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COLLEGE OF ENGINEERING
Department of Naval Architecture and Marine Engineering

Technical Report

ON MOTIONS, FORCES, AND MOMENTS ON A CATAMARAN IN REGULAR SEAS

Ryo Tasaki

Project Director: R. B. Couch

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NOMENCLATURE

A_{we}	waterplane area
b	maximum half-beam of each hull
(c+L)	subscript designating centerbody and both hulls
$(c+L)^{1,2}$	subscript designating port and starboard halves of the centerbody and hull respectively
d_m	mean draft of underwater form; $d_m = \nabla/A_{we}$
g	gravitational constant
h	elevation or depression of free surface
h_o	wave amplitude
$h_e[x,(-1)^{p-1}k,t]$	effective wave height or subwave height
$H_{c,s}$	wave factors
I_x	moment of inertia about x-axis
I_y	moment of inertia about y-axis
2k	distance between centerlines of two hulls
L	length of the catamaran
(L)	subscript designating both hulls
$(L)^{1,2}$	subscript designating port and starboard hulls respectively
m	sectional virtual mass at section x in the direction of z-axis
N	sectional damping coefficient
p	superscript designates either port or starboard
1,2	superscripts designate port and starboard respectively

T	draft of each hull
V	advance speed of catamaran
W	weight of catamaran
$w_o(x,y)$	weight distribution of catamaran
$w_{1,2}(x)$	weight distribution of port hull and starboard hull respectively
(X,Y,Z)	coordinate system fixed on the surface of the sea. X-direction coincides with the direction of advance of the catamaran
(x,y,z)	coordinate system fixed at the center of the mass which is assumed to be in the waterplane of the ship at rest
$y(x)$	half-beam of each hull at x
$Z(x)^{1,2}$	vertical displacement of the section x for port hull and starboard hull respectively. Dots indicate the derivatives with respect to time
Z_G	displacement in heave
$dF_z^{1,2}/dx$	component of force in the direction of z-axis per unit length at the section x, port hull and starboard hull respectively
dM_x/dx	moment per unit length of the body at the section x about x-axis
dM_y/dx	moment per unit length of the body at the section x about y-axis
θ	angular displacement in roll
ψ	angular displacement in pitch
κ	wave number
λ	wave length
μ	angular frequency of wave

$\mathcal{M}_x^{+,-}$ starboard and port moment respectively

$\mathcal{M}_x$ total bending moment at the centerline

ρ density of water

$\tau_y^{+,-}$ twisting moments acting at the port and starboard portions of the longitudinal centerplane respectively

τ_y total twisting moment at the longitudinal centerplane

ω angular encounter frequency = $\mu - \kappa V \cos \chi_e$

χ_e direction of wave propagation with respect to x-direction

I. INTRODUCTION

In this paper motions of a catamaran in regular waves are calculated under certain simplifying assumptions. It is felt that the results will justify certain general conclusions about the motions and strength of a catamaran in waves.

1. ASSUMPTIONS

(a) It is assumed that the beam of each hull of the catamaran is so small in comparison with the wavelength that the transverse restoring force caused by the inclination of water surface at the transverse section of each hull may be neglected.

(b) When the hydrostatic force and moments which are out of balance with the weight of the body are calculated, yielding the exciting force and moment, they are done so in accordance with the Froude-Krylov hypotheses. That is, the wave is assumed not to be interfered with by the body. But the Smith effect is taken into consideration in that the hydrostatic pressure is corrected for the pressure change due to the inertia forces of the rotating water particles.

(c) It is assumed that surging, yawing and swaying motions are small enough that the gyroscopic coupling terms in the equations of motions may be ignored.

(d) The relative motion between each hull and the water surface generates waves propagating from one hull to the other hull. In other words, there is the interference effect between two wave systems created by the relative motion between each hull and the water surface. In this paper, these effects are neglected for simplicity.

(e) The lateral forces caused by waves are not taken into consideration in this paper because motions caused by the lateral forces—yawing and swaying—are neglected. These forces could be calculated by the same method as will be utilized, and should be, because they could have serious effects on the bending moment on the central body. If this were done, the bending moment caused by these forces could be superposed on the results of the present calculations.

2. COORDINATE SYSTEMS

Before entering into the calculation, the coordinate systems to be used will be defined. There are two systems: both are right handed, and so directed the z-axes are vertical and the x-axes are in the direction of the ship's heading. In this case, where the swaying and yawing motions are neglected, the direction of the ship's heading coincides with the ship's course.

(a) Fixed Coordinate System Oriented with Respect to the Moving Ship.— The origin of this system is at a fixed point on the surface of the sea. These coordinates are described by (X,Y,Z).

