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ON MEAN MERIDIONAL CIRCULATIONS IN THE ATMOSPHERE

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ABSTRACT

ON MEAN MERIDIONAL CIRCULATIONS IN THE ATMOSPHERE

By Anandu D. Vernekar

The purpose of the study is to make a detailed investigation of mean meridional circulations forced by given eddy transports of heat and momentum and to describe the vertical variation of the energy conversions for the zonally averaged flow.

A nonhomogeneous second-order partial differential equation for the vertical p-velocity, ω, is obtained from the quasi-geostrophic vorticity and thermodynamic equations. The method of separation of variables is used to solve the zonally averaged form of this equation such that zonally averaged vertical p-velocity, ω_z, is expressed as a series of Legendre polynomials. The boundary conditions used are that ω_z is zero at the top of the atmosphere and that at the surface it is equal to that value of ω_z which is produced by the topography of the earth. After the solution for ω_z is obtained, the mean meridional velocity is determined from the zonally averaged continuity equation.

The diabatic heating in the meridional plane is estimated from the zonally averaged steady state thermodynamic equation. Computations of the zonal available potential energy and the conversion from zonal available potential energy to zonal kinetic energy are made using the distributions of diabatic heating, the vertical p-velocity and the temperature in the meridional plane.

The general conclusions which can be drawn on the basis of the calculations are:

(i) Three-cell meridional circulations are produced by the eddy transport processes in the atmosphere.

(ii) The eddy transport of momentum is twice as effective as the eddy transport of heat in forcing the meridional circulations.

(iii) The influence of the planetary scale motion on the circulation is predominant during winter whereas that of the baroclinically unstable waves dominates the forcing mechanism during the other seasons.

(iv) The seasonal variation of the meridional circulations shows that the circulation cells move toward the pole and undergo a decrease in their intensity from winter to summer.

(v) The net diabatic heating in the meridional plane is positive south of 40°N and negative north of that latitude during winter months. In the upper troposphere, the heating decreases gradually with height in the region of net heating whereas the cooling decreases sharply in the region of net cooling.

(vi) The generation of zonal available potential energy is maximum in the lower troposphere, decreases sharply with height and becomes negative in the lower stratosphere.

(vii) The conversion from zonal available potential energy to zonal kinetic energy is positive in the lower troposphere and negative in the upper troposphere.

1. INTRODUCTION

1.1 A BRIEF HISTORY OF MERIDIONAL CIRCULATIONS

The axially-symmetric flow in the atmosphere is a flow in the west-east direction which carries the bulk energy of the atmosphere, and a secondary circulation in the meridional plane. Long before any details were known about the primary flow in the west-east direction, a theoretical explanation was sought for the secondary flow in the meridional plane by Hadley (1735). The earth and the atmosphere as a system absorb more solar radiation than they emit to the space by long wave radiation in the low latitudes, whereas the opposite is true in the higher latitudes. Hence the density gradient set up by the differential heating induces a direct solenoidal circulation. If such a circulation should exist on a rotating earth the air particles in the upper troposphere which began to move toward the pole would be deflected to the east by the Coriolis force and the particles in the lower troposphere would be deflected to the west. The energy released by the direct solenoidal circulation would increase the kinetic energy of the zonal flow. If this explanation were true the easterly flow in the lower troposphere at all latitudes would reduce the rate of rotation of the earth by the frictional drag. Furthermore the poleward flow of warm air aloft and the equatorward flow of cold air near the ground would reduce the vertical temperature gradient. As a result the stability of the atmosphere would increase and therefore reduce convection especially in the middle and high latitudes.

More than a century later, Ferrel (1859) pointed out the existence of the indirect cell in the middle latitudes. Since then the meridional circulation maintained a central position in explaining the mechanism of the general circulation of the atmosphere.

Rossby (1941) gave the following qualitative explanation for a three cell meridional circulation in each hemisphere: the warm air ascending in the equatorial region moves toward the poles in the upper troposphere where it loses heat very quickly primarily by radiation and starts to descend in the horse latitudes (30°N or 30°S) of high pressure belts. The descending air is again warmed up due to compression. The frictional convergence in the low pressure belts (near the equator and 60°N or S) and the divergence from the high pressure belts cause part of the descending air to move toward the equator and part of it to move toward the poles.

The poleward branches eventually meet the cold air sliding down from the polar regions. The warmer air is forced to ascend and hence releases the latent heat of condensation. One cell extends from about the equator to 30°N (or 30°S) and the other from about 60°N (or 60°S) to the north pole (or south pole). These cells are called the direct cells in the sense that they carry heat from the source to the sink, hence converting the available potential energy to kinetic energy. The middle latitude cell is an indirect one extending from 30°N (or 30°S) to 60°N (or 60°S).

1.2 ROLE OF MERIDIONAL CIRCULATIONS IN THE GENERAL CIRCULATION

The atmosphere gains angular momentum from the earth in the low and high latitudes and it loses angular momentum to the earth in the middle latitudes due to the frictional drag at the ground. Hence there must exist a net positive flux of angular momentum into the middle latitudes from the tropical and the polar regions. This was first pointed out by Jeffreys (1926) and later verified by numerous observational studies (Widger (1949), Buch (1954), Mintz (1955), Starr and White (1955)). These studies revealed that the maximum angular momentum is found in the upper troposphere. Since the frictional dissipation of the angular momentum takes place mainly through the ground friction, the angular momentum accumulated in the upper troposphere by the horizontal eddy transports must be brought down to compensate the frictional dissipation near the ground. The baroclinically unstable disturbances in the atmosphere are capable, to a certain extent (Kuo (1952)), of transporting the angular momentum from the upper troposphere to the lower levels but some of it is done by mean meridional circulations.

Considering the heat budget of the atmosphere, there should be a positive flux of sensible heat from the low to the high latitudes in order to maintain the mean temperature gradient in the atmosphere. The transport of sensible heat is carried out mainly by two processes in the atmosphere, namely, the horizontal eddy transport processes and the meridional circulations. The former plays a significant role in the middle and high latitudes, whereas the latter cannot be ignored in the middle latitudes and by no means in the lower latitudes (Palmen and Alaka (1952)).

With regard to the maintenance of jet streams in the upper part of the troposphere, Mintz (1951) has pointed out that jet streams would have a general tendency to move toward the higher latitudes if the convergence of eddy transfer of zonal momentum were the only process in operation. Such a tendency is also seen in the short range numerical prediction (Thompson (1961)). Hence other effects such as the vertical transport of momentum, mean meridional circulations and the friction effects must counteract the convergence of the momentum in order to maintain the jet position.

In addition to the above considerations, the meridional circulations play an important role in the energetics of the atmosphere in explaining the energy (kinetic, internal heat or potential) transfer across the latitude wall and the conversion of zonal available potential energy to zonal kinetic energy.

1.3 REVIEW OF METHODS OF COMPUTATIONS

The methods of computing mean meridional circulations are essentially of two types. The first is usually called the direct method in the literature. This method uses maps of meridional component of the wind averaged for a season or a year. These maps are analyzed and values are read at grid points around latitude circles to obtain zonal averages. The zonally averaged meridional component at different levels is used to obtain a vertical profile of north-south mass flow across the latitude wall. In the steady-state conditions such a profile should satisfy the mass balance requirement across the latitude wall. This may be expressed symbolically as:

$$M^*(\varphi) = \frac{2\pi a \cos \varphi}{g} \int_0^{p_o} v_z(\varphi, p)\, dp = 0 \qquad \ldots(1.1)$$

where $M^*(\varphi)$ is the mass flux across the latitude φ, a is the radius of the earth, p is the pressure, p_o is the value of p at the surface of the earth, g is the acceleration of gravity and v_z is the zonal average of meridional component of the wind. However such a requirement is imposed by applying the correction to the computed values. The second method, called the indirect method, considers the internal dynamics of the atmosphere. Several investigators computed the meridional circulations necessary to balance the angular momentum in the atmosphere. Others on the other hand computed it as a forced motion, where the forcing functions are either known or computed from the observed data.

(a) Direct Methods

Riehl and Yeh (1950) computed the mean meridional circulations from ship observations in the zone between 40°N and 40°S and were able to establish the existence of the direct tropical cells in both hemispheres. They found an average equatorward flow of 0.5 to 2.5 m sec^{-1} near the surface of the earth during the winter. The maximum value was found around 13°N. But there was no indication of an indirect cell even up to 40°N. The inhomogeneity of periods of observations might affect the results very much.

Starr and White (1951), on the other hand, used an objective method to compute the mean meridional mass flow across the latitude wall at 30°N. Their results, based on 16 observation stations along the latitude circle, indicate that the variability in the meridional circulations is so large that they are not statistically significant from zero.

Buch (1954) made a more extensive statistical study of the meridional circulations from the wind statistics. This study shows that the standard deviation of the meridional component of wind increases considerably from equator toward the pole, and from lower to higher levels in the atmosphere. This supports the conclusions drawn by Starr and White.

