

ENGINEERING RESEARCH INSTITUTE
UNIVERSITY OF MICHIGAN
ANN ARBOR

A NEW METHOD FOR CALCULATING
WATER-DROPLET TRAJECTORIES ABOUT STREAMLINED BODIES

By

M. TRIBUS

Project M992-E

AIR RESEARCH AND DEVELOPMENT COMMAND, USAF
CONTRACT AF 18(600)-51, E.O. No. 462 Br-1

December 1951

SUMMARY

Methods hitherto available for calculating the trajectories of water droplets have used a differential analyzer of the Bush (mechanical) type. Since only three such analyzers are available in the U.S. and their rental is high, relatively few trajectory calculations have been made. The proposed method utilizes an electronic differential analyzer (of the Reeves type) of which over two hundred are available at various universities and airplane companies in the United States. An electrical conducting sheet on a standard plotting table permits feeding back air velocity data during the integration. The plotting table thus draws trajectories at a rate limited only by the speed of the servomechanisms. No operators are required during the automatic integration.

A NEW METHOD FOR CALCULATING
WATER-DROPLET TRAJECTORIES ABOUT STREAMLINED BODIES*

INTRODUCTION

Langmuir and Blodgett¹ have demonstrated that a differential analyzer of the Bush type may be used to calculate the trajectories of small water drops about streamlined bodies. Later Bergrun² and Janssen, Guibert and Robbins³ made calculations for various airfoil shapes.

Unfortunately, the use of a differential analyzer of the Bush type requires several operators, and, while the machine is quite flexible and accurate, the cost of the calculations has thus far prevented the compilation of as many cases of practical interest as the aircraft designers desire.

The equations to be solved for the two-dimensional incompressible case are:

$$\frac{x}{L} = \frac{x_0}{L} + \int_0^{u\tau/L} \left\{ \frac{1}{K} \int_0^{\tau} \frac{C_D R_p}{24} \left[\frac{u_a(x,y) - \dot{x}}{U} \right] d\tau' \right\} d\tau$$

and

$$\frac{y}{L} = \frac{y_0}{L} + \int_0^{u\tau/L} \left\{ \frac{1}{K} \int_0^{\tau} \frac{C_D R_p}{24} \left[\frac{v_a(x,y) - \dot{y}}{U} \right] d\tau' \right\} d\tau .$$

*This method was suggested in part by Dr. Rauch of the Department of Aeronautical Engineering, University of Michigan.

The instantaneous position ($x/L, y/L$) locates the drop in the dimensionless flow field ($L =$ significant dimension of streamlined body). The parameter K is a ratio between the "range" of the drop and the significant dimension of the streamlined body if Stoke's law were obeyed. The term $C_D R_p / 24$ is the ratio between the actual drag coefficient C_D and the Stoke's law drag coefficient $24/R_p$, where $R_p =$ local relative Reynolds modulus between the drop and air; i.e.,

$$R_p = \frac{2a\rho_a |\vec{U}_a - \vec{U}_d|}{\mu_a} = \frac{2a\rho_a P}{\mu_a},$$

where

- $2a =$ drop diameter;
- $\rho_a =$ air density;
- $\vec{U}_a =$ air vector velocity;
- $\vec{U}_d =$ drop vector velocity;
- $\mu_a =$ air viscosity; and
- $P =$ relative velocity.

During the integration the components of the air velocity u_a and v_a must be instantly available as a function of the dimensionless position in the field ($x/L, y/L$).

THE COMPONENTS OF THE COMPUTER

Because the users of drop-trajectory data are usually unfamiliar with computers, it will be helpful to most of the readers of this paper to explain the operation of the few components which make up the analog computer to be used for these calculations. The computer uses electrical voltages varying with time to represent velocities, forces, positions, etc.

