MODERN ICING TECHNOLOGY

LECTURE NOTES

by

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FOREWORD

These notes are the outgrowth of a seminar on icing information given at the University of Michigan in the Fall of 1951. They are intended to be useful to persons interested in entering the icing field who require a single source where either the basic information or suitable references may be found.

This first edition was rather hurriedly put together for use in a second seminar in February, 1952. Corrections and suggested additions will be most welcome.

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CHAPTER I

AN INTRODUCTION TO THE ICING PROBLEM

The type of icing with which we will be concerned in this and subsequent chapters occurs when an airplane flies through a region containing supercooled water droplets. These supercooled water droplets are in an unstable thermodynamic state and upon impinging upon the airplane turn to ice. The resultant ice formation clings tenaciously to the airplane surfaces, adding weight and deforming the aerodynamic shape. Our interest lies in the details of the ice formation process and the methods available for protecting the airplane from its harmful effects. The carburetor icing problem is not considered in this work.

In the first chapter we will consider the circumstances which bring about an icing condition in the atmosphere, typical ice formations on aircraft, and some of the schemes which have been proposed or are used to eliminate the ice. This chapter is intended to serve as an introduction to the field of airplane icing.

Water may be cooled below its freezing point to a surprising degree. Figure I-1 shows the results of experiments carried out by Dorsch and Hacker of the NACA.1 This figure shows clearly that small drops of water do not always freeze at 32°F but tend to remain liquid to quite low temperatures. Frank2 reports supercooled water at -72°C. Once the small drops have turned to ice, however, the melting always occurs at 32°F as reported by Dorsch and Hacker. Figure I-2 shows some typical observed values of supercooled water drops found in the atmosphere.

*Superior figures in the text refer to the references at the end of each chapter.
Figure I-1. Curve from Reference 3 showing average freezing temperature of various sized drops observed in Reference 1.

We have a fairly satisfactory qualitative view of why the supercooling occurs. In a drop of water below the freezing point the most stable thermodynamic state for the water molecules (plus their surroundings) is the solid state, wherein the water molecules are oriented in an ice crystal structure. However, in order to form this structure it is necessary for several water molecules to come together in the proper orientation and with the proper energy. The odds against this happening are rather large, though from the Second Law of Thermodynamics point of view we should expect that eventually the drop would freeze. We may, however, help matters along by putting something in the water to act as a nucleus upon which the ice crystals may grow. Such a nucleus will be effective to the degree that it matches the ice crystal structure or permits the crystal to start at a corner or surface defect. Thus, the best nucleus for ice, is a small crystal of ice. Another suitable nucleus is a silver iodide crystal since the structure of silver iodide resembles that of ice. There are enough kinds of materials which may nucleate supercooled water, that ordinary water usually contains sufficient nuclei to prevent undercooling. Not all nuclei are expected to be equally effective. Those which resemble the ice crystal only weakly are expected to be less effective than those which are identical. In the presence of these "weaker" nuclei the freezing point should be lower than 32°F. This concept of the specific nature of a nucleating substance is supported by observations which show that the freezing point is unchanged by successive melting and freezing of an individual drop. Levine has examined the statistical problem of distributing a finite number of specific nuclei in a large volume of water and then dividing the water into a large
Figure I-2. Liquid water content as related to mean effective drop diameter for flight observations between 5000 and 10,000 feet pressure altitude in the temperature range 15° to 20°F.

number of small droplets. The results of his analysis indicate that the smaller the drop the smaller the probability it will contain a particular nucleating substance. The statistical treatment renders plausible the linear relation between freezing temperature depression and logarithm of diameter (Figure I-1), though the slope and intercept are not predictable by this method. More work certainly remains to be accomplished on the nature of ice nuclei.

In view of the above experimental evidence, then, it is not surprising that clouds of supercooled water droplets are often encountered in the atmosphere. When these water droplets are intercepted by the leading edges of the wings and empennage, as well as all the other components, nuclei present on the surface initiate the transition from liquid to solid. Once a thin film of ice is deposited, succeeding water droplets strike an ice surface and freeze at a rate limited by the heat transfer process rather than the process of initiating an ice crystal.
Before taking up the details of the ice formation and removal, we might, in passing, note that atmospheric icing conditions may be dispelled by providing ice nuclei to "trigger" the unstable supercooled cloud. Silver iodide crystals introduced into a cloud of supercooled droplets will convert the water droplets into snow. The water will condense on these nuclei and form ice crystals. The vapor pressure of ice is lower than that of water, hence the snow grows at the expense of the water drops. Modern rainmaking operations\textsuperscript{4} are based upon this principle.

It is now worthwhile to look at some typical ice formations on aircraft. Figures 3, 4, 5, 6, and 7 show typical examples of airplane icing.

Airplane icing is serious for several reasons:

1) The airplane is increased in weight. An ice accretion of 30 lbs per minute on a DC-3-type airplane is not unusual.

Figure I-3. Ice formations on an airliner forced down by a freezing rain.
2) The airfoil contour is changed. The lift coefficient can be very rapidly reduced by deformation of the wind profile.

3) Ice breaking off the wing may damage tail surfaces. Propellers cast off ice which may damage the fuselage.

4) Air intakes may become clogged. Oil coolers, radiators, jet engine inlets may be stopped up.

5) Ice loads may break the antenna wires.

6) The ice coating on the windshield obstructs vision.

7) Instruments which rely upon air pressure become unreliable. Ice may close over pitot tubes or interfere with pitot-tube readings. Fuel vents may be closed over.

8) The ice loads may set up vibrations in antenna masts.
Figure I-5. A typical "mild" icing condition on the wing root of a B-24-type bomber. (175 mph, 6000 ft, -3°C, drop diameter = 17 microns (avg), 0.2 grams of liquid water per cubic meter of air.

9) Ice formations on various protuberances on the airplane increase the overall drag of the airplane. This effect is more serious in older-style airplanes where the rivet heads stick out from the skin and serve as individual ice collectors.

The system used for ice protection will depend not only upon the particular component but also upon the degree of ice protection desired. Any one airplane is in icing conditions only a small fraction of the time, and, depending on the bias of the designer and the demands of the customer, either emergency or "complete" protection will be provided.

The earliest type of wing de-icers were the rubber "inflatable-shoe" type. This de-icer is alternately inflated and deflated to crack the ice loose. These were practically the only de-icers used in the U. S. and Europe until 1938.5

In the latter part of 1938, the Germans brought out the Ju 88G, which had the wings heated with hot air. Gradually, the Germans replaced their
"rubber-shoe" equipped planes with heated wings, and at the close of World War II they used thermal means exclusively.

Experiments with thermal ice protection were conducted by the NACA as early as 1931, when Theodorsen and Clay built a small steamheated model Clark Y airfoil, mounted it on a Fairchild monoplane, and flew it both with water sprays and in natural icing conditions. Some of the predictions in Reference 7 on the behavior of thermal de-icing equipment have been accurately borne out in the intervening twenty years, as will be discussed later.

Several experimental airplanes have been built and flown by the NACA. The first U. S. mass-produced heated-wing-equipped airplane was the Consolidated
Figure I-7. Ice formations on an unheated jet engine after 36 minutes of icing (Reference 10).

B-24 bomber, which reached production at San Diego March 6, 1944. Since then most new aircraft have featured one form or other of thermal ice-prevention system.

The British equipped some of their bombers with leading edges containing porous metal. Alcohol or glycerol was pumped out these porous leading edges to depress the freezing point of the water and thus release the ice. The system was not satisfactory.

The heated wing was by far the most successful system and today most new aircraft employ air-heated wings for ice protection. Figures 8 and 9 show typical heated-wing systems. The heat supply in an airplane such as the B-24 employed exhaust-gas heat exchangers. In the DC-6 heated-wing separate gasoline-burning combustion heaters are used, as a weight saving has been found through simplification of the exhaust system without heat exchangers. In some jet airplanes the hot air is obtained from the last stage of the compressor. This source of air is expensive, particularly since bleeding 1 per cent of the engine air produces approximately a 2 per cent loss in thrust, but the simplifications in control, installation, maintenance, and fabrication have made the method attractive to the airplane designers, and the overall loss in performance has been acceptable.
Figure I-8. Typical flow system for a hot-air-type heated wing.

Towards the end of World War II the Germans began to experiment with de-icing methods which were expected to be more economical in their operation. Figures 10 and 11 show two of these schemes. In this country some interest has recently been shown in the intermittently heated systems, as will be seen in a later chapter.

Propeller ice-protection systems have received their share of attention. Early systems of ice protection utilized alcohol "feed shoes". These were dispensing systems which distributed the alcohol along the rotating blades. The alcohol systems were a nuisance but did give protection. Great care is required in the preparation of the slinger rings and shoes lest the alcohol be wasted. If the alcohol vapors are taken into the ventilating system, there is a danger of pilot intoxication. In battle the alcohol tanks are a fire hazard. The alcohol feed shoes have generally been replaced by electrically heated
Figure I-9. Typical double-skin construction.

Figure I-10. German proposal for an "economical" de-icer. Ice formations outside the wide gap were expected to be shed periodically (Reference 5).
propellers. The ice is allowed to form and then is shed when the heaters are intermittently energized. As will be shown later, this intermittent operation requires much less energy than continuous heating. Experimental hot-air-heated propellers have been built but none have been reported in operation.11-13

A helicopter hot-air system has been described by Hatzenberger of Sikorsky Aircraft.6 The principles of this system are the same as for the heated wing. The rotating mechanism introduces a difficulty in getting the air into the blades without leakage. The rotor acts as a pump to draw air through the heater and duct system. Figure 12 shows a photograph of the test installation made by Sikorsky.

The windshields of aircraft require ice protection. Prior to 1941, most airplane windshields were protected by alcohol spray systems which operated in conjunction with the windshield wipers. United Air Lines pioneered the use of double-paned windshields with hot air flowing between the two layers of glass. These double-paned windshields introduce multiple reflections, particularly during night landings, and are difficult to keep clean, but have otherwise been satisfactory.

Recently the trend has been toward glass with a thin layer of metal laid over to the surface to act as an electrical resistance-heating element.
The important problem here is one of thermal shock. As an airplane suddenly enters or leaves a cloud the thermal stresses may crack the glass.

Jet engines are particularly susceptible to icing difficulties. A jet engine inlet may become iced over a few minutes. The accident in 1951, in which five jet fighters were lost, attests to the seriousness of the jet-engine icing problem. The details of jet-engine anti-icing systems have not been published; however, NACA reports have described both electrical and hot-air systems for engine inlets.

Figure I-12. Experimental installation of hot-air heating system on the hollow-bladed rotor of a helicopter. The box on the top is a temperature recorder. Hot air enters the rotating collector and is ducted to the individual blades by flexible ducting (Reference 6).
REFERENCES


13. Mulholland, D. R., and Perkins, P. J., "Investigation of Effectiveness of Air Heating a Hollow Steel Propeller for Protection Against Icing".
CHAPTER II

TRAJECTORIES OF WATER DROPS AROUND STREAMLINED BODIES

The Icing of a Cylinder

The first step in an attack upon the icing problem is the analysis of the icing rates to be expected upon a particular surface.

Consider first the icing of right circular cylinder placed transverse to the air stream. Figure II-1 shows the streamlines for an incompressible non-viscous fluid flowing past a cylinder. A spherical water drop in the fluid stream will not follow the streamlines, but due to its inertia will follow a trajectory which is less curved. Figure II-2 shows a typical water-drop trajectory.

The drop trajectory shown in Figure II-2 was calculated as follows: Consider a drop in air, let

\[ u_a, v_a = \text{components of air velocity} \]
\[ u_d, v_d = \text{components of drop velocity}. \]

Then let

\[ (u_a - u_d), (v_a - v_d) = \text{components of \( F \),} \]
\[ \text{the relative drop velocity.} \]

The relative velocity of the droplet gives rise to a drag force
Figure II-1. Flow field ahead of a cylinder.

\[ F_x = \frac{1}{2} \rho_a \pi a^2 C_D P (u_a - u_d) \]
\[ F_y = \frac{1}{2} \rho_a \pi a^2 C_D P (v_a - v_d) \]

where

- \( a \) = drop radius
- \( C_D \) = drag coefficient
- \( \rho_a \) = air density

These components of drag give rise to accelerations according to Newton's equation.
Figure II-2. Typical water drop trajectory drawn by Differential Analyzer (Reference 2).

\[ F_x = m a_x = \frac{4}{3} \pi a^3 \rho_d \frac{d v_d}{dt} \]

\[ F_y = m a_y = \frac{4}{3} \pi a^3 \rho_d \frac{d v_d}{dt} \]

where

\[ \rho_d \] = drop density.

Equating the forces yields:

\[ \frac{d v_d}{dt} = \frac{x}{\rho_a \rho_d a} \frac{P(u_n - u_d)}{a} C_D \]

\[ \frac{d v_d}{dt} = \frac{x}{\rho_a \rho_d a} \frac{P(v_a - v_d)}{a} C_D \]
The trajectory of the drop is obtained by integrating the following equations simultaneously:

\[
(x - x_0) = \int \left\{ \int \frac{3}{8} \frac{\rho_a}{\rho_d} (u_a - u_d) \frac{P C_D}{a} \ dt \right\} \ dt \\
(y - y_0) = \int \left\{ \int \frac{3}{8} \frac{\rho_a}{\rho_d} (v_a - v_d) \frac{P C_D}{a} \ dt \right\} \ dt .
\]

The first integration yields the drop velocity and the second integration the drop displacement. To lessen the number of calculations required the equations are rendered dimensionless as follows:

\[
\frac{X}{C} = \frac{x_0}{C} + \left\{ \left( \frac{9 \rho_a C}{\rho_d a} \right) \left( \frac{C_D P}{24 U} \right) \left( \frac{u_a - \dot{x}}{U} \right) d \frac{U}{C} \right\} d \frac{U}{C} \\
\frac{Y}{C} = \frac{y_0}{C} + \left\{ \left( \frac{9 \rho_a C}{\rho_d a} \right) \left( \frac{C_D P}{24 U} \right) \left( \frac{v_a - \dot{y}}{U} \right) d \frac{U}{C} \right\} d \frac{U}{C} .
\]

Now let

\[
\psi = \frac{9 \rho_a C}{\rho_d a} , \quad R_P = \frac{2a \rho_a P}{\kappa_a} , \quad R_U = \frac{2a \rho_a U}{\kappa_a} .
\]

\[
\kappa_a = \text{air viscosity} \\
C = \text{a significant dimension in the flow field} \\
U = \text{free stream velocity}
\]

\[
\frac{X}{C} = \frac{x_0}{C} + \left\{ \frac{\psi}{R_U} \left( \frac{C_D R_P}{24} \right) \left( \frac{u_a - \dot{x}}{U} \right) d \frac{U}{C} \right\} d \frac{U}{C} \\
\frac{Y}{C} = \frac{y_0}{C} + \left\{ \frac{\psi}{R_U} \left( \frac{C_D R_P}{24} \right) \left( \frac{v_a - \dot{y}}{U} \right) d \frac{U}{C} \right\} d \frac{U}{C} .
\]

The integration of these equations must be accomplished step-wise, graphically, or via a differential analyzer, since \( C_D \) is a function of \( R_P \) and, except for very simple cases, \( \frac{u_a}{U} \) and \( \frac{v_a}{U} \) are rather complicated functions of \( \frac{X}{C} \) and \( \frac{Y}{C} \).

For the cylinder, then, it follows that the trajectories, i.e., the loci of \( \frac{X}{C} , \frac{Y}{C} \) from these equations, are functions of \( \psi \) and \( R_P \).
In the special case that \( \left( \frac{C_D R_p}{24} \right) = 1 \), the region of Stokes' law, the term in \( R_u = R_p (U/F) \) does not appear inside the integral and the trajectories are functions of one parameter only, i.e., \( \psi / R_u \).

Albrecht\(^1\) was the first to calculate the trajectories around a cylinder for the case of the Stokes' law regime. Later Langmuir and Blodgett, using a differential analyzer, calculated trajectories for the case where \( C_D \) varies with \( R_p \) as determined experimentally for large spheres.\(^2\) In this same report they calculated trajectories past ribbons and spheres, the results of which will be considered later.

