asks if the structural model replicates little-to-no impact of the JOBS Bank on the weekly layoff decision. The paper finds that it does, when the parameters are selected to be consistent with certain other salient moments of the data. I then investigate the parameter configurations under which the JOBS Bank effect is more substantial. The section concludes with a discussion of these counterfactual experiments.[53]

To begin the quantitative investigation, though, it is helpful to first reduce the number of structural parameters to be estimated. In the next subsection, I show how to set a number of them directly. After that, I proceed to estimation.

2.5.1 Calibration

As noted above, this paper abstracts from several details of the product market in order to focus attention on the JOBS program. As a result, with regard to the parameter that represents the price elasticity of demand, I defer to other work that has modeled the product market in more detail. For instance, Copeland and Hall (2009) apply a discrete-choice model of household demand to transaction-level data on prices and sales of individual vehicles. The authors estimate (own-price) elasticities of demand for eight market segments, such as mid-size cars and light pick-up trucks. I set φ to equal the average elasticity of 3.5 reported by Copeland and Hall. (See Table 7 for a complete list of the calibrated parameters.)

The wage-related parameters are calibrated directly based on information in the UAW agreements where possible and publicly available data on compensation in the unionized manufacturing sector where needed. The wage-related parameters include w, which is straight-time weekly pay; h, weekly non-cash benefits; b, the *share* of straight-time salary paid by the employer to a laid-off worker; and C, the non-labor component of the cost of production. Fortunately, there is a normalization available. Inspection of the Bellman equations of Section 3.2.2 and the cost of operation, (2.7), reveals that one may normalize by $\bar{W} = W + C$ and re-interpret parameters appropriately. For instance, B is now interpreted as non-JOBS unemployment compensation *relative* to the total cost of operation, $\bar{W} = W + C$, at an active plant. Likewise, W is the labor's *share* in total cost. In addition, λ is expressed as a share of $\bar{W}$ (which will help explain why its estimate in Table 7 is so low). Lastly, z is scaled by $1/\bar{W}$.

[53] An added benefit of the simulations is that, by documenting the effect of varying certain parameters on the simulated moments, they help illustrate the sources of variation in the data that identify the structural coefficients.

It remains to calibrate the two shares, B and W. After the normalization by $\bar{W}$, these parameters are constructed as follows:

$$\frac{B}{\bar{W}} = \frac{B}{W+C} = \frac{bw+h}{W+C}, \qquad \frac{W}{\bar{W}} = \frac{W}{W+C} = \frac{w+h}{W+C} \ .$$

Researchers tend to set b to the sum of two components: actual supplemental unemployment benefit paid by the Detroit Three plus an additional amount that represents the tax paid by private firms to state unemployment insurance agencies for layoffs at their plants. However, as I argued above, the Detroit Three is likely very often at the maximum tax rate, which means that the marginal tax cost of an additional layoff is approximately zero. Therefore, I will set $b = 0.4$, as the Detroit Three pay 40 percent of straight-time pay in supplemental unemployment compensation to a laid off worker (Ford, 2007). The results are robust to choices of b in between 0.4 and 0.65 , as set in Hall (2000).

Next, Ford (2007) and Chrysler (2007) provide time series data on straight-time hourly pay, w. I calculate that the average wage rate for a typical assembly plant worker over the period 1990-2006 was \$22.37.[54] Both also include data on non-cash benefits, but this is not the proper analogue to h. The reason is that the implied average hourly non-cash benefit rate reported by Ford and Chrysler includes a number of components that have no counterpart in the model, such as health insurance premiums paid on behalf of retirees. I require a more direct measure of non-cash benefits for active workers, which is what h is intended to represent. Data in this area is scarce, unfortunately. I assume that the benefit share of total compensation for the average hourly UAW worker is the same as that reported for the average (active) blue-collar union manufacturing worker. BLS reports[55] that the latter was roughly constant at 38 percent from 1993 through 2003. As anticipated, this is less than two companies' reported share of about 50 percent.[56] When benefits are included, average hourly compensation, $W = w + h$, comes to \$36.075.

The final parameter needed to calculate B and W is the non-labor cost, C. The key question here is how to treat transfers of parts (i.e., engines, powertrain) from Detroit Three parts plants to assembly facilities. On the one hand, layoffs in the theoretical model are treated as the counterpart

[54]This includes straight-time base pay plus cost of living adjustments. It does not include overtime pay. I have not been able to locate a comparable time series for GM. However, based on scattered observations of GM's base wage rate, it appears, not surprisingly, to be nearly identical to the rates paid by the other two.

[55]These data are from the Employer Cost of Employee Compensation, series ID CCU230000406000P.

[56]Researchers affiliated with MIT's International Motor Vehicle Program reported that one quarter of compensation in 1995 was paid out in non-cash benefits (Artzner and Whitney, 1997, p. 10). They relied on survey responses from 11 engine plants located around the world. The authors observe the smallest benefit shares at the four European plants and the largest shares at the two plants in less developed economies. The remainder of the plants were in North America, and the authors' figure (see p. 11) suggests that the average share at these plants was in the neighborhood of one third. Based on the authors' discussion, it appears that most of these plants were unionized.

to observed layoffs at assembly plants. This seems to suggest that one should treat the "plant" in the model as quite literally an assembly facility. On the other hand, it is possible to interpret the actor in the theoretical model as the manager of a generic facility that produces parts and final products. In the model, the manager decides layoffs across both divisions (parts and final vehicles) in the plant simultaneously, but in the data, I observe only outcomes among assembly workers. In this case, one should not treat all parts transfers as materials purchases. Instead, it is necessary to estimate the labor share in production costs across parts and final products. This was the approach of Luria (1996), who found it to be about 38 percent on average across the Detroit Three. Thus, as a baseline, I set $W = 0.38$; $C = \frac{0.62}{0.38} \times W = \$58.86$; and $B = \frac{bw+h}{W+C} = \frac{0.4\times 22.37+13.71}{36.075+58.86} = 0.152$.[57]

The duration of the contract is also calibrated. I set contract length in the model, T, to be 36 months. Consistent with UAW agreements of this length, I set the allotment, $\bar{L}$, to be 36 weeks. Over my sample, these parameters do change, but the ratio of the latter to the former remains virtually constant. So in the model, I simply fix $T = 36$ months and $\bar{L} = 36$ weeks for the duration of the 17 years over which the model is simulated.[58]

To conclude, I set values for the discount factor, β; the number of workers per facility, N; and output per worker, A. The choice of discount rate is not an inconsequential matter, since the impact of JOBS depends on the *expected* costs associated with *future* violations of the allotment, $\bar{L}$. Macroeconomic studies often calibrate β to be the inverse of one plus the risk-free Treasury bill rate. One might argue for that based on the fact that these corporations are owned by households and, therefore, ought to discount at a rate consistent with the representative household's (steady-state) optimal savings policy. On the other hand, the little direct evidence that is available suggests they do not: corporations often discount at rates appreciably above the risk-free rate.[59] I attempt to strike a balance, and to hew a little more closely to current research for comparability. I set β to $\frac{1}{1+r_b}$ where r_b is the average implied monthly corporate interest rate, since the model is set at a monthly frequency. Over my sample, $\beta = 0.9938$.[60] To select N, I use the annual Harbour reports that provide estimates of the number of hourly workers at each vehicle assembly plant in North America. I calculate that the average plant hourly workforce over my sample period was N =2540.

[57]Luria developed his estimates based on visits to dozens of assembly and parts facilities in the middle of the 1990s. The estimate cited in the main text refers to the variable cost of production, which is the proper analogue to $W + C$ in the model. It does not include other components such as research and development.

[58]If I vary the contract parameters, I must re-solve the model several times in order to generate 17 years of data. To reduce computational time, I fix $\bar{L}$ and T.

[59]Based on responses from 228 Fortune 1000 firms, Poterba and Summers (1995) calculate that the average annual real discount rate applied by these firms to future cash flows from an investment project was 12.2 percent. Their discussion suggest that this corresponds to an average annual nominal rate of between 16 and 17 percent. To put this in context, the Baa corporate bond rate in the fall of 1990, when the survey was administered, was around 10.5 percent, and the 30-year Treasury rate was in the neighborhood of 8.5 percent.

[60]I use the Baa rate tabulated and published by the Federal Reserve Board.

Lastly, I use Wards data on plant-level production and Harbour report workforce estimates to calculate average weekly output per worker to be A =1.545. It follows that output, $y = AN(4-\ell)$, may take on one of five values, $\{0,\ 3924,\ 7849,\ 11773,\ 15697\}$.

2.5.2 Indirect Inference

There remain five unknown structural parameters: α and λ of the inventory storage function and three parameters governing the evolution of idiosyncratic demand, namely, the mean (μ), variance (s^2) and persistence (ρ). I estimate these parameters by indirect inference. This involves a few steps:

- First, specify a set of moment conditions that summarize, or reflect on, key features of the optimal policy.
- Second, given a guess for the structural parameters, solve the plant's problem; simulate the endogenous variables; and calculate the moments of interest.
- Third, iterate on the structural parameters to minimize the distance between the empirical moments obtained in step two (off simulated data) and the model-generated moments obtained in step one (off actual data).[61]

More formally, I arrange the structural parameters into a 5x1 vector, $\chi \equiv (\alpha, \lambda, \mu, \rho, s)$, and let $\Gamma(\chi)$ denote the vector of empirical moments induced by the structural model. Define the empirical counterpart of $\Gamma(\chi)$ to be $\hat{\Gamma}$. Indirect inference selects χ to minimize $\left(\Gamma(\chi) - \hat{\Gamma}\right)' \Omega^{-1} \left(\Gamma(\chi) - \hat{\Gamma}\right)$, where Ω is the empirical variance-covariance matrix of the moments (Smith, 1993). The deviations of the model from its empirical counterpart are thus weighted, roughly speaking, by the inverse of the variance of the empirical moment.

I deal with each of these steps in turn. The first, the choice of moments, requires the most discussion. I then briefly describe the solution and simulation of the model and the actual algorithm used to optimize the quadratic criterion function.

The choice of moments

While I am principally interested in the structural model's ability to fit the reduced-form equation (2.14), the regression coefficients are not the only informative moments with respect to the

[61] In other words, the estimated coefficients from step one are taken as the moments in what Smith (1993) refers to as "extended method of moments". Indirect inference is now the more commonly used name for this procedure.

structural parameters. The addition of other moments will sharpen the identification of the parameters. For instance, in the canonical linear-quadratic model of inventory dynamics, where output is divisible, the steady-state inventory-sales ratio is proportional to α, suggesting that the average inventory-sales ratio in the data is likely informative with regard to this parameter. Of course, this result will not map perfectly into the model of Section 3.2, but the moment is still likely to be valuable: since the plant is penalized for deviations of I_{-1} from αx, it should, whether output is discrete or divisible, keep inventory in the neighborhood of αx on average.

With this in mind, I begin by proposing the following moments, where the numbers on the right-hand side are sample estimates:

$$\begin{aligned} \text{M.1} &: \quad \mathrm{E}\left[I/x\right] = 2.9 \\ \text{M.2} &: \quad \mathrm{Var}\left(\log I\right) = 0.042 \\ \text{M.3} &: \quad \mathrm{Corr}\left(\log I, \;\; \log I_{-1}\right) = 0.87 \\ \text{M.4} &: \quad \mathrm{E}\left[\ell\right] = 0.172. \end{aligned}$$

The interpretations of the first three are self-evident. The fourth states that the average number of weeks of layoff per month is 0.17, which corresponds to a weekly probability of layoff of 4.25 percent. These equations are estimated on model-generated and actual data, as directed in step two.

There is a reasonably intuitive correspondence between these moments and certain structural parameters. The inventory-sales ratio has already been discussed. The variance of inventory, $(M.2)$, is likely to be particularly informative with regard to the choice of λ, as the latter determines the cost of fluctuations in inventory away from αx. The autocorrelation of inventory speaks to ρ. Indeed, in the standard linear-quadratic model, it is uniquely identified by the autocorrelation of inventory. The probability of layoff, $(M.4)$, intuitively maps to the mean, μ, and variance, s^2, of $\log z$.

The final five moments are taken to be the coefficient estimates of a compressed version of the reduced-form regression introduced in Section 4:

$$\text{M.5} - \text{M.9} : \ell = 0.196\ell_{-1} + 0.22 \log I_{-1} - 0.166 \log x_{-1} - 0.014 L_{-1} + 0.1\tau + \xi.$$

This equation summarizes the dynamics of weeks of layoff in a very parsimonious way. Because it reflects on the dynamic response of ℓ to $\log I_{-1}$ and $\log x_{-1}$, it represents a valuable addition to the set of conditions $(M.1) - (M.4)$, none of which speaks to the interactions among the endogenous variables. I simplify the original regression equation (2.14) in large part because I have found that the higher-order lags add relatively little information for the purposes of identification.[62]

The coefficients of the linear regression are estimated by merging the pre and post 1990 samples. I again define a dummy variable equal to one if an observation is realized after October 1990 and zero otherwise. This dummy variable is interacted with each of the regressors identified above. This means that the effect of each of the five covariates is allowed to vary across the sample and allows one to isolate what is potentially a true JOBS Bank effect.

To be sure, it is not necessarily invalid to put forward the baseline regression estimates of Table 2 as the moments to be targeted. Even though it is doubtful that the JOBS Bank effects estimated there are causal, that need not be a problem. In principal, a wide variety of moment conditions are acceptable, provided the model may be simulated to generate them. I have chosen a different approach since the analysis of Section 4 potentially vacates the economic content of those JOBS Bank estimates. As a result, statistical identification may obtain, but it is hard to see *how* it would obtain if those estimates were taken as the moments. What would enable the structural model, in other words, to simultaneously match the JOBS Bank effects and other moments if the model is properly specified and the former are, in reality, nearly zero? The absence of any strong intuition for the mapping of moments to parameters argues against pursuing this approach.

To conclude this section, it is necessary to issue one caveat with regard to how $(M.1) - (M.9)$ are estimated on the actual panel. The caveat is that there are many factors that influence the movement of the endogenous variables in the data that have no counterpart in the model. For instance, in the data, a significant fraction of the variance of log inventory at the plant level is related to product turnover. Consider a plant that switches from the production of an unpopular large sedan to a light truck that is in high demand. The inventory level corresponding to that plant will rise abruptly to support the increase in sales. Such movements have no place in the structural model. Therefore, to place the data on the same basis as the model, it is necessary to regression-adjust the variables. Specifically, I regress $\log I$, $\log x$, and ℓ on the set of control variables introduced in Section 4, namely, a set of seasonal dummies, quarterly time dummies, vehicle fixed effects, and vehicle-specific time trends. The residuals from these preliminary regressions are taken as the regression-adjusted values of the variables and used to estimated the conditions, $(M.1) - (M.9)$.[63]

[62]The mapping between $(M.5)$ and structural parameters is admittedly less transparent than that relating $(M.1) - (M.4)$ to the parameters. It will become clearer, though, in the course of the simulation analysis performed later.

[63]The exception is $(M.4)$: I do not regression-adjust ℓ before this equation is estimated. This assumes that the mean number of weeks of layoff per month is not affected by the covariates listed in the text; the "raw" empirical mean is, in fact, the appropriate analogue to the mean generated by the structural model.

The use of regression adjustments is fairly standard in applications of indirect inference and method of simulated moments,[64] but the implementation of it here does raise a concern. These adjustments are valid if the controls, such as vehicle fixed effects, are *strictly exogenous* to the evolution of the endogenous variables. Otherwise, if they are endogenous, then their omission from the model constitutes a specification error, in which case the structural parameters cannot be consistently estimated. The sequence of product decisions would have to be simulated along with the rest of the outcomes and their implications for the endogenous variables made explicit at every point in time. Of course, strict exogeneity is unlikely to hold exactly in my sample.[65] Since a complete treatment of product choice is beyond the scope of this paper, I will proceed under the assumption that strict exogeneity is at least approximately true. An integrated treatment of product choice and labor demand remains an important topic for future work.

Solution, simulation and optimization

The model is solved by value function iteration on a grid of values for the four state variables, $(I_{-1}, L_{-1}, \tau, z)$. The grids for the JOBS-related variables come naturally: L_{-1} takes any integer between zero and $\bar{L} = 36$, whereas τ lies on a grid of integers that stretch from one to 36. The demand disturbance, z, is discretized into a 22×1 vector and the transition matrix is computed in accordance with Tauchen's (1990) suggested quadrature procedure.[66] Inventory lies on a 160×1 grid where each increment is roughly equal to 1/8 of a week's worth of output.

Since the Bellman equation of the plant defines a contraction mapping on the first-period-of-the-contract value function, the model is solved by iterating on this value function until convergence. The process is speeded along by using the solution of the no-JOBS model as the initial guess.[67]

Once the model is solved, it must be simulated. In order to reduce the sensitivity of the simulation results to the fineness of the discretization, I interpolate the ℓ-specific value functions and the inventory policy rule in order to calculate the layoff and inventory decisions at each point in time. (I found that simple bi-linear interpolation worked quite well; more time-intensive multi-dimensional splines did not improve the performance appreciably.) Fifty plants are simulated for

[64] See Cooper, Haltiwanger, and Willis (2005) and Guvenen and Smith (2009).

[65] Fortunately, it does not have to hold in order to interpret the reduced-form estimates of section 4. Consistency in that context requires only weak exogeneity (pre-determinedness) with respect to the choice of weeks of layoff per month. Only structural estimation is complicated by violations of strong exogeneity since it requires simulation of the entire time series.

[66] Adda and Cooper provided code for the arithmetic AR(1) process on the website for their 2003 textbook. It is straightforward to adapt the program to a geometric autoregressive process.

[67] The solution of the no-JOBS model is, in turn, accelerated by use of the McQueen-Porteus bounds technique. See the derivation in Bertsekas (1976) and the discussion in Rust (1994). I'm grateful that Dave Ratner pointed me to Rust's paper.

six 36-month contracts, and the first contract is discarded in order to reduce the influence of initial conditions. I repeat the simulations 25 times, which means that simulation error present in the structural parameter estimates will be on the order of $1/25 = 4$ percent (see Smith, 1993).

Lastly, I describe the search procedure used to identify the structural parameter. The presence of $(M.5)$ in particular poses a challenge to Newton-based methods of optimization because ℓ is a discretely-valued variable. As a result, perturbations in the parameters generate discontinuous movements in this series. It is now standard practice to eschew gradient-based solvers in this sort of context (see Keane and Smith (2003) and Nagypal (2007)). Instead, I use the Nelder-Mead simplex-based routine.[68]

2.5.3 Estimation Results

This section reports the results of estimation. To begin, Table 6 reports the empirical moments to be targeted and the success of the model in targeting them. The fit of the model is quite good along various dimensions, such as the inventory-to-sales ratio and the persistence of log inventory. The model also implies a response of layoffs to $\log I_{-1}$ and $\log x_{-1}$ that fits the data well. The model appears to struggle in two dimensions. First, it understates the mean level of weeks of layoff per month and the persistence of weekly layoffs. Second, it fails to capture the contract time effect. We return to the latter below. I confine remarks here to the former.

To investigate why the model misses the probability of layoff, it is useful to begin with a simple observation: the model should be able to improve on the result in Table 6 if the standard deviation, s, of the demand shock were increased. The question is, what moments conditions prevent this increase in s? It must be, in other words, that the model's performance degrades along some dimensions if a higher s is considered. It turns out that the principal restriction on a higher variance is the linear regression on which moments $(M.5) - (M.9)$ are based. To see this, I discard these moments and re-estimate the model. In this just-identified case, the model is able to precisely replicate the empirical moments, $(M.1) - (M.4)$. The most appreciable difference between the parameter estimates obtained in this case relative those reported in Table 6 lies in s, which rises by

[68] In larger problems, this algorithm has been found to get "stuck" at non-optima, but it should have little difficulty in a five-dimensional problem judging from the test problems solved in Torchen (1989). Improvements on Nelder-Mead have been developed, but I am not aware of any that are suitable to the problem analyzed in this paper. Torczon (1989), for instance, introduced a simplex-based algorithm that displays the same powerful convergence properties as line-search-augmented Newton methods. No actual derivatives are required to implement the search, but the theoretical convergence result relies on the continuous differentiability of the objective function. Kelley (1998) provides a modification to Nelder-Mead that accommodates functions which are not necessarily continuously differentiable everywhere, but the "noise" contributed by these non-differentiabilities must be bounded in a particular way that is hard to verify in practice. The application of Kelley's algorithm did not yield appreciably different results in my problem.

20 percent to 0.326. It is no surprise that such an increase pushes up the probability of a layoff. If I then estimate $(M.5)$ on the simulated data, given $s = 0.326$, the coefficients attached to ℓ_{-1} and $\log I_{-1}$ are much higher (0.33 and 0.56 , respectively) while the coefficient on $\log x_{-1}$ falls to -0.48. Each of these differ significantly from their empirical analogues.

The reason for these discrepancies helps illuminate the role of $(M.5)-(M.9)$ in the identification of the structural parameters. To begin, note that, for given (L_{-1}, τ, z), it is possible to invert the function, $Z_{\ell,\ell+1}$, and interpret the inverse, $\log I_{-1} = Z_{\ell,\ell+1}^{-1}(L_{-1}, \tau, z)$, as a threshold in inventory. When s increases, the amplitude of inventory fluctuations also rises and, as a result, inventory tends to be nearer to these bounds more often. An inspection of the data will therefore suggest that a given increase in $\log I_{-1}$ triggered an adjustment in output more frequently, which shows up in the least squares regression as a larger estimated coefficient on $\log I_{-1}$. Similarly, a given increase in $\log z_{-1}$, as proxied by $\log x_{-1}$, will result in a breach of the threshold $Z_{\ell,\ell+1}(I_{-1}, L_{-1}, \tau)$ more often and trigger adjustments in weeks worked. This is a channel through which $(M.5)-(M.9)$ informs the choice of s.

Table 7 provides the point estimates and standard errors of the structural parameters.[69] Unfortunately, there are few estimates in the literature to which to compare these. One exception is the estimate of ρ. Copeland and Hall (2008) also estimate an geometric AR(1) shock to sales. Since their model is set at a weekly frequency, I translate their estimate of $\rho = 0.937$ into a monthly figure of $0.937^4 = 0.771$. This is significantly higher than the estimate of 0.57 reported in the table. There are several differences in the moment conditions, and in the model, considered here and in Copeland and Hall, so it is difficult to trace the discrepancy in the estimated value of ρ to any single cause. One point to bear in mind, though, is that Copeland and Hall do not target the dynamics of weekly layoffs. In fact, weekly layoffs do not appear independently anywhere in their moment conditions. The simulations below make plain that, as ρ increases, the fit of the model with regard to $(M.5)-(M.9)$ in particular unravels.

The implication of these parameter estimates for the optimal layoff policy function are revealed in Figure 14, which traces out the effect of accumulated layoffs. For this exercise, I evaluate the threshold function at I_{-1} equal to one standard deviation above the mean and τ equal to one-half of the length of the contract. For each of the four thresholds, I calculate the probability that z

[69]The standard errors are calculated as the square root of the diagonal of,

$$\nabla\Gamma(\chi)\,\hat{\Omega}^{-1}\nabla\Gamma(\chi)',$$

where $\nabla\Gamma(\chi)$ is the 5×9 Jacobian of the moment vector and $\hat{\Omega}$ is the 9×9 bootstrapped empirical variance-covariance matrix of the moments. The latter is computed as follows. I draw 50 plants with replacement from the panel data set and estimate the coefficients in Γ. I repeat this 100 times and compute the covariance matrix of these coefficient estimates. This procedure follows Cooper, Haltiwanger, and Willis (2005) and Bloom (2009).

falls below it. For instance, the threshold Z_{01} is the value of z such that the plant operates for the full month if $z > Z_{01}$. It follows that $\Pr(z \leq Z_{01}) = \Phi[(Z_{01} - \mathrm{E}[\log z])/s]$ is the probability that the plant does at least one week of layoff per month. Similarly, the probability that the plant does at least two weeks of layoff is given by $\Phi[(Z_{12} - \mathrm{E}[\log z])/s]$, where Z_{12} is the threshold. It is these probabilities that the figure traces out for different values of L_{-1}. The plot reveals that the estimated effect of accumulated layoffs is essentially zero until a plant reaches 3/4 of the allotment. This seems consistent with the simulation results reported in Section 3.1. The contract time effect is suppressed simply because the model implies virtually zero effect. I will return to the contract time effect in the simulation analysis below.

2.5.4 Counterfactual Analysis

The model appears unable to rationalize any JOBS Bank effect. This naturally raises the question, under what parameter configurations would JOBS exert a strong influence on the weekly layoff decision? This section focuses in particular on ρ and s , which parallels the analysis of Section 3.1.[70] I perturb each parameter and consider its effect through two lenses. The first is the linear regression and its associated moments, $(M.5) - (M.9)$. I identify values of ρ and s that, within the context of this linear model, imply a significant response of weekly layoffs to movements L_{-1} and τ. When I measure the JOBS Bank effect in this context, moreover, it is possible to highlight how variation in these parameters increases the estimated effect of the JOBS program at the expense of the overall goodness of fit. This, in turn, sheds light on the identification of the structural parameters. Second, I directly inspect the nonlinear optimal policy function and contrast it with the policy displayed in Figure 14.

The top panel of Table 8 summarizes the effects on the model-generated moments induced by changes in ρ. The estimated effect of the JOBS Bank variables rises noticeably as ρ approaches 0.9. The estimated coefficient on L_{-1} falls to -0.26 and the coefficient on τ rises to 0.09, which are in the vicinity of the baseline estimates. To be sure that this is due to the JOBS Bank, and does not reflect the specification error in the linear model, the bottom row shows the analogous results for the model without JOBS. It is evident that movements in ρ contribute to insignificant changes in the estimated JOBS effects when these effects are not in fact there, which is reassuring.

[70] I have also considered the effect of perturbations to λ and α. Variations in α degrade the model's ability to fit the average inventory-to-sales ratio but have almost no impact on the strength of the JOBS Bank effect. Increases in λ constrain the ability of the plant to use inventory as a buffer for variations in demand. The increased reliance on layoffs raises the importance of the JOBS Bank, but the quantitative impact is slight: the model still fails to generate a significant JOBS effect. Moreover, this comes at a price: the ability of the model to replicate the variance of inventory is damaged substantially.

The results appear to be consistent with the discussion of Section 3.1 and likely driven by the same mechanism identified there: if a plant receives an adverse shock and is exposed to a very persistent demand process, it is cautious about its use of weekly layoffs since any future adverse shocks will occur when z is likely still at a low level.

An increase in the persistence of the process also affects a number of other moments. For instance, the responsiveness of weekly layoffs to fluctuations in inventory and sales rises. This is due, at least in part, to a point stressed by Ramey and Vine (2004, 2006), who argued that a plant will pay the discrete cost to raise weeks worked more often if the positive shock to sales is persistent. Otherwise, the plant will absorb the innovation by allowing the time-varying inventory target, αx, to deviate from I_{-1}, the cost of which will be relatively small (by virtue of the quadratic functional form of the inventory storage cost) for modest changes in demand.

The changes in these regression coefficients lead them to diverge substantially from their empirical analogues, particularly for ρ near 0.9. This suggests that it would be very difficult to rationalize the baseline regression results in the context of the structural model: further increases in ρ would deliver JOBS Bank effects found in Table 2, but would also contribute to a further deterioration of the fit along these dimensions.[71]

I now turn my attention to s. The effect of variations in this parameter on the model-simulated moments is reported in the bottom panel of Table 8. Again, the analogous results for the no-JOBS model are shown in parentheses. The estimated JOBS Bank effect is seen to increase, although the change is much more modest than that induced by increases in ρ. This indicates that, for a given degree of persistence, it would take a very substantial amount of volatility to produce a strong JOBS effect. Consider that when s reaches 0.43, the variance of log inventory is more than double the empirical estimate, and yet the estimated effect of accumulated layoffs is just -0.04 and the estimated coefficient attached to contract time is just 0.02. In short, a s of 0.43 remains small relative to the length of the contract: even large innovations dissipate too quickly to affect the management of weekly layoffs in a contract of 36 months.

The effect of an increase in the innovation on the other moments should not be too surprising.

[71] The effect of ρ on the volatility and persistence of inventory appears to actually be non-monotonic. This remains somewhat of a puzzle, and the interpretation offered here is tentative.

My starting point is the inventory accounting identity, $I = Y + I_{-1} - x$, from which it follows that $var\,(I) = var\,(Y) + var\,(I_{-1} - x) + 2cov\,(Y, I_{-1} - x)$. The effect of increasing ρ is determined by a "horserace" among these three terms. Layoffs become more responsive to changes in the state as ρ increases, so the first term increases. By the same token, though, that implies inventory, spurred by large adjustments in output, generally "keeps up" with movements in sales. As a result, the variance of the second second term declines, at least over a certain range of ρ. The third term, the covariance, is negative, as increases in output accompany increases in sales (and reductions in $I_{-1} - x$). The absolute value of this covariance also increases over a range of values of ρ, as the variance of Y rises relative to x.

The discussion above argued that an increase in the volatility of demand naturally shows up as an increase in the estimated co-movement between ℓ and $\log I_{-1}$ and $\log x_{-1}$, as captured effectively by the linear regression. The impact on the average inventory-to-sales ratio likely reflects the incentive for a plant to hold a larger buffer stock of inventory in anticipation of larger impulses to vehicle demand.

To conclude this section, I compare the layoff policy function calculated under some of the alternative parameterizations with the baseline estimate shown in Figure 14. To begin, I consider the effect of the accumulated layoffs. In particular, I focus here on the effect of L_{-1} on the probability that the plant performs at least one week of layoff.[72] I again fix I_{-1} at one standard deviation above its mean and set $\tau = 0.5$. The top panel of Figure 15 shows the effect of L_{-1} for three different pairs of (ρ, s): one pair consists of the point estimates reported in Table 7; another leaves s unchanged but raises ρ to 0.9; and the other leaves ρ at its estimated value of 0.57 but raises s to 0.43. Since these pairs of estimates induce very different steady state layoff probabilities (even in the absence of JOBS), I normalize the probability of layoff when $L_{-1} = 0$ under each parameterization to one and trace out the effect of increases in L_{-1}. This presentation allows one to more sharply compare the JOBS Bank effect specifically across the different parameterizations.

The thresholds react strongly to the alternative parameterizations. Recall that the estimates in Table 7 implied almost no effect of accumulated layoffs until L_{-1} reached 3/4 of the allotment. By that point, the probability of a week of layoff under $\rho = 0.9$ has already declined by 20 percent due to the accumulation of past layoffs. The effect of a higher s is actually rather similar, although the linear regression estimates (see Table 8) did not hint at such a strong effect. The reason is that, with $\rho = 0.9$, there are more plants that, in the wake of an adverse shock, faced subdued demand for an extended period. These plants actually "bump" up against the allotment, and this provides for sharper identification of the JOBS Bank effect in the context of the linear regression.

The bottom panel of the figure shows the contract time effect. For this exercise, I set I_{-1} and L_{-1} to one standard deviation above their respective means. As mentioned above, the contract time effect is literally estimated to be zero, so for the baseline ($\rho = 0.57, s = 0.268$), the threshold is a horizontal line. When $\rho = 0.9$, the probability of layoff rises fairly smoothly for most of the agreement, with the effect of contract time reaching 10 percent by about the 21st month or so (or 60 percent of the way through the contract). The effect of greater volatility is qualitatively somewhat similar but appreciably smaller.

[72] Inclusion of the other thresholds will clutter the diagram and not necessarily reveal that much more information.

2.6 Conclusions

This paper has investigated a unique and observable employment friction that prevailed within the unionized U.S. motor vehicle industry for most of the last two decades. It argued that the JOBS program's effects on labor demand, or weeks of layoff in this case, could be estimated as the change in the layoff decision due to two measurable series: "contract time", or the share of the current labor contract completed, and the share of the allotment of weeks on layoff used up.

The paper introduced a framework in which consider the effect of these JOBS-related variables. The model indicated that contract time causes an increase in the probability of layoff because the passage of time, and the resolution of uncertainty over future vehicle demand, degrades the option value of (layoff) deferment. The model also predicted that past layoffs reduce the probability of downtime because, as layoffs accumulate, the plant increasingly values the option to deploy its remaining allotted layoffs during future periods when demand may be lower than now.

Reduced-form regression analysis initially appeared to corroborate the model's qualitative implications. However, a series of robustness tests diminished and then largely overturned this tentative conclusion. The most severe result was the falsification test of Section 4.5, whereby the same reduced-form regression was run on data from the pre-JOBS era. If the original estimate reflected the causal effect of the JOBS program, the significance of the JOBS-related variables should vanish when the regression model is taken to pre-1990 data. They did not. The estimated of accumulated weeks of layoff was virtually identical across the periods, which cast doubt on a causal interpretation of the baseline result. Rather, it was argued that L_{-1} likely functioned as a proxy for various costs of adjusting weekly employment that are unrelated to the JOBS program but which deter consecutive weekly downtime. The contract time effect, in contrast, was statistically significantly larger in the post 1990 sample than in the pre-treatment period. The presence of a contract time effect, in the absence of any impact of accumulated layoffs, presents a puzzle that has not been fully resolved and awaits additional study.

The paper followed the reduced-form study with a more structural analysis. It asked two questions. First, is the reduced-form result consistent with (in the sense that it may be induced by) the structural model of Section 3? If so, this strengthens the interpretation of the false experiment as suggesting that the true JOBS effect on weekly layoffs was nearly zero. To pursue this, the structural model was estimated by indirect inference. The paper uncovered estimates that induced moments similar to those observed in the actual data, including the regression moments taken from the reduced-form regression.

In light of this finding, the paper conducts a series of counterfactual simulations to explore the set of parameter configurations that would generate a strong JOBS effect. In particular, it highlights the role of the parameters governing the vehicle demand process. It concludes that, in the data, the innovations to household demand appear to be too small and too transient to generate the strong option-value effects associated with a robust JOBS program.

I conclude with a discussion of how one might still resurrect an employment effect of the JOBS Bank. The principal point of departure from the framework above is the specification of uncertainty. Suppose the most important source of uncertainty that confronts a plant is not due to *stationary* innovations in demand. Rather, the dominant source of uncertainty relates to the "drift" in the mean level of demand for particular vehicles. The vehicle fixed effect of Section 4, in other words, is not in fact fixed. But once the new level is revealed, and that uncertainty resolved, there is relatively little else that matters: the size of the subsequent stationary disturbances to demand may be trivial by comparison. As a result, if the arrival rate of new vehicle models is sufficiently low, the plant faces a simple choice once its "fixed effect" is revealed: run the factory or not. Under JOBS, more plants are left open. But the benefit to the plant of optimally managing weekly layoffs over the contract is small since the die, in a sense, has been cast. In other words, the marginal losses due to less-than-optimal management of temporary layoffs pale in comparison to the expected losses that the plant accepted by its decision to keep the plant open at all. Under this scenario, variation in L_{-1} and τ across plants do not help identify the JOBS Bank effect.

This thought experiment suggests that perhaps a more promising source of identification may lie in vehicle prices. Since the plant must sell vehicles in order stay open, one suspects that it must reduce prices more drastically in response to adverse movements in household "taste" for their product than in the pre-JOBS era. Because these innovations are likely due to perceived changes in product quality, it may be possible to identify them via publicly available measures of vehicle quality compiled by industry analysts, such as JD Power.

To take a structural approach to this would require a richer model of household demand than that presented here in order to evaluate the JOBS effects. The payoff to this is that an exploration into the effect of the JOBS Bank on prices has the potential to further the general understanding of the relation between real frictions and nominal prices.

Table 2.1: The Effect of the JOBS Bank in a Model Without Inventory

		Weekly Layoff Probability		Effect of L_{-1} on Prob(Layoff)[a]		Effect of τ on Prob(Layoff)[a]	
ρ	*s*	W/out JOBS	With JOBS	Low L_{-1} plant	High L_{-1} plant	Start of contract	End of contract
0.6	0.268	0.0407 [0.0023]	0.04065 [0.0023]	-0.0115	-0.052	0.0055	0.0024
0.6	0.326	0.0849 [0.0037]	0.083 [0.0035]	-0.672	-2.043	0.244	0.157
0.8	0.268	0.04065 [0.0038]	0.0383 [0.0034]	-0.17	-0.391	0.051	0.071
0.8	0.326	0.0846 [0.0061]	0.073 [0.0048]	-1.194	-3.122	0.293	0.429
0.9	0.268	0.04065 [0.0053]	0.033 [0.0039]	-0.296	-0.636	0.087	0.206
0.9	0.326	0.0848 [0.0082]	0.063 [0.0055]	-1.319	-3.512	0.311	0.591

a Expressed in percentage points.

NOTE: This presents the simulations results for the model (section 2.3.1) without inventory. The first two columns report the calibration. The first of these gives the persistence, ρ, of the demand innovation. (As in the main text, ρ is quoted on a monthly basis.) The second of these gives the standard deviation of the log of demand. The next two columns report the weekly layoff probability with and without JOBS. Standard errors of the probabilities are calculated based on 100 simulations and reported in brackets. The next two columns report the effect of a change in L_{-1}, the number of weeks on layoff since the start of the agreement, on the probability of layoff, given that the plant is midway through the contract. The first of these computes the change in the weekly layoff probability when accumulated layoffs, L_{-1}, increases from its mean to one standard deviation above its mean. The second column computes the change in the probability due to an increase in L_{-1} from one standard deviation above the mean to two standard deviations above mean. The final two columns give the effect of a change in contract time (the period of the contract) on the probability of layoff, given that L_{-1} is at its mean. The first column considers a change in contract time from 9 months to 18 months. The second considers a change from 18 to 27 months.

Table 2.2: Baseline Reduced-Form Estimates

Independent Variable	No model trends	With model-specific trends
ℓ_{-1}	0.207*** [0.023]	0.174*** [0.0217]
ℓ_{-2}	0.115*** [0.019]	0.093*** [0.019]
ℓ_{-3}	0.0469** [0.0215]	0.0316 [0.0217]
ℓ_{-4}	-0.007 [0.018]	-0.021 [0.018]
log I_{-1}	0.078* [0.0475]	0.12** [0.051]
log I_{-2}	0.048 [0.067]	0.041 [0.069]
log I_{-3}	0.077 [0.073]	0.076 [0.073]
log I_{-4}	-0.033 [0.052]	0.017 [0.05]
log x_{-1}	-0.18*** [0.036]	-0.16*** [0.034]
log x_{-2}	-0.019 [.038]	-.018 [0.039]
log x_{-3}	0.076** [0.0365]	-.081** [0.035]
log x_{-4}	0.0418* [0.023]	0.042 [0.021]
L_{-1}	-0.063 [0.07]	-0.394*** [0.119]
τ	0.083*** [0.0326]	0.161*** [0.035]
No. of obs.	8404	8404
R-squared	0.3164	0.3515

* Significant at 10%, ** at 5%, *** at 1%
Dependent variable is weeks of layoff per month at each plant.
Robust (clustered at plant-level) standard errors are reported in brackets.

Table 2.3: Baseline Reduced Form With Alternative Robust Standard Errors

Independent Variable	Coefficient	Standard Error		
		Cluster by plant	Driscoll-Kraay	Thompson
ℓ_{-1}	0.174	0.0217	0.019	0.021
ℓ_{-2}	0.093	0.019	0.027	0.023
ℓ_{-3}	0.0316	0.0217	0.023	0.028
ℓ_{-4}	-0.021	0.018	0.021	0.021
$\log I_{-1}$	0.12	0.051	0.059	0.059
$\log I_{-2}$	0.041	0.069	0.083	0.067
$\log I_{-3}$	0.076	0.073	0.089	0.077
$\log I_{-4}$	0.017	0.05	0.057	0.053
$\log x_{-1}$	-0.16	0.034	0.039	0.038
$\log x_{-2}$	-.018	0.039	0.042	0.041
$\log x_{-3}$	-.081	0.035	0.041	0.034
$\log x_{-4}$	0.042	0.021	0.025	0.020
L_{-1}	-0.394	0.119	0.130	0.159
τ	0.161	0.035	0.061	0.063

NOTE: Plant-clustered standard errors allow for serial correlation of the residuals within plant but assume independence across plants. Driscoll-Kraay and Thompson standard errors correct for correlation across time and across plants.

Table 2.4: Reduced-Form Estimates With Nonlinear JOBS Variables

Independent Variable	Baseline	With second-order JOBS variables	
L_{-1}	-0.394***	-0.057	0.326
	[0.119]	[0.277]	[0.405]
L_{-1}^2		-0.46	-0.34
		[0.34]	[0.37]
τ	0.161***	1.07**	0.995**
	[0.035]	[0.43]	[0.414]
τ^2		-0.907**	-0.809**
		[0.41]	[0.39]
$L_{-1}\tau$			-0.56
			[0.42]
p-value of F-statistic: $\tau = \tau^2 = 0$		0.0009	0.0009
p-value of F-statistic: $L_{-1} = L_{-1}^2 = 0$		0.0062	0.5532

* Significant at 10%, ** at 5%, *** at 1%
Dependent variable is weeks of layoff per month at each plant.
Robust (clustered at plant-level) standard errors are reported in brackets.

Table 2.5: Reduced-Form Estimates For Pre- and Post-1990 Samples

Independent Variable	Oct. 1990-Aug. 2007 Sample	Oct. 1973-Sept. 1990 Sample	Merged Sample
ℓ_{-1}	0.174*** [0.0217]	0.234*** [0.023]	0.2*** [0.0154]
ℓ_{-2}	0.093*** [0.019]	0.06*** [0.023]	0.078*** [0.0148]
ℓ_{-3}	0.0316 [0.0217]	0.038 [0.0253]	0.0352** [0.0166]
ℓ_{-4}	-0.021 [0.018]	-0.036 [0.015]	-0.029** [0.012]
log I_{-1}	0.12** [0.051]	0.124*** [0.026]	0.11*** [0.026]
log I_{-2}	0.041 [0.069]	0.194*** [0.039]	0.173*** [0.0367]
log I_{-3}	0.076 [0.073]	0.0168 [0.03]	0.0285 [0.0296]
log I_{-4}	0.017 [0.05]	-0.036 [0.029]	-0.028 [0.027]
log x_{-1}	-0.16*** [0.034]	-0.213*** [0.028]	-0.197*** [0.023]
log x_{-2}	-.018 [0.039]	-0.052 [0.034]	-0.032 [0.025]
log x_{-3}	-.081** [0.035]	-0.016 [0.031]	-0.041* [0.025]
log x_{-4}	0.042 [0.021]	0.059 [0.027]	0.056*** [0.019]
L_{-1}	-0.394*** [0.119]	-0.365** [0.147]	-0.374** [0.145]
τ	0.161*** [0.035]	0.06* [0.034]	0.05 [0.032]
L_{-1} * 1[time > Oct. 1990]			-0.017 [0.215]
τ * 1[time > Oct. 1990]			0.124*** [0.045]
No. of obs.	8404	8192	16596
R-squared	0.3515	0.3576	0.3548

* Significant at 10%, ** at 5%, *** at 1%
Dependent variable is weeks of layoff per month at each plant.
Robust (clustered at plant-level) standard errors are reported in brackets.

Table 2.6: Empirical and Model-generated Moments

MOMENTS	DATA	MODEL
1st- and 2nd-order moments		
M.1 : Average inventory-sales ratio	2.9	2.87
	[0.011]	[0.018]
M.2 : Variance of log inventory	0.042	0.0483
	[0.0012]	[0.0027]
M.3 : Autcorrelation of log inventory	0.87	0.9
	[0.005]	[0.0038]
M.4 : Average number of weeks of layoff / mo	0.172	0.076
	[0.006]	[0.0059]
Regression moments		
M.5 : ℓ_{-1}	0.196	0.114
	[0.022]	[0.02]
M.6 : log(Inventory$_{-1}$)	0.22	0.236
	[0.03]	[0.018]
M.7 : log(Sales$_{-1}$)	-0.166	-0.169
	[0.028]	[0.022]
M.8 : L_{-1}	-0.014	0.08
	[0.225]	[0.071]
M.9 : τ	0.1	-0.004
	[0.045]	[0.016]

NOTE: This presents the empirical moments and their model counterparts. The standard errors of (M.1)-(M.4), presented in brackets, are obtained from least squares regression on the post-Oct. 1990 sample. For instance, log inventory is regressed on its own lag and a set of seasonal and time dummies, vehicle fixed effects, and vehicle-specific time trends. The coefficient on lagged inventory is taken to be the moment and the standard error of the least squares estimate appears in brackets. Moments (M.5)-(M.9) are obtained by linear regression on the merged sample. The dependent variable associated with each moment is listed in the table.

