ENGINEERING RESEARCH INSTITUTE
THE UNIVERSITY OF MICHIGAN
ANN ARBOR

Progress Report

INVESTIGATION ON MINIMIZING THE AMOUNT OF PRESSURIZING GAS REQUIRED FOR EMPTYING LOX CONTAINERS DURING MISSILE FLIGHT

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Project 2646

DEPARTMENT OF THE ARMY
DETROIT ORDNANCE DISTRICT
DETROIT, MICHIGAN
CONTRACT NO. DA-20-018-ORD-15316

September, 1957

ABSTRACT

Analytical work has been completed for the case of emptying a tank filled with saturated liquid. Effects of tank size, emptying time, and inlet gas temperature were investigated. An experimental apparatus has been designed and construction of this apparatus has been essentially completed.

INTRODUCTION

During the period July 1 to August 15 approximately 900 man hours have been spent on the project. The work has progressed along two general but correlated lines, namely, analytical and experimental. The progress along each of these lines is described in this report.

ANALYTICAL WORK

Equations have been derived which give the final amount of gas in the tank after the emptying process has been completed. It is evident that one of the variables is the heat-transfer rate during the emptying process, because the greater the heat transfer, the greater the boil-off and the less the amount of pressuring gas. Therefore, one important step in arriving at an analytical answer is to determine the heat-transfer rate during the emptying process.

Essentially three mechanisms of heat transfer through the walls have been considered.

The first, which is designated Analysis I, assumes a container made of zero thermal resistance material and no resistance to heat flow between the wall and the liquid. Thus the only resistance is the outside film co-efficient and both the heat flux and area are assumed to be constant.

In Analysis II an infinitely thin wall is assumed and thus the only heat transfer considered is in the direction perpendicular to the walls of the container. Further, heat transfer to the vapor is neglected, and this means that the area for heat transfer is considered to be only the area in contact with the liquid. Thus constant heat flux per unit area, but variable area, is assumed.

The main reason for selecting these two cases is that they represent two limiting cases. For a given outside coefficient the heat transfer as calculated by Analysis I will yield the maximum heat transfer, and Analysis II gives the minimum heat transfer. It is anticipated that the actual heat transfer will be between these two extremes.

As a more realistic approach to this problem, Analysis III has been carried out. In this analysis consideration is given to the fact that heat can be transferred from the ambient air to the outside wall above the liquid level, which heat will then be transferred longitudinally down the cylinder to the level of the liquid, and the transferred to the liquid. The calculation of the longitudinal conduction was based on the conductivity of the tank material and the wall thickness. For this calculation the heat transfers to the bottom and top plates were neglected.

Figure 1 qualitatively illustrates the differences in heat transfer for the three analyses. In this graph the normalized heat flux is shown as a function of the normalized level in the tank. When the tank is full, all three analyses yield the same results.

Several assumptions are common to all analyses made thus far. These are as follows:

(1) All vapor encountered is treated as an ideal gas.
(2) The vapor region above the liquid is thermally insulated.
(3) The heat-transfer coefficient related to the outside film is small as compared with the coefficient between the inside wall and the liquid.
(4) The pressure in the container is maintained constant at a prescribed value.
(5) Where applicable, gas introduced into the container is of constant prescribed temperature.
(6) Connections, fittings, and attachments provide no augmentation of heat flux.
(7) The liquid temperature is constant.
(8) The heat capacity of the containing vessel is neglected where applicable.
(9) The top cover plate is neglected insofar as heat transfer is concerned.
(10) For purposes of initial calculation, ullage has been taken as zero.

The following nomenclature is used in the equations which follow.

A	square feet, heat-transfer area
D	feet, tank diameter
e	base of natural logarithms
L	feet, liquid level
m	pounds mass
P	pounds per square foot, pressure
q	Btu/Hr, heat transfer rate to liquid
R	$\frac{\text{Ft-lb}_f}{\text{Lb}_m\,°\text{F}}$, specific gas constant
T	°R, absolute temperature

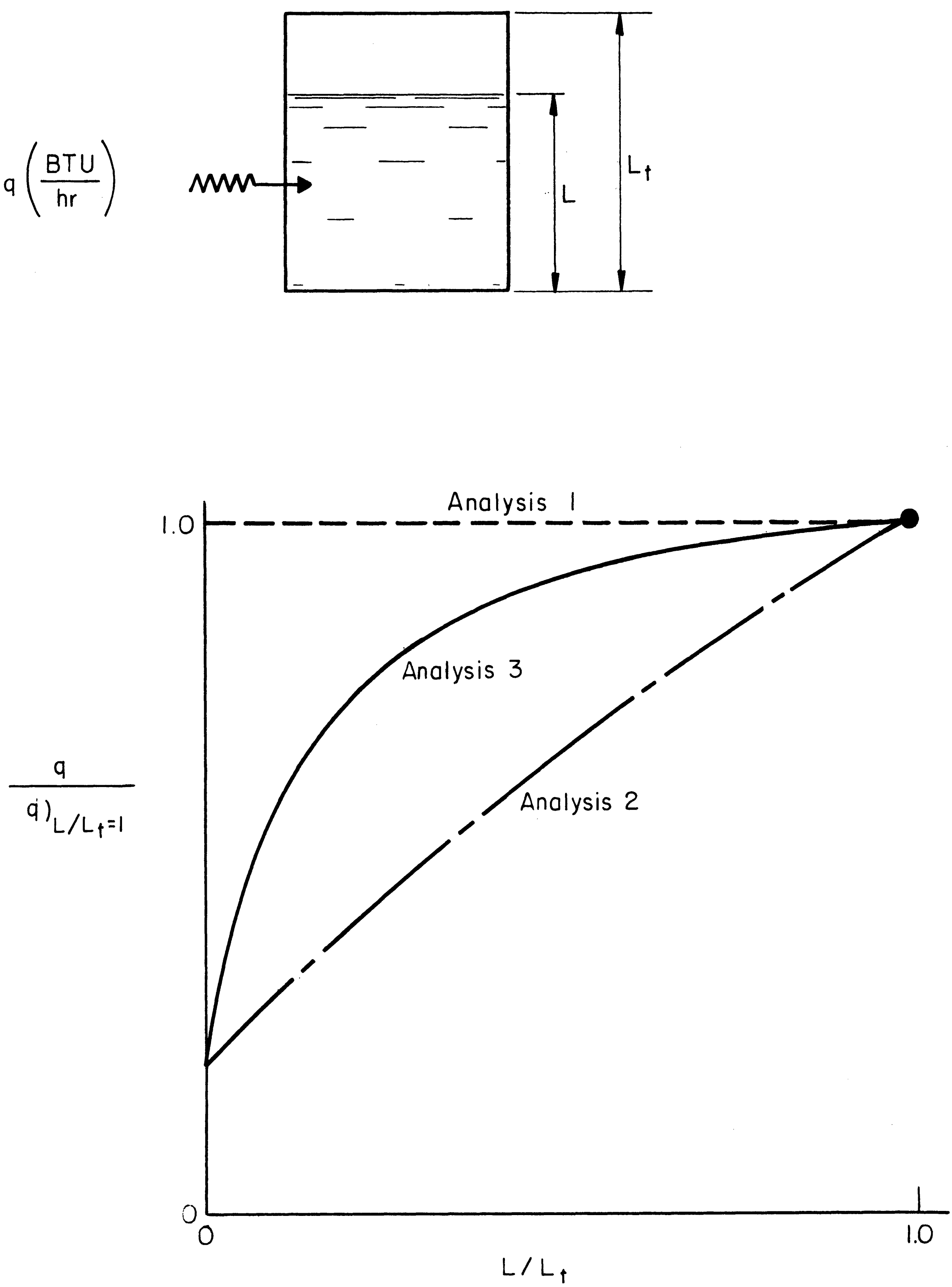

q (BTU/hr)
L
L_t
Analysis 1
1.0
Analysis 3
Analysis 2
q / q)_{L/L_t=1}
0
0
1.0
L / L_t

Fig. 1

w	lb_m/Hr, mass rate, $dm/d\Theta$
α	$\dfrac{4q/A}{\lambda\rho_L D}$, Hr^{-1}
Θ	Hr, time
λ	Btu/lb mass, latent heat of vaporization
ρ	lb mass/ft^3, mass density

Subscripts

f	final conditions (when tank is empty)
g	gas
i	inlet gas
ℓ	liquid
o	outlet, liquid
s	initial conditions (when tank is full of liquid)
t	tank

Superscript

*	condition of gas which would completely fill the tank at the liquid temperature

