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ON MODES OF OPERATION OF AUTOMATED WATER-SLUG GUN

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ABSTRACT

This is the second report on an apparatus developed for studying erosion by water jet impact. It describes modifications made to the device since the last report, new equipment developed for studying the performance of the device, and a mathematical model of the device.

ACKNOWLEDGMENTS

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I. INTRODUCTION

The impact of a high velocity liquid drop on a solid surface produces stresses of sufficient magnitude to damage the surface. This phenomena occurs when raindrops strike a high velocity aircraft and in the low pressure end of steam turbines. Since different materials have different resistances to liquid drop impact erosion, it is desired to develop a laboratory scale test for ranking erosion resistance.

A device for conducting such a test has been developed in this laboratory. It does so by impacting material specimens with high velocity liquid jets. Fig. 1 is a schematic of this apparatus. The operation of the device can best be explained by momentum considerations. A large mass moving at a low velocity (the bolt) gives up a part of its momentum to a small mass (the working fluid) which exits from the device at a high velocity. A detailed description of this apparatus was given previously (1).

This report deals with the work done on this apparatus since December 1968.

II. DEVELOPMENT OF A MATHEMATICAL MODEL OF THE JET GUN'S MODE OF OPERATION

A. List of Symbols

a	proportionality constant
A_o	orifice area
A_p	piston area
B	liquid bulk modulus
D_o	orifice diameter
D_p	piston diameter
g	gravitational acceleration
h	height above a datum plane
K	liquid compressibility
P	pressure
ρ	density
t	time from bolt impact
t_m	stroke time
V	velocity
w	frequency factor = $2\pi f$
y	piston displacement
f	frequency
V_c	chamber volume

B. General Discussion of Modeling Problems

Due to a possible interest in further increasing the velocity of the jets produced by this "jet gun", it was decided to formulate a mathematical model of the mechanism of liquid jet production so that realistic requirements could be evaluated if higher velocities than presently attainable (about 550 m/s) were to be reached. This section of this report contains two mathematical models of this process, of increasing realism although still of considerable simplicity, and the results obtained from them.

The following typical parameters for the gun were chosen for the development of a model:

Orifice Diameter = 0.0625"

Chamber Included Angle = 120^{o}

Chamber Base Diameter = 1"

Bolt Velocity at Impact = 950 cm/sec

Bolt Mass = 0.76 lb.

Resultant Jet Tip Velocity = 500 m/s

Volume Expelled Per Shot = 0.13 cm^3

From this data the most simple estimate may be made of the chamber pressure necessary to achieve a jet velocity of 500 m/s by applying Bernoulli's equation, i.e., assuming the jet expulsion can be approximated as a quasi-steady-state phenomena.

Assuming that the water in the chamber is incompressible:

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gh_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gh_2 \qquad \text{- - - (1)}$$

with subscripts 1 referring to the condition in the chamber and subscripts 2 referring to the condition in the jet outside the chambers in the atmosphere. Neglecting gravitational effects, atmospheric pressure, and liquid velocity in the chamber, eq. (1) becomes:

$$\frac{P_1}{\rho} = \frac{V_2^2}{2}$$

or, dropping the subscripts,

$$P = \rho \frac{V^2}{2} \qquad \text{- - - (2)}$$

For H_2O (ρ = 1 gr/cm^3) with a velocity of 500 m/s

$$P = \frac{(1)\ (5 \times 10^4)^2}{2}$$

$$P = 1.25 \times 10^9 \text{ dyn/cm}^2 \cong 18,000 \text{ psi}$$

Since this magnitude of chamber pressure is typical, it is obvious that some account of liquid compressibility must be taken. A more realistic model is discussed in the following section.

C. Jet Gun Mathematical Model (No. 1)

The diaphragm of the jet gun presents an obstacle to the formulation of a precise mathematical model. Thus a model in which the diaphragm was replaced by a piston driven by an appropriate forcing function was used. Fig. 2 is a schematic of the model.

The forcing function of the piston was chosen to be of the form y = a sin wt, which is consistent with the known gun performance. The constants a and w were evaluated from the following conditions:

1. The volume of water expelled per shot is 0.13 cm^3

2. The bolt velocity as it strikes the diaphragm and hence the initial diaphragm (piston) velocity, since the mass of the diaphragm is much less than the mass of the bolt, is 950 cm/sec.

Applying condition 1

with $y = a \sin wt$,

the displaced volume $= A_p y$,

so that the maximum displaced volume occurs when $wt=\pi/2$

or $\sin wt = 1$.

$$0.13 = A_p a\ (1)$$

$$a = \frac{0.13}{A_p}$$

Setting the piston diameter equal to the base diameter of the chamber, $A_p = 5.07\ cm^2$.

$$a = \frac{0.13}{5.07} = 2.565 \times 10^{-2}\ cm$$

Applying condition 2

$$\text{at } t = 0,\ \frac{dy}{dt} = 950\ cm/sec$$

$$y = a \sin wt$$

$$\frac{dy}{dt} = aw \cos wt$$

$$950 = aw\ (1)$$

$$w = \frac{9.5 \times 10^2}{2.565 \times 10^{-2}} = 3.71 \times 10^4\ sec^{-1}$$

Knowing w, the time required to discharge $0.13\ cm^3$ may be computed.

$$wt_{max} = \pi/2$$

$$t_m = \pi/2w = \frac{\pi}{2(3.71 \times 10^4)}$$

$$t_m = 4.24 \times 10^{-5}\ sec = 42.4\ \mu s$$

If the water is assumed incompressible, direct liquid displacement may be applied. A mass balance around the chamber then gives:

$$\frac{d(\rho V_c)}{dt} = \text{mass flux out}$$

$$\text{mass flux out} = -\rho A_o V$$

that is,

$$\frac{d}{dt}(\rho V_c) = -\rho A_o V \qquad \text{--- (3)}$$

since ρ is constant

$$\frac{dV_c}{dt} = -A_o V \qquad \text{--- (4)}$$

with y = a sin wt

$$V_c = V_{c_o} - A_p a \sin wt, \quad \text{where } V_{c_o} = \text{initial chamber volume} \qquad \text{-- (4-a)}$$

$$\frac{dV_c}{dt} = -A_p aw \cos wt$$

so that eq. (4) becomes

$$A_p aw \cos wt = A_o V$$

or

$$V = \frac{A_p aw}{A_o} \cos wt \qquad \text{--- (5)}$$

Thus, V_{max} occurs at t = 0 with $V_{max} = A_p aw/A_o = 2.55 \times 10^3$ m/s. Fig. 3 is a plot of this result.