Tucker (1957) used the surface weather maps to show the evidence of speculated meridional circulations. In spite of the fact that the surface observations are seriously affected by local and orographic effects, his results agree well with those of Riehl and Yeh, and also indicate the existence of a three cell circulation.

However, the positions of the cells are somewhat further north than speculated. It is possible that the subjectiveness in the analysis might favor his results. Tucker (1959) extended his studies to cover higher levels. He considered data on a 200-degree arc containing the European and North American continents so that the results are not necessarily representative of the entire Northern Hemisphere. The correction for the mass flow across the 200-degree arc was rather large, sometimes as large as the magnitude of the computed meridional velocities, indicating the uncertainty in the computed values. Nevertheless, the results indicated the existence of three cell circulations. Hence he also computed the vertical velocities in the meridional plane from the continuity equation. The results show an unusual pattern in which the vertical velocity increases almost linearly in the vertical, attaining the maximum value at the tropopause level.

Palmen and Vuorela (1963) made an extensive study of mean meridional circulations over the Northern Hemisphere using the upper-wind statistics charts. Their study indicates a strong circulation in the tropics. The maximum equatorward flow is 3.5 m sec^{-1} around 12°N very close to the ground and maximum reverse flow of 2.5 m sec^{-1} near the

200 mb level. The indirect circulation cell is centered at 55°N having a maximum poleward flow of 0.75 m sec^{-1} near the ground and maximum equatorward flow of 0.5 m sec^{-1} around 250 mb. In the tropics the correction for the mass balance across the latitude wall was very small as compared to the computed mass flow, but the correction was of the same order of magnitude as the computed meridional flow in the middle latitudes. Vuorela and Tuominen (1964) used the same computational procedure and type of data as above in computing the mean meridional circulation for the summer months. The results show that the circulation is much weaker in the summer than in the winter and the whole system of cells shifts toward the north. Further, the direct tropical cell from the Southern Hemisphere enters in the equatorial region of the Northern Hemisphere. However, the result in the higher latitudes are very uncertain because of the large correction which had to be applied for the mass balance across the latitude wall.

The results of the 'direct' method may be summarized as follows. Except in very low latitudes the mean meridional wind is a very small residue of the sum of terms of opposite signs. Hence, as indicated by the statistical studies, the standard deviation of the meridional component is large in middle and high latitudes. The variable in question ranges from 10 m sec^{-1} to -10 m sec^{-1} whereas the zonal average is less than 1 m sec^{-1}. It is likely that the errors involved in these computations have the same magnitude as that of the estimate. Hence it is necessary to devise an indirect method to compute the meridional circulations.

(b) Indirect Methods

Mintz and Lang (1955) considered the geostrophic angular momentum balance in the atmosphere to compute the mean meridional circulations. The two assumptions which they made in the computations may be serious. Firstly, they assumed that the vertical eddy momentum flux is zero above 700 mb, and secondly the flow below 700 mb is equal to the flow above 700 mb but in the opposite direction. Holopainen (1966a) used the same method as above but the angular momentum was computed from the observed wind statistics. Gilman (1965) also considered the angular momentum balance to compute the mean meridional circulations in the Southern Hemisphere. He used the method of characteristics to solve the differential equations involved. His results show that the Southern Hemisphere also has similar circulation cells of almost the same intensity as that of the Northern Hemisphere.

Kuo (1956) showed that the meridional circulations are forced motions and the conditions for the free motions are not satisfied for the large scale motions in the atmosphere. He concluded that the dominant forcing functions are the derivatives of eddy transfer of zonal momentum and eddy transfer of sensible heat but his quantitative estimates of the circulations were based on the most representative forcing functions.

1.4 AIMS OF THE STUDY

The aims of this study are:

(1) To construct a diagnostic model for meridional circulations considering the balance of momentum and heat budgets of the atmosphere.

(2) To compute the meridional circulations to satisfy the momentum and heat balance jointly and separately.

(3) To compute the meridional circulations for different scales of motions in the atmosphere.

(4) To compute the seasonal variation of the meridional circulations.

(5) To compute the daily variations of the meridional circulations.

(6) To compute the vertical variation of the generation of available potential energy and the conversion between zonal available potential energy and zonal kinetic energy.

2. FORMULATION AND METHOD OF SOLUTION

2.1 FORMULATION

Charney (1960) has derived simplified equations on the basis of dimensional analysis to justify the quasi-geostrophic assumption for large scale planetary flow. Since the present formulation is based on these equations we shall state the assumptions involved:

1. The Froude number is less than unity so that the flow is on a large planetary scale and carries the bulk energy of the atmosphere.
2. The Rossby number is small as compared to unity so that the flow can be considered as quasi-geostrophic.
3. The Richardson number is large as compared to unity so that the flow is highly stable gravitationally.
4. Since the characteristic horizontal scale of motion is large in comparison with the vertical scale of motion, the third equation of motion may be replaced by the hydrostatic equation. Hence the motion is hydrostatic everywhere and at all times.

It is then convenient to write the equations of motion with pressure as the vertical coordinate. Further, to be consistent with the spherical configuration of the earth we shall use the equations of motion in spherical coordinates. The equation for the vertical component of vorticity can be obtained by taking the curl of the first two equations of motion. The simplified form which is consistent with the above assumptions of the flow and satisfying the integral constraints (Wiin-Nielsen (1959a)) can be written as follows:

$$\frac{\partial \zeta}{\partial t} + \vec{v}.\nabla\eta = f_o \frac{\partial \omega}{\partial p} + \vec{k}.\nabla x\vec{F} \qquad ...(2.1)$$

where $\vec{v} = \vec{k}x\nabla\Psi$ is the non-divergent velocity vector, Ψ is the stream function,

$$\zeta = \frac{1}{a^2\cos^2\varphi}\frac{\partial^2\Psi}{\partial\lambda^2} + \frac{1}{a^2\cos\ \varphi}\frac{\partial}{\partial\varphi}(\cos\ \varphi\ \frac{\partial\Psi}{\partial\varphi}) = \nabla_s{}^2\Psi,$$

λ is the longitude, $\eta = \zeta+f$, f is the Coriolis parameter, f_o is the value of f at 45°N, $\omega = \frac{dp}{dt}$, and $\vec{F}$ is the viscous force per unit mass.

In equation (2.1) vertical advection of absolute vorticity and the twisting term are neglected, the absolute vorticity is advected by nondivergent winds and $\eta\frac{\partial\omega}{\partial p}$ is replaced by $f_o \frac{\partial\omega}{\partial p}$. The equation therefore satisfies the integral constraints and is also consistent with the quasi-nondivergent theory.

Normally the first two equations of motion are replaced by the vorticity and the divergence equation. The divergence equation is reduced to the balance equation by putting the divergence identically zero. The balance equation is then solved for the stream function Ψ, knowing the geopotential field. A numerical solution of the nonlinear balance equation can be found by a laborious method suggested by Shuman (1957a). Instead we shall use the so-called geostrophic stream function given by the relation $f_o\Psi = \Phi$ where Φ is the geopotential. It is then possible (Phillips (1958)) to write the thermodynamic equation in the form

$$\frac{\partial}{\partial t}\left(\frac{\partial \Psi}{\partial p}\right) + \vec{v}.\nabla\left(\frac{\partial \Psi}{\partial p}\right) + \frac{\bar{\sigma}\omega}{f_o} = - \frac{R}{C_p p f_o} H \qquad \ldots(2.2)$$

where $\sigma = -\alpha \frac{\partial \ln \theta}{\partial p}$, a measure of static stability, the bar over σ indicates the average over a (λ, φ) surface, α is the specific volume, θ is the potential temperature, R is the gas constant for dry air, C_p is the specific heat at constant pressure, H is the rate of change with time of diabatic heating per unit mass.

