Technical Appendix
Reliability Adjustment of Hospital Mortality Rates

We run a hierarchical logistic regression model with dependent variable $Y_{i,j}$
First level is patient characteristics (with patients $i$ nested in hospitals $j$) $x_{i,j}$
Second level, subscripted $j$ is the hospital random effect (identifier variable: hospid)

1. Estimate hierarchical logistic regression (random effects logit)

$$
\text{logit}[Pr(Y_{i,j} = 1| x_{i,j}, \zeta_j)] = \beta_1 + \beta_2 x_{i,j} + \zeta_j + \varepsilon_{i,j}
$$

(1)

where $x_{i,j}$ is a vector of individual characteristics, $\zeta_j \sim N(0, \psi)$ is a hospital random effect (independent across hospitals), $\varepsilon_{i,j} \sim N(0, \theta)$ is the within-hospital residual (independent across hospitals and patients).

2. Predict the random effect for each hospital, $\hat{\zeta}_j$ using empirical Bayes prediction, to update our priors (density of the random effect) with the likelihood of response given $x_{i,j}, \zeta_j$. The prior assumption treats $\psi$ as the true population parameter so that $\zeta_j \sim N(0, \psi)$.

$$
Posterior(\zeta_j | y_{1,j}, ..., y_{n,j}, X_j) \propto Prior(\zeta_j) \times Likelihood(y_{1,j}, ..., y_{n,j}|X_j, \zeta_j)
$$

(2)

The empirical bayes estimate of $\hat{\zeta}_j$ is the mean of the posterior distribution of the random intercept for cluster $j$ evaluated at the model parameter estimates:

$$
\hat{\zeta}_j = \int \zeta_j Posterior(\zeta_j | y_{1,j}, ..., y_{n,j}, X_j)d\zeta_j
$$

Estimates of $\hat{\zeta}_j$ can be obtained using numerical integration.

3. This is basically the risk-adjusted difference in mortality between the hospital and the average, so we need to add back the average to get a hospital-level rate. Calculate mean value of $\bar{x}b$ for the sample $\bar{x}b = \frac{1}{n} \sum_{i=1}^{n} x_i b$

4. Then add mean $\bar{x}b$ to the random effect $\hat{\zeta}_j$

$$
\text{hosplogodd} = \bar{x}b + \hat{\zeta}_j
$$

5. Finally, we take the inverse logit of the random effect + mean($\bar{x}b$), which yields the reliability adjusted outcome rate:

$$
Y = \text{logit}^{-1}(\text{hosplogodd}) = \text{logit}^{-1}(\bar{x}b + \hat{\zeta}_j)
$$