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Technical Report

A NOTE ON WAVE-MAKING RESISTANCE OF CATAMARANS

Ryo Tasaki

Project Director: R. B. Couch

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I. INTRODUCTION

For years, many sailing catamarans have been built, and because of their good reputation their numbers are increasing. Few large sea-going catamarans have been built, however, and very few are actually in service.[1-3] This does not mean that interest in sea-going catamarans has been meager. The catamaran has certain recognized advantages, such as a high degree of stability, large deck area, etc., and from time to time independent studies appear in the literature. For instance, W. H. Michel discusses the features and feasibility of ocean-going catamarans from a practical point of view, and offers a particular design of a catamaran for an oceanographic research vessel.[4]

A number of theoretical papers discussing the resistance of catamarans are also available. In 1946, M. Kinoshita and S. Okata calculated the wave-making resistance of a catamaran of Weinblum's mathematical ship form.[5] In 1950, K. Yokoo and R. Tasaki carried out theoretical calculations of simple ship forms, varying draft and distance between two hulls for speeds up to a Froude number of 0.5.[6,7] This study was followed by an experiment in which models corresponding to the theoretical ship forms were used. The study showed fairly good agreement between the theory and the experiment and it shed some light on the interference effect between two hulls. This experimental study also included hull forms which were asymmetrical with respect to their centerlines, i.e., the waterlines were half-moon shaped. Asymmetrical hull forms of this type were found to be inferior to the symmetrical hull forms, however.[6]

In 1955, K. von Eggers carried out theoretical and experimental studies on catamarans as well as on two ships in tandem, etc.[8] His studies included the effect of shallow water and remain the only source of information available for such an effect.

In previous papers by Yokoo and the present author,[6,7] the general characteristics of the wave-making resistance of a catamaran have been presented. The present paper attempts to describe these characteristics in further detail. It will be shown that the interference effect between two hulls can be divided into two parts: (1) the monotonically increasing part, and (2) the part which oscillates with respect to the increasing speed or to the decreasing distance between the two hulls. Numerical examples of these components as well as of total resistance are given for simple hull forms.

In previous papers, fairly good agreement between theory and experiment has been established for the models corresponding to the theoretical forms.[7,8] These forms may not be altogether suitable for a practical catamaran, but a theoretical representation of practical hull forms and a calculation based on such a representation are not yet at hand. At this stage, therefore, an independent experimental study of practical ship forms, guided by theoretical studies such as the present one, is a necessity for design purposes. Experiments on such conventional hull forms are currently underway at The University of Michigan and the result will be available shortly.

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II. WAVE-MAKING RESISTANCE OF CATAMARANS

1. COORDINATE SYSTEM, NOTATIONS, AND NON-DIMENSIONAL EXPRESSIONS

A. Coordinate System

The coordinate system to be used is defined in Fig. 1. The origin of the coordinate system is at the midship, the still-water plane, and a center plane equidistant from both hulls. The positive x-axis is in the direction of advance and the positive z-axis is directed upward.

B. Notations

The notations to be used are defined as follows:

2ℓ	length of ship; $2\ell = L$
$2k\ell$	distance between the centerlines of the two hulls
V	advance speed of the catamaran
$V\,m(\xi,\eta,\zeta)$	source distribution, strength of source (out-flow per unit tim) at a point (ξ,η,ζ)
T	depth of source distribution
R_{2W}	wave-making resistance of the catamaran
R_{OW}	wave-making resistance of each hull of the catamaran
$2R_W$	increase in wave-making resistance due to the interference between the wave systems of both hulls
θ	direction of propagation of elementary waves; zero angle is on the negative x-direction.

C. Non-Dimensional Expressions

$$\xi = x/\ell, \qquad \zeta = y/\ell, \qquad t = T/\ell$$

$$K_o = \frac{g\ell}{V^2} = \frac{1}{2F^2} \qquad \text{where F is Froude number}$$

$$C'_{2W} = \frac{R_{2W}}{\frac{1}{2}\rho V^2 L^2}, \qquad C'_{OW} = \frac{R_{OW}}{\frac{1}{2}\rho V^2 L^2}, \qquad C'_W = \frac{R_W}{\frac{1}{2}\rho V^2 L^2} .$$

The first approximation to the wave-making resistance of a catamaran is obtained by superimposing the velocity potentials or free wave patterns of the two hulls, as shown in the Appendix, and assuming that the boundary condition is not disturbed by the presence of the other hull. This approximation shows fairly good agreement with experimental results.[1,2] In fact, the presence of the other hull has only a secondary effect on the wave-making resistance, as is explained in the following section.

One method of correcting the disturbance on one hull caused by another hull is to superimpose upon the original source distribution a suitable distribution of doublets oriented perpendicularly to the direction of advance. Since Lagally's theorem shows that no additional force in the direction of advance is generated by these doublets, their influence is necessarily of secondary nature.

2. FORMULAS FOR ESTIMATING WAVE-MAKING RESISTANCE

The total wave-making resistance, R_{2W}, is the sum of twice the resistance of each hull, R_{OW}, and the resistance increase due to interference between wave systems of both hulls, $2R_W$ (see the Appendix). Thus:

$$R_{2W} = 2R_{OW} + 2R_W \tag{1}$$

where

$$R_{OW} = \frac{\rho K_o V^2 L^2}{4\pi} \int_0^{\pi/2} (P^2 + Q^2) \sec^3\theta d\theta \ , \tag{2}$$

$$R_W = \frac{\rho K_o^2 V^2 L^2}{4\pi} \int_0^{\pi/2} \cos(2K_o k \tan\theta \sec\theta)(P^2 + Q^2) \sec^3\theta d\theta \tag{3}$$

$$\begin{matrix} P \\ \\ Q \end{matrix} = \int_{-t}^{0}\int_{-1}^{1} m(\xi,\eta,\zeta) \begin{matrix} \cos \\ \\ \sin \end{matrix} (K_o\xi \sec\theta) d\xi d\zeta \ . \tag{4}$$

3. FORMULAS FOR EVALUATING WAVE-MAKING RESISTANCE FOR SYMMETRIC SHIP FORMS

In the case where the sources are distributed on the center plane of each hull, antisymmetrically with respect to the midship section and uniformly in the direction of depth, the wave-making resistance is given by the following formulas. This distribution of sources represents the wall-sided ship form of Michel's theory; waterlines are symmetric with respect to the midship section. When the new expressions

$$Z = 1 - e^{-K_o t \sec^2\theta} \tag{5}$$

and

$$M = \int_o^1 m(\xi)\sin(K_o\xi \sec^2\theta)\,d\xi \tag{6}$$

are introduced, Eqs. (1) through (4) become

$$R_{ow} = \frac{\rho V^2 L^2}{\pi} \int_o^{\pi/2} M^2 Z^2 \cos\theta d\theta \tag{7}$$

$$R_w = \frac{\rho V^2 L^2}{\pi} \int_o^{\pi/2} \cos(2K_o k \tan\theta \sec\theta) M^2 Z^2 \cos\theta d\theta\ . \tag{8}$$

III. EVALUATION OF FORMULAS

The following expression of M^2 in a power series of $(K_o \sec \theta)$ is obtained by successive partial integration of M, which has been applied to the evaluation of Eq. (7) by T. Inui.[9]

$$M^2 = \frac{1}{2}\frac{1}{K_o^2 \sec^2\theta}\sum_{n=o}^{\infty}\sum_{(2n=i+j)}(-1)^{n+i}\frac{m_i m_j}{(K_o \sec\theta)^2}$$

$$+\frac{1}{2}\frac{\cos(2K_o \sec\theta)}{K_o^2\sec^2\theta}\sum_{n=o}^{\infty}\sum_{(2n=i+j)}(-1)^{n}\frac{m_i m_j}{(K_o\sec\theta)^2}$$

$$-\frac{1}{2}\frac{\sin(2K_o\sec\theta)}{K_o^2\sec^2\theta}\sum_{n=o}^{\infty}\sum_{(2n+1=i+j)}(-1)^{n+1}\frac{m_i m_j}{(K_o\sec\theta)^{2n+1}} \tag{9}$$

where $m_i = m^{(i)}(1)$; i.e., m_i is the value of the ith derivative at the bow, $\xi = 1$.