We shall operate by $-\frac{\partial}{\partial p}$ on equation (2.1) and by $\nabla_s{}^2$ on equation (2.2) and add to obtain a diagnostic equation for ω, which can be written as follows:

$$\bar{\sigma}\nabla_s{}^2\omega + f_o{}^2 \frac{\partial^2\omega}{\partial p^2} = f_o \frac{\partial}{\partial p}(\vec{v}.\nabla\eta) - f_o \frac{\partial}{\partial p}(\vec{k}.\nabla x\vec{F})$$
$$- f_o\nabla_s{}^2\left[\vec{v}.\nabla\left(\frac{\partial \Psi}{\partial p}\right)\right] - \frac{R}{C_p p} \nabla_s{}^2 H \qquad \ldots(2.3).$$

The axially-symmetric flow in the atmosphere is greatly influenced by the presence of large scale eddy motion. We shall define this flow as a zonal average of a physical quantity x

$$x_z(\varphi,p,t) = \frac{1}{2\pi}\int_0^{2\pi} x(\lambda,\varphi,p,t)d\lambda \qquad \ldots(2.4)$$

so as to obtain an integrated effect of the eddy motion. The equation (2.3) can be written after taking the zonal average [see Appendix A] in the form

$$\frac{1}{\cos\varphi}\frac{\partial}{\partial\varphi}(\cos\varphi\frac{\partial\omega_z}{\partial\varphi}) + \frac{f_o{}^2a^2}{\bar{\sigma}}\frac{\partial^2\omega_z}{\partial p^2} =$$

$$-\frac{f_o}{\bar{\sigma}}\frac{\partial}{\partial p}\{\frac{1}{\cos\varphi}\frac{\partial}{\partial\varphi}[\frac{1}{\cos\varphi}\frac{\partial(uv)_z\cos^2\varphi}{\partial\varphi}]\}$$

$$+\frac{f_o a}{\bar{\sigma}\cos\varphi}\frac{\partial}{\partial p}(\frac{\partial F_{\lambda,z}\cos\varphi}{\partial\varphi})$$

$$-\frac{g}{(\Delta p)^2}\frac{R\ln(\frac{p_2}{p_1})}{2\pi a^2C_p\bar{\sigma}}\frac{1}{\cos\varphi}\frac{\partial}{\partial\varphi}\{\cos\varphi\frac{\partial}{\partial\varphi}[\frac{1}{\cos\varphi}\frac{\partial TH_{\Delta p}(\varphi)}{\partial\varphi}]\}$$

$$-\frac{R}{\bar{\sigma}C_p p}\frac{1}{\cos\varphi}\frac{\partial}{\partial\varphi}(\cos\varphi\frac{\partial H_z}{\partial\varphi}) \qquad \ldots(2.5)$$

where $TH_{\Delta p}(\varphi) = \frac{\Delta pC_p a\cos\varphi}{g}\int_0^{2\pi}\tilde{T}\tilde{v}\,d\lambda$

is the horizontal eddy transfer of sensible heat in a layer of thickness Δp, p_1 and p_2 are the pressures at the bottom and top of this layer. $\tilde{T}$ and $\tilde{v}$ are, respectively, the temperature and the meridional component of the wind vector, representative for the layer. The forcing function in equation

(2.5) consists of four terms. The first two terms are the vertical variation of eddy transfer of zonal momentum and viscous forces to account for the balance of angular momentum in the atmosphere. The third and the fourth terms are the horizontal variations of eddy transfer of sensible heat and diabatic heating, respectively, to account for the balance of the heat budget of the atmosphere.

Further it may be noted here that the eddy transfer of zonal momentum is proportional to the degree of eastward tilt of the trough in the lower latitudes and westward tilt in the higher latitudes on the isobaric surfaces (Starr (1951)). Similarly the eddy transfer of sensible heat is proportional to the degree of westward tilt of the trough in the vertical. The meridional circulation is thus to some extent produced by the eddy processes in the atmosphere.

The differential equation for the vertical motion is usually solved using the simplified boundary conditions that the vertical motion is zero at the top and bottom of the atmosphere. But one might consider the vertical motion at the lower boundary due to the roughness of the earth's surface and lifting on the slopes of the mountains. In the frictional layer of the atmosphere, the vorticity equation for the balanced frictional flow is written as:

$$\frac{\partial\omega}{\partial p} = \frac{1}{f_o} \frac{g}{a \cos \varphi} \frac{\partial}{\partial p}\left(\frac{\partial \tau_\varphi}{\partial \lambda} - \frac{\partial \tau_\lambda \cos \varphi}{\partial \varphi}\right) \quad \ldots(2.6)$$

where τ_φ and τ_λ are the shearing stresses in φ and λ directions. Integration of equation (2.6) from bottom to the top of the frictional layer on the assumption that the shearing stresses vanish at the top of the frictional layer, gives

$$\omega_m - \omega_f = \frac{g}{f_o a \cos \varphi}\left(\frac{\partial \tau_{\varphi,o}}{\partial \lambda} - \frac{\partial \tau_{\lambda,o} \cos \varphi}{\partial \varphi}\right) \quad \ldots(2.7)$$

Here ω_m and ω_f are the vertical velocities at the bottom and top of the frictional layer, respectively. $\tau_{\varphi,o}$ and $\tau_{\lambda,o}$ are the shearing stresses at the lower boundary of the frictional layer.

The magnitude of the surface shear stresses can be estimates by different methods. Mintz (1955) computed the mean zonal average of the surface shear stresses by integrating the steady state first equation of motion in the vertical. Phillips (1956) evaluated the stress using a semi-empirical formula of the form

$$\vec{\tau} = - C\rho |\vec{v}| \vec{v}_o \quad \ldots(2.8)$$

where C = 0.003 is a non-dimensional constant, the drag coefficient, ρ is the density at the surface of the earth, $|\vec{v}| = 10 \text{ m sec}^{-1}$ and $\vec{v}_o$ is the geostrophic velocity at the anemometer level. In view of equation (2.8) the equation (2.7) can be written as

$$\omega_m - \omega_f = -\kappa\zeta_o \qquad ...(2.9)$$

where $\kappa \approx 3$ cb.

Equation (2.9) becomes after taking the zonal average

$$\omega_{mz} - \omega_{fz} = \frac{\kappa}{a \cos\varphi} \frac{\partial}{\partial\varphi}(u_{o,z} \cos\varphi) \qquad ...(2.10)$$

where ω_{mz} is the vertical velocity at the bottom of the atmosphere due to the sloping terrain. If p_m is the surface pressure of the underlying topography of the earth, the vertical velocity due to the forced lifting is given by the formula

$$\omega_m = \vec{v}_o . \nabla p_m \qquad ...(2.11),$$

or

$$\omega_{mz} = \frac{1}{a \cos\varphi} \frac{\partial}{\partial\varphi}(\cos\varphi (p_m v_o)_z) \qquad ...(2.12).$$

Finally the vertical velocity at the top of the frictional layer is

$$\omega_{fz} = \frac{1}{a \cos\varphi} \frac{\partial}{\partial\varphi}(\cos\varphi (p_m v_o)_z) - \frac{\kappa}{a \cos\varphi} \frac{\partial}{\partial\varphi}(u_{o,z} \cos\varphi) \qquad ...(2.13).$$

The time dependent and the time averaged axially-symmetric distribution of the vertical velocity can be computed, knowing the forcing function and the boundary conditions.

2.2 COMPUTATION OF THE FORCING FUNCTIONS

It is evident from equation (2.5) that meridional circulations are secondary processes in the atmosphere forced by the large scale eddy processes, diabatic heating and the viscous forces. The estimation of such secondary processes depends entirely on the accuracy with which the forcing functions are measured or evaluated.

The second term on the right hand side of the equation (2.5) is a function of viscous forces in the atmosphere. The uncertainty in the estimates of the coefficient of eddy viscosity, however, makes it difficult to obtain reliable estimates of this forcing function. Holopainen (1964) computed the frictional forces by an indirect method, as a residual from the equations of motion in a dense data region. The accuracy of these results especially at the jet level is questionable due to the errors involved in the observed data. Kung (1966) used a similar procedure to estimate the dissipation of kinetic energy over North America for six months. His results show that the dissipation in the free atmosphere is twice as large as that in the boundary layer. Holopainen (1964) pointed out that the dissipation term, when computed as a residual, has a large standard deviation in the upper troposphere even when the seasonal values are considered and hence the estimates are not realistic. In view of this, no attempt was made to compute the viscous forces in the present study.

The fourth term in the forcing function is a function of diabatic heating in the atmosphere. The dominant diabatic heating processes in the atmosphere are radiation, latent heat release due to the phase change of water and the turbulent transfer of sensible heat. The radiational effects in the troposphere can be estimated from the vertical distribution of temperature, water vapor and clouds. The latent heat released by the phase change of water can be determined assuming that all condensed water precipitates immediately. The turbulent transfer of heat from the earth to the atmosphere can be computed assuming a reasonable value for the coefficient of eddy diffusivity. Several investigations have been made (e.g. by Baur and Phillips (1935), Jacobs (1951), Houghton (1954) and London (1957)) by the heat balance method to establish the major features of the heat budget in the atmosphere. These methods are not adequate to compute the vertical distribution of the diabatic heating. However, the vertical distribution of diabatic heating can be estimated by an indirect method used by Wiin-Nielsen and Brown (1960), Brown (1964) and Holopainen (1964), computing it as a residual from the thermodynamic equation. As far as our problem is concerned, the evaluation of diabatic heating by this method is not an independent forcing function, but let us consider the problem in equilibrium conditions as follows: Let us assume that one can compute the forcing functions depending on the eddy processes. Then one might consider computing ω_z (say $\omega_z^{(1)}$) by solving equation (2.5) for an adiabatic and frictionless flow. $\omega_z^{(1)}$ may be used to compute the diabatic heating from the zonally averaged steady state thermodynamic equation, and also the viscous forces from the steady state zonally averaged vorticity equation as a first approximation. The computed diabatic heating and the friction can then be used to compute a new vertical velocity, $\omega_z^{(2)}$, from equation (2.5). Next $\omega_z^{(2)}$ may be used to compute new diabatic heating and viscous forces from thermodynamic and vorticity equations respectively as a second approximation. The same process may be repeated until the convergence is attained in vertical velocity, diabatic heating and viscous forces.

