Technical Appendix
Reliability Adjustment of Hospital Mortality Rates

We run a hierarchical logistic regression model with dependent variable $Y_{i,j}$
First level is patient characteristics (with patients i nested in hospitals j) $x_{i,j}$
Second level, subscripted j is the hospital random effect (identifier variable: hospid)

1. Estimate hierarchical logistic regression (random effects logit)

$$logit[Pr(Y_{i,j} = 1|x_i, \zeta_j)] = \beta_1 + \beta_2 x_{i,j} + \zeta_j + \varepsilon_{i,j} \tag{1}$$

   where $x_{i,j}$ is a vector of individual characteristics, $\zeta_j \sim N(0, \psi)$ is a hospital random effect (independent across hospitals), $\varepsilon_{i,j} \sim N(0, \theta)$ is the within-hospital residual (independent across hospitals and patients).

2. Predict the random effect for each hospital, $\widehat{\zeta_j}$ using empirical Bayes prediction, to update our priors (density of the random effect) with the likelihood of response given $x_{i,j}, \zeta_j$. The prior assumption treats $\hat{\psi}$ as the true population parameter so that $\zeta_j \sim N(0, \hat{\psi})$.

$$Posterior(\zeta_j|y_{1,j}, ..., y_{n,j}, X_j) \alpha Prior(\zeta_j) \times Likelihood(y_{1,j}, ..., y_{n,j}|X_j, \zeta_j) \tag{2}$$

   The empirical bayes estimate of $\widehat{\zeta_j}$ is the mean of the posterior distribution of the random intercept for cluster j evaluated at the model parameter estimates:

$$\widehat{\zeta_j} = \int \zeta_j Posterior(\zeta_j|y_{1,j}, ..., y_{n,j}, X_j) d\zeta_j$$

   Estimates of $\widehat{\zeta_j}$ can be obtained using numerical integration.

3. This is basically the risk-adjusted difference in mortality between the hospital and the average, so we need to add back the average to get a hospital-level rate. Calculate mean value of $\widehat{xb}$ for the sample $\overline{\widehat{xb}} = \frac{1}{n} \sum_{i=1}^{n} \widehat{xb}$

4. Then add mean $\widehat{xb}$ to the random effect $\widehat{\zeta_j}$

   hosplogodd $= \overline{\widehat{xb}} + \widehat{\zeta_j}$

5. Finally, we take the inverse logit of the random effect + mean($\hat{xb}$), which yields the reliability adjusted outcome rate:

   Y $= logit^{-1}$(hosplogodd) = $logit^{-1}(\overline{\widehat{xb}} + \widehat{\zeta_j})$