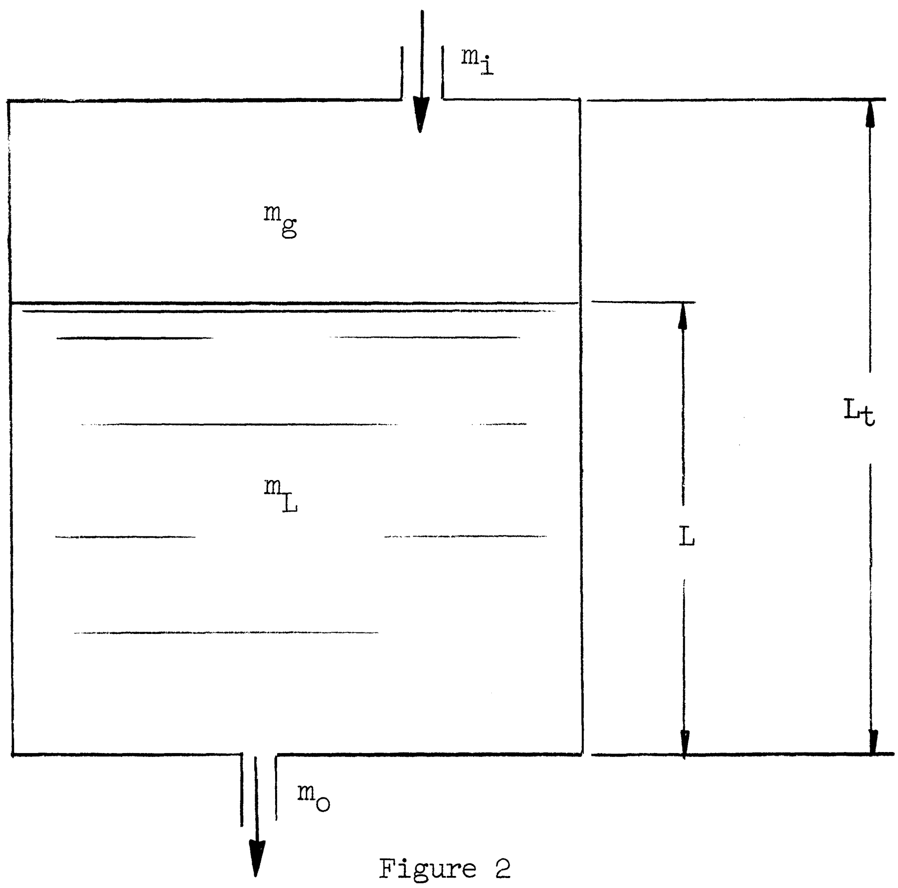


Figure 2

Computations have been made by Analyses I and II for the following quantities:

a. $\frac{w_o}{w_o max}$ as a function of time with no pressurizing gas. w_o is the rate of liquid flow out of the bottom of the tank, and w_o max is the rate of flow of liquid out of the bottom of the tank with the tank full.

b. $\left(\frac{m_i}{m_g}\right)_f$ as a function of emptying time, a constant rate of flow. (This is equivalent to varying the rate of outflow.) The significance of this parameter is that it gives the fraction of the final mass of gas in the tank which was pressurizing gas. $1-\left(\frac{m_i}{m_g}\right)_f$ is the fraction of the gas which evaporated.

c. $\frac{m_{g_f}}{m_g*}$ as a function of time to empty. m_g* is the mass of gas in the tank at the emptying pressure and the corresponding saturation temperature. If no pressurizing gas were used, all the vapor would be saturated vapor and then this ratio would be 1.

These three variables have been calculated and plotted for two heat fluxes, namely, q/A = 1000 (ft²)(hr) and q/A = 10000 Btu/(ft²)(hr); for three inlet-gas temperatures, namely, 161°R (saturation temperature), 540°R, and 1460°R; for two differently sized tanks (both of L/D = 3), one one of 1-foot diameter (which corresponds to our experimental set-up), and the other of 5-foot diameter. All calculations were for a tank pressure of 50 psia.

These results are shown on the following graphs, which are essentially self-explanatory.

The equations which have been derived for these quantities and which are the basis of the plots, are as follows:

<u>Analysis I</u>

(1) $$w_o/w_o\,max = 1 \text{ where } w_o\,max = \frac{\pi D^2\, q/A}{4\,\lambda}\left(1+\frac{4L_t}{D}\right)\left(\frac{\rho_L}{\rho_g*}-1\right)$$

(2) $$\left(\frac{m_i}{m_g}\right)_f = \left\{\frac{1}{1+\dfrac{1}{\dfrac{P}{R\rho_L\,T_i}-\dfrac{T_L}{T_i}+\left(\dfrac{\lambda\,P}{RT_i\;q/A\;(1/L_t+4/D)}\right)\times\dfrac{1}{\theta_f}}}\right\}$$

$$(3) \quad \frac{m_{gf}}{m_g*} = \frac{\rho_L}{\rho_g*}\left\{\frac{\left(1 + \frac{P}{\rho_L RT_i} - T_L/T_i\right)\theta_f + \frac{\lambda P}{RT_i q/A(1/L_t + 4/D)}}{\theta_f + \left(\frac{\lambda\rho_L}{q/A(1/L_t + 4/D)}\right)}\right\}$$

Analysis II

$$(4) \quad \left.\frac{w_o}{w_o}\right)_{max} = e^{-\left(\frac{4\ q/A}{\lambda\ \rho_g*D}\right)\theta} \quad \text{where } w_{o\,max} = \frac{\pi D^2}{4\lambda}\ q/A\left(1 + \frac{4L_t}{D}\right)\left(\frac{\rho_L}{\rho_g*} - 1\right)$$

$$(5) \quad \left.\frac{m_i}{m_g}\right)_f = \frac{1}{1 + \left\{\frac{1 - \frac{w_o}{\alpha m_{Ls}}\ln\left(1 + \frac{1}{D/4L_t + \frac{w_o}{\alpha m_{Ls}}}\right)}{\frac{P}{\rho_L RT_i} - T_L/T_i + T_L/T_i\frac{w}{\alpha m_{Ls}}\ln\left(1 + \frac{1}{D/4L_t + \frac{w_o}{\alpha m_{Ls}}}\right)}\right\}}$$

$$(6) \quad \frac{m_{gf}}{m_g*} = \frac{\rho_L}{\rho_g}\left\{(1 - T_L/T_i + P/\ {}_L RT_i) - (1 - T_L/T_i)\frac{w_o}{\alpha m_{Ls}}\ln\left(1 + \frac{1}{D/4L_t + \frac{w_o}{\alpha m_{Ls}}}\right)\right\}$$

If the tank empties quickly enough so that the total liquid mass leaving the tank is large compared with that which evaporates during the emptying process, then $\theta_f \approx M_{Ls}/w_o$ and θ_f is small so that

$$\alpha\ \theta_f \ll 1 \quad \text{and} \quad \alpha\theta_f \ll 4\ L/D \quad .$$

If these inequalities are applied to Equations (5) and (6) they become

$$(7) \quad \left(\frac{m_i}{m_g}\right)_f \approx \frac{1}{1 + \frac{1}{\left(\frac{\lambda PD}{2RT_i\ q/A} \times \frac{1}{2_f}\right) - T_L/T_i}}$$

and

$$(8) \quad \frac{m_{gf}}{m_g*} \approx T_L/T_i + \frac{2RT\ q/A}{\lambda PD}(1 - T_L/T_i)\ \theta_f \quad .$$

These equations and curves are considered to be the most important results of our analytical work. It is also possible to determine a number of other variables; for example, the final temperature of the gas in the tank.