A. The Integrator

The heart of the computer is the integrating amplifier. This unit consists of an amplifier whose output signal is fed back to the input through a condenser. Symbolically we represent the hookup as follows (the second wire, or "ground" is not shown):

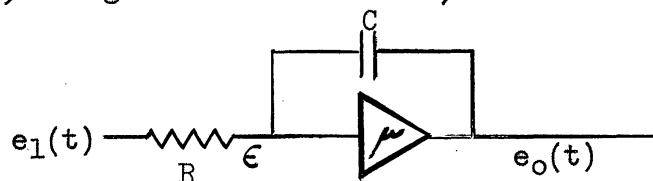


Fig. 1

ENGINEERING RESEARCH INSTITUTE • UNIVERSITY OF MICHIGAN

The current flow to the amplifier is zero and a balance of currents at the input yields:

$$\text{current from resistor} + \text{current from condenser} = 0$$

$$\frac{e_i(t) - \epsilon}{R} + C \frac{d}{dt} \left\{ e_o(t) - \epsilon \right\} = 0$$

Now let the amplifier have amplification such that

$$\mu \epsilon = -e_o \quad ,$$

where μ is a large number.

Then:

$$e_i(t) + \frac{1}{\mu} e_o(t) = + RC \frac{d}{dt} \left\{ e_o(t) \left[1 + \frac{1}{\mu} \right] \right\} .$$

If μ is very large, say of the order of 10^6 , then as an excellent approximation

$$e_i(t) = + RC \frac{d}{dt} \left\{ e_o(t) \right\}$$

or

$$e_o(t) = + \frac{1}{RC} \int_0^t e_i(t') dt' + e_o(0) .$$

Note that even if μ is not constant or accurately known, the accuracy is good so long as μ is large, and R or C do not change.

B. The Multiplier

Multiplication of two signals is accomplished by means of two potentiometers which are driven by a servomechanism. Given voltages e_a and e_b , we desire a voltage proportional to $e_a \cdot e_b$. One voltage, say e_a is impressed across the first potentiometer. The shaft is connected to a "self-balancing potentiometer" (as in a Brown Temperature Recorder, for example) which balances e_b against a fixed voltage, $E > e_b$. If the potentiometers are linear, they adopt a position e_b/E along their travel. The other potentiometer adopts this same position, but since there is a voltage e_a across it, the signal picked off is $e_a e_b/E$ and is therefore proportional to $e_a \cdot e_b$.

We represent the multiplier by the symbol

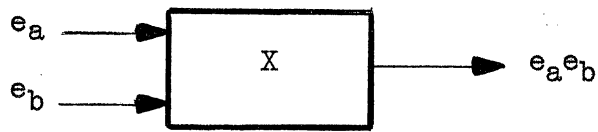


Fig. 2

C. The Square Rooter

Square roots may be taken by connecting the multiplier across the amplifier as shown below:

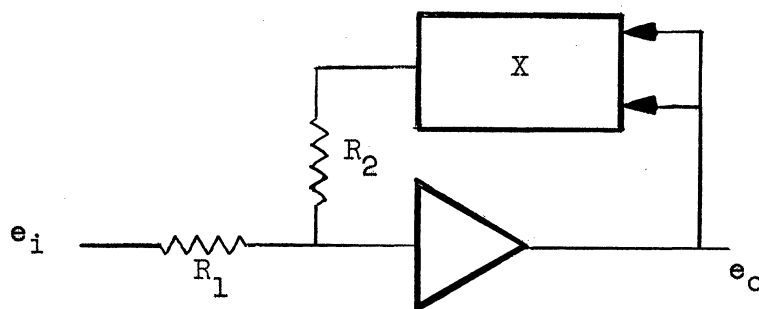


Fig. 3

A balance of currents at the amplifier input will show that if the gain of the amplifier is high enough and the resistors R_1 and R_2 are related by

$$E R_1 = R_2 ,$$

then $e_o = \sqrt{e_i}$ to an excellent approximation.

D. The Adder

If two voltages are to be added they are connected to the amplifier as shown in Fig. 4.