(b) Moving Coordinate System Oriented with Respect to the Vessel.—The origin of this system is at the ship's center of mass, which is assumed to be in the waterplane of the ship at rest. These coordinates are described by (x,y,z).

3. SHIP DESCRIPTION

The following symbols are used:

L	length of the catamaran
2k	distance between centerlines of two hulls
y(x)	half-beam of each hull at x
V	speed of catamaran
b	maximum half-beam of each hull
T	draft of each hull

Variables and constants concerning the port and starboard hulls are designated by the superscripts 1 and 2, respectively.

4. WAVE DESCRIPTION

When referred to a fixed system of coordinates (X,Y,Z) and the time variable, t, the free surface of the waves is described by the expression:

$$h(X,Y,t) = h_o \cos\{\kappa(X \cos \chi + Y \sin \chi) - \mu t\}, \qquad (1\text{-}1)$$

where h(X,Y,t) is the height of the sea surface above or below the X-Y plane, h_o is the wave amplitude, μ is the angular frequency of the wave, χ is the direction of wave propagation, $\kappa = 2\pi/\lambda$, and λ is the wavelength.

The expression of the wave for the moving coordinate system fixed to the ship is

$$h(x,y,t) = h_o \cos\{\kappa(x \cos \chi_e + y \sin \chi_e) - \omega t\}, \qquad (1\text{-}2)$$

where

$$\omega = \mu - \kappa V \cos \chi_e, \tag{1-3}$$

We should notice the subwave or effective wave, the mean depth of which is d_m, and designate it by $h_e(X,Y,t)$, where

$$h_e(X,Y,t) = h_o e^{-\kappa d_m} \cos\{\kappa(X \cos \chi + Y \sin \chi) - \mu t\}, \tag{1-4}$$

or

$$h_e(x,y,t) = h_o e^{-\kappa d_m} \cos\{\kappa(x \cos \chi_e + y \sin \chi_e) - \omega t\} . \tag{1-5}$$

If the ship were flat-bottomed, the force and moment would depend upon the effect of the subsurface at the depth of the keel. For a normal ship form, however, the effective subsurface will be somewhat nearer the free surface. T. H. Havelock[3] has suggested that the effective subsurface is that at the mean draft, d_m, of the underwater form, defined by ∇/A_{we}, the volume of displacement divided by the waterplane area. This assumption is adopted in the present calculations.

II. SHIP MOTION

1. EQUATIONS OF MOTION

Following the usual methods for the mechanics of rigid bodies acted upon by exciting forces, the motions are described by the differential equations.

$$\frac{W}{g}\ddot{Z}_G = \int_L \frac{dF_z}{dx}\,dx = \int_L dF_z$$

$$\frac{I_x}{g}\ddot{\theta} = \int_L dM_x, \quad \text{and}$$

$$\frac{I_y}{g}\ddot{\psi} = \int_L dM_y. \tag{2-1}$$

In these equations, Z_G, θ, and ψ denote displacements in heave, roll, and pitch, respectively. W is the weight of catamaran, and I_x and I_y are the moments of inertia about the x-axis and the y-axis, respectively. dF_z/dx is the component of force in the direction of the z-axis per unit length at the section x. dM_x/dx and dM_y/dx are the moments per unit length of the body at the section x about the x-axis and the y-axis, respectively.

We may denote the force in the direction of the z-axis per unit length of the portside body by dF_z^1/dx, and that of the starboard body by dF_z^2/dx. The force and moments per unit length of either body may be described as follows:

$$\frac{dF_z}{dx} = \frac{dF_z^1}{dx} + \frac{dF_z^2}{dx},$$

$$\frac{dM_x}{dx} = k\left(\frac{dF_z^1}{dx} - \frac{dF_z^2}{dx}\right), \quad \text{and}$$

$$\frac{dM_y}{dx} = -x\left(\frac{dF_z^1}{dx} + \frac{dF_z^2}{dx}\right) = -x\,\frac{dF_z}{dx}. \tag{2-2}$$

dF_z^p/dx, where p = 1 and 2, is in turn divided into three parts,

$$\frac{dF_z^p}{dx} = \frac{dF_{z1}^p}{dx} + \frac{dF_{z2}^p}{dx} + \frac{dF_{z3}^p}{dx}. \tag{2-3}$$

In this expression,

$$\frac{dF^p_{z1}}{dx} = -2\rho g b\{Z^p(x) - h_e(x,(-1)^{p+1}k,t)\} ,$$

$$\frac{dF^p_{z2}}{dx} = -N\{\dot{Z}^p(x) - \dot{h}_e(x,(-1)^{p+1}k,t\}, \quad \text{and} \tag{2-4}$$

$$\frac{dF^p_{z3}}{dx} = -\frac{d}{dt}\left[m\{\dot{Z}^p(x) - \dot{h}_e(x,(-1)^{p+1}k,t\}\right] ,$$

where ρ is the density of water, $Z^p(x)$ is the vertical displacement of section x due to the ship motions, N is the sectional damping coefficient, m is the sectional virtual mass at section x in the direction of the z-axis, and h_e is the subsurface wave profile or effective wave.

