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A METHOD FOR CALCULATING THE RATE OF EVAPORATION
AND THE CHANGE IN DROP SIZE DISTRIBUTION
FOR PURE SPRAYS INJECTED INTO UNSATURATED AIR

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SUMMARY

In this report the equations for the evaporation of a single water drop, a spray of uniform drops and a spray of nonuniform drops are presented and solved. It is shown that when wet-bulb depression is used as a parameter, considerable simplification occurs and the solutions may be obtained in closed form for the first two cases and may be found rapidly by a graphical method for the third. It is also shown that, given an initial drop-size spectrum, the successive spectra may be deduced almost immediately, without reference to the evaporation equations. Sample calculations are given for water sprays. Limitations and extensions of the analyses are discussed.

SYMBOLS AND NOMENCLATURE

- B = barometric pressure, lbs/ft²
- D = diffusion coefficient for mass transfer ft²/hr
- k = thermal conductivity of the air or gas through which diffusion occurs, Btu/hr-ft²(°F/ft)
- L_v = heat of vaporization, Btu/lb
- M = molecular weight, lbs/mol
- N = drop distribution function defined in Eq. 15, ft⁻²
- n = drop distribution function defined in Eq. 12, ft⁻¹
- P = pressure, lbs/ft²
- r = drop radius, ft
- R = universal gas constant, ft-lbs/mol-°R
- S = r², ft²
- T = temperature, °R
- W = weight of spray, lbs
- ξ = radius ratio
- ρ = drop density, lbs/ft³
- θ = time, hr

Subscripts

- s = adiabatic saturation
- wb = wet bulb
- db = dry bulb
- l = liquid
- v = vapor
- o = initial value at θ = 0.
- max = largest drop

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INTRODUCTION

The impetus to this study was given by a problem arising in the prevention of ice accretions on aircraft. In the simulation of icing conditions it is customary to spray water droplets into the air ahead of the wind-tunnel inlet. To reduce the turbulence level in the tunnel, the drops are sprayed into the air a good distance upstream. During the travel from the nozzles to the tunnel it is necessary to know the evaporation from the spray in order to control both humidity and drop size in the tunnel test section.

The solutions resulting from this study are applicable to a number of problems of technical interest. For example problems of fire-fighting, insecticide spraying, and paint spraying all require knowledge of the evaporation of droplets. Spray cooling towers utilize water-drop evaporation. In the case where the spray is not a pure liquid, the data of Reference 1 should be incorporated to allow for the effect of dissolved materials on the vapor pressure of the solvent. Only pure liquids are treated in the present report.

FORMULATION OF THE PROBLEM

As shown in References 1 and 2, the following idealizations are accurate representations of evaporation from small drops:

1. For small Reynolds and Grashof Moduli, the evaporation and heat transfer occur by diffusion.
2. During evaporation the drops are at the wet-bulb temperature.
3. The steady-state diffusion equation describes the heat and mass transfer.

In Reference 3 it is shown that when the air is saturated surface-tension effects are appreciable and cause small drops to evaporate and condensation to occur on the larger drops. When the air is not saturated, surface tension has negligible effect on evaporation.

When a small water drop is projected into still air, its velocity falls very rapidly to a low value. Therefore, for drops less than 100 microns in diameter, such as occur in icing problems and fogs, the major portion of the droplet life will be at a low Reynolds Modulus. For steady-state diffusion from a drop, a mass balance yields:

$$-4\pi r^2 \frac{dr}{d\theta} = \frac{4\pi D M r}{BR} \left(\frac{P_d}{T_d} - \frac{P_\infty}{T_\infty} \right) \quad (1)$$

In cases where the heat of vaporization is large, the omission of the thermal capacitance of the drop yields negligible error. For such a case, an energy balance yields:

$$4\pi r^2 \rho L_v dr/d\theta = 4\pi k r (T_d - T_\infty) \quad (2)$$

The simultaneous solution of Eqs. 1 and 2 yields the droplet radius, temperature, and vapor pressure as functions of time and the psychrometric conditions far from the droplet. The solution of Eqs. 1 and 2, however, leads to the conventional equation for wet-bulb depression, and considerable simplification of the calculations is effected if Postulate 2 above is adopted and Eq. 2 utilized alone.

A SINGLE DROP IN A LARGE ATMOSPHERE

If the ambient temperature and humidity are unchanged, integration of Eq. 2 leads to:

$$r^2 - r_0^2 = \frac{2k}{\rho_d L_v} (T_{wb} - T_{db}) \theta \quad (3)$$

Letting $\xi = r/r_0$,

$$\xi^2 = 1 - \frac{2k(T_{db} - T_{wb})\theta}{\rho_d L_v r_0^2} . \quad (4)$$

Figs. 1, 2, and 3 show the drop radius as a function of time for several conditions. Note the changed time scale (ten to one) for the higher temperatures. The right-hand scale of each figure is a cubic scale which shows the percentage of droplet volume remaining.

In Reference 1, part II, Fig. 8 is a graph of the measured diameter of a single water drop plotted against time. The drop was initially over 10^3 microns in diameter, and therefore did not have a negligibly small value of Grashof Modulus. Nevertheless, the drop size squared plotted against time yielded a straight line whose slope was only 10 per cent larger than predicted by Eq. 4.

A UNIFORM SPRAY OF DROPS IN A LIMITED ATMOSPHERE

During the evaporation of a spray of drops, the ambient humidity and temperature will change. The integration of 2 then requires that the variation in wet-bulb depression be specified.

Consider a spray of uniform drops having radius r_0 . Let $\xi = r/r_0$. Then if $W_{l,0}$ is the initial spray density, the spray density at any time is given by

$$W_l = W_{l,0} \xi^3 . \quad (5)$$

If $W_{v,0}$ is the initial water-vapor density, the water-vapor density at any time is given by