Table 2.7: Structural Parameters

Calibration		
Parameter	Description	Value
φ	Price elasticity of demand	3.5
$\overline{W}$	Total cost of operation	1
C	Interm. input share in total cost	0.62
W	Labor share in total cost	0.38
B	non-JOBS cost of layoff	0.152
A	Output per worker / week	1.545
N	Size of workforce	2540
L	Layoff allotment under JOBS	36 weeks
T	Length of contract	36 months
β	Discount factor	0.9938

Estimation		
Parameter	Description	Value [Std. Error]
α	Inventory/sales target	2.782 [0.085]
λ	Cost of deviation of inventory from target	2.33 e-06 [4.08 e-07]
μ	Mean of demand	14.32 [0.204]
ρ	Persistence of demand	0.573 [0.064]
s	Std. dev. of log(z)	0.268 [0.018]

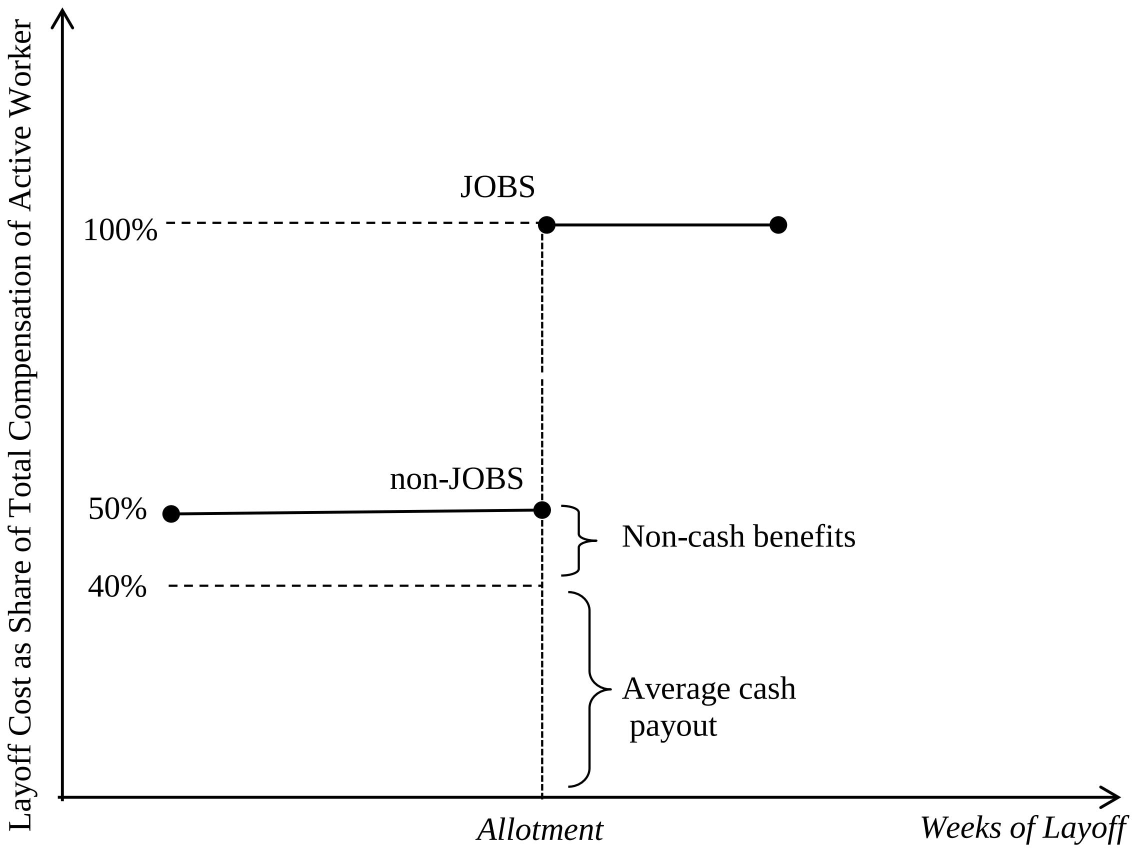
NOTE: The top panel presents the calibration of those parameters which were not estimated. The bottom panel presents the indirect-inference estimates. Standard errors are given in brackets.

Table 2.8: Counterfactual Simulations – Strengthening the JOBS Effect

	Data	ρ=0.70	ρ=0.80	ρ=0.90
Avg. inventory-sales ratio	2.904	2.8250 *2.8251*	2.7784 *2.7807*	2.7130 *2.7194*
Variance of log inventory	0.042	0.0393 *0.0393*	0.0330 *0.0339*	0.0430 *0.0707*
Autocorr. of log inventory	0.871	0.9009 *0.9009*	0.8763 *0.8755*	0.8847 *0.9088*
Avg. wks. on layoff / mo	0.172	0.1865 *0.1867*	0.3101 *0.3229*	0.3713 *0.4667*
ℓ_{-1}	0.196	0.1748 *0.1744*	0.2794 *0.2803*	0.3843 *0.3855*
log(Inventory$_{-1}$)	0.220	0.4478 *0.4478*	0.7081 *0.7060*	1.2360 *1.2633*
log(Sales$_{-1}$)	-0.166	-0.5061 *-0.5076*	-0.9597 *-0.9825*	-1.8429 *-1.9121*
L_{-1}	-0.014	0.0414 *0.0465*	-0.0938 *-0.0090*	-0.2622 *-0.0448*
τ	0.100	-0.0016 *-0.0028*	0.0341 *0.0094*	0.0912 *0.0225*

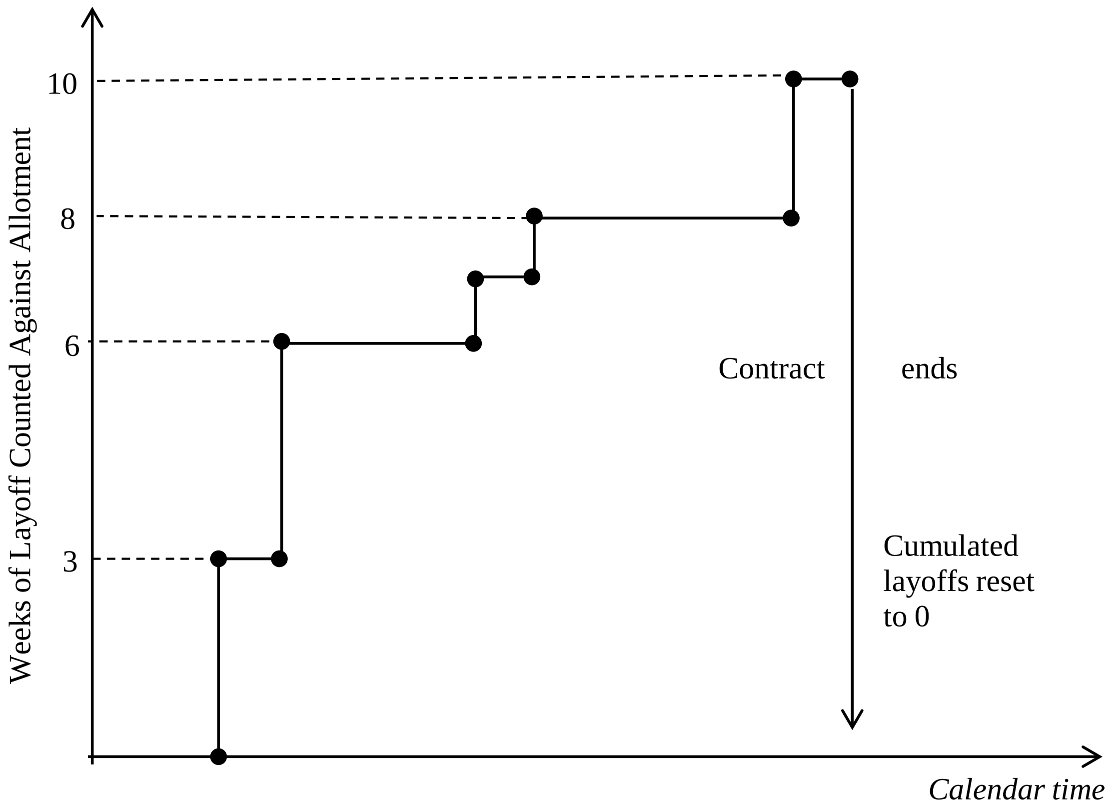
	Data	*s* = 0.32	*s* = 0.37	*s* = 0.43
Avg. inventory-sales ratio	2.904	2.9798 *2.9798*	3.1302 *3.1308*	3.3334 *3.3384*
Variance of log inventory	0.042	0.0698 *0.0698*	0.0906 *0.0905*	0.1085 *0.1080*
Autocorr. of log inventory	0.871	0.8880 *0.8880*	0.8758 *0.8757*	0.8646 *0.8640*
Avg. wks. on layoff / mo	0.172	0.1678 *0.1678*	0.2675 *0.2688*	0.3630 *0.3699*
ℓ_{-1}	0.196	0.1301 *0.1301*	0.1626 *0.1626*	0.1767 *0.1784*
log(Inventory$_{-1}$)	0.220	0.3338 *0.3338*	0.3973 *0.3972*	0.4464 *0.4478*
log(Sales$_{-1}$)	-0.166	-0.2778 *-0.2778*	-0.3266 *-0.3283*	-0.3547 *-0.3595*
L_{-1}	-0.014	0.0537 *0.0544*	0.0102 *0.0304*	-0.0414 *0.0270*
τ	0.100	-0.0033 *-0.0034*	0.0043 *-0.0011*	0.0212 *-0.0027*

NOTE: The top panel presents simulation results for various ρs. All other parameters are set equal to their point estimates (see Table 2.7). For each moment, the value induced by the model with JOBS is shown in the top row, and the value generated by the no-JOBS model is shown in italics below. The bottom panel presents results for various values of the standard deviation of log(z), *s*. Again, all other parameters are held fixed. Observe that the notation with regard to the regression moments (the final 5 moments listed in the top and bottom panels) follows that in Table 2.6.



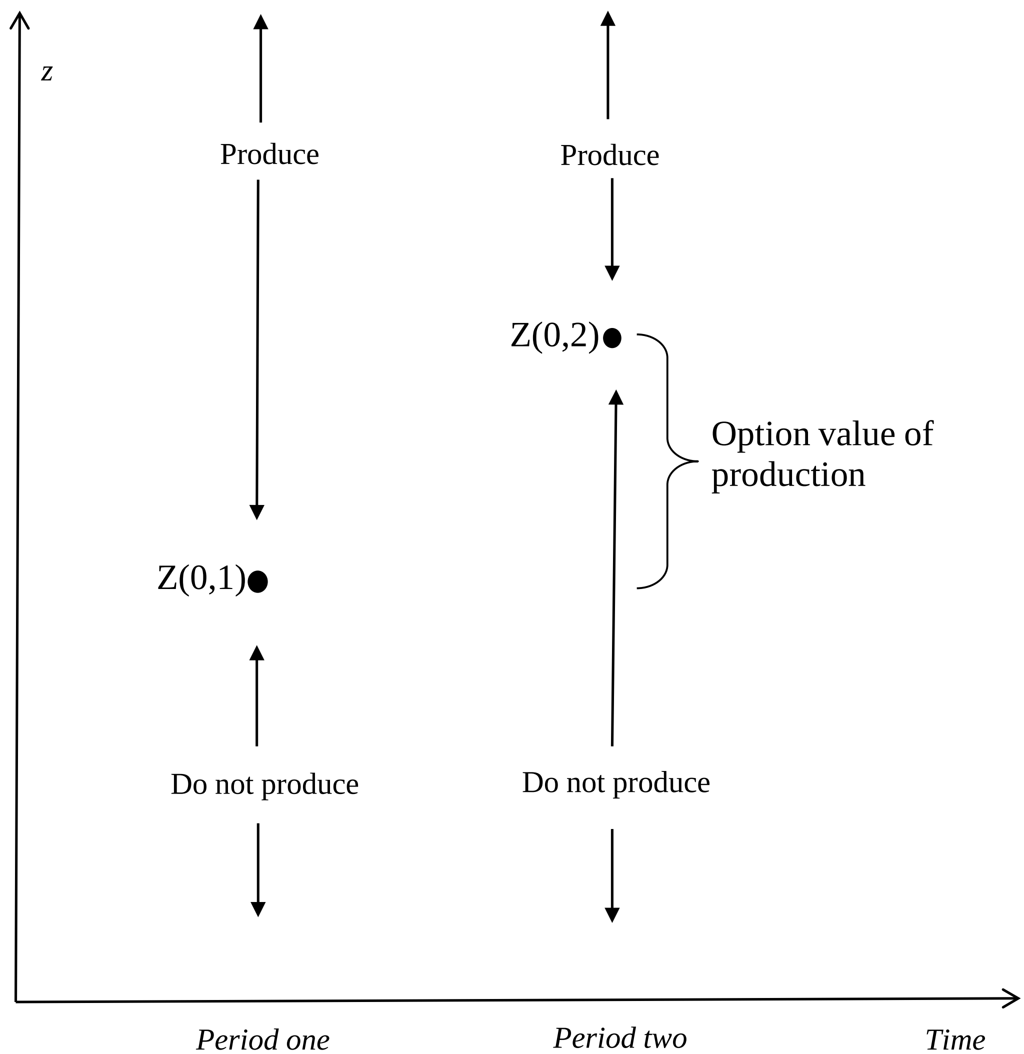
NOTE: This shows the schedule of payments that a Detroit Three firm makes to an unemployed worker. For each week until the allotment, it pays the Supplemental Unemployment Benefit, which amounts to 40 percent of the worker's (pre-tax) salary (this is the "average cash payout") plus all contractually negotiated non-cash benefits, such as health care. Once the allotment is reached, unemployment compensation rises to full salary.

Figure 2.1: The Cost of Unemployment Compensation to the Detroit Three



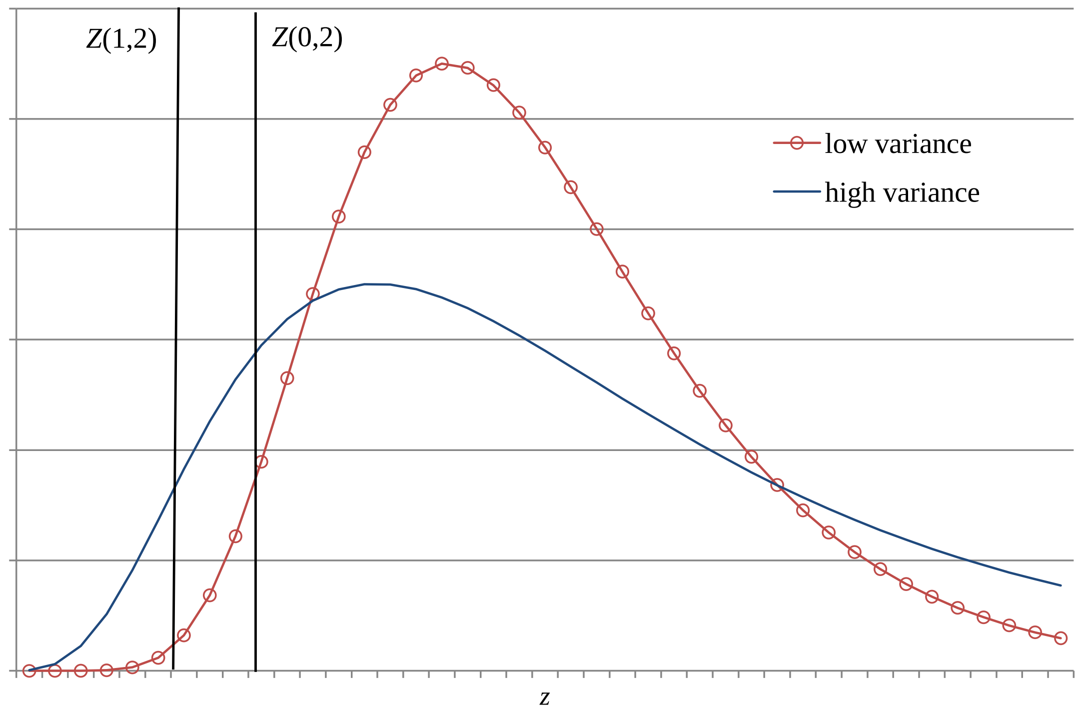
NOTE: This shows a hypothetical path of weekly layoffs for a particular worker. In this case, the worker amasses 10 weeks of layoff by the end of the contract, which means he never becomes JOBS-eligible. When the contract concludes, cumulated layoffs reset to zero, and the next contract begins.

Figure 2.2: Cumulated Layoffs Reset at End of Contract



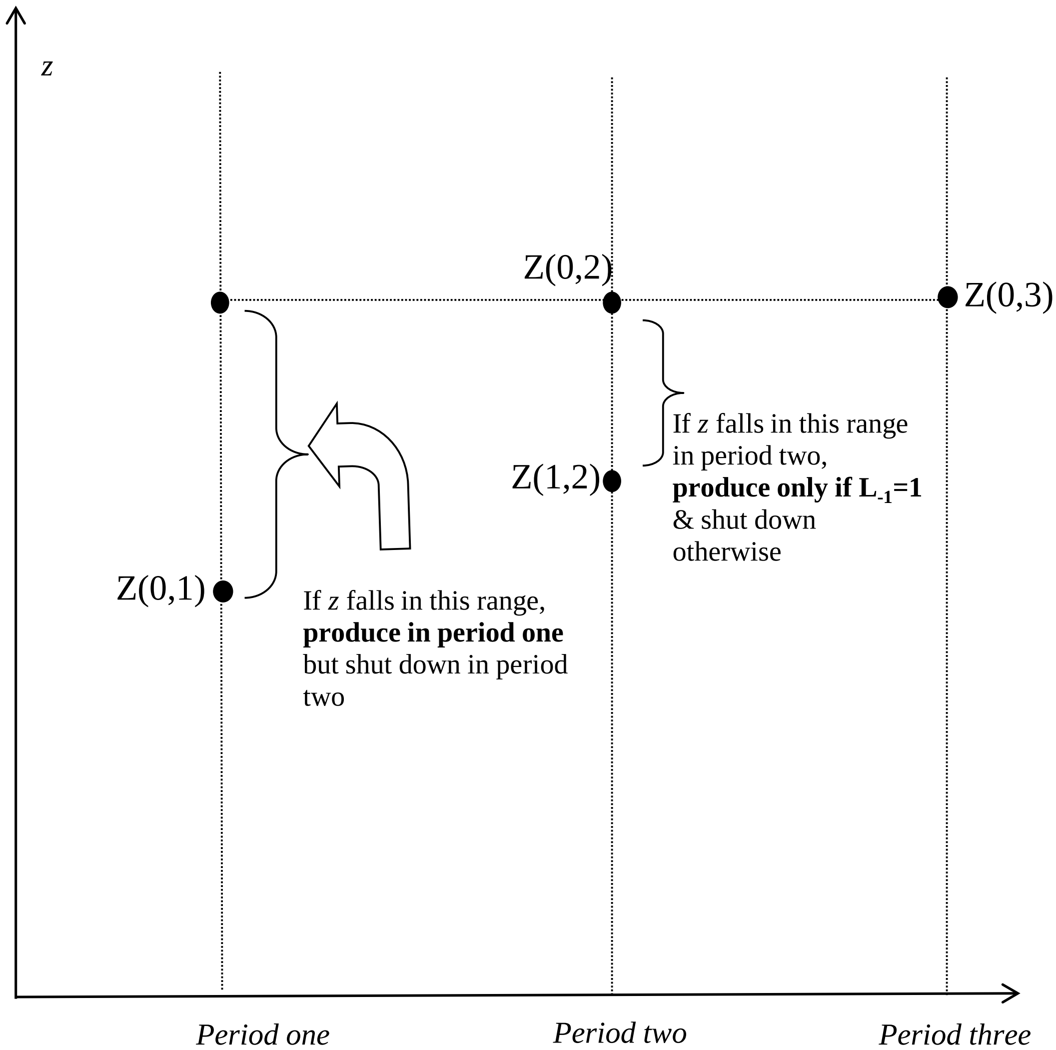
NOTE: This presents the solution to the two-period problem discussed in section 2.3.1. $\pi^Y(z)$ is profit if the plant produces, where z is the demand shock; $\pi^0(0)$ is profit if the plant shuts down with cumulated layoffs equal to zero; $Z(0,2)$ is the value of z above which the plant produces in the second period of the contract if cumulated layoffs equal zero; $Z(1,2)$ is the threshold in the second period if cumulated layoffs equal one; and $Z(0,1)$ is the value of z above which the plant produces in the first period.

Figure 2.3: The Effect of the JOBS Bank in a Two-Period Model



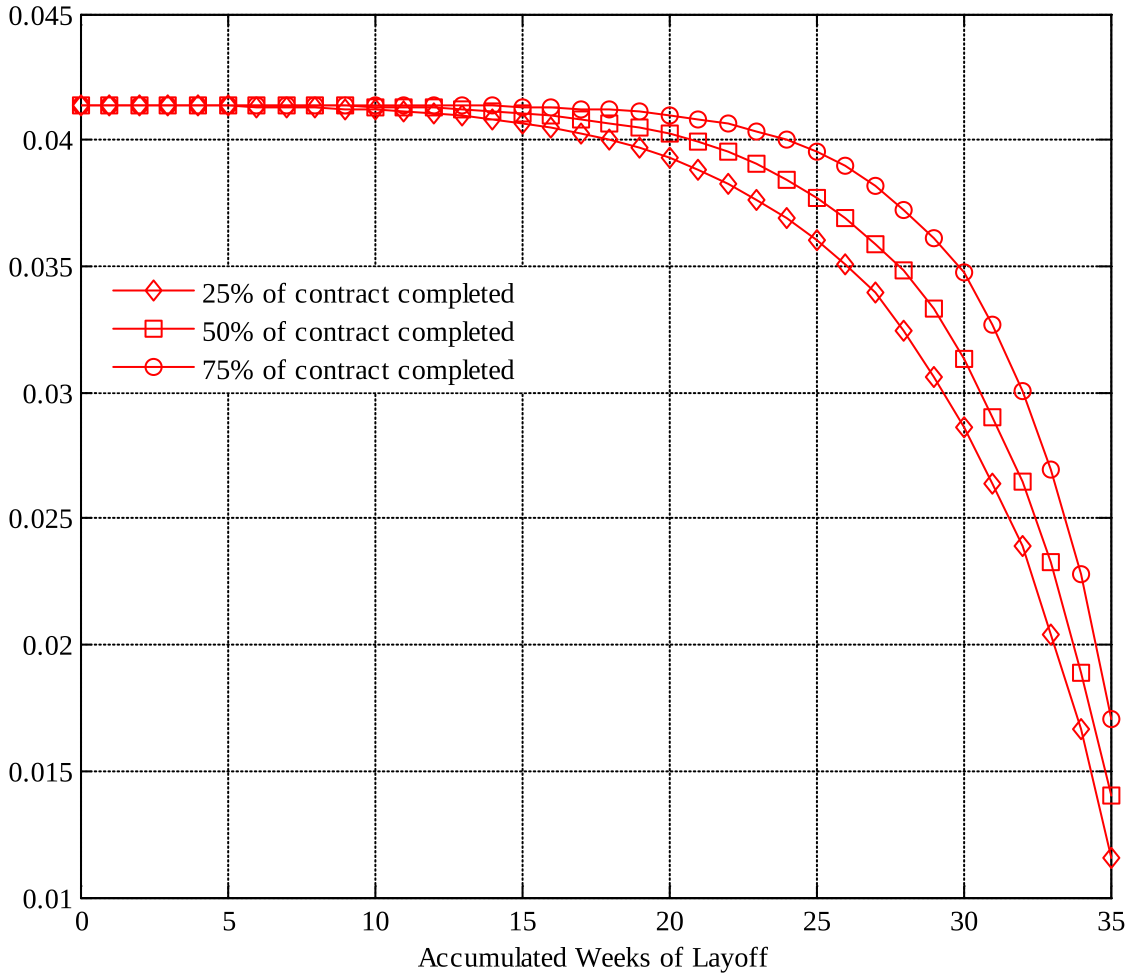
NOTE: An increase in uncertainty pushes more of the mass to the left of the thresholds. This implies a greater option value of deferment since it raises the probability that the second layoff in the allotment will be needed in the final period when demand may be particularly low.

Figure 2.4: The Effect of Uncertainty on the Option Value of Production in a Two-Period Model



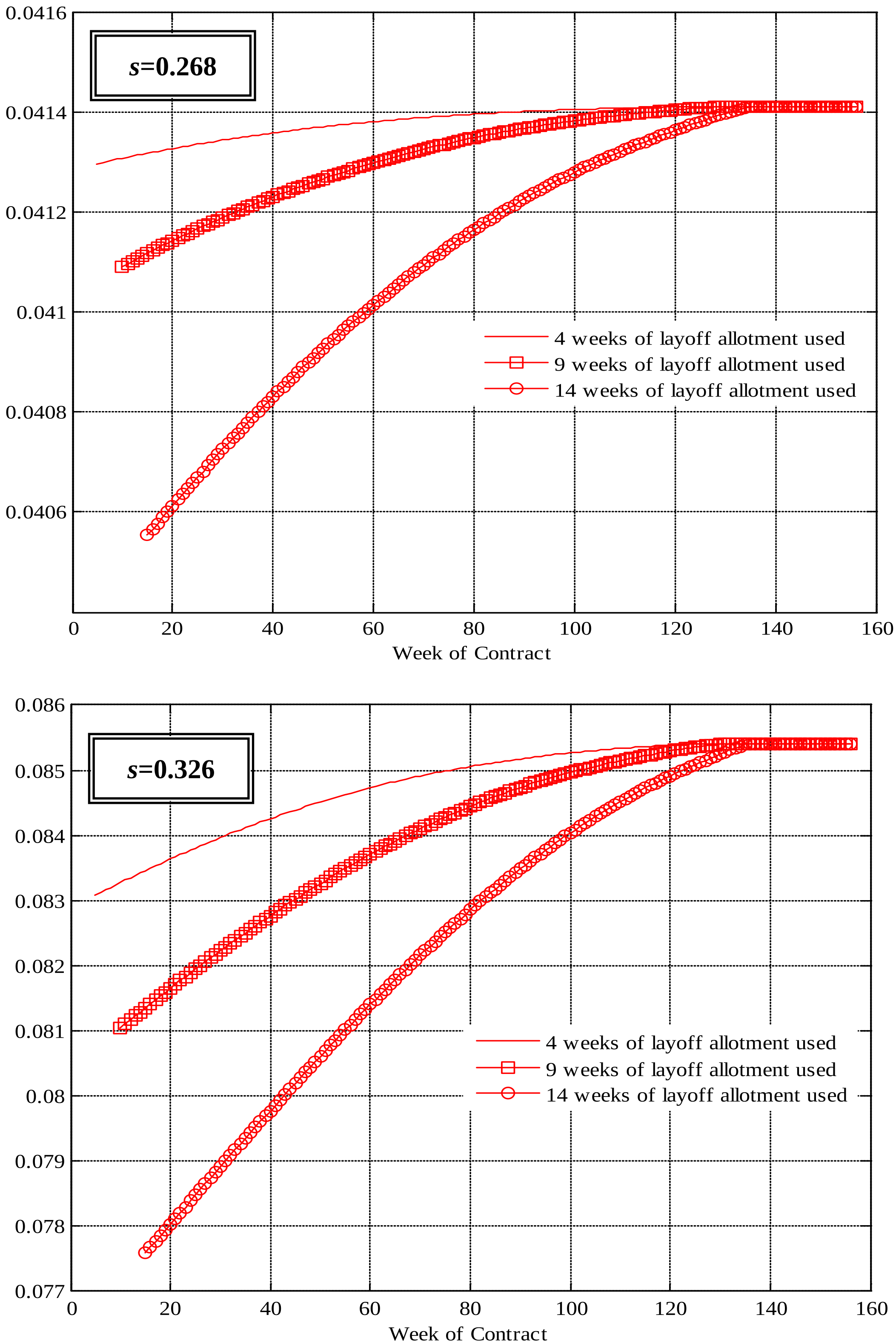
NOTE: This presents the solution to the three-period problem discussed in section 2.3.1. The notation follows Figure 2.3: $Z(L_{-1},\tau)$ represents the value of z such that the plant operates if $z > Z(L_{-1},\tau)$ and L_{-1} represents the number of accumulated weeks of layoff as of the start of the period and τ is the period of the contract. The wedge between $Z(0,1)$ and $Z(0,2)$ represents the contract time effect (with cumulated layoffs held fixed at zero). The gap between Z (0,2) and $Z(1,2)$ represents the effect of cumulated layoffs (with contract time held fixed at two). It should be noted that the model does not indicate whether Z (0,1) is in fact smaller than Z (1,2), as shown here; the order of these two may be reversed. The model, rather, makes predictions with regard to the relationship between these two thresholds and Z (0,2).

Figure 2.5: The Effect of the JOBS Bank in a Three-Period Model



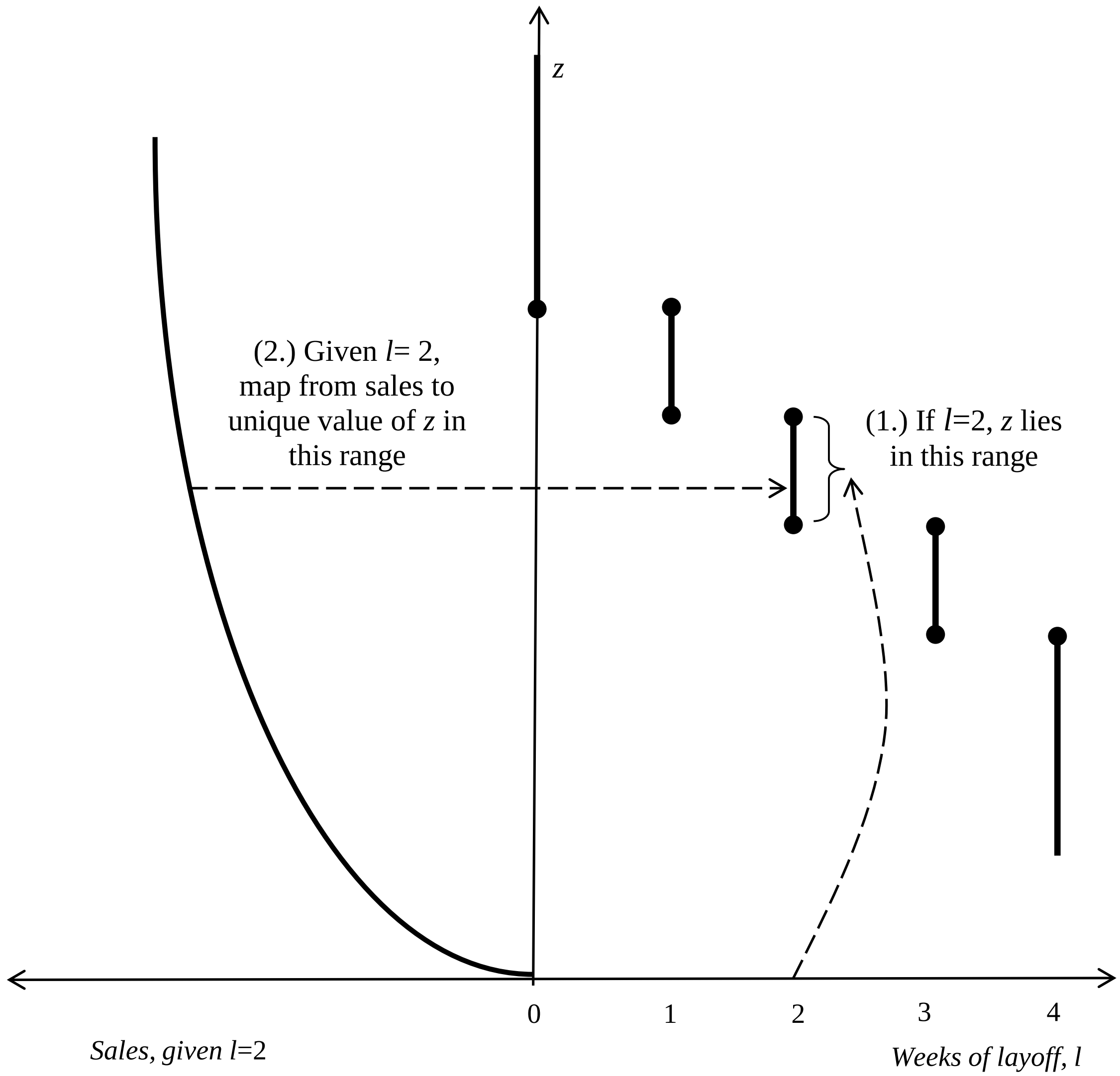
NOTE: This traces out the effect of accumulated weeks of layoff on the weekly probability of layoff in the model of section 2.3.1, which omits inventory. The length of contract is 36 months, and the layoff allotment is 36 weeks. See the text for the remainder of the calibration.

Figure 2.6: The Effect of Accumulated Layoffs on the Probability of Layoff



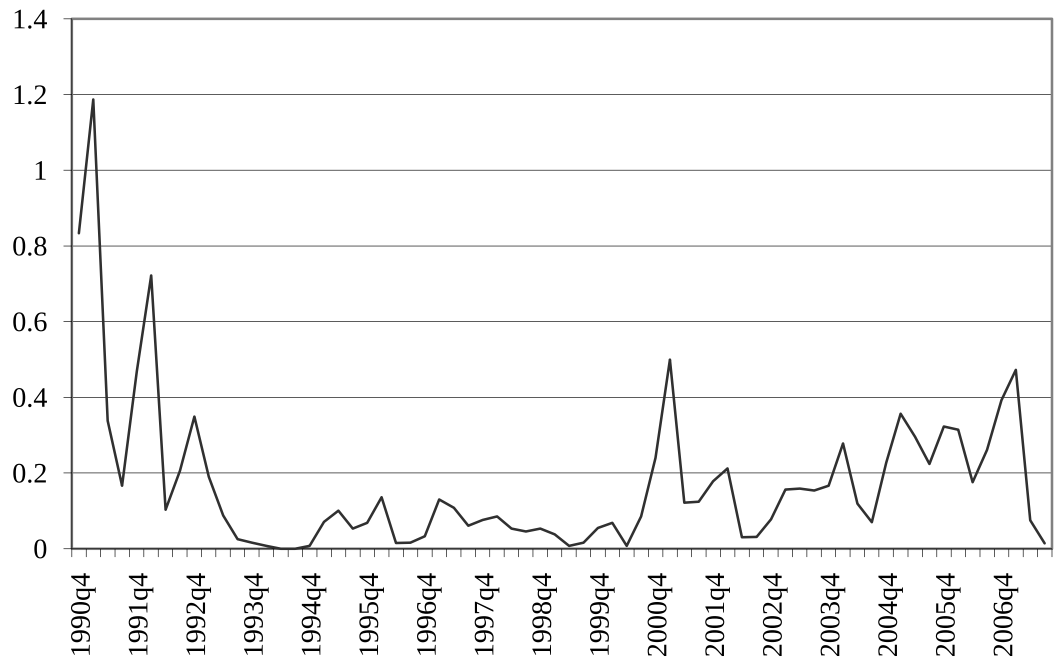
NOTE: This traces out the effect of contract time on the weekly probability of layoff in the model of section 2.3.1, which omits inventory. The length of contract is 36 months, and the layoff allotment is 36 weeks.

Figure 2.7: The Effect of Contract Time on the Probability of Layoff



NOTE: The right-hand side of the figure shows the inverse layoff policy function. The left-hand side presents the sales policy function given that *l*=2.

Figure 2.8: The Plant's Optimal Policy Functions Suggest a Proxy for z



NOTE: Weeks on layoff are summed up across U.S. assembly plants in each month and divided by the number of active plants at that time. The figure presents the quarterly average of this monthly series.

Figure 2.9: Weeks of Layoff Per Month Per U.S. Assembly Plant

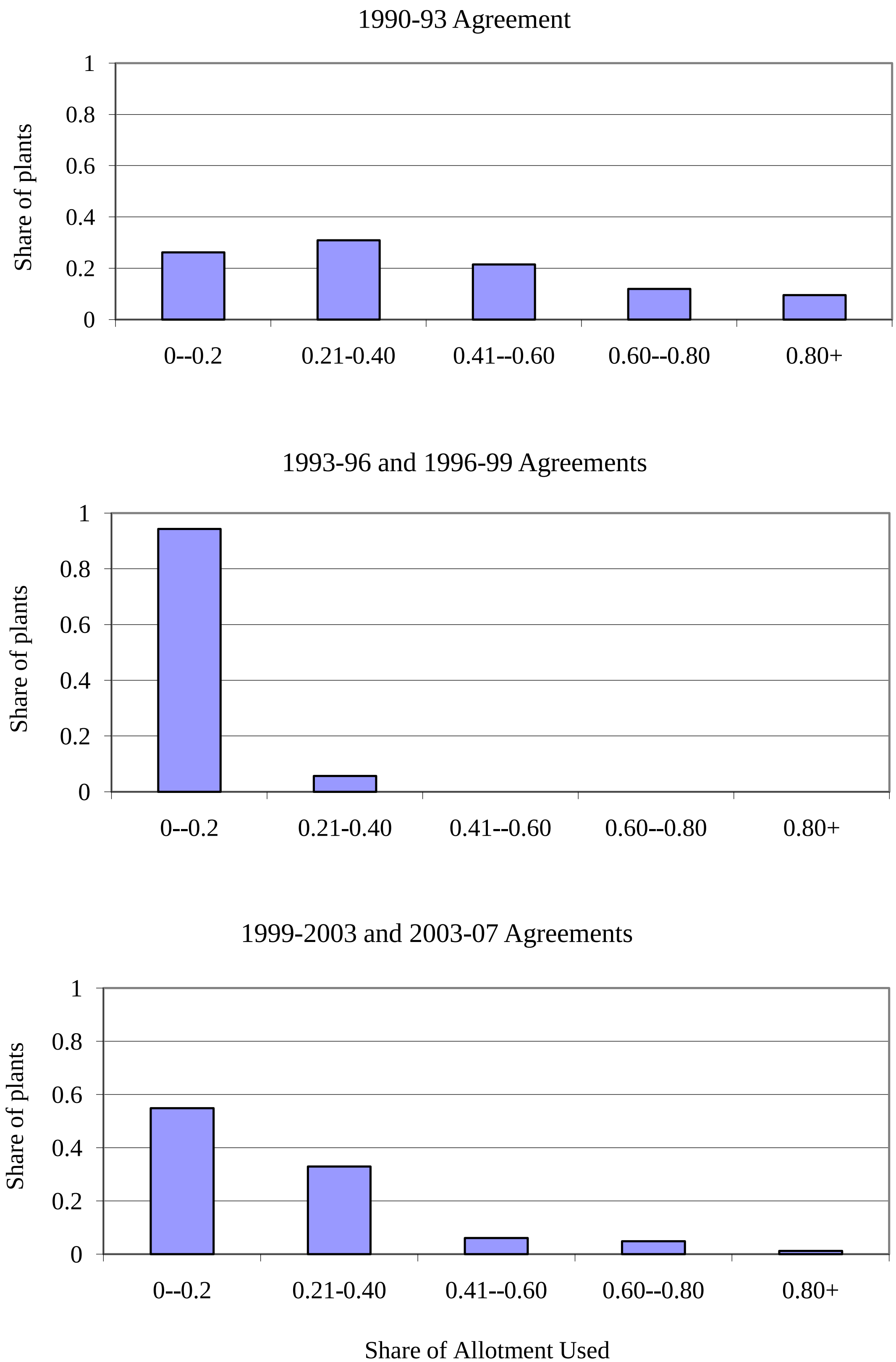
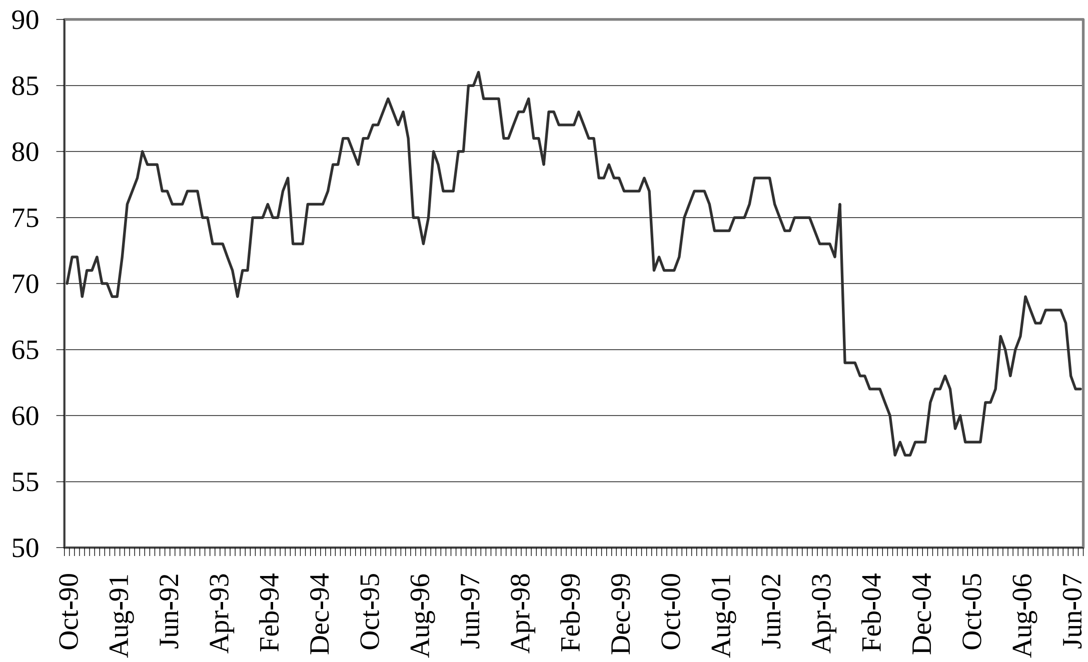
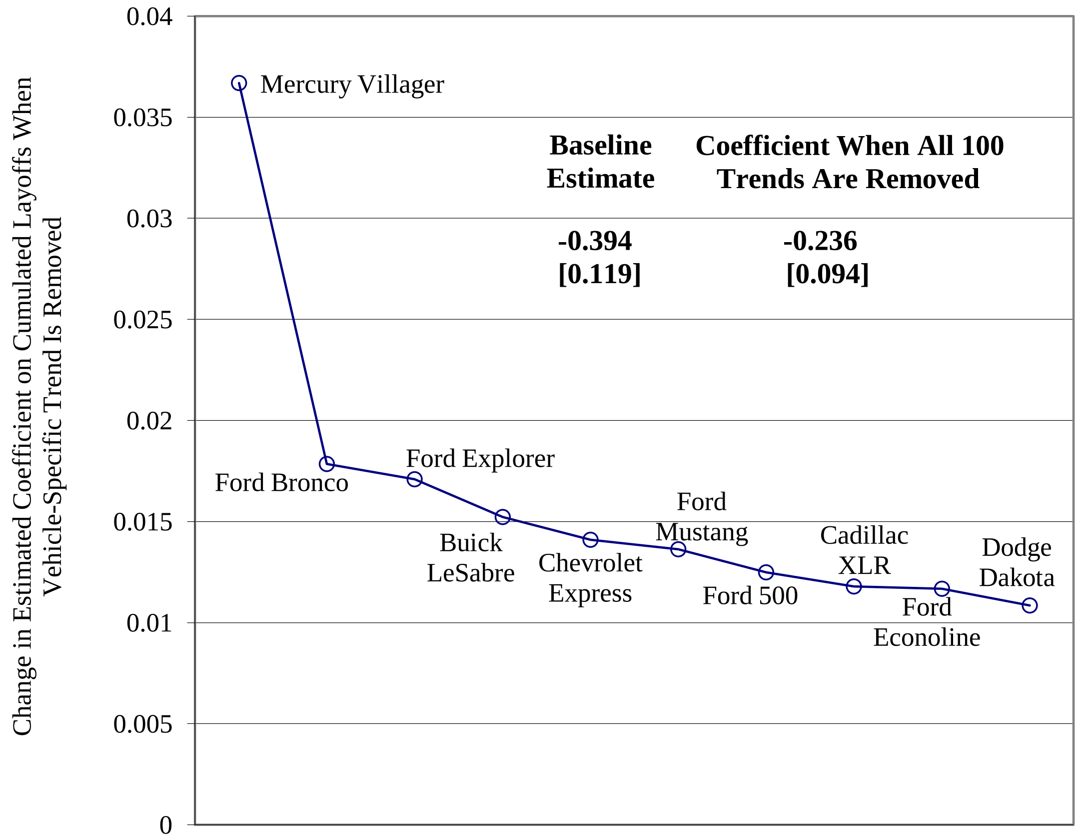


Figure 2.10: Distribution of Weeks on Layoff Within Labor Agreements



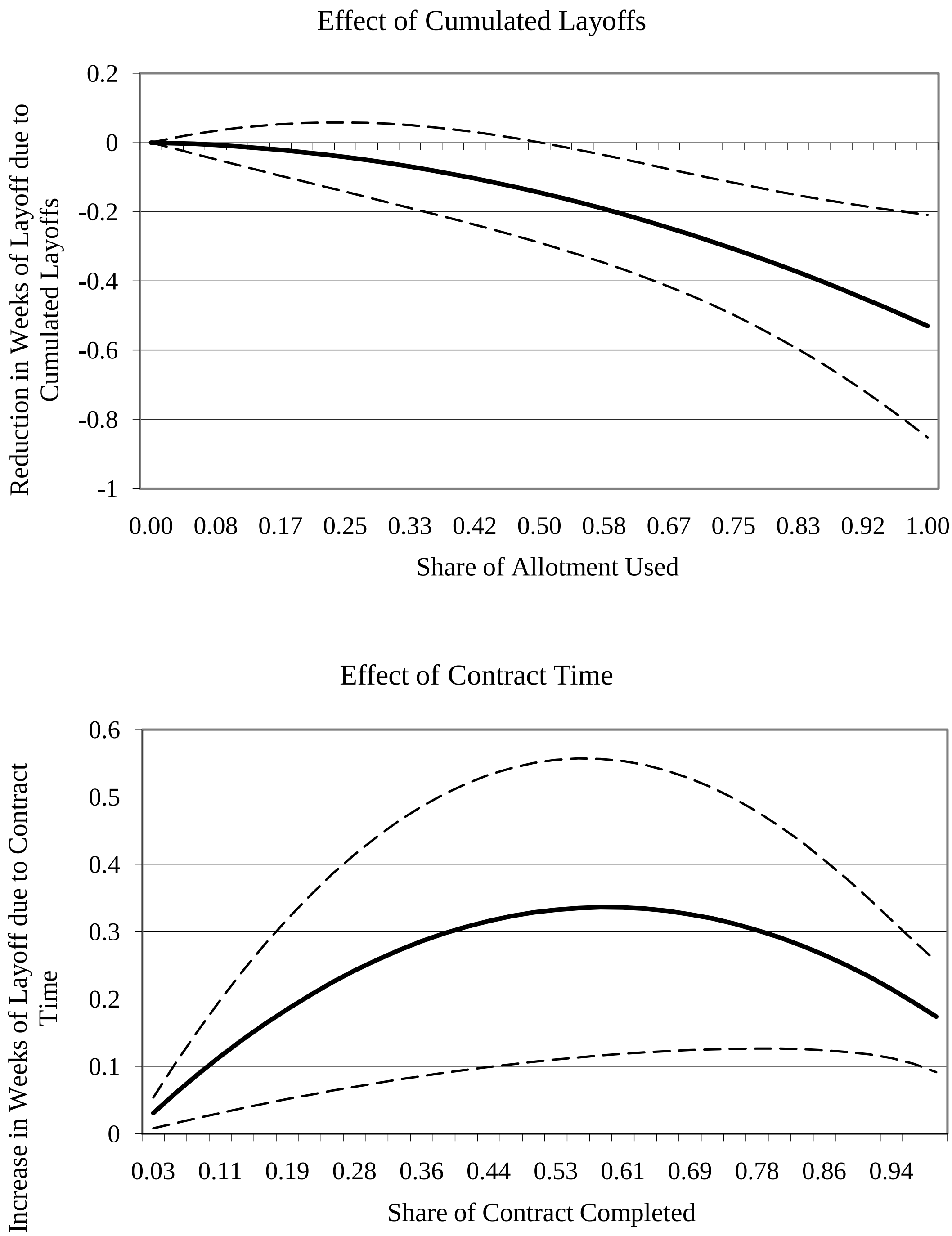
NOTE: This shows the total number of vehicle models in production in each month of the sample. There is a sudden decline in the number of models in the fall of 2003. This occurs because eight plants were closed over the life of the 2003-2007 labor agreement and so the models made at these facilities were excluded from the analysis (see Appendix A for more) .