D. Quasi-Steady State Model for Gun (Mathematical Model No. 2)

From eq. (2) it was found that the chamber pressure must approximate 2 x 10^4 PSI. Thus the effect of compressibility of water must be considered. Starting with the mass balance, eq. (3)

$$\frac{d}{dt}(\rho V_c) = -\rho A_o V \qquad \text{--- (3)}$$

and differentiating

$$\rho \frac{dV_c}{dt} + V_c \frac{d\rho}{dt} = -\rho A_o V \qquad \text{--- (6)}$$

An equation of state is now needed. For liquids like water, a permissible approximation to the equation of state is:

$$B = \rho \frac{dP}{d\rho}$$

where B is the bulk modulus of the liquid, and B = 1/K, K being the liquid compressibility. Then,

$$1/K = \rho \frac{dP}{d\rho} \qquad \text{--- (7)}$$

Separating variables,

$$\frac{d\rho}{\rho} = KdP$$

and integrating from a reference pressure ($P_o = 0$) and the appropriate density to P and the corresponding ρ assuming K reasonably constant over this range:

$$\int_0^P Kdp = \int_{\rho_o}^{\rho} \frac{d\rho}{\rho} \qquad \text{--- (8)}$$

$$KP = \ln \frac{\rho}{\rho_o}$$

so that

$$\rho = \rho_o e^{KP} \qquad \text{- - - (9)}$$

and

$$\frac{d\rho}{dt} = \rho_o e^{KP} \frac{d(KP)}{dt}$$

$$\frac{d\rho}{dt} = \rho_o K e^{KP} \frac{dP}{dt} \qquad \text{- - - (10)}$$

Substituting (9) and (10) into (6):

$$\rho_o e^{KP} \frac{dV_c}{dt} + V_c \rho_o K e^{KP} \frac{dP}{dt} = -\rho A_o V \qquad \text{- - - (11)}$$

A rate relation is necessary in order to relate the chamber pressure P to the jet velocity V. Although this is actually an unsteady flow problem, the actual jet has a relatively large L/D so that a quasi-steady-state analysis may give a reasonable approximation. With this assumption, eq. (2) is applicable, and V in this form may be substituted into eq. (11), along with ρ from eq. (9):

$$\rho_o e^{KP} \frac{dV_c}{dt} + V_c \rho_o K e^{KP} \frac{dP}{dt} = -\rho_o e^{KP} A_o \left(\frac{2P}{\rho_o e^{KP}}\right)^{1/2} \qquad \text{- - - (12)}$$

Now, from eq. (4-a)

$$\frac{dV_c}{dt} = -A_p \, aw \cos wt$$

and combining with eq. (12)

$$-\rho_o e^{KP} A_p aw \cos wt + (V_{c_o} - A_p a \sin wt) \rho_o K e^{KP} \frac{dP}{dt} = -\rho_o e^{KP} A_o \left(\frac{2P}{\rho_o e^{KP}}\right)^{1/2} \qquad \text{- - - (13)}$$

dividing through by $k \rho_o e^{KP}$;

$$\frac{-A_p aw}{K} \cos wt + (V_{c_o} - A_p a \sin wt)\frac{dP}{dt} = \frac{-A_o}{K}\left(\frac{2P}{\rho_o e^{KP}}\right)^{1/2} \qquad \text{--- (14)}$$

which, upon rearrangement yields,

$$\frac{dP}{dt} = \left[\frac{\frac{A_p aw}{K} \cos wt - \frac{A_o}{K}\left(\frac{2P}{\rho_o e^{KP}}\right)^{1/2}}{V_{c_o} - A_p a \sin wt}\right] \qquad \text{--- (15)}$$

Results were computed from this equation using the digital computer. The solution curve found is shown in Fig. 4.

The maximum chamber pressure calculated from equation (15) was 20,500 psi which compares quite closely with the 18,000 psi chamber pressure computed for a steady-state 500 m/s jet when compressibility is neglected. That this agreement is obtained from these simplistic models tends to indicate that the models do reasonably represent the jet gun behavior. It thus seems reasonable to assume that the jet gun operates essentially by a straight-forward compressible displacement principle and that complicating factors such as shock wave reflections in the chamber can be neglected to obtain at least a first-order approximation of jet gun behavior. These conclusions could be further substantiated by the use of a pressure transducer in the chamber, if this is deemed desirable at a later date.

III. MEASUREMENT OF JET VELOCITY

As previously reported (1), the velocity of the jet tip was determined from high speed motion picture sequences taken with a Beckman and Whitley 2×10^6 f/s model 330 camera. Once the jet shape was known from this method it was decided to find a more convenient and less expensive means of determining the jet tip velocity. The method adopted consisted of focusing two light beams across the path of the jet, and then focusing these beams on a high speed photodiode. The output of the photodiode is fed to an oscilloscope which gives a photographic record of the transit time of the jet tip between the two light beams and hence the mean jet velocity. Fig. 5 is a schematic of the velocity measuring apparatus, and Fig. 6 is a typical oscilloscope trace generated by this velocity measuring device. It has been found that velocities measured with this device agree quite well with velocities as measured by high speed photography.