From these trajectories certain specific items of information emerge. The first is known as the "percentage catch" and sometimes the "collection efficiency". Figure II-3 illustrates the physical significance of this term. All the

![Diagram of droplet trajectories around a cylinder](image)

\[
\psi = \text{Scale Factor} = \frac{3 C_D}{4 a_0} \\
R_u = \text{Reynolds Modulus} = \frac{2 a_0 U_s}{\mu a_0} \\
E_M = \text{Percentage Catch} = \frac{\text{Area of Impingement}}{\text{Frontal Area}}
\]

Figure II-3. Graphical representation of parameters used in trajectory work.

drops which lie within the limiting trajectories strike the cylinder. Those drops which lie outside these trajectories do not strike the cylinder but are blown around it. The ratio of the original \( y \) coordinate of the "tangent trajectory" to the cylinder radius defines the "percentage catch" which, expressed as a fraction, is given the symbol \( E_M \).
Another property of interest is the area over which the cylinder is wetted. The angle measured from the stagnation line to the tangent trajectory is given the symbol $\Theta_m$. All drops which strike the cylinder do so within $\pm \Theta_m$.

A third property of trajectories is the intensity of catch at the stagnation line, $\beta_o$. (The symbols are all from Langmuir and Blodgett.)

The catch rate may be computed from the percentage catch by

$$ W = E_M U (2C) L w , $$

where

$\dot{W} = \text{catch rate (lbs/sec)}$

$E_M = \text{"fractional catch"}$

$U = \text{velocity (ft/sec)}$

$C = \text{cylinder radius (ft)}$

$L = \text{cylinder axial length (ft)}$

$w = \text{liquid water content of atmosphere (lbs/ft}^3\text{)}$

The intensity of catch along the stagnation line is given by

$$ \frac{dW}{dA} = \beta_o U w $$

$$ dW/dA = \text{intensity of catch, lbs/sec ft}^2. $$

Because of certain end uses for their data, Langmuir and Blodgett plotted the quantities $E_M$, $\Theta_m$, and $\beta_o$ versus two parameters defined as follows:

$$ K = Ru/\psi \quad \phi = Ru \cdot \psi $$

Figures II-4a to II-6b show graphs of $E_M$, $\Theta_m$, $\beta_o$, and relative velocity $V_1$ versus $K$ and $\phi$. The relative velocity is the fraction of free stream velocity possessed by the drops as they strike the stagnation point, $\Theta = 0$.

For values of $K \leq 1/\phi$, $E_M = 0$ on a cylinder. Since $K$ contains the cylinder radius $C$ in the denominator, it follows that for a given icing condition there is a maximum size of cylinder which will collect ice! This effect of size was not always appreciated. The author once participated in a test flight where the heat supply to the wings was being continuously diminished to find the
Velocity $v$ at $\epsilon = 0$ vs. $x$ for cylinders with ideal flow.
minimum heat required to prevent ice. When the heat supply was reduced to zero, the wings still remained ice-free. Meanwhile the antenna wires and mast were icing rapidly. It is now known that this effect may occur when the drops are small. The wing, having a large radius of curvature tended to deflect the drops while the antenna mast had, in effect, a high value of $K$ and therefore a high collection efficiency.

Experimental Verification

Figures II-8a and II-8b show ice accretions on cylinders and spheres which verify qualitatively the trajectory data given by Langmuir and Blodgett. The best check of the calculations may be made using a cylinder, since the cylinder may be rotated and will remain cylindrical while icing.

A quantitative verification of the trajectory data has been obtained by mounting cylinders on a common axis and exposing them to the icing windstream. These cylinders are of different sizes and are rotated to preserve the cylindrical shape during the icing. The unknown properties of the windstream are the liquid-water content ($w$) and the drop diameter ($2a$). If the weight of ice catch on each cylinder is divided by the cylinder frontal area, wind velocity and exposure duration and the quotient plotted versus cylinder radius, $C$, using logarithmic coordinates, we get the results shown in Figure II-9. A graph of $E_M$ versus $1/K$ with $\phi$ as a parameter is shown in Figure II-10. An attempt is then made to fit the points of Figure II-9 to the curves of Figure II-10. Since a shift along either logarithmic coordinate corresponds to multiplication by a constant, it is not necessary to know the liquid-water content or drop diameter to do the curve fitting. Figure II-11 shows an example of the "fit". The amount of vertical and horizontal shift permits calculation of the unknown liquid water content and of the drop radius.

Various modifications of the above curve-fitting process have been used, all representing the same basic method. Charts have been prepared in which the curves of $E_M$ versus $1/K$ have two parameters, one the droplet Reynolds Modulus, $R_u$, and the other a characterization of the droplet-size distribution. Such curves are given in Reference 3. By matching experimental points to these curves, one obtains not only the liquid-water content and mean drop size but also the approximate drop-size distribution.* Figure II-12 shows an example of such a

* Lewis and Hocker, NACA TN 1904 "Observations of Icing Conditions Encountered in Flight During 1948" comment that the rotating-cylinder method when used in the manner described yields "indications of drop-size distribution (that) are so unreliable that they are of little or no value". However, the Lewis data under discussion was obtained partly from two-cylinder and partly from four-cylinder apparatus. Such data can often be fitted to any of several curves. Data taken, using six or more cylinders at Mt. Washington, generally can be fitted to one curve only. It seems evident that, if one is attempting to measure three variables, more than two measurements should be made.
Figure II-8a. Ice accretion obtained by exposure on Mt. Washington, N. H. (Photo, courtesy Victor Clarke, Mt. Washington, Observatory) 15-minute exposure, 7°F, 47 mph, average drop size 12.2 microns diameter, 0.21 grams water/cubic meter of air. April 8, 1946.

fit. The computed drop radius is used to check the correctness of the Ru curve. If an error has been made, the data are fitted to another Ru curve. The convergence of the method is rapid.
A Discussion of the Dimensionless Groups Used to Present Trajectory Information

In the initial derivation of the equations for the droplet trajectories, two dimensionless groups appeared, namely, $R_u$ and $\phi'$. The plotting versus $K$ ($= r_u/\nu$) permitted comparison to the Stokes' law solutions. The zero impingement at a critical value of $K$ becomes apparent. The parameter $\phi$ is useful in work where the droplet diameter is not known ($\phi = R_u \cdot \nu$).
Figure II-9. Experimental points obtained on rotating cylinders.

Figure II-13	extsuperscript{h} shows a graph of $E_M$ versus $R_u$ with $\wp$ as a parameter. According to this figure, even at very large values of velocity, $E_M$ does not necessarily approach unity. Plotting with these two parameters permits one to see the effect of velocity.

Drell and Valentine\textsuperscript{5} have plotted the data in yet another fashion. Figure II-14 shows a graph of $E_M\wp$ versus $\wp$ for several values of $R_u$. The product $E_M\wp$ is proportional to catch rate (for given speed, liquid-water content, drop size) and $\wp$ is proportional to cylinder size. This graph shows that for a given icing condition and speed there is one size of cylinder which ices more heavily than any other. Larger cylinders have low-percentage catch; smaller cylinders have less frontal area.

Spheres and Ribbons

Calculations have been made for shapes other than cylinders. Langmuir and Blodgett have published curves showing trajectory information for flow.
Figure II-10. Replot of data from Reference 2.

Figure II-11. Comparative fit, Figures II-9 and II-10.
Figure II-12. Comparison between data of Reference 2 and flight measurement (Reference 3).

Figure II-13. Replot of data from Reference 2 to show effect of velocity on catch (Reference 4).
past spheres and ribbons. The sphere data look very much like the cylinder calculations, as illustrated in Figure II-8, showing ice accretions on three spheres exposed atop Mt. Washington, N. H.

The trajectories past ribbons or flat plates perpendicular to the airstream have a different characteristic. The maximum ice accretion does not occur at the stagnation line of the ribbon but occurs on either side of the centerline at a distance \( Y/b = 0.8 \) (\( 2b = \) ribbon width). Figure II-15 shows ice accretions obtained on a flat piece of wood placed transverse to the windstream atop Mt. Washington. The board has a gentle taper which illustrates the effect of size rather well.

The Langmuir and Blodgett report not only presents data on the important parameters \( E_M, \theta_M \) and \( \beta \) but also contains data on the velocity of the drops upon impact, the angle of impact and the variation of \( \beta \) (= \( dE/d\theta \)) around the cylinder, sphere or ribbon \( \beta = dE/d(y/b) \). A few results for the case of a viscous fluid (instead of an idealized nonviscous fluid) flowing around a sphere are also given. The serious worker in the icing field should make it a point to study carefully this classical report.
Figure II-15. Icing of different-width "ribbons" is illustrated on tapered board placed flat side towards wind. Note that the maximum ice accretion occurs to either side of the stagnation region. (Photo, courtesy Victor Clarke, Mt. Washington Observatory) 15-minute exposure, 10°F, 67 mph, average drop size 9.4 microns diameter, 0.28 grams liquid water/cubic meter of air. April 7, 1946.
Airfoils

The trajectories about airfoils have been studied by a number of workers. Bergrun\textsuperscript{6} has calculated the trajectories about a 12 per cent thick Joukowski profile using a numerical method. He presents curves which are useful if one is forced to resort to hand calculations, though even with these curves, the calculations are long and tedious. Figure II-16 shows some of the results obtained by Bergrun.

Guibert, Janssen, and Robbins\textsuperscript{7} have calculated the trajectories for a symmetrical 15 per cent thick Joukowski airfoil at 0, 2 and 4° angle of attack. They also calculated the trajectories for a cambered 15 per cent thick Joukowski profile at 0° angle of attack. In this report\textsuperscript{7} data are also given showing the velocity of the drop at impingement.

In work with airfoils most workers have used the chord length of the airfoil as the significant parameter in the dimensionless modulus. Drell and Valentine\textsuperscript{5} have shown, however, that if one uses the airfoil thickness as the significant dimension, the data for the various airfoils are brought closer together and the problem of interpolating between airfoils is less difficult. The results are still far from satisfactory, as shown in Figure II-17 but the improvement from the point of view of a designer is significant. In Figure II-17,
following Drell and Valentine, the product $E_M \psi$ is plotted against $\psi$ since one is more interested in total catch than in fractional catch.

The Supersonic Wedge

Before leaving the trajectory problem, two analytical solutions are of interest. One unpublished solution pertains to the trajectories in supersonic flow about an infinite wedge. The calculation of these trajectories is simple and straightforward.

A wedge travelling at supersonic speeds has a field of flow about it such as shown in Figure II-18. Before the shock wave the flow is uniform. After the shock wave the flow is again uniform but parallel to the wedge surface. Ahead of the shock wave the drops are at rest with respect to the undisturbed air. If the shock wave is treated as a surface of discontinuity and there is no disruption due to the aerodynamic forces, the drop emerges from the shock wave with a finite velocity relative to the flow behind the shock. Now, since the field
$M_1 = 1.767$
$P_1 = 16.88''Hg$
$|V_1| = 1802\text{ ft/sec}$
$\rho_{a1} = 0.00160\text{ slugs/ft}^3$
$T_1 = 435^\circ R$

$M_2 = 1.417$
$P_2 = 27.9''Hg$
$|V_2| = 1555\text{ ft/sec}$
$\rho_{a2} = 0.00228\text{ slugs/ft}^3$
$T_2 = 504^\circ R$

Figure II-18

of flow in front of and behind the shock wave is uniform, we may change the frame of reference and consider a coordinate system which moves with the velocity $V$ of the air stream behind the shock wave. In this frame of reference the air is stationary, and we have then the problem of a drop projected into still air with an initial velocity $U$.

From Newton's law,

$$F = m \frac{dU}{dt},$$

where

$$m = \frac{4}{3} \pi a^3 \rho_d = \text{mass of drop}$$

$$F = -\frac{1}{2} \rho a u^2 C_D \pi a^2 = \text{drag force}$$

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\[ \rho_w = \text{water-drop density} \]
\[ \rho_a = \text{air density} \]
\[ a = \text{drop radius} \]
\[ C_D = \text{drag coefficient, drop to air} \]

The drop velocity as a function of time is given by
\[ \frac{dU}{dt} = -\frac{3}{8} \frac{\rho_a}{\rho_w} \frac{U^2}{a} C_D. \]

The drag coefficient, \( C_D \), is a function of the Reynolds Modulus, \( R \); hence, introducing the Reynolds Modulus
\[ R = \frac{2 U a \rho_a}{\mu_a} \]
\( \mu_a = \text{viscosity of air}, \)
\[ \frac{dR}{dt} = -\frac{3}{8} \left( \frac{\rho_a}{\rho_w} \right) \frac{C_D}{a} \frac{a^2 R^2}{2a \rho_a}. \]

Finally, then,
\[ \int_{R_0}^R \frac{dR'}{C_D R'^{\frac{3}{2}}} = -\frac{3}{16} \frac{\mu_a t}{a^2 \rho_w}. \]

The right-hand term is a dimensionless measure of the time required for a sphere at initial Reynolds modulus \( R_0 \) to drop to the speed given by \( R \). The integral may be evaluated graphically from a graph of \( C_D^{-1} \) versus \( 1/R \).

After the velocity has dropped low enough for Stokes' law to be valid, the equation above may be integrated directly.
\[ C_D = \frac{24}{R} \]
\[ \frac{9}{2} \frac{U(t - t_0)}{a^2 \rho_w} = \int \frac{dR}{R} = \ln \frac{R}{R_0}. \]
The above equation indicates that the drop never comes to rest! Langmuir has pointed out, however, that the drop travels only a finite distance. When the velocity becomes sufficiently low, Brownian movement predominates, and the above equations are no longer descriptive of the motion.

Figure II-19 shows a graph of \( R \) versus \( U(t-t_o)/4 a^2 \rho_w \) for \( R_o = 1800 \) at \( t = t_o \).

A second integration is required to obtain the "range" of the spherical droplet

\[
R = \frac{2a \rho_a}{\mu a} \frac{dx}{dt}
\]

\[
dx = \frac{R \sqrt{a}}{2a \rho_a} \ dt.
\]

Dividing both sides of the equation by \( \rho_w a \) and transposing \( \rho_a \),

\[
\frac{\rho_a}{\rho_w} d \left( \frac{x}{a} \right) = R \ d \left( \frac{u(t - t_o)}{a^2 \rho_w} \right).
\]

The integration of the above equation may be accomplished since \( R \) is known as a function of the dimensionless time. (Curve A, Figure II-19). The results of the second integration are shown as curve B of Figure II-19. The data from these curves may be used to compute the droplet trajectory as follows:

Consider the flow field defined in Figure II-18. The following conditions prevail:

| Wedge half angle | 10° |
| Mach number      | 1.767 (ahead of shock) |
| Altitude         | 15,000 ft |
| Air temperature  | -25°F |
| Drop diameter    | 20 microns = 6.56 x 10^{-5} ft |
| Mach number after shock | 1.417 |
| Pressure ratio across shock | 1.655* |
| Air-density ratio across shock | 1.428* |
| Velocity of sound ahead of shock | 1021 ft/sec |
| Shock wave angle | 45° |
| Temperature ratio across shock | 1.156* |
| Ratio of velocities across shock | 0.862* |

The relative velocity of the drop after the shock is 379 ft/sec calculated from the vector difference between \( V_1 \) and \( V_2 \). The droplet initial Reynolds modulus is

\[
R = \frac{2a U \rho a}{\kappa a} = 154
\]

The dimensionless time for a drop to come to this value of \( R \) from an initial value \( R_0 = 1800 \) is read from Figure II-19, curve A as \( 1.25 \times 10^{-2} \). Curve B shows that the drop would travel a dimensionless distance (from \( R = 1800 \) to \( R = 154 \)) of 6.10. The maximum range of the drop (from \( R = 1800 \) to \( R = 0 \)) is read from curve B as 8.63. Therefore the range \( \lambda \) of the drop initially at \( R = 154 \) is

\[
\lambda = \frac{2.53 \times 2a \mu}{\rho a}
\]

\[
\lambda = 0.141 \text{ ft}
\]

The time required to travel half this distance is calculated by entering curves A and B in reverse order. Table I was constructed in this fashion:

<table>
<thead>
<tr>
<th>Distance Traveled by Droplet Relative to Airstream (dimensionless)</th>
<th>Time Required to Traverse this Distance (dimensionless)</th>
<th>Time (seconds)</th>
<th>Distance Relative to Airstream (ft)</th>
<th>Distance Moved by Airstream Parallel to Wedge Surface in this Time (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.53</td>
<td>3.3 \times 10^{-2}</td>
<td>7.46 \times 10^{-4}</td>
<td>0.141</td>
<td>1.16</td>
</tr>
<tr>
<td>1.90</td>
<td>1.4</td>
<td>3.16</td>
<td>0.106</td>
<td>1.16</td>
</tr>
<tr>
<td>1.29</td>
<td>0.58</td>
<td>1.31</td>
<td>0.070</td>
<td>0.49</td>
</tr>
<tr>
<td>0.63</td>
<td></td>
<td></td>
<td>0.035</td>
<td>0.20</td>
</tr>
</tbody>
</table>

The above trajectories may be transformed from the moving coordinate to a coordinate system at rest with respect to the airfoil by straightforward geometrical considerations. Thus we obtain Table II.
TABLE II

<table>
<thead>
<tr>
<th>Initial Distance of Droplet relative to Wedge Centerline (ft)</th>
<th>Position at which Drop Strikes Wedge (ft)</th>
<th>Average Rate of Ice Accretion if Liquid Water Content of Air is 1 gram/m$^3$ (lbs/hr)</th>
<th>Icing Rate, Average between Nose and this Point (inches/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.035</td>
<td>0.24</td>
<td>60.0</td>
<td>0.20</td>
</tr>
<tr>
<td>0.071</td>
<td>0.56</td>
<td>51.0</td>
<td>0.18</td>
</tr>
<tr>
<td>0.107</td>
<td>1.27</td>
<td>34.0</td>
<td>0.12</td>
</tr>
<tr>
<td>0.143</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Later considerations will show that aerodynamic heating will prevent ice at these conditions, but if the temperature were -40°F, the icing would occur despite the frictional heating.

