A RE-EVALUATION OF WEBER'S LAW AS APPLIED TO PURE TONES

TECHNICAL REPORT NO. 47

Electronic Defense Group
Department of Electrical Engineering

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Project 2262
TASK ORDER NO. EDG-3
CONTRACT NO. DA-36-039 sc-63203
SIGNAL CORPS, DEPARTMENT OF THE ARMY
DEPARTMENT OF ARMY PROJECT NO. 3-99-04-042
SIGNAL CORPS PROJECT NO. 194B

August, 1958
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF ILLUSTRATIONS</td>
<td>iv</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>v</td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. EXPERIMENTAL PROCEDURE</td>
<td>7</td>
</tr>
<tr>
<td>III. ANALYSIS OF DATA</td>
<td>9</td>
</tr>
<tr>
<td>IV. DISCUSSION OF RESULTS</td>
<td>10</td>
</tr>
<tr>
<td>V. CONCLUSION</td>
<td>23</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>24</td>
</tr>
<tr>
<td>DISTRIBUTION LIST</td>
<td>25</td>
</tr>
</tbody>
</table>
LIST OF ILLUSTRATIONS

Figure 1  Illustration of Amplitude Difference as Basis for Discrimination in the Case of Pure Tones. (A) Simple Detection Situation, (B) An Amplitude Discrimination  

Figure 2  $d'$ Versus $\Delta V$ for Observer 1, $N_g = 0$  

Figure 3  $P(c)$ Versus $V_o \times 10^5$ for Observer 1  

Figure 4  $P(c)$ Versus $V_o \times 10^4$ for Observer 1  

Figure 5  $P(c)$ Versus $V_o \times 10^3$ for Observer 1  

Figure 6  $P(c)$ Versus $V_o \times 10^2$ for Observer 1  

Figure 7  $d'$ Versus $\Delta W$ for Observer 2, $N_g = 0$  

Figure 8  $P(c)$ Versus $V_o \times 10^5$ for Observer 2  

Figure 9  $P(c)$ Versus $V_o \times 10^4$ for Observer 2  

Figure 10  $P(c)$ Versus $V_o \times 10^3$ for Observer 2  

Figure 11  $P(c)$ Versus $V_o \times 10^2$ for Observer 2  

Figure 12  $d'$ Versus $\Delta W$ for Observer 3, $N_g = 0$  

Figure 13  $P(c)$ Versus $V_o \times 10^5$ for Observer 3  

Figure 14  $P(c)$ Versus $V_o \times 10^4$ for Observer 3  

Figure 15  $P(c)$ Versus $V_o \times 10^3$ for Observer 3  

Figure 16  $P(c)$ Versus $V_o \times 10^2$ for Observer 3  

Figure 17  Data for Observer 1 for a Signal Which is a Sample of White Gaussian Noise  

Figure 18  Data for Observer 2 for a Signal Which is a Sample of White Gaussian Noise  

Figure 19  Data for Observer 3 for a Signal Which is a Sample of White Gaussian Noise
ABSTRACT

The fact that Weber's law appears to apply in the same way both to intensity discrimination for pure tones and to intensity discrimination for white noise poses a theoretical paradox: in the case of pure tones, the human observer becomes less efficient as the intensity of the tone is increased, while in the case of white noise he exhibits a constant efficiency independent of intensity.

An inventory of the various possible noise sources which may exist is made, and the way in which these may be expected to affect the detectability of a signal leads to the equation

$$(d')^2 = \eta \frac{E}{N_G + N_E + kV^2}$$

where $\eta$ is the individual observers efficiency, $N_G$ is the noise introduced by the experimenter, $N_E$ is the uncontrolled noise present in the experimental situation, and $k$ is a constant indicating that small amplitude variation in the oscillator constitute a noise source proportional to the power of the lower of two signals to be discriminated.