By inserting Eq. (9) into Eqs. (7) and (8), the wave-making resistance coefficient of the catamaran is given as follows:

$$C'_{2w} = 2C'_{ow} + 2C'_{w} \tag{10}$$

$$C'_{ow} = \frac{2}{\pi K_o^2}\sum_{n=o}^{\infty}(-1)^n\frac{1}{K_o^{2n}}\left[\sum_{(2n=i+j)} m_i m_j\int_o^{\pi/2} Z^2\cos^{2n+3}\theta d\theta\right.$$

$$+\sum_{(2n=i+j)}(-1)^i m_i m_j \int_o^{\pi/2} Z^2 \cos(2K_o\sec\theta)\cos^{2n+3}\theta d\theta$$

$$\left.-\frac{1}{K_o}\sum_{(2n+1=i+j)} m_i m_j\int_o^{\pi/2} Z^2\sin(2K_o\sec\theta)\cos^{2n+4}\theta d\theta\right] \tag{11}$$

$$C_w' = \frac{2}{\pi K_o^2} \sum_{n=o}^{\infty} (-1)^n \frac{1}{K_o^{2n}}$$

$$\times \left[\sum_{(2n=i+j)} m_i m_j \int_o^{\pi/2} Z^2 \cos(2K_o k \tan\theta \sec\theta) \cos^{2n+3}\theta d\theta \right.$$

$$+ \sum_{(2n=i+j)} (-1)^i m_i m_j \int_o^{\pi/2} Z^2 \cos(2K_o k \tan\theta \sec\theta) \cos(2K_o \sec\theta) \cos^{2n+3}\theta d\theta$$

$$\left. - \frac{1}{K_o} \sum_{(2n+i=i+j)} m_i m_j \int_o^{\pi/2} Z^2 \cos(2K_o k \tan\theta \sec\theta) \sin(2K_o \sec\theta) \cos^{2n+4}\theta d\theta \right] \quad (12)$$

These expressions show that in this case the wave-making resistance is easily calculated, if the following integrals are known.

$$\int_o^{\pi/2} Z^2 \cos^{2n+3}\theta d\theta, \qquad n = 0,1,\ldots, \quad (13)$$

$$\int_o^{\pi/2} Z^2 \cos(2K_o \sec\theta) \cos^{2n+3}\theta d\theta, \qquad n = 0,1,\ldots, \quad (14)$$

$$\int_o^{\pi/2} Z^2 \sin(2K_o \sec\theta) \cos^{2n+4}\theta d\theta, \qquad n = 0,1,\ldots, \quad (15)$$

and

$$I_1(n;\ K_o,t,k) = \int_o^{\pi/2} Z^2 \cos(2K_o k \tan\theta \sec\theta) \cos^{2n+3}\theta d\theta, \qquad n = 0,1,\ldots, \quad (16)$$

$$I_2(n;\ K_o,t,k) = \int_o^{\pi/2} Z^2 \cos(2K_o k \tan\theta \sec\theta) \cos(2K_o \sec\theta) \cos^{2n+3}\theta d\theta, \ n = 0,1,\ldots, \quad (17)$$

$$I_3(n;\ K_o,t,k) \quad = \int_o^{\pi/2} Z^2\cos(2K_ok \tan\theta \sec\theta)$$

$$\times \sin(2K_o\sec\theta)\cos^{2n+4}\theta d\theta, \qquad n = 0,1,\ldots \tag{18}$$

1. RESISTANCE OF EACH HULL

The first part of the wave-making resistance, the sum of the resistance of each hull, has the same characteristics as the wave-making resistance of a single hull. The integral in Eq. (13) is the so-called fundamental term of the wave-making resistance and is evaluated by the following formulas.[9]

$$U_{n+1}(K_ot) \quad = \frac{1}{C_{n+1}} \int_o^{\pi/2} Z^2\cos^{2n+3}\theta d\theta$$

$$= \quad 1 - 2E_n(K_ot) + E(2K_ot) \tag{19}$$

where

$$C_{n+1} \quad = \int_o^{\pi/2} \cos^{2n+3}\theta d\theta \quad = \frac{2\cdot4\cdot6\ \cdots\ (2n+2)}{3\cdot5\cdot6\ \cdots\ (2n+3)} \tag{20}$$

$$E_{n+1}(K_o,t) \quad = \frac{1}{C_{n+1}} \int_o^{\pi/2} e^{(K_ot\ \sec^2\theta)}\cos^{2n+3}\theta d\theta \tag{21}$$

The integrals in Eqs. (14) and (15) are the interference terms.[9] (In the following discussion they are called the oscillating terms to distinguish them from the resistance increase due to the interference between two hulls.)

For $2K_o > 5$ or $F < 0.45$, the method of stationary phase leads to good approximations as follows:

$$\int_o^{\pi/2} Z^2\cos^{2n+3}\theta\ \cos(2K_o\sec\theta)d\theta \sim (\pi/4K_o)^{\frac{1}{2}}\ Z_o{}^2\cos(2K_o + \tfrac{\pi}{4}) \tag{22}$$

$$\int_o^{\pi/2} Z^2\cos^{2n+4}\theta\ \sin(2K_o\sec\theta)d\theta \sim (\pi/4K_o)^{\frac{1}{2}}\ Z_o{}^2\sin(2K_o + \tfrac{\pi}{4}) \tag{23}$$

2. RESISTANCE INCREASE DUE TO INTERFERENCE: MONOTONICALLY INCREASING TERM AND OSCILLATING TERM

In the author's previous paper[6,7] the resistance increase due to the interference effect given by Eq. (8) was calculated graphically and was not separated into three integrals I_1, I_2, and I_3, as it is in this paper. These integrals show the general characteristics of the interference effect in the wave-making resistance of catamarans.

The integral I_1 corresponds to the integral (13) but is not represented in an explicit form such as Eq. (19). The integrals I_2 and I_3 correspond to (14) and (15), respectively, and are transformed as follows:

$$I_2(n;\ K_o,t,k) = \frac{1}{2}(I_{21} + I_{22})$$

where

$$I_{21}(n;\ K_o,t,k) = \int_0^{\pi/2} Z^2\cos(2K_o h_1)\cos^{2n+3}\theta d\theta,$$

$$I_{22}(n;\ K_o,t,k) = \int_0^{\pi/2} Z^2\cos(2K_o h_2)\cos^{2n+3}\theta d\theta \tag{24}$$

and

$$I_3(n;\ K_o,t,k) = \frac{1}{2}(I_{31} + I_{32})$$

where

$$I_{31}(n;\ K_o,t,k) = \int_0^{\pi/2} Z^2\cos(K_o h_1)\cos^{2n+4}\theta d\theta,$$

$$I_{32}(n;\ K_o,t,k) = \int_0^{\pi/2} Z^2\cos(K_o h_2)\cos^{2n+4}\theta d\theta \tag{25}$$

with

$$h_1 = (1 + k\tan\theta)\sec\theta$$

$$h_2 = (1 - k\tan\theta)\sec\theta \tag{26}$$

In these expressions $h_1(\theta)$ does not have any stationary point, but $h_2(\theta)$ has two stationary points between $\theta = 0$ and $\pi/2$ when $k \neq 0$ and $k < 1/2\sqrt{2} = 0.354$.

Therefore, it seems possible to apply the method of stationary phase to the evaluation of the integrals I_2 and I_3. Unfortunately, however, the stationary phase method does not give good approximations for practical values of K_o. For example, I_2 and I_3 have large values for the practical values of K_o even when $k > 1/2\sqrt{2}$, as shown in Fig. 3. In this paper, therefore, graphical integrations were carried out for several values of K_o, t, and k.

Now, for simplicity, Eqs. (11) and (12) are examined for the linear distribution of sources; that is,

$$m(\xi) = a_1\xi$$

This expression corresponds to the wall-sided ship form, with parabolic water lines, of Michel's theory. In this case,

$$m_1 = a_1, \quad \text{and} \quad m_2 = m_3 = \cdots = 0$$

Equations (11) and (12) now become

$$C'_{ow} = \frac{4}{\pi} a_1^2F^4[\{C_3U_3 + (\sqrt{2}\ F)^4C_5U_5\} + (\frac{\pi}{2})^{\frac{1}{2}} Z_o^2F\{1 - (\sqrt{2}\ F)^4\}\cos(2K_o + \frac{\pi}{4})$$

$$- (\frac{\pi}{2})^{\frac{1}{2}} Z_o^2F(\sqrt{2}\ F)^2\sin(2K_o + \frac{\pi}{4})\ ] \tag{26}$$

$$C'_w = \frac{4}{\pi} a_1^2F^4[\{I_1(n = 0) + (\sqrt{2}\ F)^4\ I_1(n = 1)\} + \{I_2(n = 0)$$

$$- (\sqrt{2}\ F)^4\ I_2(n = 1)\} - \{2(\sqrt{2}\ F)^2\ I_3(n = 0)\}] \tag{27}$$

Equation (27) shows that examination of the integrals $I_1(n = 0)$, $I_2(n = 0)$, and $I_3(n = 0)$ gives some general information about the resistance increase due to interference when the Froude number, F, is not large.

The integral $I_1(n = 0; K_o,t,k)$ is a monotonic function which decreases with an increase in $(K_o k)$. The numerical value of the integral for infinite draft, that is, $I_1(n = 0; K_o,t = \infty,k)$ is shown in Fig. 2. The integral I_1 corresponds to the fundamental term of the wave-making resistance of the single-hull ship and is same as the fundamental term of each hull when k is equal to zero. It should be noticed that this term increases rapidly when the distance between two hulls, k, decreases, or when the speed of the ship, $F = 1/\sqrt{2K_o}$, increases.

