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QUARTERLY REPORT NO. 3

STUDY OF ALGEBRAS WITH RADICAL

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Project 2200

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INTRODUCTION

During most of this period, the project has been operating at full capacity. Two technical reports have been issued (2200-R-5 and 2200-R-6; see below under Jans and Feit for details) and further research and study has been carried out as explained in the individual progress reports which follow.

PROGRESS REPORT OF ROBERT M. THRALL

The major individual effort has been on the study of algebras without unity element. A paper is just ready for typing and will soon be submitted to the Canadian Journal of Mathematics. Work has begun on two other papers, one of which should be completed during the next month. The balance of time has been spent in consultation and direction of the research of the other workers on the project.

PROGRESS REPORT OF ROBERT L. DAVIS

Investigations of the structure of the free Lie ring over a modular field will rest on two kinds of preliminaries: the theory of modular tensor representations and the Lie ring theory which has been worked out for characteristic zero. The past seven weeks have been devoted to studying these preliminaries. Three papers by Thrall and Nesbitt form the essential background in modular tensor theory, while five papers by Thrall, Brandt, and Wever give the background for nonmodular Lie rings.

PROGRESS REPORT OF WALTER FEIT

Since June 21, the date of initial employment, the dissertation on character theory has been completed and has been submitted as a technical report.\* This piece of work consists of two separate topics, both of which depend heavily on the well-known theorem of R. Brauer on the characterization of characters. The first part consists in giving a bound for the number of characters in a P-block of defect d, in terms of P and d. The second part is a generalization of a theorem of Frobenius on the existence of normal subgroups in a given group. Further work is being done in some closely related problems. A group theoretic problem has been reduced to a question involving the magnitude of certain algebraic numbers. This has been solved in some special cases and will continue to be investigated in the remaining time on the project.

PROGRESS REPORT OF JOHN H. WALTER

It became apparent early in this three-month period that the method of attack on the problem of commutative algebras was not likely to lead to a successful conclusion. As a consequence, at the suggest of Professor Thrall, the structure of the principal indecomposable representations of defect 2 of group algebras have been investigated with particular attention to the modular algebra of the symmetries group on  $2p$  ciphers. Some results have been obtained and it is expected that this topic will be investigated further during the remaining weeks on this project.

\*Topics in the Theory of Group Characters, Engineering Research Institute, Report Number 2200-6-T, University of Michigan, Ann Arbor, Michigan.

PROGRESS REPORT OF JAMES P. JANS

The summary of the past three months' activities is as follows. First, the dissertation has been completed and submitted as a technical report.\* Second, the structure of a group algebra over a field of characteristic dividing the group order has been investigated. The results of this investigation are some lower bounds on the dimension and exponent of the radical of such an algebra. If the group has a normal Sylow  $p$ -subgroup, the exponent and a basis of the radical can be given in terms of the structure of that subgroup. During the time remaining, extension of the above results will be the main objective.

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\*On the Indecomposable Representations of Algebras, Engineering Research Institute, Report Number 2200-5-T, University of Michigan, Ann Arbor, Michigan.

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