Data for three observers over four noise levels is described by this equation sufficiently well to suggest that the hypothesis that Weber's law is merely a reflection of the oscillator noise ($kV^2$) is plausible.
A RE-EVALUATION OF WEBER'S LAW AS APPLIED TO PURE TONES

I. INTRODUCTION

This study is conducted within the framework of the theory of signal detectability. It is recognized that a signal in noise has a capacity to lead to detection. This capacity is utilized as a standard against which to compare the performance of an observer in a psycho-acoustical experiment.

The subject matter of this study is the application of Weber's Law to two types of signals: pure tones, and samples of white Gaussian noise. That Weber's law should apply to intensity discrimination behavior for both types of signal is somewhat of a paradox within the framework of the theory of signal detectability, because an observer whose data follows Weber's law is rated as becoming less efficient as the intensity of a pure tone is increased, but is rated as having a constant efficiency at all intensities for a white noise signal.

For the case in which two bursts of pure tones are presented to the observer, the tones being identical in every respect except amplitude, the capacity of these tones to lead to discrimination is expressed in terms of the energy of the difference signal, as illustrated in Figure 1. The uppermost of the three graphs is a voltage waveform as a function of time $V_o(t)$. The middle graph illustrates a voltage waveform $V_1(t)$. These are the two signals upon which the discrimination must be based. The bottom graph is the waveform of the difference signal, $\Delta V(t)$, obtained by subtracting point for point in time the voltage $V_o$ from the voltage $V_1$. 
FIG. I. ILLUSTRATION OF AMPLITUDE DIFFERENCE AS BASIS FOR DISCRIMINATION IN THE CASE OF PURE TONES.

(A) SIMPLE DETECTION SITUATION.

(B) AN AMPLITUDE DISCRIMINATION.
The energy of the difference signal $\Delta V(t)$ is given by:

$$E_\Delta = \int_{t=0}^{T} \Delta V^2(t) \, dt,$$

where the observation interval begins at $t = 0$ and ends at $t = T$. The capacity of these two signals to lead to discrimination can be expressed in terms of $(d')^2$, described in a previous paper (Ref. 1).

$$(d'_{\text{opt}})^2 = \frac{2E_\Delta}{N_0}$$

where $N_0$ is the noise power per unit bandwidth.

Since the two waveforms are assumed perfectly correlated, the following voltage relation holds:

$$\Delta V = V_1 - V_0'$$

where all of the voltages are expressed as r.m.s. voltages.

Ignoring corrections introduced to handle irregularities for those signals near the absolute limits of hearing, Weber's law states that, to lead to a constant level of performance, the following relation holds,

$$\frac{\Delta T}{T_0} = \text{constant},$$
where $I_0$ is the intensity of the lower of the two signals, and $\Delta I$ is the difference in the intensity of the two signals. In terms of voltages, used above

\[
I_0 \propto V_0^2 \\
I_1 \propto V_1^2 \\
I_1 - I_0 = \Delta I \propto V_1^2 - V_0^2
\]

since

\[
V_1 = V_0 + \Delta V
\]

then

\[
\Delta I \propto V_0^2 + 2V_0 \Delta V + \Delta V^2 - V_0^2
\]

and

\[
\frac{\Delta I}{I} \propto 2 \frac{\Delta V}{V_0} + \frac{\Delta V^2}{V_0^2}
\]

Thus, if equation (4) holds then for a constant level of performance, it is also true that

\[
\frac{\Delta V^2}{V_0^2} = \text{constant} = C. \tag{5}
\]

In the previous paper, efficiency is defined by the following relation

\[
(d')^2 = \eta \frac{2E_A}{N_0} \tag{6}
\]

Further, let the constant level of performance usually defined as "threshold" in the two alternative forced-choice advocated by Harris (Ref. 2) and Blackwell (Ref. 3) be the case where $d' = 1$. Incorporating the relation $\Delta V^2 = C V_0^2$ from equation (5) into equation (6), the statement of Weber's law is given by:
\[ l = \eta \frac{c v_{0}^2 t}{N_{o}} \quad (7) \]

and \( \eta \) is found to vary as \( \left( \frac{1}{V_{0}} \right)^2 \). \( \eta \) thus decreases as the reciprocal of \( I_{o} \).