Figure 2.11: Number of Models in Production



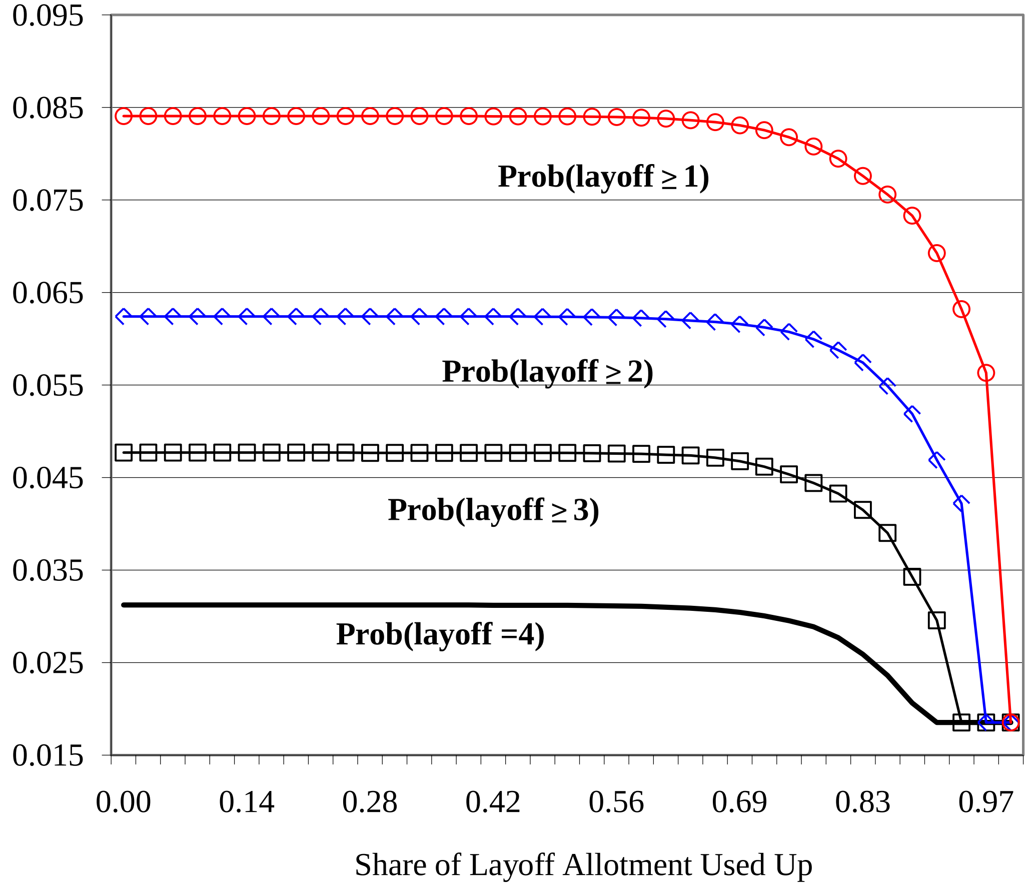
NOTE: This shows the change in the estimated coefficient on cumulated layoffs, L_{-1} when the vehicle-specific time trends above are removed from the list of covariates in the linear, reduced-form regression. For instance, when the trend associated with the Mercury Villager is removed, and all other trends are left in, the estimated coefficient on L_{-1} rises from -0.394 to -0.358. The text in the figure also reports the effect when all 10 trends are removed. Plant-clustered standard errors are given in brackets. See text (section 2.4.5) for more.

Figure 2.12: Sensitivity of Reduced-Form Estimate to Changes in Sample



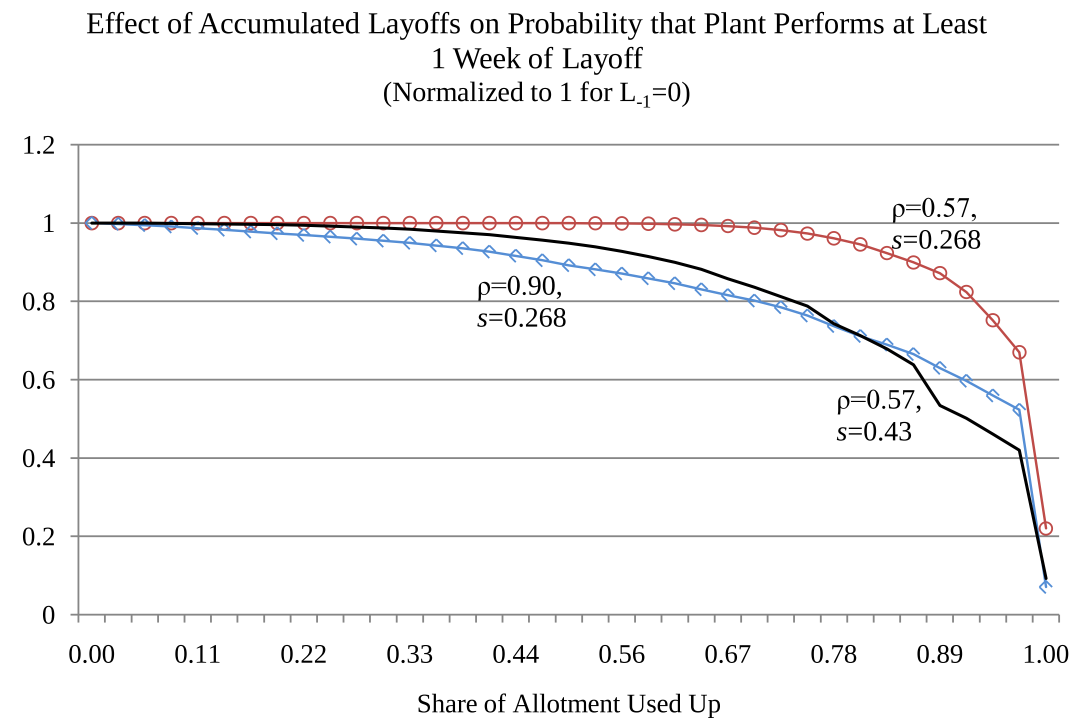
NOTE: The solid lines trace out the estimated effects from the reduced-form regression model of section 2.4.5 with quadratic terms in L_{-1} and τ. The dashed lines are two-standard error bands calculated from the plant-clustered covariance matrix.

Figure 2.13: Reduced-Form Estimates of Nonlinear JOBS Effects



NOTE: This shows the estimated effect of accumulated layoffs on the probability of layoff, i.e., the effect consistent with the parameter estimates reported in Table 2.7. The schedule labeled "Prob(layoff $\geq$ 1)" indicates the probability that at least one week of layoff is executed. The other schedules are read in the same way.

Figure 2.14: Estimated Effect of Accumulated Layoffs



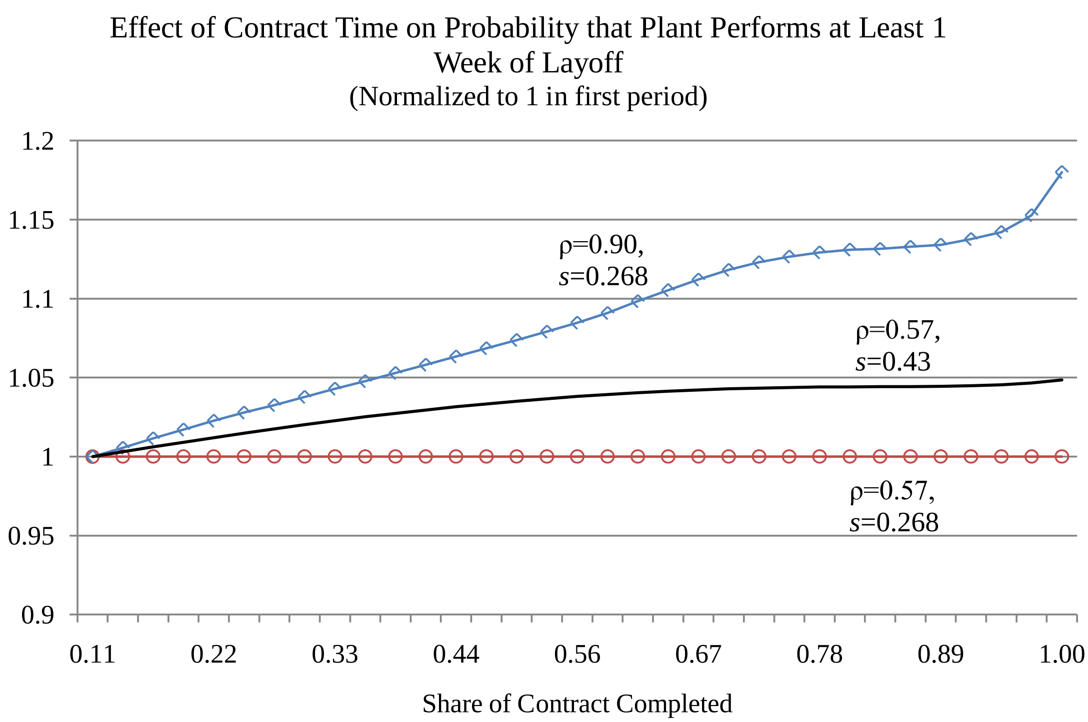


Figure 2.15: JOBS Bank Effects Under Alternative Parameters

2.7 Appendix A: The Plant-level Panel Dataset

2.7.1 Weeks of Layoff

While I do focus on inventory-induced layoffs, a few remarks are warranted regarding the others. To frame this discussion, it is helpful to organize "non-volume" shutdowns by duration: short, long, and permanent. Short shutdowns are due to brief model changeovers or a supply shortage. In the data, I classify a shutdown as "short" if it lasts no more than two months. To accommodate these in the regression analysis, I generate a single dummy variable equal to one if a plant experiences a brief shutdown in a given month and zero otherwise.

Long, but strictly temporary, shutdowns are often associated with the conversion of a plant from car to light truck or van production. As a result, it is often not possible to assign models to these plants for the period in which it is shut down.[73] It is necessary, then, to simply drop these months from the sample. In fact, in some cases, I drop the *entire contract* in which the shutdown occurs. To be specific, I drop a plant for an entire contract if it was shut down for at least one-fourth of the length of a (three- or four-year) labor agreement. In the most extreme case, it may be arithmetically impossible for a plant to place its workers on $\bar{L}$ weeks of inventory-induced layoff over the short time it is in operation over the life of that labor agreement. More generally, if plant is open for such an abbreviated period, it is not subject to the same constraints under the JOBS program as other facilities.

Permanent shutdowns occur if the firm closes the facility and indefinitely lays off all workers. In these cases, I drop the entire contract in which the closure occurs. This means that the estimates I present should be interpreted as the effects of L_{-1} and τ on the decision of how many weeks to operate per month, *conditional on* the choice to keep the plant open for the duration of the agreement.

2.7.2 The One Plant-One Product Assumption

The model of Section 3 assumed that one plant produced on (differentiated) product. There are three features of the plant-level data in particular that are at odds with the theoretical model presented in Section 3 and with the regression specification (2.15) motivated by it. They are: multi-product plants; model turnover within plants; and overlap across plants in terms of vehicles produced.

[73] This problem does not arise if a plant is shutdown for a month to prepare for a new vintage of the same model. This is what we see in "short" shutdowns discussed above.

The presence of multi-product plants, by itself, does not pose a significant challenge. Multi-product facilities may be reconciled with the theory if we re-interpret x as a "bundle" of products. They also do not raise any difficulties for the empirical analysis if each plant is the *sole* producer of its bundle. In that case, the sum of (lagged) inventory across models may be taken as a measure of I_{-1} and paired with that facility (and likewise, for sales).

Rather, it is the *interaction* between multi-product plants and overlap of production that poses the greatest challenge. To incorporate overlap, one must drop the artifice of a "bundle" and model the production of individual products. This complicates the analysis even if plants are assumed to continue to act independently (despite the presence of overlap). The reason is that each plant faces a distinct demand for each product and must carry separate inventories to meet sales of each product. As a result, the state space of the plant's problem equals the number of products under production, which in the data can be as high as four or five. In addition, if plants do coordinate – or, equivalently, if there is a central actor that assigns production schedules to each facility – the problem becomes substantially harder.[74] Unfortunately, the theoretical extensions needed accommodate product overlap lie outside the scope of this paper.

I would stress, however, that while overlap is a real presence in the data, it is not overwhelming. Slightly more than half of the plant-month observations involve no overlap at all, and the vast majority of observations involve overlap across no more than two plants. This may alleviate some concern regarding the construction of the inventory and sales series for each plant, as outlined above. But it does remain likely that simple aggregation across models introduces noise into the measured regressors.

I now turn my attention to model turnover within plants. Table A.2 depicts this turnover, as it reports all of the models produced at each assembly plant throughout the period October 1990 - September 2007. On average, a plant makes four vehicles over the time it spends in the sample. That there is turnover is evidence enough that there is an omitted margin of adjustment – namely, product selection – from our analysis of the plant's problem. Since that analysis motivated the estimation equation, this raises the concern that (2.15) is mis-specified.

[74]If there are just a handful of actors, the market resembles a dynamic differentiated oligopoly, which can quickly turn into a computationally intractable problem. If these central actors are assumed to act monopolistically, there is still no free lunch: it remains likely that the actor's choice of layoffs for any plant j will depend on the inventory of each model under that plant's production *and* on the inventory of each model under production at plants whose product portfolio overlaps with plant j. The intuition for this is straightforward. Suppose there is a manufacturer that makes three products across two plants. In particular, assume plant j makes models one and two and plant k makes models two and three. Assume that each plant either operates in a given week or shuts down for that period. Now imagine the start-of-period inventory of model three is very high. The firm's optimal response is to shut down plant k and run plant j so that the latter compensates for the lost production of model two.

To assess this, it is useful to ask how we might amend the theoretical model to accommodate turnover and what that would mean for (2.15). Assume that, in any period, a plant is, in principle, able to adopt a new model. In particular, imagine the plant may select a "better" distribution from which to draw zs, where, by "better", I mean a distribution with a higher mean, μ. The selection of a new model involves, however, a number of costs. First, assume the plant may select a new model now but it cannot begin production until D periods later. This delay reflects the time needed to convert the plant to a different model. Assume the plant foregoes all revenue while it is down for conversion. In addition, suppose the plant must pay an adjustment cost (over and above foregone revenue) that is proportional to the increase in μ, which reflects research and development costs associated with the introduction of a better model.[75]

The plant's decision process would likely proceed as follows: (i) for given model and given ℓ, select I; (ii) then, for given model, select ℓ; and, finally, (iii) select the model. This suggests that the *form* of the optimal inventory and layoff policies of section 3 hold for a given model. Moreover, the presence of a time delay indicates that model turnover, in an econometric sense, is weakly exogenous with respect to weeks on layoff per month. These two observations appear to justify the estimation of (2.15) augmented with model effects. Nonetheless, the policy rule governing model selection will depend, in part, on L_{-1} and τ: if the expected presented discounted costs of the JOBS program are large, the plant is more likely to undertake a model changeover in order to avoid triggering JOBS. Therefore, when the plant's problem is fully solved out, one anticipates that the optimal response of layoffs to L_{-1} and τ will reflect this, that is, the coefficients on these objects in the reduced-form specification would be affected. In other words, while the form of the policy is preserved, the coefficients of the reduced-form solution will reflect new information impounded in the optimal model selection policy. For this reason, the effect of model turnover is ultimately an empirical question: if the effect of the JOBS program were minimal, model selection might provide one interpretation of that result.

2.8 Appendix B: Proof of Proposition

Throughout, I assume the following.

Assumption 1 The transition function F has the Feller property.

Assumption 2 Let $z \in \mathcal{Z}$ and $I \in \mathcal{I}$, where $\mathcal{Z}$ and $\mathcal{I}$ are compact sets.

Assumption 2 requires that one impose some upper limit on the range of feasible inventory

[75] One should also include an additional term in the adjustment cost that reflects the price paid to acquire and install the new machines needed to produce a new vehicle. But it is not needed for this discussion.

levels. This is a standard way to bound an otherwise unbounded domain. To be more rigorous, one may introduce depreciation into the model, which ensures the existence of a maximal (finite) maintainable level of inventory, $\bar{I}$. From the law of motion for inventory, this level is defined by $\bar{I} = (1-\delta)\,\bar{I} + \max\{y\}$, where δ is the depreciation rate, $\max\{y\}$ corresponds to four weeks worked per month, and sales, x, are set to zero so as to maximize inventory. This yields $\bar{I} = \max\{y\}/\delta$. While this provides a foundation for Assumption 2, it adds little economic content to the problem and is unnecessary from a practical point of view. The numerical solution of the model requires that *some* bounds be placed on inventory. Provided these are selected so that they bind rarely, if at all, the enforcement of boundedness does not affect the calculation of the optimal policy.

The proof of the proposition may now be stated.

Proof of Proposition The proof is inductive. Begin with the terminal period of an agreement, in which case the value of the plant is

$$\begin{aligned} V_T(z, I_{-1}, L_{-1}) &= (Mv)(z, I_{-1}, L_{-1}, T) \\ &= \max_{\ell}\left\{\max_{I}\left[\pi(I, \ell;\ z, I_{-1}) - \omega(\ell, 0) + \beta\int v(z', I)\,dF(z'|z)\right]\right\}. \end{aligned}$$

Under the recursion hypothesis, v is bounded and (piecewise) continuous. Under Assumptions 1 and 2, boundedness and continuity are preserved under expectation. It follows that the function to be maximized is bounded and continuous. Its maximum, V_T, is therefore bounded, and by the Theorem of the Maximum, it is also continuous. As one moves to period $T-1$, this argument is repeated, where V_T takes the place of v. Inductively, it follows that the operator, M^T, maps the space of bounded and continuous functions into itself.

It remains to verify Blackwell's conditions for a contraction. Mononicity is apparent, since the maximized value of a function must rise if the function to be maximized is uniformly higher. Discounting follows from the fact that, for a constant c, $\left(M^T(v+c)\right)(z, I_{-1}, L_{-1}, T) = \left(M^T v\right)(z, I_{-1}, L_{-1}, T) + \beta^T c < \left(M^T v\right)(z, I_{-1}, L_{-1}, T) + c$.

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CHAPTER III

Marginal Jobs, Heterogeneous Firms, and Unemployment Flows

3.1 Introduction

The study of the macroeconomics of labor markets has been dominated by two influential approaches in recent research: the development of search and matching models (Pissarides, 1985; Mortensen and Pissarides, 1994) and the empirical analysis of establishment dynamics (Davis and Haltiwanger, 1992).[1] This paper provides an analytical framework that unifies these approaches by introducing a notion of firm size into a search and matching model with endogenous job destruction. The outcome is a rich, yet analytically tractable framework that can be used to analyze a broad set of features of both the cross section and the dynamics of the aggregate labor market. In a set of quantitative applications we show that the model can provide a coherent account of a) the salient features of the distributions of employer size, and employment growth across establishments; b) the amplitude and propagation of cyclical fluctuations in flows between employment and unemployment; c) the negative comovement of unemployment and vacancies in the form of the Beveridge curve; and d) the dynamics of the distribution of employer size over the business cycle.

A notion of firm size is introduced by relaxing the common assumption that firms face a linear production technology.[2] Though conceptually simple, incorporating this feature is not a trivial exercise. The existence of a non-linear production technology, and the associated presence of multi-worker firms, complicates wage setting because the surplus generated by each of the employment relationships within a firm is not the same—"the" marginal worker generates less surplus than infra-marginal workers. In section 1, we apply the bargaining solution of Stole and Zwiebel (1996) to derive a very intuitive wage bargaining solution for this environment, something that has been

[1] This chapter was co-written with Michael Elsby.

[2] In its simplest form, this manifests itself in a one firm, one job representation, as in Pissarides (1985) and Mortensen and Pissarides (1994). For the present paper, we remain agnostic on the source of diminishing returns, which may arise due to decreasing returns to scale, short-run fixed factors of production, or imperfect product market competition. For a model with the latter feature, but with exogenous separations, see Rotemberg (2006).

considered challenging in recent research (see Cooper, Haltiwanger and Willis, 2007; and Hobijn and Sahin, 2007). The solution is a very natural generalization of the wage bargaining solution in standard search and matching models. The simplicity of our solution is therefore a useful addition to the literature.[3]

The wage bargaining solution enables us to characterize the properties of the optimal labor demand policy of an individual firm in the presence of idiosyncratic firm heterogeneity. We demonstrate that the labor demand solution is analogous to that of a model of kinked hiring costs in the spirit of Bentolila and Bertola (1990), but where the hiring cost is endogenously determined by frictions in the labor market. This yields an analytical solution for the optimal labor demand policy, summarizing microeconomic behavior in the model.

In section 2, we take on the task of aggregating this behavior to the macroeconomic level. This is a challenge because the presence of a non-linear production technology and idiosyncratic heterogeneity imply that a representative firm interpretation of the model doesn't exist. To address this, we develop a method that allows us to solve analytically for the equilibrium distribution of employment across firms (the firm size distribution). In turn, this allows us to determine the level of the aggregate (un)employment stock, which is implied by the mean of that distribution. We also provide a related method that allows us to solve for aggregate unemployment flows (hires and separations) implied by microeconomic behavior. Together, these characterize the aggregate steady state equilibrium of the model economy.

In section 3 we explore the dynamics of the model by introducing aggregate shocks. A difficulty that arises in the model is that, out of steady state, individual firms must forecast future wages, which involves forecasting the future path of the distribution of employment across firms, an infinite-order state variable. A useful feature of our analytical solution for optimal labor demand is that it allows us to simplify part of this problem. In particular, we are able to derive an analytical approximation to a firm's optimal labor demand policy in the presence of aggregate shocks, obviating the need for a numerical solution. Using this, we employ an approach that mirrors the method proposed by Krusell and Smith (1998) to solve for the transition paths for the unemployment stock and flows in the presence of aggregate shocks.

These results form the basis of a series of quantitative applications, which we turn to in section

[3]Bertola and Caballero (1994) solve a related bargaining problem by taking a linear approximation to the marginal product function and specializing productivity to a two-state Markov process. The present paper relaxes these restrictions. More recent research that models endogenous separations has set worker bargaining power to zero in order to derive wages (Cooper et al., 2007; Hobijn and Sahin, 2007). In the presence of exogenous separations, Acemoglu and Hawkins (2006) characterize wages, but focus instead on a time to hire aspect to job creation, which leads to a more challenging bargaining problem. The wage bargaining solution for models with exogenous job destruction has been characterized by Smith (1999), Cahuc and Wasmer (2001), and Krause and Lubik (2007).

4. An attractive feature of the model is that, by incorporating both a notion of firm size as well as idiosyncratic heterogeneity, it delivers important cross sectional implications. We show that the model can be used to match key features of the distribution of firm size, and of employment growth across establishments. This is achieved through two aspects of the model. First, due to the existence of kinked hiring costs, optimal labor demand features a region of inaction whereby firms choose neither to hire nor fire workers. This matches a key property of the distribution of employment growth—the existence of a mass point at zero establishment growth—noted at least since the work of Davis and Haltiwanger (1992).[4] Second, informed by the well-known shape of the distribution of firm size, we adopt a Pareto specification for idiosyncratic firm productivity. A surprising outcome of this approach is that the Pareto specification also provides a very accurate description of the tails of the distribution of employment growth, something that cannot be achieved using more conventional lognormal specifications of heterogeneity.

We then use these steady-state features of the model to provide a novel perspective on the cyclical dynamics of worker flows implied by the model. It is well-known that the cyclical amplitude of unemployment, and of the job-finding rate in particular, relies critically on the size of the surplus to employment relationships (Shimer, 2005; Hagedorn and Manovskii, 2007). Intuitively, small reductions in aggregate productivity can easily exhaust a small surplus, and lead firms to cut back substantially on hiring. The presence of large and heterogeneous firms in our model opens up a new approach to calibrating the payoff from unemployment, and thereby the match surplus. Because the model is capable of matching the observed cross-sectional distribution of employment growth, we obtain a sense of the plausible size of idiosyncratic shocks facing firms. Given this, a higher payoff from unemployment implies a smaller surplus, so that jobs will be destroyed more frequently, raising the rate of worker turnover. We discipline the model by choosing the payoff from unemployment that matches the empirical rate at which employed workers flow into unemployment.

Applying this approach to an otherwise standard calibration reveals that our generalized model can replicate both the observed procyclicality of the job finding rate, as well as the countercyclicality of the employment to unemployment transition rate in the U.S.[5] We show that this is a substantial improvement over standard search and matching models. As shown by Shimer (2005), these are unable to generate enough cyclicality in job creation. To overcome this, the standard model must reduce the size of the surplus, which in turn yields excessive employment to unem-

[4] Earlier work by Hamermesh (1989), which analyzed data from seven manufacturing plants, also drew attention to the "lumpy" nature of establishment-level employment adjustment.

[5] For evidence on the countercyclicality of employment to unemployment flows, see Perry (1972); Marston (1976); Blanchard and Diamond (1990); Elsby, Michaels, and Solon (2007); Fujita and Ramey (2007); Pissarides (2007); Shimer (2007); and Yashiv (2006).

ployment transitions.[6] The generalized model does not face this tension between reproducing the cyclicality of job creation and the rate of worker turnover. Due to the diminishing marginal product of labor, the model generates simultaneously a large average surplus and a small marginal surplus to employment relationships. The former allows the model to match the rate at which workers flow into unemployment, the latter the volatility of the job-finding rate over the cycle.[7]

A potential concern in models that incorporate countercyclical job destruction, such as the model in this paper, has been that they often cannot generate the observed procyclicality of vacancies (Shimer, 2005; Mortensen and Nagypal, 2007b). Importantly, we find that our model makes considerable progress in this regard: Our calibration of the model generates most of the observed comovement between vacancies and output per worker. As a result, it reproduces a key stylized fact of the U.S. labor market: the negative comovement between unemployment and vacancies in the form of the Beveridge curve. The model therefore provides a coherent and quantitatively accurate picture of the joint cyclical properties of both flows of workers in and out of unemployment, as well as the behavior of unemployment and vacancies.

A less well-documented limitation of the standard search and matching model relates to the propagation of the response of the job finding rate to aggregate shocks to labor productivity. The job finding rate is a jump variable in the standard model, responding instantaneously to aggregate shocks, while it exhibits a sluggish response in U.S. data. An appealing feature of the generalized model is that it delivers a natural propagation mechanism: The job finding rate is a function of the distribution of employment across firms, which we show is a slow-moving state variable in our model. Simulations reveal that this aspect of the model can help account for the persistence of the decline in job creation following an adverse shock.

In the closing sections of the paper, we push the model harder by evaluating its implications for a number of additional cross-sectional outcomes. First, recent literature has emphasized empirical regularities in the cyclical behavior of the cross-sectional distribution of establishment size: While the share of small establishments with fewer than 20 workers rises during recessions, the shares of larger firms decline (Moscarini and Postel-Vinay, 2009). The model replicates this observation: For

[6] This formalizes the intuition of recent research that has argued that the average surplus required for the standard model to match the observed cyclicality of the job finding rate is implausibly small (Mortensen and Nagypal, 2007a). A small average surplus also jars with widespread evidence for the prevalence of long term employment relationships in the US economy, which researchers have taken to imply substantial rents to ongoing matches (Hall, 1982; Stevens, 2005).

[7] One might imagine that a symmetric logic holds on the supply side of the labor market if there is heterogeneity in workers' valuations of leisure so that "the" marginal worker obtains a low surplus from employment. Interestingly, Mortensen and Nagypal (2007a) argue that this is not the case. They show that if firms cannot differentiate workers' types when making hiring decisions, they will base their decision on the average, rather than the marginal, valuation of leisure among the unemployed. The same is unlikely to be true of the model studied here, since firms presumably know their production technology when making hiring decisions.

each establishment size class considered, it broadly matches the comovement with unemployment over the business cycle observed in U.S. data. Given that these implications of the model are venturing farther afield from the moments it was calibrated to match, we view these results as an important achievement.

In our final quantitative application, we evaluate the model's ability to account for the observation that workers employed in larger firms are often paid higher wages—the employer size-wage effect (Brown and Medoff, 1989). A distinctive attribute of the model is that, by incorporating large firms with heterogeneous productivities, it can speak to this empirical regularity. The magnitude of the size-wage effect implied by the model is mediated by two competing forces, as noted by Bertola and Garibaldi (2001). On the one hand, the existence of diminishing returns in production might lead one to anticipate a negative relation between employer size and wages. On the other, larger firms also tend to be more productive. Quantitatively, the latter dominates, generating one quarter of the empirical size-wage effect.

The remainder of the paper is organized as follows. Section 1 describes the set-up of the model, and characterizes the wage bargaining solution together with the associated optimal labor demand policy of an individual firm. Given this, section 2 develops a method for aggregating this microeconomic behavior up to the macroeconomic level, and uses it to characterize the steady state equilibrium of the model. Section 3 introduces aggregate shocks to the analysis. It presents an approach to computing the out of steady state dynamics of the model through the use of analytical approximations. We then use the model in section 4 to address a wide range of quantitative applications. Finally, section 5 summarizes our results, and draws lessons for future research.

3.2 The Firm's Problem

In what follows we consider a model in which there is a mass of firms, normalized to one, and a mass of potential workers equal to the labor force, L.[8] In order to hire unemployed workers, firms must post vacancies. However, frictions in the labor market limit the rate at which unemployed workers and hiring firms can meet. As is conventional in the search and matching literature, these frictions are embodied in a matching function, $M = M(U, V)$, that regulates the number

[8] Assuming a fixed number of firms is important for the model to depart from the standard search model. Free entry would yield an economy of infinitesimal firms that converges to the Mortensen and Pissarides (1994) limit. In principle, one could allow for costly firm entry as a middle ground. We abstract from this in part for simplicity. But our choice is also informed by evidence in Davis and Haltiwanger (1992). They find that, in manufacturing, while births and deaths account for around 15 percent of establishment growth, they account for a very small fraction of employment growth. The simple reason is that births and deaths are dominated by the behavior of small establishments that account for a small fraction of total employment. For models that explore the impact of firm entry on job creation, see Garibaldi (2006) and Hobijn and Sahin (2007).

of hires, M, that the economy can sustain given that there are V vacancies and U unemployed workers. We assume that $M(U,V)$ exhibits constant returns to scale.[9] Vacancies posted by firms are therefore filled with probability $q = M/V = M(U/V,1)$ each period. Likewise, unemployed workers find jobs with probability $f = M/U = M(1,V/U)$. Thus, the ratio of aggregate vacancies to aggregate unemployment, $V/U \equiv \theta$, is a sufficient statistic for the job filling (q) and job finding (f) probabilities in the model. Taking these flow probabilities as given, firms choose their optimal level of employment, to which we now turn.

3.2.1 Labor Demand

We consider a discrete time, infinite horizon model in which firms use labor, n, to produce output according to the production function, $y = pxF(n)$ where $F' > 0$ and $F'' \leq 0$. The latter is a key generalization of the standard search model that we consider: When $F'' < 0$, the marginal product of labor will decline with firm employment, and thereby will generate a downward sloped demand for labor at the firm level. p represents the state of aggregate labor demand, whereas x represents shocks that are idiosyncratic to an individual firm. We assume that the evolution of the latter idiosyncratic shocks is described by the c.d.f. $G(x'|x)$.

A typical firm's decision problem is completely analogous to that in Mortensen and Pissarides (1994), and is as follows. Firms observe the realization of their idiosyncratic shock, x, at the beginning of a period. Given this, they then make their employment decision. Specifically, they may choose to separate from part or all of their workforce, which we assume may be done at zero cost. Any such separated workers then join the unemployment pool in the subsequent period. Alternatively, firms may hire workers by posting vacancies, $v \geq 0$, at a flow cost of c per vacancy. If a firm posts vacancies, the matching process then matches these up with unemployed workers inherited from the previous period. After the matching process is complete, production and wage setting are performed simultaneously.

It follows that we can characterize the expected present discounted value of a firm's profits, $\Pi(n_{-1},x)$, recursively as:[10]

$$\Pi(n_{-1},x) = \max_{n,v} \left\{ pxF(n) - w(n,x)\,n - cv + \beta \int \Pi(n,x')\,dG(x'|x) \right\}, \quad (3.1)$$

[9] See Petrongolo and Pissarides (2001) for a summary of empirical evidence that suggests this is reasonable.
[10] We adopt the convention of denoting lagged values with a subscript, $_{-1}$, and forward values with a prime, $'$.

where $w(n,x)$ is the bargained wage in a firm of size n and productivity x. A typical firm seeks a level of employment that maximizes its profits subject to a dynamic constraint on the evolution of a firm's employment level. Specifically, firms face frictions that limit the rate at which vacancies may be filled: A vacancy posted in a given period will be filled with probability $q < 1$ prior to production. Thus, the number of hires an individual firm achieves is given by:

$$\Delta n \mathbf{1}^{+} = qv, \tag{3.2}$$

where Δn is the change in employment, and $\mathbf{1}^{+}$ is an indicator that equals one when the firm is hiring, and zero otherwise. Substituting the constraint, (3.2), into the firm's value function, we obtain:

$$\Pi(n_{-1},x) = \max_{n} \left\{ pxF(n) - w(n,x)\,n - \frac{c}{q}\Delta n \mathbf{1}^{+} + \beta \int \Pi(n,x')\,dG(x'|x) \right\}. \tag{3.3}$$

Note that the value function is not fully differentiable in n: There is a kink in the value function around $n = n_{-1}$. This reflects the (partial) irreversibility of separation decisions in the model. While firms can shed workers costlessly, it is costly to reverse such a decision because hiring (posting vacancies) is costly. In this sense, the labor demand side is formally analogous to the kinked employment adjustment cost model of the form analyzed in Bentolila and Bertola (1990), except that the per–worker hiring cost, $c/q(\theta)$, is endogenously determined.

In order to determine the firm's optimal employment policy, we take the first-order conditions for hires and separations (i.e. conditional on $\Delta n \neq 0$):

$$pxF'(n) - w(n,x) - w_n(n,x)\,n - \frac{c}{q}\mathbf{1}^{+} + \beta D(n,x) = 0, \text{ if } \Delta n \neq 0, \tag{3.4}$$

where $D(n,x) \equiv \int \Pi_n(n,x')\,dG(x'|x)$ reflects the marginal effect of current employment decisions on the future value of the firm. Equation (3.4) is quite intuitive. It states that the marginal product of labor $(pxF'(n))$ net of any hiring costs $(\frac{c}{q}\mathbf{1}^{+})$, plus the discounted expected future marginal benefits from an additional unit of labor $(\beta D(n,x))$ must equal the marginal cost of labor $(w(n,x) + w_n(n,x)\,n)$. To provide a full characterization of the firm's optimal employment policy, it remains to characterize the future marginal benefits from current employment decisions, $D(n,x)$, and the wage bargaining solution, $w(n,x)$, to which we now turn.

3.2.2 Wage Setting

The existence of frictions in the labor market implies that it is costly for firms and workers to find alternative employment relationships. As a result, there exist quasi-rents over which the firm and its workers must bargain. The assumption of constant marginal product in the standard search model has the tractable implication that these rents are the same for all workers within a given firm. It follows that firms can bargain with each of their workers independently, because the rents of each individual employment relationship are independent of the rents of all other employment relationships.

Allowing for the possibility of diminishing marginal product of labor $F''(n) < 0$, however, implies that these rents will depend on the number of workers within a firm. Intuitively, the rent that a firm obtains from "the" marginal worker will be lower than the rent obtained on all infra–marginal hires due to diminishing marginal product. An implication of the latter is that the multilateral dimension of the firm's bargain with its many workers becomes important: The rents of each individual employment relationship within a firm are no longer independent.

To take this into account, we adopt the bargaining solution of Stole and Zwiebel (1996) which generalizes the Nash solution to a setting with diminishing returns.[11] Stole and Zwiebel present a game where the bargained wage is the same as the outcome of simple Nash bargaining over the *marginal* surplus. The game that supports this simple result is one in which a firm negotiates with each of its workers in turn, and where the breakdown of a negotiation with any individual worker leads to the renegotiation of wages with all remaining workers.[12]

In accordance with timing of decisions each period, wages are set after employment has been determined. Thus, hiring costs are sunk at the time of wage setting, and the marginal surplus, which we denote as $J(n, x)$, is equal to the marginal value of labor gross of the costs of hiring:

$$J(n, x) = pxF'(n) - w(n, x) - w_n(n, x)\,n + \beta D(n, x)\,. \tag{3.5}$$

The surplus from an employment relationship for a worker is the additional utility a worker obtains from working in her current firm over and above the utility she obtains from unemployment. The

[11] This approach was first used by Cahuc and Wasmer (2001) to generate a wage equation for the exogenous job destruction case.

[12] The intuition for the Stole and Zwiebel result is as follows. If the firm has only one worker, the firm and worker simply strike a Nash bargain. If a second worker is added, the firm and the additional worker know that, if their negotiations break down, the firm will agree to a Nash bargain with the remaining worker. In this sense, the second employee regards herself as being on the margin. By induction, then, the firm approaches negotiations with the nth worker as if that worker were marginal too. Therefore, the wage that solves the bargaining problem is that which maximizes the marginal surplus.

value of employment in a firm of size n and productivity x, $W(n,x)$, is given by:

$$W(n,x) = w(n,x) + \beta E\left[sU' + (1-s)\, W(n',x')\, | n,x\right]. \tag{3.6}$$

While employed, a worker receives a flow payoff equal to the bargained wage, $w(n,x)$. She loses her job with (endogenous) probability s next period, upon which she flows into the unemployment pool and obtains the value of unemployment, U'. With probability $(1-s)$, she retains her job and obtains the expected payoff of continued employment in her current firm, $W(n',x')$. Likewise, the value of unemployment to a worker is given by:

$$U = b + \beta E\left[(1-f)\, U' + fW(n',x')\right]. \tag{3.7}$$

Unemployed workers receive flow payoff b, which represents unemployment benefits and/or the value of leisure to a worker. They find a job next period with probability f, upon which they obtain the expected payoff from employment, $W(n',x')$.

Wages are then the outcome of a Nash bargain between a firm and its workers over the marginal surplus, with worker bargaining power denoted as η:

$$(1-\eta)\left[W(n,x) - U\right] = \eta J(n,x). \tag{3.8}$$

Given this, we are able to derive a wage bargaining solution with the following simple structure:

Proposition 1 *The bargained wage, $w(n,x)$, solves the differential equation*[13]

$$w(n,x) = \eta\left[pxF'(n) - w_n(n,x)\, n + \beta f \frac{c}{q}\right] + (1-\eta)\, b. \tag{3.9}$$

The intuition for (3.9) is quite straightforward. As in the standard search model, wages are increasing in the worker's bargaining power, η, the marginal product of labor, $pxF'(n)$, workers' job finding probability, f, the marginal costs of hiring for a firm, c/q, and workers' flow value of leisure, b. There is an additional term, however, in $w_n(n,x)\, n$. To understand the intuition for this term, consider a firm's negotiations with a given worker. If these negotiations break down, the firm will have to pay its remaining workers a higher wage. The reason is that fewer workers imply that the marginal product of labor will be higher in the firm, which will partially spillover

[13] An interesting feature of this solution is its similarity to the solution obtained by Cahuc and Wasmer (2001) for the exogenous job destruction model. It is also consistent with Acemoglu and Hawkins' (2006) Lemma 2, except that it holds both in and out of steady state.

into higher wages ($w_n n < 0$). The more powerful this effect is (the more negative is $w_n n$), the more the firm loses from a given breakdown of negotiations with a worker, and the more workers can extract a higher wage from the bargain.

In what follows, we will adopt the simple assumption that the production function is of the Cobb-Douglas form, $F(n) = n^{\alpha}$ with $\alpha \leq 1$. Given this, the differential equation for the wage function, (3.9), has the following simple solution:

$$w(n,x) = \eta \left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} + \beta f \frac{c}{q} \right] + (1-\eta) b. \tag{3.10}$$

Setting $\alpha = 1$ yields the discrete time analogue to the familiar wage bargaining solution for the Mortensen and Pissarides (1994) model.

3.2.3 The Firm's Optimal Employment Policy

Now that we have obtained a solution for the bargained wage at a given firm, we can combine this with the firm's first–order condition for employment and thereby characterize the firms optimal employment policy, which specifies the firm's optimal employment as a function of its state, $n(n_{-1}, x)$. Thus, combining (3.4) and (3.9) we obtain:

$$(1-\eta) \left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b \right] - \eta\beta f \frac{c}{q} - \frac{c}{q} \mathbf{1}^{+} + \beta D(n,x) = 0. \tag{3.11}$$

Given (3.11) we are able to characterize the firm's optimal employment policy as follows:

Proposition 2 *The optimal employment policy of a firm is of the form*

$$n(n_{-1},x) = \begin{cases} R_v^{-1}(x) & \text{if} \quad x > R_v(n_{-1}), \\ n_{-1} & \text{if} \quad x \in [R(n_{-1}), R_v(n_{-1})], \\ R^{-1}(x) & \text{if} \quad x < R(n_{-1}), \end{cases} \tag{3.12}$$

where the functions $R_v(\cdot)$ *and* $R(\cdot)$ *satisfy*

$$(1-\eta) \left[\frac{pR_v(n)\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b \right] - \eta\beta f \frac{c}{q} + \beta D(n, R_v(n)) \equiv \frac{c}{q}, \tag{3.13}$$

$$(1-\eta) \left[\frac{pR(n)\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b \right] - \eta\beta f \frac{c}{q} + \beta D(n, R(n)) \equiv 0. \tag{3.14}$$

The firm's optimal employment policy will be similar to that depicted in Figure 1. It is characterized by two reservation values for the firm's idiosyncratic shock, $R(n_{-1})$ and $R_v(n_{-1})$. Specifically, for sufficiently bad idiosyncratic shocks ($x < R(n_{-1})$ in the figure), firms will shed workers until the first-order condition in the separation regime, (3.14), is satisfied. Moreover, for sufficiently good idiosyncratic realizations ($x > R_v(n_{-1})$ in the figure), firms will post vacancies and hire workers until the first-order condition in the hiring regime, (3.13), is satisfied. Finally, for intermediate values of x, firms freeze employment so that $n = n_{-1}$. This occurs as a result of the kink in the firm's profits at $n = n_{-1}$, which arises because hiring is costly to firms, while separations are costless.

To complete our characterization of the firm's optimal employment policy, it remains to determine the marginal effect of current employment decisions on future profits of the firm, $D(n,x)$. It turns out that we can show that $D(n,x)$ has the following recursive structure:

Proposition 3 *The marginal effect of current employment on future profits,* $D(n,x)$, *is given by*

$$D(n,x) = d(n,x) + \beta \int_{R(n)}^{R_v(n)} D(n,x')\, dG(x'|x), \tag{3.15}$$

where

$$d(n,x) \equiv \int_{R(n)}^{R_v(n)} \left\{(1-\eta)\left[\frac{px'\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta f\frac{c}{q}\right\} dG(x'|x) + \int_{R_v(n)}^{\infty} \frac{c}{q} dG(x'|x). \tag{3.16}$$

Equation (3.15) is a contraction mapping in $D(n,\cdot)$, *and therefore has a unique fixed point.*

The intuition for this result is as follows. Because of the existence of kinked adjustment costs (costly hiring and costless separations) the firm's employment will be frozen next period with positive probability. In the event that the firm freezes employment next period ($x' \in [R(n), R_v(n)]$), the current employment level persists into the next period and so do the marginal effects of the firm's current employment choice. Proposition 3 shows that these marginal effects persist into the future in a *recursive* fashion. Propositions 2 and 3 thus summarize the microeconomic behavior of firms in the model.[14]

To get a sense for how the microeconomic behavior of the model works, we next derive the response of an individual firm's employment policy function to changes in (exogenous) aggregate productivity, p, and the (endogenous) aggregate vacancy–unemployment ratio, θ. To do this, we

[14] It is straightforward to show that equations (3.10) to (3.16) reduce down to the discrete time analogue to the Mortensen and Pissarides (1994) model when $\alpha = 1$.

assume that the evolution of idiosyncratic shocks is described by:

$$x' = \begin{cases} x & \text{with probability} \quad 1-\lambda, \\ \tilde{x} \overset{c.d.f.}{\sim} \tilde{G}(\tilde{x}) & \text{with probability} \quad \lambda. \end{cases} \tag{3.17}$$

Thus, idiosyncratic shocks display some persistence ($\lambda < 1$) with innovations drawn from the distribution function $\tilde{G}$. Given this, we can establish the following result:

Proposition 4 *If idiosyncratic shocks, x, evolve according to (3.17), then the effects of the aggregate state variables p and θ on a firm's optimal employment policy are*

$$\frac{\partial R_v}{\partial p} < 0; \frac{\partial R}{\partial p} < 0; \frac{\partial R_v}{\partial \theta} > 0; \text{ and } \frac{\partial R}{\partial \theta} > 0 \iff n \text{ is sufficiently large.} \tag{3.18}$$

The intuition behind these marginal effects is quite simple. First, note that increases in aggregate productivity, p, shift a firm's employment policy function downwards in Figure 1. Thus, unsurprisingly, when labor is more productive, a firm of a given idiosyncratic productivity, x, is more likely to hire workers, and less likely to shed workers. Second, increases in the vacancy–unemployment ratio, θ, unambiguously reduce the likelihood that a firm of a given idiosyncratic productivity will hire workers (R_v increases for all n). The reason is that higher θ implies a lower job–filling probability, q, and thereby raises the marginal cost of hiring a worker, c/q. Moreover, higher θ implies a tighter labor market and therefore higher wages (from (3.9)) so that the marginal cost of labor rises as well. Both of these effects cause firms to cut back on hiring. Finally, increases in the vacancy–unemployment ratio, θ, will reduce the likelihood of shedding workers for small firms, but will raise it for large firms. This occurs because higher θ has countervailing effects on the separation decision of firms. On the one hand, higher θ reduces the job–filling probability, q, rendering separation decisions less reversible (since future hiring becomes more costly), so that firms become less likely to destroy jobs. On the other hand, higher θ implies a tighter labor market, higher wages, and thereby a higher marginal cost of labor, rendering firms more likely to shed workers. The former effect is dominant in small firms because the likelihood of their hiring in the future is high.