The first expression of (2-3) is the force due to the hydrostatic pressure in the waves under the Froude-Krylov assumption. The effective wave height includes the Smith effect. The second expression is the force due to the damping effect, and the third one is the force due to the hydrodynamic effect.

$Z^p(x)$ can be represented by the following equations:

$$\begin{aligned} Z^1(x) &= Z_G + k\Theta - x\psi , \\ Z^2(x) &= Z_G - k\Theta - x\psi . \end{aligned} \tag{2-5}$$

In order to evaluate the right-hand side of the Eq. (2-3), we must calculate the difference between vertical displacements of the two hulls, their sum, and their derivatives with respect to time. These are:

$$\begin{aligned} Z^1(x) + Z^2(x) &= 2(Z_G - x\psi), \\ \dot{Z}^1(x) + \dot{Z}^2(x) &= 2(\dot{Z}_G - x\dot{\psi} + V\psi), \\ \ddot{Z}^1(x) + \ddot{Z}^2(x) &= 2(\ddot{Z}_G - x\ddot{\psi} + 2V\dot{\psi}), \end{aligned} \tag{2-6}$$

and

$$\begin{aligned} Z^1(x) - Z^2(x) &= 2k\Theta, \\ \dot{Z}^1(x) - \dot{Z}^2(x) &= 2k\dot{\Theta}, \\ \ddot{Z}^1(x) - \ddot{Z}^2(x) &= 2k\ddot{\Theta}. \end{aligned} \tag{2-7}$$

The effective waves, $h_e(x,(-1)^{p+1}k,t)$ and their derivatives with respect to time, are related as follows:

$$\begin{aligned} h_e(x,k,t) + h_e(x,-k,t) &= 2H_c\cos(\kappa x \cos \chi_e - \omega t), \\ \dot{h}_e(x,k,t) + \dot{h}_e(x,-k,t) &= 2\mu H_c\sin(\kappa x \cos \chi_e - \omega t), \\ \ddot{h}_e(x,k,t) + \ddot{h}_e(x,-k,t) &= -2\mu^2 H_c\cos(\kappa x \cos \chi_e - \omega t); \end{aligned} \tag{2-8}$$

$$\begin{aligned} h_e(x,k,t) - h_e(x,-k,t) &= -2H_s\sin(\kappa x \cos \chi_e - \omega t), \\ \dot{h}_e(x,k,t) - \dot{h}_e(x,-k,t) &= 2\mu H_s\cos(\kappa x \cos \chi_e - \omega t), \\ \ddot{h}_e(x,k,t) - \ddot{h}_e(x,-k,t) &= 2\mu^2 H_s\sin(\kappa x \cos \chi_e - \omega t); \end{aligned} \tag{2-9}$$

where

$$\begin{aligned} H_c &= h_{eo}\cos(\kappa k \cos \chi_e), \\ H_s &= h_{eo}\sin(\kappa k \sin \chi_e), \quad \text{and} \\ h_{eo} &= h_e\, e^{-\kappa d_m} \end{aligned} \tag{2-10}$$

H_s and H_c are called wave factors.

From these relations, and Eqs. (2-2) and (2-3), the forces and moments per unit length of the body may be obtained as follows:

$$\begin{aligned} \frac{dF_{z1}}{dx} &= -4\rho g b\{(Z_G - x\psi) - H_c\cos(\kappa x \cos \chi_e - \omega t)\}, \\ \frac{dF_{z2}}{dx} &= -2N\{(\dot{Z}_G - x\dot{\psi} + V\psi) - \mu H_c\sin(\kappa x \cos \chi_e - \omega t)\}, \\ \frac{dF_{z3}}{dx} &= -2\{m(\ddot{Z}_G - x\ddot{\psi} + 2V\dot{\psi}) - Vm'(\dot{Z}_G - x\dot{\psi} + V\psi) + \mu^2 m H_c\cos(\kappa x \cos \chi_e \\ &\quad -\omega t) + \mu V m' H_c\sin(\kappa x \cos \chi_e - \omega t)\}. \end{aligned} \tag{2-11}$$