$$W_v = W_{v,0} + W_{l,0} (1 - \xi^3) . \quad (6)$$

It is convenient to define

$$\xi_s = \frac{W_{l,0} + W_{v,s} - W_{v,0}}{W_{l,0}} , \quad (7)$$

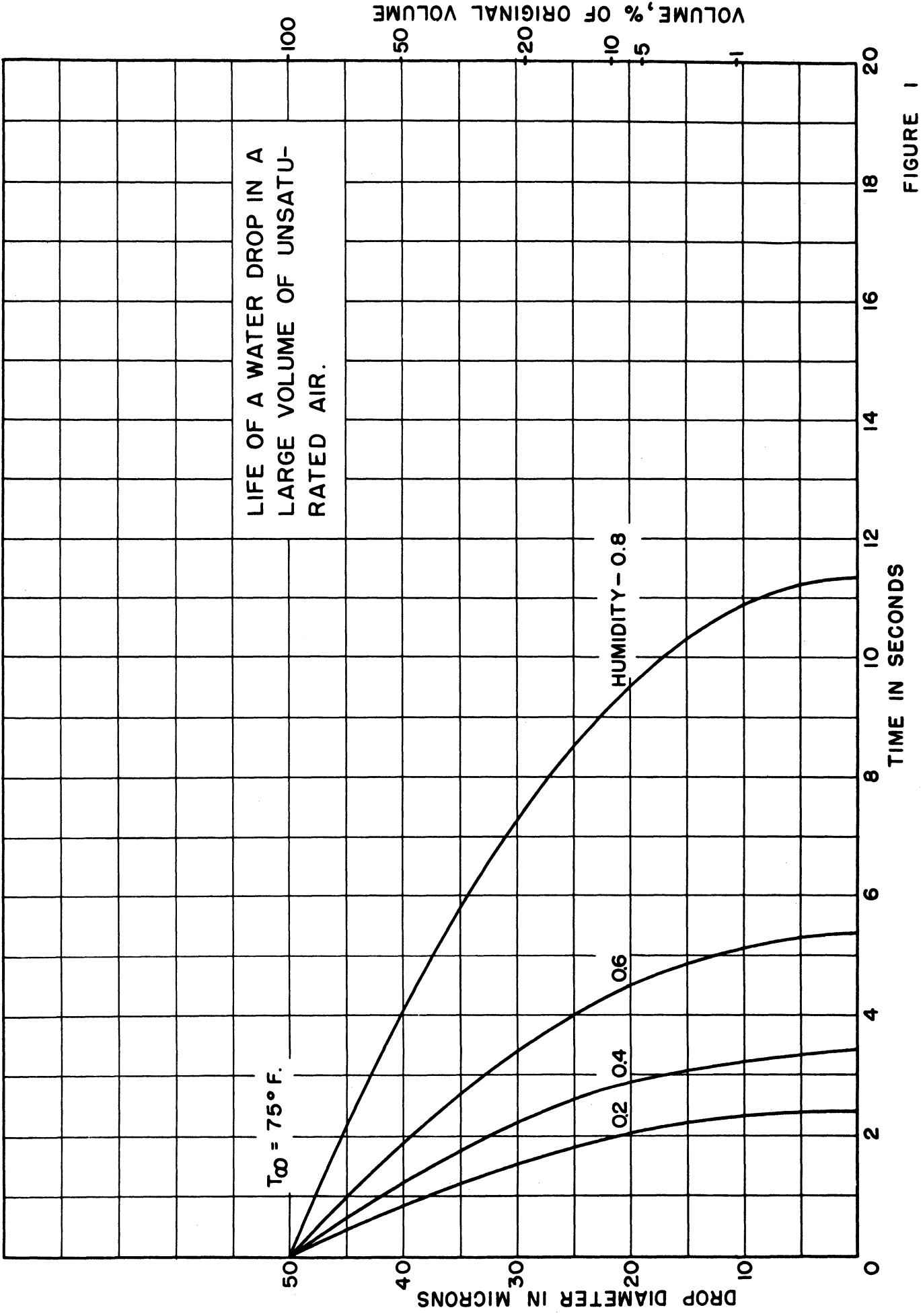


FIGURE 1

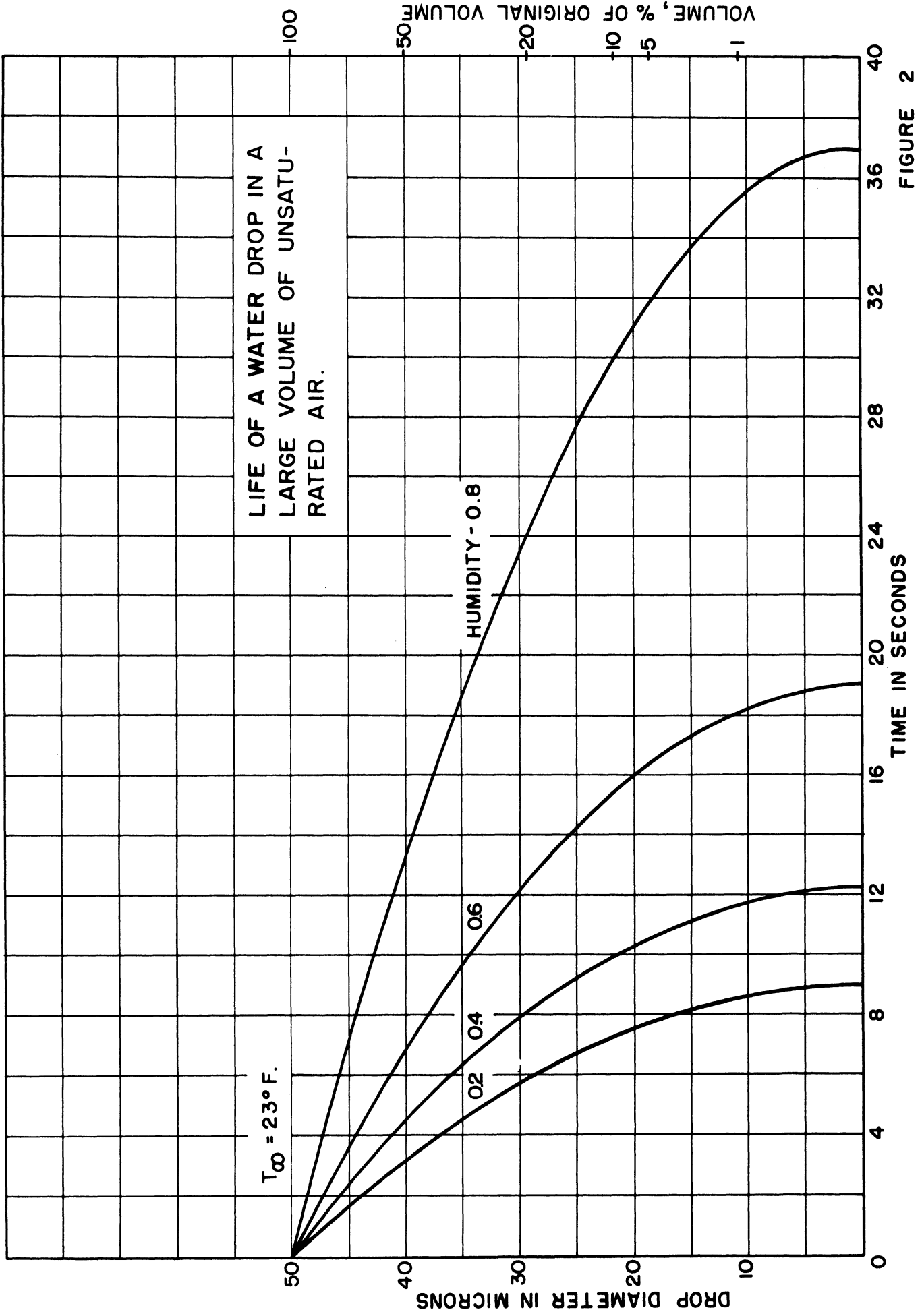
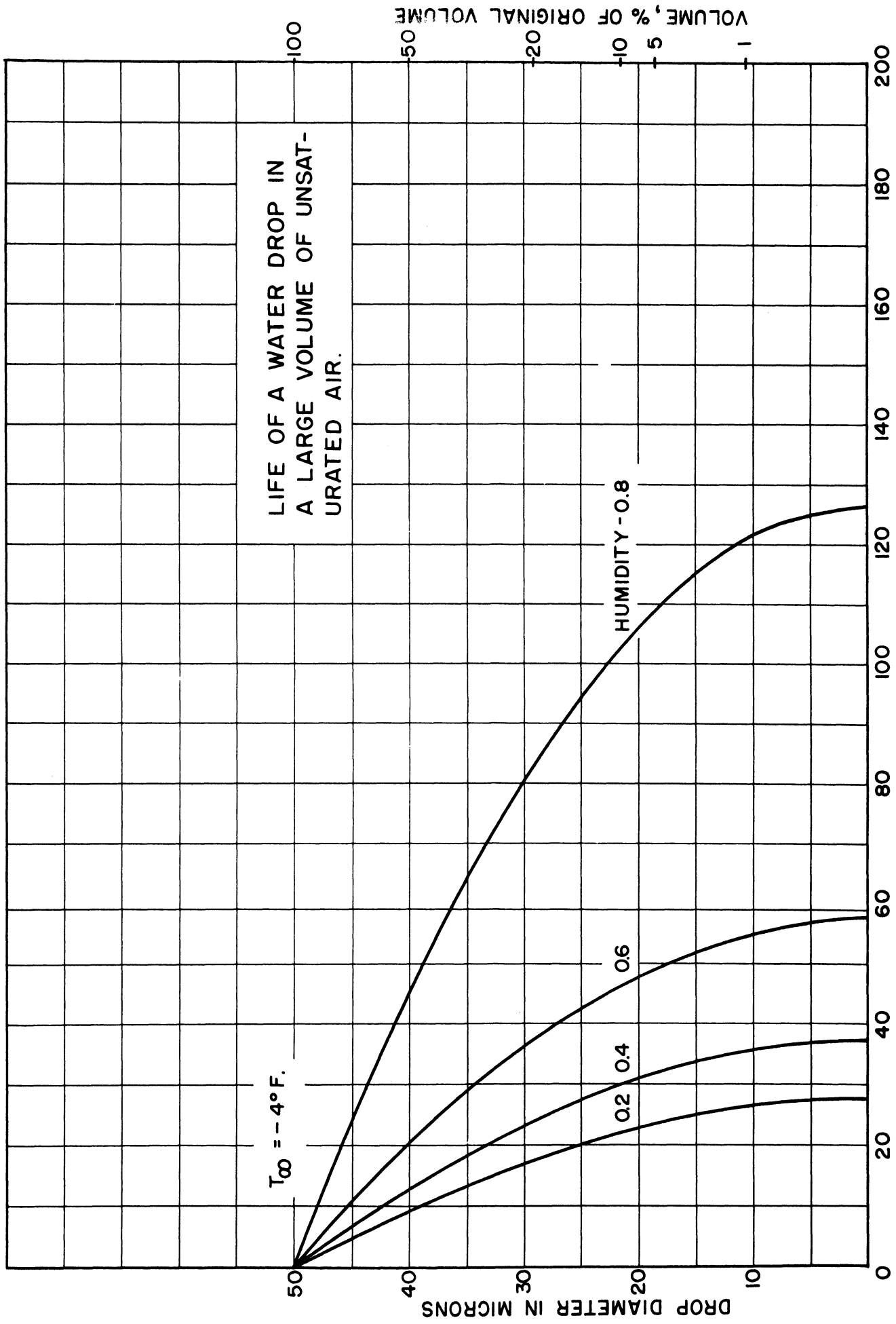