3.3 Aggregation and Steady State Equilibrium

3.3.1 Aggregation

Since we are ultimately interested in the equilibrium behavior of the aggregate unemployment rate, in this section we take on the task of aggregating up the microeconomic behavior of section 1 to the macroeconomic level. This exercise is non–trivial because each firm's employment is a non–linear function of the firm's lagged employment, n_{-1}, and its idiosyncratic shock realization, x. As a result, there is no representative firm interpretation that will aid aggregation of the model.

To this end, we are able to derive the following result which characterizes the steady state aggregate employment stock and flows in the model:

Proposition 5 *If idiosyncratic shocks, x, evolve according to (3.17), the steady state c.d.f. of employment across firms is given by*

$$H(n) = \frac{\tilde{G}[R(n)]}{1 - \tilde{G}[R_v(n)] + \tilde{G}[R(n)]}. \quad (3.19)$$

Thus, the steady state aggregate employment stock is given by

$$N = \int n dH(n), \quad (3.20)$$

and the steady state aggregate number of separations, S, and hires, M, is equal to

$$S = \lambda \int [1 - H(n)] \tilde{G}[R(n)] dn = \lambda \int H(n) \Big(1 - \tilde{G}[R_v(n)]\Big) dn = M. \quad (3.21)$$

Proposition 5 is useful because it provides a tight link between the solution for the microeconomic behavior of an individual firm and the macroeconomic outcomes of that behavior. Specifically, it shows that once we know the optimal employment policy function of an individual firm (that is, the functions $R(n)$ and $R_v(n)$) then we can directly obtain analytical solutions for the distribution of firm size, and the aggregate employment stock and flows.

The three components of Proposition 5 are also quite intuitive. The steady state distribution of employment across firms, (3.19), is obtained by setting the flows into and out of the mass $H(n)$ equal to each other. The inflow into the mass comes from firms who reduce their employment from above n to below n. There are $[1 - H(n)]$ such firms, and since they are reducing their employment, it follows from (3.12) that each firm will reduce its employment below n with probability equal to $\Pr[x < R(n)] = \lambda \tilde{G}[R(n)]$. Thus, the inflow into $H(n)$ is equal to $\lambda [1 - H(n)] \tilde{G}[R(n)]$.

Similarly, one can show that the outflow from the mass is equal to $\lambda H(n)\left(1-\tilde{G}[R_v(n)]\right)$. Setting inflows equal to outflows yields the expression for $H(n)$ in (3.19).[15] Given this, the expression for aggregate employment, (3.20), follows directly.

The intuition for the final expression for aggregate flows in Proposition 5, (3.21), is as follows. Recall that the mass of firms whose employment switches from above some number n to below n is equal to $\lambda[1-H(n)]\tilde{G}[R(n)]$. Equation (3.21) states that the aggregate number of separations in the economy is equal to the *cumulative* sum of these downward switches in employment over n. To get a sense for this, consider the following simple discrete example. Imagine an economy with two separating firms: one that switches from three employees to one, and another that switches from two employees to one. It follows that two firms have switched from >2 employees to ≤ 2 employees, and one firm switched from >1 to ≤ 1 employee. Thus, the cumulative sum of downward employment switches is three, which is also equal to the total number of separations in the economy.

3.3.2 Steady State Equilibrium

Given (3.19), (3.20), and (3.21), the conditions for aggregate steady state equilibrium can be obtained as follows. First note that each firm's optimal policy function, summarized by the functions $R(n)$ and $R_v(n)$ in Proposition 2, depends on two aggregate variables: The (exogenous) state of aggregate productivity, p; and the (endogenous) ratio of aggregate vacancies to aggregate unemployment, $V/U \equiv \theta$, which uniquely determines the flow probabilities q and f.

In the light of Proposition 5, we can characterize the aggregate steady state of the economy for a given p in terms of two relationships. The first, the *Job Creation condition*, is simply equation (3.20), which we re–state here in terms of unemployment, making explicit its dependence on the aggregate vacancy–unemployment ratio, θ:

$$U(\theta)_{JC} = L - \int n dH(n;\theta). \tag{3.22}$$

(3.22) simply specifies the level of aggregate employment that is consistent with the inflows to (hires) and outflows from (separations) aggregate employment being equal as a function of θ. The second steady state condition is the *Beveridge Curve* relation. This is derived from the difference

[15] This mirrors the mass-balance approach used in Burdett and Mortensen (1998) to derive the equilibrium wage distribution in a search model with wage posting.

equation that governs the evolution of unemployment over time:

$$\Delta U' = S(\theta) - f(\theta)U. \tag{3.23}$$

(3.23) simply states that the change in the unemployment stock over time, $\Delta U'$, is equal to the inflow into the unemployment pool – the number of separations, S – less the outflow from the unemployment pool – the job finding probability, f, times the stock of unemployed workers, U. In steady state, aggregate unemployment will be stationary, so that we obtain the steady state unemployment relation:

$$U(\theta)_{BC} = \frac{S(\theta)}{f(\theta)}. \tag{3.24}$$

The steady state value of the vacancy–unemployment ratio, θ, is co–determined by (3.22) and (3.24).

3.4 Introducing Aggregate Shocks

The previous section characterized the determination of steady state equilibrium in the model. However, in what follows, we are interested in the dynamic response of unemployment, vacancies and worker flows to aggregate shocks. To address this, we need to characterize the dynamics of the model out of steady state. The latter is not a trivial exercise in the context of the present model. Out of steady state, firms in the model need to forecast future wages and therefore, from equation (3.9), future labor market tightness. Inspection of the steady state equilibrium conditions (3.22) and (3.24) reveals that, in order to forecast future labor market tightness, firms must predict the evolution of the entire distribution of employment across firms, $H(n)$, an infinite order state variable.

Our approach to this problem mirrors the method proposed by Krusell and Smith (1998). We consider shocks to aggregate labor productivity that evolve according to the simple random walk:

$$p' = \begin{cases} p + \sigma_p & \text{w.p.} \quad 1/2, \\ p - \sigma_p & \text{w.p.} \quad 1/2. \end{cases} \tag{3.25}$$

Following Krusell and Smith, we conjecture that a forecast of the *mean* of the distribution of employment across firms, $N = \int n dH(n)$, provides an accurate forecast of future labor market tightness. We then exploit the fact that shocks to aggregate labor productivity, denoted by σ_p in equation (3.25), are small in U.S. data.[16] This allows us to approximate the evolutions of aggregate employment, N, and labor market tightness, θ, around their steady state values N^* and θ^* as follows:

$$\begin{aligned} N' &\approx N^* + \nu_N (N - N^*) + \nu_p (p' - p), \\ \theta' &\approx \theta^* + \theta_N (N' - N^*) + \theta_p (p' - p), \end{aligned} \tag{3.26}$$

for $\sigma_p \approx 0$. Under these conditions, we can approximate the optimal employment policy of an individual firm out of steady state. To see how this might be done, note from the first order conditions (3.13) and (3.14) that to derive optimal employment in the presence of aggregate shocks, one must characterize the marginal effect of current employment decisions on future profits, $D(\cdot)$, out of steady state.

Proposition 6 *If a) aggregate shocks evolve according to (3.25); b) a forecast of N provides an accurate forecast of future θ; c) aggregate shocks are small ($\sigma_p \approx 0$); and d) idiosyncratic shocks evolve according to (3.17), then the marginal effect of current employment on future profits is given by*

$$D(n, x; N, p, \sigma_p) \approx D(n, x; N^*, p, 0) + D_N^* (N - N^*), \tag{3.27}$$

where D_N^* is a known function of the parameters of the forecast equation (3.26) and the steady state employment policy defined in (3.13) and (3.14).

Proposition 6 shows that, in the presence of aggregate shocks, the forward looking component to the firm's decision, $D(n, x; N, p, \sigma_p)$, is approximately equal to its value in the absence of aggregate shocks, $D(n, x; N^*, p, 0)$, plus a known function of the deviation of aggregate employment from steady state, $D_N^* (N - N^*)$. Practically, Proposition 6 allows us to derive analytically an approximate solution for the optimal policy function in the presence of aggregate shocks, for given values of the parameters of the forecast equation (3.26).

To complete our description of the dynamics of the model, we need to aggregate the microeconomic behavior summarized in the employment policies of individual firms. A simple extension of the result of Proposition 5 implies that the aggregate number of separations and hires in the

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[16] Examples of other studies that have exploited the fact that aggregate shocks are small include Mortensen and Nagypal (2007) and Gertler and Leahy (2008).

economy at a point in time are respectively given by:

$$S(N,p) = \lambda \int [1 - H_{-1}(n_{-1})] \tilde{G}[R(n_{-1};N,p)]\, dn_{-1},$$
$$M(N,p) = \lambda \int H_{-1}(n_{-1}) \left(1 - \tilde{G}[R_v(n_{-1};N,p)]\right) dn_{-1}, \tag{3.28}$$

where $H_{-1}(n_{-1})$ is the distribution of lagged employment across firms. Notice that the timing is emphasized in the out of steady state case.

A number of observations arise from this. First, the aggregate flows depend on the level of aggregate employment, N. Recalling the accumulation equation for N yields:

$$N = N_{-1} + M(N,p) - S(N,p). \tag{3.29}$$

It follows that, to compute aggregate employment, all one need do is find the fixed point value of N that satisfies equation (3.29). This allows us to compute equilibrium labor market tightness by noting that

$$f(\theta) = M/(L - N). \tag{3.30}$$

A second observation from equation (3.28) is that, in order to compute the path of aggregate unemployment flows, and hence employment, we need to describe the evolution of the distribution of employment across firms, $H(n)$. It turns out that the evolution of $H(n)$ can be inferred by a simple extension of the discussion following Proposition 5. Recall that the change in the mass $H(n)$ over time is simply equal to the inflows less the outflows from that mass. Following the logic of Proposition 5 provides a difference equation for the evolution of $H(n)$:

$$H(n) = H_{-1}(n) + \lambda \tilde{G}[R(n;N,p)][1 - H_{-1}(n)] - \lambda \left(1 - \tilde{G}[R_v(n;N,p)]\right) H_{-1}(n). \tag{3.31}$$

This allows us to update the aggregate flows $S(N,p)$ and $M(N,p)$ over time, and hence derive the evolution of equilibrium employment.

The previous results allow us to compute the evolution of aggregate employment and labor market tightness for a given configuration of the parameters of the forecast equations (3.26). This of course does not guarantee that those parameters are consistent with the behavior that they induce. To complete our characterization of equilibrium in the presence of aggregate shocks, we follow Krusell and Smith and iterate numerically over the parameters $\{\nu_N, \nu_p, \theta_N, \theta_p\}$ to find the fixed point. In the simulations of the model that follow, the fixed point of the conjectured forecast equations in (3.26) provides a very accurate forecast in the sense that the R^2s of regressions based on (3.26) exceed 0.999.

3.5 Quantitative Applications

The model of sections 2 and 3 yields a rich set of predictions for both the dynamics and the cross-section of the aggregate labor market. In this section we draw out these implications in a range of quantitative applications, including the cross sectional distributions of establishment size and employment growth, the amplitude and propagation of unemployment fluctuations, the relationship between vacancies and unemployment in the form of the Beveridge curve, the dynamics of the distribution of establishment size, and the employer size-wage effect.

3.5.1 Calibration

Our calibration strategy proceeds in two stages. The first part is very conventional, and mirrors the approach taken in much of the literature. The time period is taken to be equal to one week, which in practice acts as a good approximation to the continuous time nature of unemployment flows. The dispersion of the innovation to aggregate labor productivity σ_p is set to match the standard deviation of the cyclical component of output per worker in the U.S. economy of 0.02.

We assume that the matching function is of the conventional Cobb-Douglas form, $M = \mu U^{\phi} V^{1-\phi}$, with matching elasticity ϕ set equal to 0.6, based on the estimates reported in Petrongolo and Pissarides (2001).[17] A weekly job finding rate of $f = 0.1125$ is targeted to be consistent with a monthly rate of 0.45. As in Pissarides (2007), we target a mean value of the vacancy–unemployment ratio of $\theta = 0.72$. Noting from the matching function that $f = \mu\theta^{1-\phi}$, the latter implies that the matching efficiency parameter $\mu = 0.129$ on a weekly basis.

Vacancy costs c are targeted to generate per worker hiring costs c/q equal to 14 percent of quarterly worker compensation. This is in accordance with the results of Silva and Toledo (2007), who use the Saratoga Institute's (2004) estimate of the labor costs of posting vacancies. In the context of the model, this implies a value of c approximately equal to 0.27 of the average worker's wage.[18]

[17] An issue that can arise when using a Cobb–Douglas matching function in a discrete time setting is that the flow probabilities f and q are not necessarily bounded above by one. This issue does not arise here due to the short time period of one week.

[18] We want to equate the per worker hiring cost c/q to 14 percent of quarterly wages, $0.14 \cdot [12 \cdot E(w)]$. Note that

To pin down worker bargaining power η we target the elasticity of average wages of newly hired workers with respect to output per worker to be equal to 0.94, based on the results of Haefke et al. (2007).[19] Inspection of the wage bargaining solution in (3.10) reveals that increased worker bargaining power leads to greater comovement between the bargained wage and aggregate labor productivity, and hence more cyclical wages.

The production function parameter α is determined by targeting an aggregate labor share based on the estimates reported in Gomme and Rupert (2007). These suggest a labor share for market production of 0.72. To complete the first part of our calibration, we choose the size of the labor force L to match a mean unemployment rate of 6.5 percent. Given the remainder of the calibration that follows, this is equivalent to choosing the labor force to match a weekly job-finding rate of 0.1125.

Idiosyncratic Shocks and the Value of Unemployment A more distinctive feature of our strategy is the calibration of the evolution of idiosyncratic firm productivity and the flow payoff from unemployment to a worker. For the former, we modify slightly the production function in sections 2 and 3 to incorporate time invariant firm specific productivity, denoted by φ, so that $y = p\varphi x F(n)$. Firm specific fixed effects φ are introduced to reflect permanent heterogeneity in firm productivity that is unrelated to the uncertainty that individual firms face over time in the form of the innovation x.

An important feature of the model of sections 2 and 3 is that it allows a flexible specification of the distribution of shocks. This is useful because conventional parameterizations, such as log-normal shocks, fail to capture the well-known Pareto shape of the cross sectional distribution of firm size. Reacting to this, we set $\varphi \sim Pareto(\varphi_m, k_\varphi)$ and $x \sim Pareto(x_m, k_x)$.[20] The minimum value of the fixed effect φ_m is chosen to yield a minimum establishment employment level of one worker, and its shape coefficient k_φ is chosen to match a mean establishment size of 17.25, based on data from the Small Business Administration for the years 1992 to 2006.[21] Innovations to idiosyncratic

the implied weekly job filling probability is given by $q = \mu\theta^{-\phi} = 0.129 \cdot 0.72^{-0.6} = 0.16$. Piecing this together yields $c/E(w) = 0.16 \cdot 0.14 \cdot 12 = 0.27$.

[19] We target the elasticity of the wages of newly hired workers rather than the elasticity of wages of all workers for two reasons. First, it is well known that it is the flexibility of wages of new hires that is relevant to the cyclicality of the job finding rate implied by search and matching models of the labor market (Shimer, 2004; Hall, 2005; Hall and Milgrom, 2008). Second, it is also well known that the wages of workers in ongoing relationships are rigid (see among others Card and Hyslop, 1997), which is at odds with the assumption of Nash wage setting that we employ here. Our target of an elasticity of 0.94 lies at the upper end of the range of estimates presented in Haefke et al. Our choice to target this number is therefore conservative, in the sense that it limits the amplitude of the cyclicality that the model can generate.

[20] A Pareto distributed random variable z is parameterized by a minimum value z_m and a "shape" parameter k, and has a density function given by kz_m^k/z^{k+1}.

[21] The data can be obtained from http://www.sba.gov/advo/research.

productivity x are normalized so that the mean innovation is equal to one. This implies that $x_m = 1 - k_x^{-1}$. Given this, we solve for firms' optimal employment policy using the results in sections 2 and 3 above (see Appendix A for details). An important outcome is that we can derive the steady state distribution of employment growth:

Proposition 7 *For a given time-invariant productivity,* φ, *the steady state density of employment growth,* $\delta = \Delta \ln n$, *across firms is given by:*

$$h_\Delta(\delta|\varphi) = \begin{cases} \lambda \int e^\delta n \tilde{G}' \left[R'\left(e^\delta n\right)\right] dH(n|\varphi) & \text{if} \quad \delta < 0, \\ \lambda \int \left(\tilde{G}\left[R_v(n)\right] - \tilde{G}\left[R(n)\right]\right) dH(n|\varphi) & \text{if} \quad \delta = 0, \\ \lambda \int e^\delta n \tilde{G}' \left[R_v'\left(e^\delta n\right)\right] dH(n|\varphi) & \text{if} \quad \delta > 0, \end{cases} \tag{3.32}$$

where $H(n|\varphi)$ *is the distribution of employment* n *conditional on fixed firm productivity* φ *derived in Proposition 5. The unconditional employment growth density is* $h_\Delta(\delta) = \int h_\Delta(\delta|\varphi) \, d\Phi(\varphi)$, *where* Φ *is the (known) c.d.f. of* φ.

Proposition 7 provides us with a novel approach to calibrating the remaining parameters of the process of idiosyncratic shocks, λ and k_x. There is abundant evidence on the properties of the cross sectional distribution of employment growth $h_\Delta(\delta)$ since the seminal work of Davis and Haltiwanger (1992). Empirically, this distribution is characterized by a dominant spike at zero employment growth, with relatively symmetric tails corresponding to job creation and job destruction (see, for example, Figure 1.A in Davis and Haltiwanger, 1992). Note that this is exactly the form of the employment growth distribution implied by the model in Proposition 7.

In practice, we choose λ to match the spike at zero in this distribution, and k_x to match the dispersion of employment growth. Intuitively, the cross sectional distribution of employment growth is a manifestation of the idiosyncratic shocks x across firms. The more often these shocks arrive (the higher is λ in the model), the more likely a firm is to alter its employment, and the smaller is the implied spike at zero employment growth. Likewise, the greater the dispersion of the innovations x the larger the implied adjustment that firms will make, hence determining the tails of the distribution. In practice, we target an annual spike of 37.2 percent and an annual standard deviation of employment growth of 0.416 based on data for continuing establishments from the Longitudinal Business Database for the years 1992 to 2005.[22]

The latter calibration of the process governing the evolution of idiosyncratic shocks is crucial for our calibration of workers' flow payoff from unemployment b. Since the work of Hagedorn and

[22] Thanks to John Haltiwanger, Ron Jarmin, and Javier Miranda for providing us with the tabulations from the LBD that allowed us to make these calculations.

Manovskii (2007), it has been recognized that the value of b plays a central role in determining the cyclical volatility of aggregate unemployment, and specifically the job-finding rate. Intuitively, higher values of b lead to a smaller surplus to employment relationships. As a consequence, small reductions in aggregate productivity can easily exhaust that surplus, and lead firms to cut back substantially on hiring. Since one of the quantitative applications we consider is the cyclical volatility of worker flows, the parameterization of b is key.

Our model suggests a novel approach to calibrating the payoff from unemployment: For a given level of dispersion in idiosyncratic shocks implied by our calibration of the evolution x, a higher value of b reduces the surplus and implies that jobs will be destroyed more frequently, raising the inflow rate into unemployment s. Thus, we choose b in such a way as to yield employment rents that match the empirical unemployment inflow rate of $s = 0.0078$ on a weekly basis, consistent with estimates reported in Shimer (2007).

The parameter values implied by our calibration are summarized in Table 1. In what follows, we summarize the implications of the calibrated model for a range of cross-sectional and aggregate outcomes.

3.5.2 Establishment Size and Employment Growth Distributions

An important component of our calibration strategy is to match key properties of the cross-sectional distributions of employment and employment growth across firms. The model's implications for these two outcomes are summarized in Propositions 5 and 7 above. In this section, we compare the steady-state distributions implied by the model with their empirical counterparts.

Figure 2 plots the distribution of establishment size in the calibrated model and recent data. Both axes are on a log scale to emphasize the Pareto shape of the distributions. The dots plot the empirical establishment size distribution using pooled data from the Small Business Administration on employment by firm size class for the years 1992 to 2006. The dashed line indicates the analogue implied by the calibrated model. Figure 2 reveals that the model accounts well for the empirical establishment size distribution. While this outcome is not surprising given the Pareto shocks fed through the model, it does highlight the benefit of using a flexible form for the distribution of idiosyncratic productivity in the model of sections 2 and 3.

What is perhaps more surprising is that the model also does a remarkable job of matching the distribution of employment growth across establishments. The dotted line in Figure 3 illustrates the empirical employment growth distribution using data for continuing establishments from the

Longitudinal Business Database. As noted above, this displays the classic features of a mass point at zero employment growth, and relatively symmetric tails. The dashed line overlays the employment growth distribution implied by the calibrated model. This bears a very close resemblance to the empirical distribution. This is more noteworthy than it might at first appear: While the use of Pareto shocks was informed by the character of the establishment size distribution in Figure 2, the message of Figure 3 is that it also provides a remarkably good account of the employment growth distribution, something that has not been emphasized in the literature on establishment dynamics.

3.5.3 The Cyclicality of Worker Flows

It is now well-known that standard search models of the aggregate labor market cannot generate enough cyclical amplitude in unemployment, and in particular the job finding rate, to match that observed in U.S. data (Shimer, 2005). A natural question is whether the generalized model analyzed here can alleviate this problem. To address this, we feed through a series of shocks to aggregate labor productivity using equation (3.25), and simulate the implied dynamic response of the model using the results of section 3. Following Mortensen and Nagypal (2007a), we then use these simulated time series to compute the model-implied elasticities of labor market stocks and flows with respect to output per worker, and compare them with their empirical counterparts.

Model Outcomes Panel A of Table 2 summarizes the results of this exercise. Outcomes in brackets are moments that the model is calibrated to match: The mean levels of the job-finding rate f, the unemployment inflow rate s, and the vacancy-unemployment ratio θ. The aim of the exercise is to draw out the implications of the model for the outcomes that the model is *not* calibrated to match, i.e. the cyclical elasticities of these outcomes with respect to output per worker.

The results in Table 2.A are remarkably encouraging: On all dimensions, the model-implied elasticities lie in a neighborhood close to the cyclicality observed in the data. Specifically, the model implies an elasticity of the job finding rate of 2.75, a little above its empirical analogue of 2.65.[23] In addition, the model-generated cyclical elasticity of the unemployment inflow rate of -1.68 lies only a little below the magnitude observed in the data.

Comparison with Mortensen and Pissarides (1994) These results make substantial progress relative to the standard Mortensen and Pissarides (1994) model. To see this, panels

[23]The cyclical elasticities reported in Table 2 differ slightly from those implied by Shimer (2005) and Mortensen and Nagypal (2007a) as a result of revisions to U.S. GDP data.

B and C of Table 2 provide two comparison exercises.[24] First, taking as given the process for idiosyncratic shocks implied by the distribution of employment growth derived above, we calibrate the standard model to match the mean levels of the job finding rate f and the unemployment inflow rate s, as well as the elasticity of s with respect to output per worker implied by the generalized model in panel A. This allows the model to speak to the implied elasticity of the job finding rate, and thereby the elasticities of vacancies and labor market tightness. The outcomes in panel B confirm what Shimer (2005) demonstrated: that the standard model is unable to generate enough cyclicality in job creation.[25] The model-implied elasticity of the job-finding rate is 1.29, less than one half of the empirical elasticity. In contrast, the generalized model studied in the present paper can account for all of the observed cyclical comovement between f and output per worker.

The Role of the Payoff from Unemployment Panel C of Table 2 provides a new perspective on the standard model's inability to match the cyclicality of unemployment flows. In this case, we again take as given the process for idiosyncratic shocks implied by the empirical distribution of employment growth. However, instead of targeting the mean level of the inflow rate into unemployment s, we now allow the standard model to match the elasticity of the job finding rate f generated by the model of sections 2 and 3, and then draw out the implications for s. Panel C reveals that the model must dramatically overstate the magnitude of unemployment inflows in order to match the cyclical comovement of f: The implied weekly inflow rate of 0.0185 is more than double that observed in the data.

This result sheds light on a recent debate in the literature. In order to match the cyclical variation in the job finding rate, the standard model requires a small surplus to employment relationships, a point emphasized by Hagedorn and Manovskii (2007) and Mortensen and Nagypal (2007a).[26] Mortensen and Nagypal further argue that the required surplus is unrealistically small. The results of Table 2 formalize this intuition: For realistic variation in idiosyncratic shocks to firms, a surplus small enough to match the cyclicality of f implies an employment to unemployment transition rate that is more than double what is observed empirically. Intuitively, a small surplus implies that small idiosyncratic shocks to employment relationships are enough to exhaust

[24]In practice, we use the version of the Mortensen and Pissarides model developed by Mortensen and Nagypal (2007b). This modifies the original model to allow for a distribution of idiosyncratic shocks without an upper bound, such as the Pareto shocks we use in the generalized model.

[25]Shimer's (2005) calibration of the standard model with exogenous job destruction yields an elasticity of f equal to 0.48. Mortensen and Nagypal (2007a) favor a different calibration that yields an elasticity of f equal to 1.56 (see their section 3.2). Pissarides' (2007) calibration of the standard model with endogenous job destruction obtains an elasticity of f equal to 1.54.

[26]A common diagnostic for the size of the flow surplus is the ratio between worker's payoff from unemployment, b, and the average product of labor. For panel A of Table 2, this ratio equals 0.514; for panel C, it equals 0.632. Thus, the Mortensen and Pissarides model demands a smaller surplus to match the volatility of the job finding rate.

the surplus and lead to destruction of a match. Consequently, realistic dispersion in idiosyncratic shocks generates excessive worker turnover.

Thus, the Mortensen and Pissarides model faces a tension: To match plausible levels of unemployment inflows, the model must generate a sufficiently large surplus at the expense of matching the cyclicality of the job finding rate. Conversely, to generate the cyclical variation in the job finding rate, the surplus must be small, which in turn yields excessive employment to unemployment transitions.

Understanding Amplification The results in Table 2 raise the question of why the generalized model yields amplification of the response of job creation to cyclical shocks. The following result provides a sense for where this amplification comes from by approximating the steady-state response of job creation to a change in aggregate labor productivity:

Proposition 8 *For small* λ, *the shift in the Job Creation condition (3.22) induced by a change in aggregate productivity* p *is given approximately by*

$$\left.\frac{d\ln\theta}{d\ln p}\right|_{JC} \approx \frac{(1-\eta)\,\tilde{p}}{\omega\phi\left[(1-\eta)\,(\tilde{p}-b)-\eta\beta c\theta\right]+\eta\beta c\theta}, \tag{3.33}$$

where ω *is the steady state employment share of hiring firms, and* $\tilde{p}\equiv\rho\overline{apl}+(1-\rho)\,\overline{mpl}$ *where* $\overline{apl}$ *and* $\overline{mpl}$ *are respectively the average and marginal product of labor of the average–sized firm, and* $\rho\equiv\frac{\alpha\eta}{1-\eta(1-\alpha)}$.

Corollary *The elasticity of the vacancy–unemployment ratio to aggregate productivity in the* $\alpha=1$ *case (Mortensen and Pissarides, 1994) is approximately equal to*

$$\frac{d\ln\theta}{d\ln p} \approx \frac{(1-\eta)\,p}{\phi\left[(1-\eta)\,(p-b)-\eta\beta c\theta\right]+\eta\beta c\theta}. \tag{3.34}$$

Equation (3.34) echoes results presented in Mortensen and Nagypal (2007a,b): The cyclical response of the vacancy-unemployment ratio θ is amplified when the average flow surplus to employment relationships, $p-b$, is small. Equation (3.33) generalizes this result to the model studied here. Inspection of (3.33) and (3.34) reveals that there are two channels through which the generalized model yields amplification of the cyclicality of labor market tightness. First, the effective surplus that matters for amplification is now given by $\tilde{p}-b$, a weighted average of the average and marginal flow surpluses. This lies below the average flow surplus as a result of the diminishing marginal product of labor in the model.

This provides a sense for why the generalized model is able simultaneously to match the rate at which workers flow into unemployment s, as well as the volatility of the job-finding rate f over the cycle: The former requires a large average surplus; the latter requires a small marginal surplus.[27] The standard Mortensen and Pissarides model cannot achieve this because of its inherent linearity.

Equation (3.33) also suggests that there is an additional effect at work in the form of the variable ω, the steady state employment share of hiring firms. To understand the significance of this term, note that in the standard Mortensen and Pissarides model where $\alpha = 1$, ω is equal to one: With a linear technology, a firm that reduces its employment will shed all of its workers since, if one worker is unprofitable at a firm, all workers are unprofitable. As a result, all surviving firms at a point in time are hiring firms in the standard model. In contrast, in the generalized model, shedding firms do not reduce their employment to zero because reducing employment replenishes the marginal product of labor. Hence ω will be less than unity, and inspection of (3.33) and (3.34) reveals that this will lead to greater amplification relative to the standard model.[28] Intuitively, it is as if the economy has to rely on a smaller mass firms to hire workers from the unemployment pool, which in turn leads to a larger increase in unemployment in times of recession.

3.5.4 The Beveridge Curve

Until now, we have been concentrating on the cyclicality of worker flows implied by the generalized model. Readers of Shimer (2005), however, will recall that the standard search and matching model also fails to match the observed cyclical volatility in the vacancy rate in the U.S., and especially so if one allows job destruction to move countercyclically.

Table 2 reiterates this message: While vacancies are markedly procyclical, with an empirical elasticity with respect to output per worker of 2.91, calibration of the Mortensen and Pissarides model yields a *counter*cyclical vacancy elasticity.[29] Shimer (2005) has emphasized that this feature of the standard search and matching model in turn leads to a dramatic failure to account for a key stylized fact of the U.S. labor market: the negative relation between vacancies and unemployment, known as the Beveridge curve.

[27] Mortensen and Nagypal (2007a) favor an average flow match surplus of $[b/(Y/N)]^{-1} - 1 = \frac{1}{0.73} - 1 = 37$ percent. The corresponding value implied by our calibration is $\frac{1}{0.61} - 1 = 64$ percent. The worker's surplus in our simulation is also substantial: Workers obtain a $(E[w] - b)/b = 18$ percent flow surplus from employment over unemployment.

[28] The reader may worry whether $(1-\eta)(\tilde{p} - b) - \eta\beta c\theta$ is positive or not. To see that it is, note that we can rewrite it as $(1-\eta)\left(\frac{p\alpha x n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right) - \eta\beta c\theta$, and observe from equations (3.13) and (3.14) that it is, in fact, the marginal flow surplus of a firm, and therefore must be positive.

[29] This arises because countercyclical job destruction leads to an offsetting increase in hires in times of recession to maintain balance between unemployment inflows and outflows, and thereby stymies the procyclicality of vacancies (Shimer, 2005; Mortensen and Nagypal, 2007b).

Figure 4 plots the Beveridge curve relation from model-generated data (the hollow circles), and compares it with the empirical analogue using vacancy data from the Job Openings and Labor Turnover Survey (the dots). While the model-generated Beveridge curve has a slightly shallower slope, it nonetheless lies very close to the array of observations witnessed in recent data. This can be traced to the results in Table 2: The cyclicality of vacancies in the model lies very close to its empirical counterpart, with a cyclical elasticity of 2.75 lying only a little below the value of 2.91 in the data.

What emerges from Table 2 and Figure 4 is a coherent and quantitatively accurate picture of the joint cyclical properties of both flows of workers in and out of unemployment, as well as the behavior of unemployment and vacancies. In addition to providing a plausible mechanism for the cyclical amplitude of the job-finding rate, the model also presents an environment in which this can be reconciled with the cyclical behavior of job destruction and vacancy creation.

3.5.5 Propagation

A less well–documented limitation of the standard search and matching model relates to the propagation of the response of equilibrium labor market tightness to aggregate shocks to labor productivity. In the Mortensen and Pissarides model, the vacancy-unemployment ratio is a jump variable and therefore moves contemporaneously with changes in labor productivity. Empirically, however, the vacancy–unemployment ratio displays sluggish behavior, and is much more persistent than aggregate labor productivity, a point emphasized by Shimer (2005) and Fujita and Ramey (2007).

An appealing feature of the model presented in sections 2 and 3 is that it admits a natural channel for the propagation of the response of the vacancy-unemployment ratio to aggregate shocks. The determination of θ over time depends on the evolution of the distribution of employment across establishments $H(n)$. Inspection of equation (3.31), the law of motion for the distribution of employment across firms, reveals that $H(n)$ is not a jump variable, but is instead a slow moving state variable in the model. In particular, rewriting (3.31) yields

$$H(n) - H_{-1}(n) = -\vartheta(n)\left[H_{-1}(n) - H^{*}(n)\right], \tag{3.35}$$

where $\vartheta(n) \equiv \lambda\left(1 - \tilde{G}\left[R_v(n)\right] + \tilde{G}\left[R(n)\right]\right)$,[30] and $H^{*}(n)$ is the steady state distribution that sets

[30]For notational simplicity, we suppress the dependence of R, R_v, and H on the aggregate state variables N and p.

$H(n) - H_{-1}(n) = 0$.

Equation (3.35) provides an important source of intuition for what factors are likely to drive propagation in the model. In particular, the rate of convergence to steady state $\vartheta(n)$ is determined by two factors. First, less frequent idiosyncratic shocks, as implied by a lower value of λ, will cause fewer firms to adjust employment, and thereby slow the reallocation of employment across firms. Second, the size of adjustment costs will determine the gap between $R_v(n)$ and $R(n)$ in a firm's optimal employment policy function in Figure 1. Larger adjustment costs will widen this gap, reducing $\vartheta(n)$ in equation (3.35), and slowing the dynamics of $H(n)$. This suggests that the magnitude of labor adjustment costs has important implications for the propagation of the response of unemployment to aggregate shocks.

Figure 5 plots the dynamic response of unemployment, labor market tightness, the job-finding rate, and the unemployment inflow rate following a permanent one percent decline in aggregate labor productivity using simulated data from the model. This confirms that the generalized model yields some propagation of the response of unemployment and labor market tightness (and thereby the job-finding rate). It takes around 20 months for unemployment to adjust to the shock in the model, and 9 months for the response of θ and f to dissipate.

This is a substantial improvement over the instantaneous response of θ and f implied by the standard search model. However, the magnitude of the propagation implied by the model is not quite enough to account fully for the persistence of the vacancy-unemployment ratio observed in the data. In their detailed analysis of the empirical dynamics of labor market tightness, Fujita and Ramey (2007) show that θ takes around five quarters, or 15 months, to adjust to an impulse to aggregate labor productivity in U.S. data.

An additional message of Figure 5 concerns the dynamics of the unemployment inflow rate s. Panel D reveals that s spikes upward instantaneously following a reduction in aggregate labor productivity, subsides in the immediate aftermath of the shock, and then converges toward a new steady state value over the course of the next two years. In the wake of a recessionary shock a discrete mass of jobs becomes unprofitable and is destroyed immediately, mirroring the implications of the standard Mortensen and Pissarides model (Mortensen, 1994). Following the shock, the inflow rate begins rising again as firms receive productivity shocks at rate λ.

Viewed together, the joint dynamics of the job-finding and inflow rate in the model bear a remarkable resemblance to the qualitative features of the response of f and s over the cycle: Recessions are characterized by a wave of inflows which then recedes and is accompanied by persistent declines in rates of job-finding, just as observed in empirical worker flows in U.S. data (see, for example, Elsby, Michaels and Solon, 2009).

3.5.6 Extensions

The previous sections have shown that the model derived in sections 2 and 3 can account for many of the cross-sectional and cyclical features of the U.S. labor market. In this section, we push the model harder. We consider two additional outcomes which the model can speak to, but was not designed to account for: the cyclical dynamics of the cross-sectional distribution of establishment size, and the employer size-wage effect.

Cyclical Dynamics of the Employer Size Distribution Until now, we have focused separately on the implications of the model for the cross section of employers and for the aggregate dynamics of labor stocks and flows. Recent literature, however, has sought to understand the joint dynamics of the cross section. Moscarini and Postel-Vinay (2009) in particular emphasize empirical regularities in the cyclical behavior of the cross-sectional distribution of establishment size: The share of small establishments rises during recessions, while the shares of larger firms decline. Figure 6 reiterates this finding. It uses annual data from County Business Patterns for the years 1986 to 2007 on the number of establishments by employer size, and plots the log deviations from trend of the establishment size shares against the unemployment rate. The dots plot the data, and the dot-dashed lines the corresponding least squares regression lines. The share of establishments with 1 to 19 workers rises with unemployment, while the shares of establishments with more than 20 employees decline as unemployment rises.

The blue dashed lines in Figure 6 plot the analogous relationships implied by simulations of the model of sections 2 and 3. The results are very encouraging: The model replicates the observation that the share of establishments with fewer than 20 employees increases in times of recession, while the shares of the larger size classes decline. In addition, the model also provides a reasonable account of the magnitude of the cyclicality of the establishment shares. It comes very close to replicating the cyclical sensitivities for the 20 to 99 employee and the 1000+ employee groups, and implies around one half of the cyclical sensitivity of the 1 to 19 and 100 to 999 employer size classes.

The observation that the share of smaller establishments rises in a recession in both the model and the data is not in itself a surprising fact, since aggregate employment, the mean of the distribution of establishment size, falls during recessions. What is noteworthy about the model is that it replicates the position in the distribution—at around 20 employees—at which this effect takes hold, as well as the magnitude of the cyclicality in some of the size classes. Given that these implications of the model are venturing even farther afield from the moments it was calibrated to match, we view the results of Figure 6 as an important achievement.

Employer Size-Wage Effect Our final application relates to the observation that workers employed in larger firms are often paid higher wages—the employer size-wage effect noted in the influential study by Brown and Medoff (1989). An attractive feature of the model is that, by incorporating a notion of employer size, and by modeling the wage bargaining process between a firm and its many workers, it can speak to such issues.

Casual inspection of the wage equation (3.10) is not heartening in this respect, however: Due to the diminishing marginal product of labor in the model, one might anticipate that the model predicts a negative correlation between wages and employer size, in direct contrast to Brown and Medoff's observation. Further consideration of equation (3.10), though, reveals that such a conclusion would be premature: While the diminishing marginal product of labor does set in for larger employers, it is also the case that larger employers will be those with higher idiosyncratic productivity x. The implications of the model for the employer size-wage effect depend on which of these forces dominates.[31]

Figure 7 illustrates the relationship between average log wages and log employment implied by the model. It takes simulated data based on the calibration in Table 1, and plots the results of a nonparametric locally weighted (LOWESS) regression of log firm wages on log firm employment. This reveals that the model does in fact predict a positive employer-size wage effect, qualitatively in line with the results of Brown and Medoff. As it turns out, the effect of higher idiosyncratic productivity outweighs the effect of diminishing marginal product.

Figure 7 also provides a sense of the magnitude of the size-wage effect. Brown and Medoff (1989) report that, controlling for observable and unobservable measures of labor quality and for differences in workplace conditions, a worker moving from an establishment with log employment one standard deviation below average to an establishment with log employment one standard deviation above average would receive a wage increase of around 10 percent. As shown in Figure 7, the counterpart implied by the model is a wage premium closer to 2.5 percent.

Thus, while the model yields a positive size-wage effect, it generates only around one quarter

[31] Bertola and Garibaldi (2001) also include a discussion of this point. Their model is somewhat less general than that presented in this paper, however. Like Bertola and Caballero (1994), the authors analyze a linear approximation to the marginal product function. In addition, a more restrictive process for productivity shocks is used: It is assumed that, if a firm receives a negative disturbance, its productivity reverts to the minimum of the distribution's support. Since these transitions are the only means by which the model generates inflows into unemployment, all firms that receive a negative shock must shed workers; inaction is, by assumption, never an optimal response for these employers.

of the magnitude of the effect observed in the data. We do not view this necessarily as a problem, however. The mechanism that accounts for the size-wage effect in the model—the interaction of surplus sharing with heterogeneity in employer productivity—is only one of a large number of proposed channels. Oi and Idson (1999) present a summary of these, including efficiency wages, market power, specific human capital, among others. In a model of wage posting, Burdett and Mortensen (1998) demonstrate that on-the-job search in the presence of labor market frictions also can generate a positive employer size-wage effect, as higher-paying firms recruit and retain more workers. The results of Figure 7 suggest that the model presented in this paper leaves room for these additional explanations.

3.6 Summary and Discussion

In this paper, we have introduced a notion of firm size into a search and matching model with endogenous job destruction. This yields a rich, yet analytically tractable framework. In a series of quantitative applications, we show that the model provides a useful laboratory for analyzing the salient features of both the dynamics and the cross section of the aggregate labor market. Specifically, a calibrated version of the model provides a coherent account of the distributions of establishment size and employment growth; the amplitude and propagation of the cyclical dynamics of worker flows; the Beveridge curve relation between unemployment and vacancies; and the dynamics of the distribution of firm size over the business cycle.

A number of avenues arise naturally in the light of this. First, the model has a well-defined concept of a firm and so lends itself to estimation using establishment level data. As a result, the analytical framework developed here will complement recent research efforts that have sought to solve and estimate search models using numerical methods (e.g. Cooper, Haltiwanger and Willis, 2007).

Second, our interpretation of the standard search and matching model as a model of kinked adjustment costs raises the question of the aggregate implications of other forms of adjustment costs in the labor market. Recent research has emphasized the importance of fixed adjustment costs in explaining the empirical properties of labor demand at the micro level (see for example Caballero, Engel, and Haltiwanger, 1997, and Cooper, Haltiwanger, and Willis, 2004). Incorporating these adjustment costs into the model will provide a unification of the joint insights of the two dominant approaches to the modelling of aggregate labor markets—the search and matching framework, and models of adjustment costs.

A final extension relates to the nature of wage setting. An attractive feature of incorporating firm size into models of the labor market is that an assessment of the multilateral dimension to wage bargaining between a firm and its many workers becomes feasible. This has been of particular interest in recent literature that has emphasized the importance of rigidities in the structure of wages within a firm, as well as of individual wages over time, for determining the volatility of unemployment (Bewley, 1999; Hall, 2005). While the wage bargaining solution derived in the present paper seeks to improve upon approaches in previous work, it is in many ways an idealized environment in which the wages of all workers can be renegotiated costlessly. This idealized setting, however, provides a fruitful benchmark for analyzing the implications of rigidities in renegotiation of wages within a firm, as well as across time.

Table 3.1: Calibrated Model Parameters

Parameter	Meaning	Value	Reason
ϕ	Matching elasticity	0.600	Petrongolo and Pissarides (2001)
μ	Matching efficiency	0.129	Pissarides (2007)
α	$F(n)=n^{\alpha}$	0.590	Labor share = 0.72
β	Discount factor	0.999	Quarterly interest rate = 0.012
b	Value of leisure	0.385	Mean inflow rate = 0.0078
c	Flow vacancy cost	0.120	Hiring cost = 14% quarterly wage
η	Worker bargaining power	0.443	Cyclicality of new hire's wage
L	Labor force	18.50	Mean job-finding rate = 0.1125
λ	Arrival rate of x	0.043	LBD data: $\Pr(\Delta \ln n = 0) = 0.372$
$\overline{x}$	Mean of x	1.000	Normalization
σ_x	Std. dev. of x	0.250	LBD data: $\sigma(\Delta \log n) = 0.416$
$\overline{\varphi}$	Mean of φ	1.177	Mean employment = 17.25
σ_{φ}	Std. dev. of φ	1.015	Minimum employment = 1

NOTE: Consistent with the timing of the model, flow parameters are reported at a weekly frequency. First and second moments of fixed firm productivity φ and the innovation to firm productivity x are reported (rather than the parameters of the respective Pareto distributions) for ease of interpretation.

Table 3.2: Cyclicality of Worker Flows: Model vs. Data

Model / Outcome	**Mean Level**		**Elasticity w.r.t. output per worker**	
	Data	*Model*	*Data*	*Model*
A. Generalized				
Job Finding Rate, *f*	[0.1125]	[0.1125]	2.65	2.75
Inflow Rate, *s*	[0.0078]	[0.0078]	-1.89	-1.68
Vacancies, *V*	--	--	2.91	2.75
Tightness, $\theta = V/U$	[0.72]	[0.72]	6.44	6.88
B. MP (*i*)				
Job Finding Rate, *f*	[0.1125]	[0.1125]	2.65	1.29
Inflow Rate, *s*	[0.0078]	[0.0078]	-1.89	[-1.68]
Vacancies, *V*	--	--	2.91	-0.478
Tightness, $\theta = V/U$	[0.72]	[0.72]	6.44	2.29
C. MP (*ii*)				
Job Finding Rate, *f*	[0.1125]	[0.1125]	2.65	[2.75]
Inflow Rate, *s*	[0.0078]	0.0184	-1.89	[-1.68]
Vacancies, *V*	--	--	2.91	-0.032
Tightness, $\theta = V/U$	[0.72]	[0.72]	6.44	3.76

NOTE: Outcomes reported in brackets are calibrated. Non-bracketed outcomes are implied by the respective model. Flow outcomes are reported on a weekly basis. Empirical elasticities for *f* and *s* are computed using quarterly averages of the job-finding rate and the unemployment inflow rate from 1948Q1 to 2007Q1 derived in Shimer (2007). Following Shimer (2005), series are detrended using a Hodrick-Prescott filter with smoothing parameter 10^5. Following Mortensen and Nagypal (2007a), elasticities with respect to output per worker are obtained by regressing the log deviation from trend of *f* and *s* on the log deviation from trend of non-farm business output per worker obtained from the Bureau of Labor Statistics. Outcomes for the Mortensen and Pissarides model in panels B and C are generated from Mortensen and Nagypal's (2007b) modification of the model to allow for unbounded idiosyncratic shocks.