$$\frac{dM_{x1}}{dx} = -4\rho gkb\{k\theta + H_s \sin(\kappa x \cos \chi_e - \omega t)\},$$

$$\frac{dM_{x2}}{dx} = -2kN\{k\dot{\theta} - \mu H_s \cos(\kappa x \cos \chi_e - \omega t)\}, \text{ and} \tag{2-12}$$

$$\frac{dM_{x3}}{dx} = -2k\{km\ddot{\theta} - kVm'\dot{\theta} - \mu^2 m H_s \sin(\kappa x \cos \chi_e - \omega t) + \mu V m' H_s \cos(\kappa x \cos \chi_e - \omega t)\};$$

$$\frac{dM_y}{dx} = -x\,\frac{dF_z}{dx} \tag{2-2}$$

Inserting the values dF_z/dx, dM_x/dx, and dM_y/dx, obtained from the above equations, into Eq. (2-1), and transferring the terms not directly affected by the wave motions to the left-hand side, the following set of three equations is obtained:

$$a_{31}\ddot{Z}_G + a_{32}\dot{Z}_G + a_{33}Z_G + a_{34}\ddot{\psi} + a_{35}\dot{\psi} + a_{36}\psi = F_z,$$

$$A_{21}\ddot{\psi} + A_{22}\dot{\psi} + A_{23}\psi + A_{24}\ddot{\psi} + A_{25}\dot{\psi} + A_{26}\psi = M_y, \text{ and} \tag{2-13}$$

$$A_{11}\ddot{\theta} + A_{12}\dot{\theta} + A_{13}\theta = M_x .$$

2. COEFFICIENTS OF EQUATIONS

The coefficients of these equations are represented by the following equations:

$$\begin{aligned}
a_{31} &= \frac{w}{g} + 2\int_L m\,dx \\
a_{32} &= 2\int_L N\,dx \\
a_{33} &= 4\rho g\int_L b\,dx \\
a_{34} &= -2\int_L mx\,dx \\
a_{35} &= -2\int_L Nx\,dx + 2V\int_L m\,dx \\
a_{36} &= -4\rho g\int_L bx\,dx + 2V\int_L N\,dx = -4\rho g\int_L bx\,dx + Va_{32} ;
\end{aligned} \tag{2-14}$$

$$A_{21} = \frac{I_y}{g} + 2\int_L mx^2 dx$$

$$A_{22} = 2\int_L Nx^2 dx$$

$$A_{23} = 4\rho g\int_L bx^2 dx + VA_{25}$$

$$A_{24} = -2\int_L mxdx = a_{34} \qquad (2\text{-}15)$$

$$A_{25} = -2\int_L Nxdx - 2V\int_L mdx$$

$$A_{26} = -4\rho g\int_L bxdx;$$

$$A_{11} = \frac{I_x}{g} + 2k^2\int_L mdx$$

$$A_{12} = 2k^2\int_L Ndx = k^2 a_{32} \qquad (2\text{-}16)$$

$$A_{13} = 4\rho g k^2\int_L bxdx .$$

3. EXCITING FORCES AND MOMENTS

F_z, M_y, and M_x in Eq. (2-13) are the force and moments caused by the waves and may be represented by the following expressions:

(a) Heaving Force.—

$$F_z = F_z^c \cos \omega t + F_z^s \sin \omega t, \qquad (2\text{-}17)$$

and

$$\begin{pmatrix} F_z^c \\ F_z^s \end{pmatrix} = \begin{pmatrix} f_1+f_2+f_3 \\ f_1'+f_2'+f_3' \end{pmatrix} = F_{zo}\begin{pmatrix} \cos \alpha_{F_z} \\ \sin \alpha_{F_z} \end{pmatrix} \qquad (2\text{-}18)$$

where

$$F_{zo}^2 = {F_z^c}^2 + {F_z^s}^2, \text{ and}$$

$$\tan \alpha_{F_z} = \frac{F_z^c}{F_z^s} \qquad (2\text{-}19)$$

$$\begin{pmatrix} f_1 \\ f'_1 \end{pmatrix} = 4\rho g H_c \int_L b \begin{bmatrix} \cos(\kappa x \cos \chi_e) \\ \sin(\kappa x \cos \chi_e) \end{bmatrix} dx$$

$$\begin{pmatrix} f_2 \\ f'_2 \end{pmatrix} = \pm 2\mu H_c \int_L N \begin{bmatrix} \sin(\kappa x \cos \chi_e) \\ \cos(\kappa x \cos \chi_e) \end{bmatrix} dx, \text{ and} \tag{2-20}$$

$$\begin{pmatrix} f_3 \\ f'_3 \end{pmatrix} = -2\mu\omega H_c \int_L m \begin{bmatrix} \cos(\kappa x \cos \chi_e) \\ \sin(\kappa x \cos \chi_e) \end{bmatrix} dx$$