FIGURE 2



LIFE OF A WATER DROP IN
A LARGE VOLUME OF UNSAT-
URATED AIR.

FIGURE 3

where $W_{v,s}$ = water vapor density at adiabatic saturation and ξ_s = radius ratio required to make $W_v = W_{v,s}$ in Eq. 6.

An inspection of the psychrometric charts shows that T_{wb} and T_{db} vary linearly with the absolute humidity during adiabatic saturation. (For water the variation in T_{wb} is almost zero.) Thus one may write:

$$T_{db} = T_{db,o} + (T_{db,s} - T_{db,o}) \left\{ \frac{1 - \xi^3}{1 - \xi_s^3} \right\} \quad (8)$$

and

$$T_{wb} = T_{wb,o} + (T_{wb,s} - T_{wb,o}) \left\{ \frac{1 - \xi^3}{1 - \xi_s^3} \right\} \quad (9)$$

Substitution of Eqs. 8 and 9 into 2, noting that $T_{wb,s} = T_{db,s}$, yields

$$\xi \frac{d\xi}{d\theta} = - \frac{k(T_{db,o} - T_{wb,o})}{r_o^2 \rho L_v} \left\{ \frac{\xi^3 - \xi_s^3}{1 - \xi_s^3} \right\} \quad (10)$$

The integral of Eq. 10 (for $\xi = 1$ at $\theta = 0$) is:

$$\frac{k(T_{db,o} - T_{wb,o})\theta}{r_o^2 \rho L_v} = \frac{1 - \xi_s^3}{3 \xi_s} \left\{ \frac{1}{2} \ln \frac{(\xi_s^2 + \xi\xi_s + \xi^2)(1 - \xi_s)^2}{(\xi_s^2 + \xi_s + 1)(\xi - \xi_s)^2} - \sqrt{3} \tan^{-1} \frac{2\xi + \xi_s}{\sqrt{3} \xi_s} + \sqrt{3} \tan^{-1} \frac{2 + \xi_s}{\sqrt{3} \xi_s} \right\} \quad (11)$$

A graph of Eq. 11 is shown in Fig. 4. The parameter on each curve is $(1 - \xi_s^3)^{-1}$, or the ratio of the weight of initial spray to the weight of water required for saturation. Figs. 5 and 6 show specific solutions to spray equations of interest in icing work.