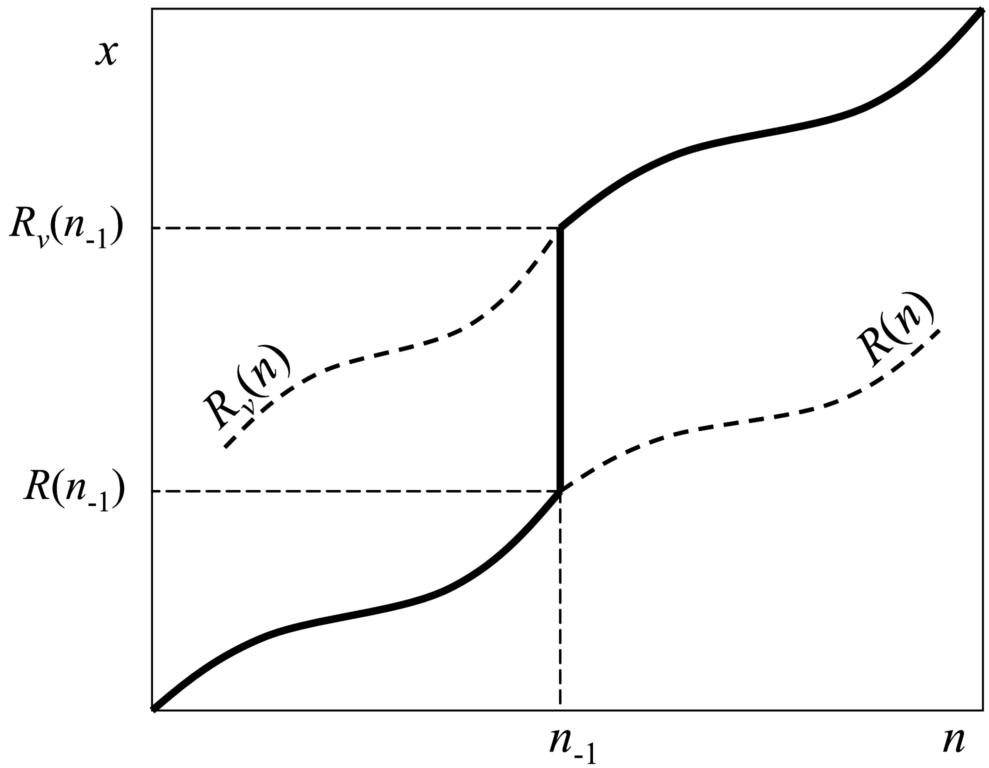
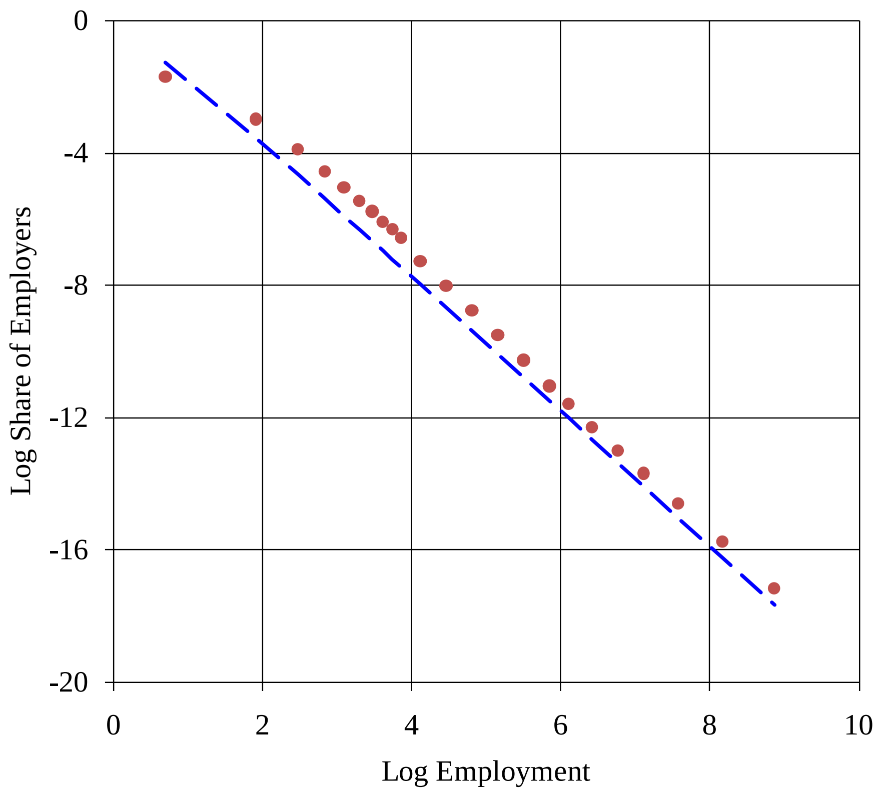
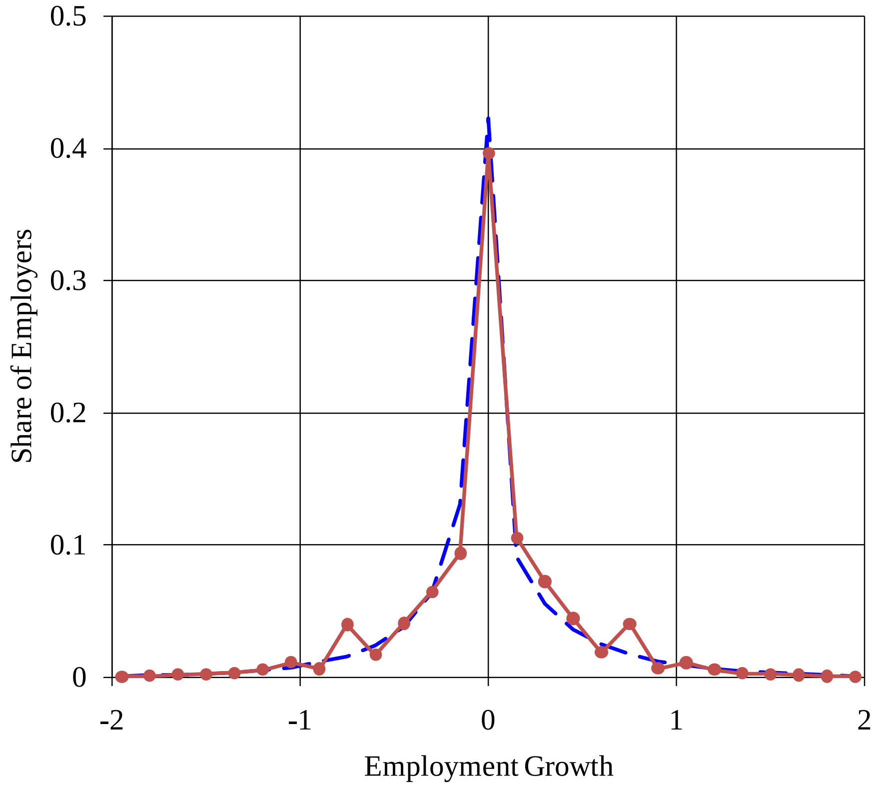


Figure 3.1: Optimal Employment Policy of a Firm



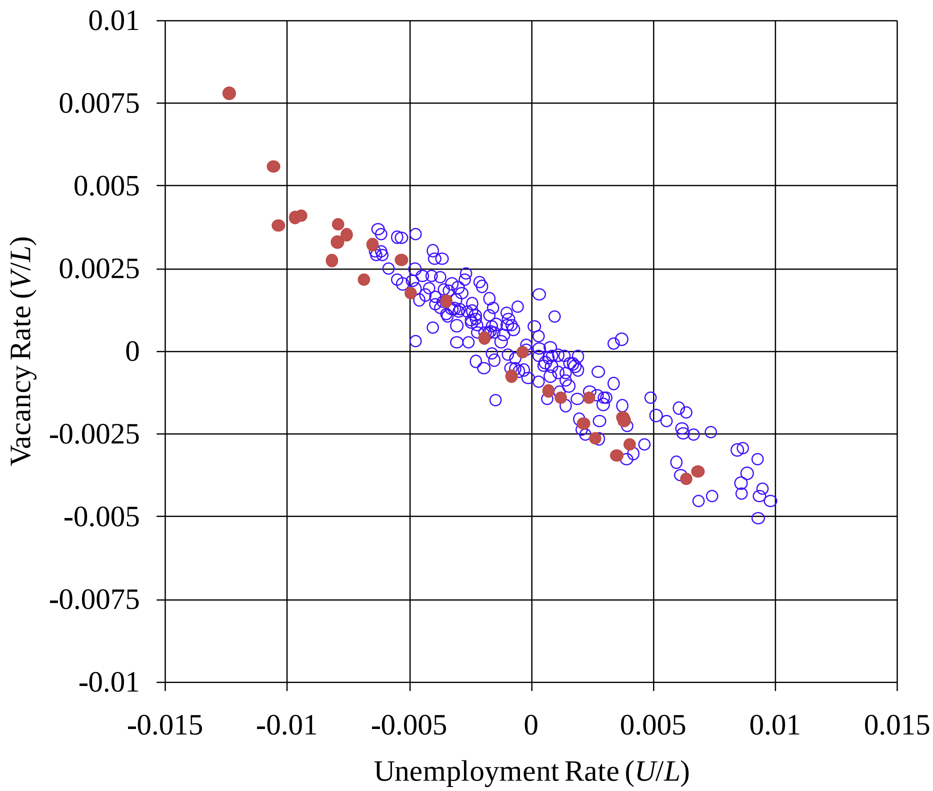
NOTE: The dots plot data on the shares of firms in successive employment categories for the years 1992 to 2006 based on data on employment by firm size class from the Small Business Administration. The dashed line plots the steady state distribution of employment across firms implied by the generalized model using the parameters reported in Table 3.1.

Figure 3.2: Employer Size Distribution: Model vs. Data



NOTE: The dotted line plots the cross sectional distribution of employment growth based on data for continuing establishments from the Longitudinal Business Database pooled over the years 1992 to 2005. The dashed line plots the steady state distribution of employment growth in the model using the parameters reported in Table 3.1.

Figure 3.3: Employment Growth Distribution: Model vs. Data



NOTE: The dots plot job openings as a fraction of the labor force against the unemployment rate using quarterly averaged data from the Job Openings and Labor Turnover Survey and the Bureau of Labor Statistics from 2001Q1 to 2007Q4. The hollow circles plot the analogous series using simulated data from the model. Series are plotted as deviations from their temporal means.

Figure 3.4: Beveridge Curve: Model vs. Data

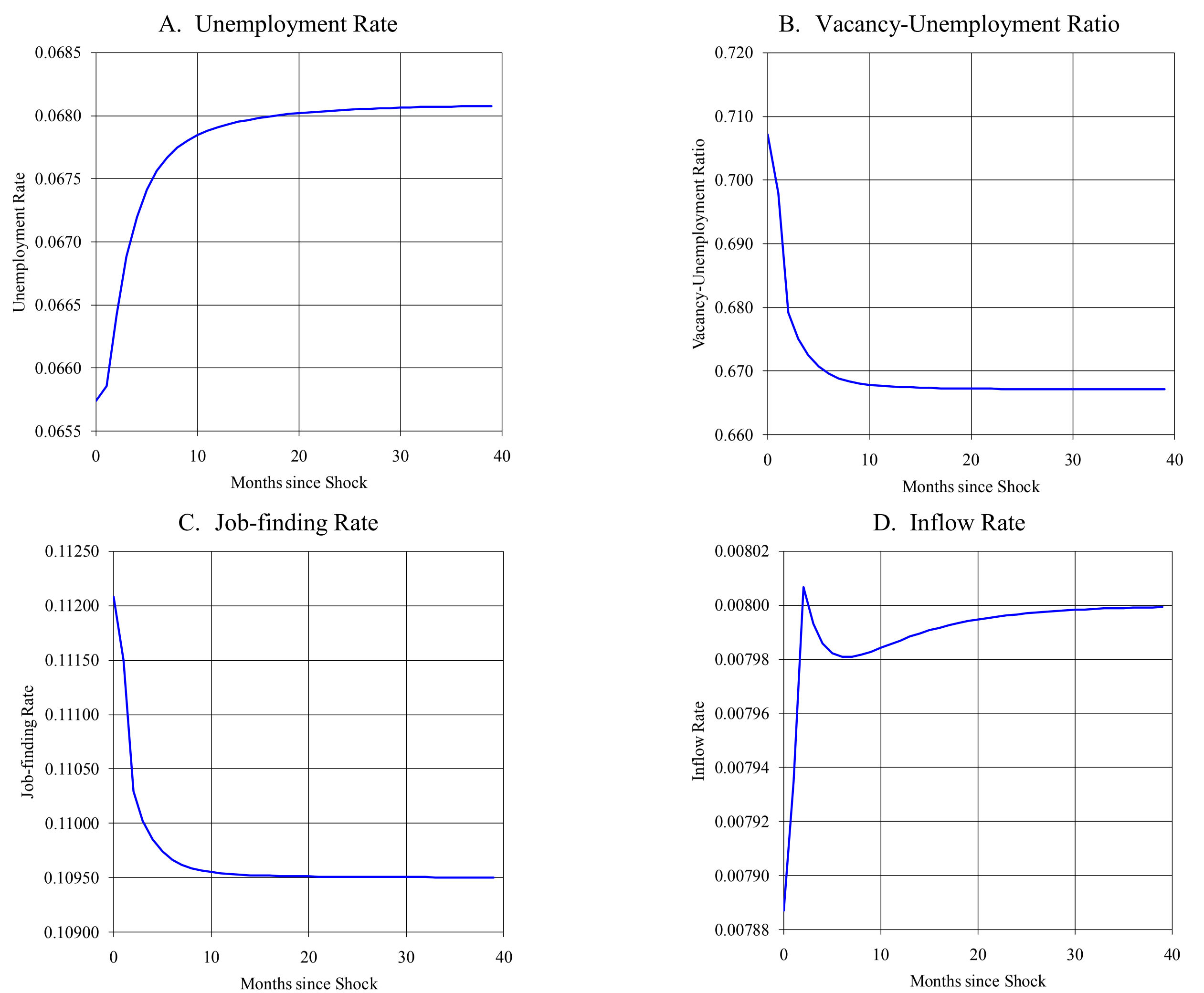
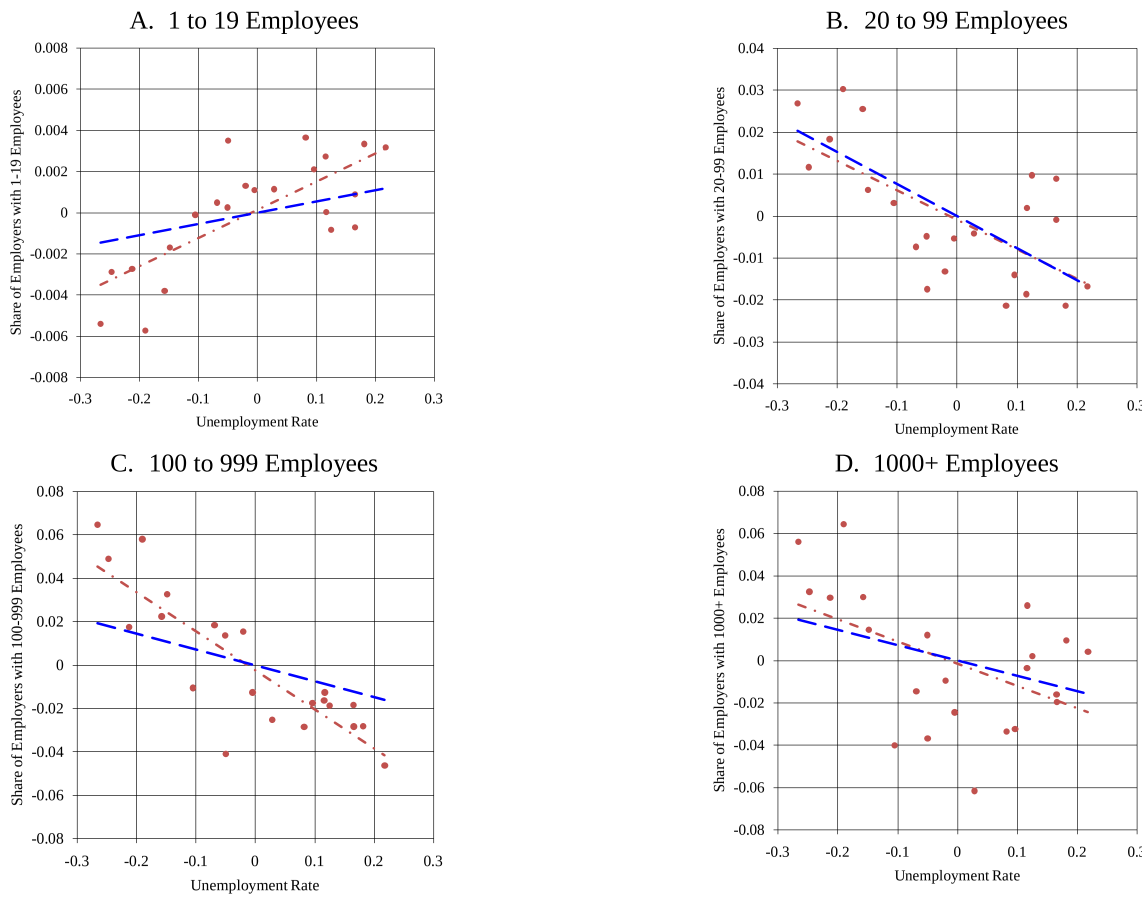
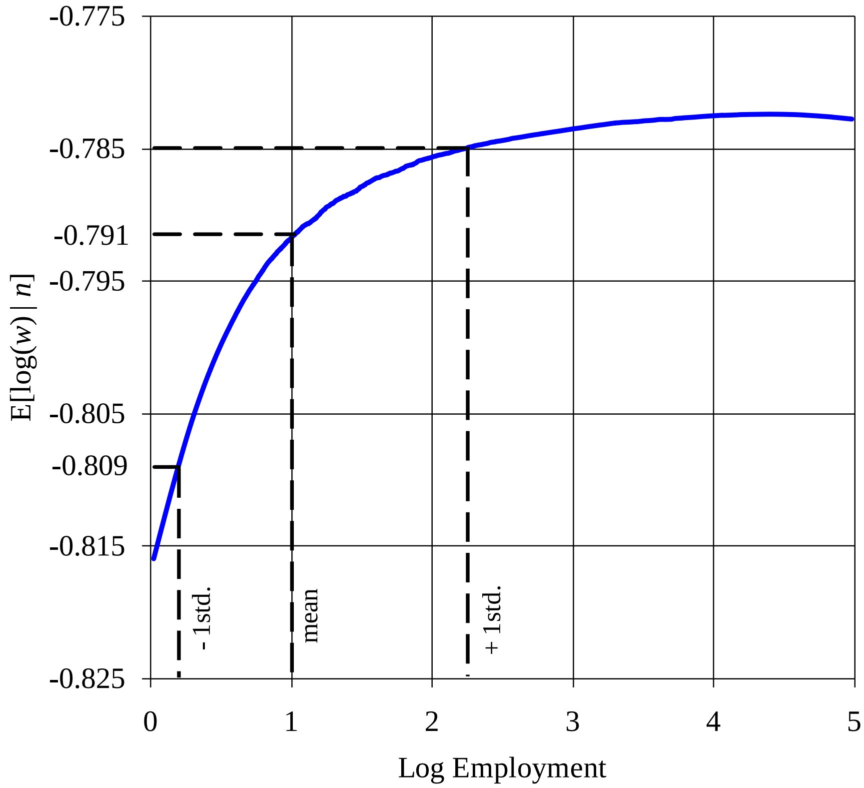


Figure 3.5: Model Impulse Responses to a Permanent 1% Decline in Aggregate Labor Productivity, p



NOTE: Dots plot the log deviation from trend of each employer size class share against the log deviation from trend of the unemployment rate. The dot-dash lines are fitted least squares regression lines based on the data. The dashed line is the analogous relationship implied by the calibrated model. Employer size data are taken from County Business Patterns for the years 1986 to 2007. Annual unemployment rate data are from the BLS. Given the short time series, simple linear time trends are used.

Figure 3.6: Cyclical Dynamics of the Employer Size Distribution: Model vs. Data



NOTE: The solid line plots the mean log wage, conditional on employer size, as a function of log employer size derived from simulations of the steady state of the model using the parameters reported in Table 3.1. Simulated data were generated for 208 periods (four years); plotted series are based on the final period.

Figure 3.7: Employer Size-Wage Effect implied by the Model

3.7 Appendix

3.8 Solution of the Simulated Model

Here we present technical details of the solution to the model in sections 2 and 3 for the purposes of the quantitative applications in section 4. For simplicity, we present the solution approach for a given fixed firm productivity φ, so we supress this notation in what follows. Aggregation across φ is achieved simply by integrating over the known distribution φ.

Steady State Optimal Employment Policy Idiosyncratic shocks evolve according to (**??**) with $x \sim Pareto\left(1-k_x^{-1}, k_x\right)$. Denoting the distribution function of x as $\tilde{G}$, we can rewrite the recursion for the function $D\left(n,x\right)$ in Proposition 3 as:

$$\begin{aligned} D\left(n,x\right) &= (1-\lambda)\,\chi\left(x\right)+\lambda\int_{R(n)}^{R_v(n)}\chi\left(x'\right)d\tilde{G}\left(x'\right)+\lambda\int_{R_v(n)}^{\infty}\frac{c}{q}d\tilde{G}\left(x'\right) \\ &\quad +\beta\left(1-\lambda\right)D\left(n,x\right)+\beta\lambda\int_{R(n)}^{R_v(n)}D\left(n,x'\right)d\tilde{G}\left(x'\right), \end{aligned} \tag{3.36}$$

where $\chi\left(x\right)\equiv\left(1-\eta\right)\left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)}-b\right]-\eta\beta f\frac{c}{q}$. We conjecture that the function $D\left(n,x\right)$ is of the form $D\left(n,x\right)=d_0+d_1\chi\left(x\right)$. Substituting this into the latter, and equating coefficients, we obtain the following solution for $D\left(n,x\right)$:

$$\begin{aligned} D\left(n,x\right) &= \frac{1-\lambda}{1-\beta\left(1-\lambda\right)}\chi\left(x\right) \\ &\quad +\frac{\lambda}{1-\beta\left(1-\lambda\right)}\frac{\tilde{G}\left[R_v\left(n\right)\right]-\tilde{G}\left[R\left(n\right)\right]}{1-\beta\left(1-\lambda\right)-\beta\lambda\left(\tilde{G}\left[R_v\left(n\right)\right]-\tilde{G}\left[R\left(n\right)\right]\right)}\mathcal{Q}\left(n\right) \\ &\quad +\frac{1-\tilde{G}\left[R_v\left(n\right)\right]}{1-\beta\left(1-\lambda\right)-\beta\lambda\left(\tilde{G}\left[R_v\left(n\right)\right]-\tilde{G}\left[R\left(n\right)\right]\right)}\lambda\frac{c}{q}, \end{aligned} \tag{3.37}$$

where $\mathcal{Q}\left(n\right)\equiv E\left(\chi\left(x'\right)|x'\in\left[R\left(n\right),R_v\left(n\right)\right]\right)$. Substituting into the first order conditions for hires and separations (3.13) and (3.14) yields two nonlinear equations in the optimal employment policy $R\left(n\right)$ and $R_v\left(n\right)$ that are straightforward to solve numerically. The aggregate employment stock and flows are then obtained directly from applying the results of Proposition 5.

Average Product and Average Marginal Product The average product of labor implied by the model is given by $APL = E\left[pxn^{\alpha-1}\right]$. Note that:

$$E\left[xn^{\alpha-1}\right] = \int \left[\int xdG\left(x|n\right)\right] n^{\alpha-1} dH\left(n\right).$$

Moreover, the optimal employment policy implies that, given n, x must lie in the interval $[R(n), R_v(n)]$, but is otherwise independently distributed. Thus:

$$\int xdG\left(x|n\right) = \frac{\int_{R(n)}^{R_v(n)} xdG\left(x\right)}{G\left[R_v\left(n\right)\right] - G\left[R\left(n\right)\right]} = \frac{1}{2}\left[R\left(n\right) + R_v\left(n\right)\right], \tag{3.38}$$

where the last equality follows from the assumption of uniform idiosyncratic shocks in the simulation. Thus:

$$APL = E\left[pxn^{\alpha-1}\right] = p\int \frac{1}{2}\left[R\left(n\right) + R_v\left(n\right)\right] n^{\alpha-1} dH\left(n\right). \tag{3.39}$$

The average marginal product of labor is simply given by $E\left[MPL\right] = E\left[px\alpha n^{\alpha-1}\right] = \alpha APL$.

Average Wages It follows from equation (3.9) that the average wage across firms is given by:

$$\bar{w}_f = \frac{\eta}{1-\eta\left(1-\alpha\right)} E\left[MPL\right] + \eta\beta f\frac{c}{q} + \left(1-\eta\right) b. \tag{3.40}$$

To obtain the average wage across workers, which we denote $\bar{w}_w$, note that $\bar{w}_w = E\left[\frac{n}{E(n)} w\left(n,x\right)\right]$ where $w\left(n,x\right)$ is the wage in a given firm defined in (3.9). That is, it is the employment-weighted average of wages across firms. Thus:

$$\bar{w}_w = \frac{\eta}{1-\eta\left(1-\alpha\right)} \frac{1}{E\left(n\right)} E\left[px\alpha n^{\alpha}\right] + \eta\beta f\frac{c}{q} + \left(1-\eta\right) b. \tag{3.41}$$

This has a very similar structure to the average wage across firms. It follows that:

$$\bar{w}_w = \frac{\eta p\alpha}{1-\eta\left(1-\alpha\right)} \frac{1}{E\left(n\right)} \int \frac{1}{2}\left[R\left(n\right) + R_v\left(n\right)\right] n^{\alpha} dH\left(n\right) + \eta\beta f\frac{c}{q} + \left(1-\eta\right) b. \tag{3.42}$$

Finally, the average wage of new hires, which we denote $\bar{w}_m$, is equal to a hiring–weighted average of wages across hiring firms. Noting from (3.12) that idiosyncratic productivity of hiring firms is given by $x = R_v(n)$, we have that:

$$\bar{w}_m = E\left[E\left(w(n,x)\,|n > n_{-1}, n_{-1}\right)\right] = \int \int_{n_{-1}} w\left(n, R_v(n)\right) \frac{dG\left[R_v(n)\right]}{1 - G\left[R_v(n_{-1})\right]} dH(n_{-1})\,. \tag{3.43}$$

3.9 Proofs

Conjecture *The optimal employment policy function is of the form specified in (3.12).*

We will later verify in the proof of Proposition 2 that the Conjecture is consistent with the solution for the wage equation obtained in Proposition 1.

Proof of Proposition 1 Note first that, under the Conjecture, we can write the marginal surplus to a firm recursively as:

$$J(n,x) = pxF'(n) - w(n,x) - w_n(n,x)\,n + \beta \int_{R_v(n)}^{\infty} \frac{c}{q} dG(x') + \beta \int_{R(n)}^{R_v(n)} J(n,x')\,dG(x')\,. \tag{3.44}$$

In addition, we can write the value to a worker of unemployment as:

$$U = b + \beta \left\{ (1-f)\,U' + f \int_0^{\infty} \int_{R_v(n)}^{\infty} W\left(R_v^{-1}(x'), x'\right) \frac{dG(x')}{1 - G\left(R_v(n)\right)} dH(n) \right\}. \tag{3.45}$$

Upon finding a job, which occurs with probability f, the new job must be in a firm which is posting vacancies. This implies that the idiosyncratic productivity of the firm, $x' > R_v(n)$, and that the level of employment in the hiring firm, $n' = R_v^{-1}(x')$. Moreover, since firms differ in size, there is a distribution of employment levels, $H(n)$, over which an unemployed worker will take expectations when evaluating the expected future benefits of being hired.[32] It is useful to rewrite the worker's value of unemployment as:

$$U = b + \beta \left\{ U' + f \int_0^{\infty} \int_{R_v(n)}^{\infty} \left[W\left(R_v^{-1}(x'), x'\right) - U'\right] \frac{dG(x')}{1 - G\left(R_v(n)\right)} dH(n) \right\}. \tag{3.46}$$

Then note that, due to Nash sharing, the worker's surplus in an expanding firm, $W\left(R_v^{-1}(x'), x'\right) - U' = \frac{\eta}{1-\eta} J\left(R_v^{-1}(x'), x'\right)$, and moreover that, by the first-order condition for a hiring firm (see (3.4)), $J\left(R_v^{-1}(x'), x\right) = c/q$. Thus, we obtain the simple result:

$$U = b + \beta U' + \beta f \frac{\eta}{1-\eta} \frac{c}{q}. \tag{3.47}$$

[32] The reader may wonder why the integral in (3.45) is not taken over the joint distribution of n and x'. The reason is that, conditional on $x' > R_v(n)$, n provides no additional information on x'; see the optimal employment policy function (3.12).

The value of employment to a worker can be written as:

(3.48)
$$W(n,x) = w(n,x) + \beta \left\{ \int_0^{R(n)} \left[\tilde{s}U' + (1-\tilde{s}) W\left(R^{-1}(x'), x'\right)\right] dG(x') + \int_{R(n)}^{R_v(n)} W(n,x')\, dG(x') + \int_{R_v(n)}^{\infty} W\left(R_v^{-1}(x'), x'\right) dG(x') \right\}.$$

An employed worker's expected future payoff can be split into three regimes. If the firm sheds workers next period ($x' < R(n)$) then the worker may separate from the firm. We denote by $\tilde{s}$ the probability that a worker separates from a firm conditional on the firm shedding workers. If the worker separates, she transitions into unemployment and receives a payoff U'. Otherwise she continues to be employed in a firm of size $n' = R^{-1}(x')$. Note that Nash sharing implies that $W\left(R^{-1}(x'), x'\right) - U' = \frac{\eta}{1-\eta} J\left(R^{-1}(x'), x'\right)$, and that, by the first-order condition, $J\left(R^{-1}(x'), x'\right) = 0$. Thus, $W\left(R^{-1}(x'), x'\right) = U'$. In the event that a firm freezes employment next period ($x' \in [R(n), R_v(n)]$) then Nash sharing implies that $W(n,x') - U' = \frac{\eta}{1-\eta} J(n,x')$. Finally, in the event that the firm hires next period, $W\left(R_v^{-1}(x'), x'\right) - U' = \frac{\eta}{1-\eta}\frac{c}{q}$. Thus, we have that:

(3.49)
$$W(n,x) = w(n,x) + \beta U' + \beta \frac{\eta}{1-\eta} \int_{R_v(n)}^{\infty} \frac{c}{q} dG(x') + \beta \frac{\eta}{1-\eta} \int_{R(n)}^{R_v(n)} J(n,x')\, dG(x').$$

Subtracting the value of unemployment to a worker from the latter, we obtain the following description of the worker's surplus:

(3.50)
$$W(n,x) - U = w(n,x) - b + \beta \frac{\eta}{1-\eta} \int_{R_v(n)}^{\infty} \frac{c}{q} dG(x') + \beta \frac{\eta}{1-\eta} \int_{R(n)}^{R_v(n)} J(n,x')\, dG(x') - \beta f \frac{\eta}{1-\eta} \frac{c}{q}.$$

Under Nash, this must be equal to $\frac{\eta}{1-\eta} J(n,x)$, where $J(n,x)$ is as derived in (3.44) so that we have:

(3.51)
$$w(n,x) = \eta \left[pxF'(n) - w_n(n,x)\, n + \beta f \frac{c}{q} \right] + (1-\eta)\, b,$$

as required.

Proof of Proposition 2 Given the wage function in (3.9), it follows that the firm's objective, (3.3), is continuous in (n_{-1}, x) and concave in n. Thus, it follows from the Theorem of the Maximum that the firm's optimal employment policy function is continuous in (n_{-1}, x). Given this, it follows that the employment policy function must be of the form stated in Proposition 2. This verifies that the Conjecture stated at the beginning of the appendix holds.

Proof of Proposition 3 First, note that one can re-write the continuation value conditional on each of the three possible continuation regimes:

$$\Pi(n,x') = \begin{cases} \Pi^{-}(n,x') & \text{if} \quad x' < R(n), \\ \Pi^{0}(n,x') & \text{if} \quad x' \in [R(n), R_v(n)], \\ \Pi^{+}(n,x') & \text{if} \quad x' > R_v(n), \end{cases} \tag{3.52}$$

where superscripts $^{-/0/+}$ refer to whether their are separations, a hiring freeze, or hires tomorrow. Thus we can write[33]:

$$\int \Pi(n,x')\,dG(x'|x) = \int_0^{R(n)} \Pi^{-}(n,x')\,dG + \int_{R(n)}^{R_v(n)} \Pi^{0}(n,x')\,dG + \int_{R_v(n)}^{\infty} \Pi^{+}(n,x')\,dG. \tag{3.53}$$

Taking derivatives with respect to n, recalling the definition of $D(\cdot)$, and noting that, since $\Pi(n,x')$ is continuous, it must be that $\Pi^{-}(n,R(n)) = \Pi^{0}(n,R(n))$ and $\Pi^{0}(n,R_v(n)) = \Pi^{+}(n,R_v(n))$, yields:

$$D(n,x) = \int_0^{R(n)} \Pi_n^{-}(n,x')\,dG + \int_{R(n)}^{R_v(n)} \Pi_n^{0}(n,x')\,dG + \int_{R_v(n)}^{\infty} \Pi_n^{+}(n,x')\,dG. \tag{3.54}$$

Finally, using the Envelope conditions in Lemma 1 below, and substituting into (3.54) we obtain (3.15) and (3.16) in the main text:

$$\begin{aligned} D(n,x) &= \int_{R(n)}^{R_v(n)} \left\{ (1-\eta)\left[\frac{px'\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta f\frac{c}{q} \right\} dG(x'|x) \\ &\quad + \int_{R_v(n)}^{\infty} \frac{c}{q} dG(x'|x) + \beta \int_{R(n)}^{R_v(n)} D(n,x')\,dG(x'|x) \\ &\equiv (\mathbf{C}D)(n,x). \end{aligned} \tag{3.55}$$

To verify that $\mathbf{C}$ is a contraction mapping, we confirm that Blackwell's sufficient conditions for a contraction hold here (see Stokey and Lucas, 1989, p.54). To verify monotonicity, fix $(n,x) = (\bar{n},\bar{x})$, and take $\hat{D} \geq D$. Then note that:

$$\int_{R(\bar{n})}^{R_v(\bar{n})} \hat{D}(\bar{n},x')\,dG(x'|\bar{x}) - \int_{R(\bar{n})}^{R_v(\bar{n})} D(\bar{n},x')\,dG(x'|\bar{x}) = \int_{R(\bar{n})}^{R_v(\bar{n})} \left[\hat{D}(\bar{n},x') - D(\bar{n},x')\right] dG(x'|\bar{x}) \geq 0. \tag{3.56}$$

[33] Henceforth, "dG" without further elaboration is to be taken as "$dG(x'|x)$".

Since $(\bar{n}, \bar{x})$ were arbitrary, it thus follows that $\mathbf{C}$ is monotonic in D. To verify discounting, note that:

$$(3.57) \qquad [\mathbf{C}(D+a)](n,x) = (\mathbf{C}D)(n,x) + \beta a \left[G(R_v(n)|x) - G(R(n)|x)\right] \leq (\mathbf{C}D)(n,x) + \beta a.$$

Since $\beta < 1$ it follows that $\mathbf{C}$ is a contraction. It therefore follows from the Contraction Mapping Theorem that $\mathbf{C}$ has a unique fixed point.

Lemma 1 *The value function defined in (3.3) has the following properties:*

$$(3.58) \qquad \begin{aligned} \Pi_n^-(n,x') &= 0, \\ \Pi_n^0(n,x') &= (1-\eta)\left[\frac{px'\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta f\frac{c}{q} + \beta D(n,x'), \\ \Pi_n^+(n,x') &= c/q. \end{aligned}$$

Proof of Lemma 1 First, note that standard application of the Envelope Theorem implies that $\Pi_n^-(n,x') = 0$ and $\Pi_n^+(n,x') = c/q$. It is only slightly less obvious what happens when $\Delta n' = 0$, i.e. when the employment is frozen next period. In this case, $n' = n$ and this implies that:

$$(3.59) \qquad \Pi^0(n,x') = px'F(n) - w(n,x')n + \beta\int \Pi(n,x'')\,dG(x''|x').$$

It therefore follows that:

$$(3.60) \qquad \Pi_n^0(n,x') = px'F'(n) - w(n,x') - w_n(n,x')n + \beta\int \Pi_n(n,x'')\,dG(x''|x').$$

Since, by definition $D(n,x') \equiv \int \Pi_n(n,x'')\,dG(x''|x')$, the statement holds as required.

Proof of Proposition 4 First note that if x evolves according to (3.17), then we can rewrite the recursion for $D(n,x)$ as:

$$(3.61) \qquad \begin{aligned} D(n,x) = {} & \frac{1-\lambda}{1-\beta(1-\lambda)}\chi(x) + \frac{\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} \chi(x')\,d\tilde{G}(x') \\ & + \frac{\lambda}{1-\beta(1-\lambda)}\int_{R_v(n)}^{\infty} \frac{c}{q}\,d\tilde{G}(x') + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} D(n,x')\,d\tilde{G}(x'), \end{aligned}$$

where $\chi(x) \equiv (1-\eta)\left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta c\theta$. It follows that the LHS of the first–order conditions, (3.13) and (3.14) are increasing in x, because $\chi(x)$ is increasing in x. Thus, to establish that $\partial R_v/\partial p < 0$ and $\partial R/\partial p < 0$, simply note that the function $D(n,x)$ is also increasing in p and thus the LHS of (3.13) and (3.14) are increasing in p.

To ascertain the marginal effects of θ we first need to establish the marginal effect of θ on the function $D(n,x)$. Rewriting $f/q = \theta$ and $q = q(\theta)$ in (3.61), differentiating with respect to θ, and using the first–order conditions, (3.13) and (3.14), to eliminate terms we obtain:

$$D_\theta = -\eta\beta c \frac{1-\lambda\left(1-p^0\right)}{1-\beta\left[1-\lambda\left(1-p^0\right)\right]} - \frac{c}{q}\frac{q'(\theta)}{q}\frac{\lambda p^+}{1-\beta\left[1-\lambda\left(1-p^0\right)\right]}, \tag{3.62}$$

where $p^0 \equiv \tilde{G}(R_v(n)) - \tilde{G}(R(n))$, $p^+ \equiv 1-\tilde{G}(R_v(n))$, and $p^- \equiv \tilde{G}[R(n)]$. Note that D_θ is independent of x. Differentiating the first–order condition for a hiring firm, (3.13), with respect to θ we obtain:

$$-\eta\beta c + \frac{c}{q}\frac{q'(\theta)}{q} + \beta D_\theta = -\frac{\eta\beta c}{1-\beta\left[1-\lambda\left(1-p^0\right)\right]} + \frac{c}{q}\frac{q'(\theta)}{q}\frac{1-\beta\left(1-\lambda p^-\right)}{1-\beta\left[1-\lambda\left(1-p^0\right)\right]} < 0 \tag{3.63}$$

since $q'(\theta) < 0$. Thus it follows that $\partial R_v/\partial\theta > 0$. Likewise, differentiating the first–order condition for a shedding firm, (3.14), with respect to θ we obtain:

$$-\eta\beta c + \beta D_\theta = -\frac{\eta\beta c}{1-\beta\left[1-\lambda\left(1-p^0\right)\right]} - \beta\frac{c}{q}\frac{q'(\theta)}{q}\frac{\lambda p^+}{1-\beta\left[1-\lambda\left(1-p^0\right)\right]}. \tag{3.64}$$

Thus, $\partial R/\partial\theta > 0 \iff n > R_v^{-1}\tilde{G}^{-1}\left(1+\frac{\eta}{\varepsilon_{q\theta}}\frac{f}{\lambda}\right)$ where $\varepsilon_{q\theta} \equiv \frac{d\ln q}{d\ln\theta}$.

Proof of Proposition 5 *Proof of (3.19) and (3.20):* See main text.

Proof of (3.21): First note that a necessary condition for a firm to shed workers is that it receives an idiosyncratic shock, which occurs with probability λ. In this event, the number of separations in a firm that is shedding workers is equal to $n_{-1} - R^{-1}(x)$, since separating firms set employment, $n = R^{-1}(x)$. Now imagine, counterfactually, that all firms shared the same lagged employment level, n_{-1}. Then, the aggregate number of separations in the economy would equal:

$$\Lambda(n_{-1}) = \lambda\int_{x_{\min}}^{R(n_{-1})}\left[n_{-1} - R^{-1}(x)\right]d\tilde{G}(x), \tag{3.65}$$

where $x_{\min}$ is the lower support of idiosyncratic productivity. Using the change of variables,

$x=R(n)$, and integrating by parts:

$$\Lambda(n_{-1})=\lambda\int_{n_{\min}}^{n_{-1}}(n_{-1}-n)\frac{d\tilde{G}[R(n)]}{dn}dn=\lambda\int_{n_{\min}}^{n_{-1}}\tilde{G}[R(n)]\,dn. \tag{3.66}$$

Now, of course, the true aggregate number of separations is equal to $S=\int\Lambda(n_{-1})\,dH(n_{-1})$, where $H(\cdot)$ is the c.d.f. of employment. Denoting $n_{\max}$ as the upper support of $H(\cdot)$, further integration by parts reveals that:

$$S=\Lambda(n_{\max})-\lambda\int\tilde{G}[R(n_{-1})]\,H(n_{-1})\,dn_{-1}=\lambda\int[1-H(n)]\,\tilde{G}[R(n)]\,dn, \tag{3.67}$$

as required. A similar method reveals that the aggregate number of hires in the economy, $M=\lambda\int H(n)\left(1-\tilde{G}[R_v(n)]\right)dn$. It follows from the steady state condition for the distribution for employment, (3.19), that separations, S, are equal to hires, M.

Proof of Proposition 6 Given that aggregate shocks evolve according to (3.25), and denoting the forecast equations for N and θ in (3.26) as $N'(N,p)$ and $\theta'(N',p)$ respectively, we can write the marginal effect of current employment on future profits as

$$D(n,x,N,p;\sigma_p)=\frac{1}{2}d(n,x,N'(N,p+\sigma_p),p+\sigma_p)+\frac{1}{2}d(n,x,N'(N,p-\sigma_p),p-\sigma_p), \tag{3.68}$$

where

$$\begin{aligned} d(n,x,N'(N,p'),p') &= \int_{R(n,N',p')}^{R_v(n,N',p')}\chi(n,x',N',p')\,dG(x'|x)+\int_{R_v(n,N',p')}^{\infty}c\left[\theta'(N',p')\right]^{\phi}dG(x'|x) \\ &\quad +\beta\int_{R(n,N',p')}^{R_v(n,N',p')}D(n,x',N'(N,p\prime),p')\,dG(x'|x), \end{aligned} \tag{3.69}$$

and $\chi(n,x,N,p)=(1-\eta)\left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)}-b\right]-\eta\beta cE\left[\theta'(N',p')\,|p\right]\equiv\chi_0+\chi_1 px+\chi_2 E\left[\theta'(N',p')\,|p\right]$. Taking a Taylor series approximation to $D(n,x,N,p;\sigma_p)$ around $\sigma_p=0$ we obtain

$$D(n,x,N,p;\sigma_p)\approx D(n,x,N^*,p;0)+D_{\sigma_p}(n,x,N^*,p;0)\,\sigma_p+D_N^*(N-N^*), \tag{3.70}$$

where $D_N^*\equiv D_N(n,x,N^*,p;0)$. It is straightforward to show that $D_{\sigma_p}(n,x,N^*,p;0)=0$, and that $D_N^* = d_{N'}(n,x,N^*,p)\,\nu_N$. Under the conjectured forecast equations in (3.26), we can write[34]

$$\begin{aligned} d_{N'}\left(n,x,N'\left(N,p'\right),p'\right) &= \int_{R(n,N',p')}^{R_v\left(n,N',p'\right)} \chi_{N'}\left(n,x',N',p'\right) dG\left(x'|x\right) \\ &\quad + \int_{R_v(n,N',p')}^{\infty} c\phi\left[\theta'\left(N',p'\right)\right]^{\phi-1}\theta'_{N'}\left(N',p'\right) dG\left(x'|x\right) \\ &\quad + \beta\int_{R(n,N',p')}^{R_v\left(n,N',p'\right)} D_N\left(n,x',N'\left(N,p'\right),p'\right) dG\left(x'|x\right). \end{aligned} \tag{3.71}$$

Evaluating at $N = N^*$ and $p' = p$, and noting that $\chi_{N'}(n,x,N',p) = \chi_2\theta_N\nu_N$, and $\theta'_{N'}(N^*,p) = \theta_N$, we obtain

$$\begin{aligned} d_{N'}\left(n,x,N^*,p'\right) &= \chi_2\theta_N\nu_N\int_{R(n,N^*,p)}^{R_v(n,N^*,p)} dG\left(x'|x\right) + c\phi\theta_N{\theta^*}^{\phi-1}\int_{R_v(n,N^*,p)}^{\infty} dG\left(x'|x\right) \\ &\quad + \beta\int_{R(n,N^*,p)}^{R_v(n,N^*,p)} D_N\left(n,x',N^*,p\right) dG\left(x'|x\right). \end{aligned} \tag{3.72}$$

Recall from above that $D_N^* = d_{N'}(n,x,N^*,p)\,\nu_N$. Putting this together yields

$$\begin{aligned} D_N^* &= \chi_2\theta_N\nu_N^2\int_{R(n,N^*,p)}^{R_v(n,N^*,p)} dG\left(x'|x\right) + c\phi\theta_N\nu_N{\theta^*}^{\phi-1}\int_{R_v(n,N^*,p)}^{\infty} dG\left(x'|x\right) \\ &\quad + \beta\nu_N\int_{R(n,N^*,p)}^{R_v(n,N^*,p)} D_{N'}\left(n,x',N^*,p;0\right) dG\left(x'|x\right). \end{aligned} \tag{3.73}$$

Under the form of idiosyncratic shocks in (idiosync. eq.) we obtain:

$$\begin{aligned} D_N^* &= \theta_N\nu_N^2\chi_2\frac{(1-\lambda)+\lambda\left(\tilde{G}\left[R_v^*(n)\right]-\tilde{G}\left[R^*(n)\right]\right)}{1-\beta\nu_N\left[(1-\lambda)+\left(\tilde{G}\left[R_v^*(n)\right]-\tilde{G}\left[R^*(n)\right]\right)\right]} \\ &\quad + \theta_N\nu_N\lambda c\phi{\theta^*}^{\phi-1}\frac{1-\tilde{G}\left[R_v^*(n)\right]}{1-\beta\nu_N\left[(1-\lambda)+\left(\tilde{G}\left[R_v^*(n)\right]-\tilde{G}\left[R^*(n)\right]\right)\right]}, \end{aligned} \tag{3.74}$$

where $R_v(n,N^*,p) \equiv R_v^*(n)$ and $R(n,N^*,p) \equiv R^*(n)$ summarize the steady state employment policy function.