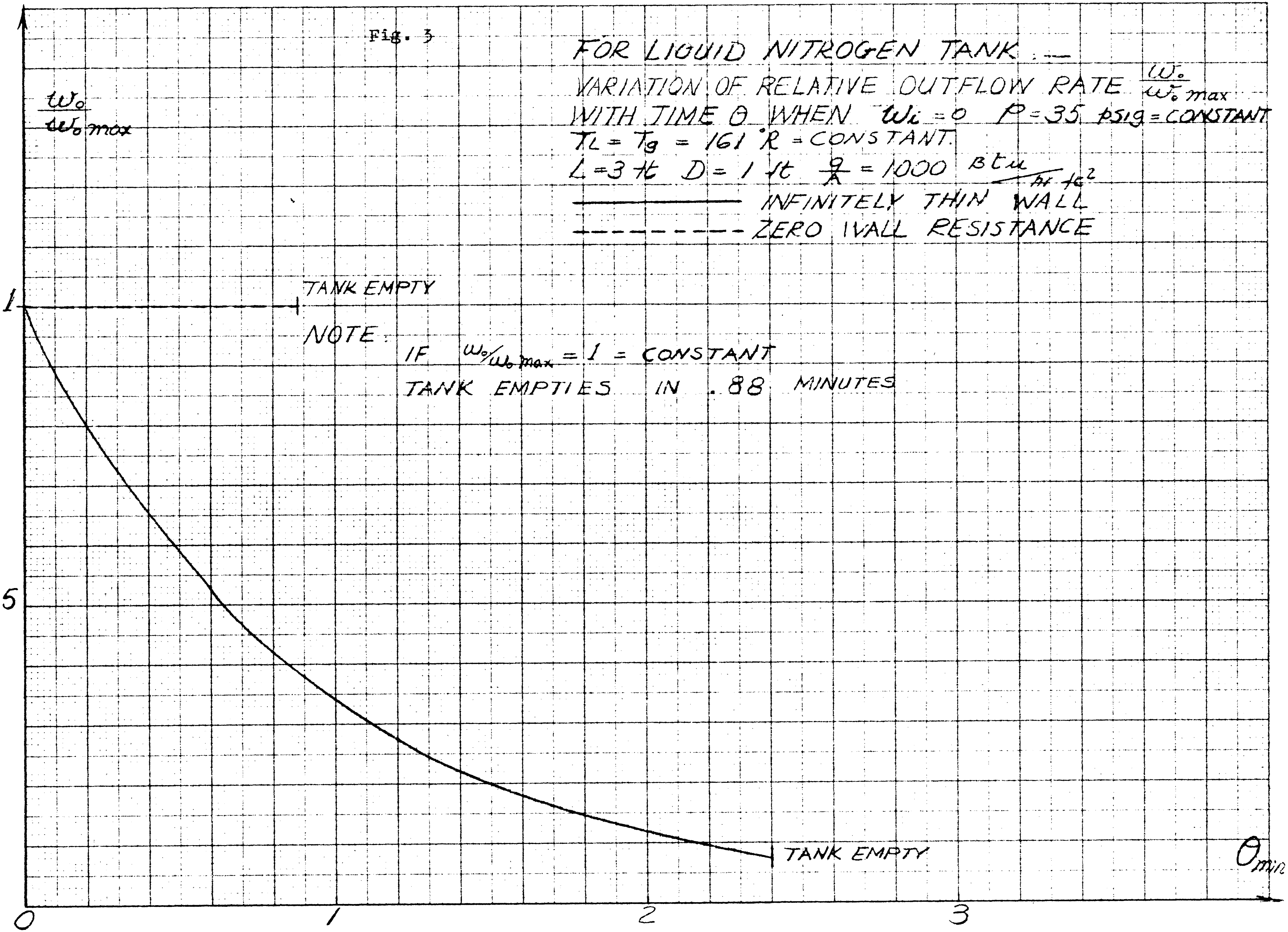
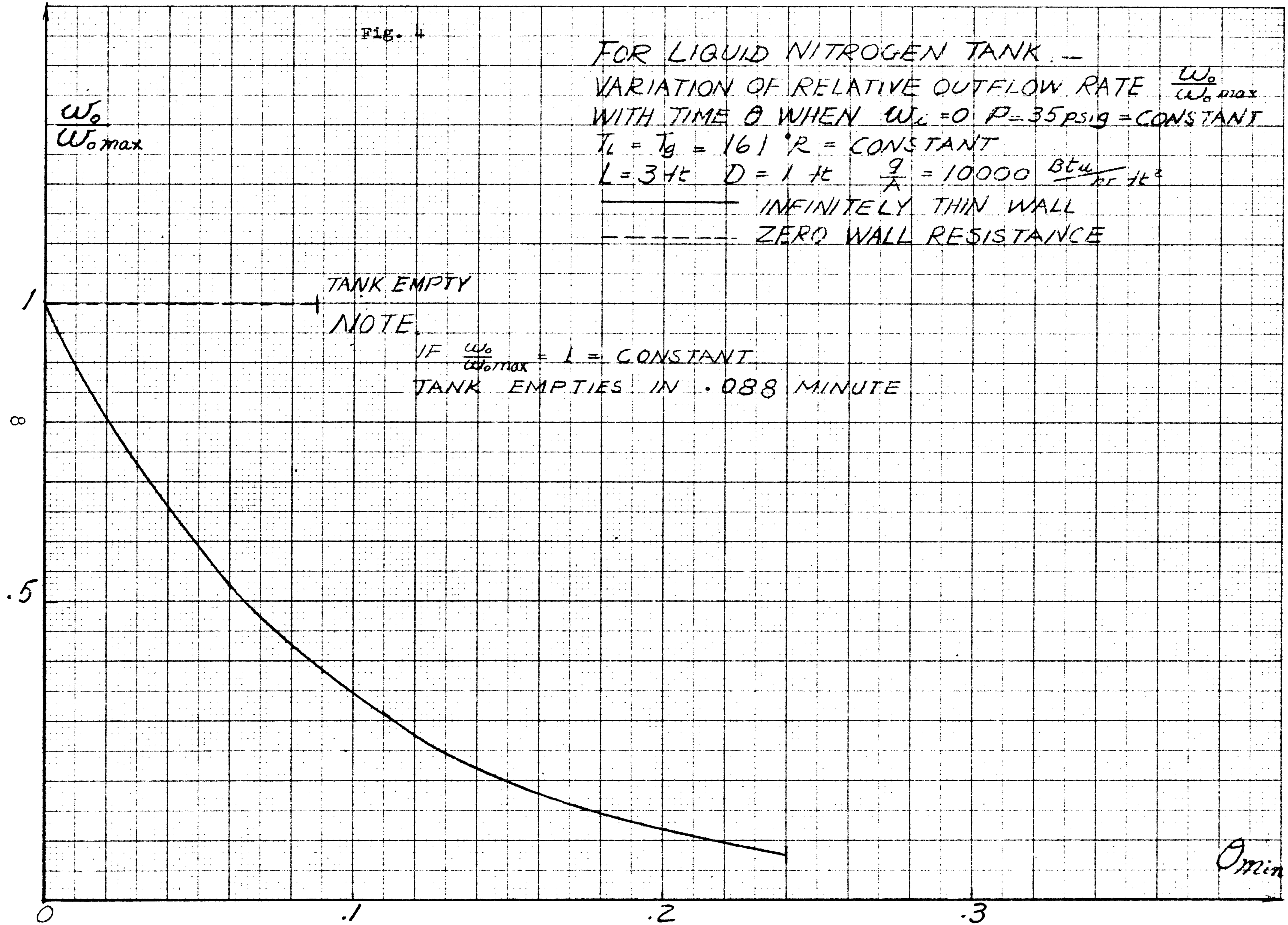

FOR LIQUID NITROGEN TANK :—
VARIATION OF RELATIVE OUTFLOW RATE $\frac{w_o}{w_{o\,max}}$
WITH TIME θ WHEN $w_i = 0$ $P = 35$ psig = CONSTANT
$T_L = T_g = 161\,°R$ = CONSTANT
$L = 3$ ft $D = 1$ ft $\frac{q}{A} = 1000\ \frac{Btu}{hr\,ft^2}$
INFINITELY THIN WALL
ZERO WALL RESISTANCE
TANK EMPTY
NOTE: IF $w_o/w_{o\,max} = 1$ = CONSTANT
TANK EMPTIES IN .88 MINUTES
TANK EMPTY
$\frac{w_o}{w_{o\,max}}$
1
.5
0
1
2
3
θ_{min}

Fig. 3

Fig. 4



FOR LIQUID NITROGEN TANK. —
VARIATION OF RELATIVE OUTFLOW RATE $\frac{\omega_o}{\omega_{o\,max}}$
WITH TIME θ WHEN $\omega_i = 0$ $P = 35$ psig = CONSTANT
$T_L = T_g = 161\,^\circ R$ = CONSTANT
$L = 3$ ft $D = 1$ ft $\frac{q}{A} = 10000 \frac{Btu}{hr\ ft^2}$
INFINITELY THIN WALL
ZERO WALL RESISTANCE
$\frac{\omega_o}{\omega_{o\,max}}$
TANK EMPTY
NOTE.
IF $\frac{\omega_o}{\omega_{o\,max}} = 1$ = CONSTANT
TANK EMPTIES IN .088 MINUTE
1
.5
0
.1
.2
.3
θ_{min}