For the case of a signal which is a sample of Gaussian noise, the capacity to lead to discrimination is expressed as:

\[ d'_{\text{opt}} = \sqrt{\text{wt}} \frac{S_{o}}{N_{o}} \sqrt{\frac{1}{\frac{1}{2} \left( \frac{S_{o}}{N_{o}} \right)^2 + \frac{S_{o}}{N_{o}} + 1}} \quad (8) \]

where \( N_{o} \) is power measurement linearly related to \( I_{o} \), and \( N_{o} + S_{o} \) bears the same linear relation to \( I_{1} \). For Weber's law to apply \([\text{equation } (4)]\), then

\[ \frac{S_{o}}{N_{o}} = \text{constant.} \quad (9) \]

It is the relative efficiency which is of interest here. By the same argument above, it can be observed that, if, in the relation

\[ d' = \eta \sqrt{\text{wt}} \frac{S_{o}}{N_{o}} \sqrt{\frac{1}{\frac{1}{2} \left( \frac{S_{o}}{N_{o}} \right)^2 + \frac{S_{o}}{N_{o}} + 1}} \quad (10) \]

\( \frac{S_{o}}{N_{o}} \) is constant, then \( \eta \) is also constant. For the case of signals which are samples of white Gaussian noise, the Weber's law observer exhibits an efficiency which is independent of \( I_{o} \) or \( N_{o} \) in the equation.
A possible solution to the paradox can be reached by considering a number of sources of noise which combine to make up the effective value of $N_o$ employed in the experiment. One such source is that noise generated by a noise generator and introduced by the experimenter, designated $N_G$. A second source is the residual noise of the experiment designated $N_E$. A third source can arise from amplitude and frequency variations in the output of the audio oscillator employed. While noise of this type may not be large, it is narrow band around the frequency of the signal, and is all relevant to the experiment. Since in the experiments reported below the oscillator always has the same output and the voltages are obtained by attenuation, this noise is designated by $kV_o^2$. Since the three sources of noise are assumed to be independent, and thus additive, equation (6) becomes

$$(\bar{a}')^2 = \eta \frac{2E \Delta}{N_G + N_E + k V_o^2},$$

(11)

Since previous experiments supporting the Weber’s law application to pure tones were carried out in silence ($N_G = 0$), one would expect Weber’s law to apply as soon as $k V_o^2 \gg N_E$. If the experimenter introduces noise, then Weber’s law should apply only after $k V_o^2 \gg N_G + N_E$.

It is obvious that there must be noise of the type introduced by the audio oscillator. The only question is one of magnitude. Is it sufficiently great to account for the experimental results which appear to support Weber’s law?

The problem of determining the magnitude of this noise, particularly in term of an $N_o$ type of measure, is severe from the standpoint of physical mea-
McPherson* has observed amplitude variation in the output of the Hewlett-Packard type audio oscillator used in the experiments reported below. The major source of this noise is a tube. While both the amplitude and spectrum of this noise can vary widely from tube to tube, quantitative measurement is difficult since it is so low relative to the over-all output level of the oscillator. It is further very difficult to measure since the noise is a cycle to cycle variation, and any measure depending on time averages is not sensitive to the variation.

Since direct physical measurement of $N_E$ and $k$ are difficult, these constants will be estimated by fitting equation (11) to the data. Since the constants $N_E$ and $k$ are conceived to characterize the equipment, the attempt is to achieve this fit with a single value for these constants for the three observers, while a value of $\eta$ will be assigned to each observer.

II. EXPERIMENTAL PROCEDURE

In the experiments conducted in this study of Weber's law, the two alternative forced-choice technique was employed throughout. In this technique the two signals are presented in random order and the observer is asked to state which of the two is of the greater amplitude, the first or the second. The three observers, University of Michigan undergraduates, listened through PDR-8 earphones connected in parallel. In each of their booths there was a series of lights which informed them of the progress of the experiment. The flash of the first light warns the observers to ready themselves for the obser-

* Personal communication
vation. A second light flashes twice marking the position in time of the two signals. A third light flashes indicating that it is time to designate their choice by pressing one of two buttons. On the flash of a fourth light they are presented with the correct answer information. At this point, the data for this trial are recorded on an IBM card. The apparatus then recycles for the next trial. The entire trial occupies approximately 3 1/2 seconds.