Trajectories Near a Stagnation Point

G. I. Taylor$^9$ has found an analytical solution for the case of a drop in the vicinity of a stagnation point and small enough to follow Stokes' law. The solution shows clearly some of the results which occur in the more complicated cases. The equations are:

$$\frac{4}{3} \pi a^3 \rho_d \frac{du_d}{dt} = \frac{1}{2} \pi a^2 C_D P (u_a - u_d) \rho_a$$

$$\frac{4}{3} \pi a^3 \rho_v \frac{dv_a}{dt} = \frac{1}{2} \pi a^2 C_D P (v_a - v_d) \rho_a$$

where

- $a$ = drop radius
- $\rho_d$ = drop density
- $u_d$ = drop x component of velocity
- $t$ = time
- $C_D$ = drag coefficient, spherical drop to air
- $P$ = relative velocity between drop and air
- $u_a$ = air x component of velocity
- $\rho_a$ = air density
- $v$ = y component of velocity

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If Stokes' law applies,

\[ C_D = \frac{2h}{R} \]

\[ R = \frac{2 a \rho d P}{\mu a} \]

\[ R = \text{Reynolds modulus} \]

\[ \mu a = \text{air absolute viscosity} \]

Substituting for the drag coefficient,

\[ \frac{2}{9} \frac{\rho w a^2}{\mu a} \frac{d u_d}{dt} + u_d = u_a \]

\[ \frac{2}{9} \frac{\rho w a^2}{\mu a} \frac{d v_d}{dt} + v_d = v_a \]

These equations are integrable separately if \( u_a \) and \( v_a \), respectively, are functions of \( x \) and \( y \) only. The velocity field must satisfy the continuity relation

\[ \frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y} = 0 \]

Hence, if we seek a solution such that

\[ u_a = u_a(x) \quad \text{and} \quad v_a = v_a(y) \]

the only possible solutions are of the form

\[ u_a = C_u - C_1 x \]

\[ v_a = C_v + C_1 y \]

These components correspond to streamlines which are right hyperbolae. Figure II-20 shows the flow field which is similar to the flow against a corner or a large flat plate. The flow field is also a good approximation to the immediate vicinity of a stagnation point.

Now let

\[ \dot{x} = \frac{d u_d}{dt} \quad \text{and} \quad \ddot{y} = \frac{d v_d}{dt} \]
Figure II-20. Flow field in a corner or near stagnation point.

then

\[ \frac{2}{9} \frac{\rho a^2}{K_a} \dot{x} + \dot{x} = C_u - C_1 x \]

\[ \frac{2}{9} \frac{\rho_w a^2}{K_a} \dot{y} + \dot{y} = C_v + C_1 y . \]

Now let

\[ x = x - \frac{C_u}{C_1} , \quad y = y + \frac{C_v}{C_1} \quad \text{and} \quad Z = \frac{9 K_a t}{2 \rho_w a^2} , \]

\[ \frac{d^2 x}{dz^2} + \frac{d x}{dz} = \frac{2}{9} \frac{C_1 \rho_w a^2}{K_a} x , \]
\[
\frac{d^2 Y}{d Z^2} + \frac{d Y}{d Z} = \frac{2}{9} \frac{C_1 \rho \omega a^2}{\mu a} Y
\]

If the initial position is \(X_o, Y_o\) and the initial velocity (Z scale of time) is \(X_o', Y_o'\), then the solutions to the above equations are

\[
X = e^{\frac{Z}{2}} \left[ \frac{2X_o' + X_o}{\alpha} \sinh \left( \frac{\alpha}{2} Z \right) + X_o \cosh \left( \frac{\alpha}{2} Z \right) \right]
\]

\[
Y = e^{\frac{Z}{2}} \left[ \frac{2Y_o' + Y_o}{\beta} \sinh \left( \frac{\beta}{2} Z \right) + Y_o \cosh \left( \frac{\beta}{2} Z \right) \right]
\]

where

\[
X_o' = \frac{dX}{dZ} \text{ at } Z = 0
\]

\[
X_o = X \text{ at } Z = 0
\]

\[
Y_o' = \frac{dY}{dZ} \text{ at } Z = 0
\]

\[
Y_o = Y \text{ at } Z = 0
\]

\[
\alpha = \sqrt{1 - \frac{8}{9} \frac{C_1 \rho \omega a^2}{\mu a}}
\]

\[
\beta = \sqrt{1 + \frac{8}{9} \frac{C_1 \rho \omega a^2}{\mu a}}
\]

For the case \(C_1 > 0\) (Figure II-19) the equation in \(Y\) is unaffected by whether \((8/9)(C_1 \rho \omega a^2/\mu a)\) is greater or less than unity. The equation in \(X\), however, shows two different behaviors. For small values of drop size, \(\alpha\) is real but always less than unity. The equation for \(X\) displays an exponential decrease but remains positive for all finite \(Z\), i.e., the drops do not reach the plane \(X = 0\) unless the initial velocity \(X_o'\) is sufficiently large. For large drops \(\alpha\) becomes imaginary and the hyperbolic functions become sine and cosine. The drop then exhibits an oscillatory damped motion as in Figure II-21. (The equations do not recognize the existence of the wall at \(X = 0\).)
Returning now to the case of small drops ($\alpha$ real) when $Z$ is large,

\[
\sinh \frac{\alpha Z}{2} \rightarrow \frac{1}{2} e^{\frac{\alpha Z}{2}} \quad \cosh \frac{\alpha}{2} Z \rightarrow \frac{1}{2} e^{\frac{\alpha Z}{2}}
\]

\[
X = e^{\left(\frac{1}{2}\frac{\alpha}{Z}\right)Z} \left[ \frac{2X_0' + X_0}{\alpha} + X_0 \right].
\]

If the initial position of the drop is a negative coordinate, the initial value $X_0'$ must be positive and at least equal to

\[
X_0' = -\frac{\alpha + 1}{2} X_0,
\]

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if the drop is ever to reach \( X = 0 \). Rewriting the above in terms of the physically significant parameters,

\[
\frac{dx}{dt} \text{ \text{critical}} = \frac{2}{4} \frac{\kappa_a}{\rho w u^2} \left( 1 + \sqrt{1 - \frac{3}{9} \frac{C_l \rho w u^2}{\kappa_a}} \right) \left( x_0 - \frac{C_n}{C_l} \right).
\]

The above velocity is the critical one required for a drop to just reach the plane \( X = 0 \) from initial position \( x_0 \).

Summary of Available Trajectory Data

Table III gives a summary of various trajectory data currently available in the literature.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Source</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinders</td>
<td>Albrecht, Ref. 1</td>
<td>Stokes' law regime</td>
</tr>
<tr>
<td>Cylinders</td>
<td>Glauert, Ref. 11</td>
<td>Stokes' law regime</td>
</tr>
<tr>
<td>Cylinders</td>
<td>Kantrowitz, Ref. 10</td>
<td>Oseen's law</td>
</tr>
<tr>
<td>Cylinders</td>
<td>Langmuir, Ref. 2</td>
<td>Experimental ( C_D ) data</td>
</tr>
<tr>
<td>Spheres</td>
<td>Langmuir, Ref. 2</td>
<td>Experimental ( C_D ) data</td>
</tr>
<tr>
<td>Ribbons</td>
<td>Langmuir, Ref. 2</td>
<td>Experimental ( C_D ) data</td>
</tr>
<tr>
<td>Stagnation region</td>
<td>Taylor, Ref. 9</td>
<td>Stokes' law</td>
</tr>
<tr>
<td>Airfoils</td>
<td>Glauert, Ref. 11</td>
<td></td>
</tr>
<tr>
<td>Uncambered, 15% thick</td>
<td>Guibert, Ref. 7</td>
<td>0, 2, and 4° angle of attack</td>
</tr>
<tr>
<td>Joukowski, Cambered, 15% thick</td>
<td>Guibert, Ref. 7</td>
<td>0° angle of attack</td>
</tr>
<tr>
<td>Joukowski, NACA 652-015</td>
<td>Guibert, Ref. 7</td>
<td>0° angle of attack</td>
</tr>
<tr>
<td>Uncambered, 12% thick</td>
<td>Bergrun, Ref. 6</td>
<td>0° angle of attack</td>
</tr>
<tr>
<td>Supersonic wedge</td>
<td>Tribus, Ref. 8</td>
<td></td>
</tr>
</tbody>
</table>
REFERENCES


CHAPTER III

REMOVING THE ICE

In the previous chapter we have considered the mechanism of ice deposition on streamlined bodies. There still remain important trajectory problems to be considered, such as new methods of obtaining trajectory data, the influence of compressibility on ice accretion, and the icing characteristics of the new types of airfoils. The previous material, even though it is incomplete, should serve as an introduction and guide to the current literature where new data are constantly being presented.

Detailed studies of what happens to a drop of water just as it strikes an airfoil surface have not been accomplished. Langmuir\(^1\) has suggested that "bounce off" may play an important role, though thus far experiments have not revealed the effect. For a while it was thought that the drops of water might blow off. However, unpublished experiments by Tracy B. Gardner indicate no "blowoff". Gardner used an airfoil with sintered-metal porous strips in the nose region and also inserted in the airfoil surface about one foot back from the leading edge. Water was pumped out through the leading edge and permitted to run along the surface and be collected in the second porous metal strip. The collected water was drained off by suction and compared in quantity with the ejected water. No blowoff was indicated. Figure III-1 shows the apparatus used by Gardner.

Boelten, Johnson, Sanders, and Rubesin\(^2\) have made detailed analyses of the forces which tend to remove a drop from the surface. They have also considered the relation between rate of evaporation and water catch as affecting the percentage of "wetted" area of a wing. The studies were exploratory but did not reveal net forces which would be expected to remove water droplets.

NACA experiments\(^3\), wherein blueprint paper was laid over the wing to reveal those areas which were wetted, showed that in the area of water
impingement the water covered the entire surface, but aft of this region the water flowed in rivulets. No blowoff was indicated in these traces, either. See Figures III-2a and III-2b.

This evidence has given rise to the general belief that once the drops of water impinge upon the airfoil, they remain on the surface. The drops may evaporate, freeze upon impact, roll along the surface individually (until they blowoff the trailing edge), flow in rivulets, or they may collect in depressions. If the airfoil is heated in one region and not in another, the water may flow back to the unheated regions and cause "runback" or "freeze-back". It must be noted in passing that "bounce off", if it does occur, has thus far not been found.
Figure III-2. Records of runback obtained by NACA (a) in flight and (b) in the tunnel. The records were obtained by wrapping a water-sensitive film around the wing leading edge. The wetted areas are clearly visible. The increased turbulence in the tunnel as compared to free flight is evident from the jagged appearance of the rivulets aft of the region of primary impingement (Reference 13).

The designer of an anti-icing system may elect to design his equipment to function in one of several ways:

(a) The entire airfoil surface may be heated above 32°F. The water will then run along the surface and off the trailing edge. This method is most suited to small objects such as jet-engine inlet guide vanes, pitot-tube masts, antennae, and helicopter blades. The method is also used for windshields, though the resultant ice accumulations at the window frames are often objectionable.

(b) The area of water impingement and a short region directly after it may be heated so intensely that all the impinging water is evaporated. This method is used in conventional heated wings. At moderate speeds (200 to 300 mph) the heat requirements are large but not excessive. At higher speeds the energy requirements place a severe burden upon the heat source. The heat may be supplied by hot air, exhaust gases, or electrical resistance heaters.

(c) The surface may be heated intermittently, permitting the ice to gather during the "off" period and melting the interface during the "on" period. This method is especially attractive on
propeller blades where the centrifugal forces are available to release the ice. The method is also used for wings if a "parting strip" at the leading edge is kept warm to prevent "capping". Two methods of heating a wing intermittently have been proposed in Germany and are reported in Reference 4. Electrical energy is usually used, but hot air may be employed intermittently.

(d) The surface may be unheated and the ice permitted to form until removed by mechanical means. The B. F. Goodrich Company markets a inflatable rubber "shoe" which fits over the wing surface and is operated pneumatically. Mechanical systems have an unfortunate history of not operating cleanly; i.e., fragments of ice often stick to the surface. Recently this company has announced some improvements which include a treatment of the rubber to lower the adhesion. Another type of mechanical de-icer is described by Robert Smith-Johannsen. Smith-Johannsen describes the very greatly diminished ice adhesion which occurs when the ice is frozen quickly. He also shows how the presence of impurities will markedly lower the ice adhesion. The "shoeshine de-icer" or "peel-off de-icer", as it was called, consisted of a very thin, less than 5 mm-thick, stainless-steel sheet which was wrapped around the leading edge of the wing and then pulled parallel to the wing surface in much the same fashion as one pulls a rag while shining a shoe. The movement was approximately a half inch. In trials on Mt. Washington the operation was reported as quite successful. The metal was considered preferrable to rubber since the thermal conduction helped remove the heat of fusion of ice and therefore aided quick freezing. Coatings containing special impurities to lower ice adhesion could also be incorporated. The de-icer has not been developed sufficiently for judgment on its efficacy to be made yet.

(e) A melting point depressant may be released in the area of water impingement. This last method has fallen into disfavor. Hardy has shown how the evaporation of the alcohol causes a refrigerating effect and thereby diminishes the alcohol effectiveness. Similarly, special pastes and finishes have been proposed from time to time, but without much success.

(f) The leading edge may be made of a porous material and hot air forced from the interior of the wing. A blanket of warm air is thus created over the icing region. A careful analysis of the method has not yet been made, but preliminary results indicate less hot air is needed than for conventional hot-air anti-icers. This system would not be advisable for a large wing because of the adverse effect
upon the boundary layer, but for a radar housing or other small object it might be very economical. Too few data are now available for adequate appraisal.

(g) Theodorsen and Clay\(^5\) suggested that the leading edge impingement area be heated to just above the freezing point and the "runback" collected in a small trough or slot. Tessman\(^5\) fifteen years later, made the same suggestion except that porous metal strips were to replace the trough or slot. Insofar as is known the method has never been tried except by Theodoreson and Clay, who reported success.

Heat Transfer Media

Of the various media suitable for transferring heat to the exterior surfaces, air has been the one found most practical. In 1931 the NACA\(^5\) demonstrated a system utilizing alcohol and water. Boiling occurred at the engine exhaust heater and condensation occurred in a small wing model. The alcohol prevented freezing. Systems wherein boiling may occur are usually too heavy for aircraft use because of the high pressures which may occur. Similarly liquids are not practical, particularly since such systems must be leak proof.

The sources of the hot air depend upon the particular airplane. In conventional propeller-driven aircraft with reciprocating engines the source of heat may be the exhaust gases. Although this sort of heat source looks attractive, particularly since the heat seems almost "free", close analysis has shown that considering the overall airplane performance separate gasoline-burning heaters are often superior. Each airplane has its own peculiarities, and where one airplane may utilize the exhaust gases separately for jet-propulsive assistance (as in the Douglas DC-6) and therefore may be unable to accommodate a heat exchanger, another may have a common exhaust manifold which is so located that the addition of a heat exchanger may be made quite easily (as in the Convairliner).

In jet-propelled aircraft the hot air is often taken from the last stage of compression. The hot compressed air may be carried in relatively small conduits since the pressure drop available for distributing the air is high. The performance loss in the engine is serious. An extraction of 1 per cent of the airflow causes approximately a 2 per cent loss in thrust. Some designers have considered extracting air from an early stage of compression and utilizing gasoline-burning heaters for raising the air temperature.