Now, $\omega_z^{(1)}$ is the solution of equation (2.5) when $F_{\lambda z} = H_z = 0$. The zonally averaged steady state vorticity equation when $\omega_z = \omega_z^{(1)}$ is

$$\frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}(F_{\lambda,z}\cos\varphi) = f_o\frac{\partial\omega_z^{(1)}}{\partial p}$$

$$+\frac{1}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}\left(\frac{1}{\cos\varphi}\frac{\partial(uv)_z\cos^2\varphi}{\partial\varphi}\right) \qquad \ldots(2.14)$$

or

$$\frac{f_o a}{\bar{\sigma}}\frac{1}{\cos\varphi}\frac{\partial}{\partial p}\left(\frac{\partial}{\partial\varphi}F_{\lambda,z}\cos\varphi\right) = \frac{f_o^2a^2}{\bar{\sigma}}\frac{\partial^2\omega_z^{(1)}}{\partial p^2}$$

$$+\frac{f_o}{\bar{\sigma}}\frac{\partial}{\partial p}\left\{\frac{1}{\cos\varphi}\frac{\partial}{\partial\varphi}\left(\frac{1}{\cos\varphi}\frac{\partial(uv)_z\cos^2\varphi}{\partial\varphi}\right)\right\} \qquad \ldots(2.15).$$

The zonally averaged steady state thermodynamic equation when $\omega_z = \omega_z^{(1)}$ is as follows:

$$\frac{gR\ln(\frac{p_2}{p_1})}{2\pi a^2 f_o C_p(\Delta p)^2}\frac{1}{\cos\varphi}\frac{\partial}{\partial\varphi}(TH_{\Delta p}(\varphi)) + \frac{\bar{\sigma}\omega_z^{(1)}}{f_o} =$$

$$-\frac{R}{C_p p f_o}H_z \qquad \ldots(2.16)$$

or

$$-\frac{R}{\bar{\sigma}C_p p}\frac{1}{\cos\varphi}\frac{\partial}{\partial\varphi}\left(\cos\varphi\frac{\partial H_z}{\partial\varphi}\right) =$$

$$\frac{gR\ln(\frac{p_2}{p_1})}{2\pi a^2\bar{\sigma}C_p(\Delta p)^2}\frac{1}{\cos\varphi}\frac{\partial}{\partial\varphi}\left(\cos\varphi\frac{\partial}{\partial\varphi}\left[\frac{1}{\cos\varphi}\frac{\partial TH_{\Delta p}(\varphi)}{\partial\varphi}\right]\right)$$

$$+\frac{1}{\cos\varphi}\frac{\partial}{\partial\varphi}\left(\cos\varphi\frac{\partial\omega_z^{(1)}}{\partial\varphi}\right) \qquad \ldots(2.17)$$

Substituting (2.15) and (2.17) in (2.5) we get

$$\frac{1}{\cos\varphi}\frac{\partial}{\partial\varphi}\left(\cos\varphi\frac{\partial\omega_z}{\partial\varphi}\right) + \frac{f_o^2a^2}{\bar{\sigma}}\frac{\partial^2\omega_z}{\partial p^2} =$$

$$\frac{1}{\cos\varphi}\frac{\partial}{\partial\varphi}\left(\cos\varphi\frac{\partial\omega_z^{(1)}}{\partial\varphi}\right) + \frac{f_o^2a^2}{\bar{\sigma}}\frac{\partial^2\omega_z^{(1)}}{\partial p^2} \qquad \ldots(2.18).$$

Since the boundary conditions are the same for ω_z and $\omega_z^{(1)}$ it is clear from equation (2.18) that

$$\omega_z \equiv \omega_z^{(1)}$$

It is therefore not possible to obtain the diabatic heating and the viscous forces using the ω_z obtained from an adiabatic and frictionless flow.

Hence we shall further assume that the flow is adiabatic and frictionless. This will then imply that the fluid is inviscid and hence there is no skin friction at the lower boundary. The only vertical velocity at the lower boundary is due to the irregular terrain over the surface of the earth. It will be seen later that some of the computations of vertical velocity were done assuming that the earth's surface is smooth, so that vertical velocity at the lower boundary is zero.

The first and the third term in the forcing function are functions of eddy transfer of zonal momentum and eddy transfer of sensible heat, respectively. These data were obtained from the calculations made by Wiin-Nielsen, Brown and Drake (1963) and (1964). The data for eddy transfer of zonal momentum were available for two observations per day as a function of latitude from 20°N to 87.5°N at the interval of 2.5° latitude at the isobaric levels 850, 700, 500, 300 and 200 mb for the months of January, April, July, October for

1962 and January 1963. Also the data for January 1964 were available with the three additional levels 1000, 150 and 100 mb. The data for eddy transfer of sensible heat were available for the same grid and observations for the corresponding layers 1000-850, 850-700, and so on, as shown in figure (1). Further these data were given as a function of wave number including wave numbers 1 to 15. A brief outline of their method of computation is given in Appendix B.

In addition to the eddy transfer of zonal momentum and sensible heat one would need the distribution of the static stability, σ, in the atmosphere so as to compute the forcing function. To be consistent with assumption 3 in section 2.1, σ was taken as $\bar{\sigma}(p) = \frac{\sigma_o}{p^2}$ where $\sigma_o = 0.625$ for the troposphere as shown by Wiin-Nielsen (1959b). Or one could use the average values of static stability computed by Gates (1960). It may be mentioned here that the formula suggested by Wiin-Nielsen is closely in agreement with the computed values of the static stability except near 850 mb and 200 mb.

2.3 METHOD OF SOLUTION

The following transformation of the independent variables was made to write equation (2.5) in a convenient form:

$\sin \varphi = \mu$ and $p_* = \frac{p}{p_o}$ where $p_o = 100$ cb, the surface pressure

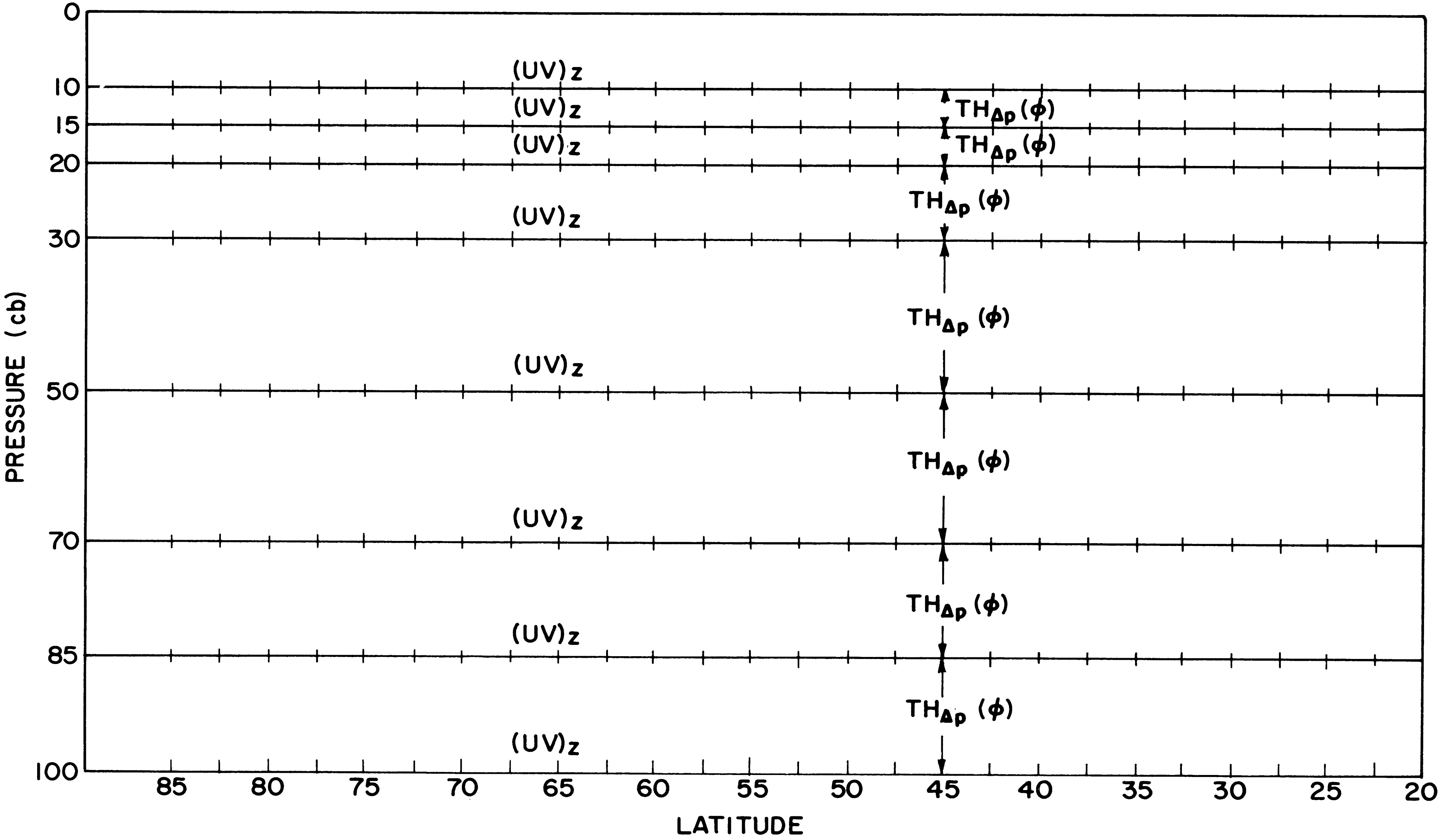

Figure 1. Schematic diagram showing the data grid for which the eddy transfer of zonal momentum and the eddy transfer of sensible heat were available for the month of January 1964.