The integrals I_2 and I_3 are oscillating functions with respect to K_o and k, which correspond to the oscillating term of the single-hull ship. In Fig. 3 the contribution of these integrals I_1, I_2, and I_3, to the resistance increase due to interference between two hulls is shown plotted against the distance between two hulls, k, for a given advance speed, $K_o = 5(F = 0.316)$. When $k = 0$, from Eqs. (16), (11), (22), and (23):

$$I_2(n = 0;\ K_o, t = \infty, k = 0)$$
$$= \int_0^{\pi/2} Z^2 \cos(2K_o \sec\theta) \cos^{2n+3}\theta d\theta \ \doteqdot\ \left(\frac{\pi}{4K_o}\right)^{\frac{1}{2}} \cdot 1 \cdot \cos\left(2K_o + \frac{\pi}{4}\right), \qquad n = 0,1,\ldots$$

and

$$I_3(n = 0;\ K_o, t = \infty, k = 0)$$
$$= \int_0^{\pi/2} Z^2 \sin(2K_o \sec\theta) \cos^{2n+4}\theta d\theta \ \doteqdot\ \left(\frac{\pi}{4K_o}\right)^{\frac{1}{2}} \cdot 1 \cdot \sin\left(2K_o + \frac{\pi}{4}\right), \qquad n = 0,1,\ldots$$

The variation of the integrals I_2 and I_3 plotted against K_o are shown in Fig. 4 for infinite draft, $t = \infty$, and for a given distance between two hulls, $k = 0.3$. That is, $I_2(n = 0;\ K_o, t = \infty, k = 0.3)$ and $I_3(n = 0;\ K_o, t = \infty, k = 0.3)$ are shown. In this figure I_{21}, I_{22}, I_{31}, and I_{32} are given by Eqs. (24) and (25); the variation of the integral I_1 is also shown for reference. When $K_o = 0$, from expressions (16), (17), and (21) the integrals I_2 and I_3 are:

$$I_2 = \int_0^{\pi/2} 1 \cdot 1 \cdot \cos^{2n+3}\theta d\theta = C_3 = 2/3$$

and

$$I_3 = \int_0^{\pi/2} 1 \cdot 0 \cdot \cos^{2n+3}\theta d\theta = 0$$

By comparing Fig. 3 with Fig. 4 it is noted that over the range of the practical values of k and K_o, the variation of the integrals I_2 and I_3 with respect to k ($2k\ell$ being the distance between two hulls) is much more gradual than the comparable variations with respect to K_o (K_o corresponding to inverse square of speed). Furthermore, it is seen in Fig. 3 that, for the small value of k, the monotonically increasing term, I_1, is dominant. For example, in this case, $k \leqq 0.27$ this limit depends on the speed parameter, K_o (see Fig. 8).

In expression (27) the oscillating term is:

$$I_2(n = 0) - 2(\sqrt{2}\ F)^2\ I_3(n = 0)$$

This expression shows that the integral I_2 is predominant in the oscillating term for small Froude numbers. Figure 4 shows that in this case the resistance increase due to the monotonically increasing term is more than offset by the negative resistance due to the oscillating term near the value of $K_o = 4.8$ and 6.9.

In Fig. 5, the contribution of these integrals to the resistance increase is shown plotted against advance speed for the case of infinite draft and for a distance between two hulls, k, equal to 0.3. In this figure $B = F^4\ I_1$ is the monotonically increasing term and $C = F^4\{I_2 - 2(\sqrt{2}\ F)^2\ I_3\}$ is the oscillating term with increasing Froude number. The maximum cancellation occurs when the amplitude of the oscillating term takes the maximum value. It is expected, therefore, from Figs. 3, 4, and 8 that the k = 0.3 would give the maximum cancellation. In reference to Fig. 5, however, it is noted that, compared with the oscillating term the monotonically increasing term takes so large a part in the resistance increase that it has a decisive effect on the wave-making resistance of the catamaran. In other words, the gain obtained by the effect of the interference is cancelled by the monotonically increasing term. In the case illustrated by Fig. 5, the range of negative increase of wave-making resistance due to the oscillating term covers Froude numbers from 0.30 to 0.36. This range is reduced to Froude numbers from 0.31 to 0.35 by the monotonically increasing term. The amplitude of the oscillating term is reduced by half by the monotonically increasing term.

In order to see the contribution of these terms to the wave-making resistance of catamarans, the wave-making resistance of each hull is shown in Fig. 6. (The scale is the same as that in Fig. 5). The monotonically increasing term, $B = F^4\ I_1$ in Fig. 5, corresponds to the fundamental term, $B = F^4\ C_3U_3$ in Fig. 6. The oscillating term corresponding to

$$C = F^4\{I_2 - 2(\sqrt{2}\ F)^2\ I_3\}$$

in Fig. 5 is the sum of

$$F^4\left(\frac{\pi}{2}\right)^{\frac{1}{2}} F \cos\left(2K_o + \frac{\pi}{4}\right)$$

and

$$F^4\left(\frac{\pi}{2}\right)^{\frac{1}{2}} F(\sqrt{2}\ F)^2 \sin\left(2K_o + \frac{\pi}{4}\right),$$

i.e., C = D+E, in Fig. 6. Comparison of Figs. 5 and 6 shows that the magnitude of each term is comparable. In the present case, the resistance decrease is expected to reach 50% of the wave-making resistance of each hull. The ratios of the resistance increase to the wave-making resistance of each hull for several values of k are shown in Figs. 9 and 10, plotted against Froude number.

It may be said, comparing Figs. 3 and 5 with Figs. 8, 9, and 10, that the results obtained by both methods show good agreement.

IV. EFFECT OF DRAFT

The integrals in expressions (13) through (15) and (29) through (34) consist of the following factors:

$$Z^2 = (1 - e^{-K_0 t \sec^2\theta})^2 \tag{a}$$

and

$$\cos^{2n+3}\theta, \qquad \cos(2K_0\sec^2\theta)\cos^{2n+3}\theta, \qquad \sin(2K_0\sec^2\theta)\cos^{2n+4}\theta;$$

$$\cos(2K_0 k \tan\theta \sec\theta)\cos^{2n+3}\theta;$$

$$\cos(2K_0 h_1)\cos^{2n+3}\theta, \qquad \cos(2K_0 h_2)\cos^{2n+3}\theta;$$

$$\sin(2K_0 h_1)\cos^{2n+3}\theta, \qquad \sin(2K_0 h_2)\cos^{2n+4}\theta \tag{b}$$

The first factor (a), represents the effect of the draft change and is shown plotted against θ with respect to several values of $K_0 t$ in Fig. 7(a). It is noted that Z^2 is constant for small values of θ and may be substituted by its value at $\theta = 0$, i.e., $Z_0{}^2$. The first of the factors in (b) is $\cos^{2n+3}\theta$. The others oscillate rapidly for values of θ close to $\pi/2$ and their envelopes are $\cos^{2n+3}\theta$ or $\cos^{2n+4}\theta$, which become small for value of θ close to $\pi/2$ (see Fig. 7(b)). In other words, the factors in (b) contribute the most to the integral for the small value of θ, for which Z^2 is also very close to $Z_0{}^2$. The integrals are, therefore, almost proportional to the value of Z^2 at $\theta = 0$, or $Z_0{}^2$. It is noted from Eqs. (25) and (26) that the ratio of the resistance increase due to the interference to the resistance of each hull, $R_W/R_{OW} = C_W/C_{OW}$, is not affected appreciably by the draft change. In Refs. 6 and 7 these ratios for the length-draft ratio $L/T = 20$ and 10 are shown against advance speed and several distances between two hulls. These figures also confirm the finding noted above (see Figs. 9 and 10).

V. CONCLUSIONS

1. The wave-making resistance of the catamaran can be divided into two parts, one the resistance of each hull and the other the resistance increase due to the interference between two hulls.

2. The resistance increase due to the interference can be further divided into two parts. The first part, the monotonic and positive term, is a monotonically decreasing function with respect to the increase of the product of speed parameter K_o and distance parameter between the two hulls, $K_o k$ or $k/2F^2$. For a catamaran with small distance between the two hulls, the wave-resistance increase due to this term is dominant over the second part. The second part, the oscillating term, which oscillates with respect to the speed parameter K_o, does not change rapidly with respect to the distance between the two hulls.

3. The ratio of the resistance increase due to the interference to the resistance of each hull is not appreciably affected by the changes in draft.