Proof of Proposition 7 Consider the c.d.f. of employment growth for a given lagged

[34] Note that the effects of N' on the limits of integration will cancel by virtue of the first order conditions for optimal hiring and firing.

employment level, n_{-1}, and for the case where employment growth is negative:

$$\begin{aligned}\Pr(\Delta \ln n < \delta | n_{-1}, \delta < 0) &= \Pr\left(\ln R^{-1}(x) - \ln n_{-1} < \delta | n_{-1}\right) \\ &= \Pr\left(x < R\left(e^{\delta} n_{-1}\right) | n_{-1}\right) \\ &= \lambda \tilde{G}\left[R\left(e^{\delta} n_{-1}\right)\right]. \end{aligned} \tag{3.75}$$

It follows that the unconditional c.d.f. of employment growth, given that $\Delta \ln n < 0$ is equal to:

$$H_{\Delta}(\delta) \equiv \Pr(\Delta \ln n < \delta) = \lambda \int \tilde{G}\left[R\left(e^{\delta} n_{-1}\right)\right] dH(n_{-1}), \tag{3.76}$$

It follows that the density of employment growth is given by $h_{\Delta}(\delta) = H'_{\Delta}(\delta) = \lambda \int \tilde{G}'\left[R'\left(e^{\delta} n_{-1}\right)\right] e^{\delta} n_{-1} dH(n_{-1})$, as stated in the Proposition. A similar method reveals that, in the case where $\Delta \ln n > 0$:

(3.77)

$$H_{\Delta}(\delta) = \lambda \int \tilde{G}\left[R_v\left(e^{\delta} n_{-1}\right)\right] dH(n_{-1}), \text{ and } h_{\Delta}(\delta) = \lambda \int \tilde{G}'\left[R'_v\left(e^{\delta} n_{-1}\right)\right] e^{\delta} n_{-1} dH(n_{-1}).$$

Finally there is a mass point at zero employment growth. Clearly that is given by:

$$h_{\Delta}(0) = H_{\Delta}\left(0^{+}\right) - H_{\Delta}\left(0^{-}\right) = \lambda \int \left(\tilde{G}\left[R_v(n_{-1})\right] - \tilde{G}\left[R(n_{-1})\right]\right) dH(n_{-1}). \tag{3.78}$$

Lemma 2 *If idiosyncratic shocks evolve according to (3.17), and the matching function is of the form $M(U,V) = \mu U^{\phi} V^{1-\phi}$, then the marginal firm surplus defined in (3.44) is given by*

$$\begin{aligned} J &= \frac{\psi p \alpha n^{\alpha-1}}{1-\beta(1-\lambda)}\left[x + \frac{\beta\lambda p^{0}}{1-\beta(1-\lambda)-\beta\lambda p^{0}}\mathcal{E}(n)\right] \\ &\quad - \frac{(1-\eta) b}{1-\beta(1-\lambda)-\beta\lambda p^{0}} - \beta\frac{c}{q}\frac{\eta f - \lambda p^{+}}{1-\beta(1-\lambda)-\beta\lambda p^{0}}, \end{aligned} \tag{3.79}$$

and the marginal effects of n, p and θ on J are given by

$$\begin{aligned} J_n &= -\frac{1-\alpha}{n}\frac{\psi p \alpha n^{\alpha-1}}{1-\beta(1-\lambda)}\left[x + \frac{\beta\lambda p^{0}}{1-\beta(1-\lambda)-\beta\lambda p^{0}}\mathcal{E}(n)\right] \\ J_p &= \frac{1}{p}\frac{\psi p \alpha n^{\alpha-1}}{1-\beta(1-\lambda)}\left[x + \frac{\beta\lambda p^{0}}{1-\beta(1-\lambda)-\beta\lambda p^{0}}\mathcal{E}(n)\right] \\ J_{\theta} &= -\beta\frac{c}{q}\frac{1}{\theta}\frac{\eta f - \phi\lambda p^{+}}{1-\beta(1-\lambda)-\beta\lambda p^{0}}, \end{aligned} \tag{3.80}$$

where $\psi \equiv \frac{1-\eta}{1-\eta(1-\alpha)}$, $\mathcal{E}(n) \equiv E\left(x' | x' \in \left[R(n), R_v(n)\right]\right)$, and p^{0}, p^{+} are as defined in the Proof *to Proposition 4.*

Proof of Lemma 2 Since firms only receive an idiosyncratic shock with probability λ each period, we can use the recursion for $J(n,x)$, (3.44), to write:

$$\begin{aligned} J(n,x) &= \frac{1}{1-\beta(1-\lambda)}\left[\psi p x \alpha n^{\alpha-1} - (1-\eta) b - \eta\beta c\theta\right] \\ &\quad + \frac{\beta\lambda}{1-\beta(1-\lambda)}\frac{c}{q}\int_{R_v(n)} d\tilde{G} + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} J(n,x')\, d\tilde{G}. \end{aligned} \tag{3.81}$$

We then conjecture that $J(n,x)$ is of the form $j_0 + j_1 x$. Substituting this assumption into the latter, and equating coefficients yields:

$$\begin{aligned} j_0 &= -\frac{(1-\eta) b}{1-\beta(1-\lambda)} - \beta\frac{c}{q}\frac{\eta f - \lambda p^{+}}{1-\beta(1-\lambda)} + \frac{\beta\lambda p^{0}}{1-\beta(1-\lambda)}\left[j_0 + j_1\mathcal{E}(n)\right], \\ j_1 &= \frac{\psi p \alpha n^{\alpha-1}}{1-\beta(1-\lambda)}.. \end{aligned} \tag{3.82}$$

Solving for j_0 we obtain the required solution for $J(n,x)$. Likewise, we can obtain recursions for the marginal effects of n and θ:

$$\begin{aligned} J_n(n,x) &= -\frac{1}{1-\beta(1-\lambda)}\frac{1-\alpha}{n}\psi p x \alpha n^{\alpha-1} + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} J_n(n,x')\, dG, \\ J_p(n,x) &= \frac{1}{1-\beta(1-\lambda)}\psi x \alpha n^{\alpha-1} + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} J_p(n,x')\, d\tilde{G}, \\ J_\theta(n,x) &= -\frac{\eta\beta c + \beta\lambda\frac{c}{q^2}q'(\theta)\int_{R_v(n)} dG}{1-\beta(1-\lambda)} + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} J_\theta(n,x')\, dG. \end{aligned} \tag{3.83}$$

Again using the method of undetermined coefficients, and noting that the Cobb Douglas matching function implies $q = \mu\theta^{-\phi} \implies \frac{c}{q^2}q'(\theta) = -\frac{c}{q}\frac{\phi}{\theta}$, yields the required solutions for J_n, J_p and J_θ.

Proof of Proposition 8 Total differentiation of the Job Creation condition, $U(\theta) = L - N(\theta)$, yields $d\theta/dp = -(\partial N/\partial p)/(\partial N/\partial\theta)$. Indexing firms by i, we can write aggregate employment as $N \equiv E(n) = \int n(n_{-1}(i), x(i); \xi)\, di$, where $n(n_{-1}, x; \xi)$ is the employment policy function that is common to all firms, which in turn depends on some parameters ξ (which includes p and θ). Differentiating yields:

$$\frac{\partial N}{\partial\xi} = \int\left[\frac{\partial n}{\partial\xi} + \frac{\partial n}{\partial n_{-1}}\frac{\partial n_{-1}}{\partial\xi}\right] di. \tag{3.84}$$

Note from the form of the employment policy function in (3.12) that $\partial n/\partial\xi = 0$ if $\Delta n(i) = 0$, and

$\partial n/\partial n_{-1} = 1$ iff $\Delta n(i) = 0$. Substitution and separation of integrals yields

$$\begin{aligned} \frac{\partial N}{\partial \xi} &= \int_{i:\Delta n_i>0} \left.\frac{\partial n}{\partial \xi}\right|_{\Delta n_i>0} di + \int_{i:\Delta n_i=0} \frac{\partial n_{-1}}{\partial \xi} di + \int_{i:\Delta n_i<0} \left.\frac{\partial n}{\partial \xi}\right|_{\Delta n_i<0} di \\ &= \mathbf{p}^{+} E\left(\frac{\partial n}{\partial \xi}\middle| \Delta n > 0\right) + \mathbf{p}^{0} E\left(\frac{\partial n_{-1}}{\partial \xi}\right) + \mathbf{p}^{-} E\left(\frac{\partial n}{\partial \xi}\middle| \Delta n < 0\right), \end{aligned} \tag{3.85}$$

where $\mathbf{p}^{+}, \mathbf{p}^{0}$, and $\mathbf{p}^{-}$ respectively denote the steady-state probabilities of raising, freezing, and cutting employment. Note further that in steady state $E(\partial n_{-1}/\partial \xi) = E(\partial n/\partial \xi) = \partial N/\partial \xi$, so that we obtain the result that:

$$\frac{\partial N}{\partial \xi} = \pi E\left(\frac{\partial n}{\partial \xi}\middle| \Delta n > 0\right) + (1-\pi) E\left(\frac{\partial n}{\partial \xi}\middle| \Delta n < 0\right), \tag{3.86}$$

where $\pi \equiv \mathbf{p}^{+}/\left(1-\mathbf{p}^{0}\right)$. Thus, we can rewrite the marginal effect of a change in p on θ as:

$$\frac{d\theta}{dp} = -\frac{\pi E\left(\frac{\partial n}{\partial p}\middle| \Delta n > 0\right) + (1-\pi) E\left(\frac{\partial n}{\partial p}\middle| \Delta n < 0\right)}{\pi E\left(\frac{\partial n}{\partial \theta}\middle| \Delta n > 0\right) + (1-\pi) E\left(\frac{\partial n}{\partial \theta}\middle| \Delta n < 0\right)}. \tag{3.87}$$

Then note that the first-order conditions for optimal labor demand set the marginal firm surplus, $J(n,x)$ as follows:

$$J(n,x) = \begin{cases} c/q(\theta) & \text{if} \quad \Delta n > 0, \\ 0 & \text{if} \quad \Delta n < 0. \end{cases} \tag{3.88}$$

It is immediate from Lemma 2 that $\partial n/\partial p = -J_p/J_n = \frac{1}{1-\alpha}(n/p)$ regardless of whether $\Delta n > 0$ or $\Delta n < 0$. It remains to derive $\partial n/\partial \theta$ in each case. Log-linearizing the function J around n, p, x, and θ, we obtain:

$$\log J \approx \varepsilon_{Jn} \log n + \varepsilon_{Jp} (\log p + \log x) + \varepsilon_{J\theta} \log \theta + const. \tag{3.89}$$

Using this and totally differentiating the first-order conditions for optimal labor demand with respect to n and θ, we obtain:

$$\varepsilon_{Jn} d\log n + \varepsilon_{J\theta} d\log \theta \approx \begin{cases} -d\log q(\theta) & \text{if} \quad \Delta n > 0, \\ 0 & \text{if} \quad \Delta n < 0. \end{cases} \tag{3.90}$$

Given the Cobb Douglas matching function assumption, $q(\theta) = \mu\theta^{-\phi}$, and it follows that $d\log q(\theta) =$

$-\phi d \log\theta$. Thus:

$$\frac{\partial n}{\partial\theta} = \frac{\partial \log n}{\partial \log\theta}\frac{n}{\theta} \approx \begin{cases} \frac{\phi-\varepsilon_{J\theta}}{\varepsilon_{Jn}}\frac{n}{\theta} & \text{if} \quad \Delta n > 0, \\ -\frac{\varepsilon_{J\theta}}{\varepsilon_{Jn}}\frac{n}{\theta} & \text{if} \quad \Delta n < 0. \end{cases} \tag{3.91}$$

Substituting this into (3.87), we obtain:

$$\left.\frac{d\log\theta}{d\log p}\right|_{JC} \approx -\frac{1}{1-\alpha}\frac{\varepsilon_{Jn}}{\omega\phi - \varepsilon_{J\theta}}, \tag{3.92}$$

where $\omega \equiv \pi E\left(n|\Delta n > 0\right)/E\left(n\right)$ is the steady state share of employment in hiring firms. In what follows, we evaluate the approximation (3.89) to the marginal surplus around mean employment, $N \equiv E\left(n\right)$, and mean productivity conditional on mean employment, $x = \mathcal{E}\left(N\right) \equiv E\left(x'|x' \in \left[R\left(N\right), R_v\left(N\right)\right]\right)$. Thus, using the results of Lemma 2 it follows that we can write:

$$J_n = -\frac{1}{N}\frac{\left(1-\alpha\right)\psi p\alpha N^{\alpha-1}}{1-\beta\left(1-\lambda\right)-\beta\lambda p^0}\mathcal{E}\left(N\right),$$

and:

$$J = \frac{\psi p\mathcal{E}\left(N\right)\alpha N^{\alpha-1} - \left(1-\eta\right)b - \beta\frac{c}{q}\left[\eta f - \lambda p^+\right]}{1-\beta\left(1-\lambda\right)-\beta\lambda p^0}, \tag{3.93}$$

where $\psi \equiv \left(1-\eta\right)/\left[1-\eta\left(1-\alpha\right)\right]$. Substituting back into the aggregate elasticity of θ with respect to p, we obtain:

$$\left.\frac{d\log\theta}{d\log p}\right|_{JC} \approx \frac{\psi p\mathcal{E}\left(N\right)\alpha N^{\alpha-1}}{\omega\phi\left[\psi p\mathcal{E}\left(N\right)\alpha N^{\alpha-1} - \left(1-\eta\right)b - \eta\beta c\theta\right] + \eta\beta c\theta - \left(1-\omega\right)\phi\beta\frac{c}{q}\lambda p^+}. \tag{3.94}$$

Noting that the marginal product of labor in the average-sized firm is equal to $p\mathcal{E}\left(N\right)\alpha N^{\alpha-1}$, and assuming λ is sufficiently small, we obtain:

$$\left.\frac{d\log\theta}{d\log p}\right|_{JC} \approx \frac{\left(1-\eta\right)\tilde{p}}{\omega\phi\left[\left(1-\eta\right)\left(\tilde{p}-b\right) - \eta\beta c\theta\right] + \eta\beta c\theta}, \tag{3.95}$$

where $\tilde{p} \equiv \rho p\mathcal{E}\left(N\right)N^{\alpha-1} + \left(1-\rho\right)p\mathcal{E}\left(N\right)\alpha N^{\alpha-1}$ and $\rho \equiv \alpha\eta/\left[1-\eta\left(1-\alpha\right)\right]$, as required.

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CHAPTER IV

The Analytics of Fixed Adjustment Costs and Myopic Rules of Thumb

4.1 Introduction

The last twenty years have seen a breakthrough in the analysis of establishment-level employment dynamics.[1] Hamermesh (1989) documented long periods of inaction, in which employment at individual plants remained fixed, punctured by bursts of large changes in the number of workers. It was apparent that a common assumption in the literature up to that time – employers make small and frequent adjustments because they face a strictly convex cost of changing the size of the workforce – may have induced plausible aggregate, but not microeconomic, dynamics.[2] Davis and Haltiwanger's (1992) analysis of employment adjustment across tens of thousands of U.S. manufacturers confirmed that this lumpy behavior is pervasive at the plant level.

One source of such lumpiness is a fixed cost of adjustment. The simplest fixed cost is a lump sum charge, C, that a plant pays whenever it makes any (net) change to the size of the workforce. Since the adjustment cost "jumps" from zero to C for even the slightest change in employment, it gives rise to the infrequent adjustment observed in plant-level data.

Our focus in this paper is the analytics of labor demand in the presence of a fixed adjustment cost. We introduce a new method to solve an optimal labor demand problem that often serves as the backbone of larger-scale models and which has, in previous research, generally been solved only numerically. The principal payoff of this is that the solution allows us to look through the "black box" that generally envelopes the numerical analysis of such problems.

In particular, the paper shows precisely how the solution to the dynamic problem nests the corresponding static policy rule as a special case. Put differently, we demonstrate how to decompose

[1] This chapter was co-written with Michael Elsby.

[2] Caballero, Engel, and Haltiwanger (1997) argue that non-smooth adjustment frictions are necessary to fully account for aggregate employment growth as well. See Cooper and Willis' (2004) critique and Caballero and Engel's (2004) rebuttal.

the solution to the dynamic programming problem into static and forward-looking components. The first represents the contemporaneous effect of the adjustment friction on the optimal policy that emerges even in a static setting, while the second summarizes the effect that expectations of the future exert on the choice of current labor demand. Since the static model is straightforward and well-known, this result goes a considerable way toward simplifying, and clarifying, what is often thought to be a complicated optimization problem.

Moreover, we find that the forward-looking component of the optimal policy is, under certain calibrations, quantitatively unimportant. The myopic, or static, rule serves as a reasonable "rule of thumb" in the sense of generating paths of employment growth that match very closely those induced by the true policy. This suggests that the myopic rule may be a good guide to understanding the mechanics of rich dynamic models.

What lies behind this approximation result is the interaction between sufficiently small adjustment frictions and sufficiently high uncertainty. It is commonly thought that a fixed cost raises the return to being forward-looking because the firm can avoid costly-to-reverse errors by considering the future consequences of its behavior. Of course, the strength of this effect is declining in the size of the friction. A perhaps less appreciated point in this literature is that, if the distribution of future outcomes is sufficiently diffuse, it is hard to improve upon the static policy since the firm's forecast of the future is relatively uninformative. In other words, a great deal of uncertainty over whether today's decision will be reversed effectively reduces the benefit of forming a state-contingent plan to avoid such reversals. Critically, the estimates in the literature do point to frictions that are small relative to the degree of idiosyncratic uncertainty.

The remainder of this paper is organized as follows. Section 2 presents the plant's optimization problem and solves for the optimal policy function under a certain regularity condition on the stochastic process for idiosyncratic productivity. It delineates the relation between the static and forward-looking policy rules and argues that the former is likely to serve as a very good approximation if the adjustment friction is relatively small and the magnitude of idiosyncratic risk is substantial. In Section 3, we relax the regularity condition on idiosyncratic productivity, which means that the forward-looking policy derived in Section 2 now functions more as an approximately optimal rule. We conduct simulation exercises to explore the quality of this analytical approximation and it usefulness as a guide to the properties of the model. The section finds that it performs very well. This raises, in turn, the question of whether the myopic rule, which bears a strong resemblance to the analytical approximation, also replicates the behavior the true forward-looking policy function. The section finds that, under the baseline calibration, it does. In Section 4, we explore the robustness of this result to alternative calibrations and stochastic processes for productivity. While certain parameter configurations do challenge the quality of the myopic rule along some dimensions, it still performs quite well on average. Section 5 concludes and outlines avenues for related research.

4.2 The Plant's Problem

4.2.1 Environment

The model is set in discrete time. At the beginning of the period, the plant's idiosyncratic productivity, x, is revealed. The plant must then decide the level of employment, n, and hours per worker, h. We assume that hours per worker are costless to adjust, in which case the hours decision is a straightforward intra-temporal problem.[3] In contrast, the plant faces a "disruption cost" if it adjusts n: if the plant makes any change to the size of the workforce, it loses a fraction, $0 < 1 - \lambda < 1$, of that period's revenue. More precisely, if we let $y(x, n, h)$ represent revenue (or output), then the plant pays a cost

$$c(x, n, h) = (1 - \lambda)\, y(x, n, h) \text{ for any } \Delta n \neq 0. \tag{4.1}$$

Notice that this cost applies to net, rather than gross, changes in the number of workers. Thus, the disruption cost is perhaps best thought of as induced by changes in the number of *jobs* rather than the number of workers per se.[4] This formulation, which is actually just a variant on the conventional fixed cost of adjustment, has been adopted by several authors who analyze employment dynamics under non-convex adjustment frictions (see, Cooper, Haltiwanger, and Willis, 2005; Bachmann, 2009; and Bloom, 2009).[5]

[3] To our knowledge, Sargent (1978) and Shapiro (1986) represent the only attempts to estimate the cost of adjusting hours per worker. Both find that it is economically negligible and statistically insignificantly different from zero. Given the available data at the time of their writing, each paper carries out estimation within a representative-plant setting. We are not aware of any work that revisits this question using recently made available plant-level data. Without further insight on this point, we follow the rest of the literature (see, e.g., Caballero, Engel, and Haltiwanger, 1997; Cooper, Haltiwanger, and Willis, 2005; and Bloom, 2009) in assuming it is costless to adjust hours per worker.

[4] To be sure, there are sources of gross adjustment costs, such as the cost of recruiting and training new hires (see Barron, Berger, and Black 1997) and, in the case of layoffs, the taxes owed under "experience-rated" unemployment insurance (UI) systems (see Anderson 1993). But these strike us as better modeled as costs that grow linearly in Δn, rather than as a fixed cost, which is the subject of this paper.

[5] In Hamermesh and Pfann's (1996) taxonomy of adjustment cost functions, the fixed adjustment cost is simply a lump-sum charge that employers pay whenever they make a change to the size of the workforce. But since large establishments "outgrow" any lump-sum cost, recent work has amended this formulation. Specifically, the dependence of the disruption cost on the level of the current workforce does not change the essential character of the cost – it remains discontinuous at the origin ($\Delta n = 0$) – and yet it guarantees that the cost remains quantitatively relevant

Disruption costs are motivated in a variety of ways in the literature. When a plant reduces its workforce, it may need to reassign the specific tasks once performed by separated workers to those who remain at the plant. This reorganization may disrupt the normal flow of work and lead to a slowdown in production. Alternatively, when the plant adds new jobs, those tasks must be incorporated into the production process. The assimilation of these jobs into the normal pattern of work may involve a good deal of "trial and error" that leads to reduced output initially.

The remainder of the plant's environment consists of a production technology, a wage agreement, and the law of motion for idiosyncratic productivity. The plant combines man-hours to produce output, y, subject to a decreasing-returns-to-scale technology given by

$$y\left(x,n,h\right)=x\left(nh\right)^{\alpha}, \tag{4.2}$$

with $0<\alpha<1$.[6]

We assume that, when the plant selects hours and workers, it takes the compensation agreement as given. This agreement relates labor input to the plant's wage bill and is assumed to take the form,

$$W\left(n,h\right)=n\left(b+\omega h^{\zeta}\right). \tag{4.3}$$

Notice that compensation is independent of the realization of idiosyncratic productivity, x. Cooper, Haltiwanger, and Willis (2007) show that this outcome emerges from a contracting problem between a risk-neutral plant and a risk-averse worker. In this context, b may be viewed as unemployment compensation and ωh^{ζ} as the disutility of labor supply, where $\omega>0$ and $\zeta>1$. Bloom (2009) also adopts this formulation.

Lastly, it is necessary to specify the stochastic process of productivity. We assume that x follows a geometric autoregressive process where the innovations are drawn from a uniform distribution centered about zero:

$$\log x'=\mu+\rho\log x+\varepsilon', \quad \varepsilon'\sim U\left[-s,s\right]. \tag{4.4}$$

even for large establishments. To see that the discontinuity is present, note that $y\left(x,n,h\right)=y\left(x,n_{-1}+\Delta n,h\right)\neq 0$ even if $\Delta n=0$ (provided $n_{-1},h>0$). Therefore, the adjustment cost, which is proportional to y if $\Delta n\neq 0$, is not continuous at the origin: $\lim_{\Delta n\to 0}\left[\left(1-\lambda\right)y\left(x,n_{-1}+\Delta n,h\right)\right]\neq 0$. This generates the same "jump" in the cost of adjustment observed in the lump-sum case.

[6] Equation (4.2) may be generalized to treat the elasticity of hours and workers separately. While this may be advisable in quantitative work (see Shapiro, 1986), it is not necessary for our purposes in this paper.

Notice that the bounds on the innovation, ε, imply bounds on the realization x' for a given x. In particular,

$$x' \in \left[x^{\rho}\mathrm{e}^{\mu-s},\ x^{\rho}\mathrm{e}^{\mu+s}\right]. \tag{4.5}$$

This feature of the stochastic process will play an important role in what follows.

The assumption of uniform innovations has generally been defended on grounds of tractability.[7] To see why, it is helpful to inspect the conditional probability density function associated with (4.4). We begin with the CDF, which is given by

$$G\left(x' \mid x\right) = \Pr\left(X < x' \mid x\right) = \frac{\log x' - \mu - \rho \log x + s}{2s},$$

and differentiate to obtain the probability density function:

$$g\left(x'|x\right) = \frac{1}{2sx'}. \tag{4.6}$$

This states that, despite the persistence in the stochastic process, the conditional pdf is actually independent of this period's x. This dramatically simplifies calculation of expected values. In addition, while tractability is the motivation for the use of uniform innovations, we work in Sections 3 and 4 to demonstrate that quantitative analysis of the plant's problem is not sensitive to this assumption; the performance of the myopic rule is similar if, for instance, we assume lognormal disturbances.

We are now prepared to state the plant's problem. The value of the plant is given recursively by the Bellman equation,

$$\Pi\left(x, n_{-1}\right) = \max_{n,h}\left\{x\left(nh\right)^{\alpha} - W\left(n,h\right) - \left(1-\lambda\right)x\left(nh\right)^{\alpha} \times \mathbf{1}\left[\Delta n \neq 0\right] + \beta \int \Pi\left(x', n\right) dG\left(x'|x\right)\right\},$$

where $\mathbf{1}\left[\Delta n \neq 0\right]$ is an indicator function equal to one if the expression in brackets is true.[8] Specifically, if the plant adjusts employment, then it pays a cost equal to $1-\lambda$ of revenue, which leaves it with receipts equal to $x\left(nh\right)^{\alpha} - \left(1-\lambda\right)x\left(nh\right)^{\alpha} = \lambda x\left(nh\right)^{\alpha}$. Otherwise, it earns $x\left(n_{-1}h\right)^{\alpha}$.

For reasons that will become apparent, it is helpful to re-write the Bellman equation as follows.

[7]Danziger (1999) and Gertler and Leahy (2008) both make use of the law of motion, (4.4). (They set $\rho = 1$, but this is not necessary.) These authors analyze a price setter's problem in the presence of a fixed cost of price adjustment. Gertler and Leahy note that it is very difficult to solve such a problem for alternative distributions.

[8]The notation here follows Cooper, Haltiwanger, and Willis (2005).

Let $\Pi^{\Delta}(x)$ represent the value of adjusting employment, and let $\Pi^{0}(x, n_{-1})$ denote the value of setting $n = n_{-1}$, i.e., the value of "doing nothing". Then $\Pi(x, n_{-1})$ is seen to be

$$\Pi(x, n_{-1}) = \max\left\{\Pi^{\Delta}(x),\ \Pi^{0}(x, n_{-1})\right\},$$

where

$$\Pi^{\Delta}(x) = \max_{n,h}\left\{x(nh)^{\alpha} - W(n,h) - (1-\lambda)x(nh)^{\alpha} + \beta\int \Pi(x', n)\,dG(x'|x)\right\}$$

and

$$\Pi^{0}(x, n_{-1}) = \max_{h}\left\{x(n_{-1}h)^{\alpha} - W(n_{-1}, h) + \beta\int \Pi(x', n_{-1})\,dG(x'|x)\right\}.$$

4.2.2 Hours Per Worker

The choice of hours per worker is a simple intra-temporal problem. This allows us to solve for the optimal choice of hours and substitute h out of the dynamic program. Since the cost of adjustment is proportional to total output, it is apparent that the optimal choice of hours depends on whether the plant adjusts employment. Specifically, the first order conditions for hours per worker are

$$\Delta n \neq 0 : h = \left(\frac{\lambda x \alpha}{\omega \zeta}\right)^{\frac{1}{\zeta-\alpha}} n^{-\frac{1-\alpha}{\zeta-\alpha}} \tag{4.7}$$

$$\Delta n = 0 : h = \left(\frac{x \alpha}{\omega \zeta}\right)^{\frac{1}{\zeta-\alpha}} n_{-1}^{-\frac{1-\alpha}{\zeta-\alpha}}. \tag{4.8}$$

These state that, for a given number of workers, the plant selects a lower level of hours per worker if $\Delta n \neq 0$. The reason is that the disruption cost acts to diminish productivity, x, which lowers the marginal product of labor (man-hours) and deters greater hours worked.

Substitution of the hours policy rule into the plant's revenue function helps to illuminate the nature of the disruption cost. Plugging (4.8) and (4.2) into the adjustment cost function (4.1), we obtain

$$\begin{aligned} c(x,n) &= (1-\lambda)\left(\frac{\lambda\alpha}{\omega\zeta}\right)^{\frac{\alpha}{\zeta-\alpha}} x^{\frac{\zeta}{\zeta-\alpha}} n^{\alpha\frac{\zeta-1}{\zeta-\alpha}} \\ &= (1-\lambda)\left(\frac{\lambda\alpha}{\omega\zeta}\right)^{\frac{\alpha}{\zeta-\alpha}} x^{\frac{\zeta}{\zeta-\alpha}} (n_{-1}+\Delta n)^{\alpha\frac{\zeta-1}{\zeta-\alpha}} \end{aligned} \tag{4.9}$$

The cost of adjustment, shown graphically in Figure 1, is (i) concave in n and (ii) discontinuous at zero net employment growth i.e., $c(x,n) = c(x, n_{-1} + \Delta n) \neq 0$ if $\Delta n = 0$. It shares (ii) with the conventional formulation of a fixed cost as a lump-sum charge, as discussed in Hamermesh and Pfann (1996). This feature induces infrequent employment adjustments. Feature (i) affects the size of the adjustment in the event that the plant decides to change employment. A plant subject to a lump-sum fixed cost faces a marginal cost of adjusting of zero, conditional on $\Delta n \neq 0$. Here, that is not true, but the concavity of $c(x,n)$ still encourages substantial changes, conditional on adjusting. Consider a plant that debates whether to set employment at n or $n + \Delta n$. The cost to add an additional Δn is relatively low because the cost function is concave. As a result, the plant might find it optimal to hire additional workers now in order to avoid the expected discounted cost associated with a small upward adjustment in the future when n and x (and thus $c(x,n)$) may be higher. Conversely, a plant that debates whether to set employment at n or $n - \Delta n$ may opt for the latter since a marginal decrease in employment generates a relatively large reduction in the cost of adjustment due to the concavity of $c(x,n)$.

When we substitute the policy rule for hours into the Bellman equation, we obtain a dynamic program expressed only in terms of n. Specifically, we have

$$\Pi^{\Delta}(x) = \max_{n} \left\{ A (\lambda x)^{\frac{\zeta}{\zeta-\alpha}} n^{\alpha \frac{\zeta-1}{\zeta-\alpha}} - bn + \beta \int \Pi(x', n)\, dG(x'|x) \right\}, \tag{4.10}$$

where $A \equiv \frac{\zeta-\alpha}{\zeta}\left(\frac{\alpha}{\omega\zeta}\right)^{\frac{\alpha}{\zeta-\alpha}}$. Analogously, the value of inaction is given by

$$\Pi^{0}(x, n_{-1}) = A x^{\frac{\zeta}{\zeta-\alpha}} n_{-1}^{\alpha \frac{\zeta-1}{\zeta-\alpha}} - bn_{-1} + \beta \int \Pi(x', n_{-1})\, dG(x'|x). \tag{4.11}$$

It remains to solve for optimal employment. This is the task to be completed over the remainder of this section.

4.2.3 The Optimal Policy: A Conjecture

We will solve the plant's problem by the method of undetermined coefficients. That is, we conjecture,and then verify, the form of the optimal policy. The motivation for the conjecture is derived from an analysis of the problem in the special case where $\beta = 0$. It is straightforward to show in this context that the plant follows an Ss-like strategy: for x within a band, $[L_o(n_{-1}),\ U_o(n_{-1})]$, where L_o and U_o are continuous monotone functions, the plant does not adjust employment. If x falls outside of this region, it resets n. In addition, one may show that the "triggers", $L_o(n_{-1})$ and $U_o(n_{-1})$, are log-linear, as is the optimal reset policy, which is often conveniently expressed as an inverse demand function, $x = \chi_o(n)$. In particular, $L_o(n_{-1}) = \mathcal{L}_o n_{-1}^{1-\alpha}$; $U_o(n_{-1}) = \mathcal{U}_o n_{-1}^{1-\alpha}$; and $x = \chi(n) = \chi_o n^{1-\alpha}$, where $\mathcal{L}_o$, $\mathcal{U}_o$, and χ_o are constants.

Our conjecture is that the form of the policy in the dynamic enviroment is identical to that in the one-period problem. It is presented graphically in Figure 2.

Conjecture *Conditional on adjustment, the plant sets n according to an optimal reset policy function, $\chi(n)$, that solves the maximization problem embedded in (4.10). The plant adjusts n only if $x \notin [L(n_{-1}), U(n_{-1})]$, where $L(n_{-1})$ and $U(n_{-1})$ are the "triggers" that satisfy the following value matching relations,*

(4.12)
$$\begin{aligned} \Pi^0(L(n_{-1}), n_{-1}) &= \Pi^\Delta(L(n_{-1})) \\ \Pi^0(U(n_{-1}), n_{-1}) &= \Pi^\Delta(U(n_{-1})). \end{aligned}$$

Otherwise, if $x \in [L(n_{-1}), U(n_{-1})]$, then $n = n_{-1}$. Lastly, the functions that constitute the optimal policy take a log-linear form,

(4.13)
$$\begin{aligned} \chi(n) &= \chi n^{1-\alpha} \\ L(n_{-1}) &= \mathcal{L} n_{-1}^{1-\alpha} \\ U(n_{-1}) &= \mathcal{U} n_{-1}^{1-\alpha}, \end{aligned}$$

where χ, $\mathcal{L}$, and $\mathcal{U}$ are constants such that $\mathcal{L} < \chi < \mathcal{U}$.

One significant implication of the Conjecture is that the reset policies of the dynamic and static solutions are perfectly correlated. That is, conditional on adjustment, the elasticity of n with respect to x is identical across the two solutions and equal to $\frac{1}{1-\alpha}$. Of course, the time path of employment derived from the two solutions will differ at least to some degree since, in general, $\mathcal{U} \neq \mathcal{U}_o$ and $\mathcal{L} \neq \mathcal{L}_o$. Still, the Conjecture suggests the possibility that the optimal policy function in a rich, dynamic problem is actually quite straightforward. The next two sections verify this Conjecture.

4.2.4 The Choice of Employment Conditional on Adjustment

Conditional on $\Delta n \neq 0$, the choice of n must satisfy the first order condition,

$$\alpha \frac{\zeta - 1}{\zeta - \alpha} A (\lambda x)^{\frac{\zeta}{\zeta-\alpha}} n^{-\zeta \frac{1-\alpha}{\zeta-\alpha}} - b + \beta D(x, n) = 0, \tag{4.14}$$

where $D(x, n)$ is the expected future marginal value of employment:

$$D(x, n) \equiv \int \Pi_n (x', n)\, dG(x'|x). \tag{4.15}$$

To uncover the optimal policy of the plant, we evidently must solve for $D(x, n)$. The next proposition indicates that, under certain circumstances, the solution to $D(x, n)$ is quite simple.

Proposition 1 *If* $\log x$ *follows (4.4) and if the lower and upper triggers,* $L(n)$ and $U(n)$, *respect the bounds on idiosyncratic productivity given in (4.5), then*

$$D(x, n) = \frac{\alpha \frac{\zeta-1}{\zeta-\alpha} A \mathcal{P}_\zeta \chi^{\frac{\zeta}{\zeta-\alpha}} - b\mathcal{P}_0}{1 - \beta \mathcal{P}_0} = \text{constant},$$

where

$$\mathcal{P}_\zeta \equiv \frac{1}{2s} \frac{\zeta - \alpha}{\zeta} \left[G_U^{\frac{\zeta}{\zeta-\alpha}} - G_L^{\frac{\zeta}{\zeta-\alpha}} \right]$$

$$\mathcal{P}_0 \equiv \frac{1}{2s} \log \left(\frac{G_U}{G_L} \right)$$

and

$$G_U \equiv \mathcal{U}/\chi, \quad G_L \equiv \mathcal{L}/\chi.$$

That D is independent x is follows directly from the two assumptions stated in the proposition, namely, that the innovations are uniformly distributed and the boundaries of idiosyncratic productivity, $x^\rho e^{\mu - s}$ and $x^\rho e^{\mu + s}$, do not bind on the inaction region, $[L(n), U(n)]$. As we saw from (4.6), the uniform distribution implies that the probability density function of x' is independent of this period's x. As a result, x matters only insofar as it determines the support of the distribution, as shown in (4.5). Suppose now that the inaction region spanned by $L(n)$ and $U(n)$ lies within this support. The plant, when it selects n this period, is only concerned with the marginal value of employment for values of x' within this inaction band; if x' falls outside of that region, it will reoptimize. Thus, the plant has to forecast the marginal value of labor for $x' \in [L(n),\ U(n)]$. As a result, the upper and lower bounds of the support of x' are not relevant. This implies that x plays no role in the formation of the expected marginal value of labor.

This raises the question of whether the boundaries of x should be expected to bind on the inaction space. Intuitively, for λ sufficiently near one, the inaction region will be small relative to the variance of x, and Proposition 1 will apply in force.[9] But we do not want to restrict λ. Thus, it is possible – and, in the simulations, we do find this – that one of the triggers extends beyond a boundary of x. In this case, the effective trigger is in fact the limit of the feasible range ($x^{\rho}\mathrm{e}^{\mu-s}$ if $L(n) < x^{\rho}\mathrm{e}^{\mu-s}$ and $x^{\rho}\mathrm{e}^{\mu+s}$ if $U(n) > x^{\rho}\mathrm{e}^{\mu+s}$). As a result, the solution that we propose in this section is better thought of as an approximate guide to the true solution of the dynamic program. If the triggers violate the bounds infrequently, and only modestly, then the approximate solution we derive here will function as a very good approximation. Indeed, this is what we find below.[10]

Returning to the first order condition, we substitute in $D(x,n)$ and apply the Conjecture, $x(n) = \chi n^{1-\alpha}$. This expression then becomes a nonlinear equation in three unknown constants, $\{\chi, G_U, G_L\}$:

$$\chi = \left[\frac{1}{\lambda^{\frac{\zeta}{\zeta-\alpha}} + \beta\left(\mathcal{P}_{\zeta} - \lambda^{\frac{\zeta}{\zeta-\alpha}}\mathcal{P}_0\right)}\frac{\zeta-\alpha}{\zeta-1}\frac{\hat{b}}{\alpha}\right]^{\frac{\zeta-\alpha}{\zeta}}, \tag{4.16}$$

where $\hat{b} \equiv b/A$. To gain some insight into this result, consider first re-writing it in the following way,

$$\chi = \chi_o \times \left[1 + \beta\left(\mathcal{P}_{\zeta}\lambda^{-\frac{\zeta}{\zeta-\alpha}} - \mathcal{P}_0\right)\right]^{-\frac{\zeta-\alpha}{\zeta}},$$

where

$$\chi_o \equiv \lambda^{-1}\left[\frac{\zeta-\alpha}{\zeta-1}\frac{\hat{b}}{\alpha}\right]^{\frac{\zeta-\alpha}{\zeta}}.$$

The term, χ_o, is the solution to first order condition if $\beta = 0$. Equivalently, χ_o solves the one-period, or static, version of the plant's optimization problem. Therefore, this formulation decomposes the solution into two parts: a contemporaneous component, χ_o, and a forward-looking one. The latter remains somewhat hard to penetrate, though. To attack this, we note that, up to a first order, $\mathcal{P} \equiv \mathcal{P}_{\zeta} \cong \mathcal{P}_0 \cong \frac{G_U - G_L}{2s}$ (where the expansion is taken around $G_U = G_L = 1$). This is a reasonable approximation if λ is not too far from one. In that case, the inaction band will be sufficiently small,

[9] In a price-setting problem with ρ set to one and quadratic revenue, Danziger (1999) derives an explicit condition on the menu cost of adjustment needed to ensure that the triggers do not breach the bounds of x.

[10] That D is independent of n is almost equally convenient but economically less interesting. It essentially follows from the log-linearity of the revenue function and the optimal policy, $\chi(n) = \chi n^{1-\alpha}$. It turns out that, after substituting in this conjecture and collecting powers in n, the exponent on employment collapses to zero.

and alternative measures of the size of that band, such as $\mathcal{P}_\zeta$ and $\mathcal{P}_0$, will converge. This yields

$$\chi \cong \chi_o \times \left[1 + \beta\mathcal{P}\left(\lambda^{-\frac{\zeta}{\zeta-\alpha}} - 1\right)\right]^{-\frac{\zeta-\alpha}{\zeta}}. \tag{4.17}$$

This expression reveals the relation between the static and forward-looking policies more clearly. In particular, it indicates that a forward-looking plant, conditional on adjusting, sets n higher than its myopic ($\beta = 0$) counterpart. To see this, simply note that, since $\lambda < 1$, it must be that $\chi < \chi_o$. Letting n_o represent the optimal level of employment that solves the static problem, this means $n_o = (x/\chi_o)^{\frac{1}{1-\alpha}} < (x/\chi)^{\frac{1}{1-\alpha}} = n$. Intuitively, a forward-looking plant that is adjusting this period recognizes that, while the current marginal product is degraded by λ, the future marginal product, conditional on not adjusting, will be higher. Formally, this intuition operates through the forward-looking component $\mathcal{P}\left(\lambda^{-\frac{\zeta}{\zeta-\alpha}} - 1\right)$. Recall that $\mathcal{P}$ represents the probability of not adjusting next period. Therefore, when either $\mathcal{P}$ is large or λ small, the discrepancy between current productivity and expected future productivity is sizable. As a result, a forward-looking plant accumulates a greater stock of workers now.[11]

It follows, then, that if the adjustment friction is relatively small (λ is large) and the probability of adjusting sufficiently high, the importance of being forward-looking declines. To gauge the quantitative difference between the forward-looking and myopic rules, consider a calibration that is consistent with the literature and on par with what we implement in Section 3. In particular, set $\lambda = 0.92$, $\zeta = \frac{5}{3}$, and $\alpha = \frac{2}{3}$. Note that the value of λ, on which there is some agreement in the literature, is in fact not far from one. Next, suppose $G_U - G_L = \frac{3}{4}$, which is broadly consistent with the simulations conducted in the next section. Lastly, we set the standard deviation, σ, of ε to $\frac{1}{2}$. The form the forward-looking component in (4.17) indicates that choice of the standard deviation is also likely to be critical: when σ is large, the probability of adjusting again next period is high ($\mathcal{P}$ is small), and so the wedge between current productivity and expected future productivity is diminished. For uniformly distributed innovations, $\sigma = s/\sqrt{3}$, in which case it follows that $s = \frac{\sqrt{3}}{2}$. Under this calibration, we calculate

[11] As this discussion suggests, it is the *interaction* of foresight and the disruption cost that generates labor hoarding relative to the static model. This result does not generalize to alternative forms of fixed adjustment costs. For instance, suppose the establishment pays a lump-sum charge to change employment. In this case, the adjustment cost does not impinge on the first order condition and, in particular, does not degrade the productivity of the employer while adjusting. Consequently, the reset policy function in this model is, to a first order, identical to that in the static version of the problem.

$$\chi \cong \chi_o \times \left[1 + \frac{1}{\sqrt{3}}\lambda^{-\frac{5}{3}}\left(1 - \lambda^{\frac{5}{3}}\right)\right]^{-\frac{3}{5}} \cong \chi_o \times 0.95.$$

That is, the myopic solution is right to within 5 percent.

We next consider the decision to adjust. To preview the discussion, we again find that the myopic rule is quite accurate.

4.2.5 The Decision to Adjust

The decision of whether to adjust is made by comparing the value of adjusting, Π^{Δ}, with the value of inaction, Π^0. Define the gap between these as

$$\Delta\left(x, n_{-1}\right) = \Pi^{\Delta}\left(x\right) - \Pi^0\left(x, n_{-1}\right).$$

We develop a recursion in $\Delta\left(x, n_{-1}\right)$ that may be solved by the method of undetermined coefficients, just as we solved $D\left(x, n\right)$. Substituting in using (4.10) and (4.11), we first obtain the following:

$$\Delta\left(x, n_{-1}\right) = \max_n \left\{ \begin{array}{c} Ax^{\frac{\zeta}{\zeta-\alpha}}\left[\lambda^{\frac{\zeta}{\zeta-\alpha}} n^{\alpha\frac{\zeta-1}{\zeta-\alpha}} - n_{-1}^{\alpha\frac{\zeta-1}{\zeta-\alpha}}\right] - bn + bn_{-1} \\ +\beta \int \Pi\left(x', n\right) dG\left(x'|x\right) - \beta\int \Pi\left(x', n_{-1}\right) dG\left(x'|x\right) \end{array} \right\}. \tag{4.18}$$

The key to formulating the recursion now lies in re-writing the forward integrals. Fortunately, the Conjecture implies that

$$\begin{aligned} & \int \Pi\left(x', n\right) dG\left(x'|x\right) - \int \Pi\left(x', n_{-1}\right) dG\left(x'|x\right) \\ = \quad & -\int_{L(n)}^{U(n)} \Delta\left(x', n\right) dG\left(x'|x\right) + \int_{L(n_{-1})}^{U(n_{-1})} \Delta\left(x', n_{-1}\right) dG\left(x'|x\right). \end{aligned}$$

The top line represents the "excess return" to adjusting this period: it is the difference between the expected value of beginning next period with a re-optimized level of employment as opposed to the level of employment inherited at the start of this period. The bottom line provides an alternative way to express this. It states, first, that the difference in continuation values depends only on the values of Δ within the inaction regions associated with n and n_{-1}. Outside of these bands, the plant will re-set employment, and the size of the workforce with which it began the period will have no implication for the value of the plant going forward. But why do the Δs appear? On the one hand, if the net value of bringing n_{-1} forward and adjusting next period is relatively high, then the net value of implementing that adjustment this period is also relatively high. This is why the term, $\Delta(x', n_{-1})$, enters positively in this expression. On the other hand, if it is profitable to bring forward a new level of employment (n) and then do nothing next period, then the net value of adjusting now must be relatively large. This is why the net value of adjusting, $\Delta(x, n_{-1})$, increases in $-\Delta(x', n) = \Pi^0(x', n) - \Pi^\Delta(x')$.