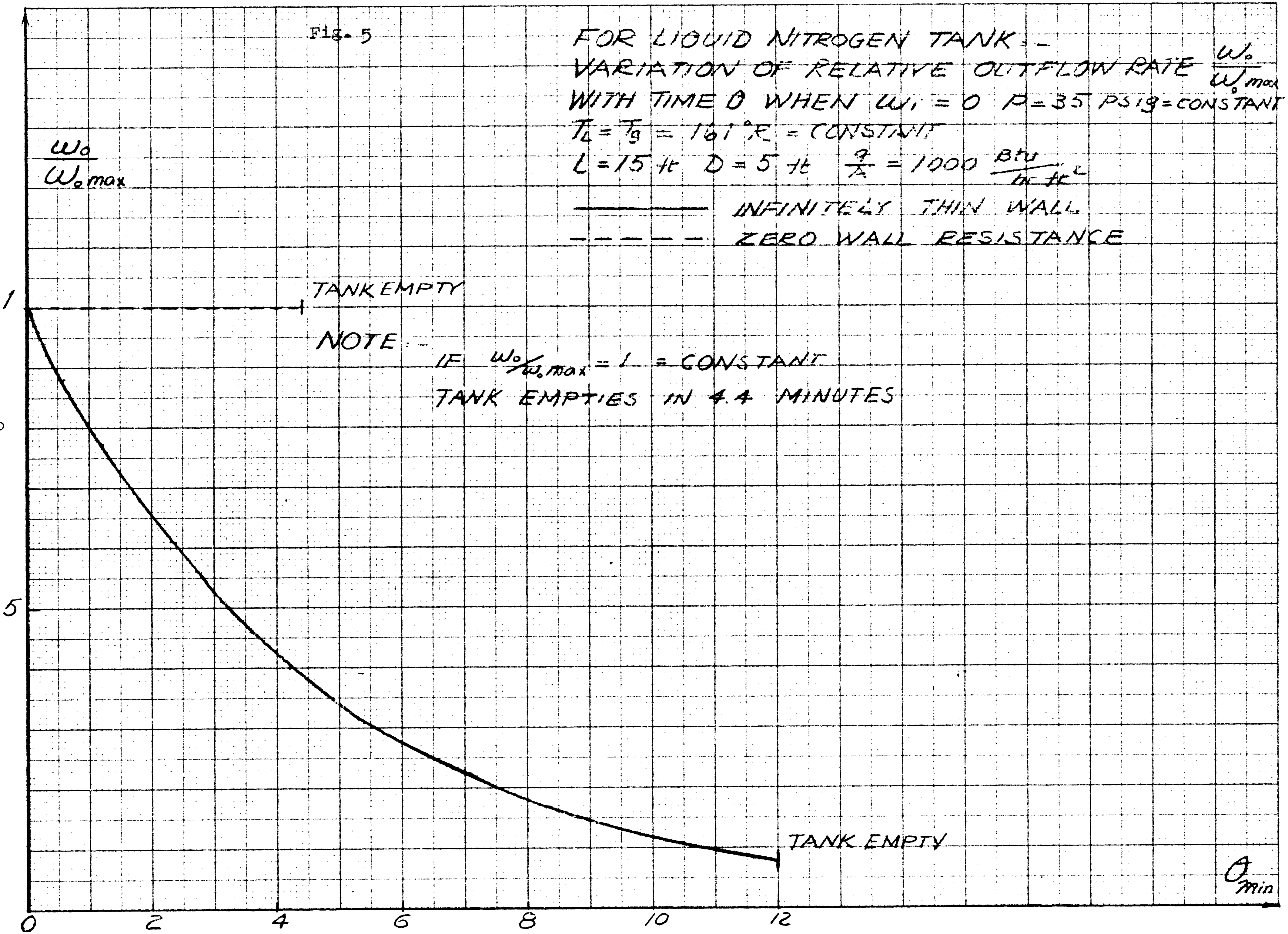
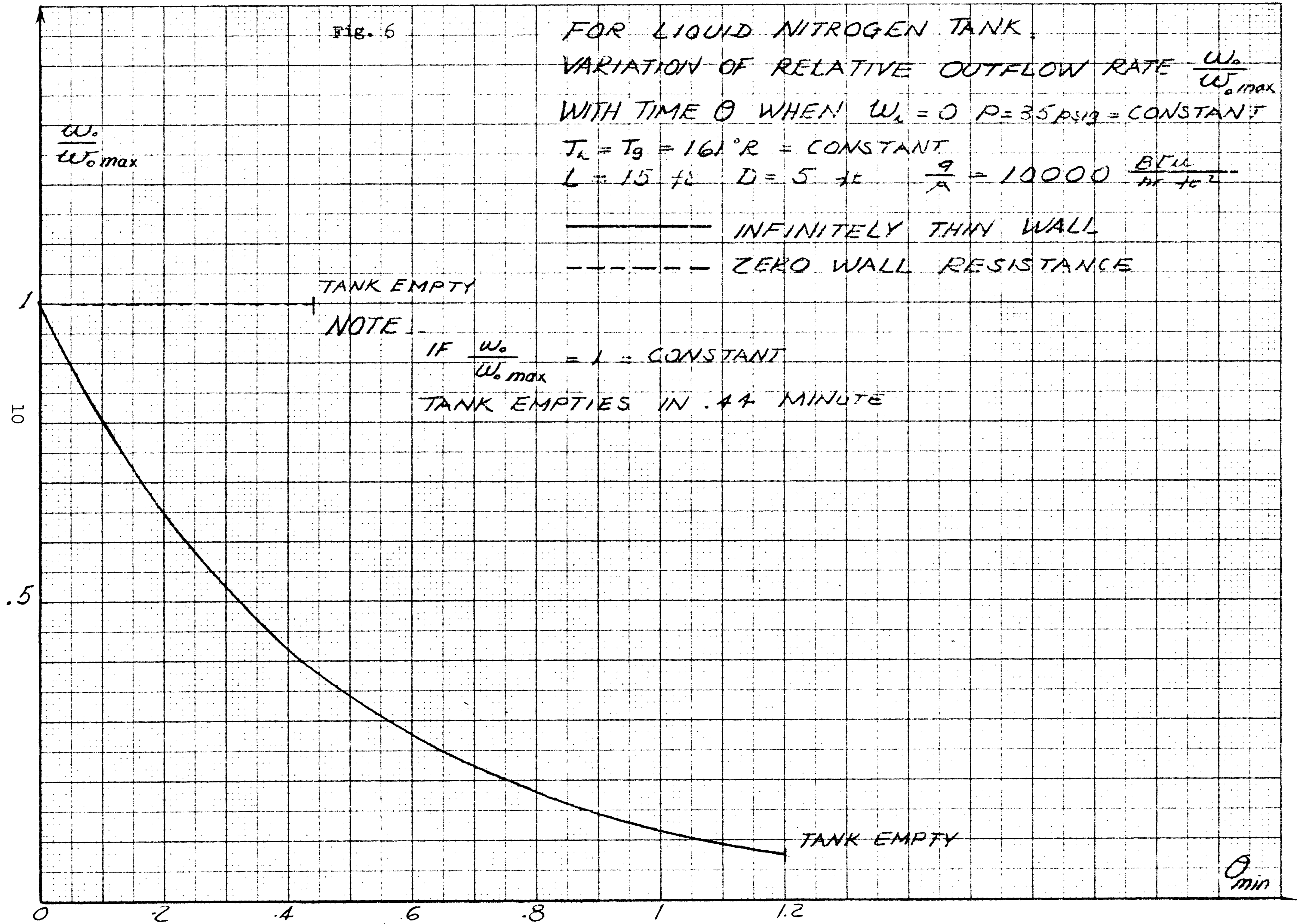
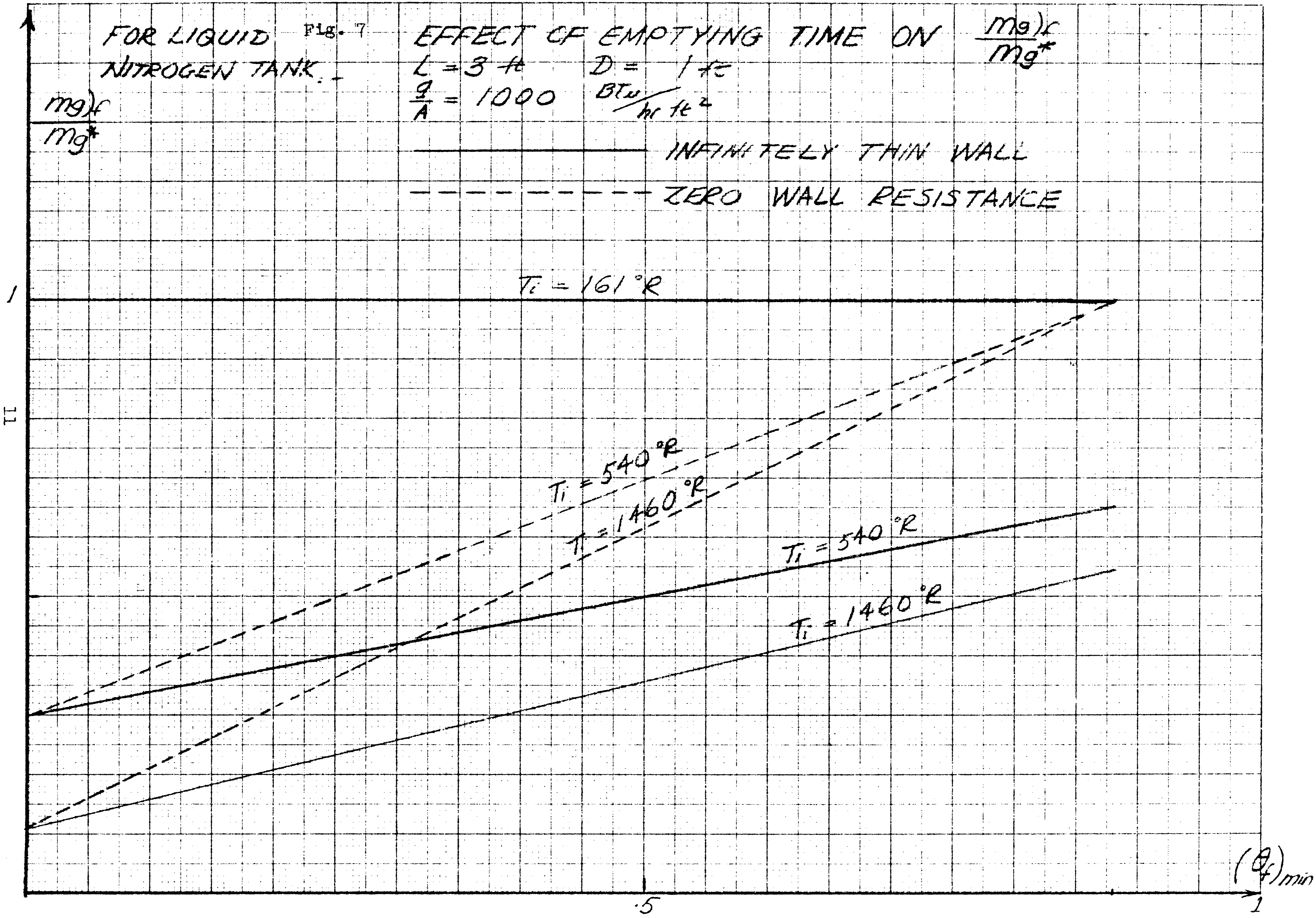

FOR LIQUID NITROGEN TANK:-
VARIATION OF RELATIVE OUTFLOW RATE $\frac{w_o}{w_{o\,max}}$
WITH TIME θ WHEN $w_i = 0$ $P = 35$ psig = CONSTANT
$T_L = T_g = 161\,°R$ = CONSTANT
$L = 15$ ft $D = 5$ ft $\frac{q}{A} = 1000 \frac{Btu}{hr\,ft^2}$
INFINITELY THIN WALL
ZERO WALL RESISTANCE
TANK EMPTY
NOTE:- IF $w_o/w_{o\,max} = 1$ = CONSTANT
TANK EMPTIES IN 4.4 MINUTES
TANK EMPTY
$\frac{w_o}{w_{o\,max}}$
1
.5
0
2
4
6
8
10
12
θ min