so that $0 \leq \mu \leq 1$ for $0 \leq \varphi \leq \frac{\pi}{2}$

and $0 \leq p_* \leq 1$ for $0 \leq p \leq 100$ cb.

Equation (2.5) for adiabatic and frictionless flow then takes the form:

$$\frac{\partial}{\partial\mu}\{(1-\mu^2)\frac{\partial\omega_z}{\partial\mu}\} + q^2 \frac{\partial^2\omega_z}{\partial {p_*}^2} = M(\mu,p_*) \qquad \ldots(2.19)$$

where
$$M(\mu,p_*) = -\frac{f_o}{\bar{\sigma}p_o}\frac{\partial}{\partial p_*}\{\frac{\partial^2}{\partial\mu^2}[(1-\mu^2)(uv)_z]\} - \frac{gR}{2\pi a^2}\frac{\ln(\frac{p_2}{p_1})}{(\Delta p)^2 C_p \bar{\sigma}}\frac{\partial}{\partial\mu}\{(1-\mu^2)\frac{\partial^2}{\partial\mu^2}(TH_{\Delta p}(\varphi))\}$$

and
$$q^2 = \frac{{f_o}^2 a^2}{\bar{\sigma}{p_o}^2}$$

It may be noted here that the forcing function $M(\mu,p_*)$ is a function of time, and therefore so is ω_z.

Since equation (2.19) is a diagnostic equation we shall drop the time dependence. Equation (2.19) can be solved by relaxation methods, but it is convenient to solve it by separation of variables to reduce the truncation errors involved in finite differencing.

Let $\omega_z(\mu,p_*)$ be expressed as a finite series of Legendre polynomials.

$$\omega_z(\mu,p_*) = \sum_{n=0}^{N} A_n(p_*)P_n(\mu) \qquad \ldots(2.20)$$

Similarly the forcing function can be expressed as

$$M(\mu,p_*) = \sum_{n=0}^{N} B_n(p_*)P_n(\mu) \qquad \ldots(2.21)$$

$P_n(\mu)$ are orthogonal functions over $-1 \leq \mu \leq 1$, that is,

$$\int_{-1}^{1} P_n(\mu)P_m(\mu)d\mu = \begin{cases} \frac{2n+1}{2} & \text{for } n=m \\ 0 & \text{for } n\neq m. \end{cases} \qquad \ldots(2.22)$$

Hence $B_n(p_*)$ can be computed knowing $M(\mu,p_*)$ over $-1 \leq \mu \leq 1$ using the orthogonal property (2.22) from the relation

$$B_n(p_*) = \frac{2n+1}{2} \int_{-1}^{1} M(\mu,p_*)P_n(\mu)d\mu \qquad \ldots(2.23).$$

It was mentioned earlier in section 2.2 that the data to compute $M(\mu,p_*)$ were available only from 20°N to 87.5°N. The data for eddy transfers were extrapolated from 20°N to the equator by a biquadratic even polynomial in φ (see Appendix C), such that there is no flux across the equator and the function and its derivatives are continuous at 20°N. The value at the pole was obtained by Newton's interpolation formula knowing the value at 87.5°N which is the same on either side of the pole. Further it was assumed that the eddy transfers are symmetric around the equator, so

$$M(\mu,p_*) = M(-\mu,p_*) \qquad 0 \leq \mu \leq 1 \qquad \ldots(2.24)$$

Equation (2.23) can then be written as:

$$B_n(p_*) = (2n+1) \int_0^1 M(\mu, p_*) P_n(\mu) d\mu \qquad \ldots(2.25)$$

here n has to be an even integer to satisfy (2.24), because $P_n(\mu) = (-1)^n P_n(-\mu)$. The Legendre polynomials $P_n(\mu)$ were computed from the recurrence relation

$$P_n(\mu) = \frac{2n-1}{n} \mu P_{n-1}(\mu) - \frac{n-1}{n} P_{n-2}(\mu) \quad \text{for } n \geq 2$$

and by definition

$$P_n(\mu) = \frac{1}{2^n} \frac{1}{n!} \frac{d^n}{d\mu^n}(\mu^2-1)^n \qquad \ldots(2.26).$$

Therefore

$$P_o(\mu) = 1$$

$$P_1(\mu) = \mu.$$

The accuracy of computing the Legendre polynomials was checked by their orthogonality property (2.22). The error never exceeded 10^{-4}, when integration was evaluated by Simpson's quadrature formula.

The time averaged eddy transports over the months are smooth functions of φ and p. The eddy transfer of zonal momentum is differentiated twice with respect to μ and once with respect to p_* and the eddy transfer of sensible heat is differentiated three times with respect to μ in equation

(2.19) to get the forcing functions. The differentiation was done numerically, using centered finite differencing. The resulting forcing function was not a smooth function and therefore it was necessary to smooth $M(\mu, p_*)$ such that its main features are retained and the order of magnitude is not affected. This was achieved by taking a smaller number of polynomials than required to represent it completely. $N = 10$ was enough to represent the main features of the forcing function without affecting its order of magnitude. The integration in equation (2.25) was evaluated by Simpson's quadrature formula.

Now, equation (2.19) can be written for a particular Legendre polynomial after substituting for $\omega_z(\mu, p_*)$ and $M(\mu, p_*)$ from equations (2.20) and (2.21), respectively, as follows:

$$A_n(p_*)\frac{d}{d\mu}\{(1-\mu^2)\frac{dP_n(\mu)}{d\mu}\} + q^2 \frac{d^2A_n(p_*)}{dp_*^{\,2}} P_n(\mu) = B_n(p_*)P_n(\mu) \qquad \ldots(2.27).$$

$P_n(\mu)$ satisfies the Legendre equation

$$\frac{d}{d\mu}\{(1-\mu^2)\frac{dP_n(\mu)}{d\mu}\} + n(n+1)P_n(\mu) = 0 \qquad \ldots(2.28).$$

Thus in view of equation (2.28) we may write (2.27) in the form

$$q^2 \frac{d^2A_n(p_*)}{dp_*^{\,2}} - n(n+1)A_n(p_*) = B_n(p_*) \qquad \ldots(2.29)$$

Equation (2.29) was solved by ordinary centered finite difference method with boundary conditions that $A_n(0) = 0$ and $A_n(1) = 0$ (i.e. $\omega_z = 0$ at the top and bottom of the atmosphere), for the months of January, April, July, October 1962 and January 1963. For January 1964, however, $A_n(1)$ was determined from the vertical velocity due to the terrain of the earth's surface. In equation (2.12) the vertical velocity produced by the orography depends on the surface pressure and meridional component of wind. The surface pressure was computed from the hydrostatic equation knowing the height of the topography which was obtained from the data compiled by Berkovsky and Bertoni (1955). For January 1964 the data was available to compute the surface geostrophic meridional component of the wind. $\omega_z(\mu, p_*)$ was then computed from equation (2.20). One should note, here, that $\omega_z(\mu, p_*)$ has to satisfy an additional physical constraint imposed by the equation of continuity. The area average of ω over any isobaric surface which does not intersect the ground is zero.