IV. DETERMINATION OF BOLT VELOCITY

In order to develop a theoretical model of the jet gun (see Appendix) it became necessary to know the bolt velocity as it impacts the diaphragm. It may be shown that for a spring-mass system of this type that the velocity of the mass is given by the expression

$$V = h(k/m)^{1/2}\cos(k/m)^{1/2}t \qquad \text{- - - - - - - - (1)}$$

where

V = velocity

t = time

h = initial spring compression

k = spring constant

$m = m_b + m_v$

m_b = mass of bolt

m_v = virtual mass of spring which is equal to half the mass of the spring

These parameters for the jet gun are:

$h = 3.49$ cm

$m_b = .73$ lb.

$m_v = 1/2(.241)$ lb

$k = 167$ lb-f/in

Since at impact $\cos(k/m)^{1/2}t = 1$, the velocity at impact becomes

$$V_o = h(k/m)^{1/2} \qquad \text{- - - - - - - - (2)}$$

or substituting numbers

$V_o = 961$ cm/sec

In order to verify this figure it was decided to measure the position of the bolt at various times after its release and thus determine V_o from the slope of this curve. This measurement was made by taking a photographic time exposure of the bolt under stroboscopic illumination. However, it was found that the bolt rebounded after impacting the diaphragm and this produced confusing pictures. Thus a means was necessary to limit the stroboscope illumination to one stroke of the bolt. To accomplish this, the circuit shown in Fig. 7 was constructed. The curve of bolt position versus time obtained in this manner is shown in Fig. 8. From Fig. 8 the bolt velocity as it impacts the diaphragm was found to be 944 cm/sec. The agreement with the theoretical frictionless value of 951 cm/sec is thus excellent.

V. ALTERATIONS IN BOLT

It was found that during repeated testing the bolt head tended to seize in the bore. Furthermore, it was found that the bolt assembly showed substantial wear after approximately 10^3 impacts. It was therefore decided to make the following alterations to the bolt assembly:

1. An oilite bronze bushing was fastened to the bolt head (type 4340 steel) to provide a better bearing surface.

2. The bolt guide bushing was changed from a ball type bushing to a simple oilite bronze bushing in order to eliminate the tendency of the former to groove the bolt shaft.

These modifications are expected to give a useful bolt life of the order of 10^5-10^6 impacts.

VI. SELECTION OF DIAPHRAGM MATERIAL

Since the diaphragm is exposed to the working fluid (water in all work done to date), corrosion is a problem. The diaphragm material initially selected for this device was type 4340 steel, heat treated to a Rockwell C hardness of 50 to 52. Rusting of this material proved to be a nuisance although not a serious problem, so that it was decided to construct a diaphragm not having this problem.

A new diaphragm was, therefore, first constructed of type 440-C stainless steel, heat treated to a Rockwell C hardness of 48-50. This diaphragm did not corrode. However, after it had been subjected to approximately 200 impacts it developed a radial pattern of cracks and it use was discontinued. A second diaphragm was constructed of type 4340 steel, heat treated to a Rockwell C hardness of 50-52. To the face of this diaphragm a sheet of 0.006" thick phosphor bronze was soft-soldered. This composite diaphragm did not corrode. It, however, failed after it had been subjected to approximately 30 impacts by delamination of the bronze sheet.

Since both of these attempts at constructing a corrosion-free diaphragm have resulted in low diaphragm life, the use of the original type 4340 steel diaphragm has been continued at least for the present. The corrosion of this diaphragm can be minimized by emptying the chamber of water and coating the diaphragm with oil when the gun is idle.

VII. CONCLUSIONS

An automated device for the projection of high speed liquid jets against a target has been further developed. The detailed performance of the perfected device has been further analyzed and some mechanical improvements made. It is therefore concluded that as soon as minor developmental modifications are completed on the apparatus to ensure its performance, the device will be ready for "production testing" of rain erosion or other materials at velocities up to about 550 m/s.

APPENDIX

Calculation of Constants

Eq. (16)

$$\frac{dp}{dt} = \frac{A_p aw}{K} \cos wt - \frac{A_o}{K}\left(\frac{2P}{\rho_o e^{KP}}\right)^{1/2}$$

$$A_p = \frac{\pi D_p^2}{4} = \frac{\pi (2.54)^2}{4} = 5.07 \text{ cm}^2$$

$$A_o = \frac{\pi D_o^2}{4} = \frac{(\pi)(1.586 \times 10^{-1})^2}{4} = 1.975 \times 10^{-2} \text{ cm}^2$$

$$V_{c_o} = \frac{\pi}{3} r^2 h = \frac{\pi}{3} (1.27)^2 (1.27 \tan 30^o) = 1.573 \text{ cm}^3$$

$$w = 3.71 \times 10^4 \text{ sec}^{-1}$$

$$a = 2.565 \times 10^{-2} \text{cm}$$

$$K = 4.58 \times 10^{-11} \text{ Dyn/cm}^2$$

so that,

$$\frac{A_p aw}{K} = \frac{(5.07)(2.565 \times 10^{-2})(3.71 \times 10^4)}{4.58 \times 10^{-4}}$$

$$= 10.53 \times 10^{13} \text{ cm}^5/\text{Dyn sec}$$

$$\frac{A_o \sqrt{2}}{K\sqrt{\rho_o}} = \frac{(1.975 \times 10^{-2})(1.414)}{(4.58 \times 10^{-11})(1)} = 6.11 \times 10^8 \frac{\text{cm}^{11/2}}{\text{Dyn G}^{1/2}}$$

$$A_p a = (5.07)(2.565 \times 10^{-2}) = 1.301 \times 10^{-1} \text{ cm}^3$$

REFERENCES

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2. Pitek, Martin T., and Hammitt, Frederick G., "Hypervelocity and Fluid Impact Studies, A Literature Review," ORA Technical Report No. 08153-1-T, Laboratory for Fluid Flow and Heat Transport Phenomena, Nuclear Engineering Department, The University of Michigan, Sept. 1966.

