ENGINEERING RESEARCH INSTITUTE
UNIVERSITY OF MICHIGAN
ANN ARBOR

CALCULATIONS ON DROP SIZE GROWTH AND SUPERSATURATION ON AIR IN AN ICING WIND TUNNEL

By

M. TRIBUS

J. KLEIN

Project M992-3

WRIGHT AIR DEVELOPMENT CENTER, U. S. AIR FORCE
CONTRACT AF 18(600)-51, E. O. NO. 462 Br-1

March, 1953

SUMMARY

Calculations have been made for one case on the growth of droplets and the amount of supersaturation occurring in an icing wind tunnel. The results show that condensation on the droplets is negligible but that a large degree of supersaturation is present at the test section of the tunnel.

CALCULATIONS ON DROP SIZE GROWTH AND SUPERSATURATION OF AIR IN AN ICING WIND TUNNEL

STATEMENT OF THE PHYSICAL PROBLEM

In the operation of an icing wind tunnel it is often desirable to suck saturated air laden with water drops through the test section. In passing through the wind tunnel the air is expanded and cooled, and the resulting thermodynamic state of the air at the test section is often of interest. In particular, it is important to know whether the air is supersaturated and also whether the water drops have grown to a size different from those present at the inlet to the wind tunnel.

In the calculations which are presented in this report, only one case has been considered. This case was computed by hand and illustrates the possibility of supersaturation in wind tunnels at rather modest velocities. The system analyzed consists of the University of Michigan icing wind tunnel and a cloud of 5-micron-radius water drops with a concentration of 1 gram/cubic meter at the inlet and an initial temperature of 32°F. The air is considered as accelerated to a velocity of 478 feet/second (Mach number = 0.45).

During the condensation process certain conservative idealizations were made which permit the calculations to be accomplished with less labor; in each case these assumptions tend to enhance condensation on the water drops. The important simplifications consist in setting both the surface tension and the heat of condensation equal to zero. In each case, therefore, the water drops are treated as though they had lower vapor pressure than is expected to occur in the actual situation.

SYSTEM UNDER ANALYSIS

For the physical problem described above the following mathematical system is analyzed. It is desired to determine certain variables as functions

of the distance x (in feet) from the tunnel inlet. These variables are:

A	cross-sectional area of tunnel	ft^2
B	pressure of air	lbs/ft^2
T	temperature of air	°F
U	velocity of air	ft/sec
$\rho_{wv,s}$	density of saturated water vapor	lbs/ft^3
ρ_{wv}	density of water vapor actually present	lbs/ft^3
W_{wv}	weight of water vapor passing a given point in 1 sec	lbs/sec
r	radius of water drops	ft
D	diffusivity of water vapor	ft^2/sec

The tunnel under consideration is of square cross section, so that A is determined from its profile as given in reference 1. Since the air flow is assumed to be isentropic, B, T, and U are determined from standard tables (reference 2) and when T is known $\rho_{wv,s}$ and D may be found (reference 3). This leaves ρ_{wv}, W_{wv}, and r as the unknown variables, with relations between these quantities determined from consideration of continuity of flow and the laws of condensation.

EQUATIONS

The continuity of flow gives at once

$$W_{wv} = AU\rho_{wv} \tag{1}$$

and from the laws of condensation we have

$$r\,dr = \frac{(\rho_{wv} - \rho_{wv,s})D}{\rho_{liq}} \frac{B_o}{B} \frac{dx}{U} \tag{2}$$

$$dW_{wv} = -4\pi rn\,(\rho_{wv} - \rho_{wv,s})D \frac{B_o}{B} \frac{dx}{U}, \tag{3}$$

where ρ_{liq} is the density of liquid water (62.4 lbs/ft^3), n is the number of drops passing a given point per second, which for the conditions described above is 2.35×10^{10} drops/sec, and B_o is 1 atmosphere (2116 lbs/ft^2). Hence

there is a system of one algebraic and two first-order differential equations to solve for the three unknown functions.

METHOD OF SOLUTION

If Equation 2 is solved for $(\rho_{wv} - \rho_{wv,s})D \frac{B_o}{B} \frac{dx}{U}$ and the result substituted into Equation 3, we have

$$dW_{wv} = -4\pi n r^2 \rho_{liq} \, dr ,$$

which is immediately integrable to give

$$W_{wv} = -\frac{4}{3} n\pi r^3 \rho_{liq} + C_2 , \tag{4}$$

where C_2 represents the total weight of water in both liquid and vapor forms passing any point of the tunnel in 1 second. This constant is readily computed from the initial conditions. Solving Equation 1 for ρ_{wv} and substituting for W_{wv} as given by Equation 4 results in

$$\rho_{wv} = \frac{1}{AU} \left(C_2 - \frac{4}{3} n\pi r^3 \rho_{liq}\right) . \tag{5}$$

This quantity may now be substituted into Equation 2 to give

$$r \, dr = \frac{\left(C_2 - \frac{4}{3} n\pi r^3 \rho_{liq} - AU \rho_{wv,s}\right)D \, B_o}{\rho_{liq} \, AU^2 \, B} \, dx , \tag{6}$$

which can be rewritten as

$$\frac{dr}{dx} = f(r,x) = \frac{\left(C_2 - \frac{4}{3} n\pi r^3 \rho_{liq} - AU \rho_{wv,s}\right)D \, B_o}{r \, \rho_{liq} \, AU^2 \, B} . \tag{7}$$

In Equation 7 all the variables in the right-hand expression are known functions of x or r and hence this equation is solvable numerically by the Runge-Kutta method (reference 4). These calculations have been made using formulas for second-order accuracy and increments in x graduated in accordance with the curvature of the tunnel profile. This method gives an accuracy comparable to the data obtained from the tables and the use of a third- or higher-order accuracy procedure would not result in any improvement in overall accuracy. Once the function r is known, W_{wv} and ρ_{wv} are computed from Equations 4 and 5 respectively to complete the solution of the problem.