As an example of a comparison between two types of heating systems, Jongeneel and David\(^5\) presented the data of Table I during a symposium on anti-icing in 1946. The data refer to the DC-6-type airplane.
### TABLE I

**WEIGHT ANALYSIS OF POSSIBLE HEAT SOURCES FOR A 4-ENGINE CARGO AIRPLANE**

<table>
<thead>
<tr>
<th></th>
<th>(a) Combustion Heaters</th>
<th>(b) Collector Ring</th>
<th>(c) Short Stack</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total heat required</td>
<td>947,000</td>
<td>316,000</td>
<td>316,000</td>
</tr>
<tr>
<td>Heat per engine (d)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuel weight for 2 hrs</td>
<td>132</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight of system, excluding fuel</td>
<td>530</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td>BHP required per engine for long-range cruise (e)</td>
<td>910</td>
<td>830</td>
<td>HP</td>
</tr>
<tr>
<td>Exhaust back pressure causes a loss per engine</td>
<td>9</td>
<td>6</td>
<td>HP</td>
</tr>
<tr>
<td>Additional power loss due to loss of exhaust jet thrust</td>
<td>0</td>
<td>40</td>
<td>HP</td>
</tr>
<tr>
<td>Total loss per engine</td>
<td>9</td>
<td>46</td>
<td>HP</td>
</tr>
<tr>
<td>Total loss to airplane</td>
<td>36</td>
<td>184</td>
<td>HP</td>
</tr>
<tr>
<td>Fuel weight, sfc = 0.45, 16 hours flight (f)</td>
<td>260</td>
<td>1325</td>
<td>1lb</td>
</tr>
<tr>
<td>Total Weight</td>
<td>662</td>
<td>860</td>
<td>1925</td>
</tr>
</tbody>
</table>

(a) Combustion heaters are mounted in the wings and tail. The engines are equipped with jet stacks.

(b) A collector ring is mounted on each engine, the exhaust gases manifold together and passed through a heat exchanger. The loss in jet thrust is not charged against anti-icing.

(c) Some of the exhaust gases are diverted to an exchanger.

(d) Supply of heat sufficient for one engine inoperative.

(e) Less HP is required if jet thrust available from engine exhaust.

(f) Two hours in icing in 16 hours flying.

The air temperature is limited by the allowable temperature (less than 300°F) of the aluminum alloys used in airfoil construction. The softening temperature of the plastic used in safety glass similarly limits the temperature in a windshield anti-icing system to less than 200°F.
Several proposals have been made to extract exhaust gases from the engine and to dilute them with fresh air. In a jet engine the relative purity of the exhaust gases and the moderate temperature make the system attractive. No announcements have been made of such systems in actual use.

Great care must be exercised in the design of the leading-edge construction of large wings. The thermal expansion at the leading edge may cause severe stresses. An early experimental Navy flying boat was reported to have had such a severe leading-edge expansion that the ailerons were locked in place as the trailing edge was compressed. More recently an experimental airplane was reported to have "popped its rivets" during a ground run-up.

Economical Aspects of Ice Protection

Most workers in the field of ice prevention like to feel that the equipment being designed will be used to enable the pilot to fly through an icing condition without affecting his schedule or course. This happy day has not yet dawned, however, and most ice protection equipment is still looked upon for emergency use only. Since neither the military nor the airlines reckon the value of a life in terms of dollars and cents, it is difficult to discuss the worth of the de-icing apparatus.

Nevertheless, some figures are available on the cost of operating anti-icing equipment in the airlines. Dave North and Dan Beard of American Airlines have made a critical comparison of the costs of operating the Douglas DC-6 (combustion heaters) and the Convairliner 240 (heat exchanger). Table II gives a summary of the data reported by North and Beard.\textsuperscript{11}

Unfortunately these data represent "debugging" or breaking-in experience so that the cost figures must be considered as tentative. On the other hand, most airplanes have short lives so that the "debugging" period is an appreciable fraction of the life span.

In the field of military aircraft a careful "operational analysis" is required. The added weight of de-icing equipment detracts from the payload or top speed which may therefore result in greater loss due to enemy action. This loss must be balanced against the probable loss by icing and the military advantages of being able to fly in ice.

The commercial airliner must balance the cost of the equipment against the loss in revenue from schedule delays and possible crashes.

These economical aspects of the de-icing problem have not been studied as thoroughly as have the scientific problems. Those few persons who have had
to face this problem have almost invariably concluded that what is needed is a removable anti-icing system, one which could be installed for special missions and left off for others.

Only the mechanical de-icer of the Goodrich type at present meets this removability requirement. Such a system also enjoys the advantage that a company may specialize in de-icer design and therefore prepare de-icers for several airplanes. The continued sale of rubber de-icers in the face of competition from other more effective systems (viz., thermal) is probably directly attributable to this latter advantage.

### TABLE II

**COMPARATIVE COSTS FOR TWO COMMERCIAL DE-ICING SYSTEMS**

<table>
<thead>
<tr>
<th></th>
<th>Douglas DC-6</th>
<th>Convair 240</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>WING AND TAIL</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heater replacement</td>
<td>$2400</td>
<td>$24,000</td>
</tr>
<tr>
<td>Heater maintenance</td>
<td>384</td>
<td>1,520</td>
</tr>
<tr>
<td>System maintenance</td>
<td>312</td>
<td>255</td>
</tr>
<tr>
<td>System replacements</td>
<td>328</td>
<td>225</td>
</tr>
<tr>
<td>Fuel cost</td>
<td>576</td>
<td>---</td>
</tr>
<tr>
<td>Fuel required to make up thrust loss</td>
<td>---</td>
<td>10,500</td>
</tr>
<tr>
<td><strong>Subtotal for wings and tail</strong></td>
<td>$4000</td>
<td>$16,000</td>
</tr>
<tr>
<td><strong>PROPELLER</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heated &quot;shoes&quot;</td>
<td>$1920</td>
<td>$2,880</td>
</tr>
<tr>
<td>Maintenance of &quot;shoes&quot;</td>
<td>320</td>
<td>240</td>
</tr>
<tr>
<td>Maintenance of system</td>
<td>160</td>
<td>120</td>
</tr>
<tr>
<td>System replacements</td>
<td>400</td>
<td>260</td>
</tr>
<tr>
<td><strong>Subtotal for propellers</strong></td>
<td>$2800</td>
<td>$3,500</td>
</tr>
<tr>
<td><strong>WINDSHIELDS</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electrically-heated glass, breakage</td>
<td>$-----</td>
<td>$1,150</td>
</tr>
<tr>
<td>System maintenance</td>
<td>----</td>
<td>115</td>
</tr>
<tr>
<td>System replacements</td>
<td>----</td>
<td>135</td>
</tr>
<tr>
<td>Hot-air windshield costs</td>
<td>400</td>
<td>-----</td>
</tr>
<tr>
<td><strong>Subtotal for windshields</strong></td>
<td>$400</td>
<td>$1,400</td>
</tr>
</tbody>
</table>

Based on a 4000-hour period, 6 per cent of flying time is in icing. The two systems are not strictly comparable. The Convairliner is lower to the ground (increased propeller abrasion) and has development problems in the relatively unconventional exhaust heater system.
REFERENCES


CHAPTER IV

THE ENERGY TRANSFERS AT AN ICING SURFACE

(Analysis of the Separate Modes of Energy Transfer)

At a nonporous* surface exposed to the impingement of super-cooled water droplets seven modes of energy transfer are possible. These modes of energy transfer are depicted in Figure IV-1, where the arrows denote the direction of energy flow. (The radiation term is negligible at conventional flight speeds in icing.) If heat is supplied from behind the icing surface, a word equation for the heat balance is:

The heat flux from the ice side of the ice-to-air interface equals the heat loss to the air boundary layer by convection the frictional heating by the air stream the energy loss via evaporation or sublimation the heat release due to fusion of ice the heat loss due to sensible heating of the water the gain from kinetic energy of the drops radiation to the surroundings.

*D. M. Patterson has called attention to the porosity of some ice formations and suggested that leakage of air under and around an ice formation may be significant. Adequate observations are not yet available.
If the surface temperature is above 32°F the term involving the heat of fusion does not appear. (References 1, 2, 3, and 4 treat the heat loss from surfaces in icing where the temperature is above the freezing point.)

Below the freezing point the formation of ice liberates heat and the surface temperature is raised until the other modes of energy transfer are sufficient to offset the heat of fusion. Figures IV-2a, b, c are taken from NACA icing wind-tunnel measurements and serve to illustrate the heat-of-fusion term in the energy balance. In each figure the curves represent records made of the time-temperature histories of propeller blades exposed to icing conditions. The curves are particularly interesting because in each case we see the temperature of the blade before and after the water sprays are turned on. The temperature rise is the greatest when the liquid-water content is large.

Energy flux when the surface temperature is below freezing is treated in References 6, 7, and 8.
Figure IV-2a. (Figure 10f, Ref. 5) 800 rpm, 5°F ambient, \(r/R = 0.33\), 0.4 grams/meter\(^3\). Heat of fusion causes 10°F temperature rise.

In References 1 through 8 the various modes of energy transfer have been treated as though they were independent of one another. Unfortunately the experimental difficulties have been too great to permit close checking upon the individual terms during icing. The best that can now be said is that the data available from the experiments conducted on surfaces above the freezing point do not contradict this view.

In this chapter we consider the separate terms in the energy balance. The treatment given here must of necessity be brief, for each mode of energy transfer is a subject worthy of study by itself. In the next chapter the terms will be put together.

The Heat Loss by Convection

The convective heat loss to the air is confined (by definition) to a boundary layer. This boundary layer begins at the stagnation region as a
Figure IV-2b. Measurements by Lewis for internally heated blade. Liquid water content of air, 0.3 grams/meter$^3$.

Figure IV-2c. Measurements by Lewis for internally heated blade. Liquid water content of air, 0.9 grams/meter$^3$. 
relatively thin laminar flowing film which increases in thickness along the surface. At some point, or more likely, over a region of the surface, the laminar layer changes to a turbulent one. The blanket of warm air in the boundary layer retards the heat flux from the regions downstream of the heated leading edge.

For a surface heated to a uniform temperature the heat flux via convection is usually calculated from the equation

\[ q_c = h_c \, A \, (t_s - T_1), \]

where

- \( q_c \) = convective heat flux, BTU/hr
- \( A \) = area of heat flux, ft\(^2\)
- \( t_s \) = surface temperature, °F
- \( T_1 \) = air temperature outside the boundary layer, °F
- \( h_c \) = unit thermal conductance, defined by above equation, BTU/hr ft\(^2\) °F.

The unit thermal conductance for an isothermal surface depends on the velocity distribution in the boundary layer and on the fluid properties. For most systems the conductance is determined experimentally, though for certain simple shapes analytical methods are available. Eckert \( ^9 \) gives a description of some of the simpler techniques. Seban \( ^{10} \) surveys the methods available for laminar flow when both the pressure and surface temperature are not constant. Martinelli \( ^{11} \) discusses turbulent flows and presents and empirical method which appears to be as accurate as any other available and considerably more rapid than most.

The case of laminar flow yields to analytical treatment more easily than does turbulent flow. For many systems, however, experimental methods are more suitable, particularly since the pressure distribution and the location of the transition point are not always known in advance. Giedt \( ^{12} \) has shown how artificially induced turbulence may change the heat transfer by more than 20 per cent. Lewis and Gelder \( ^{13} \) have shown how the turbulence from a spray bar 40 feet upstream moved the transition point on a wing model and increased the heat transfer. (We shall later see that under some circumstances this effect is not important.)

The unit thermal conductance may be measured for a particular body by conducting an experiment in which \( q, A, t_s, \) and \( T_1 \) are measured simultaneously. The conductance is then calculated from

\[ h_c = \frac{q}{A(t_s - T_1)}. \]
One technique for measuring $h_c$ is to put an electrical heating element on the surface, properly insulated from the back side, and measure the power input, surface temperature, and free-air temperature. Such an arrangement is described on page 76 of Reference 1 and also in Reference 14. In each of these references a thin nichrome ribbon heater was laid over cork insulation wrapped around the leading edge.

First approximations to the heat transfer in the laminar boundary layer may be obtained by treating the leading-edge region as a cylinder and the afterbody as a flat plate.\textsuperscript{11} (The method of Seban\textsuperscript{10} should be used where more accuracy is needed.) A suitable empirical expression for the heat flux from an isothermal cylinder ($\pm 15$ per cent) is:

$$h_c = 0.194 T_f ^{0.49} \left( \frac{U_\infty \gamma}{D} \right)^{0.5} \left[ 1 - (\phi/90)^3 \right]$$

$h_c$ = "local" unit thermal conductance, BTU/hr ft$^2$ \textsuperscript{o}F
$T_f$ = "film" or average boundary layer temperature, \textsuperscript{o}R
$U_\infty$ = air velocity far from cylinder, ft/sec
$\gamma$ = air density, lbs/ft$^3$
$D$ = cylinder diameter, ft
$\phi$ = angle measured from stagnation point, degrees ($0 \leq \phi < 60^\circ$).

For an isothermal flat plate in laminar flow an empirical equation is

$$h_c = 0.0562 T_f ^{0.5} (U_\infty \gamma)^{0.5} x^{-0.5}$$

$x$ = distance from stagnation point, ft.

Chapman and Rubesin\textsuperscript{15} have shown how the above equation may be seriously in error for nonuniform temperature systems if the flow is laminar. The criterion for the importance of the nonuniformity of the surface in the case where the surface temperature varies linearly with distance from the stagnation point is

$$x \left( \frac{dt}{dx} \right)/(t_s - T_L) \approx 1,$$

where

$x$ = distance from stagnation point
$dt/dx$ = temperature gradient (constant)
$(t_s - T_L)$ = surface to air temperature difference at $x = 0$.

Figure IV-3 shows the effect of nonuniformity of surface temperature as a function of the above parameter.\textsuperscript{16}

For the case of turbulent flow Martinelli recommends that the equation for a flat plate be used except that the local velocity should be
Figure IV-3. Heat transfer from a flat plate in laminar flow with linearly increasing temperature difference in the flow direction.

substituted for the (uniform) velocity over a flat plate. A suitable empirical expression for the unit thermal conductance over an isothermal flat plate is

\[ h_c = 0.51 T_f^{0.3} (U_1 \gamma)^{0.8} x^{-0.2} \]

\( T_f \) = "film" or average boundary layer temperature, °R
\( U_1 \) = velocity just outside boundary layer, ft/sec
\( \gamma \) = air density, lbs/ft³
\( x \) = distance back from stagnation point, ft.

For the turbulent boundary layer over a nonisothermal flat plate Scsa17 has shown how the corrections by Rubesin18 may be used to predict heat transfer in the vicinity of an abrupt change in surface temperature.

A particular case of interest is one where the surface is unheated for a distance, \( S \), and then the heating starts. Such a case arises during intermittent heating of separate wing sections. The Rubesin analysis yields a correction of the form

\[ h_c = h_c \left[ 1 - \left( \frac{S}{x} \right)^{39/40} \right] - \frac{7}{39}, \]

nonisothermal isothermal

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where

\[ x = \text{distance from leading edge} \]
\[ S = \text{length of unheated starting section}. \]

Scsela\textsuperscript{17} obtains excellent agreement between the above expression and a set of experimental data over a wide range of \((S/x)\) and Reynolds modulus.

The Heat Gain Due to Frictional Heating

The growth of the boundary layer is brought about by the viscous nature of the fluid. As the laminations of air flow past one another, the viscosity causes a drag force to be exerted. This frictional drag is an irreversible working of the fluid elements and results in an increase in temperature. The heating in any one lamination is offset by the thermal conductivity of the fluid which conducts heat from the hotter to the colder laminations. In the case of air, the net result of these two opposing effects is an increase in the temperature near an insulated wall. The temperature rise of the fluid adjacent to the surface when there is no heat flux is described by

\[ t_e - T_\text{l} = r \frac{U_1^2}{2gJCP} , \]

where

- \( t_e \) = "equilibrium" surface temperature °F
- \( T_\text{l} \) = air static temperature just outside boundary layer, °F
- \( U_1 \) = air velocity just outside boundary layer, ft/sec
- \( C_p \) = air heat capacity, BTU/lb °F
- \( g = 32.2 \text{ ft/sec}^2 \)
- \( J = 778 \text{ ft lb/BTU} \)

Experimentally, for air, it has been found that the recovery, \( r \), defined in the above equation is approximately 0.85 for laminar and 0.92 for turbulent boundary layers.