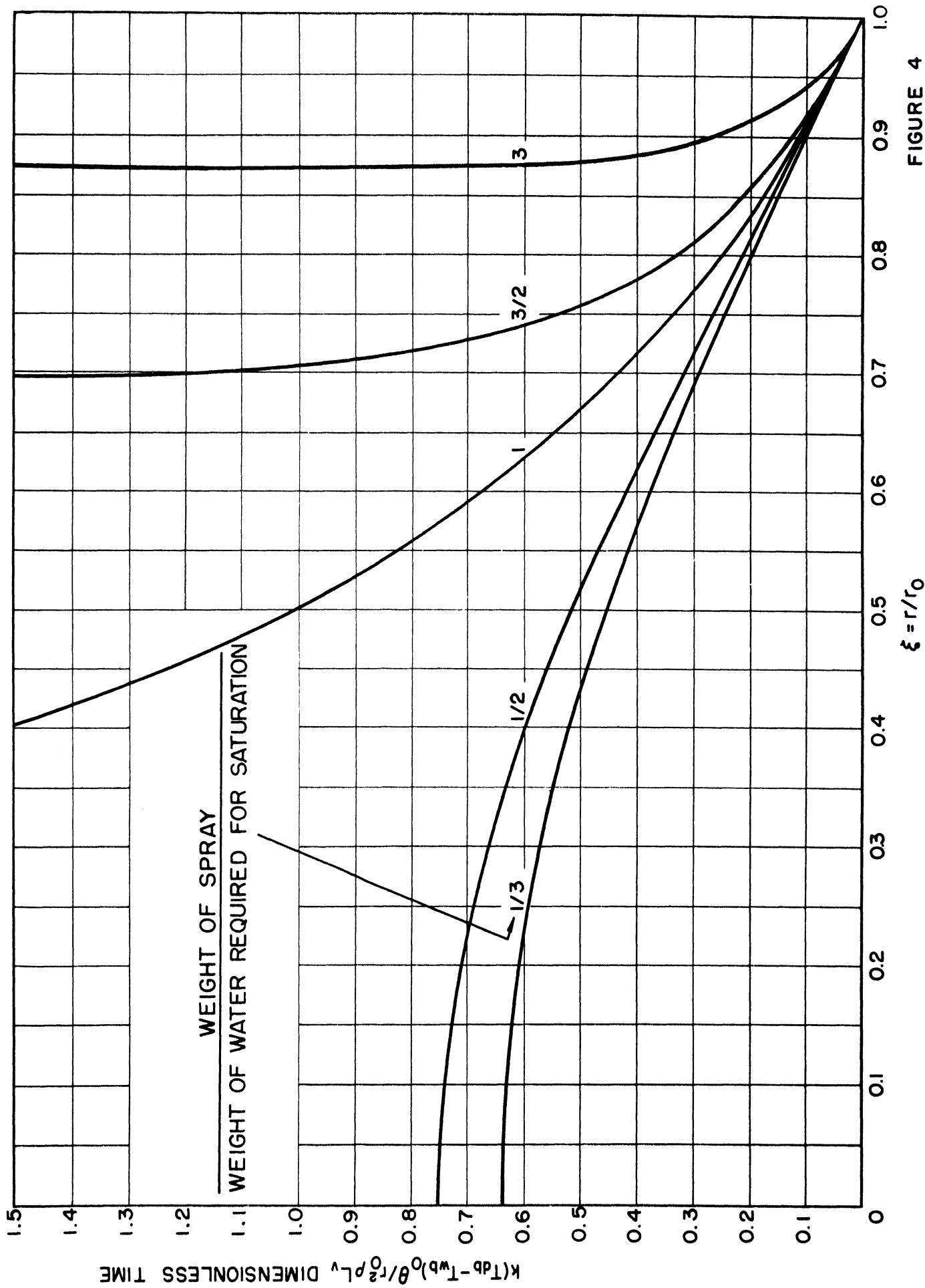


FIGURE 4

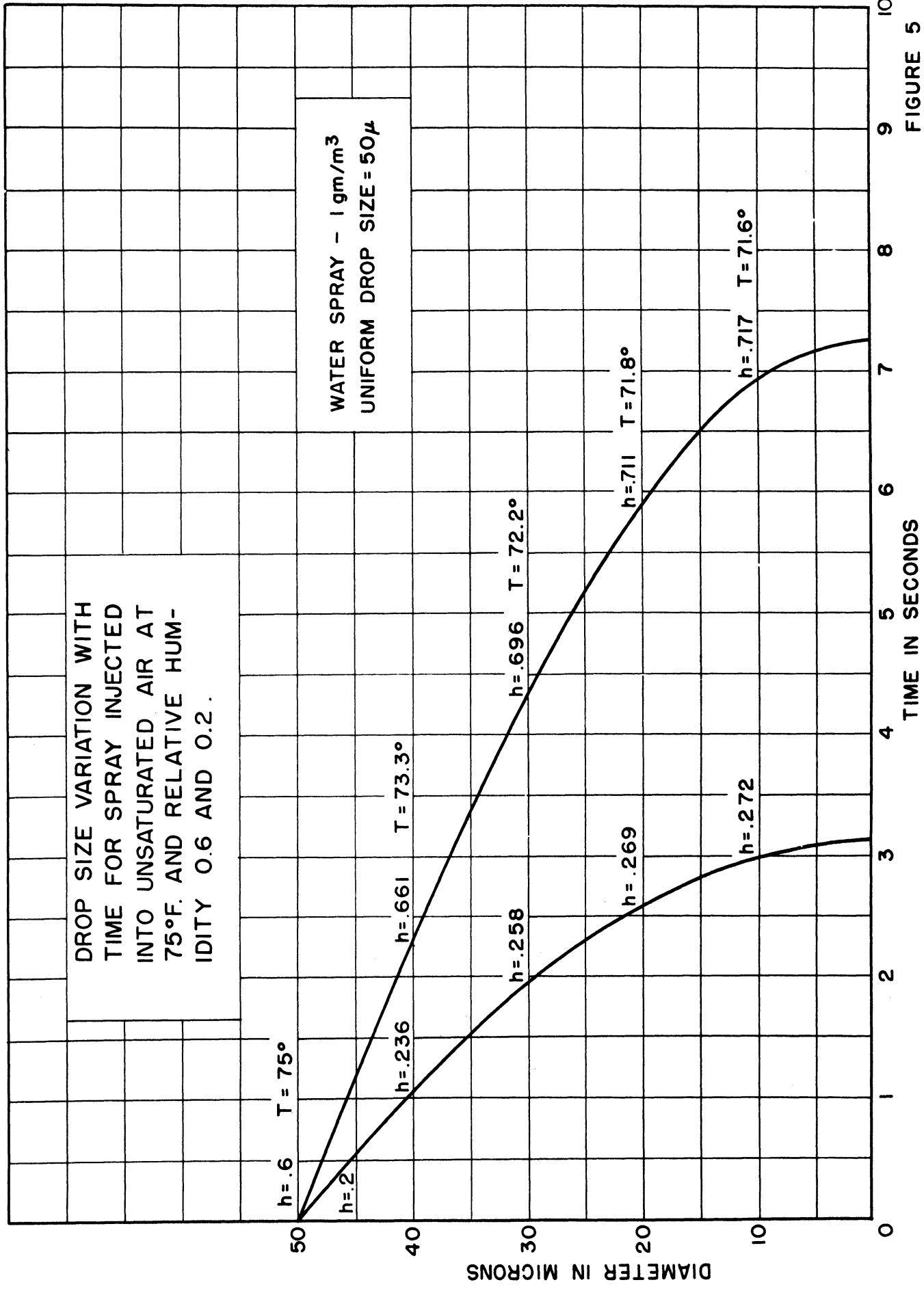


FIGURE 5

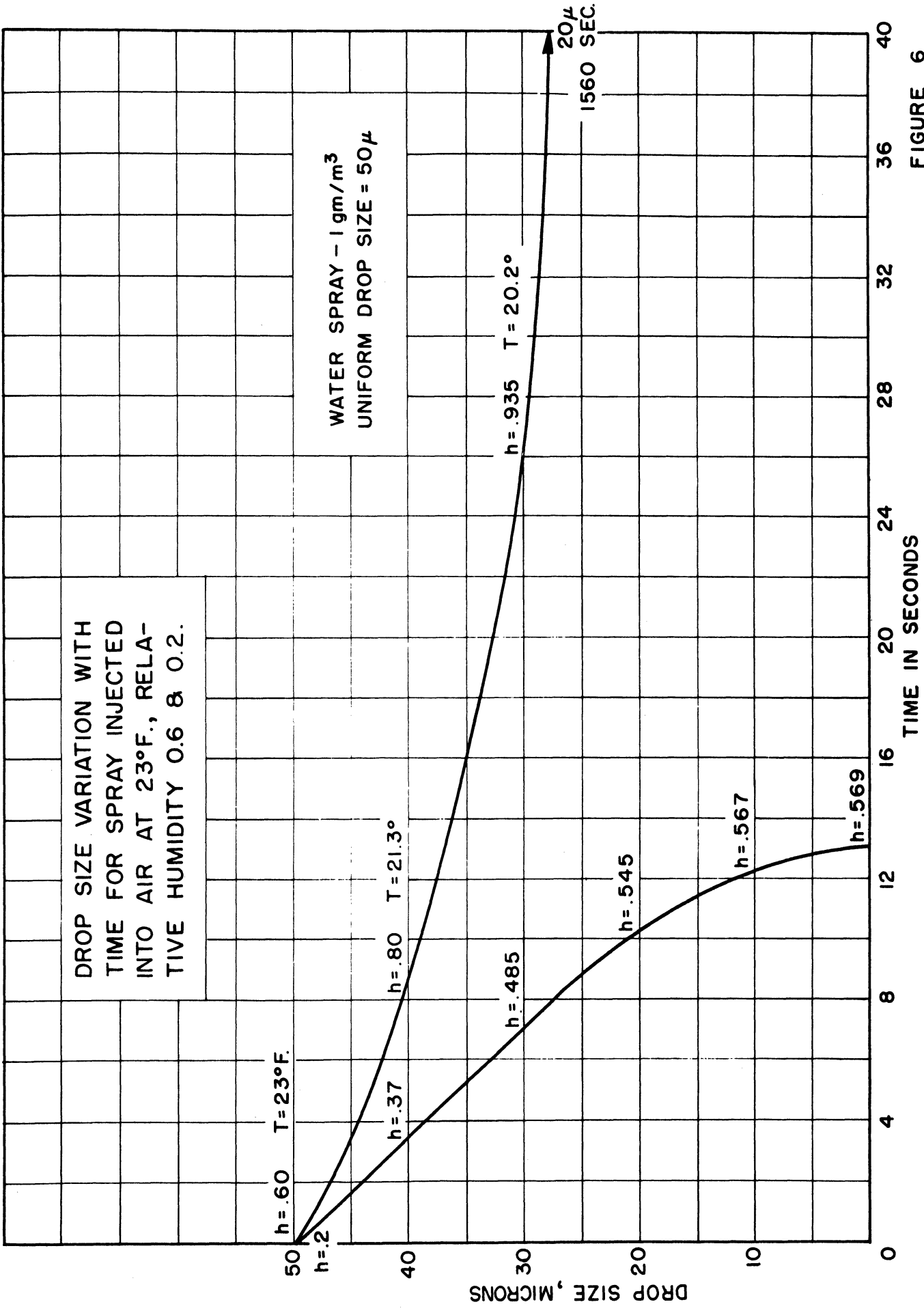


FIGURE 6

A NONUNIFORM SPRAY OF DROPS IN A LIMITED ATMOSPHERE

The conventional method for the presentation of information concerning sprays of droplets is to graph the spray distribution function $n(r)$ (Reference 4). The distribution function has the characteristic that $n(r) dr$ is the number of drops having drop radii between r and $r + dr$. From the drop distribution function, one may calculate:

$$\text{Number of drops} = \int_0^{\infty} n(r) dr \quad . \quad (12)$$

$$\text{Total droplet surface} = \int_0^{\infty} 4\pi r^2 n(r) dr \quad . \quad (13)$$

$$\text{Total volume} = \int_0^{\infty} \frac{4}{3} \pi r^3 n(r) dr \quad . \quad (14)$$

Fig. 7 shows a typical graph of $n(r)$ for a commercial spray nozzle.