The random ordering is achieved by a radiation programmed random number generator. A digital counter is driven by a 2500 cycle oscillator. A decade counter counts the output of a Geiger tube on which the average time between counts is approximately 0.1 seconds. When the decade counter completes its cycle, the digital counter is stopped indicating the interval in which the signal of large amplitude is presented.

Both signals are generated by the same audio oscillator fed through separate attenuators and gates. The voltages at the top of each attenuator are adjusted to be equal when read across the terminals of the earphones. One of the attenuators is then set to yield the value \( V_0 \), and this is gated on its first positive zero crossing in each of the observation intervals. The second attenuator is set to yield the value \( \Delta V \) and this gated on its first positive zero crossing during the observation interval determined by the random number generator. Each gate passes the same integral number of cycles. In the observation interval in which both signals are gated they are combined in a linear adding network.

Two hundred trials constitute a datum point.

Four different levels of \( N_G \) were investigated. Except for the case where \( N_G = 0 \), a single value of \( \Delta V \) was maintained throughout that noise level. When \( N_G = 0 \), two different levels of \( \Delta V \) were studied for low values of \( V_0 \), and three different levels for the higher values of \( V_0 \). In those experiments in
which a measurable value of \( N_G \) was introduced by the experimenter, the nth value of \( V_o \) was the \( (n - 1) \)st value of \( V_o \) plus \( \Delta V \).

III. ANALYSIS OF DATA

First, the data for \( N_G = 0 \) were analyzed. The first step is to fit a curve of \( d' \) as a function of \( \Delta V \) for each value of \( V_o \). It is assumed that this curve should be a straight line intersecting the origin \( (d' = 0 \) when \( \Delta V = 0 \)). From these curves, an estimate of that value of \( \Delta V \) necessary to lead to a \( d' = 1.00 \) is made.

This value of \( \Delta V^2 \) is then plotted as a function of \( V_o^2 \). Since by the relation from equation (11)

\[
\eta \Delta V^2 t = N_E + k V_o^2 \quad (12)
\]

the intercept on the ordinate is an indication of the residual noise of the equipment and the slope is an indication of \( k \). These plots were fit satisfactorily by straight lines. The intercepts and slopes were different for each of the observers.

Then the efficiencies of the three observers were determined from the experiment with noise. This permitted an estimate of a single set of parameters for the equation with only the efficiencies varying from observer to observer. Finally all of the data were compared to predictions so determined, and a single correction was made by eye. This lead to the equation.

\[
(d')^2 = \eta \frac{2E\Delta}{N_G + 5.6 \times 10^{-12} + 2 \times 10^{-4} V_o^2} \quad (13)
\]
The data for the several experiments are illustrated in Figures 2-15. The curves are those calculated using Equation (13) with values of \( \eta = .28, .21, \) and \(.17 \) for observers 1, 2, and 3 respectively.

In the white noise experiments in which \( \frac{S_0}{N_0} \) was maintained roughly constant, \( \eta \) was calculated from equation (10). In Figures 16-17-18, \( \eta \) is plotted as a function of the logarithm of \( N_0 \). For two of the observers \( \eta \) decreases slightly as \( N_0 \) is increased, while for the third it appears to be more or less constant.

IV. DISCUSSION OF RESULTS

Equation (13) agrees reasonably well with the obtained results. While certain departures are apparent (it should be noted that all of the data has been presented in a way which emphasizes these departures), none of these departures is great. In fact, had the data been treated in terms of db scales, the departure would have been hidden completely. If equation (13) is acceptable, then one has a satisfactory explanation for departure from Weber's law for low intensities. The explanation is that there is noise present even if the experimenter doesn't put it there.