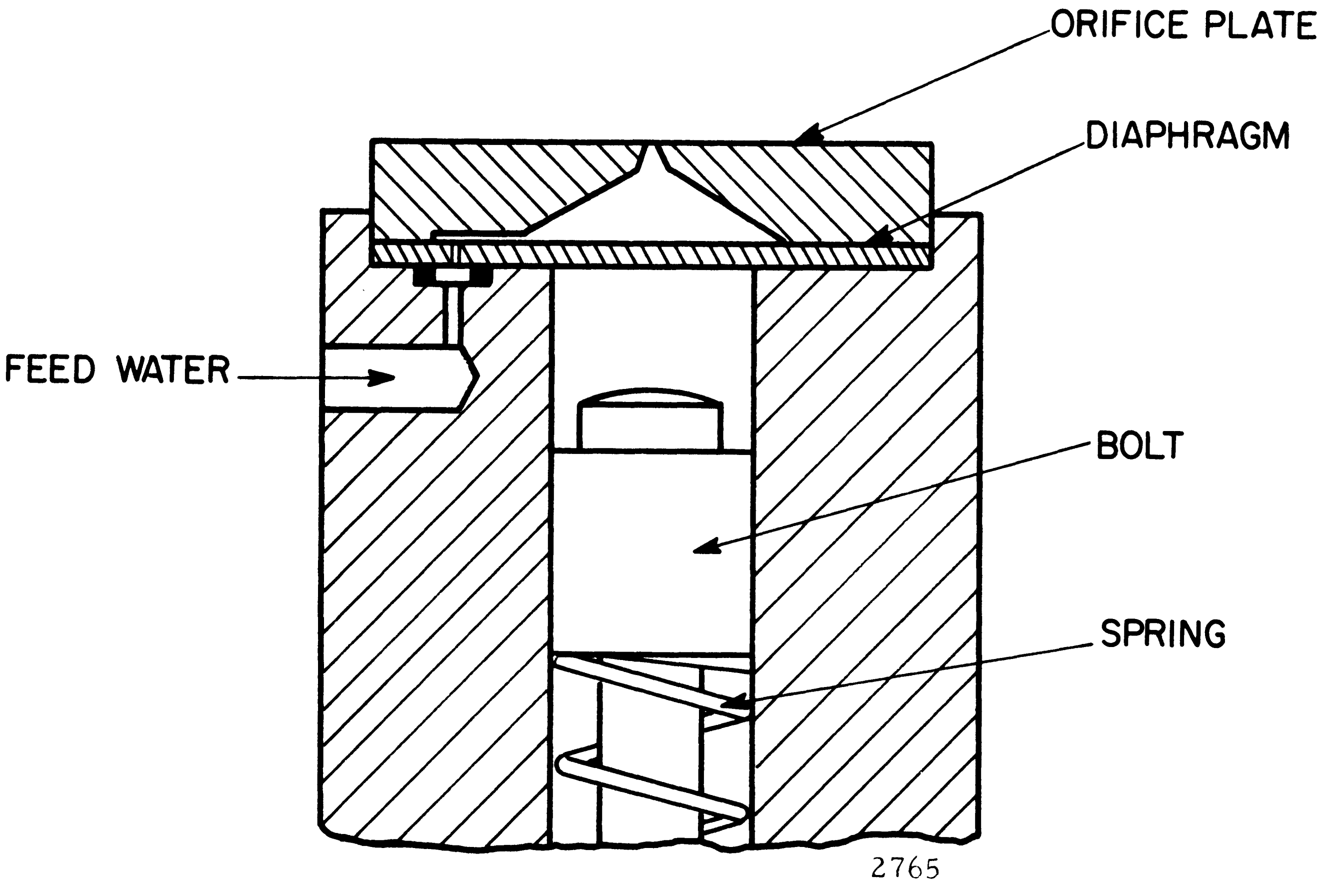


Fig. 1. Schematic of Jet Gun

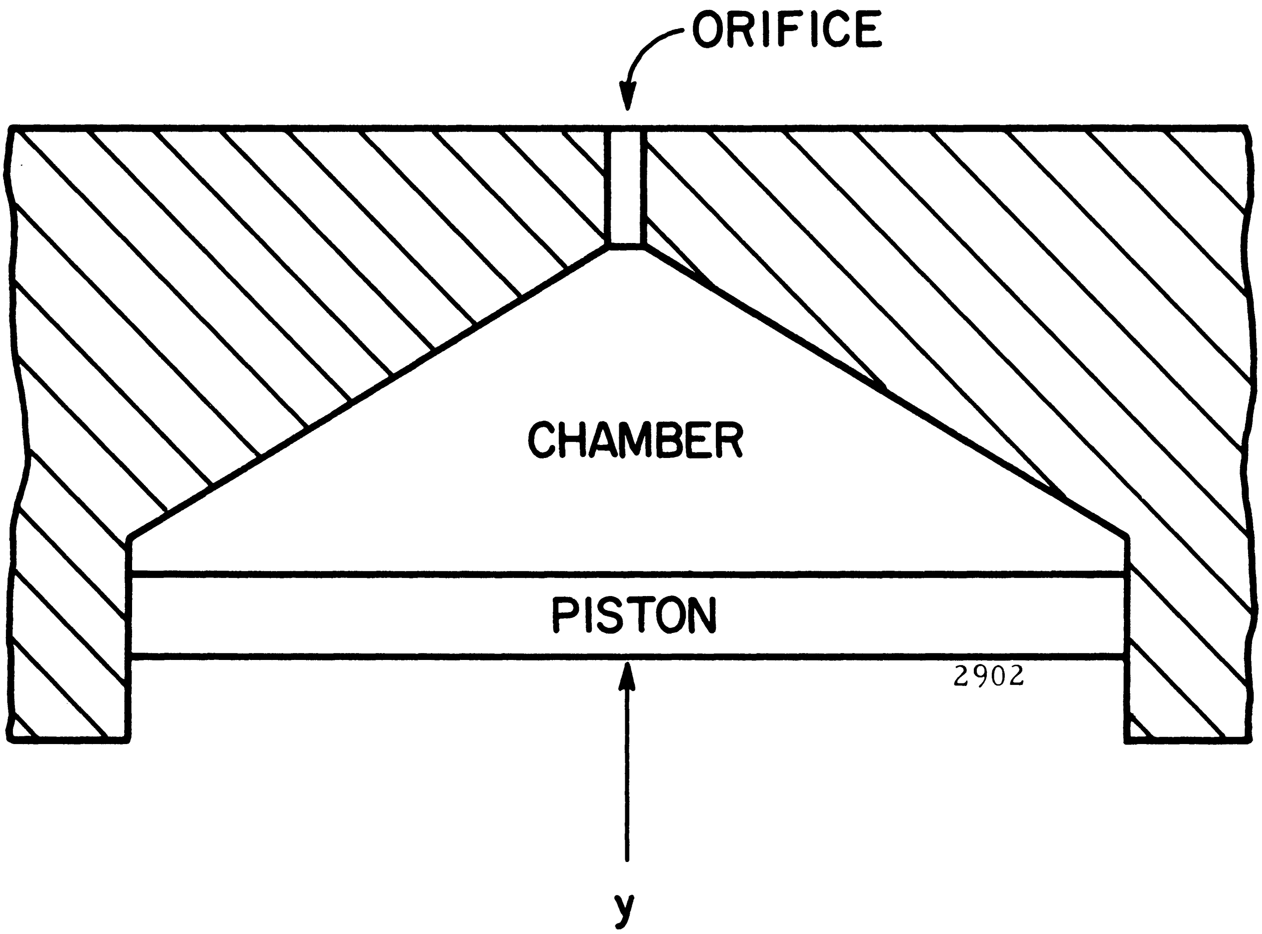


Fig. 2. Schematic of Model