Combining the forward integrals with the top line of (4.18), substituting in the policy rule $n = (x/\chi)^{\frac{1}{1-\alpha}}$, and making use of the Conjecture concerning the triggers $U(n)$ and $L(n)$, the recursion in (4.18) becomes

$$\begin{aligned}\Delta(x, n_{-1}) &= bn_{-1} - An_{-1}^{\alpha\frac{\zeta-1}{\zeta-\alpha}} x^{\frac{\zeta}{\zeta-\alpha}} + \left[A\lambda^{\frac{\zeta}{\zeta-\alpha}}\chi^{\frac{\zeta}{\zeta-\alpha}} - b\right]\chi^{-\frac{1}{1-\alpha}}x^{\frac{1}{1-\alpha}} \\ &\quad -\beta\int_{\left(\frac{\mathcal{L}}{\chi}\right)x}^{\left(\frac{\mathcal{U}}{\chi}\right)x}\Delta(x', n)\,dG(x'|x) + \beta\int_{L(n_{-1})}^{U(n_{-1})}\Delta(x', n_{-1})\,dG(x'|x).\end{aligned} \tag{4.19}$$

The form of this expression suggests the following guess for a solution to Δ:

$$\Delta(x, n_{-1}) = \delta_0 n_{-1} + \delta_1 n_{-1}^{\alpha\frac{\zeta-1}{\zeta-\alpha}} x^{\frac{\zeta}{\zeta-\alpha}} + \delta_2 x^{\frac{1}{1-\alpha}}, \tag{4.20}$$

where the δs are constants. The next proposition states that this conjecture may be verified and provides the solutions for the unknown coefficients.

Proposition 2 *Under the Conjecture (4.13), and given the law of motion for* $\log x$ *in (4.4), the solution to* $\Delta(x, n_{-1})$ *is given by (4.20) with*

$$\begin{aligned}\delta_1 &= -A \\ \delta_0 &= A\frac{\hat{b} + \beta\left[\lambda^{\frac{\zeta}{\zeta-\alpha}}\mathcal{P}_1 - \mathcal{P}_\zeta\right]\chi^{\frac{\zeta}{\zeta-\alpha}}}{1 + \beta\left[\mathcal{P}_1 - \mathcal{P}_0\right]} \\ \delta_2 &= A\chi^{-\frac{1}{1-\alpha}}\left[\lambda^{\frac{\zeta}{\zeta-\alpha}}\chi^{\frac{\zeta}{\zeta-\alpha}} - \frac{\hat{b} + \beta\left[\lambda^{\frac{\zeta}{\zeta-\alpha}}\mathcal{P}_1 - \mathcal{P}_\zeta\right]\chi^{\frac{\zeta}{\zeta-\alpha}}}{1 + \beta\left[\mathcal{P}_1 - \mathcal{P}_0\right]}\right],\end{aligned}$$

where

$$\mathcal{P}_1 \equiv \frac{1}{2s}(1-\alpha)\left[G_U^{\frac{1}{1-\alpha}} - G_L^{\frac{1}{1-\alpha}}\right].$$

We are now able to complete the solution for the optimal policy. Recall that the triggers, $L(n_{-1})$ and $U(n_{-1})$, satisfy the value matching conditions, $\Delta(L(n_{-1}), n_{-1}) = \Delta(U(n_{-1}), n_{-1}) = 0$. That is, if $g(n_{-1})$ represents an unknown function, the triggers are the roots of

$$0 = \delta_0 n_{-1} + \delta_1 n_{-1}^{\alpha\frac{\zeta-1}{\zeta-\alpha}} g(n_{-1})^{\frac{\zeta}{\zeta-\alpha}} + \delta_2 g(n_{-1})^{\frac{1}{1-\alpha}}.$$

The Conjecture states that $g(n_{-1})$ is of the form, $gn_{-1}^{1-\alpha}$ (where $g = \mathcal{L},\mathcal{U}$). Substituting this in, we find

$$0 = \delta_0 + \delta_1 \chi^{\frac{\zeta}{\zeta-\alpha}} G^{\frac{\zeta}{\zeta-\alpha}} + \delta_2 \chi^{\frac{1}{1-\alpha}} G^{\frac{1}{1-\alpha}},$$

where $G \equiv g/\chi$. Proposition 2 and the reset policy, (4.16), allow us to write this as:

$$\begin{aligned} 0 &= \left[(\lambda\chi_o)^{\frac{\zeta}{\zeta-\alpha}} - \hat{b}q\right] G^{\frac{1}{1-\alpha}} - \chi_o^{\frac{\zeta}{\zeta-\alpha}} G^{\frac{\zeta}{\zeta-\alpha}} + \hat{b}q \\ &\quad + (1-q)(\lambda\chi_o)^{\frac{\zeta}{\zeta-\alpha}} \left(1 - G^{\frac{1}{1-\alpha}}\right) \end{aligned} \tag{4.21}$$

where

$$q \equiv \frac{1 + \beta\left(\lambda^{-\frac{\zeta}{\zeta-\alpha}}\mathcal{P}_\zeta - \mathcal{P}_0\right)}{1 + \beta(\mathcal{P}_1 - \mathcal{P}_0)}$$

$$1 - q = -\beta\frac{\lambda^{-\frac{\zeta}{\zeta-\alpha}}\mathcal{P}_\zeta - \mathcal{P}_1}{1 + \beta(\mathcal{P}_1 - \mathcal{P}_0)}.$$

Setting $q = 1$ (equivalently, $\beta = 0$), this expression collapses to the value matching condition in the static, or myopic, problem.

Equation (4.21) is a cumbersome nonlinear expression. It is straightforward to solve it numerically, and, when we do so, we have invariably found two, distinct real-valued roots with one less than one and the other greater than one (G_L and G_U in terms of the notation introduced above). But it is difficult to prove this in the context of (4.21). To make progress, we must rely on an approximation introduced in Section 2.4, namely, $\mathcal{P} \equiv \mathcal{P}_\zeta \cong \mathcal{P}_0 = \frac{G_U - G_L}{2s}$. Moreover, up to a first order, it is also true that $\mathcal{P}_1 \cong \frac{G_U - G_L}{2s}$. Thus, we have that $\mathcal{P} \equiv \mathcal{P}_\zeta \cong \mathcal{P}_1 \cong \mathcal{P}_0$. Substituting this into the definition of q, using $\chi_o \equiv \lambda^{-1}\left[\frac{\zeta-\alpha}{\zeta-1}\frac{\hat{b}}{\alpha}\right]^{\frac{\zeta-\alpha}{\zeta}}$, and re-arranging gives

$$0 = \left(\frac{1-\hat{\alpha}}{\hat{\alpha}} + \right) G^{\frac{1}{1-\alpha}} - \frac{1}{\hat{\alpha}}\lambda^{-\frac{\zeta}{\zeta-\alpha}} G^{\frac{\zeta}{\zeta-\alpha}} + (1-) \tag{4.22}$$

where $\hat{\alpha} \equiv \alpha\frac{\zeta-1}{\zeta-\alpha} < 1$ is the returns to scale in employment (after concentrating out hours worked) and

$$\begin{aligned} &\equiv \beta\frac{1-\hat{\alpha}}{\hat{\alpha}}\left(\lambda^{-\frac{\zeta}{\zeta-\alpha}} - 1\right)\mathcal{P} \\ &= \beta\frac{1-\hat{\alpha}}{\hat{\alpha}}\left(\lambda^{-\frac{\zeta}{\zeta-\alpha}} - 1\right)\frac{1}{\sigma 2\sqrt{3}}[G_U - G_L] \equiv \phi[G_U - G_L] \end{aligned} \tag{4.23}$$

represents the forward-looking component of the value matching condition.[12] That is, setting

[12]Note that $\hat{b}$ vanishes from this expression. This indicates that, for a sufficiently small disruption cost (in which

recovers the value matching condition associated with the static ($\beta = 0$) problem. It is possible to characterize the solution of (4.22).

Proposition 3 *There exist two distinct, real-valued roots to equation (4.22), where one root, G_L, is less than one and the other, G_U, greater than one.*

Since $G_L \equiv \mathcal{L}/\chi$ and $G_U \equiv \mathcal{U}/\chi$, the proposition verifies the existence of the triggers that constitute the optimal policy – to the extent, that is, that (4.22) serves as a satisfactory approximation to the true value matching condition. We will find in the next section that the critical approximation needed to derive (4.22), namely $\mathcal{P} \equiv \mathcal{P}_\zeta \cong \mathcal{P}_1 \cong \mathcal{P}_0$, does work reasonably well in practice. Therefore, the proposition may be seen as confirming the conjecture stated in (4.13).

We see the same term, $\left(\lambda^{-\frac{\zeta}{\zeta-\alpha}} - 1\right)\mathcal{P}$, in (4.23) which governed the importance of the forward-looking component in the optimal reset policy of Section 2.4. Why would this be? Remember that the forward-looking term encouraged an employer who adjusted to hire more than if it behaved myopically. Intuitively, this same motive should drive employers to modify their rules for when to adjust. For instance, if expected future productivity is high relative to productivity this period, an employer may be more willing to raise employment over a wider range of xs: by doing so, he "locks in" those future gains. Now suppose a forward-loooking employer receives an x that would trigger a reduction in the workforce at a myopic establishment. The former may choose not to adjust precisely because it is not nearly as far away from the upper trigger as its myopic counterpart; the gain from adjusting downwards is diminished if the probability of reversing oneself is high. Thus, the forward-looking employer actually freezes over this lower range of xs when a myopic plant would reduce n. In short, the same reasoning behind our analysis of the reset policy leads one to suspect that triggers of a forward-looking plant would shift down relative to those of a myopic establishment. We will see in the next section that this is true.[13]

To conclude this section, we would now like to gauge the quantative importance of forward-looking behavior on the choice of triggers. Even within the context of (4.22), this is slightly difficult to do since is a function of G_U and G_L. However, under the calibration used in the prior subsection, it turns out that is relatively insensitive to plausible variations in the gap, $G_U - G_L$. The simple reason is that λ is not too far from one (0.92) and σ is relatively large ($\frac{1}{2}$). As a result, the coefficient, ϕ, pre-multiplying $G_U - G_L$ in (4.23) is fairly small, $\phi = 0.1065$. Therefore, variation

case the first order approximation of the $\mathcal{P}$s is satisfactory), the fixed component of the wage, $\hat{b}$, has virtually no effect on the choice of the triggers. It does, of course, affect the reset policy.

[13] A complete analysis of comparative statics remains difficult since $\mathcal{P}$ depends on G_L and G_U. Therefore, we restrict ourselves here to outline the intuition and then revisit this matter numerically in Sections 3 and 4.

in $G_U - G_L$ from 0.6 to 1.2 (which summarizes the relevant range) shifts from 0.064 to just 0.128. Contrast this with the other coefficients that appear alongside in (4.22), namely, $\frac{1-\hat{\alpha}}{\hat{\alpha}} = \frac{5}{4}$ and 1, and it appears that relatively little is lost if we regard $G_U - G_L$ as approximately fixed for the purposes of this discussion.

With this approximation, a few calculations reveal an important result. Given the calibration of the last subsection ($\beta = 1$, $\lambda = 0.92$, $\zeta = \frac{5}{3}$, $\alpha = \frac{2}{3}$, $\sigma = \frac{1}{2}$, and $G_U - G_L = \frac{3}{4}$), the value matching condition associated with the dynamic problem becomes

$$0 = 1.33 \times G^{\frac{1}{1-\alpha}} - 2.585 \times G^{\frac{\zeta}{\zeta-\alpha}} + 0.919,$$

whereas the corresponding condition in the static version of the model is

$$0 = 1.25 \times G^{\frac{1}{1-\alpha}} - 2.585 \times G^{\frac{\zeta}{\zeta-\alpha}} + 1.$$

Figure 3 graphs each of these expressions. The forward-looking component appears to exert relatively little influence on the solution. The logarithmic gap between the G_Ls (the smaller of the two solutions) and the G_Us (the larger of the two) is only about five points each. As we will show when we turn to simulation analysis, this translates into minor differences in the paths of employment induced by the myopic and true policy rules.

The effect of the forward-looking component may be seen another way. Re-arranging (4.22), we obtain,

$$0 = \frac{1-\hat{\alpha}}{\hat{\alpha}} G^{\frac{1}{1-\alpha}} - \frac{1}{\hat{\alpha}} \lambda^{-\frac{\zeta}{\zeta-\alpha}} G^{\frac{\zeta}{\zeta-\alpha}} + 1 + \left(G^{\frac{1}{1-\alpha}} - 1 \right). \tag{4.24}$$

We note that, again to a first order,

$$G^{\frac{1}{1-\alpha}} - 1 \cong \frac{1}{1-\alpha} (G-1).$$

Substituting this in, the forward-looking component of (4.24) becomes

$$\begin{aligned} \left(G^{\frac{1}{1-\alpha}} - 1 \right) &= \frac{\phi}{1-\alpha} (G_U - G_L)(G-1) \\ &= \frac{\phi}{1-\alpha} \left[(G_U - 1)(G-1) + (1 - G_L)(G-1) \right]. \end{aligned}$$

This expression suggests that the foresight effect is second-order given a first-order friction. That is, if $1 - G_L \cong \varepsilon \cong G_U - 1$, then the expression in brackets becomes $2\varepsilon^2$. This is what lies behind the calculations regarding the effect of the forward-looking component.[14]

4.3 Simulations

This section has two principal goals. The first is to investigate how well the log-linear form in the Conjecture, which is exact locally in the range $[L(n), U(n)] \in [x^\rho e^{\mu - s}, x^\rho e^{\mu + s}]$, fits the global policy function. The second is to ask whether the analytical results derived under the assumption of uniform innovations serve as a good guide to models in which the firm faces alternative stochastic processes. Many authors assume, for instance, that x follows a first-order Markov process with lognormal innovations. We investigate this process in particular in this and the next section.

4.3.1 Log Uniform Productivity

Analytical solutions are unavailable once we allow that the bounds on x may bind with regard to the inaction space. Therefore, we must turn to numerical analysis. Specifically, we solve the model by value function iteration. We fix a calibration and use it throughout this section and defer sensitivity analysis until Section 4.

The baseline calibration is reported in Table 1. In large part, these values are drawn from Cooper, Haltiwanger, and Willis (2005, CHW hereafter). CHW estimate a model that is very similar to that presented in Section 2. The only difference is that they replace the uniform distribution in (4.4) with the Gaussian. We find below, however, that the salient moments of the data are largely insensitive to the choice of the distribution function, and so we proceed on the basis of CHW's calibration since their model is the closest in substance to ours.[15]

[14]To derive these results, we have had to confine ourselves to a model where innovations to $\log x$ are drawn from a uniform distribution. It is possible to show that the quality of the myopic approximation is actually robust to *any* Markov process for x when the adjustment cost takes the form $c(x, n) = Cx^{\frac{1}{1-\alpha}}$. Note that c is independent of n but remains a function of x, so large employers do not "outgrow" it. This is the form of the cost adopted by Gertler and Leahy (2008). Under this alternative assumption on c, one may prove the equivalence, up to a first order, between the static and optimal policies. The proof follows directly by differentiation of (4.15) and (4.18) with respect to C.

This result does not hold in our case. This may be most easily illustrated by appealing to the first order condition. This says, loosely, that the optimal reset policy $(\chi(n))$ depends on the contemporaneous marginal product and the expected future marginal product, conditional on not adjusting next period. The presence of λ drives a wedge between these two components, since the productivity of labor is not degraded in the future if the employer does not adjust. This wedge makes the forward term salient even for λ near one. In contrast, under $c(x, n) = Cx^{\frac{1}{1-\alpha}}$, the adjustment cost appears nowhere in the first condition. As a result, for sufficiently small C, the forward term exerts only a second-order influence on the choice of employment.

[15]It should be noted that the model of Bloom (2009), whose estimate of ζ is referenced in Table 1, is substantively different from that presented in Section 2. Bloom's model includes disruption, kinked, and quadratic adjustment costs on both capital and labor. Whether the analysis of Section 2 extends to a model with such a rich array of frictions is a question we defer for future work.

CHW's report an estimate of $\lambda = 0.919$, which implies a loss of roughly eight percent of quarterly revenue whenever an adjustment is undertaken.[16] This is consistent with Bloom's (2009) estimate.[17] We set ζ, which (partially) determines the elasticity of compensation with respect to hours, to 1.59, which is the midpoint between CHW and Bloom.[18] The remainder of the parameters – the persistence of productivity (ρ), the standard deviation of the innovation (σ), and the returns to scale (α) – are simply taken from CHW. We essentially lift these parameters from the literature since our objective in this paper is not to re-estimate the model, but to explore the implications of the literature's findings.

We first assess how well the analytical approximation of the policy rule derived in Section 2 "fits" the globally valid policy function. The latter must be obtained numerically via value function iteration. The result is shown in Figure 4. Given $x_{-1} = \mathrm{E}[x]$, the dashed lines bound the range in which x this period must lie. Within this range, the approximate and true policy rules lie virtually on top of one another.[19] Evidently, while the global policy rule is not necessarily log-linear, it appears to be very nearly so. We have confirmed that the quality of the approximation does not deteriorate as x_{-1} falls or increases away from $\mathrm{E}[x]$.

The log-linearity of the global policy rule suggests that, even if the boundaries of x bind on the inaction region, $[L(n_{-1}),\ U(n_{-1})]$, the approximation errors are likely to be small. This is a critical result. In the simulations to which we turn next, the boundaries of x do in fact bind on the zone of inaction roughly 10-15 percent of the time, although the violations are relatively modest (in the sense that $U(n_{-1})$ $(L(n_{-1}))$ does not exceed $x^{\rho}\mathrm{e}^{\mu+s}$ $(x^{\rho}\mathrm{e}^{\mu-s})$) by more than 5 percent on average). Yet the approximate policy rule performs very well. This is in part due to the fact that the fundamental log-linear structure of the static model is apparently inherited by the global

[16]CHW augment the model of Section 2 with a quadratic adjustment cost. They find a modest improvement in the goodness of fit, and the estimated value of λ increases to 0.976. We abstract from quadratic adjustment costs in this analysis. As Table 3b of CHW illustrates, the most critical adjustment friction, as measured by its contribution to the model's ability to fit the data, is the disruption cost. In the interest of tractability, then, we narrow our focus to the disruption cost and set λ accordingly. We should note that this choice is conservative in the sense that the myopic approximation improves considerably as λ nears one (and the adjustment friction approaches zero).

[17]In Bloom, the disruption cost is given by $C_L^F y$, where y is output (or revenue). Comparing this with (4.1), and noting that Bloom's data are annual rather than quarterly, we see that the mapping from C_L^F to λ is $C_L^F = \frac{1}{4}(1-\lambda)$. Inverting this, and using Bloom's estimate of $C_L^F = 0.021$, gives $\lambda = 0.916$.

[18]CHW find that $\zeta = 1.09$ and Bloom estimates $\zeta = 2.093$. A lower value of ζ raises the flexibility of hours since compensation is relatively insensitive to adjustments on the intensive margin. As a result, a low value of ζ expands the region of inaction and increases the degree of inertia in plant-level employment.

This does raise an important technical point. Because $\zeta \cong 1$ generates a very wide zone of inaction, the bounds of that inaction region frequently reach beyond the feasible range of x, $\left[x_{-1}^{\rho}\mathrm{e}^{\mu-s}, x_{-1}^{\rho}\mathrm{e}^{\mu+s}\right]$. This degrades the quality of the approximation of the analytical solution obtained in Section 2. While this issue does merit further work, it is important to note that $\zeta = 1.59$ remains on the low end of the range of values found in the literature. In this sense, it is still a relatively conservative choice. Indeed, in separate work that analyzes a dynamic labor demand model with job search, CHW (2007) calibrate ζ to be 2.9.

[19]Since we solve the dynamic program by discretizing the state space, there is inevitably some error in the calculation of the "true" policy rule. Nonetheless, for the sake of brevity, we will refer to the policy rule obtained by value function iteration as the true policy.

forward-looking policy rule.

Next, we simulate the approximate and true policy rules and compare their implications for the cross-sectional and time series properties of employment and hours.[20] To begin, we simulate two panels of employment and hours data: in one panel, the plants observe the true policy and, in the other, all plants implement the approximate policy. Thus, each plant in the first panel has a counterpart in the second panel that faced the same sequence of xs but implemented the approximate rule. We calculate a quarterly time series of employment growth at the plant level . For each pair of plants (one observes the true policy and the other follows the approximation), we compute the correlation of the employment paths, and then take the average of the pairwise correlations across establishments. The results are reported in Table 2. The policy rules generate very similar paths of quarterly employment growth; the average correlation is almost 0.99.

The similarity across these simulated panels indicates that the two policy rules likely induce very similar empirical moments. For instance, the cross-sectional standard deviation of quarterly employment growth (where the latter is measured by $\Delta \log(n)$) induced by the approximate policy rule is within 0.2 percent of the standard deviation generated by the exact rule.[21] Another salient feature of the cross-sectional distribution is the share of establishments in each quarter that do not adjust employment. These establishments form a "spike" in the quarterly employment growth distribution at $\Delta \log(n) = 0$. Table 2 indicates that the two policy rules generate virtually the same mass point at zero.

When we turn to the intensive margin, the two policy rules still yield similar results, though some discrepancies emerge. On a positive note, the cross-sectional distribution of hours growth induced by each policy rule is nearly identical, as shown in Table 2. However, the correlation between the simulated paths of hours growth at the plant level is 0.9262.[22] To see why this is relatively low, consider a pair of plants – one of them observes the true policy and the other implements the approximate rule – in which each adjusts employment this period. There will be some discrepancy between the two plants' choices, in part because of approximation error in the analytical policy function and in part because the true policy rule is actually slightly contaminated by numerical

[20] We follow CHW and simulate the employment decisions of 1000 plants for 400 quarters each. Each plant is simulated for so many quarters in order to reduce the influence of initial conditions (i.e., n_{-1} and x_{-1}).

[21] We focus on the standard deviation induced by one policy rule *relative* to that induced by another, rather than the point estimate itself. CHW do not actually target moments of the employment growth distribution in their simulated method of moments exercise. (They are more interested in the *co-movement* of employment and hours changes, and their selection of moments reflects this.) Thus, we do not expect the model to perform well along these dimensions. Instead, we only seek to demonstrate that, given a calibration, the approximate policy rule generates moments that are very similar to those induced by the exact rule. Future work will take up the task of re-estimating the model.

[22] We further document this apparent discrepancy between cross-sectional moments and plant-level time series behavior in the sensitivity analysis of Section 4 and defer a full discussion of the issue until then.

error (see footnote 17). This discrepancy is not large relative to the typical change in employment (conditional on adjusting). But hours worked is appreciably less variable than employment under the baseline calibration, and so relatively small differences in $\Delta \log n$ are translated into slightly larger discrepancies in hours growth.[23]

Lastly, we find that each policy rule implies similar co-movement in hours and employment growth. The contemporaneous correlation induced by the approximate policy is within 7 percent of that generated by the exact policy function. The dynamic relationship between intensive and extensive adjustments that emerges from each of the simulations is also very similar. CHW summarize the dynamics with a bivariate first-order VAR in employment and hours growth. We estimate the VAR on the simulated data. With one exception, the coefficients from these regressions are very similar across the two policy rules. The exception relates to the response of $\Delta \log(h)$ to $\Delta \log(n_{-1})$, but the point estimate of this coefficient under each policy rule is less than 10^{-3}. We claim, then, that the discrepancy observed in the table is largely due to sampling variation and has no meaningful implication for the quality of the analytical approximation to the true policy function.

We conclude that, for this calibration, the analytical approximation derived in Section 2 is a very good guide to the "true" solution. This finding motivates us to ask whether the myopic policy rule, which bears a strong resemblance to the analytical solution of the dynamic model, also serves as a reasonable approximation to the exact solution of the dynamic problem. Figure 5 compares the two policy rules graphically. The myopic rule appears to fit the exact dynamic rule remarkably well. (As above, we set $x_{-1} = \mathrm{E}[x]$, but the goodness of fit is unaffected by variations in x_{-1}.) This is consistent with the calculations presented in the prior section. It follows that the myopic rule induces empirical moments that correspond very closely with those generated by the exact policy rule. This is shown in the right-hand column of Table 2.

The ability of the myopic policy rule to approximately replicate the key moments of plant-level employment suggests that the profit lost due to myopia is likely very small. This is confirmed in the final row of Table 2. In the course of the simulations, we calculate average contemporaneous profit generated by all three policy rules (the myopic, the approximate dynamic, and the true dynamic policy rules). A myopic firm earns just 0.6 percent less profit than a firm that solves the exact dynamic labor demand problem.

[23] CHW actually find that hours growth at the plant level is nearly as variable as employment growth. This fact is likely behind their estimate of $\zeta = 1.09$, which implies that compensation is relatively insensitive to variations in hours worked. In this paper, we split the difference between CHW's estimate and Bloom's finding of $\zeta = 2.093$. This choice of ζ substantially reduces the variability of hours growth. In future work that re-estimates the model, we will return to this question of how to reconcile disparate estimates of ζ in a manner consistent with data on hours growth.

4.3.2 Lognormal Productivity

Section 2 assumed that $\log x$ follows an AR(1) process with uniform innovations in the interest of tractability. This section assesses whether the principal result derived under the assumption of uniform innovations – the near optimality of myopic behavior – extends to models with alternative assumptions on the stochastic process of x. Perhaps the most common assumption in the literature is that x follows a geometric Gaussian autoregressive process. Indeed, CHW assume that $\log x$ is given by the AR(1) representation in equation (4.4) but with uniform innovations replaced by Gaussian ones. This section adopts CHW's specification of the law of motion of x and evaluates the performance of the myopic policy function relative to the optimal rule. The calibration of the structural parameters is unchanged (see Table 1).[24]

As above, we first compare the policy rules graphically. Since the innovation, ε, in (4.4) may lie anywhere on the real line, the support of $\log x$ is independent of $\log x_{-1}$. This differs from the log uniform process, and implies, as a result, that the policy rules shown in Figure 6 are valid for any history of past xs.

The figure suggests that at least some of the key properties of the optimal policy under uniform innovations extend to the model with Gaussian innovations. For instance, under uniform innovations, we found $\chi < \chi_o$. This implied that an employer which resets n selects a higher level of employment if it is forward looking. This finding appears to generalize to the model with Gaussian disturbances. The figure indicates, for instance, that a firm which receives the mean value of x ($\log(\mathrm{E}[x]) \cong 0.14$), and resets employment according to the forward-looking policy, selects a level of n that is 22.5 log points higher than that chosen by a firm that follows the myopic policy. This should not be too surprising: the intuition behind $\chi < \chi_o$ was that a forward-looking firm accumulates additional workers in anticipation of the marginal product increasing once the adjustment is complete. This finding was never obviously related to any assumption on the distribution of shocks.

In addition, under uniform innovations, we found that $\frac{\mathcal{U}}{\chi}$ and $\frac{\mathcal{U}_o}{\chi_o}$ were approximately the same, as were $\frac{\mathcal{L}}{\chi}$ and $\frac{\mathcal{L}_o}{\chi_o}$. Since $\chi < \chi_o$, it followed that $\mathcal{U} < \mathcal{U}_o$ and $\mathcal{L} < \mathcal{L}_o$. These results implied that one might obtain the thresholds governing the decision to adjust in the dynamic model by essentially shifting down the myopic thresholds. This qualitative finding is also evident in Figure 6. As a result, and as we found in the case of uniform innovations, the myopic and dynamic policy rules appear to be parallel to one another. This suggests that firms which follow different rules will

[24] Since analytical results of any kind are unavailable under Gaussian innovations, there is no analogue in this subsection to what we have referred to as the approximate dynamic policy function and which was derived under the assumption of uniform innovations. Therefore, in this section, whenever we refer to the dynamic policy rule, we mean the policy function solved for numerically via value function iteration.

select different employment levels at any point in time but the two employment paths will likely co-move very closely. This impression is confirmed in the simulations, to which we now turn.

We simulate the plant's behavior under the myopic and dynamic policy rules. The results are collected in Table 3, which is exactly analogous to Table 2 discussed in the prior subsection. Specifically, we compare simulations of quarterly employment growth under each policy rule. The correlation across the two policy rules is, again, 0.99. The myopic policy rule also replicates key features of the employment growth distribution induced by the dynamic rule. For instance, the myopic rule generates virtually the same probability of adjustment. The dispersion in employment growth induced by the myopic policy function is, on the other hand, slightly greater than that generated by its dynamic counterpart. Still, the standard deviation of $\Delta \log (n)$ under the myopic rule is within three percent of that under the "true" policy function. The myopic rule also performs quite well with respect to the behavior of hours worked. The correlation of hours growth across the two policy rules is actually slightly greater than that observed in the model with uniform innovations, and the dispersion in hours growth is virtually the same in the two simulations.

In some instances, the two policy rules do generate appreciably different outcomes with respect to the relationship between hours and employment growth. The contemporaneous correlation between hours and employment growth is -0.04 under the true policy and -0.106 under the myopic rule. These two results are far apart in log terms (as indicated in the table). Whether this difference is economically meaningful depends on what it portends for structural inference: does it suggest, for instance, that a myopic rule would require a different configuration of structural parameters in order to replicate the empirical correlation? We defer such questions for future work that explicitly tackles structural estimation. In regards to the simulated VAR, the major discrepancy relates to the response of $\Delta \log h$ to $\Delta \log n_{-1}$. Again, however, the point estimates under each policy rule are small (-0.0109 under the true policy, -0.014 under the myopic rule) and appear to be economically insignificant.

To conclude this subsection, we consider the losses, in terms of foregone profit, from myopia. As in the model with uniform innovations, the losses are slight: myopia costs the firm less than one third of one percent relative to the profit obtained under the true, forward-looking policy.

4.4 Sensitivity Analysis

In this section, we evaluate the performance of the myopic policy rule under alternative calibrations. To frame this discussion, it is helpful to revisit the forward-looking components of the reset policy (4.17), restated here as

$$\left[1+\beta\left(\lambda^{-\frac{\zeta}{\zeta-\alpha}}-1\right)\mathcal{P}\right]^{-\frac{\zeta-\alpha}{\zeta}},$$

and of the threshold policy (4.23),

$$\beta\left(\frac{1}{\hat{\alpha}}-1\right)\left(\lambda^{-\frac{\zeta}{\zeta-\alpha}}-1\right)\mathcal{P},$$

where $\mathcal{P}\equiv\frac{G_U-G_L}{2s}=\frac{G_U-G_L}{2\sigma\sqrt{3}}$. Since the gap G_U-G_L is endogenous, the comparative statics here are not straightforward. But it remains useful, if only heuristic, to consider the effect of changing parameters holding G_U-G_L fixed. These thought experiments will guide our simulation analysis in this section, as they suggest where the results of Section 2 may be sensitive to perturbations in the structural parameters.

Each of these expressions suggests that the importance of the forward-looking component is declining in λ and σ. The effect of lowering λ is to increase expected future productivity relative to current productivity, which raises the payoff to taking into account expected future developments in deciding n this period. Why does an in increase in σ reduce the payoff to foresight? When the environment becomes volatile, the odds of re-optimizing next period increase, and so the benefits of choosing n this period with an eye toward the future diminish.

In addition, since $1/\hat{\alpha}-1=\frac{\zeta}{\zeta-1}\frac{1-\alpha}{\alpha}$, it is also straightforward to verify that each component is decreasing in ζ. Intuitively, when ζ is low, the establishment relies more heavily on changes in hours per worker to absorb movements in x (since the elasticity of compensation with respect to hours is small). As a result, the establishment must prepare itself to maintain its current level of employment for up to several quarters, raising the payoff from being forward looking in selecting n.[25]

Notice one parameter that is not present in these expressions, namely, ρ. This reflects the fact that the probability density function of x is independent of x_{-1}. This feature of the stochastic process (4.4), we recall, is precisely what enabled us to obtain a quasi-analytical solution to the problem. But it also implies that the persistence of productivity does not affect the determination of the optimal policy. On this score, the model with uniform innovations diverges from the model with Gaussian disturbances.

[25]Inspection of these expressions reveals that the effect of α is actually ambiguous: a higher α lowers $1/\hat{\alpha}-1$ but raises $\lambda^{-\frac{\zeta}{\zeta-\alpha}}-1$ (since $\lambda<1$). In the interest of space, and since most applied work generally does not differ on the choice of α, we do not explore the effect of varying this parameter.

Motivated by this discussion, we now explore the effect of alternative calibrations of λ, σ, and ζ on the quality of the approximation afforded by the myopic policy rule.[26] We also vary the persistence parameter, ρ, when we turn to the analysis of the model with Gaussian shocks.

4.4.1 Log Uniform Productivity

We first perturb λ and leave the remainder of the calibration unchanged. The results are presented in the top panel of Table 4. (The bottom panel relates results under Gaussian innovations, which we set aside for now.) We consider values in the neighborhood of the baseline estimate, varying λ from 0.9 to 0.94. These are actually relatively sizable variations in light of the standard error (0.005) estimated by CHW. The correlation between the employment path induced by the myopic rule and the path generated by the optimal rule does decline slightly as λ falls further away from one. This deterioration is reflected, and amplified, in the correlation between the hours growth paths induced by the two policy rules. It is also reflected in the fact that the loss of profit due to myopic behavior is larger, although it remains on the order of just one percent.

The only real surprise in the top panel of Table 4 relates to the co-movement of hours and employment growth. The contemporaneous correlation induced by the myopic rule is too low when λ falls to 0.9 and too high as λ climbs to 0.94. It is important to keep these numbers in perspective, though: when $\lambda = 0.9$, the correlation generated by the optimal policy is -0.0571, whereas the correlation induced by the myopic rule is -0.0841. The log difference between these simulation results likely overstates the economic content of this discrepancy. Still, in future work that re-estimates the model, we will have to pay particular attention to how well the myopic rule is able to replicate the empirical correlation between hours and employment.

Reductions in σ and ζ also appear to have the hypothesized effect on the quality of the myopic approximation. When each of these parameters is reduced, the myopic rule's implied paths for employment and hours deviate more appreciably from those induced by the optimal policy function, as reported in top panels of Tables 5 and 6. Nonetheless, the additional loss of profit is very modest. Note that the co-movement between hours and employment growth, as summarized by the contemporaneous correlation and the VAR estimates, continues to present the greatest challenge for the myopic rule in terms of its ability to replicate the optimal policy.

Across all of the comparative statics, though, it appears that plant-level discrepancies in the paths of employment and hours growth do not, in general, undermine the ability of the myopic rule

[26] Since the approximate dynamic rule and the myopic rule coincided so closely under the baseline calibration, we simply focus our attention on the myopic rule here. We have verified, moreover, that the two do induce virtually identical moments even under the alternative calibrations considered in this section.

to replicate the salient cross-sectional moments. For instance, a myopic plant adjusts employment just as frequently as a plant that implements the optimal rule.[27] This result is due to the fact, noted in relation to Figures 4-6, that the optimal policy functions – $L(n), \chi(n)$, and $U(n)$ – are essentially parallel to their myopic counterparts. Thus, the myopic plant may adjust at slightly different times, since a given realization of x will not breach its thresholds whenever it breaches the triggers of the forward-looking plant. But the myopic establishment will, over the course of many years, still adjust approximately as often as its foresighted counterpart since the size of the inaction space is roughly the same. The same logic lies behind why the myopic rule induces virtually the same cross-sectional standard deviation of employment growth. The degree of cross-sectional dispersion depends on the size of the mass point at $\Delta n = 0$ and on the typical size of adjustment, conditional on $\Delta n \neq 0$. The form of the policy function, as seen in Figures 4-6, indicates that the typical size of adjustment be similar across the myopic and forward-looking plants since the distances between $\mathcal{L}$ and χ and between $\mathcal{U}$ and χ are roughly the same across the two policy rules, even though the individual components of the policy function differ.

4.4.2 Lognormal Productivity

This subsection investigates the performance of the myopic across various calibrations in the context of a model with Gaussian innovations to $\log x$. The results are reported in the bottom panels of Tables 4-6 so as to facilitate comparison with the case of uniformly distributed shocks.

In some important ways, the behavior of the Gaussian model across different choices of λ parallels that of the model where $\log x$ is struck by uniform innovations. In particular, the fit of employment and hours growth under the myopic rule deteriorates for smaller λ, and the loss of profit increases modestly. In addition, the failures of the myopic rule, in both models, are concentrated around the moments that involve the co-movement of hours and employment growth. For instance, the myopic rule dramatically overstates the contemporaneous correlation between hours and employment in each case. However, it should be noted that it does perform comparatively well in the Gaussian model with regard to the dynamic response of $\Delta \log h$ to $\Delta \log n_{-1}$.

The story is much the same when we turn to the comparative statics on σ and ζ. Reductions in these parameters degrade the quality of the myopic approximation, although the profit losses remain negligible. Again, the static and dynamic responses of hours growth to employment growth prove to be the most elusive moments to match. The robustness of the conclusions across the two models should not necessarily surprise us, since the economic mechanisms behind these comparative

[27]The exception to this is the case of $\sigma = 0.6$. Further investigation of this calibration will have to be undertaken.

statics – outlined in the introduction to this section – did not depend in any material way on the stochastic process of productivity. We view the simulation results as largely confirmatory of the intuition provided above.

To conclude the sensitivity checks, we now vary the persistence of productivity. In general, the results reported in Table 7 suggest that variations in the persistence does not present any systematic challenge to the quality of the myopic rule. The correlation of employment growth across the myopic and forward-looking rules, for instance, hovers around 0.99 across $\rho = 0.39$ to $\rho = 0.79$, and the loss in profit stays in the neighborhood of one third of one percent. Interestingly, the myopic rule does appear to induce patterns of co-movement between hours and employment growth that more nearly match those under the true, forward-looking policy when ρ is relatively high.

4.5 Conclusion

This paper has investigated the analytics of dynamic labor demand in the presence of a fixed cost of employment adjustment. Researchers generally resort to numerical analysis to solve this problem. We find that, given a particularly tractable driving force for idiosyncratic risk, it is possible to obtain quasi-analytical solutions. The results suggested that the substantially simpler policy rule of the corresponding static problem may serve as a remarkably good approximation, or rule-of-thumb, to the true, forward-looking policy function. We then investigated this finding more exhaustively using simulation analysis. The simulation exercises indicated that this key insight appears to largely survive variations in the structural parameters and in the form of the stochastic process governing the evolution of idiosyncratic productivity.

These results throw open the door to two avenues for further research. The first revisits the estimation of the plant's problem presented in Section 2. In this paper, we have taken the parameter estimates as given from the literature. However, as we have suggested elsewhere (see footnote 20), the simulated method of moments results may be sensitive to the targeting of alternative, but equally salient, moments of the data. It would be valuable to re-estimate both the static and dynamic models and document the implications of myopia for structural inference under various moment-matching criteria. Does estimation of the static model in fact recover approximately the same parameter estimates? This would be another way to investigate the quantitative implications of myopic rule-of-thumb behavior.

Another avenue to pursue is to enrich the model of Section 2 and then conduct additional simulation exercises to explore the quality of the myopic rule as an approximation to optimal behavior. The model considered in this paper admittedly omits additional adjustment frictions on employment and abstracts entirely from capital accumulation and price setting. It is important to determine whether the results presented here are robust to these extensions of the baseline model. If so, that holds open the possibility of developing accurate rules of thumb to guide our evaluation of a rich class of structural models.

Table 4.1: Baseline Calibration

Parameter	*Description*	*Value*	*Source*
λ	Adjustment friction	0.92	CHW
ζ	Elasticity of compensation w/ respect to hours	1.59	CHW and Bloom
α	Returns to scale in total labor input	0.64	CHW
ρ	Persistence of productivity	0.39	CHW
σ	Standard deviation of innovation to productivity	0.5	CHW

NOTE: The values drawn from Cooper, Haltiwanger, and Willis (CHW) are from the row labeled "Disrupt" in their Table3a. The value of ζ is the midpoint between CHW and the estimate given in Bloom (see the column labeled "All" in his Table 3).

Table 4.2: The Baseline Calibration with Uniform Productivity Innovations

The approximate dynamic policy rule		*The myopic policy rule*	
Corr($\Delta\log(n^a)$, $\Delta\log(n)$)	0.9875	Corr($\Delta\log(n^m)$, $\Delta\log(n)$)	0.9873
Standard deviation of $\Delta\log(n)$ / Standard deviation of $\Delta\log(n^a)$	1.0022	Standard deviation of $\Delta\log(n)$ / Standard deviation of $\Delta\log(n^m)$	1.0023
Prob($\Delta\log(n)=0$) / Prob($\Delta\log(n^a)=0$)	1.0023	Prob($\Delta\log(n)=0$) / Prob($\Delta\log(n^m)=0$)	1.0021
Corr($\Delta\log(h)$, $\Delta\log(h^a)$)	0.9262	Corr($\Delta\log(h)$, $\Delta\log(h^m)$)	0.9251
Standard deviation of $\Delta\log(h)$ / Standard deviation of $\Delta\log(h^a)$	0.9980	Standard deviation of $\Delta\log(h)$ / Standard deviation of $\Delta\log(h^m)$	0.9978
Corr($\Delta\log(n)$, $\Delta\log(h)$) / Corr($\Delta\log(n^a)$, $\Delta\log(h^a)$)	1.0684	Corr($\Delta\log(n)$, $\Delta\log(h)$) / Corr($\Delta\log(n^m)$, $\Delta\log(h^m)$)	1.0695
$\beta_{\Delta n,\Delta n}$ / $\beta^a_{\Delta n,\Delta n}$	1.0068	$\beta_{\Delta n,\Delta n}$ / $\beta^m_{\Delta n,\Delta n}$	1.0068
$\beta_{\Delta n,\Delta h}$ / $\beta^a_{\Delta n,\Delta h}$	0.9740	$\beta_{\Delta n,\Delta h}$ / $\beta^m_{\Delta n,\Delta h}$	0.9744
$\beta_{\Delta h,\Delta n}$ / $\beta^a_{\Delta h,\Delta n}$	0.3385	$\beta_{\Delta h,\Delta n}$ / $\beta^m_{\Delta h,\Delta n}$	0.3380
$\beta_{\Delta h,\Delta h}$ / $\beta^a_{\Delta h,\Delta h}$	0.9995	$\beta_{\Delta h,\Delta h}$ / $\beta^m_{\Delta h,\Delta h}$	0.9994
log($E[\pi^a]$ / $E[\pi]$)	-0.0018	log($E[\pi^m]$ / $E[\pi]$)	-0.0063

NOTE: This compares the moments induced by alternative policy rules. The superscript "a" refers to the analytical approximation to the dynamic policy rule, whereas the superscript "m" refers to the myopic policy rule. The absence of any superscript means that the moment was generated by the "true" forward-looking policy rule, which was solved numerically by value function iteration.

In many of the rows, what is presented is the ratio of the moments associated with two policy functions. For instance, the second row in the left-hand column presents the ratio of the standard deviation of employment growth induced by the "true" forward-looking policy to that induced by the analytical approximation to the forward-looking rule.

The terms, $\beta_{\Delta n,\Delta n}$ and $\beta_{\Delta n,\Delta h}$, are the coefficients from a regression of $\Delta\log(n)$ on the first lag of employment and hours growth, respectively. Similarly, the terms, $\beta_{\Delta h,\Delta n}$ and $\beta_{\Delta h,\Delta h}$, are the coefficients from a regression of $\Delta\log(h)$ on the first lag of employment and hours growth, respectively.

The term $E[\pi]$ refers to the average flow profit calculated over the simulations.

Table 4.3: The Baseline Calibration with Gaussian Productivity Innovations

Corr($\Delta\log(n^m)$, $\Delta\log(n)$)	0.9908
Standard deviation of $\Delta\log(n)$ / Standard deviation of $\Delta\log(n^m)$	0.9736
Prob($\Delta\log(n)=0$) / Prob($\Delta\log(n^m)=0$)	0.9994
Corr($\Delta\log(h)$, $\Delta\log(h^m)$)	0.9451
Standard deviation of $\Delta\log(h)$ / Standard deviation of $\Delta\log(h^m)$	1.0009
Corr($\Delta\log(n)$, $\Delta\log(h)$) / Corr($\Delta\log(n^m)$, $\Delta\log(h^m)$)	0.3716
$\beta_{\Delta n,\Delta n}$ / $\beta^m_{\Delta n,\Delta n}$	1.0326
$\beta_{\Delta n,\Delta h}$ / $\beta^m_{\Delta n,\Delta h}$	0.9667
$\beta_{\Delta h,\Delta n}$ / $\beta^m_{\Delta h,\Delta n}$	0.7848
$\beta_{\Delta h,\Delta h}$ / $\beta^m_{\Delta h,\Delta h}$	0.9968
log($E[\pi^m]$ / $E[\pi]$)	-0.0028

See Note to Table 4.2.