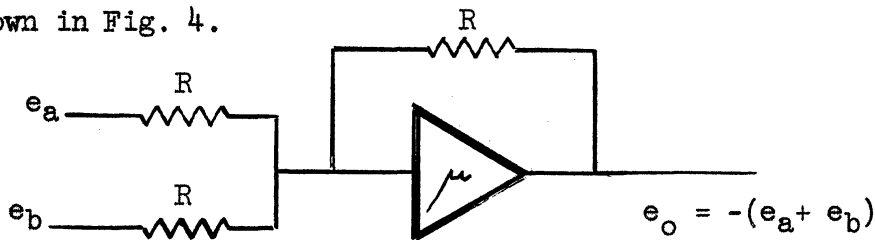


Fig. 4

E. The Output Plotter

A plotting table with x and y leadscrews driven by servomechanisms serves to position a plotting pen.

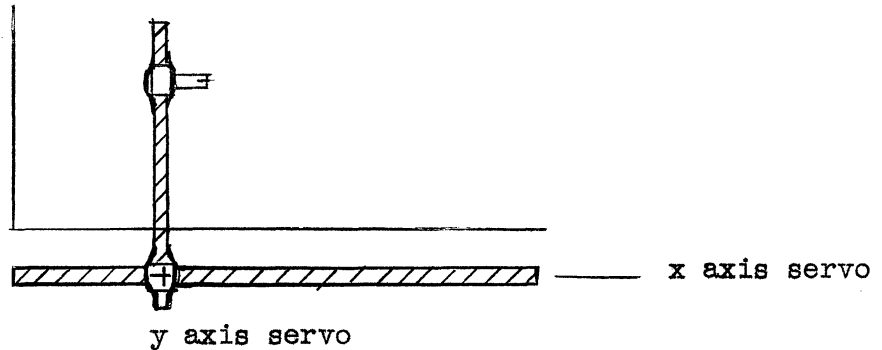


Fig. 5

F. Representing a Function

The output plotter may be used to represent a function by removing the y-axis leadscrew and replacing it by a resistance wire. A piece of solder is bent to the form of the curve to be represented and as the resistance wire is passed over it (through the action of the x-axis servo) a voltage is picked off.