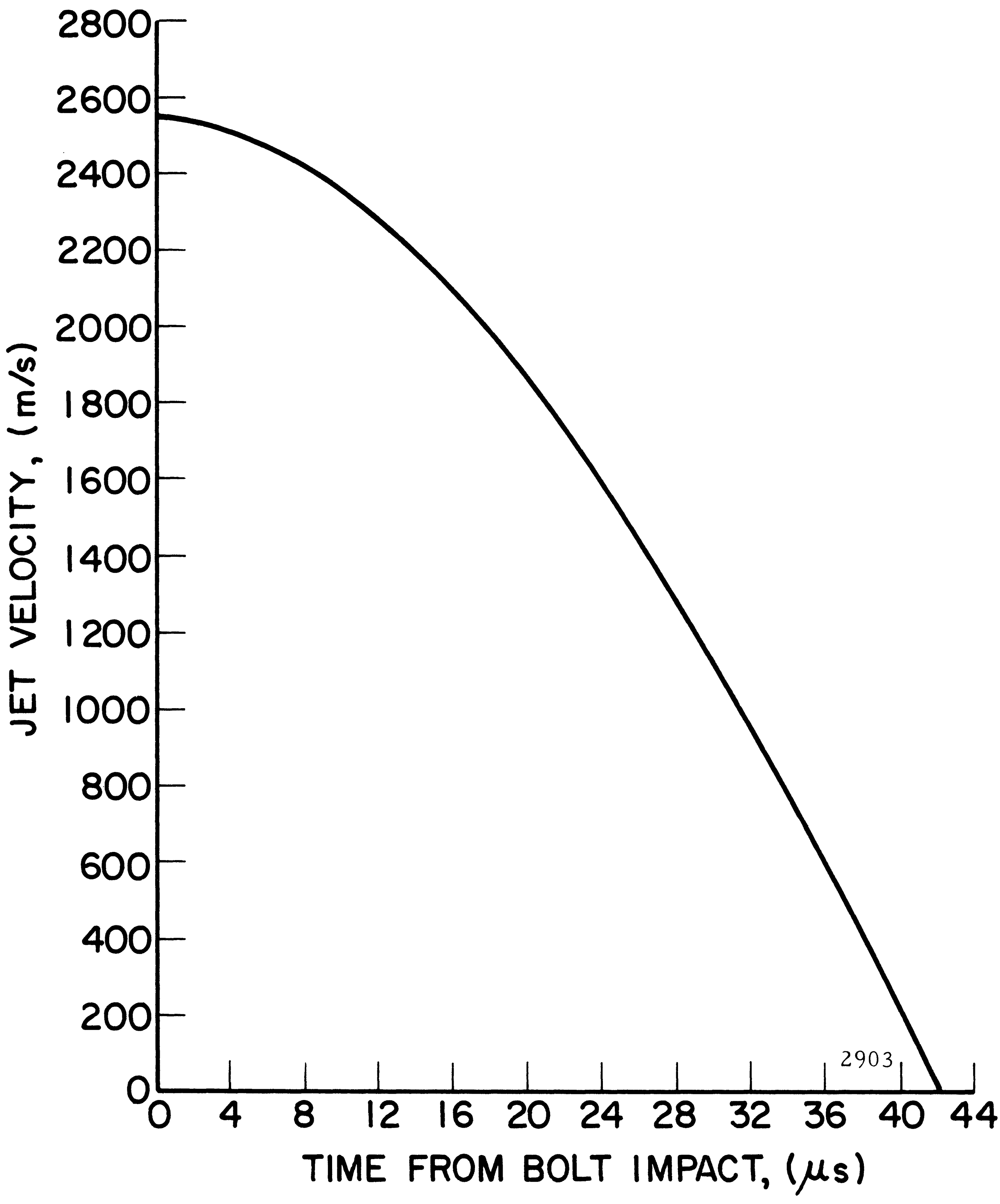


Fig. 3. Solution Curve for Limiting Case Model

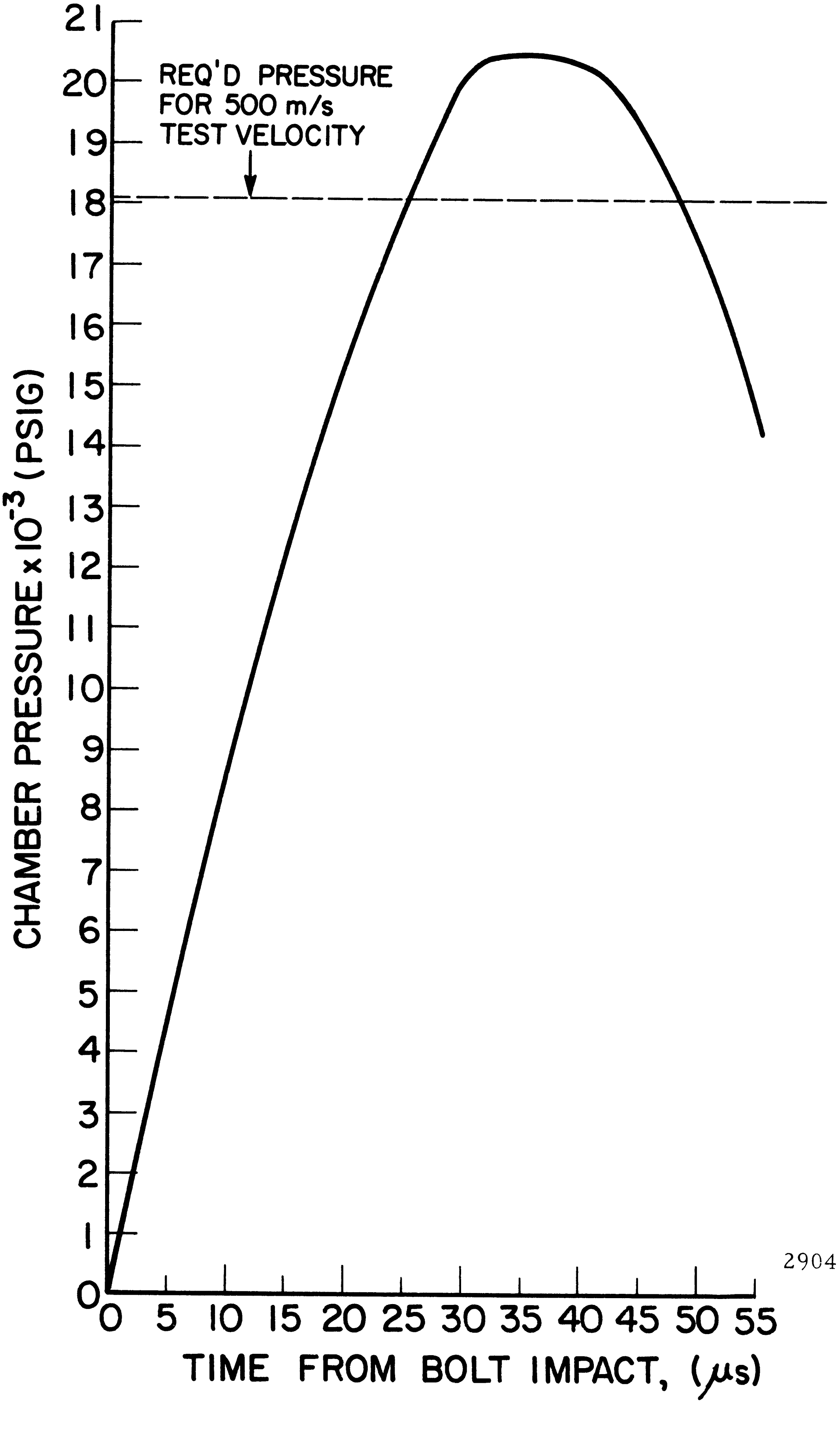