Fig. 5

Fig. 6

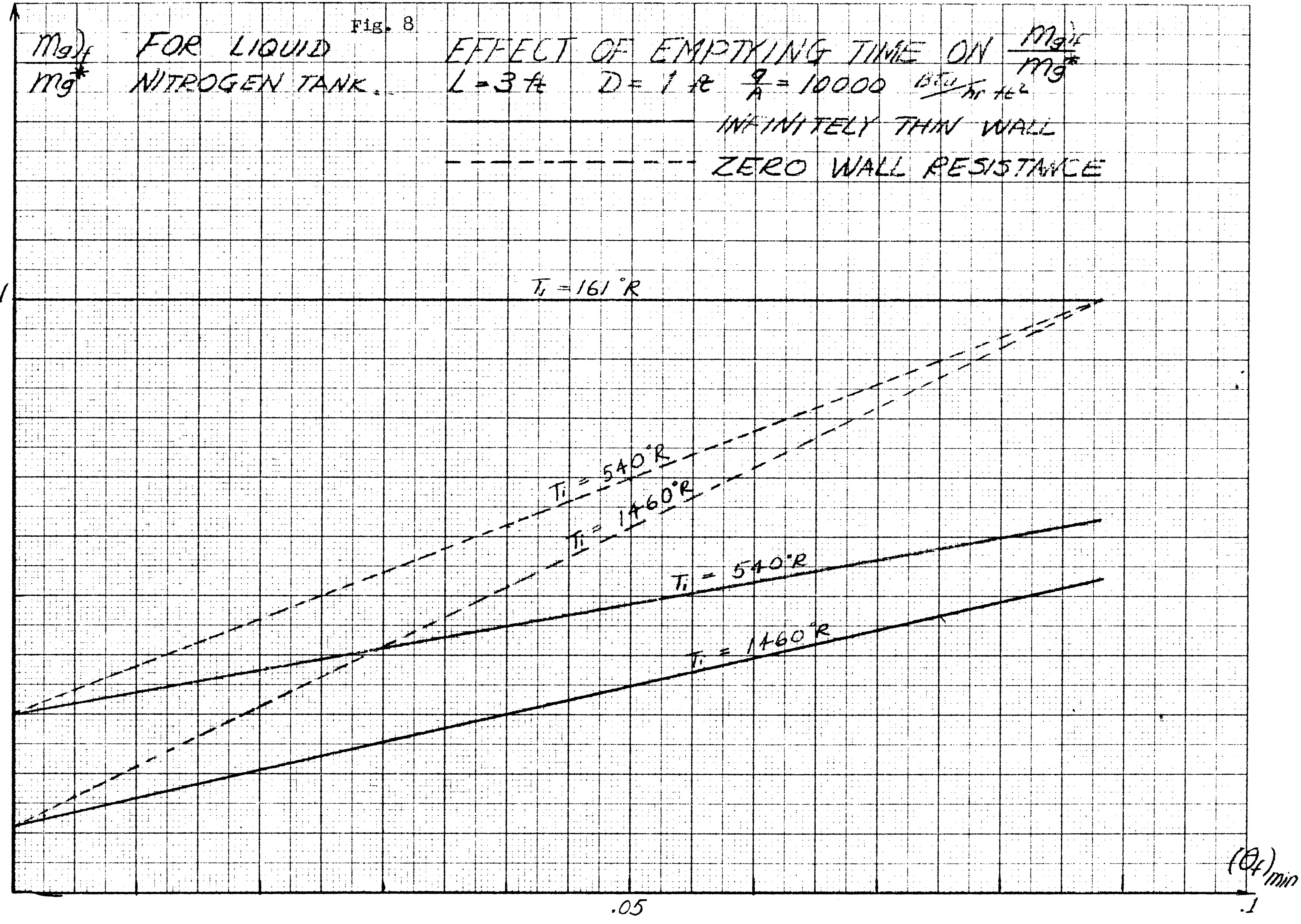


FOR LIQUID NITROGEN TANK:
VARIATION OF RELATIVE OUTFLOW RATE $\frac{w_o}{w_{o\,max}}$
WITH TIME θ WHEN $w_i = 0$ $P = 35$ psig = CONSTANT
$T_L = T_g = 161°R$ = CONSTANT
$L = 15$ ft $D = 5$ ft $\frac{q}{A} = 10000 \frac{BTU}{hr\ ft^2}$
INFINITELY THIN WALL
ZERO WALL RESISTANCE
TANK EMPTY
NOTE:
IF $\frac{w_o}{w_{o\,max}} = 1$ = CONSTANT
TANK EMPTIES IN .44 MINUTE
TANK EMPTY
$\frac{w_o}{w_{o\,max}}$
θ_{min}
1
.5
0
.2
.4
.6
.8
1
1.2



FOR LIQUID NITROGEN TANK:-
Fig. 7
EFFECT OF EMPTYING TIME ON $\frac{mg)_f}{mg^*}$
$L = 3$ ft $D = 1$ ft
$\frac{q}{A} = 1000$ $\frac{Btu}{hr\,ft^2}$
INFINITELY THIN WALL
ZERO WALL RESISTANCE
$\frac{mg)_f}{mg^*}$
$T_i = 161°R$
$T_i = 540°R$
$T_i = 1460°R$
$T_i = 540°R$
$T_i = 1460°R$
1
.5
1
$(\theta_f)_{min}$

Fig. 8



$\frac{Mg)_f}{Mg^*}$
FOR LIQUID NITROGEN TANK.
EFFECT OF EMPTYING TIME ON $\frac{Mg_f}{Mg^*}$
$L = 3$ ft $D = 1$ ft $\frac{q}{A} = 10000 \frac{Btu}{hr\ ft^2}$
INFINITELY THIN WALL
ZERO WALL RESISTANCE
$T_i = 161°R$
$T_i = 540°R$
$T_i = 1460°R$
$T_i = 540°R$
$T_i = 1460°R$
1
.05
.1
$(\theta_f)_{min}$

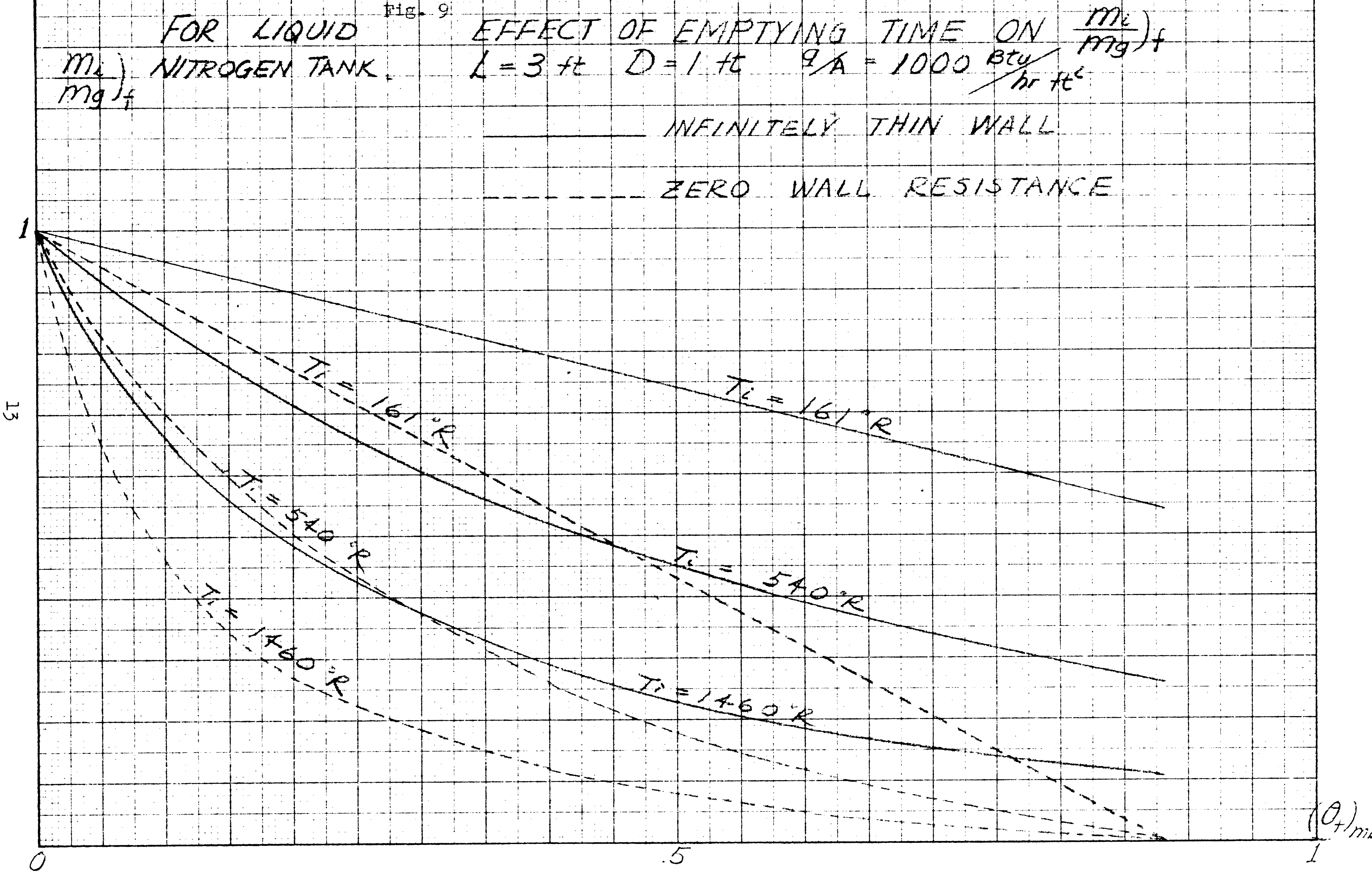
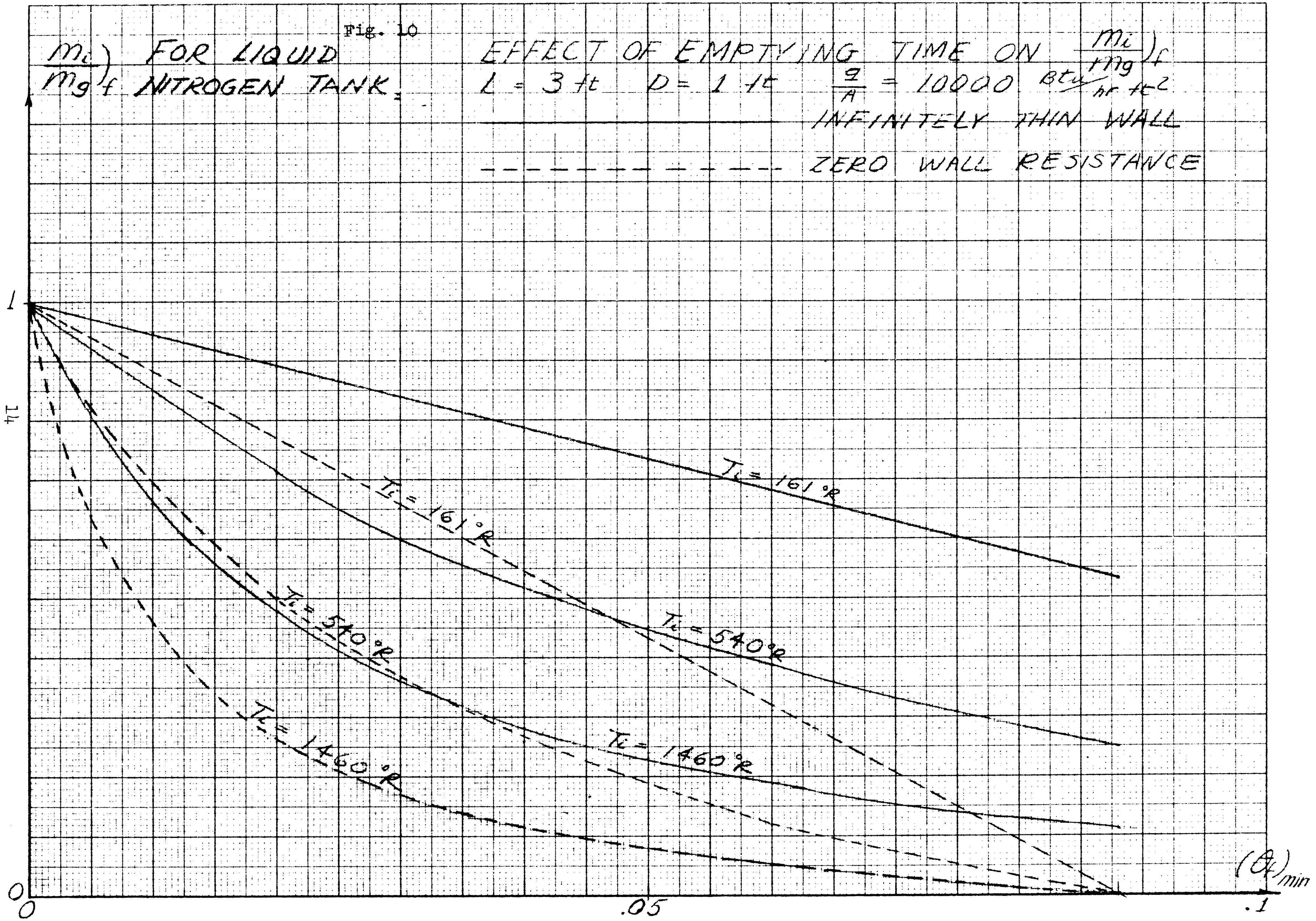