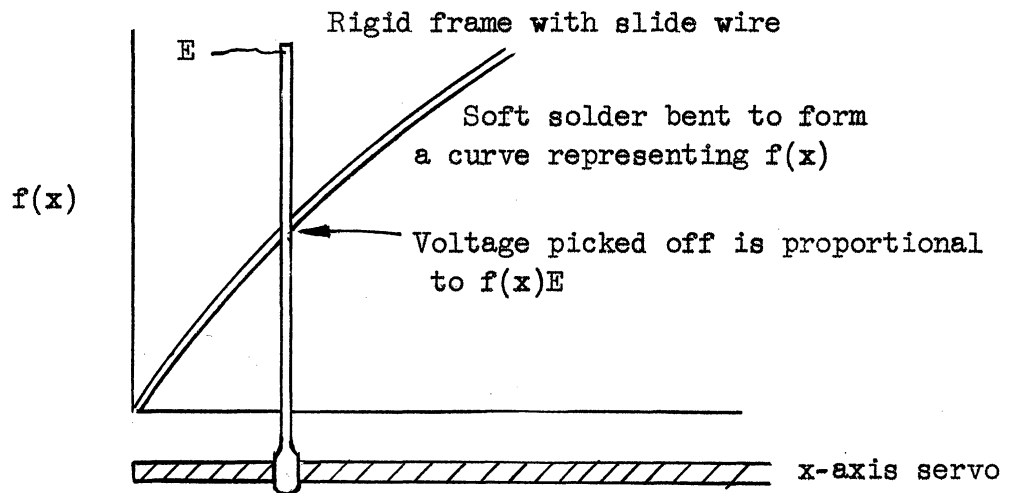


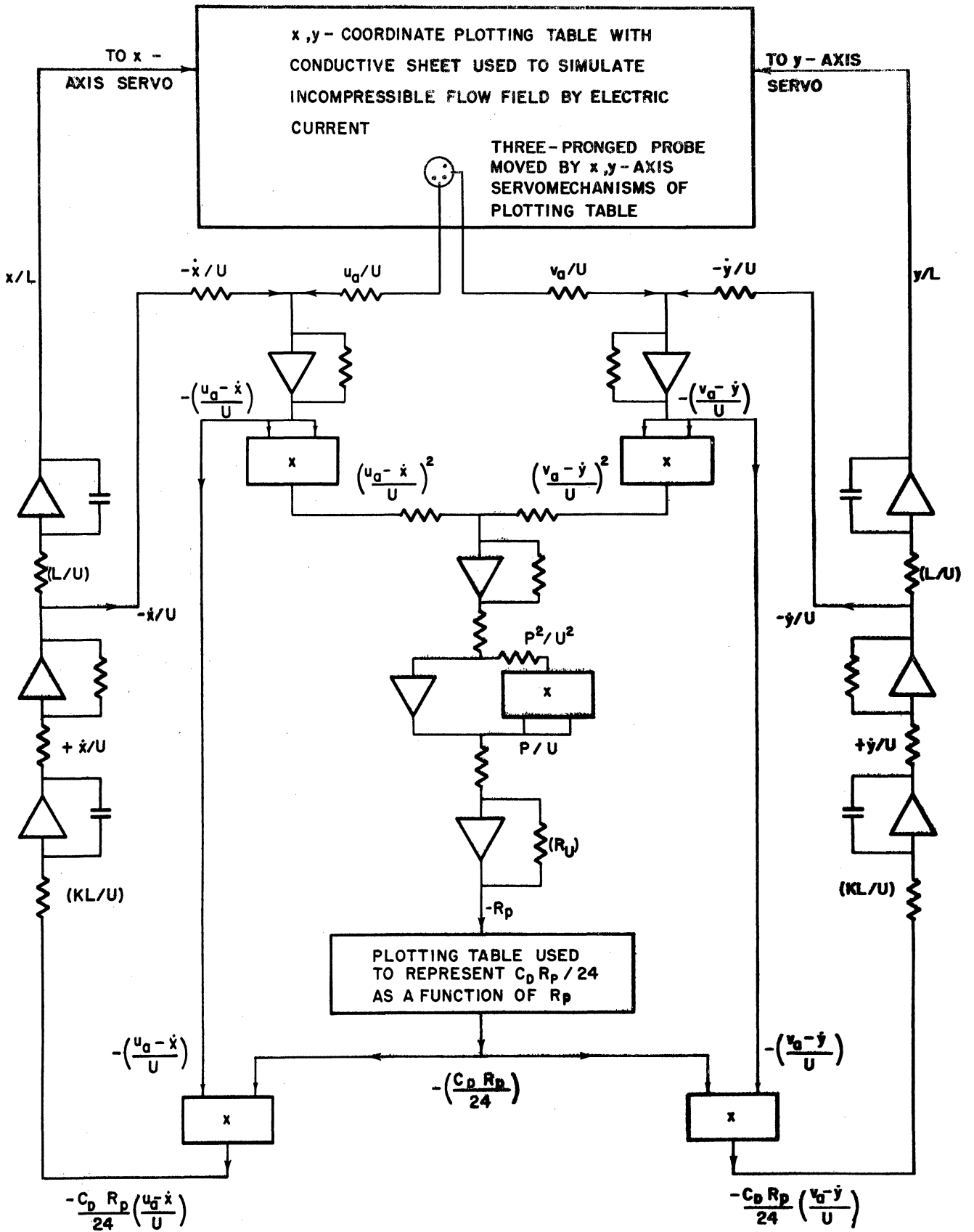
Fig. 6

THE COMPUTER ASSEMBLY

Fig. 7 shows the completed computer hookup using the items explained above but with one special element added. This element, used to represent the air velocity field, is shown at the top of Fig. 7. The device consists of a standard x,y plotting table from which the pen has been removed. The plotting paper has been replaced by a very thin sheet of stainless steel or a painted graphite film. Electrodes are positioned on this film in such a way as to produce either a current field or a potential field analogous to the air flow field. (For example, an electrode cut in the shape of an airfoil may be used.) In place of the recording pen a three-pronged probe rests on the conductive sheet. The prongs are at the corners of an isosceles right triangle with the legs parallel to the x and y axes. The distance between closest prongs is approximately 0.10 inch. At any point in the field the potential between the central prong and the others is proportional to the x and y components of air velocity. The voltages are fed back to the computer as the three-pronged probe moves across the conducting field tracing out the trajectory. Meanwhile a second plotting table draws the trajectory to provide a permanent record. To use the computer one proceeds as follows:

- A. The conductive sheet is laid on the plotting table and the appropriate potentials applied. The pen is replaced by the three-pronged probe aligned to the x and y axes. The alignment is carried out easily by placing the probe where the field is uniform and rotating it to pick off a maximum (or minimum) voltage.
- B. The resistor across the amplifier which feeds the Reynolds Modulus table is adjusted to the appropriate value of R_u .
- C. The resistors on the integrators are adjusted to set the time constant (RC) of the integrating network for a suitable ratio to the time constant (L/U) of the trajectory problem so that the plotting tables will not be overloaded. The first integrator also has its resistor adjusted proportional to K, the inertia parameter. (See sample calculation, page 8).
- D. The capacitors on the second integrators are shorted. The x,y plotting table will move to the origin. Then

ADAPTATION OF REEVES COMPUTER FOR DROP-TRAJECTORY COMPUTATIONS



a fixed voltage is impressed upon the condenser of the second y integrator to move the plotter to the initial y coordinate.

- E. During the above operation the potentials from the plotting tables are connected to the output of the first integrators; this will charge the condensers on them to voltages corresponding to initial values for $+ (\dot{x}/U)$ and $+ (\dot{y}/U)$. These operations result in the following initial conditions.

$$\frac{x}{L} = \frac{x_0}{L} = 0 \quad \text{(no charge on second x-axis integrator)}$$

$$\frac{y}{L} = \frac{y_0}{L} \quad \text{(given initial charge on second y-axis servo)}$$

$$\begin{aligned} \frac{\dot{x}}{U} &= \frac{U_a}{U} \\ \frac{\dot{y}}{U} &= \frac{v_a}{U} \end{aligned} \quad \text{(charge on first x-and y-axis integrators equal and opposite to signals from three-pronged probe on plotting table)}$$

$$R_p = 0 \quad \text{Initially the drop is at rest with respect to the air}$$

$$\frac{C_D R_p}{24} = 1.00$$

$$P = 0$$

CALCULATING SIZES OF RESISTORS AND CAPACITORS

In the electrical network the time units are measured in multiples of RC or rather t/RC . In the droplet trajectory problem the unit of time is $(U\tau/L)$

$$\frac{U\tau}{L} = \frac{t}{RC}$$

$$t = \tau \frac{U}{L} RC$$

For a typical problem:

$$\begin{aligned}U &= 200 \text{ ft/sec} \\L &= 1 \text{ ft} \\ \tau &= 0.005 \text{ sec}\end{aligned}$$

If we wish the recording pen to travel at a rate such that (in scale) the record is drawn at 0.2 ft/sec, the required time constant is

$$5 = \frac{0.005 \times 200}{1} RC$$

$$RC = 5 \text{ sec}$$

$$\text{If } C = 10 \text{ microfarad, } R = 0.5 \text{ megohm}$$

This time constant will permit the drawing of a typical trajectory in 25 sec.

All four integrator capacitors are made equal to 10 microfarads. The resistors for the x- and y-axis second integrators are set at 0.5 megohm. The first integrators are set at 0.5K megohm, where K = the dimensionless parameter appropriate to the problem ($0.1 < K < 100$).

CONCLUSION

All the necessary components for the computer have been developed and are available except the three-pronged probe and the conductive sheet. These two items do not represent a difficult development. Work on them is currently going forward.

REFERENCES

1. Langmuir, I., and Blodgett, K., "A Mathematical Investigation of Water Droplet Trajectories" AAF TR 5418, February 1946.
2. Bergrun, N.R., "A Method for Numerically Calculating the Area and Distribution of Water Impingement on the Leading Edge of an Airfoil in a Cloud", NACA TN 1397, August 1947.
3. Guibert, A., Janssen, E., and Robbins, Wm., "Determination of the Rate, the Area, and the Distribution of Impingement of Water-drops on Various Airfoils from Trajectories Obtained on the Differential Analyzer", NACA RM 9A05, February 1949.