That is,

$$\bar{\omega} = \frac{1}{2\pi} \int_0^{\pi/2} \int_0^{2\pi} \omega \cos \varphi \, d\varphi \, d\lambda$$

$$= \int_0^{\pi/2} \omega_z \cos \varphi \, d\varphi$$

$$= \bar{\omega}_z = 0. \qquad \text{...(2.30)}$$

But
$$\bar{\omega}_z = \sum_{n=0}^{N} A_{2n}(p_*) \int_0^1 P_{2n}(\mu)d\mu$$
$$= A_o(p_*) \int_0^1 P_o(\mu)d\mu + \sum_{n=1}^{N} A_{2n}(p_*) \int_0^1 P_{2n}(\mu)d\mu \qquad \ldots(2.31),$$

$$\int_0^1 P_o(\mu)d\mu = 1 \qquad \ldots(2.32),$$

and
$$\int_0^1 P_{2n}(\mu)d\mu = \frac{1}{4n+1}\int_0^1 \left(\frac{dP_{2n+1}(\mu)}{d\mu} - \frac{dP_{2n-1}(\mu)}{d\mu}\right)d\mu$$
$$= \frac{1}{4n+1}(P_{2n+1}(1)-P_{2n+1}(0)-P_{2n-1}(1)+P_{2n-1}(0))$$
$$= 0 \qquad \ldots(2.33),$$

since $P_{2n+1}(0) - P_{2n-1}(0) = 0$

and $P_{2n+1}(1) = P_{2n-1}(1) = 1.$

Hence $A_o = 0$ to satisfy equation (2.30).

Therefore

$$\omega_z(\mu,p_*) = A_2(p_*)P_2(\mu)+\ldots+A_n(p_*)P_n(\mu) \qquad \ldots(2.34).$$

The zonally averaged vertical velocity distribution was computed from equation (2.34) from the equator to the north pole at intervals of 2.5 degrees and for the levels 775, 600, 400 and 250 mb for the four months of 1962 representing the four seasons and January 1963, whereas vertical resolution was better in January 1964, and the vertical velocity was computable for levels 925, 775, 600, 400, 250, 175 and 125 mb from equation (2.34) and at 1000 mb due to the orographic lifting.

The axially-symmetric flow consists of a pure zonal flow u_z and a meridional flow with velocity $\vec{v}_m = v_z\vec{j} + \omega_z\vec{k}$ which is identical in all meridional planes. Once the ω_z is computed by the method described above, the zonally averaged meridional component can be computed from the equation of continuity.

The zonally averaged equation of continuity in spherical coordinates is

$$\frac{1}{a \cos \varphi} \frac{\partial}{\partial \varphi}(v_z \cos \varphi) + \frac{\partial \omega_z}{\partial p} = 0 \qquad \ldots(2.35).$$

Integrating (2.35) from the north pole to any latitude φ_i we get

$$\int_{\pi/2}^{\varphi_i} \frac{\partial}{\partial \varphi}(v_z \cos \varphi) d\varphi = -a \int_{\pi/2}^{\varphi_i} \frac{\partial \omega_z}{\partial p} \cos \varphi \, d\varphi \qquad \ldots(2.36).$$

Since $v_z \cos \varphi = 0$ at the north pole (i.e. $\varphi = \pi/2$),

$$v_z(\varphi_i, p) = - \frac{a}{\cos \varphi_i} \int_{\pi/2}^{\varphi_i} \frac{\partial \omega_z}{\partial p} \cos \varphi \, d\varphi \qquad \ldots(2.37).$$

$\frac{\partial \omega_z}{\partial p}$ was computed by centered finite differencing and the integration was carried out using the trapezoidal rule. The zonally averaged meridional component of velocity was obtained from the equator to the north pole at intervals of 2.5 degrees and for levels 887.5, 687,5, 500 and 325 and 125 mb in 1962 and January 1963. For January 1964 it was computed for the levels 962.5, 850, 687.5, 500, 325, 212.5, 150 and 62.5 mb.

3. PRESENTATION AND DISCUSSION OF RESULTS

It was mentioned earlier that the data were available only from 20°N to 87.5°N. It was necessary to extrapolate the data from 20°N to the equator to be able to employ a convenient method of solution. One cannot be certain of the results in the extrapolated region, so we shall present the results in the region 20°N to 87.5°N.

3.1 MEAN MERIDIONAL CIRCULATIONS IN AN ADIABATIC FRICTIONLESS ATMOSPHERE

The axially-symmetric vertical velocity distribution, as a function of latitude and pressure, for the month of January 1963 is shown in figure (2). To avoid a further assumption about the density variations in the atmosphere the results are given in the units 10^{-5} mb sec^{-1}. Hence the positive values indicate a descending motion, while the negative values represent an ascending motion. The maximum values of the ascending or the descending motion are found at the level of non-divergence in the atmosphere. Intense downward flux of mass is found around 35°N, the region of the subtropical high pressure belts, while the upward flux of mass occurs in the region of the low pressure belts near 55°N. The lines of zero vertical motion are the centers of the meridional circu-

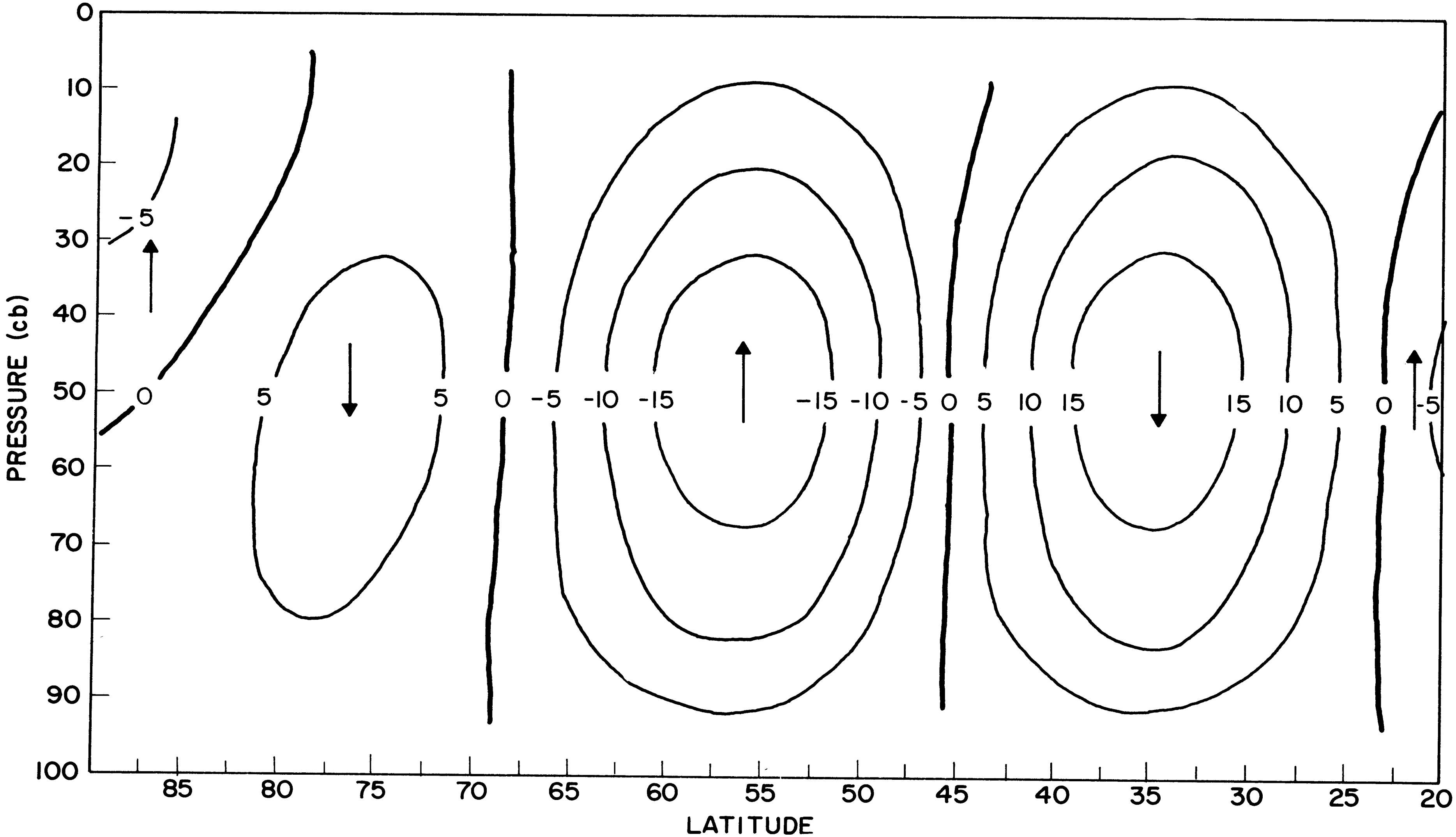

Figure 2. The zonally averaged vertical velocity, ω_z, for the month of January 1963 as a function of latitude and pressure in the unit 10^{-5}mb sec^{-1}.

lation cells. Thus, the tropical direct cell is situated around 22.5°N, the reverse middle latitude cell near 45°N and the polar direct cell around 70°N. The intensity of the cells decreases from the equator toward the pole. The rising vertical motion found in the polar region near 400 mb is an indication of opposite meridional circulations in the stratosphere. Our result is thus in agreement with a stratospheric circulation postulated by Kuo (1956). Since our data are restricted to levels below the 200 mb level such reverse circulations are not seen in the middle latitudes and the subtropical regions.