Fig. 9 EFFECT OF EMPTYING TIME ON $\left(\frac{m_i}{m_g}\right)_f$ FOR LIQUID NITROGEN TANK. $L = 3$ ft $D = 1$ ft $q/A = 1000\ \frac{Btu}{hr\ ft^2}$

Fig. 10



$\frac{m_l}{m_g})_f$ FOR LIQUID NITROGEN TANK.
EFFECT OF EMPTYING TIME ON $\frac{m_l}{m_g})_f$
$L = 3$ ft $D = 1$ ft $\frac{q}{A} = 10000$ $\frac{Btu}{hr\ ft^2}$
INFINITELY THIN WALL
ZERO WALL RESISTANCE
$T_i = 161°R$
$T_i = 540°R$
$T_i = 1460°R$
$T_i = 161°R$
$T_i = 540°R$
$T_i = 1460°R$
1
0
0
.05
.1
$(\theta_f)_{min}$

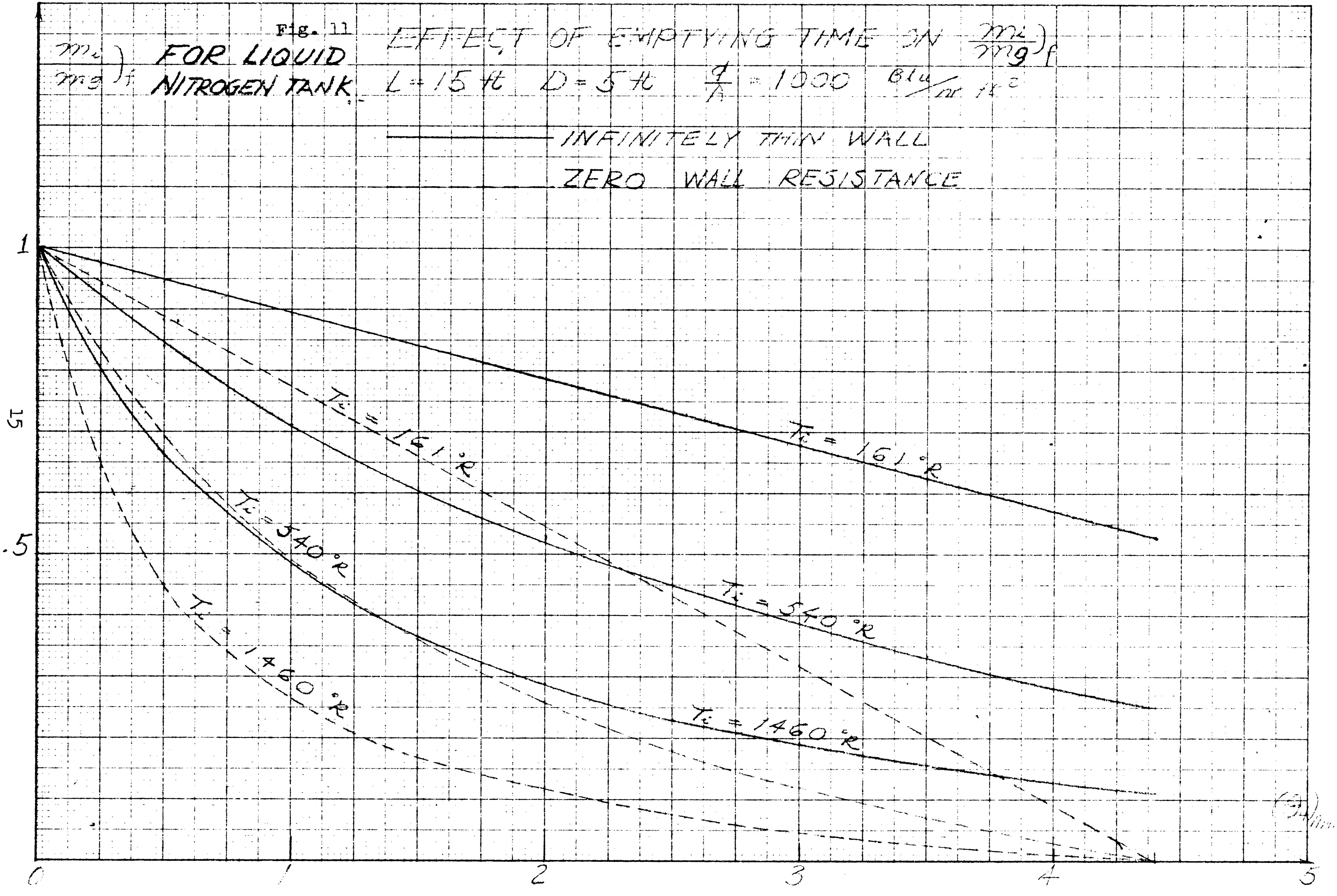


Fig. 11
$\left.\frac{m_i}{m_g}\right)_f$
FOR LIQUID NITROGEN TANK
EFFECT OF EMPTYING TIME ON $\left.\frac{m_i}{m_g}\right)_f$
$L = 15$ ft $D = 5$ ft $\frac{q}{A} = 1000$ Btu/hr ft²
INFINITELY THIN WALL
ZERO WALL RESISTANCE
1
.5
$T_i = 161°R$
$T_i = 540°R$
$T_i = 1460°R$
$T_i = 161°R$
$T_i = 540°R$
$T_i = 1460°R$
0
1
2
3
4
5
$(\theta)_{lim}$