(b) Pitching Moment.—

$$M_y = M_y^c \cos \omega t + M_y^s \sin \omega t, \tag{2-12}$$

and

$$\begin{pmatrix} M_y^c \\ M_y^s \end{pmatrix} = \begin{pmatrix} m_{y1} + m_{y2} + m_{y3} \\ m_{y'1} + m_{y'2} + m_{y'3} \end{pmatrix} = M_{y_0} \begin{pmatrix} \cos \alpha_{M_y} \\ \sin \alpha_{M_y} \end{pmatrix}, \tag{2-22}$$

where

$$M_{y_0}^2 = M_y^{c^2} + M_y^{s^2}, \text{ and} \qquad \tan \alpha_{M_y} = \frac{M_y^s}{M_y^c} ; \tag{2-23}$$

$$\begin{pmatrix} M_{y1} \\ M'_{y1} \end{pmatrix} = -4\rho g H_c \int bx \begin{bmatrix} \cos(\kappa x \cos \chi_e) \\ \sin(\kappa x \cos \chi_e) \end{bmatrix} dx$$

$$\begin{pmatrix} M_{y2} \\ M'_{y2} \end{pmatrix} = \mp 2\mu H_c \int Nx \begin{bmatrix} \sin(\kappa x \cos \chi_e) \\ \cos(\kappa x \cos \chi_e) \end{bmatrix} dx, \text{ and} \tag{2-24}$$

$$\begin{pmatrix} M_{y3} \\ M'_{y3} \end{pmatrix} = \begin{matrix} 2\mu\omega H_c \int mx \begin{bmatrix} \cos(\kappa x \cos \chi_e) \\ \sin(\kappa x \cos \chi_e) \end{bmatrix} dx \\ \mp 2\mu H_c V \int m \begin{bmatrix} \sin(\kappa x \cos \chi_e) \\ \cos(\kappa x \cos \chi_e) \end{bmatrix} dx \end{matrix}$$

(c) Rolling Moment.—

$$M_x = M_x^c \cos \omega t + M_x^s \sin \omega t \tag{2-25}$$

$$\begin{pmatrix} M_x^c \\ M_x^s \end{pmatrix} = \begin{pmatrix} m_{x1} + m_{x2} + m_{x3} \\ m'_{x1} + m'_{x2} + m'_{x3} \end{pmatrix} = M_{x_o} \begin{pmatrix} \cos \alpha_{M_x} \\ \sin \alpha_{M_x} \end{pmatrix} \tag{2-26}$$

$$M_{x_o}^2 = M_x^{c^2} + M_x^{s^2}, \text{ and}$$
$$\tan \alpha_{M_x} = \frac{M_x^s}{M_x^c} ; \tag{2-27}$$

$$\begin{pmatrix} m_{x1} \\ m'_{x1} \end{pmatrix} = \mp 4\rho g k H_s \int b \begin{bmatrix} \sin(\kappa x \cos \chi_e) \\ \cos(\kappa x \cos \chi_e) \end{bmatrix} dx$$

$$\begin{pmatrix} m_{x2} \\ m'_{x2} \end{pmatrix} = 2\mu k H_s \int N \begin{bmatrix} \cos(\kappa x \cos \chi_e) \\ \sin(\kappa x \cos \chi_e) \end{bmatrix} dx, \text{ and} \tag{2-28}$$

$$\begin{pmatrix} m_{x3} \\ m'_{x3} \end{pmatrix} = \pm 2\mu\omega k H_s \int m \begin{bmatrix} \sin(\kappa x \cos \chi_e) \\ \cos(\kappa x \cos \chi_e) \end{bmatrix} dx$$

4. SOLUTION OF EQUATIONS

The exciting force and moments in Eq. (2-13) may be expressed in complex form to facilitate the algebraic work of the solution. If it is then assumed that the ship is moving in a uniform sea, and that steady state motion is established, the transient responses will have been damped out and only a particular solution of Eq. (2-13) is required. This is the most useful solution as a response function of the ship in waves.[5]

When we utilize the following expressions,

$$\overline{F}_z = F_z^c - iF_z^s,$$

$$\overline{M}_y = M_y^c - iM_y^s, \text{ and} \tag{2-29}$$

$$\overline{M}_x = M_x^c - iM_x^s .$$

The exciting force and moments F_z, M_y and M_z are the real parts of the complex expressions, or

$$F_z = \mathbf{R}(\overline{F}_z \, e^{i\omega t})$$

$$M_y = \mathbf{R}(\overline{M}_y \, e^{i\omega t}), \text{ and} \tag{2-30}$$

$$M_x = \mathbf{R}(\overline{M}_x \, e^{i\omega t})$$

where $\mathbf{R}(\ )$ indicates that only the real part is to be taken.