Table 4.4: Sensitivity Analysis to Variations in the Size of the Adjustment Cost

	λ=0.9	λ=0.92	λ=0.94
	Uniform productivity innovations		
Corr($\Delta\log(n^m)$, $\Delta\log(n)$)	0.9616	0.9873	0.9909
Standard deviation of $\Delta\log(n)$ / Standard deviation of $\Delta\log(n^m)$	0.9890	1.0023	1.0034
Prob($\Delta\log(n)$=0) / Prob($\Delta\log(n^m)$=0)	0.9970	1.0021	1.0053
Corr($\Delta\log(h)$, $\Delta\log(h^m)$)	0.8242	0.9251	0.9204
Standard deviation of $\Delta\log(h)$ / Standard deviation of $\Delta\log(h^m)$	0.9842	0.9978	1.0062
Corr($\Delta\log(n)$, $\Delta\log(h)$) / Corr($\Delta\log(n^m)$, $\Delta\log(h^m)$)	0.6798	1.0695	1.3029
$\beta_{\Delta n,\Delta n}$ / $\beta^m_{\Delta n,\Delta n}$	1.0224	1.0068	1.0183
$\beta_{\Delta n,\Delta h}$ / $\beta^m_{\Delta n,\Delta h}$	0.9453	0.9744	0.8965
$\beta_{\Delta h,\Delta n}$ / $\beta^m_{\Delta h,\Delta n}$	0.4444	0.3380	2.3478
$\beta_{\Delta h,\Delta h}$ / $\beta^m_{\Delta h,\Delta h}$	0.9931	0.9994	1.0056
log($E[\pi^m]$ / $E[\pi]$)	-0.0119	-0.0063	-0.0029
	Gaussian productivity innovations		
Corr($\Delta\log(n^m)$, $\Delta\log(n)$)	0.9674	0.9908	0.9928
Standard deviation of $\Delta\log(n)$ / Standard deviation of $\Delta\log(n^m)$	0.9462	0.9736	0.9751
Prob($\Delta\log(n)$=0) / Prob($\Delta\log(n^m)$=0)	1.0121	0.9994	1.0083
Corr($\Delta\log(h)$, $\Delta\log(h^m)$)	0.8195	0.9451	0.9338
Standard deviation of $\Delta\log(h)$ / Standard deviation of $\Delta\log(h^m)$	1.0157	1.0009	1.0109
Corr($\Delta\log(n)$, $\Delta\log(h)$) / Corr($\Delta\log(n^m)$, $\Delta\log(h^m)$)	0.0881	0.3716	0.1658
$\beta_{\Delta n,\Delta n}$ / $\beta^m_{\Delta n,\Delta n}$	1.0225	1.0326	1.0263
$\beta_{\Delta n,\Delta h}$ / $\beta^m_{\Delta n,\Delta h}$	0.9997	0.9667	0.9566
$\beta_{\Delta h,\Delta n}$ / $\beta^m_{\Delta h,\Delta n}$	1.0168	0.7848	0.7423
$\beta_{\Delta h,\Delta h}$ / $\beta^m_{\Delta h,\Delta h}$	0.9910	0.9968	0.9988
log($E[\pi^m]$ / $E[\pi]$)	-0.0073	-0.0028	-0.0019

NOTE: This compares the moments induced by alternative policy rules for a range of values of the adjustment cost parameter, λ. The notation follows that of Table 4.2. The top panel concerns the model with uniformly distributed productivity; the bottom panel relates results for the model with Gaussian shocks.

Table 4.5: Sensitivity Analysis to Variations in the Degree of Uncertainty

	σ=0.4	σ=0.5	σ=0.6
	Uniform productivity innovations		
Corr($\Delta\log(n^m)$, $\Delta\log(n)$)	0.9294	0.9873	0.9836
Standard deviation of $\Delta\log(n)$ / Standard deviation of $\Delta\log(n^m)$	0.9813	1.0023	1.0137
Prob($\Delta\log(n)=0$) / Prob($\Delta\log(n^m)=0$)	0.9993	1.0021	0.9520
Corr($\Delta\log(h)$, $\Delta\log(h^m)$)	0.7987	0.9251	0.8430
Standard deviation of $\Delta\log(h)$ / Standard deviation of $\Delta\log(h^m)$	1.0116	0.9978	0.9683
Corr($\Delta\log(n)$, $\Delta\log(h)$) / Corr($\Delta\log(n^m)$, $\Delta\log(h^m)$)	0.7137	1.0695	1.7830
$\beta_{\Delta n,\Delta n}$ / $\beta^m_{\Delta n,\Delta n}$	1.0152	1.0068	0.9678
$\beta_{\Delta n,\Delta h}$ / $\beta^m_{\Delta n,\Delta h}$	1.0171	0.9744	1.1191
$\beta_{\Delta h,\Delta n}$ / $\beta^m_{\Delta h,\Delta n}$	0.9099	0.3380	0.0588
$\beta_{\Delta h,\Delta h}$ / $\beta^m_{\Delta h,\Delta h}$	1.0072	0.9994	1.0197
log($E[\pi^m]$ / $E[\pi]$)	-0.0074	-0.0063	-0.0052
	Gaussian productivity innovations		
Corr($\Delta\log(n^m)$, $\Delta\log(n)$)	0.9574	0.9908	0.9956
Standard deviation of $\Delta\log(n)$ / Standard deviation of $\Delta\log(n^m)$	0.9462	0.9736	0.9751
Prob($\Delta\log(n)=0$) / Prob($\Delta\log(n^m)=0$)	0.9960	0.9994	1.0088
Corr($\Delta\log(h)$, $\Delta\log(h^m)$)	0.8509	0.9451	0.9488
Standard deviation of $\Delta\log(h)$ / Standard deviation of $\Delta\log(h^m)$	0.9920	1.0009	1.0187
Corr($\Delta\log(n)$, $\Delta\log(h)$) / Corr($\Delta\log(n^m)$, $\Delta\log(h^m)$)	0.2677	0.3716	0.0869
$\beta_{\Delta n,\Delta n}$ / $\beta^m_{\Delta n,\Delta n}$	1.0516	1.0326	1.0291
$\beta_{\Delta n,\Delta h}$ / $\beta^m_{\Delta n,\Delta h}$	0.9413	0.9667	0.9373
$\beta_{\Delta h,\Delta n}$ / $\beta^m_{\Delta h,\Delta n}$	0.8598	0.7848	0.6512
$\beta_{\Delta h,\Delta h}$ / $\beta^m_{\Delta h,\Delta h}$	0.9767	0.9968	0.9998
log($E[\pi^m]$ / $E[\pi]$)	-0.0049	-0.0028	-0.0031

See Note to Table 4.4.

Table 4.6: Sensitivity Analysis to Variations in the Disutility of Labor

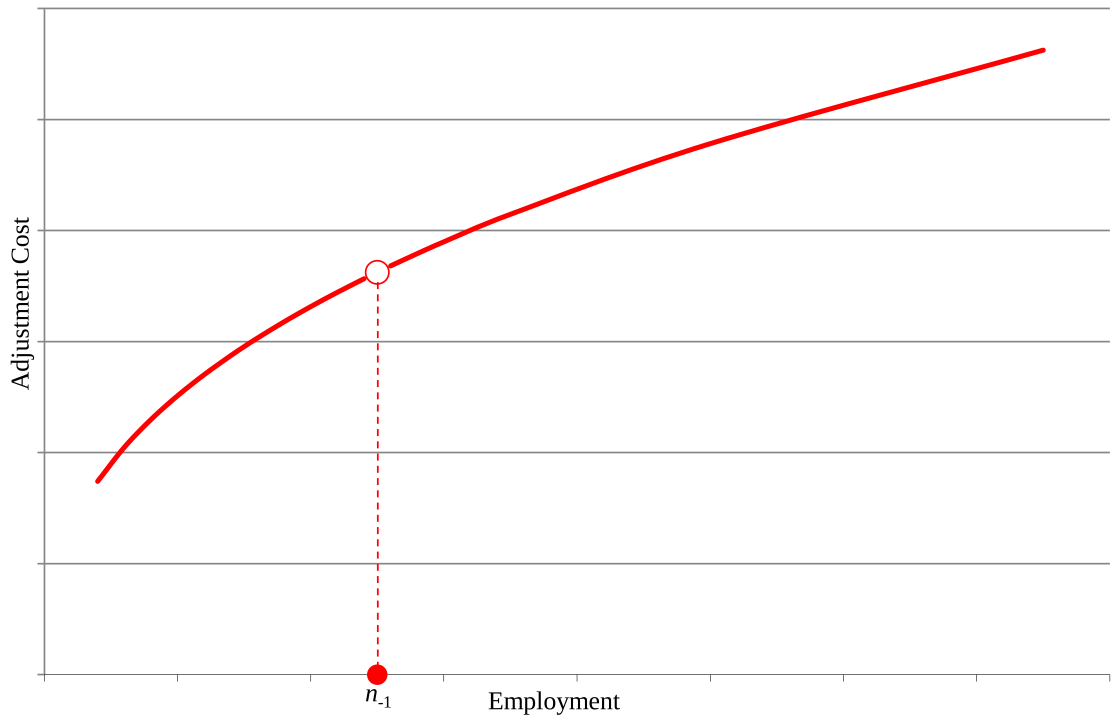
	ζ=1.34	ζ=1.59	ζ=1.84
	Uniform productivity innovations		
Corr($\Delta\log(n^m)$, $\Delta\log(n)$)	0.9674	0.9873	0.9816
Standard deviation of $\Delta\log(n)$ / Standard deviation of $\Delta\log(n^m)$	0.9679	1.0023	1.0019
Prob($\Delta\log(n)$=0) / Prob($\Delta\log(n^m)$=0)	1.0113	1.0021	1.0011
Corr($\Delta\log(h)$, $\Delta\log(h^m)$)	0.8657	0.9251	0.8781
Standard deviation of $\Delta\log(h)$ / Standard deviation of $\Delta\log(h^m)$	1.0127	0.9978	0.9854
Corr($\Delta\log(n)$, $\Delta\log(h)$) / Corr($\Delta\log(n^m)$, $\Delta\log(h^m)$)	0.3600	1.0695	1.1000
$\beta_{\Delta n,\Delta n}$ / $\beta^m_{\Delta n,\Delta n}$	1.0202	1.0068	1.0222
$\beta_{\Delta n,\Delta h}$ / $\beta^m_{\Delta n,\Delta h}$	0.9670	0.9744	1.0225
$\beta_{\Delta h,\Delta n}$ / $\beta^m_{\Delta h,\Delta n}$	0.7941	0.3380	12.5000
$\beta_{\Delta h,\Delta h}$ / $\beta^m_{\Delta h,\Delta h}$	0.9871	0.9994	1.0145
log($E[\pi^m]$ / $E[\pi]$)	-0.0085	-0.0063	-0.0055
	Gaussian productivity innovations		
Corr($\Delta\log(n^m)$, $\Delta\log(n)$)	0.9734	0.9908	0.9910
Standard deviation of $\Delta\log(n)$ / Standard deviation of $\Delta\log(n^m)$	0.9364	0.9736	0.9786
Prob($\Delta\log(n)$=0) / Prob($\Delta\log(n^m)$=0)	1.0070	0.9994	0.9983
Corr($\Delta\log(h)$, $\Delta\log(h^m)$)	0.8747	0.9451	0.9307
Standard deviation of $\Delta\log(h)$ / Standard deviation of $\Delta\log(h^m)$	1.0069	1.0009	0.9850
Corr($\Delta\log(n)$, $\Delta\log(h)$) / Corr($\Delta\log(n^m)$, $\Delta\log(h^m)$)	0.0561	0.3716	0.3615
$\beta_{\Delta n,\Delta n}$ / $\beta^m_{\Delta n,\Delta n}$	1.0536	1.0326	1.0295
$\beta_{\Delta n,\Delta h}$ / $\beta^m_{\Delta n,\Delta h}$	0.9406	0.9667	0.9754
$\beta_{\Delta h,\Delta n}$ / $\beta^m_{\Delta h,\Delta n}$	0.8648	0.7848	0.7447
$\beta_{\Delta h,\Delta h}$ / $\beta^m_{\Delta h,\Delta h}$	0.9734	0.9968	0.9944
log($E[\pi^m]$ / $E[\pi]$)	-0.0054	-0.0028	-0.0025

See Note to Table 4.4.

Table 4.7: Sensitivity Analysis to Variations in the Persistence of Productivity

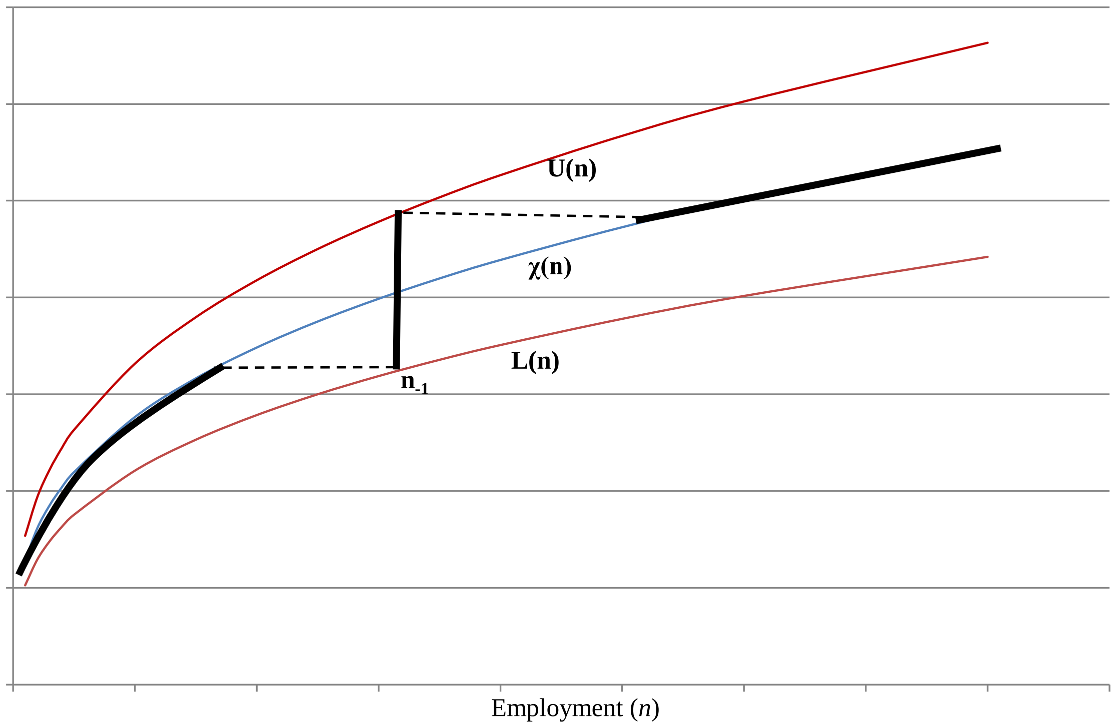
	ρ=0.39	ρ=0.59	ρ=0.79
	Gaussian productivity innovations		
Corr($\Delta\log(n^m)$, $\Delta\log(n)$)	0.9908	0.9846	0.9927
Standard deviation of $\Delta\log(n)$ / Standard deviation of $\Delta\log(n^m)$	0.9736	0.9647	1.0132
Prob($\Delta\log(n)=0$) / Prob($\Delta\log(n^m)=0$)	0.9994	1.0045	0.9987
Corr($\Delta\log(h)$, $\Delta\log(h^m)$)	0.9451	0.9023	0.9471
Standard deviation of $\Delta\log(h)$ / Standard deviation of $\Delta\log(h^m)$	1.0009	1.0111	1.0105
Corr($\Delta\log(n)$, $\Delta\log(h)$) / Corr($\Delta\log(n^m)$, $\Delta\log(h^m)$)	0.3716	0.3888	1.2549
$\beta_{\Delta n,\Delta n}$ / $\beta^m_{\Delta n,\Delta n}$	1.0326	1.0761	0.9967
$\beta_{\Delta n,\Delta h}$ / $\beta^m_{\Delta n,\Delta h}$	0.9667	0.9687	1.0469
$\beta_{\Delta h,\Delta n}$ / $\beta^m_{\Delta h,\Delta n}$	0.7848	0.7171	1.0259
$\beta_{\Delta h,\Delta h}$ / $\beta^m_{\Delta h,\Delta h}$	0.9968	0.9890	1.0149
log($E[\pi^m]$ / $E[\pi]$)	-0.0028	-0.0038	-0.0032

NOTE: This compares the moments induced by alternative policy rules for a range of values for the persistence, ρ, of the productivity shock. It relates results solely for the model with Gaussian innovations since the persistence parameter does not affect the results in the case of uniformly distributed shocks (see the main text for more).



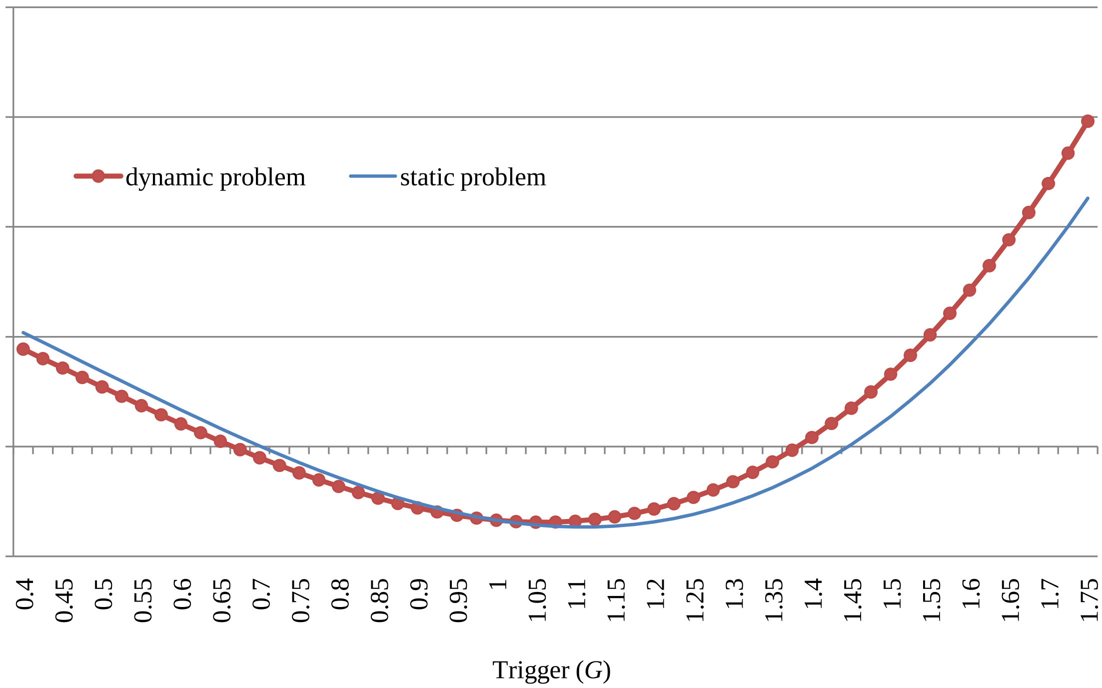
NOTE: This displays the adjustment cost function. It equals zero if employment does not change, i.e., employment this period is set to n_{-1}, which is the level of employment carried over from last period. Otherwise, the adjustment cost is a fraction of the firm's current-period revenue. The latter is, in turn, a concave function of employment.

Figure 4.1: The Adjustment Cost Function



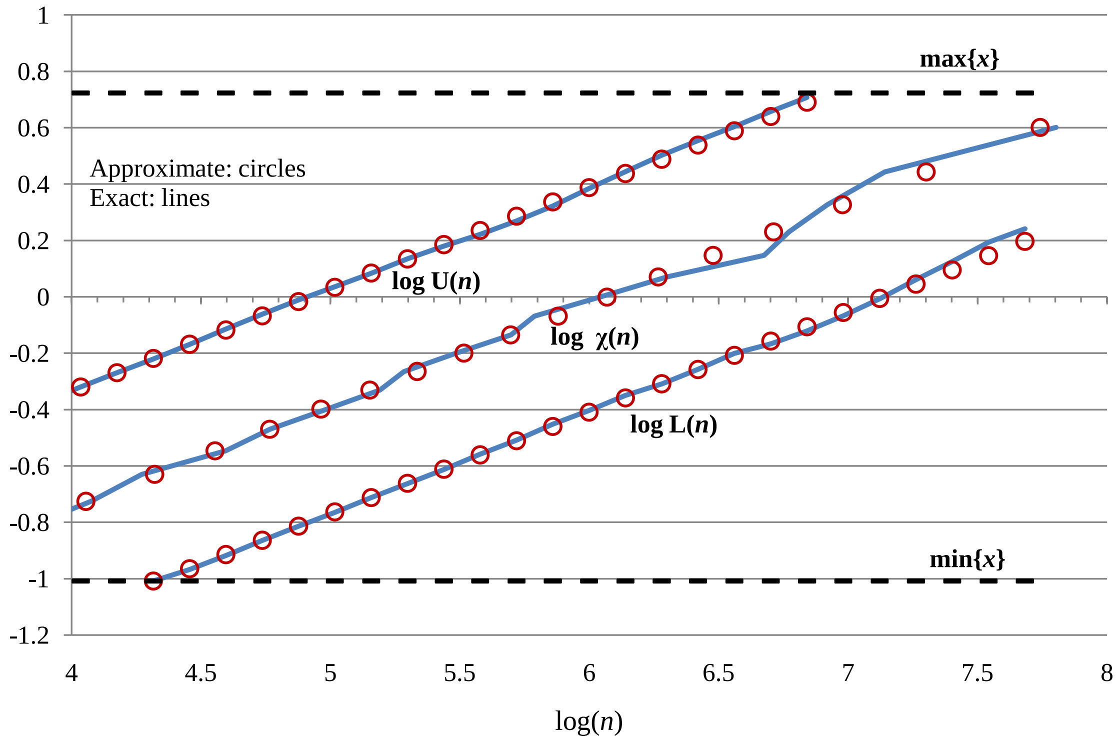
NOTE: This displays the conjectured form of the optimal policy. The thick line traces out the optimal labor demand function: for a range of x within [L(n_{-1}), U(n_{-1})], the plant maintains employment at n_{-1}. Outside of this band, it sets employment according to $x=\chi(n)$.

Figure 4.2: The Optimal Policy Function



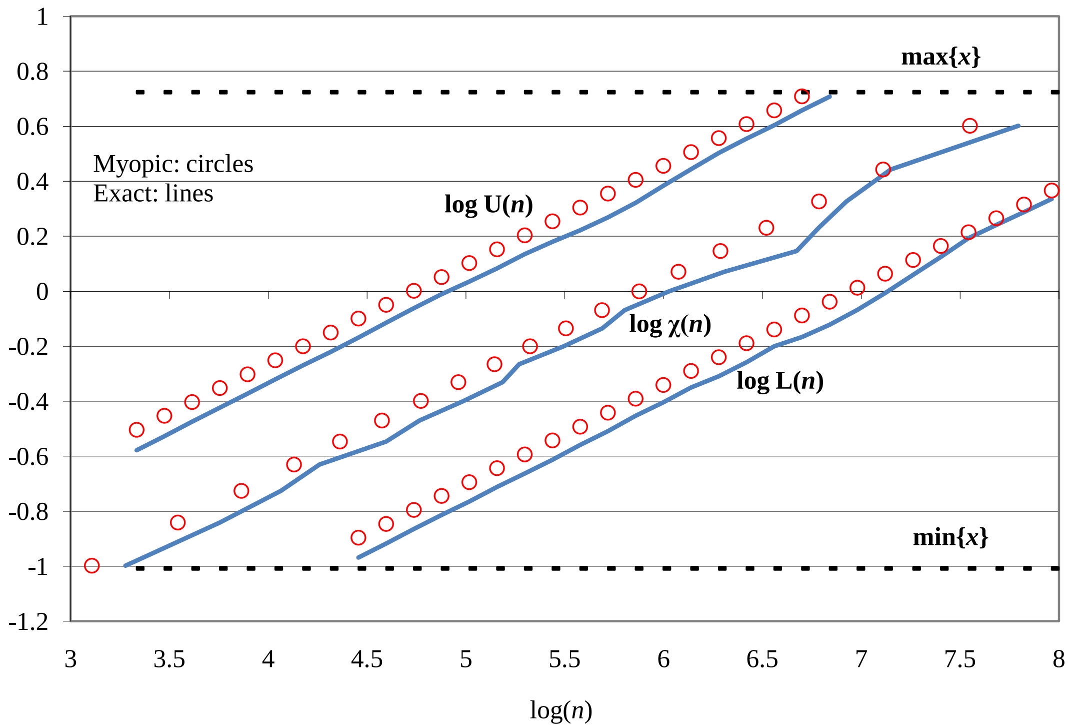
NOTE: This displays the solutions of the value matching relation in the dynamic and static labor demand problems. The ratio of the larger root to the smaller root equals the ratio of the upper trigger (U) to the lower trigger (L) (see Figure 4.2). Thus, the smaller of the two solutions is associated with the policy rule for reducing employment. The larger of the two roots is associated with the policy rule for raising employment.

Figure 4.3: Solving the Value Matching Condition in the Dynamic and Static Problems



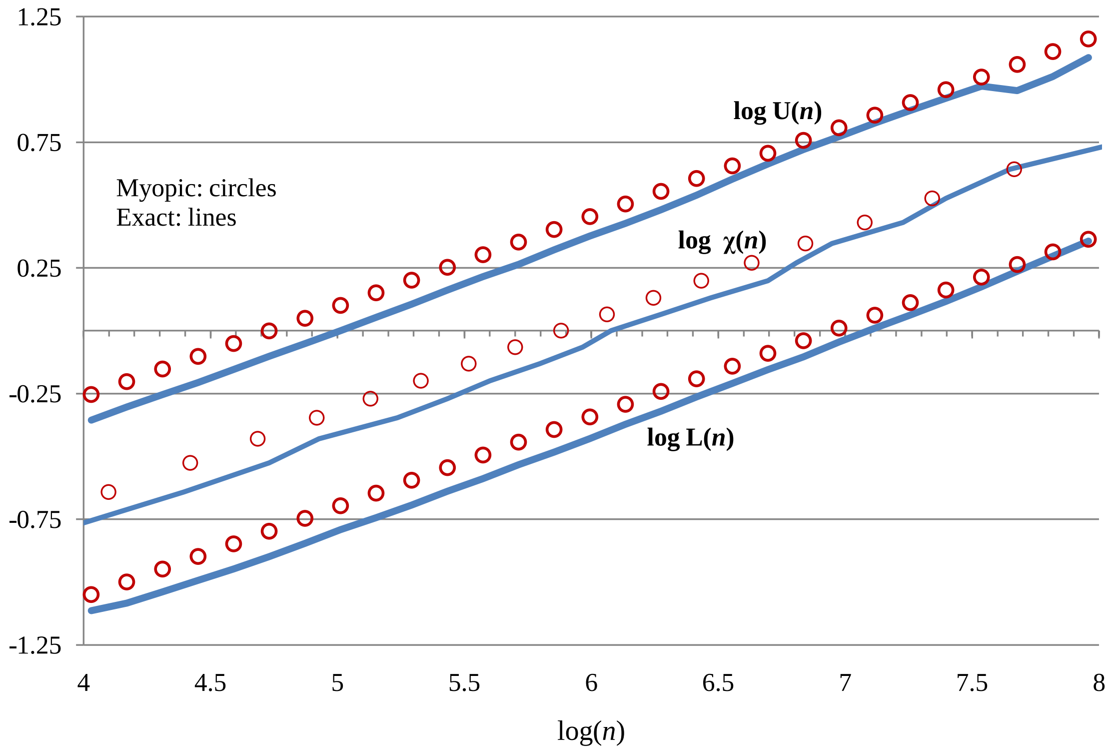
NOTE: This displays the policy function for the model with uniformly distributed productivity innovations. The approximate forward-looking policy function of the dynamic model, derived analytically in Section 4.2, is shown in circles. The exact forward-looking policy function associated with the dynamic model, which was solved numerically in Section 4.3, is shown in lines. The dashed lines at the top and bottom of the figure show the upper and lower bounds on productivity, given the parameters of the uniform shock process.

Figure 4.4: Approximate and Exact Dynamic Policy Rules under Uniform Innovations



NOTE: The policy function that solves the myopic model is shown in circles. The exact forward-looking policy function of the dynamic model with uniformly distributed productivity innovations is shown in lines. The dashed lines at the top and bottom of the figure show the upper and lower bounds on productivity, given the parameters of the uniform shock process.

Figure 4.5: Myopic and Optimal Policy Rules under Uniform Innovations



NOTE: The policy function that solves the myopic model is shown in circles. The exact forward-looking policy function of the dynamic model with Gaussian distributed productivity innovations is shown in lines.

Figure 4.6: Myopic and Optimal Policy Rules under Gaussian Innovations

4.6 Appendix: Proofs

Proof of Proposition 1 Under the Conjecture, and given the restriction $[L(n), U(n)] \subset [x^{\rho}\mathrm{e}^{\mu-s}, x^{\rho}\mathrm{e}^{\mu+s}]$, it is possible to decompose the expected future value of the plant as follows:

$$\int \Pi(x', n)\, dG(x'|x) = \int^{L(n)} \Pi^{\Delta}(x')\, dG(x'|x) + \int_{L(n)}^{U(n)} \Pi^{0}(x', n)\, dG(x'|x) + \int_{U(n)} \Pi^{\Delta}(x')\, dG(x'|x).$$

Differentiating with respect to n, and applying the value matching conditions (4.12), yields

$$\int \Pi_n(x', n)\, dG(x'|x) = \int_{L(n)}^{U(n)} \Pi_n^{0}(x', n)\, dG(x'|x).$$

Forwarding (4.11) one period and differentiating $\Pi^0(x', n)$ with regard to n then gives

$$\begin{aligned} D(x,n) &= \int_{L(n)}^{U(n)} \Pi_n^0(x', n)\, dG(x'|x) \\ &= \int_{L(n)}^{U(n)} \left[\hat{\alpha} A x'^{\frac{\zeta}{\zeta-\alpha}} n^{-\zeta\frac{1-\alpha}{\zeta-\alpha}} - b\right] dG(x'|x) + \beta \int_{L(n)}^{U(n)} D(x', n)\, dG(x'|x), \end{aligned}$$

where $\hat{\alpha} \equiv \alpha\frac{\zeta-1}{\zeta-\alpha}$.

Since innovations to $\log x$ are uniformly distributed, it is straightforward to calculate the first integral. There are two parts to it:

$$\int_{L(n)}^{U(n)} x'^{\frac{\zeta}{\zeta-\alpha}}\, dG(x'|x) = \frac{1}{2s}\frac{\zeta-\alpha}{\zeta}\left[U(n)^{\frac{\zeta}{\zeta-\alpha}} - L(n)^{\frac{\zeta}{\zeta-\alpha}}\right]$$

and

$$\int_{L(n)}^{U(n)} dG(x'|x) = \frac{1}{2s}\log\left(\frac{U(n)}{L(n)}\right).$$

Under the Conjecture, these expressions become, respectively,

$$\begin{aligned} & \frac{1}{2s}\frac{\zeta-\alpha}{\zeta}\left[\left(\frac{U(n)}{\chi(n)}\right)^{\frac{\zeta}{\zeta-\alpha}} - \left(\frac{L(n)}{\chi(n)}\right)^{\frac{\zeta}{\zeta-\alpha}}\right]\chi(n)^{\frac{\zeta}{\zeta-\alpha}} \\ = & \frac{1}{2s}\frac{\zeta-\alpha}{\zeta}\left[G_U^{\frac{\zeta}{\zeta-\alpha}} - G_L^{\frac{\zeta}{\zeta-\alpha}}\right]\chi(n)^{\frac{\zeta}{\zeta-\alpha}} \\ \equiv & \mathcal{P}_{\zeta}\chi(n)^{\frac{\zeta}{\zeta-\alpha}} \end{aligned}$$

and

$$\frac{1}{2s}\log\left(\frac{U(n)}{L(n)}\right)=\frac{1}{2s}\log\left(\frac{G_U}{G_L}\right)\equiv\mathcal{P}_0,$$

where $G_U\equiv\mathcal{U}/\chi$ and $G_L\equiv\mathcal{L}/\chi$. Substituting these results into the recursion for D, and using the Conjecture $\chi(n)=\chi n^{1-\alpha}$, yields

$$D(x,n)=\hat{\alpha}A\mathcal{P}_\zeta\chi^{\frac{\zeta}{\zeta-\alpha}}-b\mathcal{P}_0+\beta\int_{L(n)}^{U(n)}D(x',n)\,dG(x'|x)\,. \tag{4.25}$$

Notice that, under the Conjecture, $\mathcal{P}_\zeta$ and $\mathcal{P}_0$ are constants. This suggests that a natural guess for the forward term is $D(x,n)=\mathrm{d}=$*constant*. We substitute this into (4.25) and rearrange to obtain

$$D(x,n)=\frac{\hat{\alpha}A\mathcal{P}_\zeta\chi^{\frac{\zeta}{\zeta-\alpha}}-b\mathcal{P}_0}{1-\beta\mathcal{P}_0},$$

which is in fact independent of x and n.

Proof of Proposition 2 Using the conjecture (4.20), we solve the forward integrals in the recursion (4.19), obtaining

$$\begin{aligned}\int_{L(n_{-1})}^{U(n_{-1})}\Delta(x',n_{-1})\,dG(x'|x) &= \int_{L(n_{-1})}^{U(n_{-1})}\left[\delta_0 n_{-1}+\delta_1 n_{-1}^{\alpha\frac{\zeta-1}{\zeta-\alpha}}x'^{\frac{\zeta}{\zeta-\alpha}}+\delta_2 x'^{\frac{1}{1-\alpha}}\right]dG(x'|x)\\ &= \left[\delta_0\mathcal{P}_0+\delta_1\mathcal{P}_\zeta\chi^{\frac{\zeta}{\zeta-\alpha}}+\delta_2\mathcal{P}_1\chi^{\frac{1}{1-\alpha}}\right]n_{-1}\end{aligned}$$

and

$$\begin{aligned}\int_{\left(\frac{\mathcal{L}}{\chi}\right)x}^{\left(\frac{\mathcal{U}}{\chi}\right)x}\Delta(x',n)\,dG(x'|x) &= \int_{\left(\frac{\mathcal{L}}{\chi}\right)x}^{\left(\frac{\mathcal{U}}{\chi}\right)x}\left[\delta_0 n(x)+\delta_1 n(x)^{\alpha\frac{\zeta-1}{\zeta-\alpha}}x'^{\frac{\zeta}{\zeta-\alpha}}+\delta_2 x'^{\frac{1}{1-\alpha}}\right]dG(x'|x)\\ &= \left[\delta_0\mathcal{P}_0\chi^{-\frac{1}{1-\alpha}}+\delta_1\mathcal{P}_\zeta\chi^{-\frac{\alpha}{1-\alpha}\frac{\zeta-1}{\zeta-\alpha}}+\delta_2\mathcal{P}_1\right]x^{\frac{1}{1-\alpha}}.\end{aligned}$$

Substituting these results into (4.19), we now have

$$\Delta(x,n_{-1})=\delta_0 n_{-1}+\delta_1 n_{-1}^{\alpha\frac{\zeta-1}{\zeta-\alpha}}x^{\frac{\zeta}{\zeta-\alpha}}+\delta_2 x^{\frac{1}{1-\alpha}}$$

$$= \left\{ b + \beta \left(\delta_0 \mathcal{P}_0 + \delta_1 \mathcal{P}_\zeta \chi^{\frac{\zeta}{\zeta-\alpha}} + \delta_2 \mathcal{P}_1 \chi^{\frac{1}{1-\alpha}} \right) \right\} n_{-1} - A n_{-1}^{\alpha \frac{\zeta-1}{\zeta-\alpha}} x^{\frac{\zeta}{\zeta-\alpha}}$$

$$+ \left\{ \begin{array}{c} \left(A \lambda^{\frac{\zeta}{\zeta-\alpha}} \chi^{\frac{\zeta}{\zeta-\alpha}} - b \right) \chi^{-\frac{1}{1-\alpha}} \\ -\beta \left(\delta_0 \mathcal{P}_0 \chi^{-\frac{1}{1-\alpha}} + \delta_1 \mathcal{P}_\zeta \chi^{-\frac{\alpha}{1-\alpha} \frac{\zeta-1}{\zeta-\alpha}} + \delta_2 \mathcal{P}_1 \right) \end{array} \right\} x^{\frac{1}{1-\alpha}}.$$

Equating coefficients yields

$$\delta_0 = b + \beta \left(\delta_0 \mathcal{P}_0 + \delta_1 \mathcal{P}_\zeta \chi^{\frac{\zeta}{\zeta-\alpha}} + \delta_2 \mathcal{P}_1 \chi^{\frac{1}{1-\alpha}} \right)$$

$$\delta_1 = -A$$

$$\delta_2 = \left(A \lambda^{\frac{\zeta}{\zeta-\alpha}} \chi^{\frac{\zeta}{\zeta-\alpha}} - b \right) \chi^{-\frac{1}{1-\alpha}} - \beta \left(\delta_0 \mathcal{P}_0 \chi^{-\frac{1}{1-\alpha}} + \delta_1 \mathcal{P}_\zeta \chi^{-\frac{\alpha}{1-\alpha} \frac{\zeta-1}{\zeta-\alpha}} + \delta_2 \mathcal{P}_1 \right)$$

To simplify these expressions, define

$$\Omega \equiv \delta_0 \mathcal{P}_0 + \delta_1 \mathcal{P}_\zeta \chi^{\frac{\zeta}{\zeta-\alpha}} + \delta_2 \mathcal{P}_1 \chi^{\frac{1}{1-\alpha}}.$$

Then inspection reveals that

$$\delta_0 = b + \beta \Omega$$

$$\delta_2 = \left[\left(A \lambda^{\frac{\zeta}{\zeta-\alpha}} \chi^{\frac{\zeta}{\zeta-\alpha}} - b \right) - \beta \Omega \right] \chi^{-\frac{1}{1-\alpha}},$$

from which we obtain

$$\delta_2 = \left[A \lambda^{\frac{\zeta}{\zeta-\alpha}} \chi^{\frac{\zeta}{\zeta-\alpha}} - \delta_0 \right] \chi^{-\frac{1}{1-\alpha}},$$

Substituting this into the expression for δ_0 yields

$$\delta_0 = A \frac{\hat{b} + \beta \left[\lambda^{\frac{\zeta}{\zeta-\alpha}} \mathcal{P}_1 - \mathcal{P}_\zeta \right] \chi^{\frac{\zeta}{\zeta-\alpha}}}{1 + \beta \left[\mathcal{P}_1 - \mathcal{P}_0 \right]}.$$

Since δ_0, δ_1, and δ_2 are constant coefficients, this verifies the conjecture.

Proof of Proposition 3 To begin, we recapitulate equation (4.22),

$$0 = z\left(G\right) \equiv \left(\frac{1-\hat{\alpha}}{\hat{\alpha}} +\right) G^{\frac{1}{1-\alpha}} - \frac{1}{\hat{\alpha}} \lambda^{-\frac{\zeta}{\zeta-\alpha}} G^{\frac{\zeta}{\zeta-\alpha}} + (1-).$$

Take the first order condition of $z\left(G\right)$, and let G^{*} denote the value of G that satisfies $z'\left(G\right) = 0$. The strategy of the proof is to show that, at G^{*}, z is negative ($z\left(G^{*}\right) < 0$) and then to verify that z increases as one moves away from G^{*} in either direction, i.e., that

$$\begin{aligned} z'\left(\mathcal{G}\right) &< 0 \text{ for all } \mathcal{G} < G^{*} \\ z'\left(\mathcal{G}\right) &> 0 \text{ for all } \mathcal{G} > G^{*}. \end{aligned} \tag{4.26}$$

Together, these statements imply that G^{*} is the global minimum and that $z\left(G\right)$ crosses zero twice.

To proceed, we solve the first order condition to find that G^{*} is given by

$$G^{*} = \left(\frac{\lambda^{-\frac{\zeta}{\zeta-\alpha}}}{1-\hat{\alpha}+\hat{\alpha}} \times \zeta \frac{1-\alpha}{\zeta-\alpha}\right)^{\frac{1-\alpha}{\alpha}\frac{\zeta-\alpha}{\zeta-1}}.$$

Substituting this into $z\left(G\right)$, and introducing the auxiliary terms,

$$\begin{aligned} \hat{\alpha} &\equiv \alpha\frac{\zeta-1}{\zeta-\alpha} \\ \hat{\zeta} &\equiv \zeta - 1, \end{aligned}$$

we obtain the following expression for $z\left(G^{*}\right)$:

$$z\left(G^{*}\right) = -\left(\frac{1-\hat{\alpha}}{\hat{\alpha}} +\right)^{-\frac{1-\hat{\alpha}}{\hat{\alpha}}} \left(\frac{1-\hat{\alpha}}{\hat{\alpha}}\right)^{\frac{1-\hat{\alpha}}{\hat{\alpha}}} \lambda^{-\frac{\hat{\alpha}+\hat{\zeta}}{\hat{\zeta}}\frac{1}{\hat{\alpha}}} + 1 - .$$

To show that this is negative, it is sufficient to show that

$$-\left(\frac{1-\hat{\alpha}}{\hat{\alpha}}\right)^{-\frac{1-\hat{\alpha}}{\hat{\alpha}}} \left(\frac{1-\hat{\alpha}}{\hat{\alpha}}\right)^{\frac{1-\hat{\alpha}}{\hat{\alpha}}} \lambda^{-\frac{\hat{\alpha}+\hat{\zeta}}{\hat{\zeta}}\frac{1}{\hat{\alpha}}} + 1 < 0$$

since > 0 and $\hat{\alpha} < 1$. This inequality, in turn, requires

$$1 < \lambda^{-\frac{\hat{\alpha}+\hat{\zeta}}{\hat{\zeta}}\frac{1}{\hat{\alpha}}},$$

which holds since $\lambda < 1$ and $\frac{\hat{\alpha}+\hat{\zeta}}{\hat{\zeta}}\frac{1}{\hat{\alpha}} > 1$.

Next, we need to show that the derivative of z,

$$\left(\frac{1-\hat{\alpha}}{\hat{\alpha}}+\right)\frac{1}{1-\alpha}G^{\frac{\alpha}{1-\alpha}}-\frac{1}{\hat{\alpha}}\lambda^{-\frac{\zeta}{\zeta-\alpha}}\frac{\zeta}{\zeta-\alpha}G^{\frac{\alpha}{\zeta-\alpha}}$$

satisfies (4.26). Letting $G=\xi G^*$, for some positive scalar ξ, this becomes

$$\left\{\xi^{\frac{\alpha}{1-\alpha}\frac{\zeta-1}{\zeta-\alpha}}-1\right\}\gamma\xi^{\frac{\alpha}{\zeta-\alpha}},$$

where $\gamma\equiv\left(\frac{1-\hat{\alpha}}{\hat{\alpha}}+\right)\frac{1}{1-\alpha}G^{*\frac{\alpha}{1-\alpha}}=\frac{1}{\hat{\alpha}}\lambda^{-\frac{\zeta}{\zeta-\alpha}}\frac{\zeta}{\zeta-\alpha}G^{*\frac{\alpha}{\zeta-\alpha}}$ holds by virtue of the first order condition. The term in brackets is negative for $\xi<1$ and positive for $\xi>1$.

To conclude the proof, we show that one root is less than one and one root is greater than one. To do so, it is sufficient to verify that $z(1)<0$. Since z is increasing as one moves away from G^* in each direction, it must be that, if $z(1)<0$, the smaller of the two roots lies to the left of $G=1$ and the larger lies to the right. Plugging in $G=1$, we see that z is negative iff

$$1<\lambda^{-\frac{\zeta}{\zeta-\alpha}},$$

which holds since $\lambda<1$ and $\frac{\zeta}{\zeta-\alpha}>1$.

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CHAPTER V

Conclusion

This dissertation has explored the empirical and theoretical implications of a rich array of employment adjustment frictions. The adjustment costs varied from the readily observable JOBS Bank program, which operated within the U.S. motor vehicle industry, to the harder-to-quantify, though no less important, costs associated with recruiting workers and integrating them into the ebb and flow of operations at a plant. Just as the nature of these costs varied, so did their effects on labor demand.

Chapter 1 considered the consequences of the JOBS Bank. It showed that JOBS exerted an influence on labor demand by generating an option value of production: a plant wanted to safeguard the layoff weeks it was allotted for a contract in case vehicle demand declined before the labor agreement expired. The structure of the provision provided several testable predictions. Specifically, weeks of layoffs at a plant should decline as its accumulated stock of past layoffs approached the allotment. In addition, for any given level of past layoffs, the plant should defer downtime until later in the contract when the uncertainty over vehicle demand is largely resolved. This "wait-and-see" approach would allow it to conserve its allotment in case layoffs are needed when vehicle demand declines. The chapter tested these predictions on a comprehensive panel dataset that spans all assembly plants of the Detroit Three manufacturers.

The analysis found little, if any, effect of the JOBS Bank on weekly layoff behavior. Simulations of the structural model indicated what may have been behind this result. They showed that the strength of the option-value effects relied in large part on the persistence of vehicle demand shocks. The reason was straightforward: if a plant expects demand to remain subdued for some time after an adverse shock, it has an incentive to defer layoffs now and instead "wait and see" if it will need its allotment later in the contract should another negative disturbance arrive while demand is already low. The estimation results indicated that fluctuations in vehicle demand were in fact relatively short-lived, which thus weakened the JOBS Bank effect.

The second and third chapters contributed theoretical analyses of adjustment frictions that are commonly found in the literature but whose implications are often difficult to fully develop analytically. Chapter 2 presented a model that rests on three pillars: (i) firms face a cost of recruiting new hires; (ii) there are diminishing returns to labor (at least in the short run); and (iii) firms face both idiosyncratic and aggregate risk. When the model was evaluated quantitatively, it was shown to provide a coherent account of several salient features of the U.S. labor market. In particular, it replicated the distributions of employer size and employment growth across establishments; the amplitude and propagation of cyclical fluctuations in flows between employment and unemployment; the negative comovement of unemployment and vacancies; and the dynamics of the distribution of employer size.

The third chapter considered the effect of a fixed cost of adjustment on a firm's optimal labor demand rule. It uncovered an unexpected result: although a fixed cost is often thought to substantially complicate the firm's problem, the optimal forward-looking policy rule is, under reasonable parameterizations, quantitatively indistinguishable from the labor demand rule associated with the corresponding static, or myopic, model. This is surprising because the fixed cost generates an optimal degree of inaction, meaning that the firm will occasionally leave the size of its workforce unchanged in response to fluctuations in business conditions. As a result, the firm is in a long-term relationship with its workers. And yet, the employment adjustment friction does not have the effect that is often ascribed to it, that is, it does not seem to appreciably raise the importance of being foresighted. That the losses associated with myopia are small indicates that one may be able to summarize the key qualitative and quantitative features of a rich class of dynamic models with relatively simple myopic "rules of thumb". This would represent quite an advance in our understanding of the implications of employment adjustment costs.