APPENDIX

DERIVATION OF FORMULAS FOR ESTIMATING THE WAVE-MAKING RESISTANCE OF CATAMARANS

T. H. Havelock[10] showed that when the free wave pattern behind a ship is given by

$$\zeta(x,y) = \int_0^{\pi/2} (F_1 \sin A \cos B + F_2 \cos A \sin B + F_3 \cos A \cos B + F_4 \sin A \sin B)\, d\theta \tag{1}$$

where

$$A = K_0 \frac{x}{\ell} \sec \theta$$

and

$$B = K_0 \frac{y}{\ell} \sin \theta \sec^2\theta,$$

the wave making resistance of the ship is

$$R = \frac{1}{4} \pi \rho V^2 \int_0^{\pi/2} (F_1{}^2 + F_2{}^2 + F_3{}^2 + F_4{}^2) \cos^3\theta\, d\theta. \tag{2}$$

The wave pattern due to a point source at $(\xi\ell, k\ell, \zeta\ell)$ is

$$\zeta_{ws}(x,y;\ \xi\ell, k\ell, \zeta\ell) = \frac{mK_0}{\pi\ell} \int_{-\pi/2}^{\pi/2} \sec^3\theta\ e^{-K_0\zeta \sec^2\theta}$$

$$\times \cos[K_0\{(\frac{x}{\ell} - \xi)\cos\theta + (\frac{y}{\ell} - k)\sin\theta\}\sec^2\theta]\, d\theta.$$

If we put $X_1 = K_0\xi \sec\theta$ and $K = K_0 k \sin\theta \sec^2\theta$, then

$$\zeta_{ws}(x,y;\ \xi\ell,k\ell,\zeta\ell) = \frac{mK_o}{\pi\ell}\int_{-\pi/2}^{\pi/2}\sec^3\theta\ e^{-K_o\zeta\ \sec^2\theta}$$

$$\times\ [\cos K\{\cos X_1 \cos A \cos B + \sin X_1 \sin A \cos B$$

$$- \cos X_1 \sin A \sin B + \sin X_1 \cos A \sin B\}$$

$$- \sin K\{\cos X_1 \cos A \sin B + \sin X_1 \sin A \sin B$$

$$- \cos X_1 \sin A \cos B + \sin X_1 \cos A \cos B\}]d\theta.$$

Now we calculate the wave pattern due to two point sources which are symmetrical with respect to the x-z plane, that is,

$$\zeta_{ws}(x,y) = \zeta_{ws}(x,y;\ \xi\ell,k\ell,\zeta\ell) + \zeta_{ws}(x,y;\ \xi\ell,-k\ell,\zeta\ell)$$

In this calculation the terms containing sin K cancel each other since

$$\cos K(k\ell) = \cos K(-k\ell)$$

and

$$\sin K(k\ell) = -\sin K(-k\ell).$$

Then

$$\zeta_{ws}(x,y) = \frac{2mK_o}{\pi\ell}\int_{-\pi/2}^{\pi/2}\sec^3\theta\ e^{-K_o\zeta\ \sec^2\theta}\cos K$$

$$\times\ (\cos X_1 \cos A \cos B + \sin X_1 \sin A \cos B$$

$$- \cos X_1 \sin A \sin B + \sin X_1 \cos A \sin B)d\theta.$$

For the source distribution over the center plane of each hull of the catamaran, the free wave pattern is

$$\zeta_w(x,y) = \frac{2K_o\ell}{\pi}\int_{-\pi/2}^{\pi/2}\int_{-t}^{o}\int_{-1}^{1} m \cos K \sec^3\theta \; e^{-K_o\zeta \sec^2\theta}$$

$$\times (\cos X_1 \cos A \cos B + \sin X_1 \sin A \cos B$$

$$- \cos X_1 \sin A \sin B + \sin X_1 \cos A \sin B) d\xi d\zeta d\theta \,.$$

Now, $\cos K \sec^3\theta \; e^{-K_o\zeta \sec^2\theta}$, $\cos X_1$, $\sin X_1$, $\cos A$, $\sin A$, and $\cos B$ are even functions of θ, and $\sin B$ is an odd function of θ, so that the above expression is equivalent to

$$\zeta_w(x,y) = \frac{4K_o\ell}{\pi}\int_{o}^{\pi/2}\int_{-t}^{o}\int_{-1}^{1} m \cos K \sec^3\theta \; e^{-K_o\zeta \sec^2\theta}$$

$$\times (\cos X_1 \cos A \cos B + \sin X_1 \sin A \cos B) d\xi d\zeta d\theta.$$

Therefore, $F_1,\ldots,F_4$ in Eq. (1) are:

$$F_1 = \frac{4K_o\ell}{\pi} \cos K \sec^3\theta \int_{-t}^{o}\int_{-1}^{1} m \; e^{-K_o\zeta \sec^2\theta} \sin X_1 d\xi d\zeta$$

$$= \frac{4K_o\ell}{\pi} \cos K \sec^3\theta \cdot Q$$

$$F_2 = 0$$

$$F_3 = \frac{4K_o\ell}{\pi} \cos K \sec^3\theta \int_{-t}^{o}\int_{-1}^{1} m \; e^{-K_o\zeta \sec^2\theta} \cos X_1 \; d\xi d\zeta$$

$$= \frac{4K_o\ell}{\pi} \cos K \sec^3\theta \cdot P$$

$$F_4 = 0$$

Then, Eq. (2) leads to

$$R_{2W} = \frac{1}{4}\pi\rho V^2 \int_0^{\pi/2} (F_1{}^2 + F_3{}^2)\cos^3\theta d\theta$$

$$= \frac{4\rho K_0{}^2 V^2 \ell^2}{\pi} \int_0^{\pi} \cos^2 K(P^2 + Q^2)\sec^3\theta d\theta$$

$$= \frac{\rho K_0{}^2 V^2 L^2}{2\pi} \int_0^{\pi/2} (1 + \cos 2K)(P^2 + Q^2)\sec^3\theta d\theta$$

$$= 2R_{OW} + 2R_W$$

where

$$R_{OW} = \frac{\rho K_0{}^2 V^2 L^2}{4\pi} \int_0^{\pi/2} (P^2 + Q^2)\sec^3\theta d\theta$$

and

$$R_W = \frac{\rho K_0{}^2 V^2 L^2}{4\pi} \int_0^{\pi/2} \cos 2K(P^2 + Q^2)\sec^3\theta d\theta \ .$$

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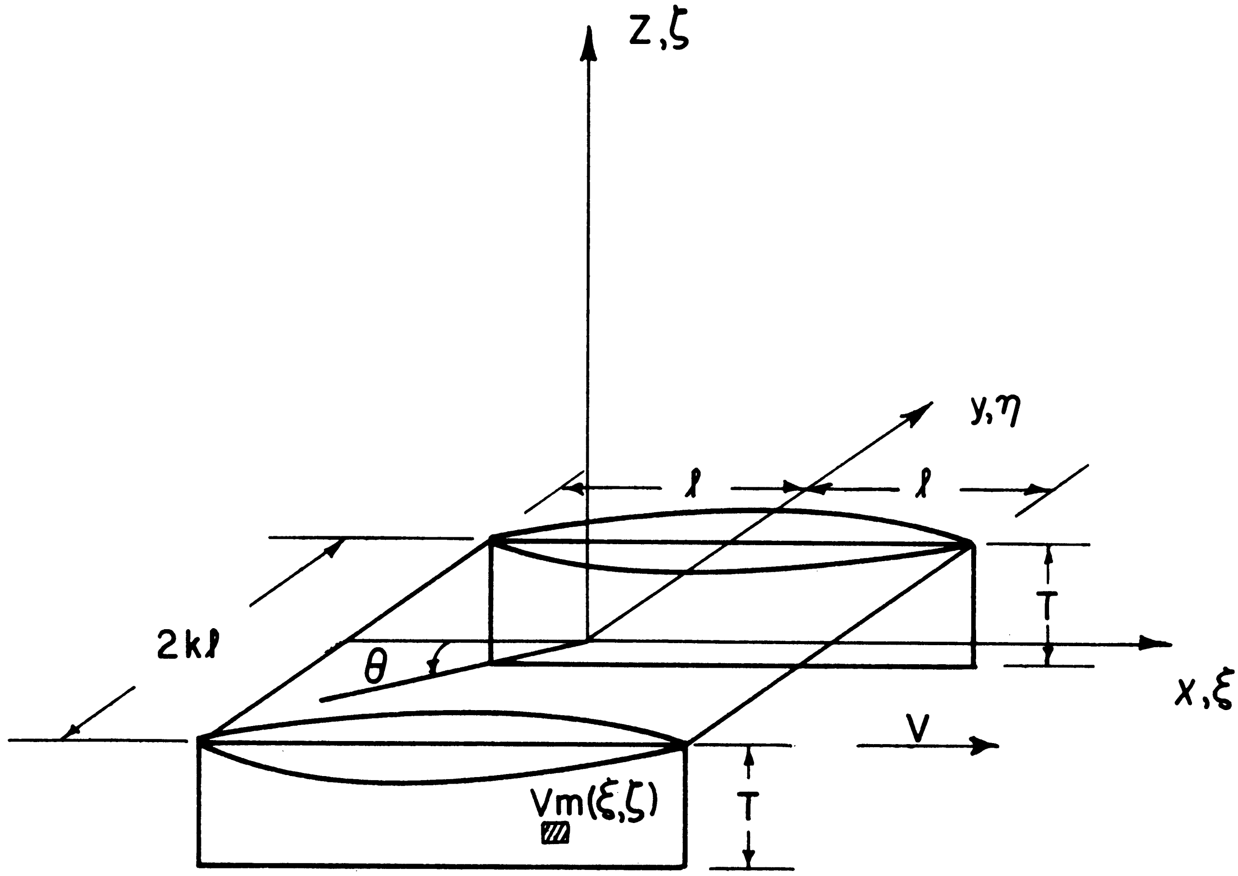


Fig. 1. Coordinate system.