The distribution of the meridional component of the axially-symmetrical flow, for the month of January 1963, is illustrated as a function of latitude and pressure in figure (3). The meridional velocities are given in the units cm sec^{-1}. The negative and the positive values indicate southward and northward flow, respectively. In the subtropical region the equatorward flow extends up to 35°N in the lower troposphere, while the poleward flow at higher levels extending up to the same latitude, describes a part of the tropical direct cell. The reverse cell in the middle latitudes has a maximum northward flow of 60 cm sec^{-1} around 47°N in the lowest layer and the maximum reverse flow of the same intensity near the

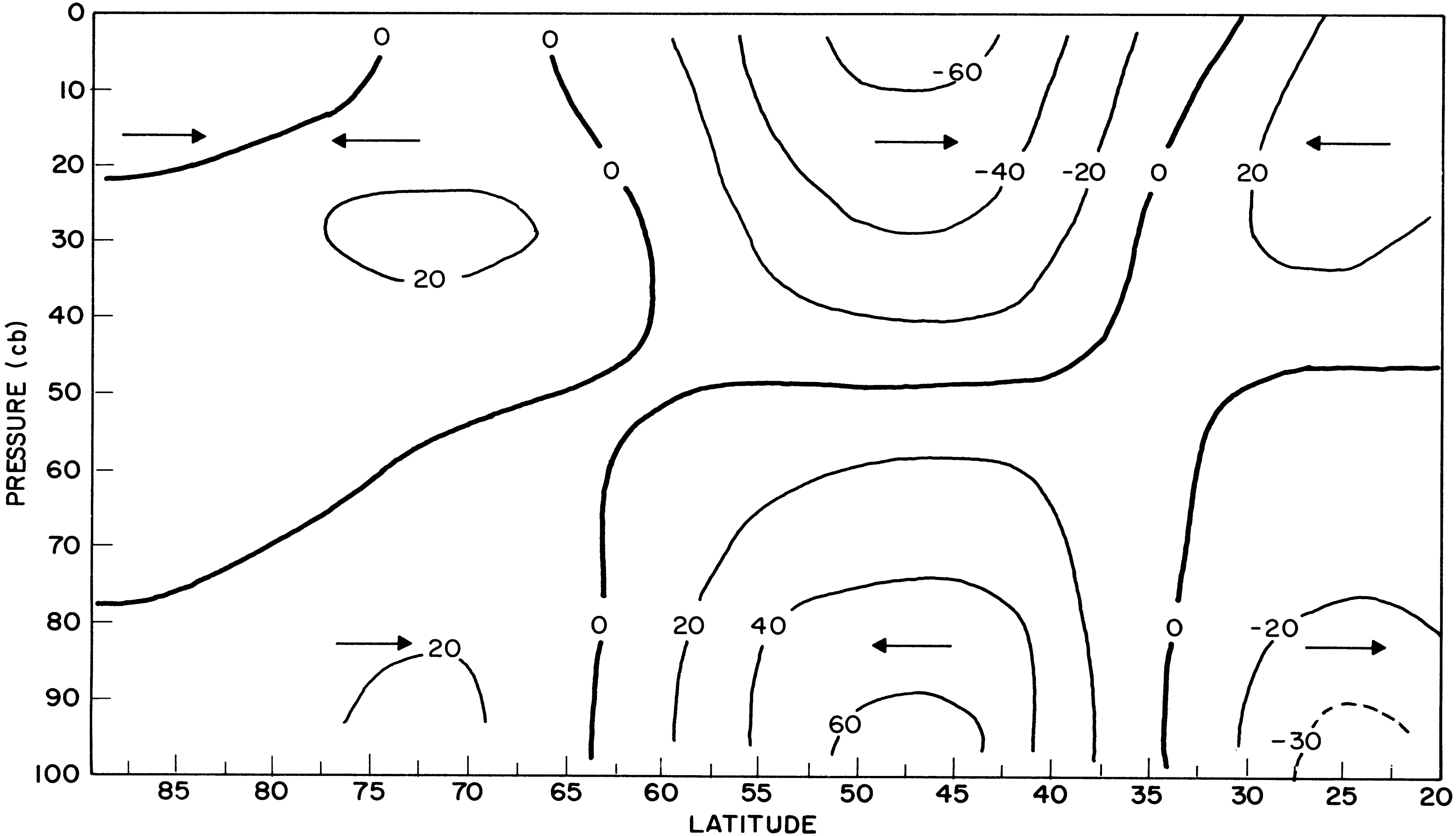

Figure 3. The zonally averaged meridional velocity, v_z, for the month of January 1963 as a function of latitude and pressure in the unit cm sec^{-1}.

top of the troposphere. The polar direct cell being much weaker than the middle latitude cell has a maximum equatorward flow of 20 cm sec^{-1} near the ground and a maximum poleward flow of the same intensity near 300 mb. The equatorward flow near 100 mb in the polar region is again an indication of the reverse cell in the stratosphere. These computations of the mean meridional circulations are in agreement with the results obtained by Mintz and Lang (1955), and Holopainen (1966a).

3.2 SEPARATE EFFECTS OF EDDY TRANSFERS OF ZONAL MOMENTUM AND SENSIBLE HEAT ON THE MEAN MERIDIONAL CIRCULATIONS

We have seen from equation (2.19) that the forcing function $M(\mu, p_*)$ is the sum of two terms. The first term is a function of eddy transfer of zonal momentum and the second is a function of eddy transfer of sensible heat. Henceforth we shall refer to the former as $f((uv)_z)$ and the latter as $f((Tv)_z)$. The role of one of the terms, in the mean meridional circulations, can be determined by letting the other be identically zero in the (μ, p_*) plane.

The distributions of the vertical and the meridional velocities due to $f((uv)_z)$ for January 1963 are shown in figures (4) and (5) respectively. The circulation pattern in these figures is very similar to the corresponding circu-

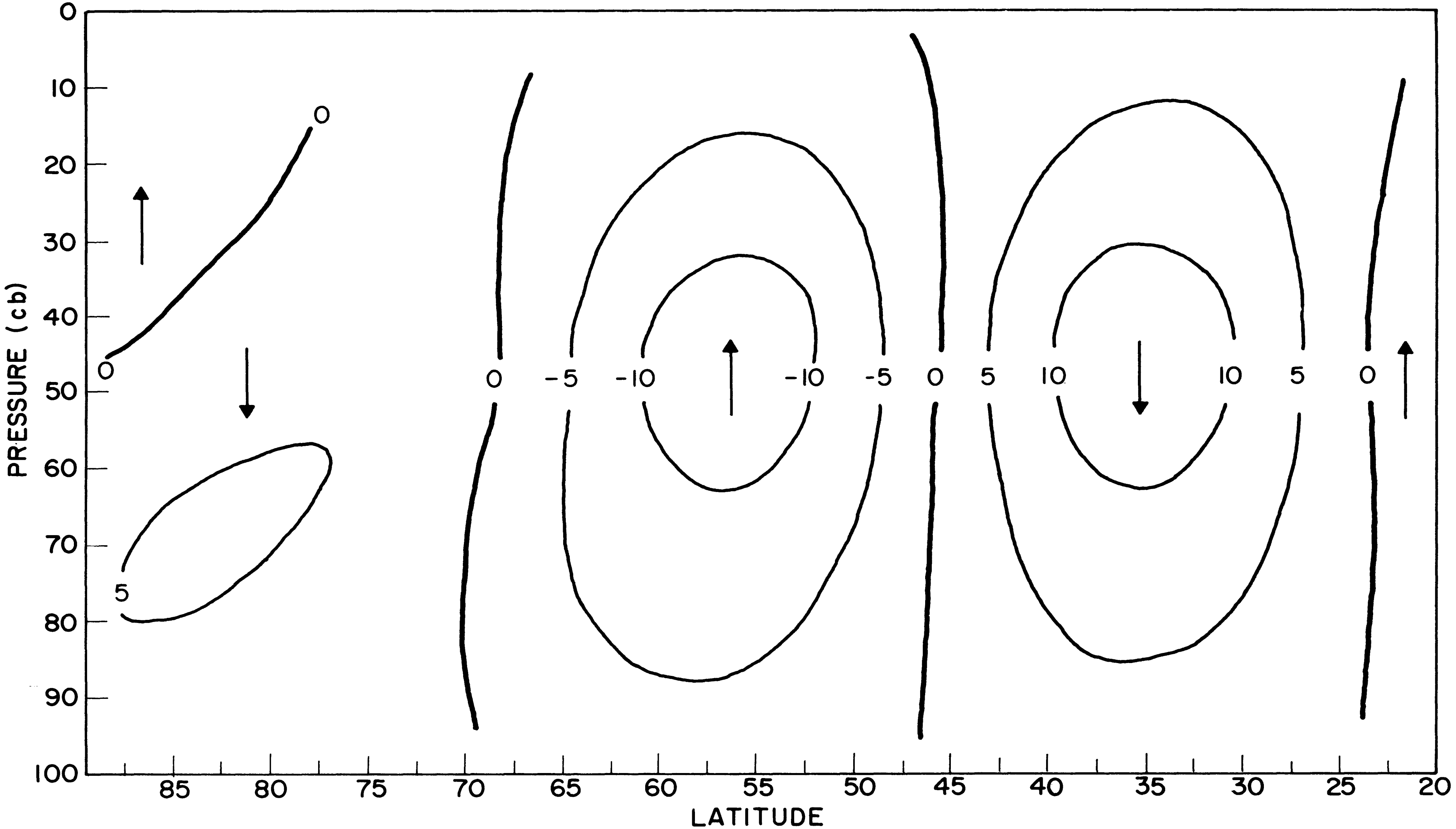

Figure 4. The zonally averaged vertical velocity, ω_z, produced by $f((uv)_z)$ for the month of January 1963. Arrangement and units as in figure 2.