Where aerodynamic heating is important, it is customary to redefine the unit thermal conductance as follows:

\[ q = hC A (t_s - t_e) , \]

or

\[ q = hC A (t_s - T_\text{l} - r \frac{U_1^2}{2gJCP}) . \]
The net effect of aerodynamic heating is therefore to supply a heat source of relative strength

\[
q = h_c A r \frac{U_1^2}{2 g J C_p}
\]

(friction)

References 19 and 20 discuss aerodynamic heating in greater detail.

Sublimation or Evaporation

The evaporation or diffusion of water from a wetted or iced surface proceeds whenever the vapor pressure at the surface exceeds the vapor pressure in the surrounding air. The diffusion is hindered by the boundary layer of water vapor which grows as the air moves downstream along the surface. The mechanism for the water boundary layer growth and diffusion is similar to the mechanism whereby heat is convected; hence it is quite natural to define a conductance for mass transfer by the equation

\[
g = h_m A (H_s - H_i)
\]

where

\[
g = \text{mass flux, lbs H}_2\text{O/hr}
\]
\[
A = \text{area of evaporation, ft}^2
\]
\[
H_s = \text{humidity at surface, lbs H}_2\text{O/lb air}
\]
\[
H_i = \text{humidity outside boundary layer lbs H}_2\text{O/lb air}
\]

These define \(h_m\) = unit mass conductance, lbs/hr ft\(^2\). The above definition is not unique. The vapor pressure or water concentration could just as well have been chosen as potentials instead of humidity.

Experiments\(^{21}\) have shown that, when defined as above, the unit mass conductance bears a simple relation to the unit thermal conductance for heat transfer, i.e., in the range of temperatures of interest in icing,

\[
\frac{h_m C_{pa}}{h_c} = 1.0,
\]

where \(C_{pa}\) = unit heat capacity of air BTU/lb \(^\circ\)F.

Because the humidity is not a convenient parameter, the working equations usually use the equation

\[
H_s - H_i = 0.62 \frac{B}{B} (P_s - P_i),
\]

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where:

\[ 0.62 = \text{ratio, molecular wt of water/mol. wt air} \]
\[ B = \text{barometric pressure, } H_g \]
\[ P_s = \text{surface vapor pressure of water, } H_g \]
\[ P_1 = \text{stream vapor pressure of water, } H_g. \]

Since each pound of water requires L BTU/lb during vaporization, the heat loss during evaporation is

\[ q_e = \frac{0.62 h_s L A}{B} \frac{h_v L A}{C_p a} (P_s - P_1) , \]

In the case where the surface temperature is nonuniform, the above relation becomes inaccurate.

Energy Transfer to or From the Impinging Water (Sensible heating and fusion)

Langmuir\textsuperscript{22} has shown that the droplets travel through the disturbed air so rapidly that they undergo negligible change in temperature during their transit time. If the rate of water impingement is \( R_w \) (lbs/hr ft\(^2\)) and the enthalpy of the water before and after impact is denoted by \( G_s \) and \( G_\infty \) (the usual symbols \( h \) or \( H \) for enthalpy were used in the previous sections for conductance and humidity, respectively), then the energy added to the water is

\[ q = R_w A (G_s - G_\infty) . \]

Water

If there is no change in phase (i.e., no ice formed) the change in enthalpy is

\[ G_s - G = C_p w (t_s - T_\infty), \]

where

\[ C_p w = \text{heat capacity of water, BTU/lb °F.} \]

The above equation applies for \( t_s > 32°F \).

If ice forms,

\[ G_s - G_\infty = C_p I (t_s - 32) - 144 + C_p w (32 - T_\infty), \]

where

\[ C_p I = \text{heat capacity of ice, BTU/lb °F} \]
\[ 144 = \text{heat of fusion of ice at } 32°F. \]
Thus we have two different equations depending upon whether the temperature is above or below the freezing point:

\[ q_{\text{sensible}} = R_w A \left( C_{p_w} t_s - T_\infty \right) \]
\[ q_{\text{sensible}} = R_w A \left[ -144 + 32 \left( C_{p_w} - C_{p_f} \right) + C_{p_f} t_s - C_{p_w} \right] \quad t_s > 32^\circ F 
+ \text{fusion} \]

\[ q_{\text{sensible}} = R_w A \left[ -144 + (32 - t_s) \left( C_{p_w} - C_{p_f} \right) \right] \quad t_s < 32^\circ F \]

This last equation is separated into two parts, as follows:

\[ q_{\text{sensible}} = R_w A \left( C_{p_w} t_s - T_\infty \right) \quad \text{all } t_s \]
\[ q_{\text{fusion}} = R_w A \left[ -144 + (32 - t_s) \left( C_{p_w} - C_{p_f} \right) \right] \quad t_s < 32^\circ F \]

The discussion of the appropriate equation when \( t_s = 32^\circ F \) will be deferred until the complete energy balance is taken up.

The term in \( R_w \) is taken from the trajectory data.

**Kinetic Energy of the Drops**

The drops arrive with velocity \( V \) and therefore have a kinetic energy

\[ \frac{V^2}{2g} \frac{J}{J} \]

with respect to the surface. The energy added to the surface thus is

\[ q_{\text{KE}} = R_w A \frac{V^2}{2g} \frac{J}{J} \]

The velocity possessed by the drops at impact may be calculated from the trajectory data. The contribution of this last term is small compared to the others, but not entirely negligible in the heat balance at high speeds. In most design work \( V \) may be taken equal to \( U_\infty \). In research work \( V \) may be taken from a calculation such as represented in Figure II-7.
REFERENCES


6. Tribus, M., "Intermittent Heating for Aircraft Ice Protection with Application to Propellers and Jet Engines". Trans ASME, November 1951.


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CHAPTER V

THE ENERGY TRANSFERS AT AN ICING SURFACE

(The Complete Energy Balance and Some Applications)

Messinger has distinguished the three important regimes of heating, as follows:

A. \( t_s > 32^\circ F \)
B. \( t_s = 32^\circ F \)
C. \( t_s < 32^\circ F \).

In the treatment which follows we shall follow the notation used by Messinger and consider each regime separately.

A. Surface Above 32^\circ F

The heat-balance equation is

\[
q = q_{\text{convection}} + q_{\text{evaporation}} + q_{\text{sensible}} - q_{\text{friction}} - q_{\text{kinetic}}
\]

or, from Chapter IV,

\[
\frac{q}{A} = h_c \left(t_s - T_o\right) - 2.9L/B (P_s - P_{\infty}) + R_w (t_s - T_o) - h_c r \frac{U_{\infty}^2}{2gJC_p} - R_w \frac{U_{\infty}^2}{2g J}.
\]
The above equation is awkward to handle, hence certain new groupings of the above variables are made which simplify (but do not clarify) calculations.

A parameter $b$ is defined

$$b = \frac{R_w C_p w}{h_c}$$

and the three functions

$$\Theta_1'' = \theta_s (1 + b) + \frac{2.91 L}{B} p_s$$
$$\Theta_2'' = \tau_\infty (1 + b) + \frac{2.91 L}{B} p_\infty$$
$$\Theta_3 = \frac{r}{C_p a} + b \left( \frac{U_\infty^2}{2 g J} \right),$$

where

$t_s$ = surface temperature, °F
$\tau_\infty$ = air temperature, °F
$L$ = heat of vaporization of water, BTU/lb
$p_s$ = saturation pressure of water at $t_s$, "Hg
$p_\infty$ = saturation pressure of water at $\tau_\infty$, "Hg
$B$ = barometric pressure, "Hg
$r$ = "recovery factor"
$U_\infty$ = free stream velocity ft/sec.

The heat flux equation is

$$(q/A) = h_c (\Theta_1'' - \Theta_2'' - \Theta_3)$$.

Note that $\Theta_1''$ depends primarily upon $t_s$, $\Theta_2''$ upon $\tau_\infty$ and $\Theta_3$ upon $U_\infty$, with $b$ as a parameter. Graphs of these functions are given in Figures V-1, V-2, and V-3. (The above equation is restricted to the case where $t_s$ is uniform and above 32°F.)

Hardy\textsuperscript{2} has suggested a different grouping of these variables

$$q = q_{\text{convection}} \left(1 + \frac{q_{\text{evaporation}}}{q_{\text{convection}}} \right) - q_{\text{friction}} + q_{\text{sensible}}$$

The ratio $(q_{\text{evaporation}}/q_{\text{convection}})$ is given the symbol $X$. From Chapter IV,

$$X = \frac{0.62 L}{B C_p a} \left[ \frac{p_s - p_\infty}{t_s - \tau_\infty} \right].$$
\[ \theta_i = t_s(1 + 0.47b) + 2.90 L_s P_s / B \]

for

\[ B = 29.92 \text{ "Hg (sea level)} \]

\[ P_s = \text{vapor pressure ice at } t_s (\text{"Hg}) \]

Figure V-1a
\[ \theta_1 = t_s (1 + 0.47b) + 2.90L_s P_s / B \]

for

\[ B = 24.89'' \text{Hg} \ (5,000 \text{ft}) \]

\[ P_s = \text{vapor pressure ice at } t_s ('' \text{Hg}) \]

Figure V-1b
\[ \theta_1 = t_s (1 + 0.47b) + 2.90 L_s \frac{P_s}{B} \]

for

\[ B = 20.58'' Hg (10,000 \text{ ft}) \]

\[ P_s = \text{vapor pressure ice at } t_s ('' Hg) \]

**Figure V-1c**
\[ \theta_I = t_s(1 + 0.47b) + 2.90L_s \frac{P_s}{B} \]

for

\[ B = 16.88'' \text{Hg (15,000 ft)} \]

\[ P_s = \text{vapor pressure ice at } t_s'' \text{Hg} \]
\[ \theta_2 = \tau_\infty (1+b) + 2.90 \frac{L_s P_\infty}{B} + 127b \]

for

\[ B = 29.92" Hg \text{ (sea level)} \]

\[ P_\infty = \text{vapor pressure } H_2 O \text{ at } \tau_\infty (" Hg) \]

Figure V-2a
\[ \theta_2 = T_\infty (1+b) + 2.90 L_s R_\infty / B + 127 b \]

for

\[ B = 24.89 \text{"Hg} \ (5,000 \text{ ft}) \]

\( R_\infty \) = vapor pressure \( H_2O \) at \( T_\infty \) ("Hg)

**Figure V-2b**
\[ \theta_2 = T_\infty (1 + b) + 2.90 L S \frac{P_\infty}{B} + 127 b \]

for

\[ B = 13.75 \text{"Hg (20,000 ft)} \]

\[ P_\infty = \text{vapor pressure H}_2\text{O at } T_\infty \text{"Hg) } \]

Figure V-2e
\[ \theta_3 = \frac{r}{c_p} + b \cdot \frac{U_{\infty}^2}{2gJ} \]

- \( r = 0.875 \)
- \( c_p = 0.241 \text{ BTU/lb. - °F.} \)
- \( g = 32.2 \text{ ft/sec}^2 \)
- \( J = 778 \text{ ft.lb./lb.} \)

Figure V-3
At low speeds (such as takeoff or landing approach) and neglecting sensible heating, the heat-flux equation is simply

\[ (q/A) = h_c (t_s - T_\infty) (1 + X). \]

The term \((1 + X)\) is thus a multiplier which shows how much greater the heat loss in wet air is as compared to dry air (if \(h_c\) is unchanged).

Tables of \(X\) are presented in Reference 3.

The term \(X/(1 + X)\) represents the heat loss which is actually used to evaporate water. Since \(X\) increases with temperature, it follows that a thermally efficient heated anti-icer has a high surface temperature.

B. Surface Below 32°F

The heat balance when \(t_s < 32°F\) is:

\[ q = q_{\text{convection}} + q_{\text{sublimation}} + q_{\text{sensible}} - q_{\text{fusion}} - q_{\text{friction}} - q_{\text{kinetic}} \]

From Chapter IV,

\[ (q/A) = h_c (t_s - T_\infty) + \frac{2.9L_s}{B} h_c \left( P_s - P_\infty \right) + R_w (t_s - T_\infty) - R_w \left[ \frac{144}{(32 - t_s)} (C_{PW} - C_{PI}) \right] - h_c r \frac{U_\infty^2}{2gJc_p} - R_w \frac{U_\infty^2}{2gJ}. \]

Again, define the \(\Theta\) functions

\[ \Theta_1 = t_s (1 + 0.47b) + 2.90L_s \frac{P_s}{B} \]
\[ \Theta_2 = T_\infty (1 + b) + 2.90L_s \frac{P_\infty}{B} + 127b, \]

where

- \(t_s\) = surface temperature, °F
- \(T_\infty\) = air temperature, °F
- \(L_s\) = heat of sublimation of ice, BTU/1b
- \(P_s\) = vapor pressure of ice at \(t_s\), "H_g
- \(P_\infty\) = vapor pressure of supercooled water at \(T_\infty\), "H_g
- \(B\) = barometric pressure, "H_g.

The heat flux equation is therefore

\[ q/A = h_c (\Theta_1 - \Theta_2 - \Theta_2) \]
Graphs of $\theta_1$ and $\theta_2$ are given in Figures V-4 and V-5.

C. Surface at 32°F

When $t_0 = 32°F$, there is present a two-phase system of ice and water, and an additional parameter is required to describe the nature of the surface conditions. Messinger$^1$ introduced the "freezing fraction", n. When $n = 1$, all the water is frozen and the surface runs dry. When $n = 0$, no ice is formed. For $0 < n < 1$, the surface is coated with ice and water.

The energy balance is written

$$ q = q_{\text{convection}} + q_{\text{evaporation}} + q_{\text{sensible}} - q_{\text{fusion}} - q_{\text{friction}} - q_{\text{kinetic}}, $$

or

$$ (q/A) = h_c \left( 32 - \theta_0 \right) + \frac{2.9L \cdot h_c}{B} \left( 0.180 - R_\infty \right) + R_w \left( 32 - \theta_0 \right) $$

$$ - n R_w 144 - h_c r \frac{U_\infty^2}{2gJ \cdot c_{pa}} - R_w \frac{U_\infty^2}{2gJ} $$

Define now the $\theta'$ functions

$$ \theta_1' = 32 \left( 1 + b \right) + \frac{2.9L \cdot P_\infty}{B} $$

$$ \theta_2' = \theta_0 \left( 1 + b \right) + \frac{2.9L \cdot P_\infty}{B} + 144 \ n \ b, $$

where

$\theta_0 = \text{free stream temperature, °F}$

$L = \text{heat of vaporization of water at 32°F}$

$P_\infty = \text{vapor pressure of water at 32°F, } ^\circ H_2$'

$B = \text{barometric pressure, } ^\circ H_2$'

The heat flux is thus

$$ (q/A) = h_c \left( \theta_1' - \theta_2' - \theta_3 \right). $$

The $\theta_1'$ and $\theta_2'$ functions are presented in Figures V-6 and V-7. Figures V-1 through V-7 follow the scheme of Reference 1 but are for all altitudes of interest.
\[ \theta_i = 32 (1 + b) + \frac{560}{B} \]

Figure V-4
\[ \theta_2' = \theta_2 (1 + b) + 3120 \frac{p}{B} + 144nb \]

for

\[ B = 29.92 \text{"Hg (sea level)} \]

\[ p = \text{vapor pressure H}_2\text{O at } T_{\text{Hg}} \]

Figure V-5a
\[ \theta_2' = T_\infty (1 + b) + 3120 P_b/B + 144 nb \]

for

\[ B = 24.89'' Hg \text{ (5,000 ft)} \]

\[ P_b = \text{vapor pressure } H_2O \text{ at } T_\infty ('' Hg) \]

Figure V-5b
\theta_2 = T_\infty (1+b) + 3120 P_\infty / B + 144 \text{ nb}

for

\quad B = 20.56'' Hg (10,000 ft)

\quad P_\infty = \text{vapor pressure H}_2\text{O at } T_\infty (''Hg)

Figure V-5c
\[ \theta_2^f = T_\infty (1 + b) + 3120 P_\infty / B + 144 \text{nb} \]

for \( B = 16.88 \text{"Hg (15,000 ft)} \)

\( P_\infty = \text{vapor pressure H}_2\text{O at } T_\infty (\text{"Hg}) \)

Figure V-5d
\[ \theta_2' = T_\alpha (1+b) + 3120 P_\alpha / B + 144 nb \]

for

\[ B = 13.75" \text{Hg} \ (20,000 \text{ft}) \]

\[ P_\alpha = \text{vapor pressure } H_2O \text{ at } T_\alpha (" \text{Hg}) \]

Figure V-5e
\[ \theta_{1}^{II} = t_{s}(1+b) + 3100 \frac{P}{B} \]

\[ \theta_{2}^{II} = T_{s}(1+b) + 3100 \frac{P}{B} \]

for

\[ B = 29.92 \text{ "Hg (sea level) } \]