It is difficult to claim at present that the factor dependent on the level of \( V_o \) is entirely due to the audio oscillator variations. From the data, it appears that the third observer might be better fit with a somewhat greater constant. Even here, considering the method of fit, the departure is not so great that the hypothesis should be rejected. A large quantity of data, over four different noise levels and three different observers is reasonably well satisfied by this single equation (13). The equation is based on a logical
The lines are calculated from Equation (13), and thus are not purported to be best fits for the individual graphs. All numbers shown on scales should be multiplied by $10^{-8}$.

FIG. 2. $d'$ VS. $\Delta v$ FOR OBSERVER NO. 1, $N_6 = 0$. 

---
FIG. 3.
$N_G = 0$

FIG. 4.
$N_G = 6.22 \times 10^{-9}$

OBSERVER NO. 1.

THE LINE IS CALCULATED FROM EQUATION (13).

$P(c)$ IS THE PERCENTAGE OF CORRECT RESPONSES.
FIG. 5.

\[ N_G = 9.73 \times 10^{-9} \]

FIG. 6.

\[ N_G = 4.92 \times 10^{-9} \]

THE LINE IS CALCULATED FROM EQUATION (13).
The lines are calculated from equation (13), and thus are not purported to be best fits for the individual graphs. All numbers shown on scales should be multiplied by 10^{-5}.

Fig. 7. $d'$ vs. $\Delta v$ for observer no. 2, $N_g = 0$.  

-14-
FIG. 8.

$N_g = 0$

FIG. 9.

$N_g = 6.22 \times 10^{-9}$

OBSERVER NO. 2.

THE LINE IS CALCULATED FROM EQUATION (13).

$P(c)$ IS THE PERCENTAGE OF CORRECT RESPONSES.
FIG. 10.

\[ N_G = 9.73 \times 10^{-9} \]

FIG. 11.

\[ N_G = 4.92 \times 10^{-9} \]

OBSERVER NO. 2.

THE LINE IS CALCULATED FROM EQUATION (13).
THE LINES ARE CALCULATED FROM EQUATION 13, AND
THUS ARE NOT PURPORTED TO BE BEST FITS FOR THE
INDIVIDUAL GRAPHS. ALL NUMBERS SHOWN ON SCALES
SHOULD BE MULTIPLIED BY $10^{-5}$.

FIG. 12. $d'$ VS. $\Delta v$ FOR
OBSERVER NO. 3, $N_g = 0$.  

—17—
FIG. 13.

$N_g = 0$

FIG. 14.

$N_g = 6.22 \times 10^{-9}$

OBSERVER NO. 3.

The line is calculated from equation (13). $P(c)$ is the percentage of correct responses.
FIG. 15.

$N_G = 9.73 \times 10^{-9}$

FIG. 16.

$N_G = 4.92 \times 10^{-9}$

OBSERVER NO. 3.

THE LINE IS CALCULATED FROM EQUATION (13).
FIG. 17
OBSERVER NO. 1.

DATA FOR OBSERVER 1 FOR A SIGNAL WHICH IS A SAMPLE OF WHITE GAUSSIAN NOISE
FIG. 18
OBSERVER NO. 2.

DATA FOR OBSERVER 2 FOR A SIGNAL WHICH IS A SAMPLE OF WHITE GAUSSIAN NOISE
FIG. 19
OBSERVER NO. 3.

DATA FOR OBSERVER 3 FOR A SIGNAL WHICH IS A SAMPLE OF WHITE GAUSSIAN NOISE
argument which suggests that the parameters might not assume the same values in different laboratories, or for that matter with different oscillators.

Our study of Weber's law for signals which are a sample of white Gaussian noise is in essential agreement with the results reported by Miller (Ref. 4). When \( \eta \) is plotted as a function of the logarithm of \( N_0 \) for each of our observers (Figures 16-17-18), there appears to be a slight fall-off in efficiency as the intensity of the signals is increased but this drop is not great. It certainly would not be noticed had db scales been used.

V. CONCLUSION

The hypothesis that Weber's law as applied to intensity discrimination of pure tones reflects a condition of the environment rather than the hearing mechanism is found plausible. There are controlled sources of noise in experiments. Even experiments performed in "silence" involve noise, and this noise may be the limiting factor of hearing.
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