Fig. 4. Solution Curve for QSS Model

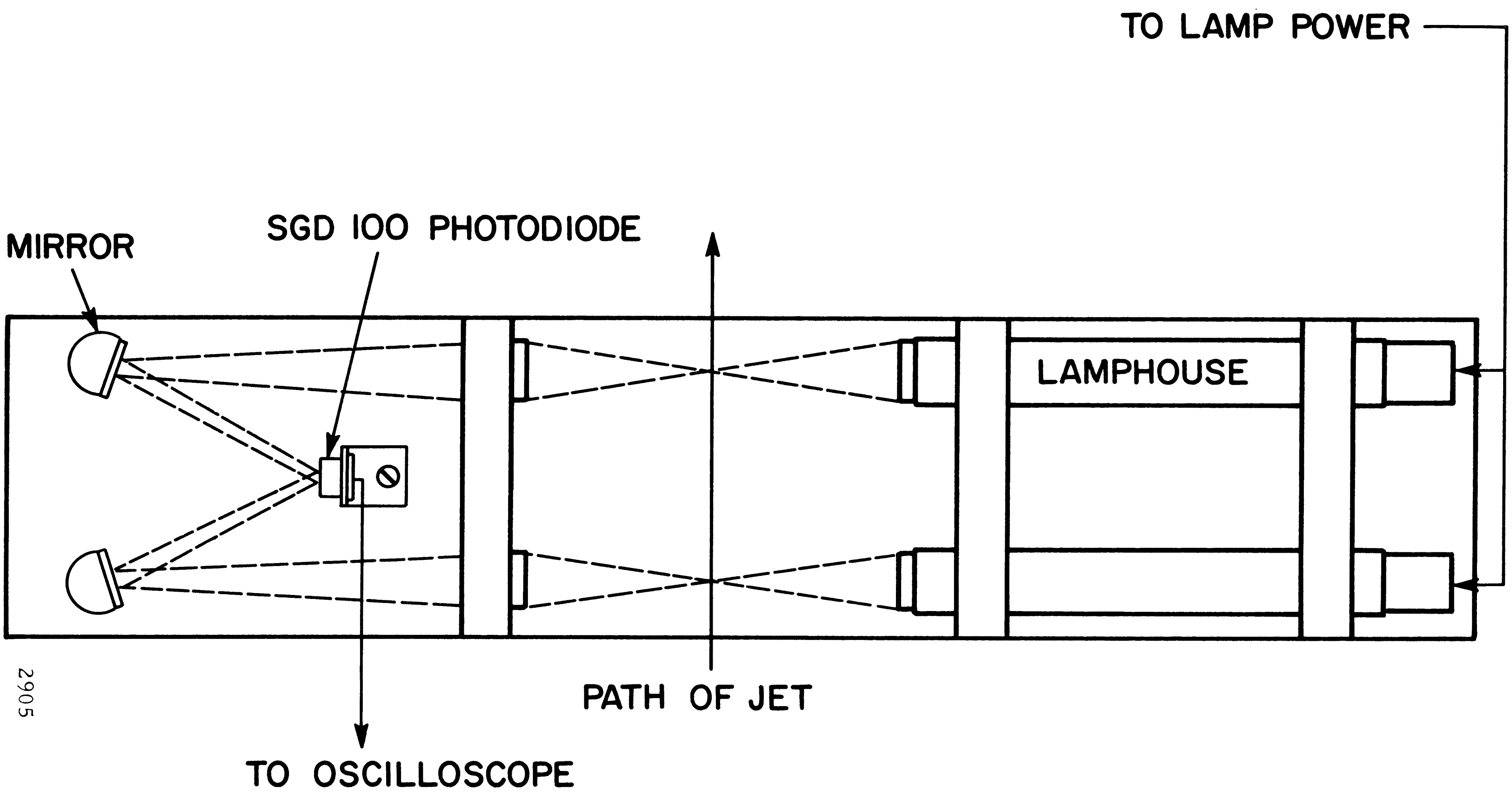
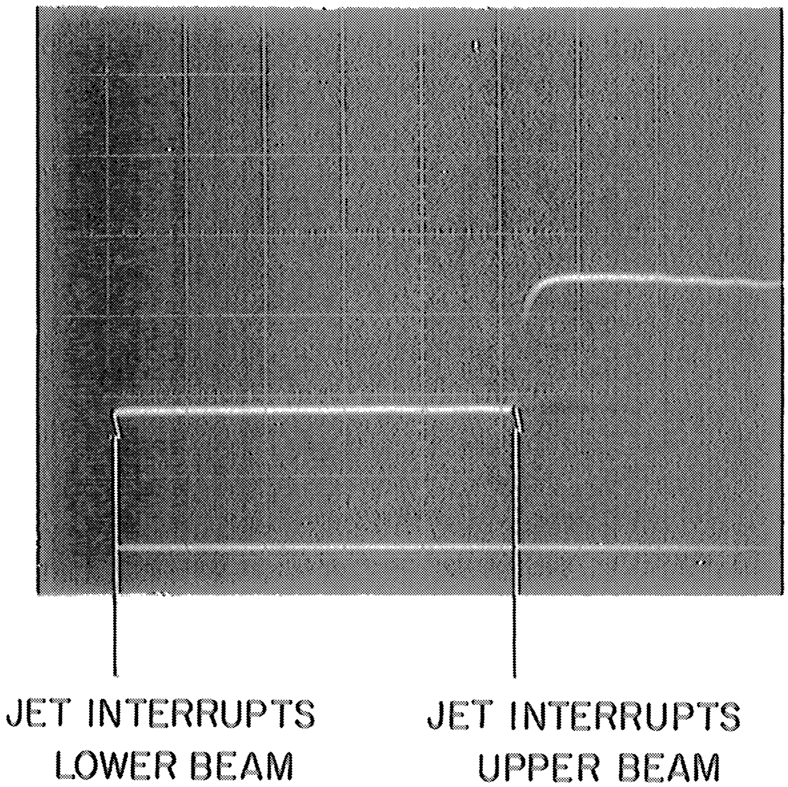


Fig. 5. Schematic of Velocity Measuring Device

VERTICAL = 20 us/cm
HORIZONTAL = 20 mv/cm



Delta t = 105.0 μs, which is equivalent to a jet velocity of 259 m/s.