Figure V-6a
\[
\theta_1'' = t_b (1+b) + 3100 \quad \frac{P_b}{B}
\]

\[
\theta_2'' = T_m (1+b) + 3100 \quad \frac{P_m}{B}
\]

for

\[B = 24.89'' \text{Hg (5,000 ft)}\]
\[ \theta'_i = t_5(1+b) + 3100 \ P_5/B \]

\[ \theta'_e = t_5(1+b) + 3100 \ P_5/B \]

for

\[ B = 20.58'' \text{Hg}(10,000 \text{ ft}) \]

Figure V-6c
\[ \theta_1'' = t_g (1 + b) + 3100 \frac{P_g}{B} \]

\[ \theta_2'' = T_a (1 + b) + 3100 \frac{P_a}{B} \]

for

\[ B = 16.88'' \text{ Hg (15,000 ft)} \]
\[ \theta''_2 = t_s (l + b) + 3100 P_s / B \]

\[ \theta''_2 = T_s (l + b) + 3100 P_s / B \]

for

\[ B = 13.75'' \text{Hg}(20,000 \text{ft}) \]

Figure V-6e
Figure V-7
Recapitulation

For

\[ t_s < 32^\circ F \quad q/A = h_c (\theta_1 - \theta_2 - \theta_3) \]
\[ t_s = 32^\circ F \quad q/A = h_c (\theta_1' - \theta_2' - \theta_3) \]
\[ t_s > 32^\circ F \quad q/A = h_c (\theta_1'' - \theta_2'' - \theta_3''). \]

Evaporation rate:

\[ g = 2.9 \ h_c \ A \ (P_s - P_m) \ B^{-1} \ \text{lbs/hr}. \]

First Application: The Temperature of an Unheated Surface. If we set \((q/A) = 0\), we have from the equations above

\[ \theta_1 = \theta_2 + \theta_3 \]
\[ \theta_1' = \theta_2' + \theta_3' \]
\[ \theta_1'' = \theta_2'' + \theta_3'' \]

For a given value of airplane speed, altitude-free air temperature, liquid-water content and effective droplet size, we may calculate the unit thermal conductance \((h_c)\), the water catch rate \((R_w)\), and, therefore, the dimensionless parameter \(b\). We then enter the graphs of \(\theta_2''\) and \(\theta_3\). The sum is equal to \(\theta_1''\), and the graph of \(\theta_1''\) is used to find \(t_s\). If \(t_s < 32\) we must use the \(\theta\) functions. Using the \(\theta_2\) and \(\theta_3\) functions we obtain \(\theta_1\), and if the resultant is above the graph we known that \(t_s = 32^\circ F\). Finally, using the \(\theta'\) functions we determine \(n\), the "freezing fraction".

These three cases will be illustrated using data from the NACA icing wind tunnel. Figure V-8 shows a sketch of the NACA experimental setup. Figure V-9 shows a typical set of data reported by Lewis^4.

Our interest centers in the temperature just before the heating cycle. Table I below gives three sets of data taken from Reference 4.

<table>
<thead>
<tr>
<th>case</th>
<th>(T_\infty)</th>
<th>(W)</th>
<th>radius</th>
<th>rpm</th>
<th>(x^*)</th>
<th>ts measured</th>
<th>Figures in Reference 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>11</td>
<td>0.46</td>
<td>1.98</td>
<td>925</td>
<td>1.8</td>
<td>32, 32, 33, 34, 31</td>
<td>25b, 26b, 27b</td>
</tr>
<tr>
<td>(\phi)</td>
<td>2</td>
<td>0.20</td>
<td>3.02</td>
<td>925</td>
<td>1.2</td>
<td>19, 21, 16</td>
<td>30b, 31b</td>
</tr>
<tr>
<td>(\beta)</td>
<td>11</td>
<td>0.50</td>
<td>1.15</td>
<td>925</td>
<td>1.2</td>
<td>28, 30, 32, 28</td>
<td>24a, 24b, 29b</td>
</tr>
<tr>
<td>(c_F)</td>
<td>g/m^3 ft.</td>
<td>925</td>
<td>1.2</td>
<td>28, 30, 32, 28</td>
<td>26b, 27b, 30b, 31b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>*distance from airfoil nose.</td>
<td>V-28</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*}
Figure V-8. Sketch of test setup used by Lewis. Since the throat of the tunnel could not accommodate the propeller, the diffuser was used.

The catch rate in the vicinity of the measuring point (x inches from the leading edge) was calculated by using the trajectory data of Reference 5. The mean drop size reported by Lewis was 50 microns. The relative velocity between propeller and air was found by adding vectorially the tunnel air speed to the propeller station tangential velocity.

Table II shows the computed data preparatory to entering the charts.

<table>
<thead>
<tr>
<th>Case</th>
<th>$U_\infty$</th>
<th>$\gamma$</th>
<th>$R_1$</th>
<th>$E_M$</th>
<th>$\Delta R/E_M$</th>
<th>$h_0$</th>
<th>$\Delta s/c$</th>
<th>$t/c$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>290</td>
<td>101</td>
<td>395</td>
<td>0.80</td>
<td>0.50</td>
<td>21.3</td>
<td>0.20</td>
<td>0.24</td>
<td>0.44</td>
</tr>
<tr>
<td>f</td>
<td>363</td>
<td>112</td>
<td>495</td>
<td>0.80</td>
<td>0.28</td>
<td>25.0</td>
<td>0.15</td>
<td>0.12</td>
<td>0.105</td>
</tr>
<tr>
<td>p</td>
<td>246</td>
<td>54</td>
<td>333</td>
<td>0.85</td>
<td>0.04</td>
<td>17.5</td>
<td>0.10</td>
<td>0.75</td>
<td>0.44</td>
</tr>
<tr>
<td>ft/sec</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\begin{align*}
\text{BTU} & \text{ hr ft} \\
\end{align*}
Figure V-9. Typical data reported by Lewis for the time-temperature variations on an intermittently heated propeller blade. In this test the cycle time was 96 seconds, tunnel temperature 2°F, 925 rpm. The heating intensity was 9 watts per square inch on the nose and 8 watts per square inch on the first four inches of the camber face, 8 watts per square inch for the first inch on the thrust face.

For example, in case d, we enter Figure V-3 and read \( \theta_2 = 5^\circ F \). Then we enter Figure V-6a (the tunnel operated at seal level) and read \( \theta_2'' = 20 \) opposite \( \gamma_\infty = 11^\circ F \). The sum is \( \theta_1'' = 25^\circ F \), which in Figure V-6a corresponds to below 32°F. Therefore, the surface temperature is at or below freezing. We next enter Figure V-2a and read \( \theta_2 = 80 \). The sum of 80 and 5 is 85 (\( = \theta_1 \)). We then read in Figure V-1a and find \( t_s \chi 32^\circ F \). Thus we know that \( t_s = 32^\circ F \). The freezing fraction is next found from the \( \theta' \) functions. In Figure V-4 we find \( \theta_1' = 66 \). Then, since \( \theta_2' = \theta_1' - \theta_3' = 66 - 5 = 61 \), from Figure V-5a, we find \( \theta_2' = 144nb = 24 \). Therefore, 144 nb = 37, and since \( b = 0.44 \), from Figure V-7, \( n = 0.45 \).

We predict therefore that at this station the temperature is 32°F and that the surface of the ice is running wet with water which flows back and freezes along the aft surface. Sixty-five per cent of the water freezes as it strikes.

Table III shows a summary of the results of calculations for these cases.
TABLE III

<table>
<thead>
<tr>
<th>Case</th>
<th>$\theta_3$</th>
<th>$\theta_2''$</th>
<th>$\theta_1''$</th>
<th>$\theta_2$</th>
<th>$\theta_1$</th>
<th>$\theta_1'$ (or $\theta_2''$-144$^\circ$)</th>
<th>n</th>
<th>$t_s$ predicted</th>
<th>$t_s$ measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>5</td>
<td>20</td>
<td>25$^#$</td>
<td>80</td>
<td>85*</td>
<td>66</td>
<td>24</td>
<td>0.65</td>
<td>32</td>
</tr>
<tr>
<td>f</td>
<td>10</td>
<td>5</td>
<td>15$^#$</td>
<td>20</td>
<td>30</td>
<td>--</td>
<td>--</td>
<td>1.0</td>
<td>18</td>
</tr>
<tr>
<td>p</td>
<td>5</td>
<td>18</td>
<td>23$^#$</td>
<td>80</td>
<td>85*</td>
<td>66</td>
<td>24</td>
<td>0.65</td>
<td>32</td>
</tr>
</tbody>
</table>

$^\#$ Below 32$^\circ$F  
* Above 32$^\circ$F

Messinger has used these functions to prepare graphs showing the effects of speed on wing temperatures. Figure V-10 is taken from Messinger's paper and shows the free stream velocity required to prevent freezing at various free air temperatures. The required speed increases with increasing altitude.

Figure V-11, also taken from Messinger's paper, shows the variations in surface temperature as a function of airspeed. Although $b$ is shown as an independent parameter, $b$ is also a function of airspeed. In laminar regions $b$ varies almost as the square root of velocity. In turbulent regions $b$ varies more nearly with the 0.2 power of velocity. When ice is forming the value of $b$ changes due to the effect of the distortion of the streamlines. Both the water catch and unit thermal conductance will thus be changed in an unpredictable fashion.

A Second Application. Energy Requirements for Heating Small Cylinders. Using the three equations for the three regimes, we may graph the heat required as a function of surface temperature. Figure V-12 shows such a graph. From Figure V-12 we see that the heat flux varies with $b$ differently above and below the freezing point.

Below 32$^\circ$F increasing $b$ requires decreasing $q$. Above 32$^\circ$F the reverse is true, but the dependence upon $b$ is not so strong.

It is of interest to calculate the energy required to keep a surface warm and dry during an icing condition. Under these circumstances the impinging moisture must be evaporated as soon as it arrives. If the rate of water catch is known and if the unit thermal conductance is known, the mass-transfer equation may be used to predict the required surface temperature for evaporation. Then the heat flux equation may be used to predict the heat loss.

Schaefer$^6$ has presented data on the energy required to keep a small "calrod" dry during exposure to an icing condition. Reference 7 presents an
analysis of Schaefer's data. Table IV presents a summary of comparisons between Schaefer's data and computed data. The data gathered by Schaefer represent observations of the heat required to keep the calrod dry and also heat required to permit the leading edge to appear slightly moist. Most of the computed heat fluxes are bracketed by Schaefer's data.

A Third Qualitative Illustration

The physical nature of the ice formation in its initial stages may be predicted from the previous equations. When the surface temperature is low (either low value of $T_0$ or low value of $b$), the drops are expected to freeze upon impact. The ice accretion should resemble a graph of the local catch rate. The ice should be rather porous, for the solid phase is being formed in much the same fashion as a porous sintered metal is made of small particles joined by melting.
<table>
<thead>
<tr>
<th>Run</th>
<th>Drop Diam. Micron</th>
<th>Liq. Wat. C/ft³</th>
<th>Air Temp °F</th>
<th>Air Speed mph</th>
<th>Heat Loss</th>
<th>Heat Lost/Frontal Area, Btu/hr ft²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\Delta_M$</td>
<td>$\Delta_o$</td>
</tr>
<tr>
<td>1</td>
<td>5.8</td>
<td>0.29</td>
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<td>41</td>
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<td>1.3</td>
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<td>0.38</td>
<td>7</td>
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<td>-2</td>
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<td>-2</td>
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<td>3.1</td>
<td>2.8</td>
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<td>-6</td>
<td>59</td>
<td>3.1</td>
<td>3.0</td>
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<td>0.36</td>
<td>3</td>
<td>70</td>
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<td>60</td>
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<td>2.6</td>
</tr>
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<td>3.1</td>
<td>2.3</td>
</tr>
<tr>
<td>9</td>
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<td>0.25</td>
<td>-6</td>
<td>64</td>
<td>3.1</td>
<td>2.5</td>
</tr>
<tr>
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<td>0.36</td>
<td>3</td>
<td>64</td>
<td>4.0</td>
<td>3.4</td>
</tr>
<tr>
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<td>3.9</td>
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<td>0</td>
<td>56</td>
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<td>4.1</td>
</tr>
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<td>3</td>
<td>80</td>
<td>5.5</td>
<td>5.2</td>
</tr>
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<td>14</td>
<td>61</td>
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<td>5.0</td>
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</tr>
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<td>0.62</td>
<td>12</td>
<td>80</td>
<td>7.6</td>
<td>4.9</td>
</tr>
<tr>
<td>17</td>
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<td>86</td>
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<td>0.65</td>
<td>30</td>
<td>62</td>
<td>5.6</td>
<td>5.1</td>
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<tr>
<td>20</td>
<td>17.7</td>
<td>1.12</td>
<td>25</td>
<td>46</td>
<td>5.6</td>
<td>5.1</td>
</tr>
<tr>
<td>21</td>
<td>5.7</td>
<td>0.12</td>
<td>28</td>
<td>71</td>
<td>5.7</td>
<td>5.4</td>
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<tr>
<td>22</td>
<td>8.8</td>
<td>0.10</td>
<td>-18</td>
<td>73</td>
<td>5.6</td>
<td>2.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Watt/cm²</th>
<th>Btu/hr, ft²</th>
<th>Watt/cm²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
When $\phi$ or $b$ are large, the surface of the ice should run wet. However, at the "edge" of the catch zone, the value of $b$ will drop to zero. We expect then, that the ice accretion will be flattened, since near the stagnation point there will be excess water which freezes near either side. Now, a flat surface catches ice near the edges, hence, the ice buildup will shift to produce the familiar "horned" shape.

Bartlett and Dickey have observed and classified ice formations on jet-engine guide vanes. Their classifications and observations are shown in Figures V-13 and V-14 and support the above description.
REFERENCES


CHAPTER VI

THE DESIGN OF A CONTINUOUSLY ANTI-ICED AIR-HEATED WING

Considerable analytical and experimental work on air-heated wing anti-icing systems has been accomplished by the NACA and the U. S. Air Force, as well as by the airplane companies. In principle, at least, the design of an air-heated wing is straightforward. Some design information remains yet to be gathered, particularly in regard to trajectory data on new airfoils at high speeds, and there are numerous practical difficulties in attempting to meet the heat requirements and yet remain within the weight limits currently considered acceptable. Nevertheless, the laying-out of a basic design can proceed today with considerable assurance based on the research efforts mentioned above.\(^1\-4\)

In this chapter the simplest and most straightforward design method will be presented in order that the important characteristics may be more clearly seen. After the general performance features have been surveyed, the calculation methods used for more accurate design will be considered.

Figures I-8 and I-9 show the general layout of a conventional air-heated wing. The basic problem in such a design is the determination of the air temperature and weight rate required to anti-ice successfully the wing under a given set of meteorological conditions. We shall defer to a later chapter the discussion of the appropriate meteorological conditions to be used for design purposes.

The preliminary design of an air-heated wing requires the simultaneous solution of three sets of equations. The first equations relate the heat transfer from the wing surface to the surrounding air stream (via convection, evaporation, etc.). The second set of equations describes the heat conduction in the wing metal, and the third describes heat transfer from the air in the double-skin gap. Once the air rate and inlet air temperature have been decided, the pressure necessary to pump the air must be calculated.
In low-speed aircraft the wing outer skin is generally as thin as possible. The chordwise conduction of heat is then small, and there is no need to solve the equation of heat conduction in the outer skin. Only two equations need be solved simultaneously. Neel\textsuperscript{5} and Neel, Bergrun, Jukoff, and Schlaff\textsuperscript{3} demonstrate a technique to be used for this case.

In high-speed aircraft the outer skin on the wing is quite thick, and the chordwise heat conduction tends to provide an isothermal surface. Under the restriction that the wing is isothermal, the simultaneous solution of the heat-flow equations is considerably simplified. For purposes of clarity this simpler system will be considered here and the more complex systems taken up later. In the analysis which follows, the wing surface is taken as isothermal.

The Heat Flux from the Heated Air to the Wing Surface

Consider now the typical heated-wing construction with double skin. Figures VI-1 and VI-2 show the heat flow paths from the air. The aluminum

![Schematic Diagram of Heat Flow From Heated Air to Outside](image)

Figure VI-1. Schematic diagram of heat flow from heated air to outside.
used to form the double skin is a good conductor of heat, and, since it is usually well bonded to the outer skin, there is considerable heat transfer from the air in the "D" duct even before it enters the double skin.