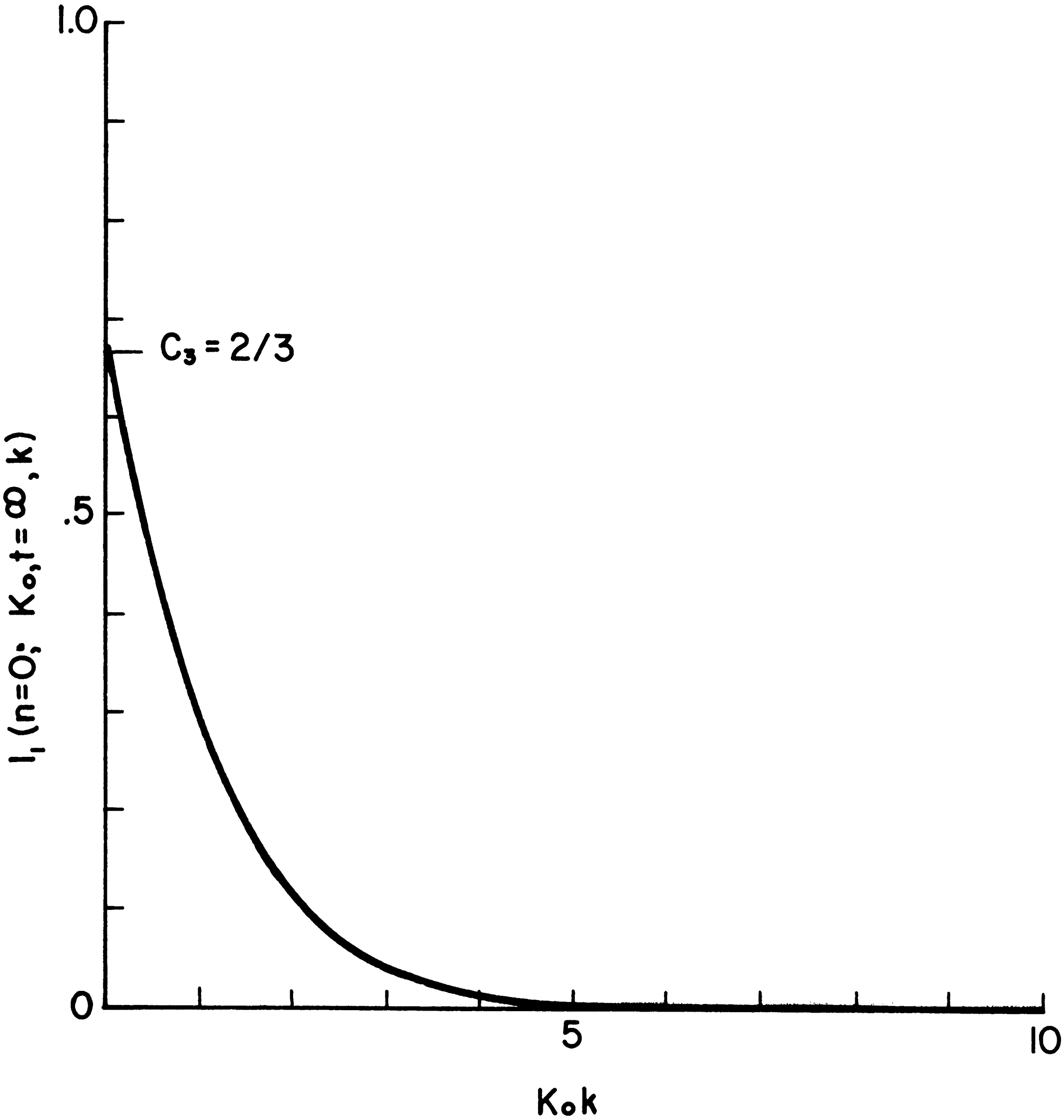


Fig. 2. Monotonically increasing term, $I_1(n = 0;\ K_o,\ t = \infty,\ k)$.

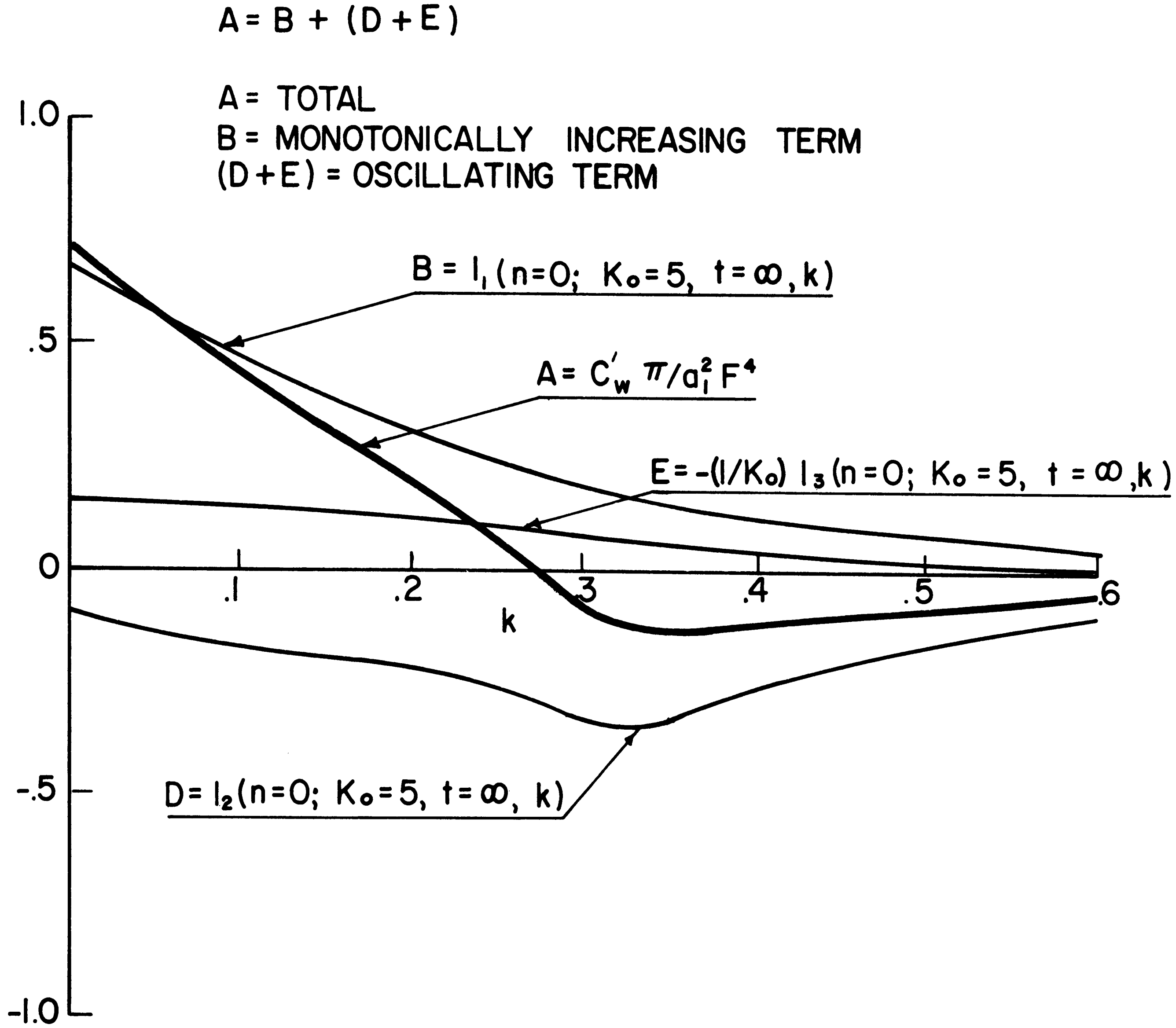


Fig. 3. Contribution of I_1, I_2, and I_3 to the resistance increase due to interference for a given advance speed.