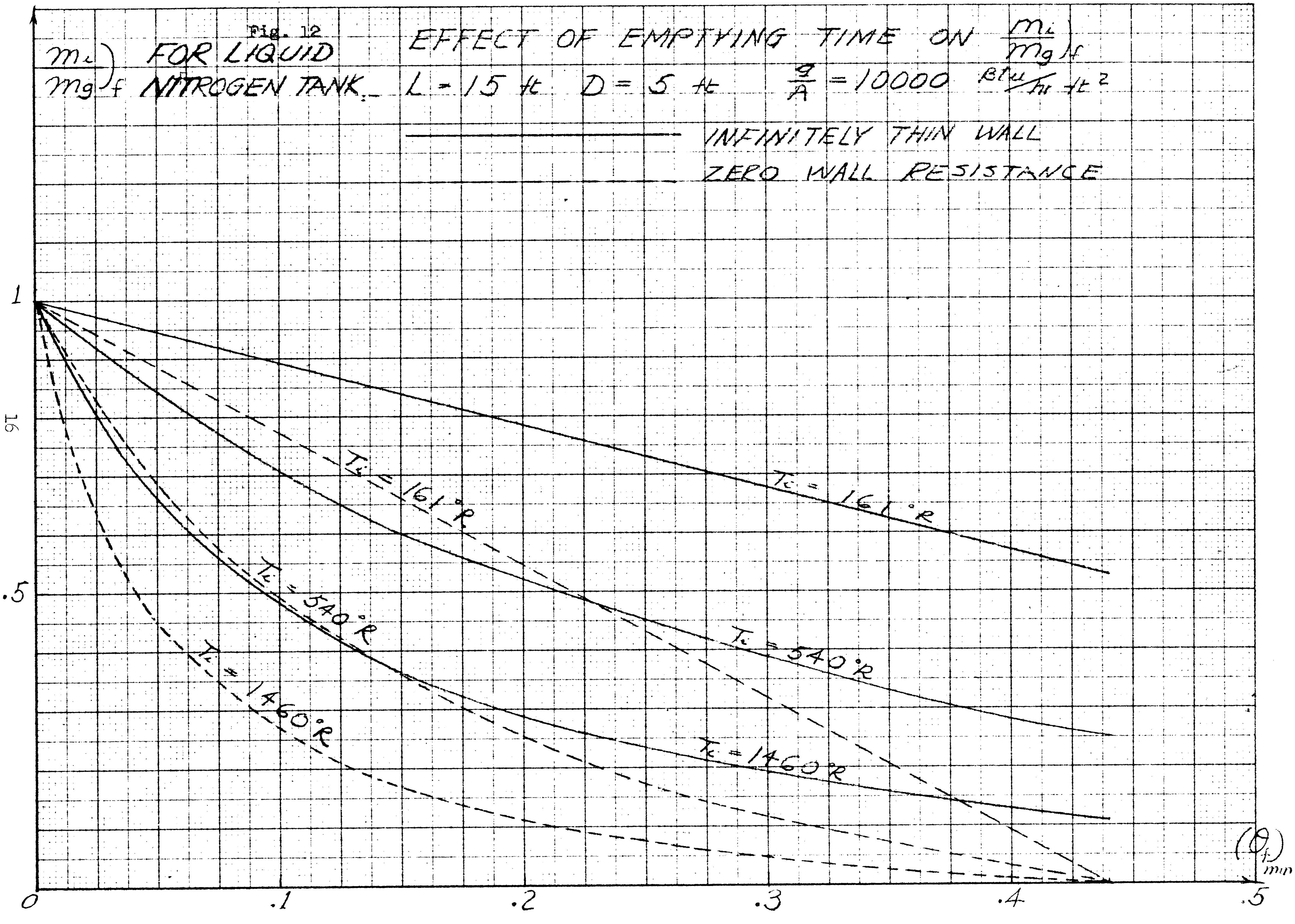
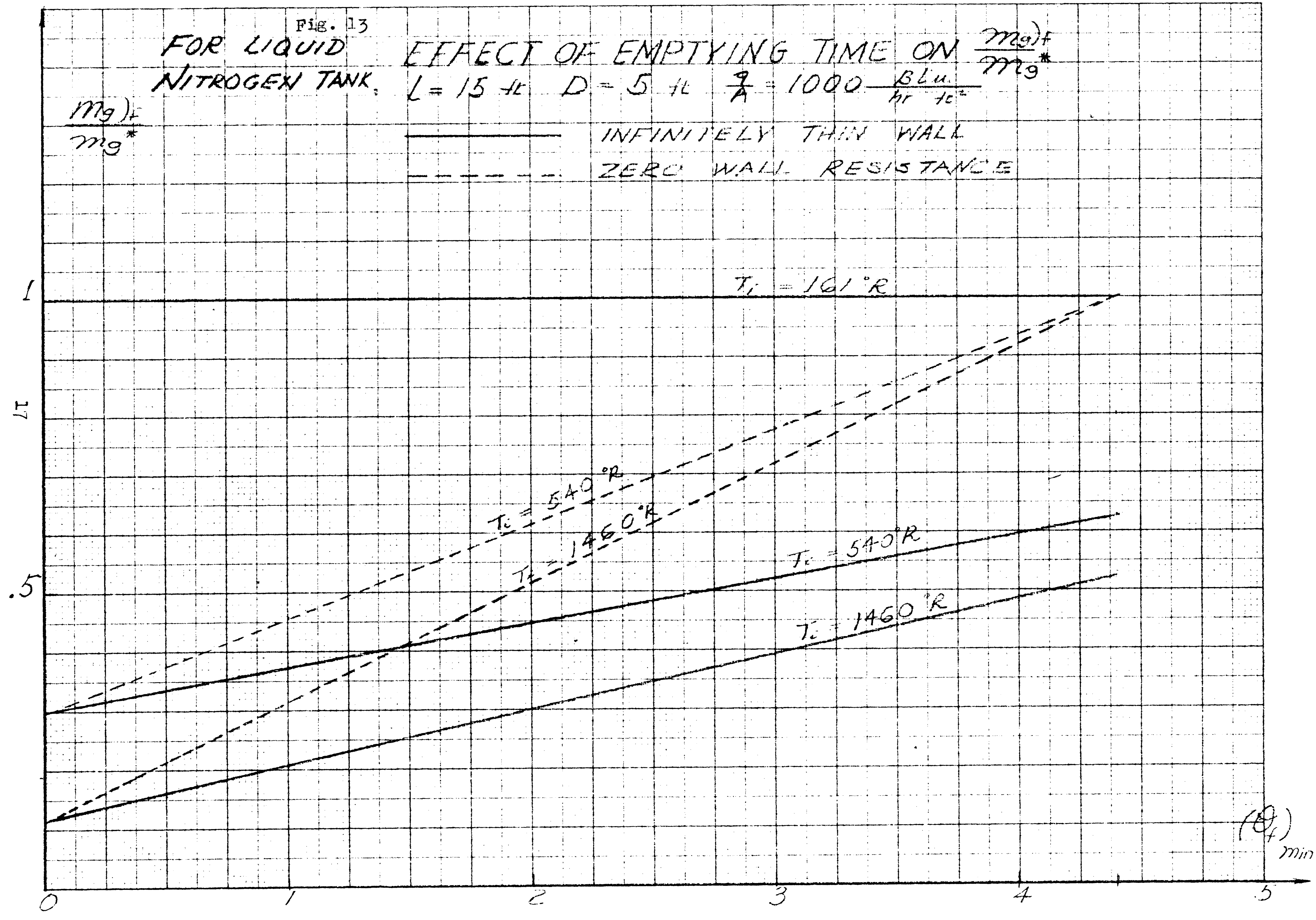
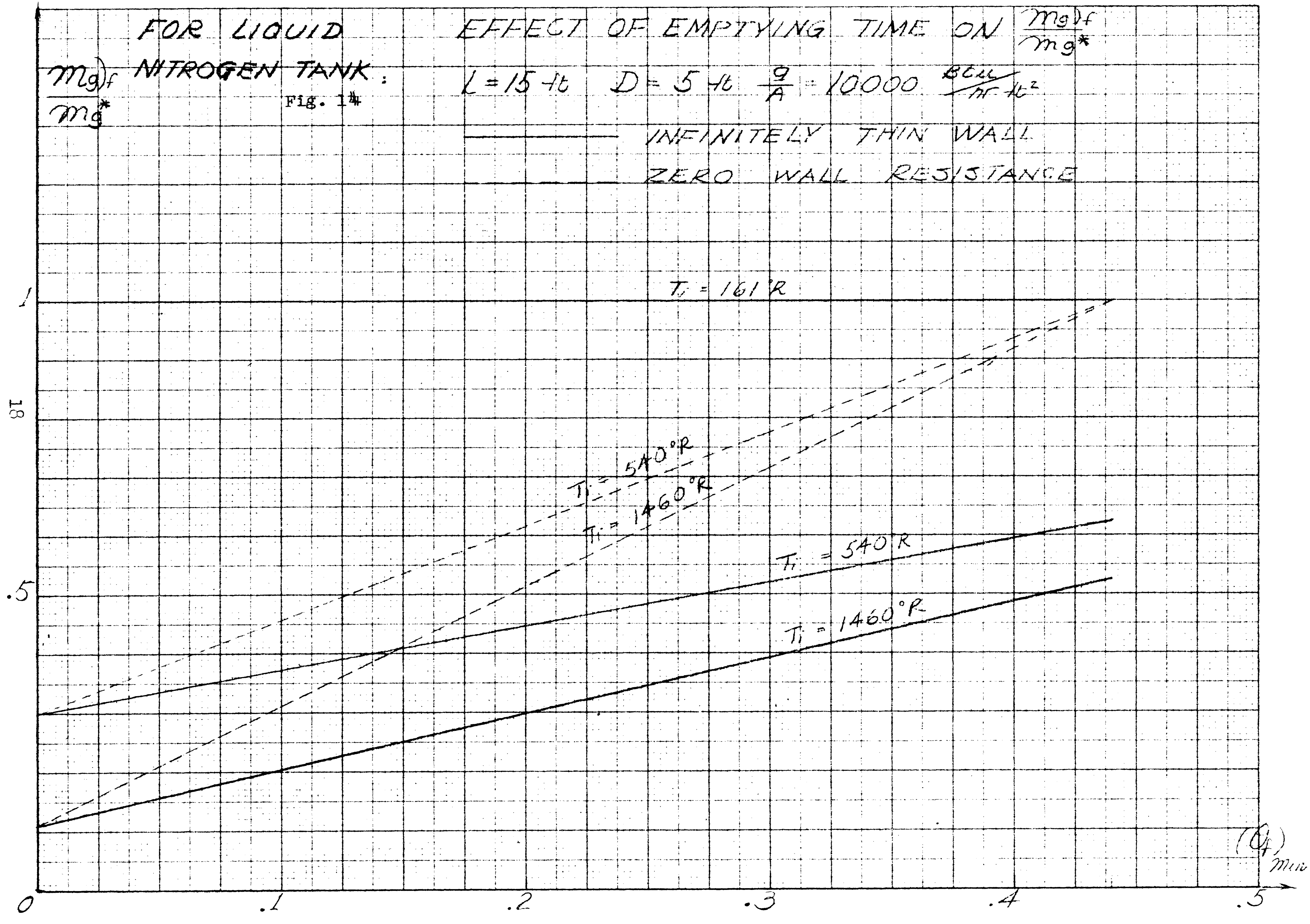