The heat transfer from air to the walls of a conduit is described by the equation

\[ q = h_i A (t_a - t_s), \]

where

- \( q \) = heat flux, BTU/hr
- \( A \) = surface area, ft\(^2\)
- \( t_a \) = "mixed mean" air temperature at any section, °F
- \( t_s \) = surface temperature, °F.

The above equation serves to define \( h_i \), the unit thermal conductance for the heat transfer from the interior of the conduit.

Figure VI-2. Schematic diagram of heat loss from heated air to inner ducting of leading edge system.
Figure VI-3 shows a typical variation in $h_1$ along the double-skin passage. As the air turns and enters the air gap, the unit thermal conductance drops from a high to a low value. About an inch from the inlet, the flow becomes turbulent and the conductance rises abruptly. The conductance then falls slowly with distance along the duct.

A typical laboratory apparatus for measuring heat transfer within the double skin construction is shown in Figures VI-4 to VI-7. Figure VI-4 shows the inner skin. Figure VI-5 shows steam-condensation partitions installed on what would normally be the airfoil outer skin. Figure VI-6 shows the complete test unit with instrumentation on the interior and the steam jacket over the outside. Figure VI-7 shows the laboratory set up with the various steam condensate tubes visible below the steam jacket. The weight of steam collected in each tube is a measure of the heat transferred locally, since the heat of condensation is known. Average conductances may be obtained from the point values of Figure VI-3 by integration and may be used when the surface temperature is uniform.
Figure VI-4

If one considers an air-heated wing system where the surface temperature is constant over the heated region and the conductance, \( h_i \), is also constant at its average value, the following heat-balance equation may be applied:

\[
dq = h_i (t_a - t_s) \, dA = - W \, C_p \, dt_a,
\]

where

\[
W = \text{air rate, lbs/hr}
\]
\[
C_p = \text{air unit heat capacity, BTU/lb °F.}
\]

Since \( h_i \), \( t_s \), \( W \), and \( C_p \) are considered constant, the above equation may be integrated to yield

\[
\frac{t_{a2} - t_{az}}{t_{ai} - t_s} = 1 - e^{-\frac{h_i A}{W C_p}}.
\]
The total heat flux for the air may then be computed from

\[ Q = W C_p (t_{a1} - t_{a2}) = W C_p (t_{a1} - t_{sa}) \left( 1 - \exp \left( - \frac{h A}{W C_p} \right) \right). \]

The above equation may be used in several ways. For example, for a given surface temperature and heat flux, the required inlet air temperature may be computed as a function of air rate.

Matching the Interior Airflow to the Heat Requirements

As an example of the design of a heated-wing air system*, consider Figure 8, which shows the water catch rate on several airfoils (treated as

*D. M. Patterson has prepared a handy set of nomographs for use in rapidly carrying out the design of air-heated wings. The use of the nomographs obscures some of the variables, hence the calculations here are carried out in greater detail (See Reference 6).
though they were cylinders).* The designer chooses the particular configuration and condition corresponding to his airfoil and prepares a table such as shown in Table I. Items 1 to 11 in Table I are self-evident entries. Item 9 may be taken from water droplet trajectory data.*

Item 12 is computed as follows:

(Consider 225 mph, sea level, 15°F, drops of 15 micron diameter, liquid water concentration of 1.0 grams/meter³)

The evaporation rate is given by \( g = h_m A (H_s - H_w) \) (see Chapter IV), which may be rearranged to yield

* The use of a cylinder as an approximation to an airfoil as suggested in Reference 2 does not yield accurate results. The data and References of Chapter II should be consulted.
Figure VI-7. Test stand used for determination of internal heat transfer coefficients and pressure drops.

\[ g = \frac{h_c}{C_p} \frac{A}{(0.62) (P_s - P_w)/B} \text{ (see Chapter IV).} \]

For this case,

\[ g = 3.03 \text{ lbs water/hr} \]
\[ A = 6.0 \text{ ft}^2 \]

(Top and bottom surfaces heated directly for 2 feet; one foot aft heated indirectly by conduction, see Reference 2).

\[ B = 29.9 \text{ inches Hg} \]
\[ C_p = 0.241 \text{ BTU/lb }^\circ\text{F}. \]

The unit thermal conductance, \( h \), may be approximated by treating the leading-edge region as a cylinder and the afterbody as a flat plate. (Much better approximations are available, but their application would cloud this
discussion). Figure VI-9 shows the nature of this idealization. Experiments with cylinders and airfoils have been predicted well by the following empirical equations:

A. For the cylinder:

\[ h_\phi = 0.194 \ T_f^{0.49} \left( \frac{U_{\infty} \gamma}{D} \right)^{0.5} \left[ 1 - (\phi/90)^3 \right], \]

where

- \( h_\phi \) = local conductance at angle \( \phi \) BTU/hr ft² °F
- \( T_f \) = "film temperature", average between surface and free stream, °R
- \( U_{\infty} \) = free stream velocity, ft/sec
- \( \gamma \) = air density, lbs/ft³
- \( D \) = cylinder diameter, ft
- \( \phi \) = angle measured from a stagnation line, degrees.
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TABLE 1

<table>
<thead>
<tr>
<th>No.</th>
<th>Item</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Altitude</td>
<td>&quot;F&quot;</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Ambient Air Temperature</td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>Water Drop Size</td>
<td></td>
<td>33</td>
</tr>
<tr>
<td>4</td>
<td>Liquid Water Content</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>Air Velocity</td>
<td>&quot;lbm/s&quot;</td>
<td>0.082</td>
</tr>
<tr>
<td>6</td>
<td>Air Density</td>
<td>&quot;lbm/s&quot;</td>
<td>0.082</td>
</tr>
<tr>
<td>7</td>
<td>Indicated Air Speed</td>
<td>&quot;lbm/s&quot;</td>
<td>0.082</td>
</tr>
<tr>
<td>8</td>
<td>True Air Speed</td>
<td>&quot;lbm/s&quot;</td>
<td>0.082</td>
</tr>
<tr>
<td>9</td>
<td>Efficiency of Water Catch</td>
<td>&quot;lbm/s&quot;</td>
<td>0.082</td>
</tr>
<tr>
<td>10</td>
<td>Rate of Water Catch</td>
<td>&quot;lbm/s&quot;</td>
<td>0.082</td>
</tr>
<tr>
<td>11</td>
<td>Effective Ambient Air Temperature</td>
<td></td>
<td>0.082</td>
</tr>
<tr>
<td>12</td>
<td>Required Wing Speed for Dep.</td>
<td>&quot;lbm/s&quot;</td>
<td>0.082</td>
</tr>
<tr>
<td>13</td>
<td>Best to Rain Water Temperature</td>
<td>&quot;lbm/s&quot;</td>
<td>0.082</td>
</tr>
<tr>
<td>14</td>
<td>Dry Best Loss</td>
<td>&quot;lbm/s&quot;</td>
<td>0.082</td>
</tr>
<tr>
<td>15</td>
<td>Best to Evaporate Water</td>
<td>&quot;lbm/s&quot;</td>
<td>0.082</td>
</tr>
<tr>
<td>16</td>
<td>Total Best Loss</td>
<td>&quot;lbm/s&quot;</td>
<td>0.082</td>
</tr>
<tr>
<td>17</td>
<td>Total Best Loss Per Unit Area</td>
<td>&quot;lbm/s&quot;</td>
<td>0.082</td>
</tr>
<tr>
<td>18</td>
<td>Best to Rain Water Temperature</td>
<td>&quot;lbm/s&quot;</td>
<td>0.082</td>
</tr>
<tr>
<td>19</td>
<td>Dry Best Loss</td>
<td>&quot;lbm/s&quot;</td>
<td>0.082</td>
</tr>
<tr>
<td>20</td>
<td>Best to Evaporate Water</td>
<td>&quot;lbm/s&quot;</td>
<td>0.082</td>
</tr>
<tr>
<td>21</td>
<td>Best to Evaporate Water</td>
<td>&quot;lbm/s&quot;</td>
<td>0.082</td>
</tr>
<tr>
<td>22</td>
<td>Besting Air Weight*</td>
<td>&quot;lbm/s&quot;</td>
<td>0.082</td>
</tr>
</tbody>
</table>

Note: All items above are for one foot of span.  
*Based on inlet air temperature of 50°F.

B. For laminar flow over a flat plate:

\[ h = 0.0562 \frac{T_r^{0.5}}{(U_\infty y)^{0.5}} \times x^{-0.5} \]

where \( x = \) distance from beginning of boundary layer, ft.

C. For turbulent flow over a flat plate:

\[ h = 0.51 \frac{T_r^{0.3}}{(U_\infty y)^{0.8}} \times x^{-0.2} \]

For the case at hand, the individual conductances are plotted for the three regimes and the average conductance obtained by graphical integration. (The charts and equations by Patterson\(^6\) enables one to find \( h_\text{average} \) rapidly).

For the case at hand the average conductance is found to be \( h = 25 \).

The required vapor pressure difference to evaporate the moisture is taken from

\[ g = \frac{h C_p A (0.62) (P_r - P_\infty)}{P_r - P_\infty} = \frac{g C_p B}{h A (0.62)} = \frac{3.05 \times 0.241 \times 29.9}{25 \times 6 \times 0.62} = 0.231 \]

VI-10
At 15°F the vapor pressure of water is 0.08 inches of mercury. From the steam tables we find that at $P_s = 0.31 \ "Hg$, that the temperature $t_s = 46^\circ F$.

Entries 13 to 15 in Table I are computed as follows:

**Item 13. Sensible heating of the water**

$$Q = 0.03 \times (46 - 15) = 94 \text{ BTU/hr};$$

**Item 14. Dry heat loss (convection minus frictional gain)**

$$\text{Convection} = h A (t_s - T_{\infty}) = 25 \times 6 \times 31 = 4,650 \text{ BTU/hr},$$

$$\text{Frictional gain} = h A \frac{U_\infty^2}{2g_c \mu c_p} = 25 \times 6 \times 7.7 = 1,150 \text{ BTU/hr},$$

$$\text{Net} = 4,650 - 1,150 = 3,500;$$
Item 15. Heat to evaporate the water:

\[ Q = 3.03 \times 1,065 = 3,200 \text{ BTU/hr} . \]

The remaining entries of Table I are self-evident with the exception of the last entry, which will be considered shortly.

The required inlet air temperature may now be computed as a function of the air rate, since both \( q \) and \( t_s \) are known. For example at 100 lbs/hr per ft of span (10 passages top plus 10 passages bottom, Figure VI-3) the average value of \( h_i = 10.0 \) for the interior. From the equation

\[ Q = \frac{W}{C_p} (t_{a_i} - t_s) (1 - e^{-h_1 A/W C_p}) \]

\[ 6960 = 100 \times 0.24 (t_{a_i} - t_s) (1 - e^{-10\times4/100\times0.24}) . \]

(Note \( A = 4 \) feet, since 2 feet are indirectly heated.)

\[ t_{a_i} - 46 = 35^\circ F \]
\[ t_{a_i} = 403^\circ F \]

On the other hand, if \( t_{a_i} \) is restricted to 250°F, the required air weight flow is 166 lbs/hr as shown in Table I.

We may compute the "dry air" wing temperature as follows:

If the surface is such a good conductor that \( t_s = \) constant, then the heat flux from the wing to the air is

\[ Q = h_o A (t_s - t_c) . \]

The heat flux from the interior air is

\[ Q = \frac{W}{C_{pa}} (t_{a_i} - t_s) (1 - e^{-h_1 A/W C_p}) . \]

Given \( W = 100 \) lbs/hr \( t_{a_i} = 400^\circ F \) \( A_i = 4 \text{ ft}^2 \) inside \( C_p = 0.241 \text{ BTU/ft}^2/\text{F} \)

\[ h_1 = 10 \text{ BTU/hr ft}^2/\text{F} \quad t_e = 22.7^\circ F \quad h_o = 25 \text{ BTU/hr ft}^2/\text{F} \quad A_o = 6 \text{ ft}^2, \text{ outside} . \]

The above equations are solved to yield \( t_s = 70^\circ F \).

In Table I some of the entries on lines 11 and 12 have been left blank. Under severe icing conditions it was found impossible to blow the required
amounts of air through the structure with the available pressures. Pressure drop considerations are not treated here since they are not peculiar to heated-wing design. See for example Reference 2, 12 and especially 13 for discussion of pressure drop.*

More Accurate Design Methods

The principal parameters which enter the design are:

\[ h_0 \]  
the unit thermal conductance from the wing exterior to the ambient air,

\[ h_i \]  
the unit thermal conductance from the air inside of the wing,

\[ R_w \]  
the local rate of water catch,

and, the wettedness, i.e., the ratio of the area wetted by water to the surface area for heat transfer.

References 9, 10, 11, 12, and 13 of Chapter V discuss the calculation of the exterior conductance.

The unit thermal conductance in the interior of the wing may be approximated by treating the flow to be similar to the inlet region of a conduit. Poppendieck has reviewed the available literature on inlet effects. For the oddly shaped inlets which occur in conventional heated wings, experimental measurements are usually superior to analytical techniques.

The local rate of water catch is calculable from the type of data in Chapter II.

The Wettedness

NACA measurements\(^3,4,5,8\) as well as experiments by Guibert\(^9\) and Gardner\(^10\) have shown that aft of the area of primary impingement (\(+\Theta_M\), see Chapter II), the water does not usually flow in a sheet but flows in rivulets (see Figures III-1 and III-2). Guibert tried using wetting and nonwetting agents with erratic results. Gardner\(^10\) and later Neel, et. al.,\(^3\) reported that the rivulet pattern was not too sensitive to the rate at which the water flowed back, provided some minimum flow was maintained.

*Reference 12 does not calculate heat requirements in the more modern fashion.
Guibert has suggested that a fraction of the area of primary water  
impingement the evaporation term in the heat balance be multiplied by the ratio,  
\( \frac{A_w}{A_s} \), the fractional wettedness. Neel, et. al., used the traces (Figure III2a)  
of the rivulets to calculate the wettedness and reported values ranging from  
15 to 35 per cent. Guibert reported wettedness from 0.08 to 1.46, basing his  
result on heat-transfer considerations. The data showing \( \frac{A_w}{A_s} \) greater than  
1.0 are obviously in error. The NACA scheme of wrapping a water sensitive  
tape about the surface should give a more accurate time average result provided  
the tape surface is not too different from the wing surface under study.

**Accuracy of Design**

In view of the uncertainties which surround each of the important  
parameters which enter the design of an anti-icing system, one may well question  
the accuracy with which the completed system's behavior may be predicted. The  
present state of the art certainly leaves much to be desired, but some of the  
uncertainties are not as bad as they may seem.

Messinger has shown that the design of an air-heated anti-icing  
system is not too greatly affected by the magnitude of the exterior unit  
thermal conductance. (This effect is also discussed in Reference 2).

The reason for this peculiar result is that for a given airflow and  
air temperature inside the wing, an increased exterior conductance results in  
a lower wing surface temperature, thus increasing the effectiveness with which  
the heat supply from the heaters may be used. The exponential variation of the  
vapor pressure with temperature tends to stabilize the evaporative system. A  
large value of the conductance combined with a low-vapor-pressure difference  
may evaporate as much water as a low conductance combined with a high vapor-  
pressure difference. The size of air ducting and heaters may well be the same  
in the two cases, though the detailed behavior (surface temperatures, air outlet  
temperatures) may be radically different.

**REFERENCES**

1. Hardy, J. K., "An Analysis of the Dissipation of Heat in Conditions of  
Icing from a Section of the Wing of a C-46 Airplane". NACA Report 831, 1945.

Heated Wings". AAF TR 4972, ADD. I, January 9, 1946.


CHAPTER VII

INTERMITTENT HEATING

In Chapter VI the design of an "anti-icing" system was considered. In this chapter we shall consider the design of a thermal "de-icing" system. The advent of higher speeds and increased knowledge about the severity of icing conditions have served to raise the known energy requirements for anti-icing. As an answer to the problem of protecting the new faster aircraft, the designers have turned to the use of intermittent heating. In an intermittently heated system the ice is permitted to form and then the ice-metal interface is heated for a short time. At 32°F the ice adhesive strength goes to zero and the wind forces pull the ice away. By keeping the water in the region of primary impingement until the ice is released, the formation of runback is avoided.

A de-icing system (as opposed to an anti-icing system) compromises the performance of the airplane, since the surfaces are no longer kept aerodynamically clean. The amounts of ice permitted to form are small, however, and it seems well worthwhile to sacrifice some performance in icing conditions for the sake of a lower installed weight. This point of view is justified in airline-type operations, for example, since icing conditions account for but 5 to 8 per cent of flying time.