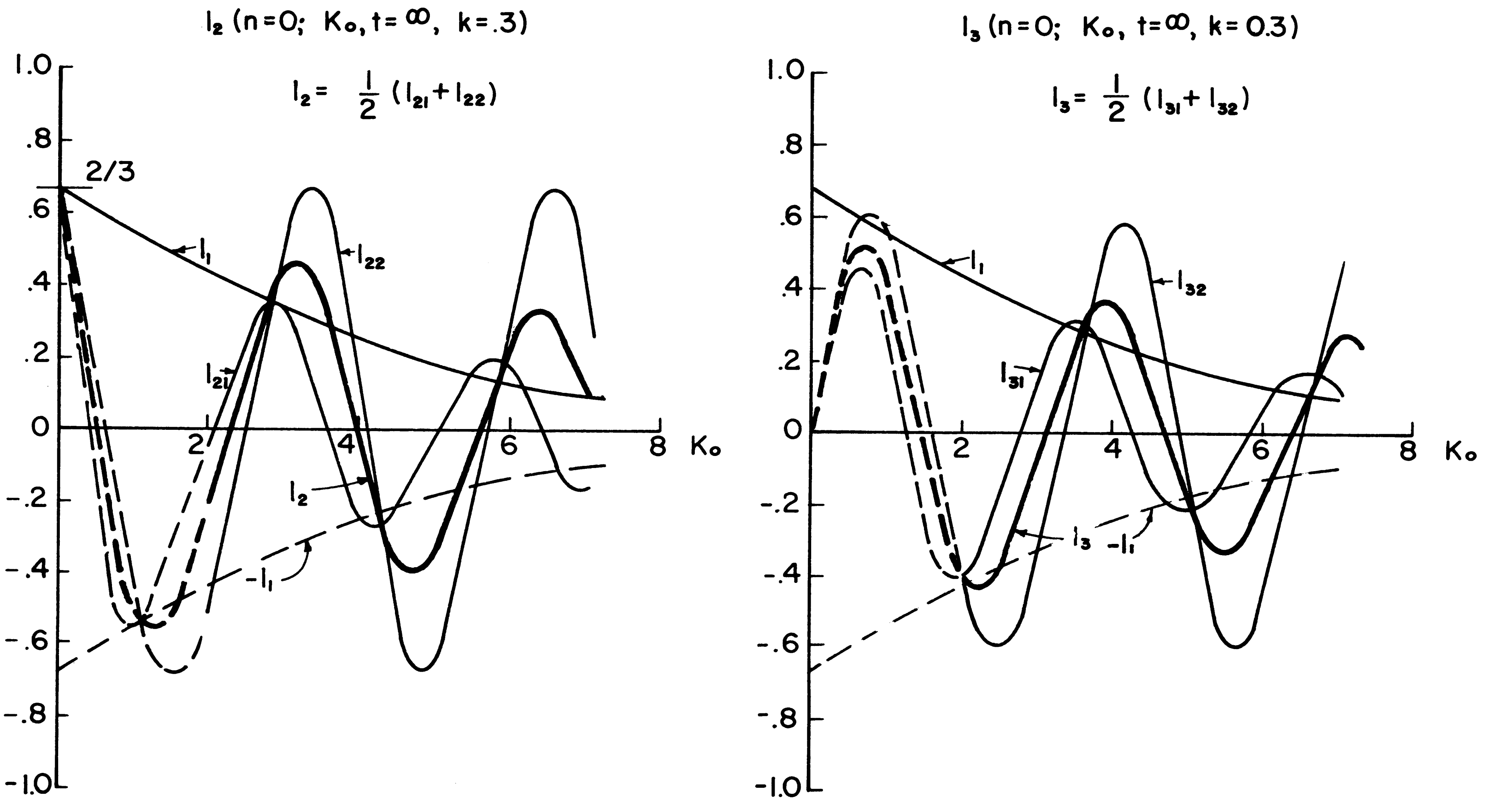


Fig. 4. Oscillating terms, $I_2(n = 0;\ K_o,\ t = \infty,\ k = 0.3)$ and $I_3(n = 0;\ K_o,\ t = \infty,\ k = 0.3)$.

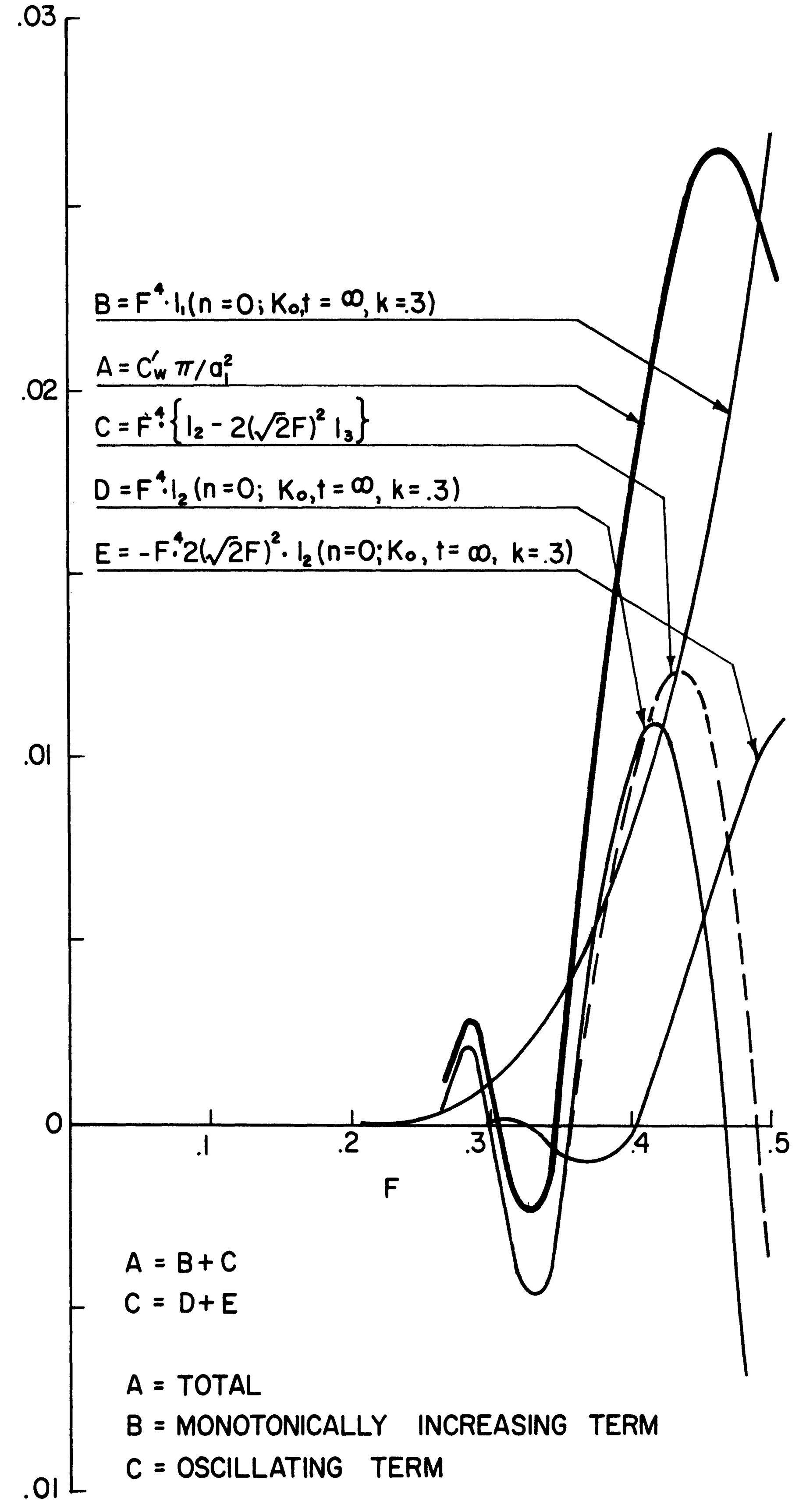


Fig. 5. Contribution of I_1, I_2, and I_3 to the resistance increase due to interference for a given distance between two hulls.