Fig. 12
EFFECT OF EMPTYING TIME ON $\frac{m_i}{m_g}\Big)_f$ FOR LIQUID NITROGEN TANK:-
$L = 15$ ft $D = 5$ ft $\frac{q}{A} = 10000$ Btu/hr-ft^2
INFINITELY THIN WALL
ZERO WALL RESISTANCE
$\frac{m_i}{m_g}\Big)_f$
$T_i = 161°R$
$T_i = 540°R$
$T_i = 1460°R$
$T_i = 161°R$
$T_i = 540°R$
$T_i = 1460°R$
1
.5
0
.1
.2
.3
.4
.5
$(\theta_f)_{min}$


FOR LIQUID NITROGEN TANK.
EFFECT OF EMPTYING TIME ON $\frac{m_g)_f}{m_g^*}$
$L = 15$ ft $D = 5$ ft $\frac{q}{A} = 1000 \frac{Btu}{hr\ ft^2}$
INFINITELY THIN WALL
ZERO WALL RESISTANCE
$\frac{m_g)_f}{m_g^*}$
$T_i = 161°R$
$T_i = 540°R$
$T_i = 1460°R$
$T_i = 540°R$
$T_i = 1460°R$
1
.5
0
1
2
3
4
5
$(\theta_f)_{min}$
Fig. 13


FOR LIQUID NITROGEN TANK:
EFFECT OF EMPTYING TIME ON $\frac{Mg)_f}{Mg^*}$
$L = 15$ ft $D = 5$ ft $\frac{q}{A} = 10000 \frac{Btu}{hr\ ft^2}$
INFINITELY THIN WALL
ZERO WALL RESISTANCE
$\frac{Mg)_f}{Mg^*}$
$T_i = 161°R$
$T_i = 540°R$
$T_i = 1460°R$
$T_i = 540°R$
$T_i = 1460°R$
$(\theta_f)_{min}$
0
.1
.2
.3
.4
.5
.5
1
Fig. 14

An additional analysis was carried out for a tank having a thermally insulated gas chamber and an unknown heat-transfer rate to the liquid, $q(\theta)$, as a function of time. A result of this analysis is the following expression for final gas mass in the tank.

$$m_{g_f} = \frac{P}{\rho_L R T_i} m_{o_f} + \frac{1}{\lambda} \left[1 + \frac{P}{\rho_L R T_i} - T_L/T_i \right] \int_0^{\theta_f} q(\theta)\, d\theta \quad .$$

The analyses mentioned up to now all assume a thermally insulated vapor region in the tank. An attempt is now being made to attack the problem in which heat transfer with the vapor region is accounted for.

EXPERIMENTAL WORK

A schematic diagram of the test apparatus is shown in Figure 15. The apparatus consists essentially of a tank in which the emptying procedures of the rocket can be simulated. The tank is so instrumented that heat-transfer rates, flow rates, and appropriate temperatures can be measured. All initial tests will be made with liquid nitrogen.

The tank consists of an aluminum cylinder 12-in. ID by 37 in. long with a wall thickness of 3/8 in. The liquid level of the nitrogen can be visually observed through a plexiglass window which covers a series of 45 1/2-in.-diameter holes (along a vertical line) in the tank. It is estimated that the presence of the plexiglass window will affect the over-all heat-transfer characteristic by no more than 3%. The tank is suspended through a load cell. The signal from the load cell is fed to a 4-channel Sanborn recorder, thus permitting the mass of nitrogen in the tank to be measured as a function of time. The nitrogen boil-off from the tank is measured by passing the boil-off through a gasometer. The tank has been pressure-tested at 100 psig. A Fisher Governor Company back-pressure regulating valve will maintain the pressure in the tank at the desired value, which in these tests is to be 35 psig.

Provision has been made for measuring the wall temperatures of the tank at six different levels. Temperatures of the liquid and vapor within the tank will be measured at approximately 16 places to determine what temperature gradients, if any, exist. Copper constantan thermocouples in conjunction with a type K-3 Leeds and Northrup potentiometer will be used to measure temperature. Transient temperatures can be recorded on the Sanborn recorder.

Three series of tests are planned. Series I will determine as accurately as possible the heat-transfer characteristics of the tank, with the liquid nitrogen at atmospheric pressure and 35 psig. Series II will deal with the transient phenomena associated with the initial pressurization of

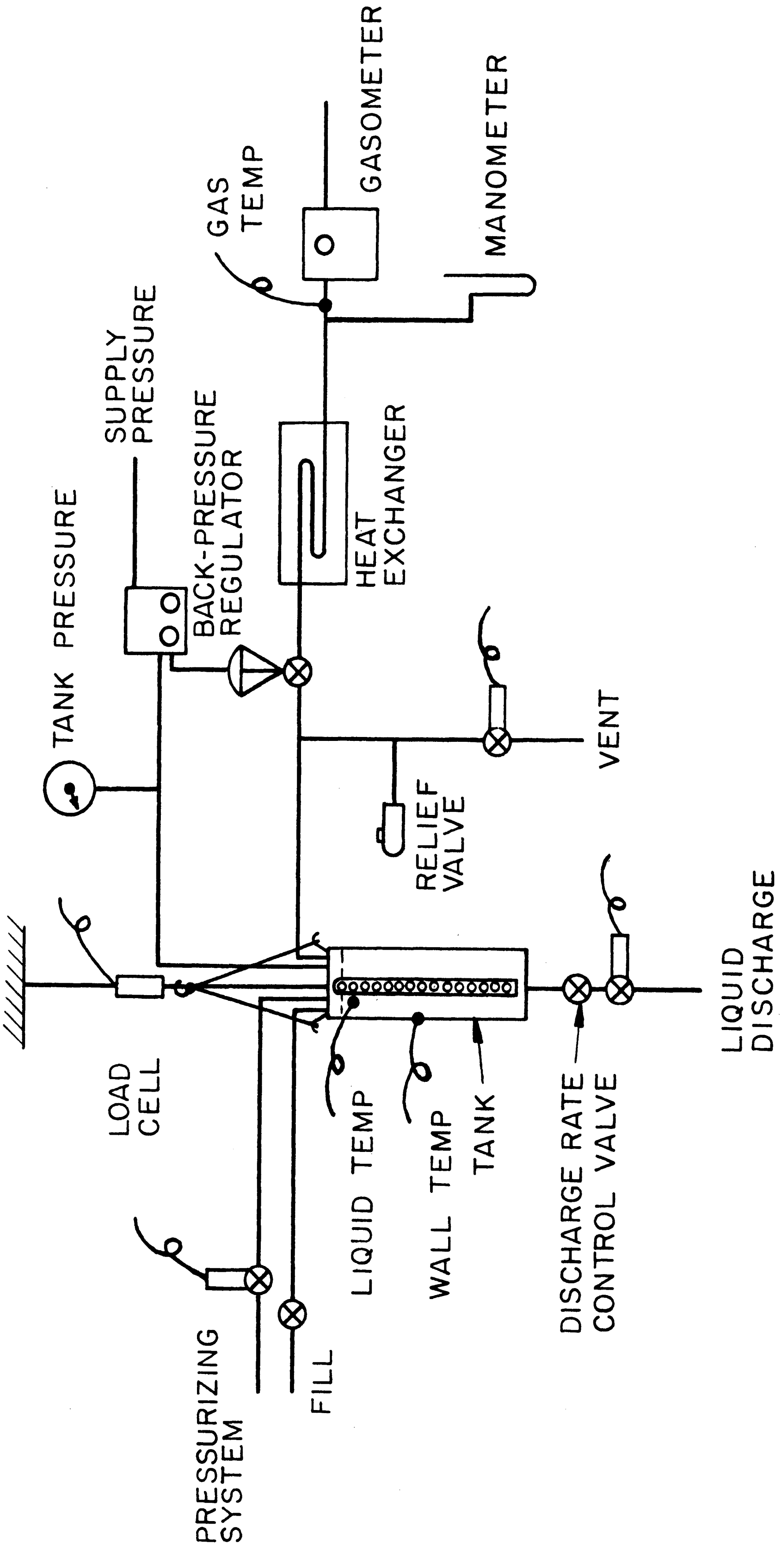


Fig. 15 Experimental Apparatus.

the tank. Series III will involve the actual discharge of liquid from the tank and will be used to check the analytical relations discussed in the previous section of this report.

The heat-transfer characteristics of the tank will be determined by measuring the rate of liquid nitrogen boil-off with the gasometer. At the same time the various temperatures of the ambient air, the wall, and the liquid and vapor will be measured. With these data the heat-transfer characteristics will be correlated with standard equations for predicting heat-transfer coefficients.

In Series II the temperature of the liquid and the wall temperature will be measured as a function of time immediately after pressurizing. Visual observation of the formation of bubbles will also be made.

In Series III a number of tests will be conducted. First of all the rate of liquid outflow as a function of time will be measured for the case of no pressurizing gas. In this test the load cell will be utilized. A number of tests will be conducted with constant discharge rates. In this case the amount of pressurizing gas will be metered and the temperatures of the vapor will be measured as a function of time. Finally, helium will be used as the pressurizing gas, and a study made of the final mass of gas when helium is used.

FUTURE WORK

The experimental apparatus has been essentially completed, and the tests concerning heat-transfer rates will be undertaken shortly. When these have been completed, the tests involving the actual discharge of liquid will be undertaken, and the experimental results compared with the analytical analysis.

The next phase of the analytical work will be to investigate the transient phenomena associated with pressurizing the tank. These phenomena include rate of temperature rise, initiation of bubble formation, and time for equilibrium to be established. Experimental studies will parallel the analytical work.

The objective in gaining this understanding is to determine what steps may be taken to reduce the amount of pressurizing gas required, which is the ultimate objective of this project.