Intermittent Thermal De-Icers

Propellers have used electrical de-icers with success. In a propeller system the centrifugal forces are available to cast off the ice. The centrifugal forces put a limitation on the allowable "off" time (the time during which the ice is permitted to form) since the ice, when released, may be cast off with considerable force and large pieces of ice may damage the fuselage.
The only intermittent de-icers for which descriptions are available are those which have been used with propellers. Figure VII-1 shows an example of an exterior de-icer \(^1\), \(^2\), \(^3\) and Figure VII-2 shows the type of construction used in a de-icer mounted in the propeller blade interior. \(^4\), \(^5\) The placing of the heating elements inside the blade reduces maintenance problems but increases the energy required for de-icing, as will be shown later.

Figure VII-1. Layer details of heaters used in propeller tests by Lewis. This construction is typical of presently used heaters.

Calculating the Performance of an Intermittent De-Icer

In an intermittently operated electric de-icer the heat balance equation must be written:

\[
\text{The Heat from the Electric Heaters} \quad = \quad \begin{cases} 
\text{The heat required to raise the structure temperature} \\
\text{the heat required to melt an ice interface} \\
\text{any heat loss through the ice and the exterior surface during the above process} 
\end{cases}
\]

\(^{VII-2}\)

References 4 and 6 present analyses of propeller de-icing systems in which the first term on the right (above) predominates, the second term is negligible, and the third term forms 30 to 40 per cent of the total.

At the exterior surface of the ice the modes of energy transfer described in Chapter V are considered to occur.

The heat transfer beneath the ice depends on the type of construction used. Two systems will be considered here in order to illustrate the method of calculation.

Because of the complexity of the interior construction of a typical intermittent de-icer it is convenient to use an electrical analog for computing the heat-flux distribution. The use of the analog will be shown for the case of a propeller, though the method is general and suitable for wings or other surfaces.
In the electrical analogy to heat flow the following correspondences are used.

**TABLE I**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Electrical</th>
<th>Thermal</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPACITANCE</td>
<td>C, Farad ( \frac{\text{coulomb}}{\text{volt}} )</td>
<td>( W \ C_p \ BTU/^\circ F )</td>
</tr>
<tr>
<td>RESISTANCE</td>
<td>( R, \text{ ohm} \ \frac{\text{volt sec}}{\text{coulomb}} )</td>
<td>( R_t \ \ ^\circ F \ \text{hr/BTU} )</td>
</tr>
<tr>
<td>POTENTIAL</td>
<td>( E, \text{ volt} )</td>
<td>( T \ \ ^\circ F )</td>
</tr>
<tr>
<td>CHARGE</td>
<td>( Q, \text{ coulomb} )</td>
<td>( Q \ \ BTU )</td>
</tr>
<tr>
<td>FLUX</td>
<td>( I, \text{ ampere} \ \frac{\text{coulomb}}{\text{second}} )</td>
<td>( q \ \ BTU/hr )</td>
</tr>
<tr>
<td>TIME</td>
<td>( t, \text{ second} )</td>
<td>( \theta \ \text{hour} )</td>
</tr>
</tbody>
</table>

The equations descriptive of heat flux and current flow bear a similarity. Thus, consider steady heat flux through a slab of thickness \( \Delta x \), thermal conductivity \( k \), and area \( A \). The flux is given by

\[
q = \frac{k A (T_2 - T_1)}{\Delta x}
\]

which may be rewritten

\[
q = \frac{T_2 - T_1}{(\Delta x/kA)} = \frac{T_2 - T_1}{R_t}
\]

Figure VII-3
The resemblance to Ohm's law

\[ I = \frac{E}{R}, \]

with the correspondence

- \( I \) \rightarrow \( q \)
- \( E \) \rightarrow \( T_2 - T_1 \)
- \( R \) \rightarrow \( R_t = \Delta x/k\lambda \)

permits one to draw a thermal network using electrical symbols. In a complicated network one may construct the analogous electrical network and determine the behavior experimentally.

If the flow is unsteady and the slab undergoes changes in temperature, the energy absorbed is given by

\[ q = WC_p \frac{dT}{d\Theta}, \]

which resembles

\[ I = C \frac{dE}{dt}, \]

with

- \( I \) \rightarrow \( q \)
- \( E \) \rightarrow \( T \)
- \( t \) \rightarrow \( \Theta \)
- \( C \) \rightarrow \( WC_p \).

If the capacity of the slab is considered as concentrated at its center, the thermal network is:

![Diagram of thermal network](image)

Figure VIII-4
If the slab is considered as composed of several "sub-slabs" a better approximation is obtained.

![Diagram of electrical network](image)

**Figure VII-5**

The sum of the resistances equals $R_t$, and the sum of capacitances equals $W C_p$ in Figure VII-5.

The behavior of the thermal network of Figure VII-5 may be found experimentally by building an analogous electrical network and measuring its behavior. If the heat flux is in more than one dimension, additional circuit elements are required.

We now turn to the examination of the thermal network representative of a propeller and its de-icer while operating in icing conditions.

Figure VII-1 shows a typical present-day de-icer attached to a conventional propeller. Figure VII-6 shows the heat-flux paths and the thermal network which approximates the propeller conduction field. A thermal analyzer was used in this case to "solve" the network behavior. The thermal analyzer contains resistors and capacitors which may be quickly combined to form a desired network. Figure VII-7 shows a view of the thermal analyzer* with associated recorders, timers, power supplies, and nonlinear elements.

The heat production at the propeller's electrical resistance elements is relatively independent of the temperature in the propeller. To simulate this behavior a constant current network is needed. A good approximation to such a network is a high-voltage source monitored by a pentode operating on the flat part of its characteristic curve.

At the surface of the propeller the heat flow is nonlinear with temperature, hence a simple combination of nonlinear elements cannot be utilized. Figure VII-8 shows a network capable of simulating the heat-flux current versus temperature (voltage) characteristic of a surface in icing. Figure VII-9 compares the thermal and electrical systems.

* University of California, Los Angeles
Figure VII-6. Network representative of the idealized heat flow indicated by the arrows in (B) above.
The heat-balance equations derived in Chapters IV and V refer to the surface where the ice is forming. The ice layer represents a resistance and capacitance which is small compared to the characteristics of the propeller section in Figure VII-6. For example an ice layer 0.15 inches thick represents a resistance approximately 16 per cent as great as the convective resistance. To simplify the network this ice resistance term has been omitted here.*

Figure V-9 shows a typical set of time-temperature variations measured on a propeller. This figure shows clearly the contributions of the various terms which enter the energy balance. At the left of the figure we see the contribution of aerodynamic heating. When the tunnel sprays are turned on, we see the heat-of-fusion contribution. Then, as the heaters are energized intermittently, we see the rise and fall of blade temperature. In Figure IV-2 similar data are shown in which the heat-of-fusion term is more pronounced.

* A network element which has a servo-controlled variable resistor has since been developed to simulate the growing ice layer. In some de-icers this approximation would be seriously in error. See, for example, the discussion at the end of Reference 4.
Figure VII-6. Non-linear network used to represent energy flux at the ice surface.

Figures VII-10a to VII-10d show comparisons between the results of the analyzer calculations and the measurements in the wind tunnel.$^4,^6$

The analyzer is a convenient tool for exploring the behavior of an intermittently operated de-icer. Figure VII-11 shows the results of operating the de-icer at constant energy but with shorter "on" times (i.e., higher powers). The reason for the increased effectiveness of shorter bursts of power is found in Figure VII-12, which shows how much of the heat flows inward to the propeller blade rather than outward to melt the ice.

A simplified view of the behavior of an intermittently heated de-icer is shown in Figure VII-13, where the entire propeller thermal capacitance is lumped in one capacitor.

The nonlinear character of the heat flux is replaced by a linear element. The resulting simple RC circuit fed by a constant current source has the behavior shown in Figure VII-14. The minimum energy occurs at short bursts
Figure VII-9. Behavior of non-linear network shown when the parameters are adjusted to approximate the heat transfer characteristics of straight dashed line segments represent the electrical network current-voltage characteristics.

of high power and is limited by the thermal capacity, i.e., from Figure VII-14, the energy input has as its lowest value

\[ q_\Theta = W C_p \Delta T \]

\[(\text{power})(\text{time}) = (\text{thermal capacity})(\text{temp. rise}) \]

The minimum energy requirements occur when the time of heating is vanishingly small and the thermal capacity of the heaters has been reduced to zero.

A second and important characteristic of intermittently operated systems is the diminished heat requirements at increasing water concentrations. Figures IV-2a, b, and c demonstrate the effect very well. In Figure IV-2c the temperature rise due to water impingement is greater than the temperature rise caused by the heaters.

VII-10
Figure VII-10a. Tunnel temperature 11°F, 925 rpm.

Figure VII-10b. Comparison between data of Lewis and Thermal Analyzer. ($T_\infty = 2^\circ\text{F}, 925 \text{ rpm}, 9 \text{ watts/in}^2 \text{ at seg 3}, 8 \text{ watts/in}^2 \text{ on 1, 2, 4, 5, } r/R = 0.81$). Points correspond to highest curve.
Figure VII-10c. Typical data obtained from thermal analyzer showing time-temperature variations on leading edge of heated propeller blade with comparison to data of Lewis (tunnel temperature 4°F, propeller rpm 800, r/R 0.33).

Figure VII-10d. Comparison between data of Lewis and thermal analyzer (tunnel temperature 5°F, propeller rpm 800, r/R 0.33.)
Figure VII-11. Effect of varying power density at constant energy.

Figure VII-12. Thermal analyzer data on heat flow inward to blade during a cycle.
Figure VII-13. Simplified view of intermittent de-icer (above) and a further simplification (below).

Figure VII-14. Energy requirements for de-icing as a function of heating time.
The Design of an Intermittent De-Icer

An examination of the above data shows that the minimum energy for de-icing is governed by the equation

\[ q\theta = W C_p (t_s - t_o) \]

In this equation, \( q\theta \) is the total energy required for de-icing, \( W \) is the thermal capacity of the heated structure (and the ice), \( t_s \) is the temperature at which the ice sheds (somewhere just above \( 32^\circ F \)), and \( t_o \) is the temperature of the surface when \( q = 0 \). The actual energy required will exceed this amount because of the loss of heat to the airstream, but as noted in Figure VII-13, the excess losses may be minimized by short bursts of power.

The allowable "off" times will be set by the maximum allowable ice buildup. At low temperatures, where icing is usually less severe, the allowable "off" times may be increased. Under these conditions also the initial temperature, \( t_o \), will be low, hence a larger value of \( q\theta \) will be needed. An automatic timer which changes the cycle times in accordance with the outside air temperature is described in Reference 7.

The lowest temperatures at which icing data have been evaluated are above \(-40^\circ F\) (Reference 8). Thus, a temperature rise of \( 32 - (-40) = 72^\circ F \) in clear air, provided the rise occurs within on tenth of the time constant of the system (as determined from cooling curves), should provide de-icing over the important range of icing conditions.

In an intermittent de-icing system the rate of water catch determines the allowable "off" periods, and the area of impingement determines the extent of heated area required. The water catch rate has a lesser influence upon the design energy requirements for ice release, since the design is established when the icing rate is low, rather than at the other end of the icing intensity scale, as is the case with anti-icing systems.

In a wing de-icer a continuously heated region at the nose must be maintained as a "parting strip" to prevent ice capping. The minimum necessary size of parting strip has not yet been determined and most probably is a function of the airfoil shape. The energy required by the parting strip may be calculated from data used in conventional anti-icer design((Chapter VI).
REFERENCES


CHAPTER VIII

METEOROLOGICAL CONDITIONS FOR DESIGN

The problem of choosing the proper meteorological conditions for the design of an ice-prevention system involves a fine balance between the allowable weight and maintenance penalties and the probability of encountering an icing storm which exceeds the design conditions. Heated wing equipment is still too new for the adequacy of the designs to be properly assessed. The gathering of data has been hindered until now by the lack of suitable instruments for measuring and reporting liquid-water contents and drop sizes encountered in the atmosphere.

The nature of the equipment used for ice prevention tends to set the design condition almost as much as do the meteorological data. For example, a heated wing anti-icing system fails to give adequate protection whenever the impinging moisture is not evaporated and runback begins to form. A small amount of runback is not dangerous, hence the design condition for an air heated wing anti-icer will be set, not only by the maximum water catch but also by the horizontal extent of the icing condition, i.e., how long the airplane will be in the icing conditions and how much runback can be tolerated.

On the other hand, an intermittent de-icer will be designed primarily on the basis of low-temperature icing.

The de-icing of a jet engine inlet will depend on the liquid-water content and temperature and only to a lesser extent on drop size, since the collection efficiencies of the engine components are usually high. A failure of the de-icing system of a jet engine is more serious and can lead to disaster more quickly than a wing anti-icer; hence, the horizontal extent of the icing condition becomes less important compared to the instantaneous severity.
Natural icing conditions vary greatly in intensity not only from point to point but also from time to time. A cloud is a dynamic thing; the water drops are constantly growing or evaporating. Indeed, in a particular region of the cloud, the small drops may be evaporating while the big ones grow. The measurement of liquid-water content and drop size is therefore made difficult, and simultaneous independent measurements can rarely be made. Figure VIII-1 shows a record made during an icing flight and shows the variations in icing intensity recorded on the leading edge of a flat disc in the icing airstream.

Most measurements of icing have been made using the multicylinder method described in Chapter II. Reference 1 presents a statistical analysis of all icing observations available up to 1950. Approximately 3600 observations were analyzed, about 30 per cent being flight observations. William Lewis, in a sequence of NACA reports 2-7 has gathered and studied flight data taken by NACA. He has sorted the observations with respect to clouds of stratoform type, which extend over large areas, and cumuloform type, which generally display a lesser horizontal extent but greater severity. In Reference 4 Lewis proposed to use a classification system for icing conditions as follows:

- Trace of Ice
- Light Ice
- Moderate Ice
- Heavy Ice

Originally these quantities were defined in terms of the ice accretion rate on a 3-inch diameter circular cylinder at 200 mph. Lewis converted the Weather Bureau definitions to zones on a graph of liquid-water content versus mean effective drop size. These classifications have apparently been accepted by the aircraft industry, though obviously the same icing cloud may produce different ice characteristics on a slow, large cargo airplane as compared to a small high-speed fighter.

Later, Lewis and Jones 6 suggested classifications for icing conditions. These classifications are best summarized by Table I, taken directly from Reference 6. At the present date, the classification appears rational though probably some of the recommended drop sizes and liquid-water contents may be expected to change as more data become available.

Figures VIII-2 and VIII-3 are taken from a recent report by Hacker and Dorsch, which shows a method of selecting the design condition for a particular heated-wing anti-icing system. The collection of water is cross-plotted on a tabulation of the number of icing observations known at each combination of drop size and liquid-water content. The stratoform clouds are considered separately from the cumuloform clouds. The designer may use such a cross plot to visualize the degree of protection afforded by various systems.
Figure VIII-1. Typical flight record as made with rotating disc instrument.
Figure VIII-2. Constant water-collection-rate curves for hypothetical airfoil superimposed on frequency distribution of icing observations: Stratiform clouds between pressure altitudes of 10,000 and 20,000 feet.
Figure VIII-3. Constant water-collection-rate curves for hypothetical airfoil superimposed on frequency distribution of icing observations: Cumuliform clouds between pressure altitudes of 10,000 and 20,000 feet.
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<th>Class</th>
<th>Item</th>
<th>Air Temp. (°F)</th>
<th>Liquid water content (g/m²)</th>
<th>Drop diameter (Microns)</th>
<th>Pressure altitude (ft)</th>
<th>Remarks</th>
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<td>5.0</td>
<td>50</td>
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VIII-6
S. S. Schaetzell has suggested a different method of calculating. In his method, for a particular airfoil, one makes a three-dimensional plot. The three variables are (1) drop size, (2) liquid-water content, and (3) rate of catch. The resultant surface is defined by an airfoil at a particular speed, altitude and angle of attack. Using the recommendations of Lewis and Jones Schaețzel postulates a relation between drop size and liquid-water content as a function of temperature. This relation permitted the plotting of isothermals on the above surface. Figure VIII-4 shows the resultant graph. These isothermals are lines of equal energy required for anti-icing; hence, the most severe design conditions can be selected readily.

Figure VIII-4. Roc* versus drop diameter and liquid water content at various temperatures. (NACA 0012 wing 12' x, 362 f.p.s. tes 20,000 ft.) The roc at any temperature is given by the vertical intercept between the temperature lines plotted on the base and the temperature line on the roc surface.

*Rate of catch.
REFERENCES