RESULTS

The computations show that the radius of the drops increases monotonically to 5.156 microns at the test section of the tunnel. Hence the drops will always be within a few per cent of their original size at any point in the tunnel, and of the water originally present in vapor form only a relatively small amount condenses on the drops. To analyze the degree of supersaturation present the quantity $(\rho_{wv} - \rho_{wv,s})/(\rho_{wv,s})$ was computed. This quantity also increases monotonically as a function of x and reaches a value of 1.118 at the test section. This implies that at this point the weight of water vapor present is over twice that required for saturation. Approximate calculations show that changing the initial liquid water content within the values which can be obtained experimentally gives results only slightly different from those obtained for this case. Hence for a tunnel of this type operating approximately under the conditions described here, it may be stated that the radius of the drops changes by at most a few per cent and a large degree of supersaturation exists over most of the tunnel length. Figure 1 illustrates the idealized system treated here and gives the pertinent results of the analysis.

REFERENCES

1. Nicholls, J. A., et al., Design of an Icing Wind Tunnel, University of Michigan, ERI report, Contract No. AF 18(600)-51, E.O. No. 462 Br-1, June 1952.

2. Emmons, H. W., Gas Dynamics Tables for Air, Dover, 1943, pp. 17-18.

3. Perry, J. H., Editor, Chemical Engineers Handbook, McGraw-Hill, 1950, pp. 768-770.

4. Hildebrand, F. B., Advanced Calculus for Engineers, Prentice-Hall, 1949, pp. 103-106.

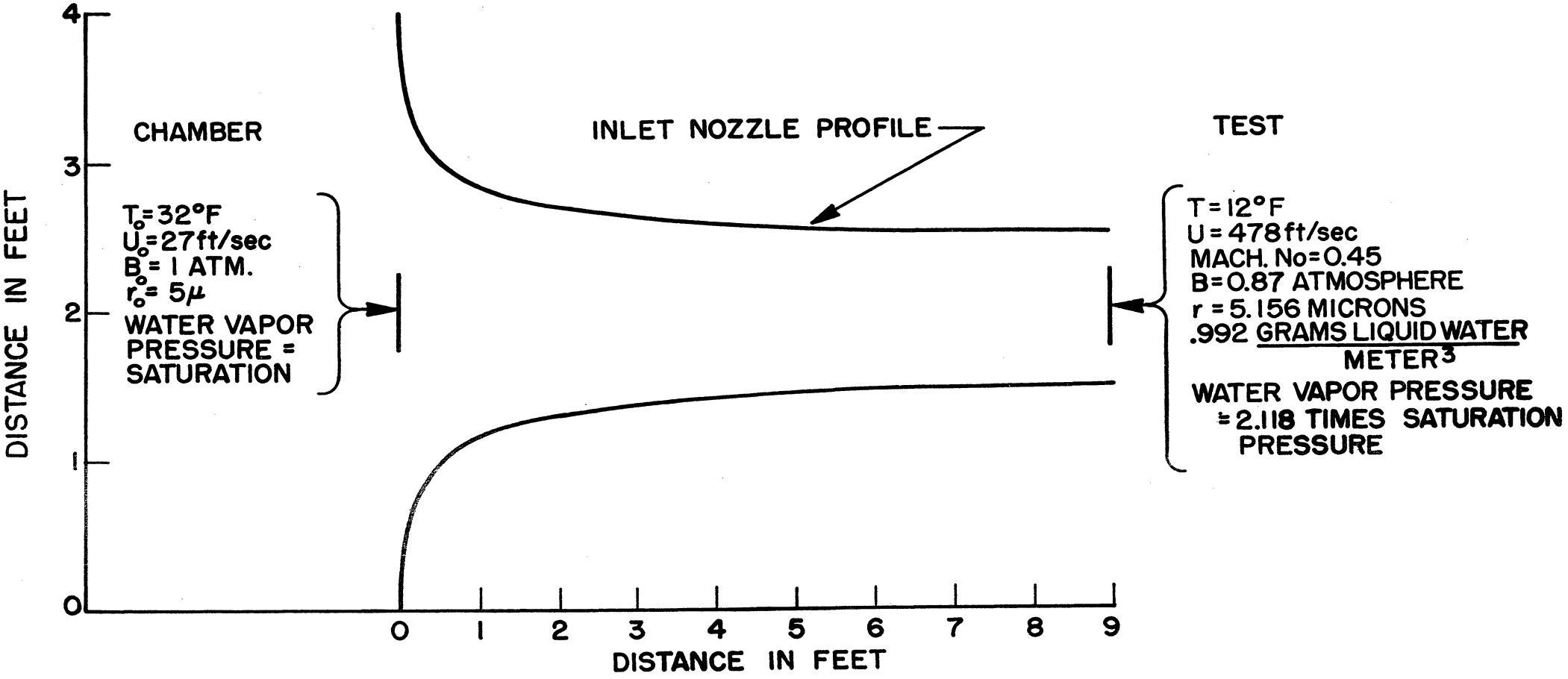

FIG. 1. THE IDEALIZED SYSTEM AND PERTINENT RESULTS OF THE ANALYSIS