Therefore, the desired solution Eq. (2-13) is the real parts of the following equations:

$$a_{31}\ddot{Z} + a_{32}\dot{Z} + a_{33}Z + a_{34}\ddot{\psi} + a_{35}\dot{\psi} + a_{36}\psi = \overline{F}_z \, e^{i\omega t}$$

$$A_{24}\ddot{Z} + A_{25}\dot{Z} + A_{26}Z + A_{21}\ddot{\psi} + A_{22}\dot{\psi} + A_{23}\psi = \overline{M}_y \, e^{i\omega t} \tag{2-31}$$

$$A_{11}\ddot{\theta} + A_{12}\dot{\theta} + A_{12}\theta = \overline{M}_x \, e^{i\omega t}$$

Since the exciting force and moments are harmonic, the solution likewise can be assumed to be harmonic or of the form

$$Z_G = \overline{Z}_G \, e^{i\omega t}$$

$$\psi = \overline{\psi} \, e^{i\omega t} \tag{2-32}$$

$$\theta = \overline{\theta} \, e^{i\omega t}$$

where $\overline{Z}_G$, $\overline{\psi}$, and $\overline{\theta}$ have the complex forms,

$$\bar{Z}_G = Z_G^c - iZ_G^s$$
$$\bar{\psi} = \psi^c - i\psi^s \qquad (2\text{-}33)$$
$$\bar{\theta} = \theta^c - i\theta^s$$

The real part of the solution of Eq. (2-31), i.e., the solution of Eq. (2-13), is

$$Z_G = Z_G^c \cos \omega t + Z_G^s \sin \omega t$$
$$\psi = \psi^c \cos \omega t + \psi^s \sin \omega t \qquad (2\text{-}34)$$
$$\theta = \theta^c \cos \omega t + \theta^s \sin \omega t$$

Substitution of the assumed solution into Eq. (2-31) leads to the following algebraic equation:

$$(-\omega^2 a_{31} + i\omega a_{32} + a_{33})\bar{Z}_G + (-\omega^2 a_{34} + i\omega a_{35} + a_{36})\bar{\psi} = \bar{F}_z$$
$$(-\omega^2 A_{24} + i\omega A_{25} + A_{26})\bar{Z}_G + (-\omega^2 A_{21} + i\omega A_{22} + A_{23})\bar{\psi} = \bar{M}_y \qquad (2\text{-}35)$$
$$(-\omega^2 A_{11} + i\omega A_{12} + A_{13})\bar{\theta} = \bar{M}_x$$

The solution in the complex form is

$$\bar{Z}_G = \frac{P+iQ}{R+iS}$$
$$\bar{\psi} = \frac{P'+iQ'}{R+iS}, \text{ and} \qquad (2\text{-}36$$
$$\bar{\theta} = \frac{M_x^c - iM_x^s}{-(\omega^2 A_{11} - A_{13}) + i\omega A_{12}}$$

Where

$$R = (\omega^2 a_{31}-a_{33})(\omega^2 a_{21}-a_{23}) - (\omega^2 a_{34}-a_{36})(\omega^2 A_{24}-A_{26})$$
$$- \omega^2 a_{32}A_{22} - \omega^2 a_{35}A_{25}$$

$$S = -\omega a_{32}(\omega^2 A_{21}-A_{23}) - \omega A_{22}(\omega^2 a_{31}-a_{33})$$
$$+ \omega a_{35}(\omega^2 A_{24}-A_{26}) + \omega^2 A_{25}(\omega^2 a_{34}-a_{36})$$

$$P = -F_z^c(\omega^2 A_{21}-A_{23}) + M_y^c(\omega^2 a_{34}-a_{36})$$
$$+ \omega F_z^s A_{22} - \omega M_y^c a_{35}$$

$$Q = F_z^s(\omega^2 A_{21}-A_{23}) - M_y^s(\omega^2 a_{34}-a_{36})$$
$$+ \omega F_z^c A_{22} - \omega M_y^s a_{35}$$

$$P' = -M_y^c(\omega^2 a_{31}-a_{33}) + F_z^c(\omega^2 A_{24}-A_{26})$$
$$+ \omega M_y^s a_{32} - \omega F_z^s a_{35}$$

$$Q' = M_y^s(\omega^2 a_{31}-a_{33}) - F_z^s(\omega^2 A_{24}-A_{26})$$
$$+ \omega M_y^c a_{32} - \omega F_z^c a_{35} \ . \qquad (2\text{-}37)$$