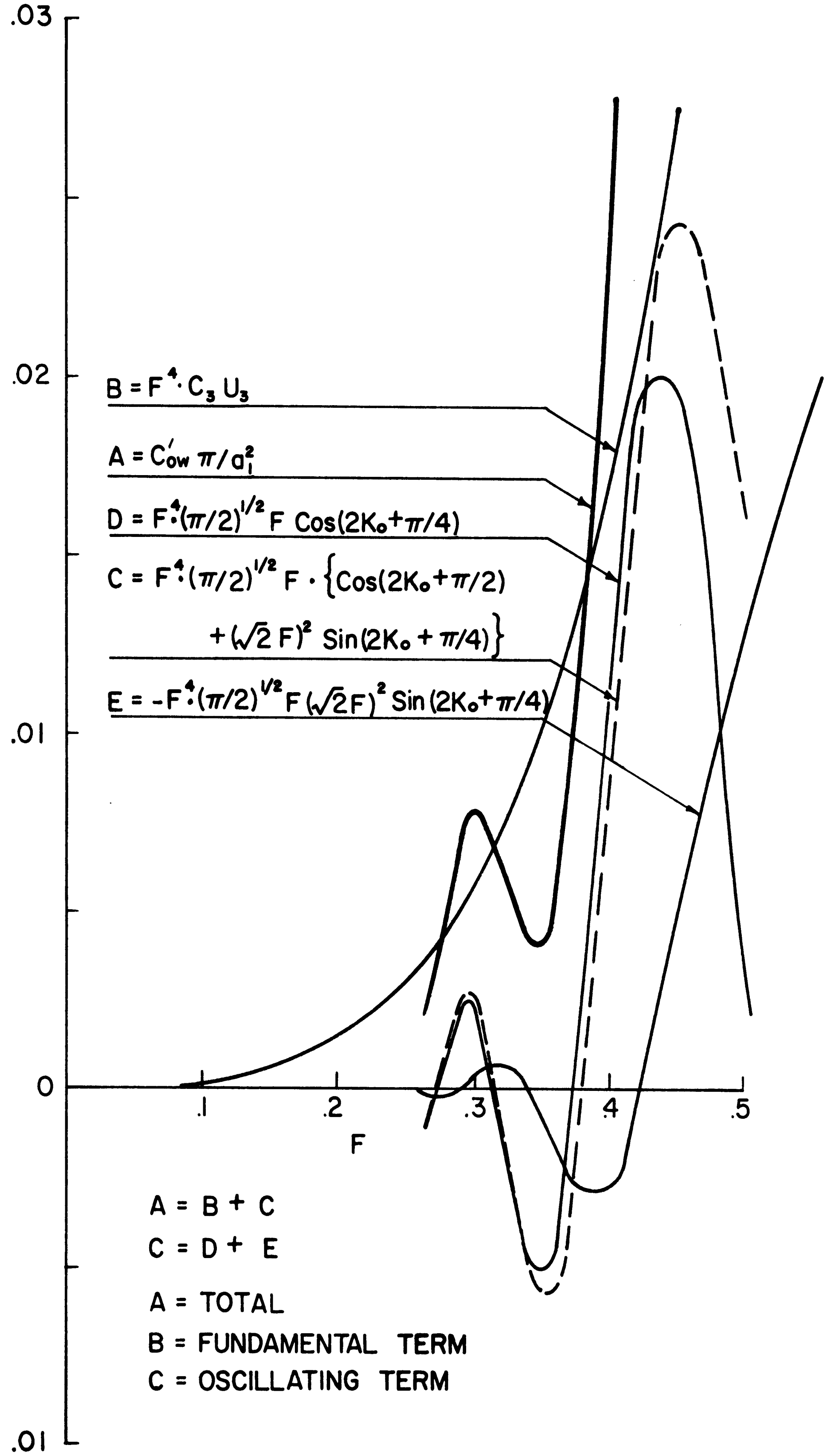
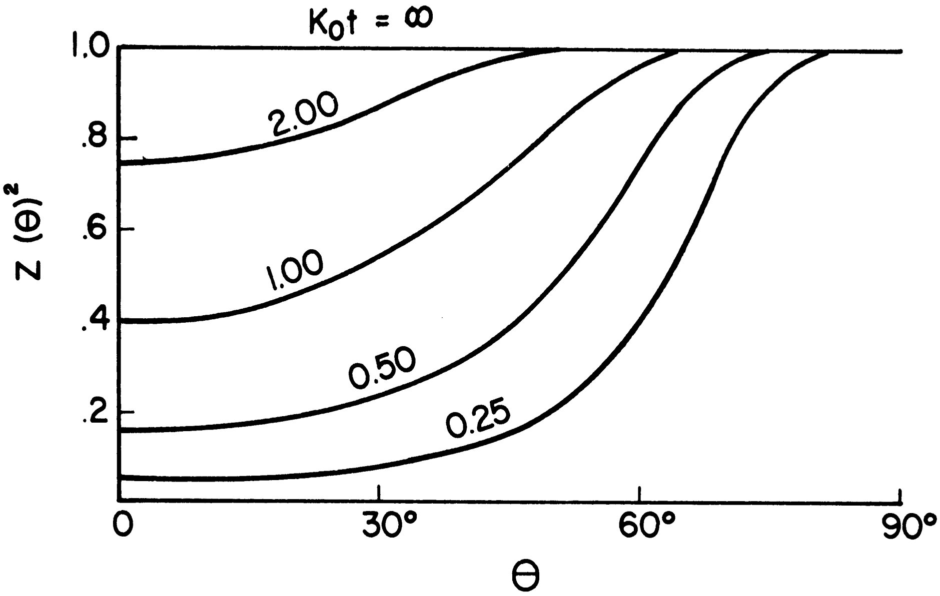
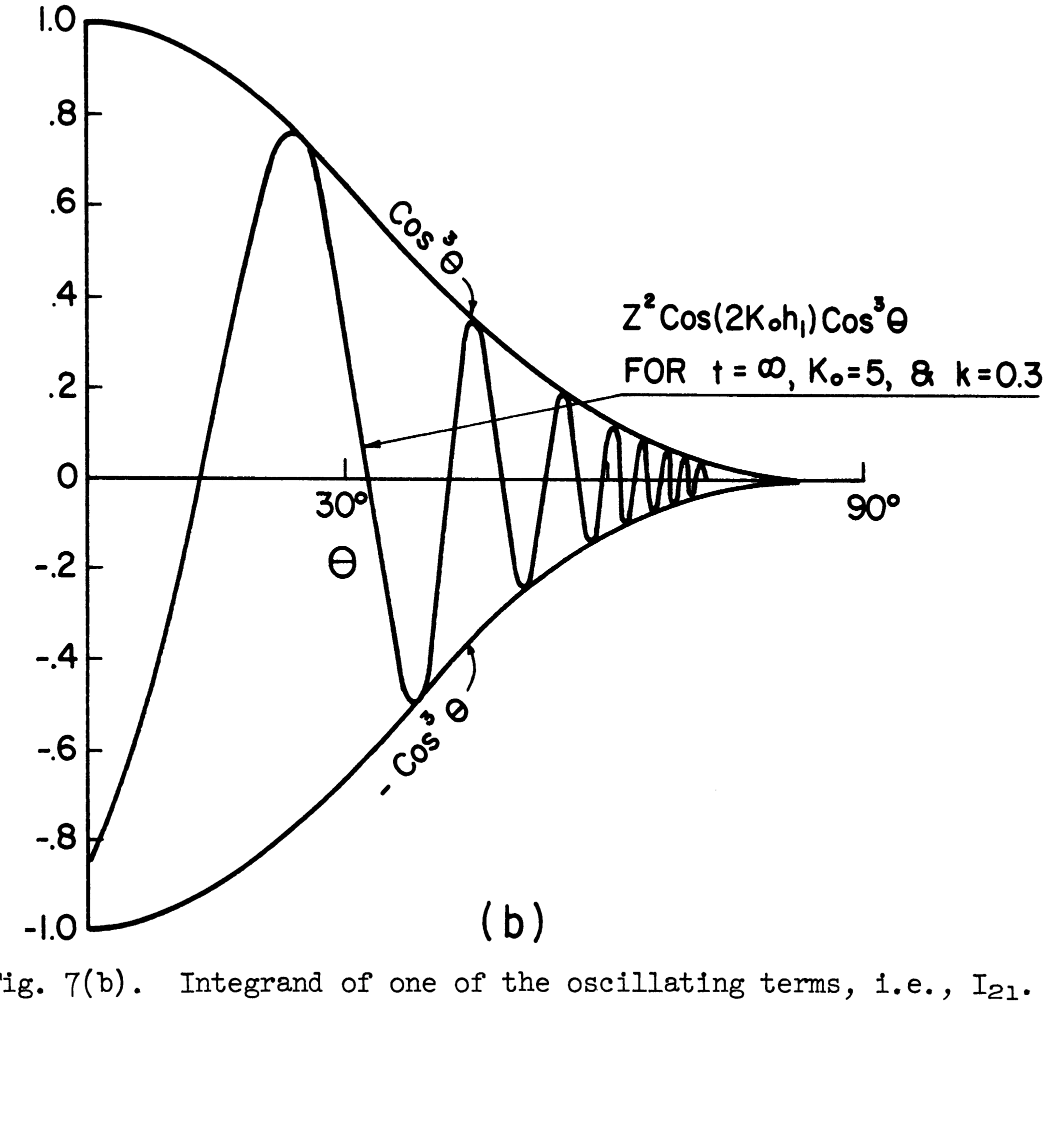


Fig. 6. Fundamental term, oscillating term, and total wave-making resistance coefficient of each hull.



(a)

Fig. 7(a). Factor representing the draft effect: $Z^2(\theta) = (1-e^{-K_0 t \sec \theta})^2$.