Fig. 6. Typical Oscilloscope Trace From Velocity Measuring Device

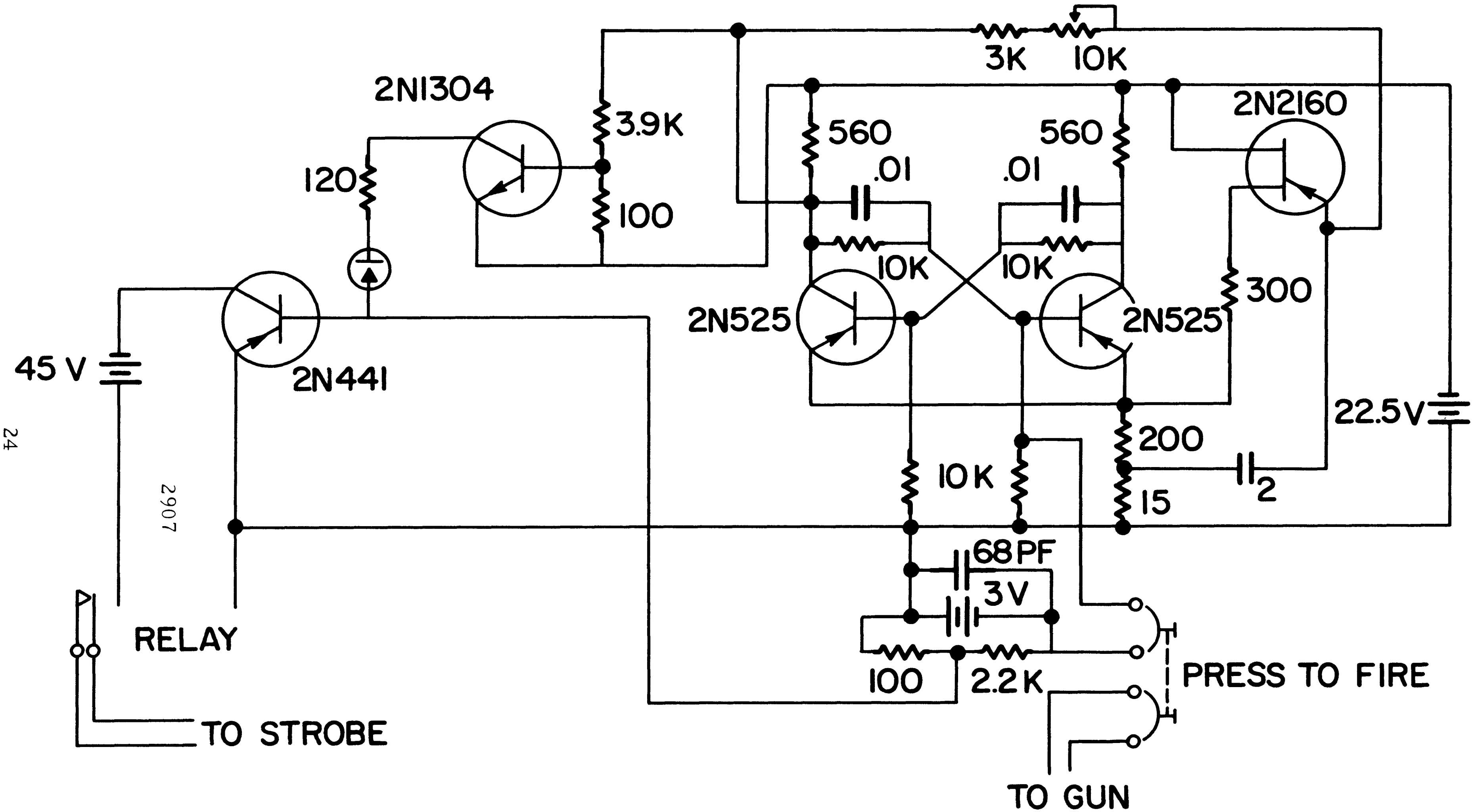


Fig. 7. Schematic of Timing Circuit

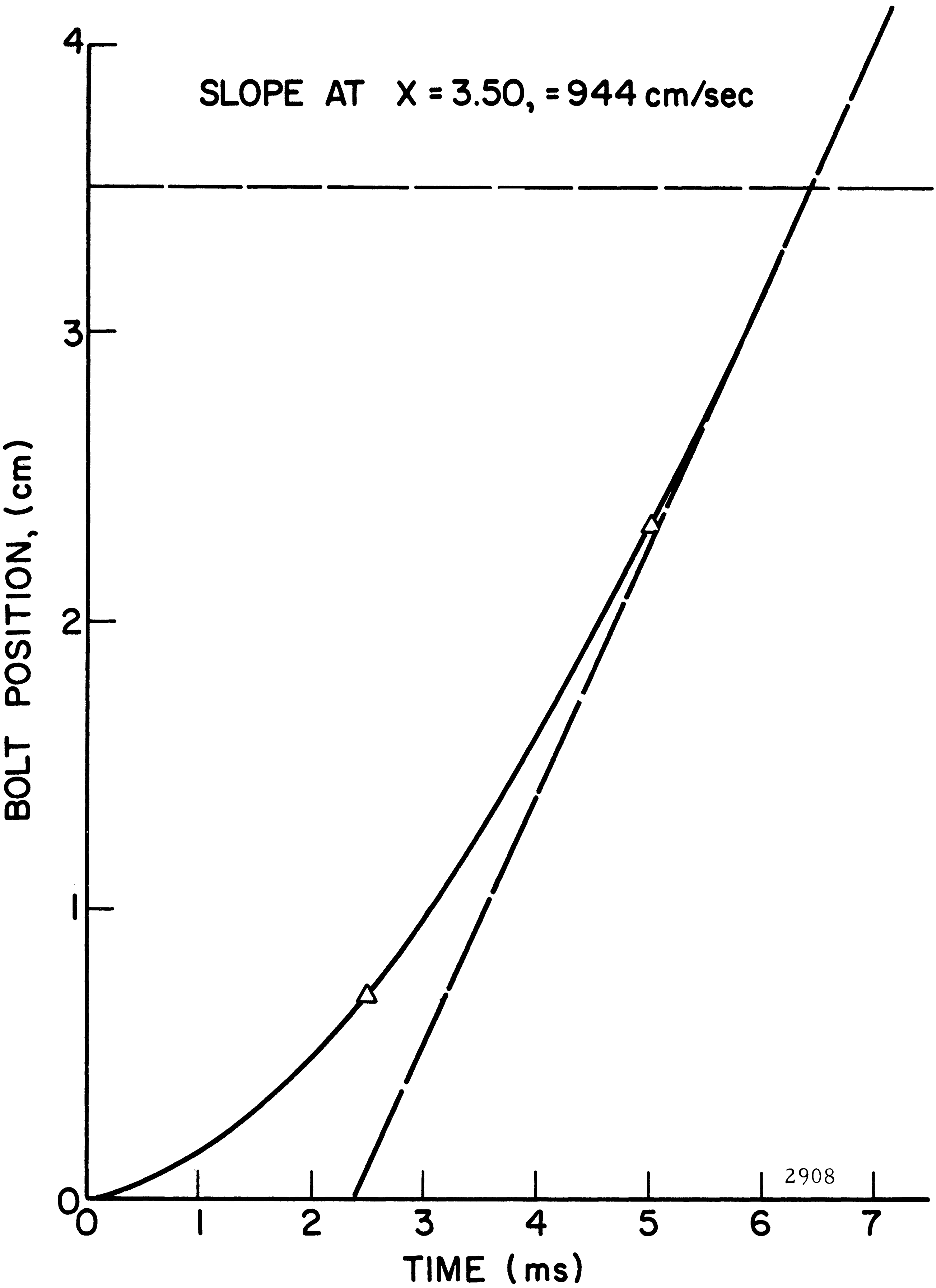


Fig. 8. Bolt Position vs. Time