As will be shown later, it is convenient to define a new parameter $S = r^2$ and a new drop distribution function $N(S)$ such that $N(S) dS$ is the number of drops having values of r^2 between S and $S + dS$. The spray characteristics are then computed as:

$$\text{Number of drops} = \int_0^{\infty} N(S) dS \quad . \quad (15)$$

$$\text{Total drop surface} = \int_0^{\infty} 4\pi S N(S) dS \quad . \quad (16)$$

$$\text{Total drop volume} = \int_0^{\infty} \frac{4}{3} \pi S^{3/2} N(S) dS \quad . \quad (17)$$

Now Eq. 2 applies to any two drops, say drops which are identified by subscripts 1 and 2. Rewriting Eq. 2 in terms of S,

$$\frac{dS_1}{d\theta} = \frac{dS_2}{d\theta} = \frac{-2k(T_{db} - T_{wb})}{\rho_d L_v} \quad . \quad (18)$$

Since the right-hand term of Eq. 18 is the same function of time for all drops (if the spray is well mixed), it follows that $dS/d\theta$ is the same for all drops.

Fig. 8 is a replot of Fig. 7, wherein the distribution function $N(S)$ is shown. It follows from the conclusion above that all SUCCESSIVE DROP DISTRIBUTIONS ARE OBTAINED FROM FIG. 8 BY A LINEAR SHIFT ALONG THE S AXIS. IN THE CASE OF COMPLETE EVAPORATION OF SOME DROPS THE CURVE TO THE LEFT OF THE ORIGIN IS OMITTED ($S < 0$).

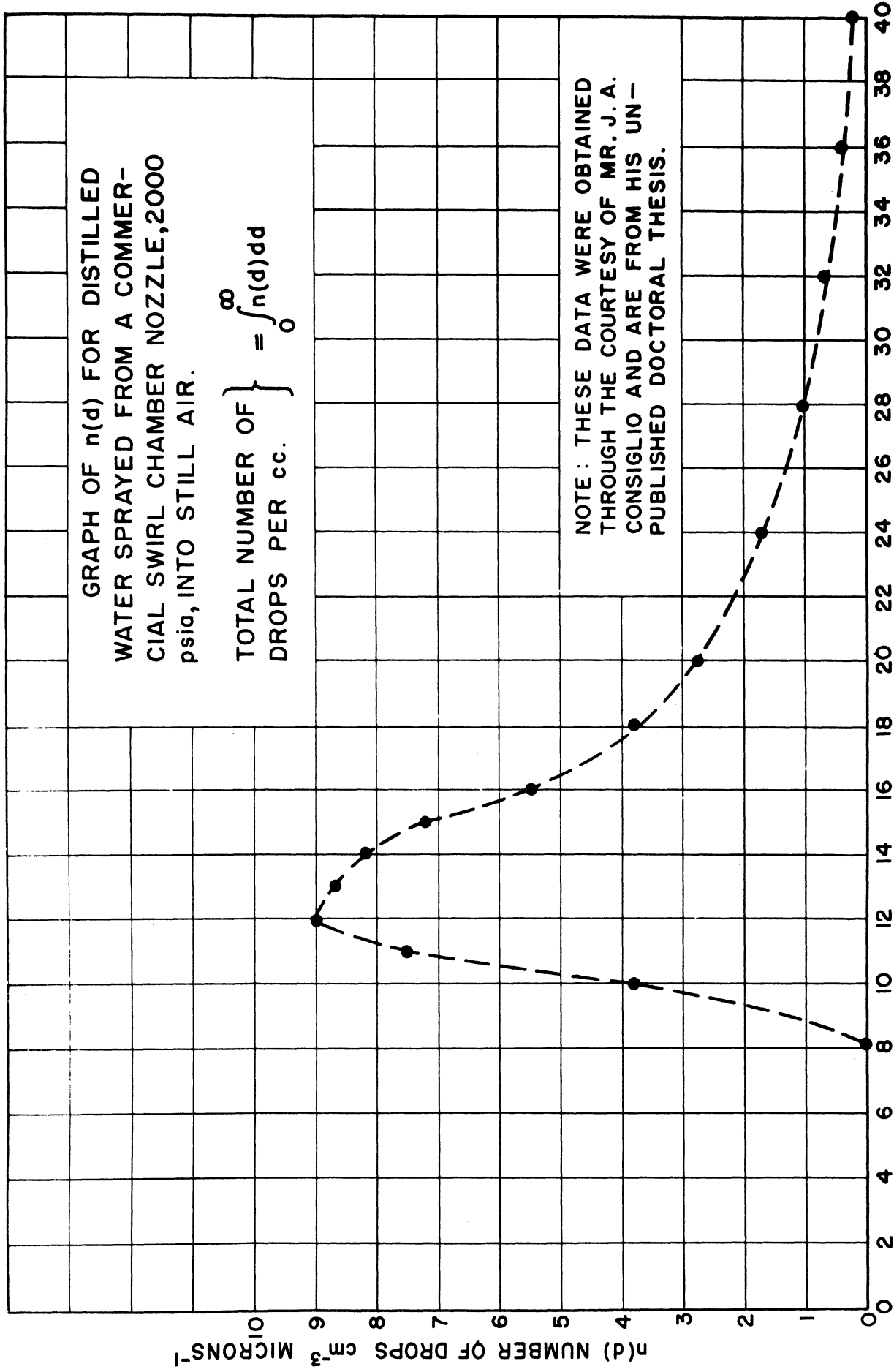
In view of the above conclusion, the rate of evaporation from the spray may be computed by calculating the variation in drop size of the largest drop and relating all other drop to this maximum drop radius.

For example, let $S_{\max,0} = r_0^2$ for the largest drop initially present in the spray. Then by graphical integration one may obtain the weight of spray as a function of S_{\max} :

$$W_l(S_{\max}) = \int_0^{S_{\max}} N(S) \frac{4}{3} \pi S^{3/2} dS \quad . \quad (19)$$

A graph of W_l versus S_{\max} for the data of Fig. 7 is shown in Fig. 9. The variation in wet-bulb depression as before is a linear function of the weight of spray which has evaporated; i.e.,

$$T_{db} - T_{wb} = (T_{wb,0} - T_{db,0}) \left\{ \frac{W_{v,s} - W_{v,0} - W_{l,0} - W_l}{W_{v,s} - W_{v,0}} \right\} \quad . \quad (20)$$