(b)

Fig. 7(b). Integrand of one of the oscillating terms, i.e., I_{21}.

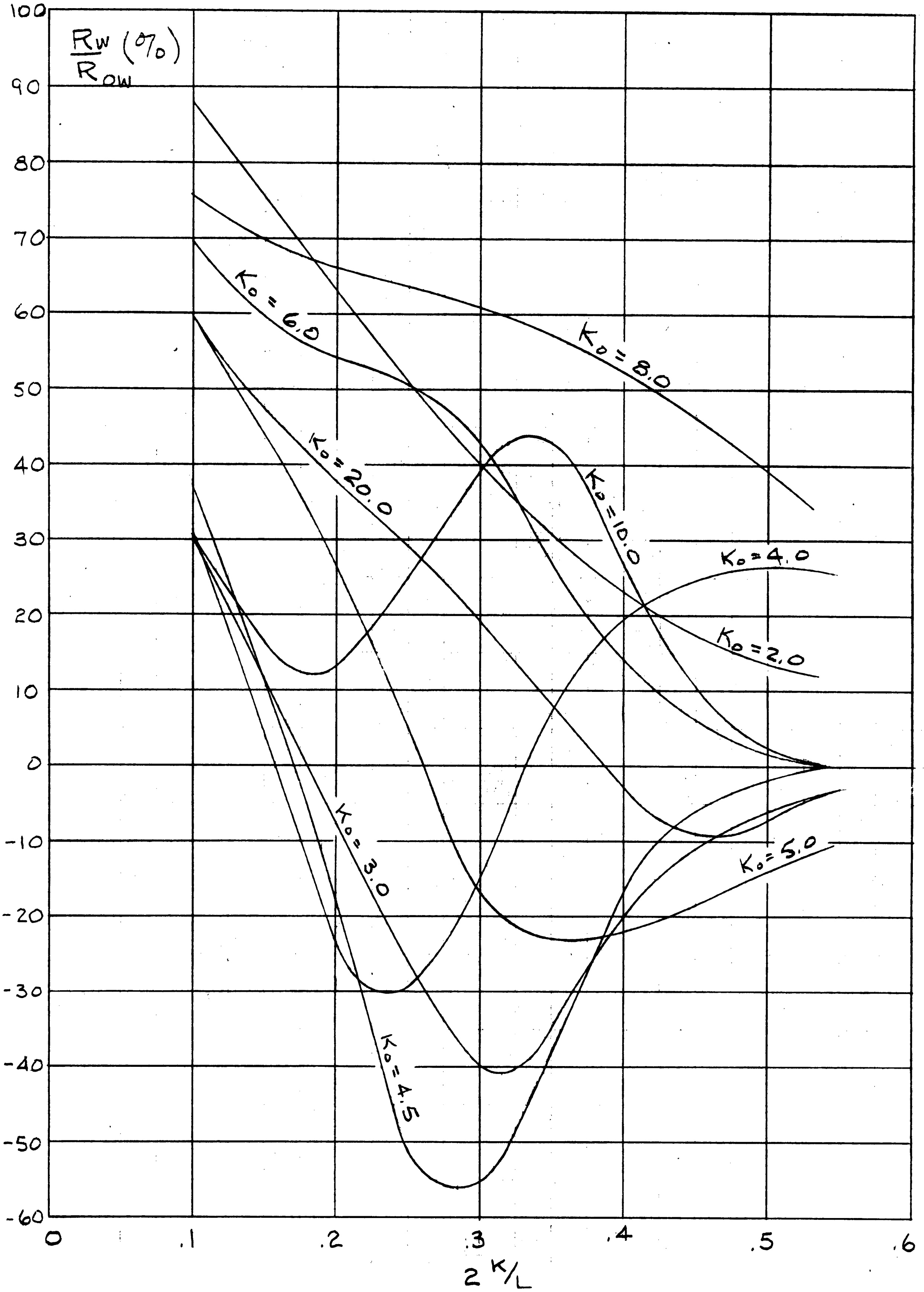


Fig. 8. Interference term plotted against the distance parameter 2k/L. (Reproduced from Fig. 4, Ref. 6. 2k/L and K_0 in the figure corresponds to k and $2K_0$ in the present paper, respectively.)

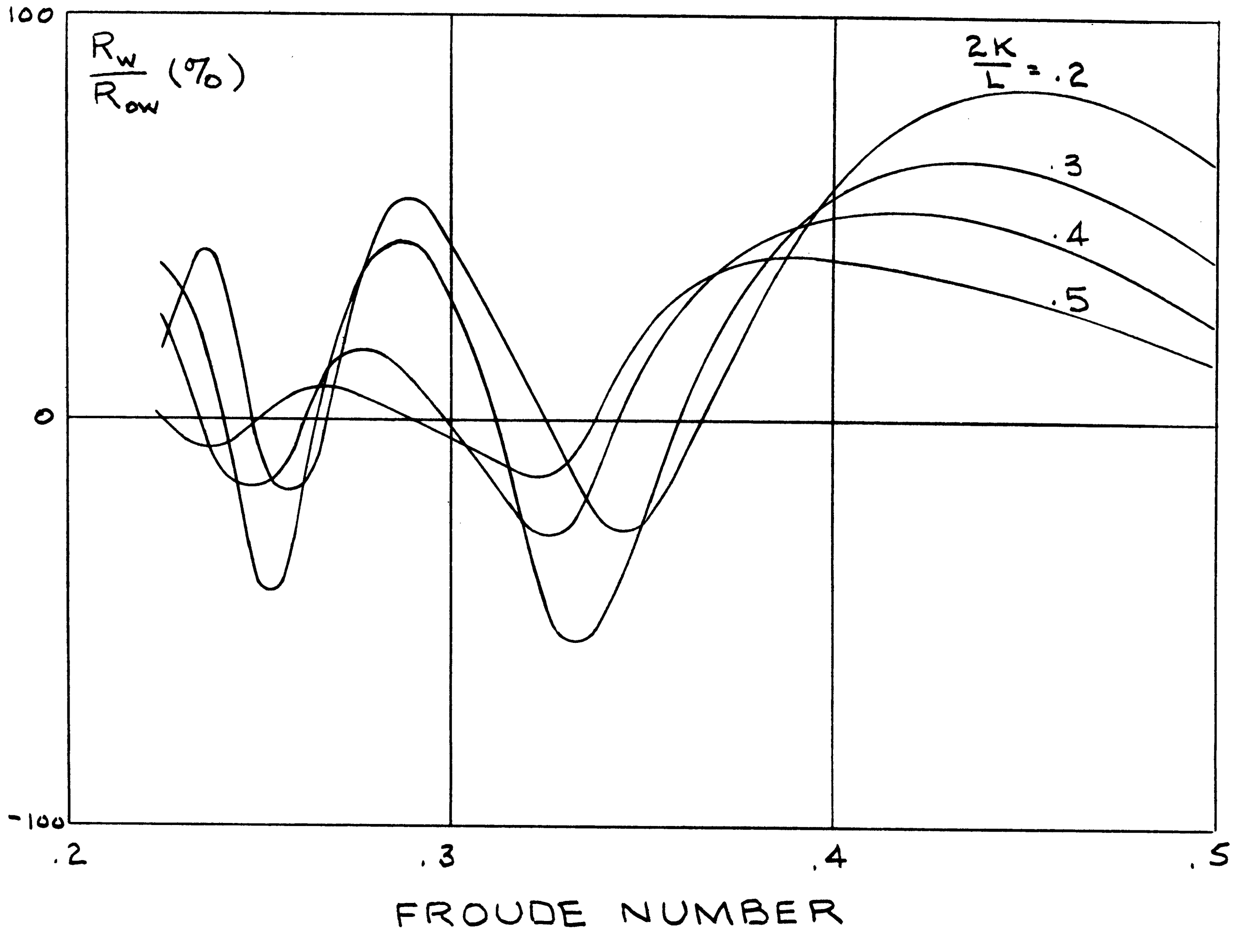


Fig. 9. Interference term plotted against Froude number for T/L = 0. (Reproduced from Fig. 5, Ref. 6. 2k/L and K_o in the figure corresponds to k and $2K_o$ in the present paper, respectively.)

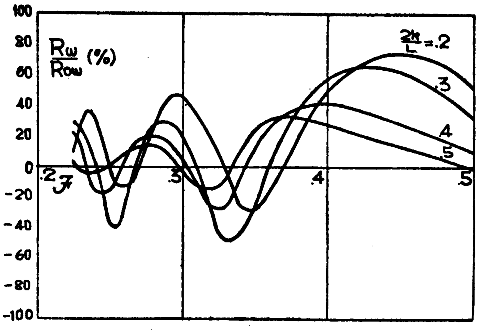


Fig. 10. Interference term against Froude number for T/L = 0. (Reproduced from Fig. 3, Ref. 7. 2k/L and K_0 in the figure corresponds to k and $2K_0$ in the present paper, respectively.)