The real part of the solution, Eq. (2-36), is the solution of Eq. (2-13) as mentioned above, that is, Z_G^c, Z_G^s, ψ^c, ψ^s, θ^c, and θ^s in Eq. (2-34) are represented by the following:

$$Z_G^c = \frac{PR+QS}{R^2+S^2}$$

$$Z_G^s = \frac{PS-QR}{R^2+S^2}$$

$$\psi^c = \frac{P'R+Q'S}{R^2+S^2}$$

$$\psi^s = \frac{P'S-Q'R}{R^2+S^2}$$

$$\theta^c = \frac{-M_x^c(\omega^2 A_{11}-A_{13})-\omega M_x^s A_{12}}{(\omega^2 A_{11}-A_{13})^2+\omega^2 A_{12}}$$

$$\theta^s = \frac{-M_x^s(\omega^2 A_{11}-A_{13})+\omega M_x^c A_{12}}{(\omega^2 A_{11}-A_{13})^2+\omega^2 A_{12}} \qquad (2\text{-}38)$$

III. HYDRODYNAMIC FORCE AND MOMENT ACTING ON CENTERBODY

The structure of a catamaran is more complicated than that of a conventional ship. The centerbody and both hulls constitute a statically indeterminate system. Therefore, in order to determine the strength of each member, it is necessary to know the distribution of forces and moments in relation to this overall configuration. This distribution of the hydrodynamic forces and moments along each hull may be calculated from the equations developed above, but the full calculation is too lengthy for estimating the strength of the centerbody at the stage of the initial design of a catamaran. In order to establish a kind of standard to estimate the strength of the centerbody of a catamaran, the bending and twisting moments may be calculated under the following assumptions.

(a) The longitudinal strength of each hull is great enough in comparison with that of the centerbody that the deflections along the connections between centerbody and each hull are neglected. In other words, each hull is regarded as a rigid boundary connected to the centerbody.

(b) If the weight distribution in the centerbody is given by $w_o(x,y)$ and the weight distribution of each hull by $w_1(x)$ and $w_2(x)$, it is assumed that $w_1(x)$ and $w_2(x)$ are concentrated along the longitudinal centerline of each hull and that $w_1(x) = w_2(x)$. The inertia force, $-\frac{w_p(x)}{g}\ddot{Z}(x)$; $p = 1,2$, is thus working at section x of each hull, and the inertia force of the centerbody at a point (x,y) is $\frac{w_o(x,y)}{g}\ddot{Z}(x,y)$. It should be recalled that Z(x,y) is the vertical displacement at (x,y) given by

$$Z = Z_G + y\Theta - x\psi, \tag{3-1}$$

and the second derivative with respect to time is therefore

$$\ddot{Z} = \ddot{Z}_G + y\ddot{\Theta} - x\ddot{\psi} \tag{3-2}$$

where x is independent of time.

1. BENDING MOMENT IN THE DIRECTION OF x-AXIS

The bending moment about the x-axis along the centerline longitudinal axis of the centerbody may be calculated as follows.

If $\mathfrak{M}_x^+$ and $\mathfrak{M}_x^-$ are used to denote the starboard and port moments, respectively, they may be expressed as

$$\mathfrak{M}_x^+ = -\iint_{(c+L)^1} y \frac{w(x,y)}{g} \ddot{Z}(x,y)\,dxdy + k\int_{(L)^1} dF_Z^1(x)$$

and

$$\mathfrak{M}_x^- = \iint_{(c+L)^2} y \frac{w(x,y)}{g} \ddot{Z}(x,y)\,dxdy + k\int_{(L)^2} dF_Z^2(x)$$

where $(c+L)^1$ and $(c+L)^2$ mean the port and starboard halves of the centerbody and hull, respectively. $(L)^1$ and $(L)^2$ designate the port and starboard hulls, respectively.

The total bending moment $\mathfrak{M}_x$ at the centerline is then given by

$$\begin{aligned} \mathfrak{M}_x &= \tfrac{1}{2}(\mathfrak{M}_x^+ + \mathfrak{M}_x^-) \\ &= -\iint_{(c+L)} (y) \frac{w(x,y)}{g} \ddot{Z}(x,y)\,dxdy \\ &\quad + k\int_{(L)} dF_Z(x) \end{aligned} \tag{3-3}$$

where $(c+L)$ means the centerbody and both hulls, and (L) designates both hulls.