DIAMETER, MICRONS

FIGURE 7

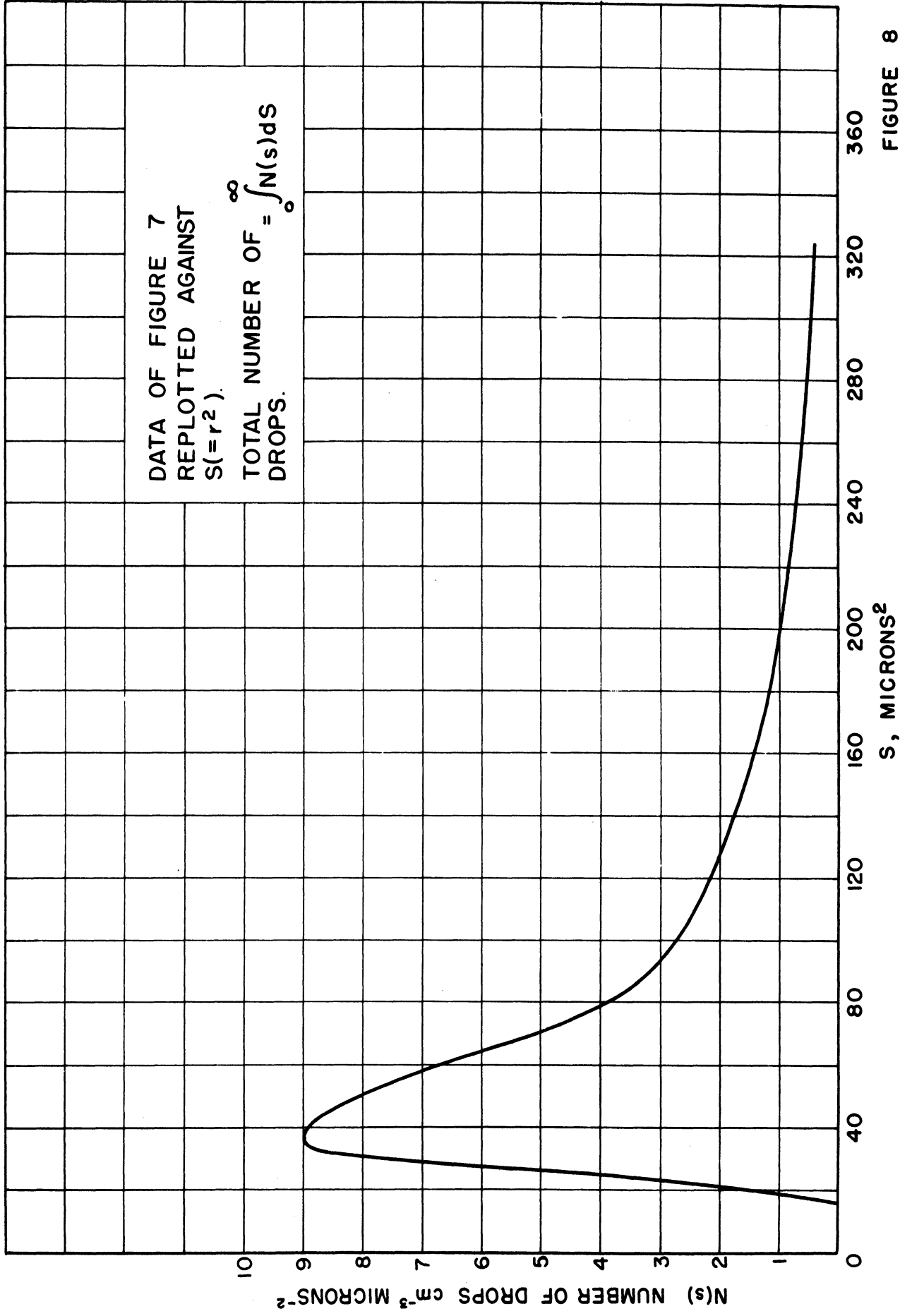


FIGURE 8

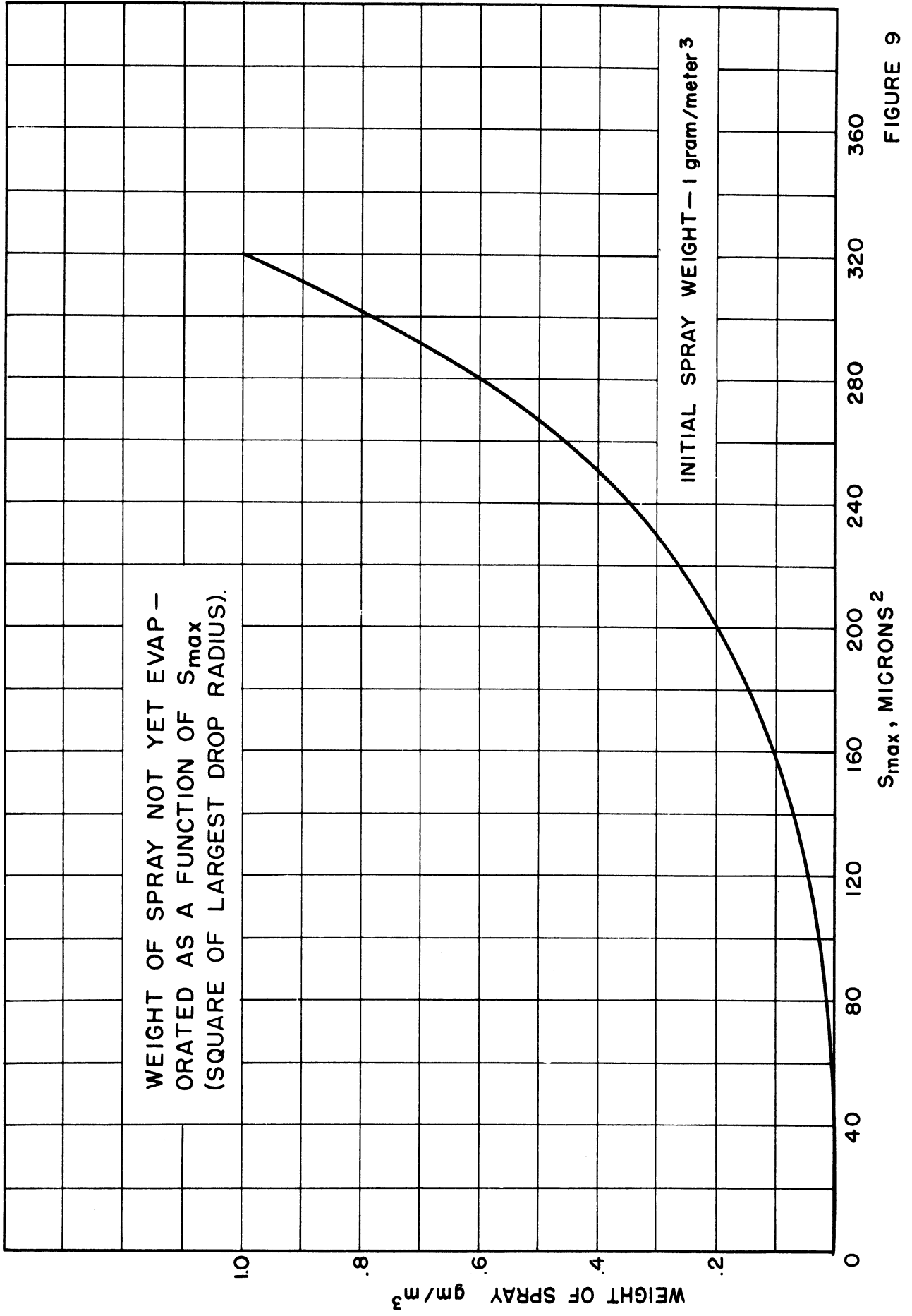


FIGURE 9

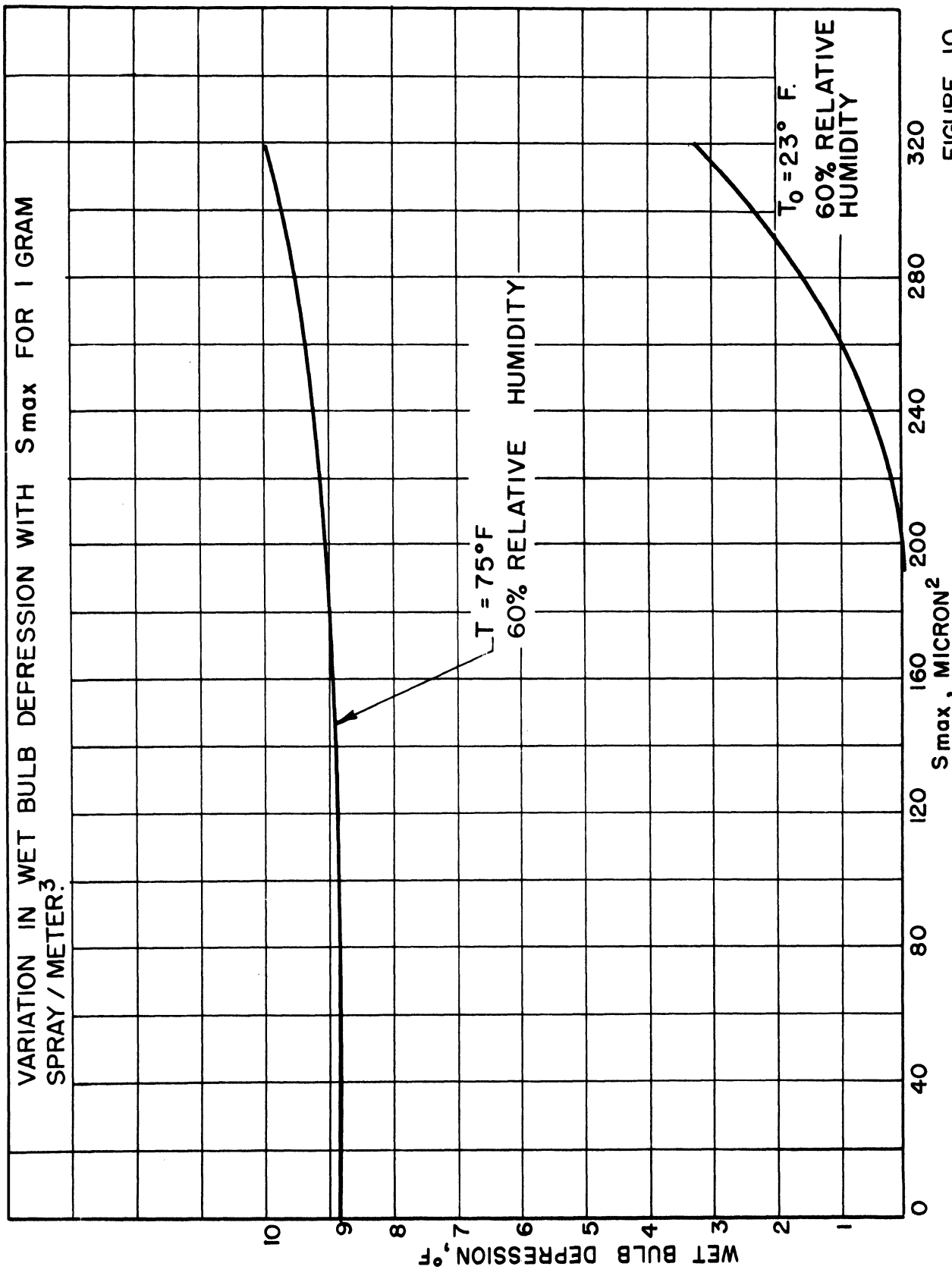


FIGURE 10

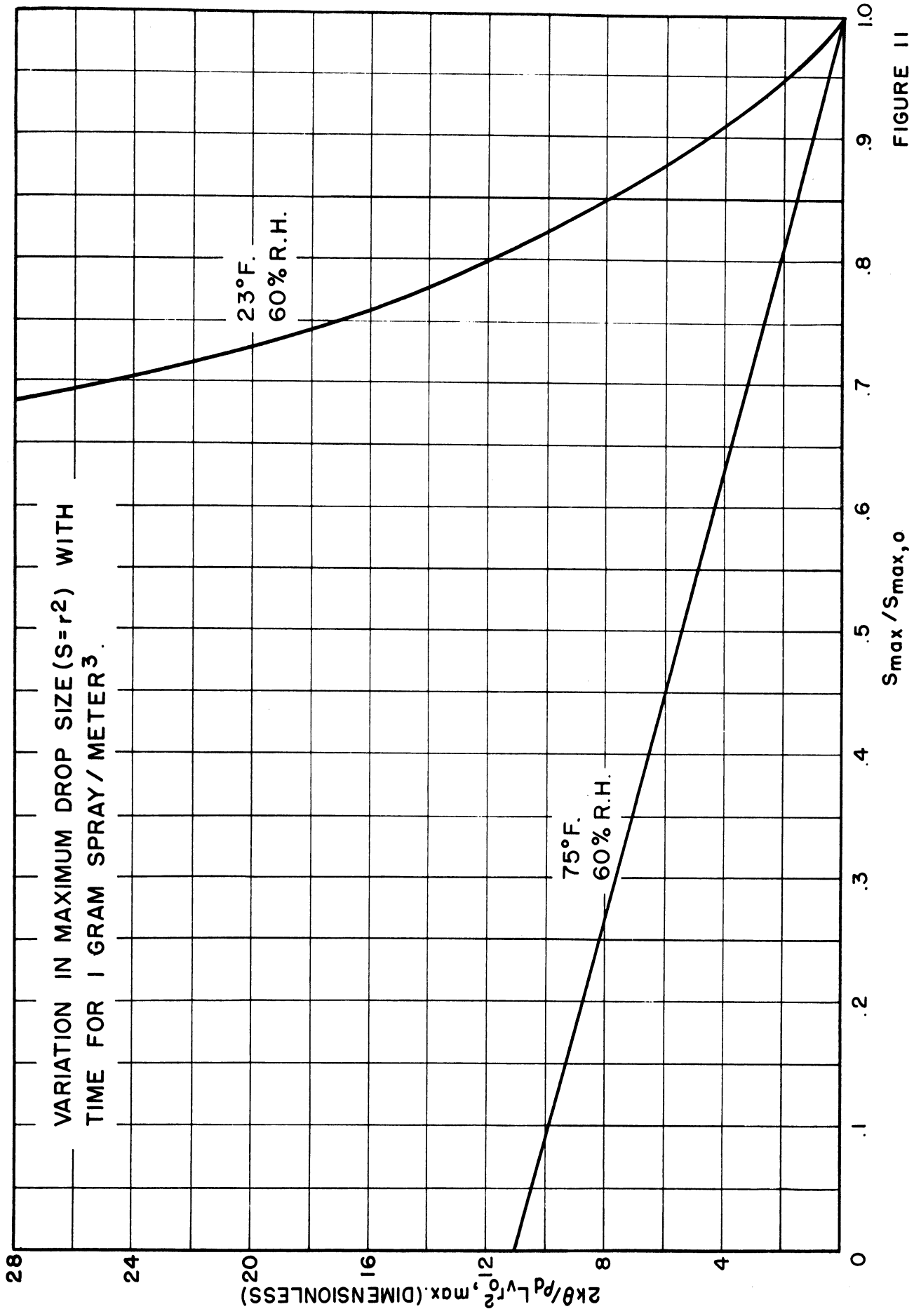


FIGURE 11

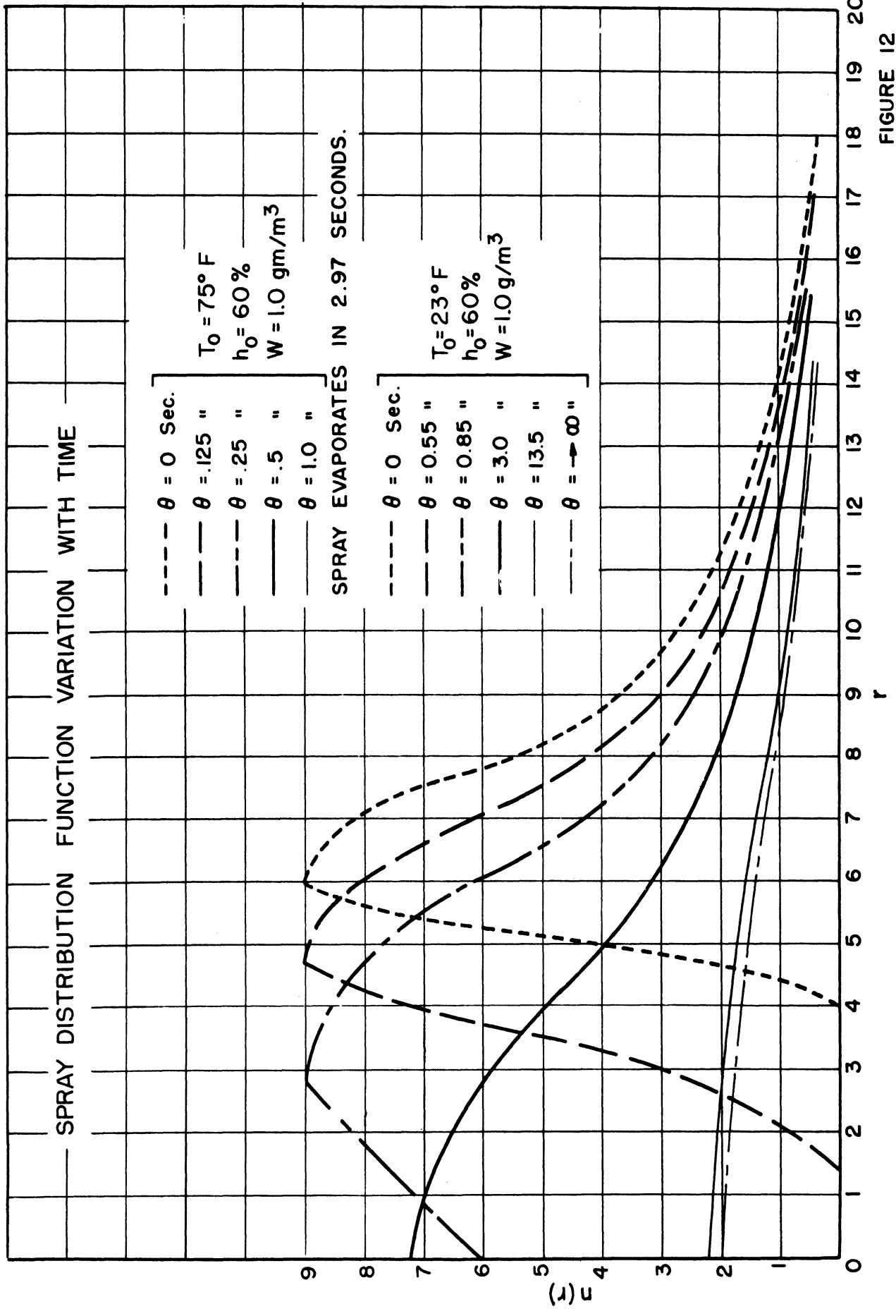


FIGURE 12

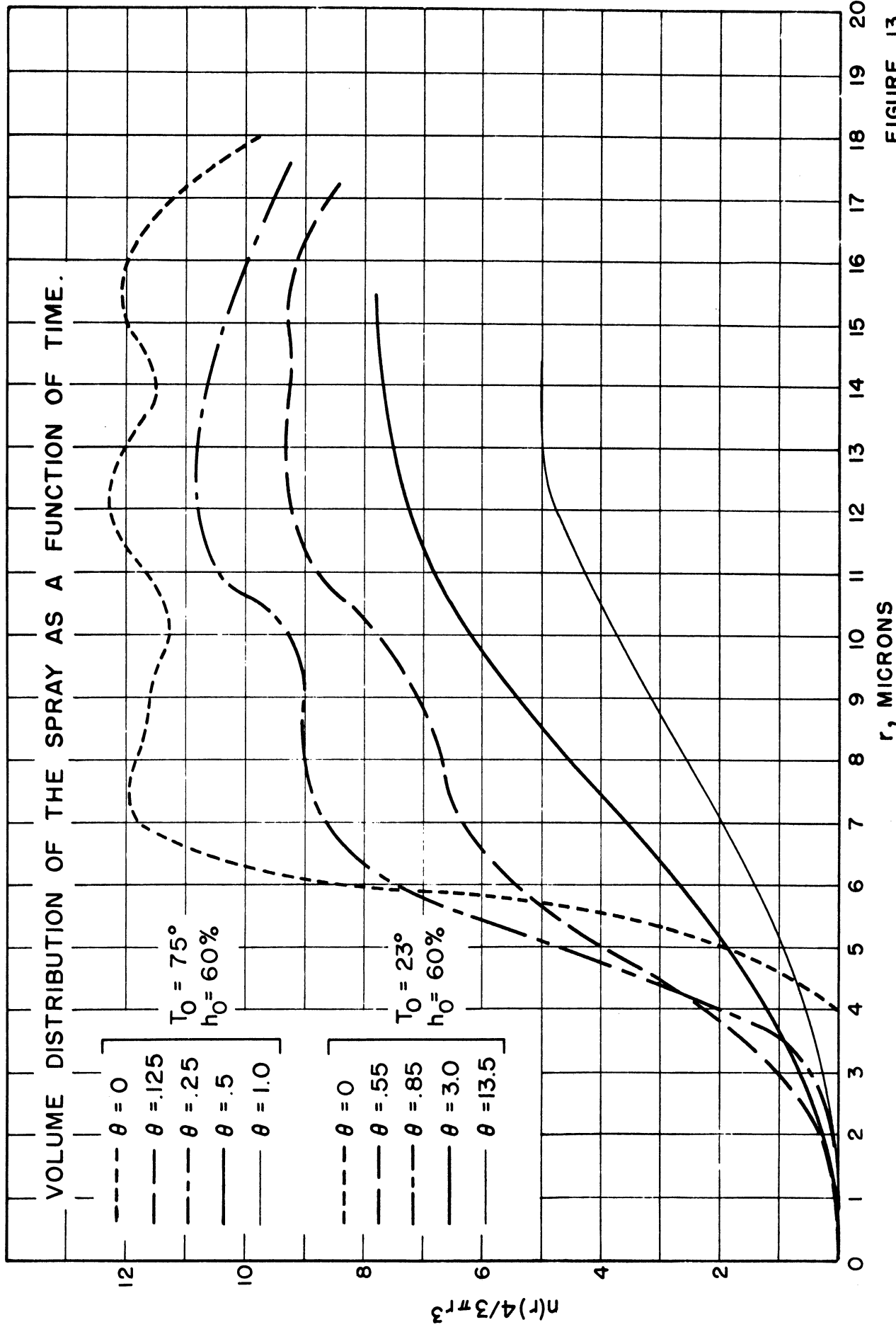


FIGURE 13

The wet-bulb depression can therefore be presented as a function of S_{\max} as shown in Fig. 10. Graphical integration of Eq. 18 in the form:

$$\int_{S_{\max,0}}^{S_{\max}} \frac{dS}{(T_{db} - T_{wb})} = - \frac{2 k \theta}{\rho_d L_v} \quad (21)$$

yields the results shown in Fig. 11. The spray distribution function can then be read from Fig. 8 by displacing the origin an amount $S_{\max} - S_{\max,0}$. The drop distribution function in the conventional representation, $n(r)$, is shown for various times in Fig. 12. Fig. 13 shows the volume distribution function $4/3\pi n(r)r^3$ versus r at successive times.

DISCUSSION

The analysis presented above does not include the effects of surface tension or electrical charge on the vapor pressure of the droplet. These effects are negligible except in cases where the drops exist in saturated or slightly supersaturated air. For such cases, Langmuir (Reference 3) has shown that the small drops tend to evaporate, while the large drops increase in size. Eventually only a few large drops would be produced by this process, were it not for other effects such as falling out of the large drops under the influence of gravity or breaking up under stresses produced by air currents. When the weight of spray introduced is sufficient to saturate the air, the problem may be divided into two parts. In the first part of the evaporation process, the analysis given here may be applied. This analysis leads to a spray distribution in saturated air. Using this latter distribution one may then proceed, with the method of analysis by Langmuir (Reference 3), to calculate the shift in droplet spectrum.

Another case often occurs in which the droplets do not consist of pure liquid but are composed of a volatile solvent and an involatile solute, as for example in paint spraying. Reference 1 shows how the appropriate wet-bulb depression must be found for each concentration of solute, if the solute serves to lower the vapor pressure of the droplet. The method of attack in this case is not as clear, since although each droplet starts at the same concentration of solute, after a short while the concentration is different for each drop.

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