Because of the assumption of symmetry of the weight distribution with respect to the longitudinal centerline, i.e., $w(x,y) = w(x,-y)$, and specifying

$$\frac{1}{w/2} \iint_{(c+L)^1} w(x,y)\,y\,dxdy = k_y' , \tag{3-4}$$

and

$$\frac{1}{w/2} \iint_{(c+L)^1} w(x,y)\,xy\,dxdy = k_{xy}' \tag{3-5}$$

we may write

$$\mathcal{M}_x = -\frac{k_y'W}{2g}\ddot{Z}_G - \frac{k_{xy}'W}{2g}\ddot{\psi} + \frac{k}{2}\int_{(L)} dF_z(x)$$

$$= \frac{W}{2g}\{(k-k_y')\ddot{Z}_G - k_{xy}'\ddot{\psi}\} \qquad (3\text{-}6)$$

If the bending moment at midships of the centerbody is represented in the form

$$\mathcal{M}_x = \mathcal{M}_x^c \cos \omega t + \mathcal{M}_x^s \sin \omega t \qquad (3\text{-}7)$$

then, from the solutions of the equations of motions (2-38), we may write.

$$\mathcal{M}_x^c = -\frac{W\omega^2}{g}\frac{\{(k-k_y')P - k_{xy}P'\}R + \{(k-k_y')Q - k_{xy}Q'\}S}{R^2 + S^2}$$

$$\mathcal{M}_x^s = -\frac{W\omega^2}{g}\frac{\{(k-k_y')P - k_{xy}P'\}S - \{(k-k_y')Q - k_{xy}Q'\}R}{P^2 + S^2} \qquad (3\text{-}8)$$

2. TWISTING MOMENT

The twisting moment caused by the hydrodynamic forces and the inertia forces may be calculated as follows.

If τ_y^+ and τ_y^- are used to denote twisting moments acting at the port and starboard portions of the longitudinal centerplane, respectively, they may be expressed as:

$$\tau_y^+ = -\iint_{(c+L)^1} \frac{w(x,y)}{g}\ddot{Z}(x,y)x\,dxdy + \int_{(L)^1} F_z^1(x,y)x\,dx,$$

and

$$\tau_y^- = \iint_{(c+L)^2} \frac{w(x,y)}{g}\ddot{Z}(x,y)x\,dxdy - \int_{(L)^2} F_z^2(x,y)x\,dx.$$

The total twisting moment at the longitudinal centerplane is then given by

$$\tau_y = \frac{1}{2}(\tau_x^+ - \tau_y^-)$$

$$= -\frac{1}{2}\iint_{(c+L)} \frac{w(x,y)}{g}\, x\, [\text{sign } y]\ddot{Z}(x,y)\,dxdy$$

$$- \frac{1}{2k}\int x dM_x \ . \qquad (3\text{-}9)$$

Using the expression given in Eq. (3-5), we may write

$$\tau_y = -\frac{W}{2g} k'_{xy}\ddot{\theta} - \frac{1}{2k}\int x dM_x \qquad (3\text{-}10)$$

If the twisting moment at the longitudinal centerplane is represented in the form

$$\tau_y = \tau_y^c \cos \omega t + \tau_y^s \sin \omega t \qquad (3\text{-}11)$$

then, using the expression of Eq. (2-12) for dM_x, we may write

$$\tau_y^c = \frac{Wk_{xy}}{g}(\omega^2\theta^c - M_y^s)$$

$$\tau_y^s = \frac{Wk_{xy}}{g}(\omega^2\theta^s - M_y^c) . \qquad (3\text{-}12)$$

REFERENCES

1. Jacobs, W. R., "The Analytical Calculation of Ship Bending Moments in Regular Waves," Journal of Ship Research, vol. 2, No. 1, June 1958, pp. 20-29.

2. Fukuda, J., "On the Midship Bending Moments of a Ship in Regular Waves," Journal of Society of Naval Architects, Japan, vol. 110, Dec. 1961, pp. 241-252.

3. Havelock, T. H., "Notes on the Theory of Heaving and Pitching," Transactions of Royal Institution of Naval Architects, 1945, pp. 109-122.

4. St. Denis, M. and Pierson, W. J., "On the Motions of Ships in Confused Seas," TSNAME, vol. 61, 1953, pp. 280-332.

5. Lewis, E. V., "Ship Speed in Irregular Seas," TSNAME, vol. 63, 1955, pp. 87-88.