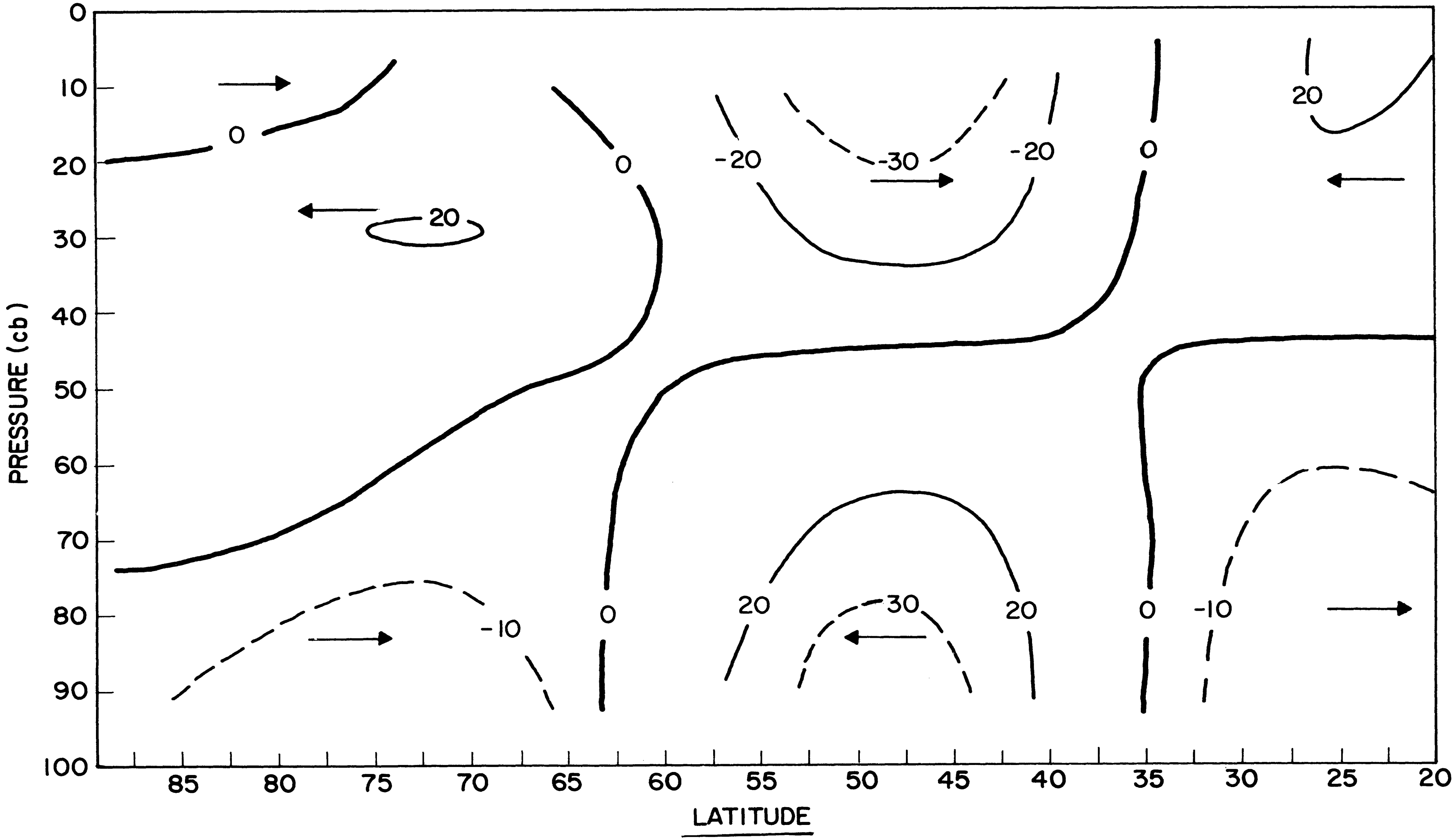

Figure 5. The zonally averaged meridional velocity, v_z, produced by $f((uv)_z)$ for the month of January 1963. Arrangement and units as in figure 3.

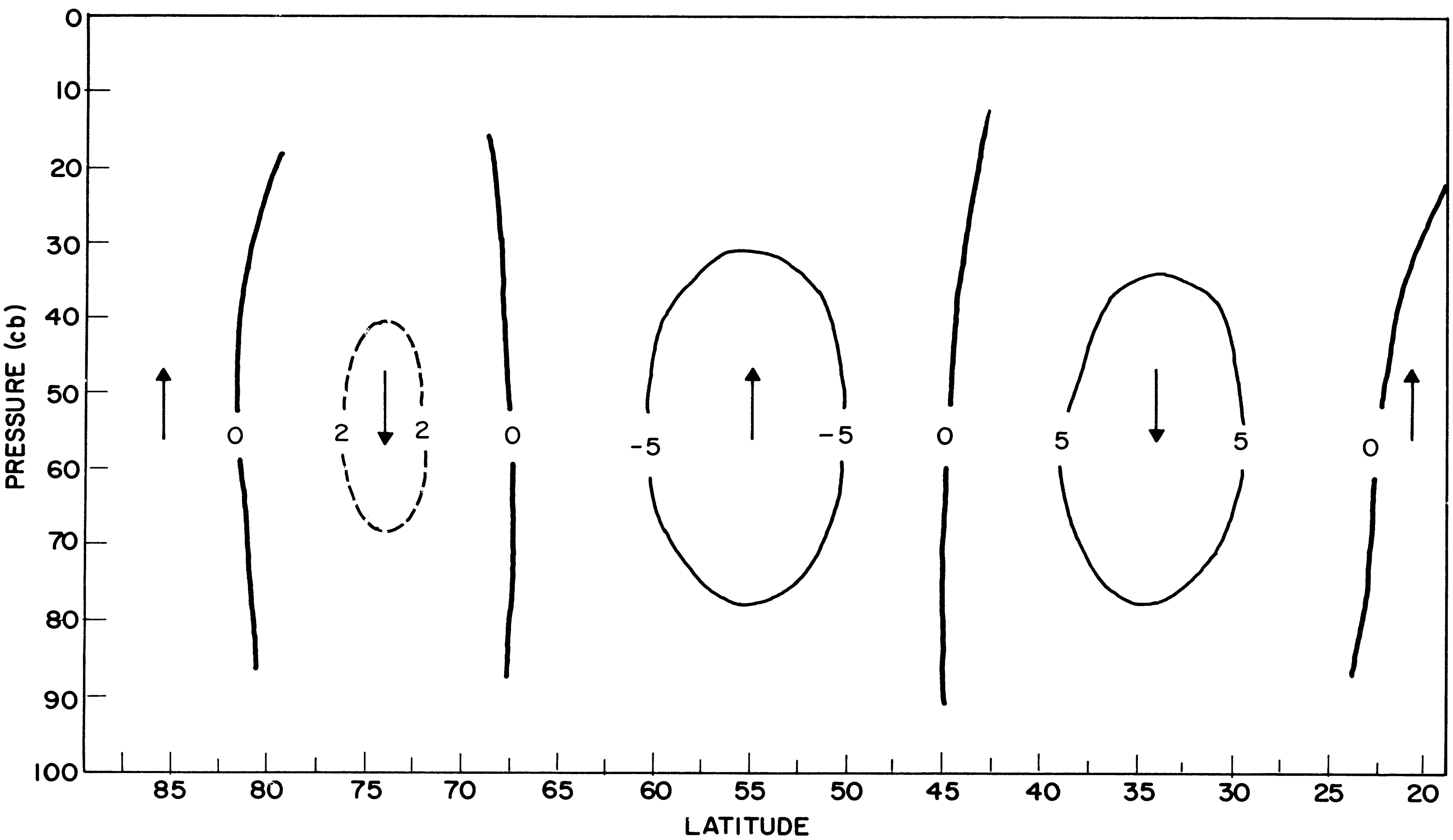

Figure 6. The zonally averaged vertical velocity, ω_z, produced by $f((Tv)_z)$ for the month of January 1963. Arrangement and units as in figure 2.

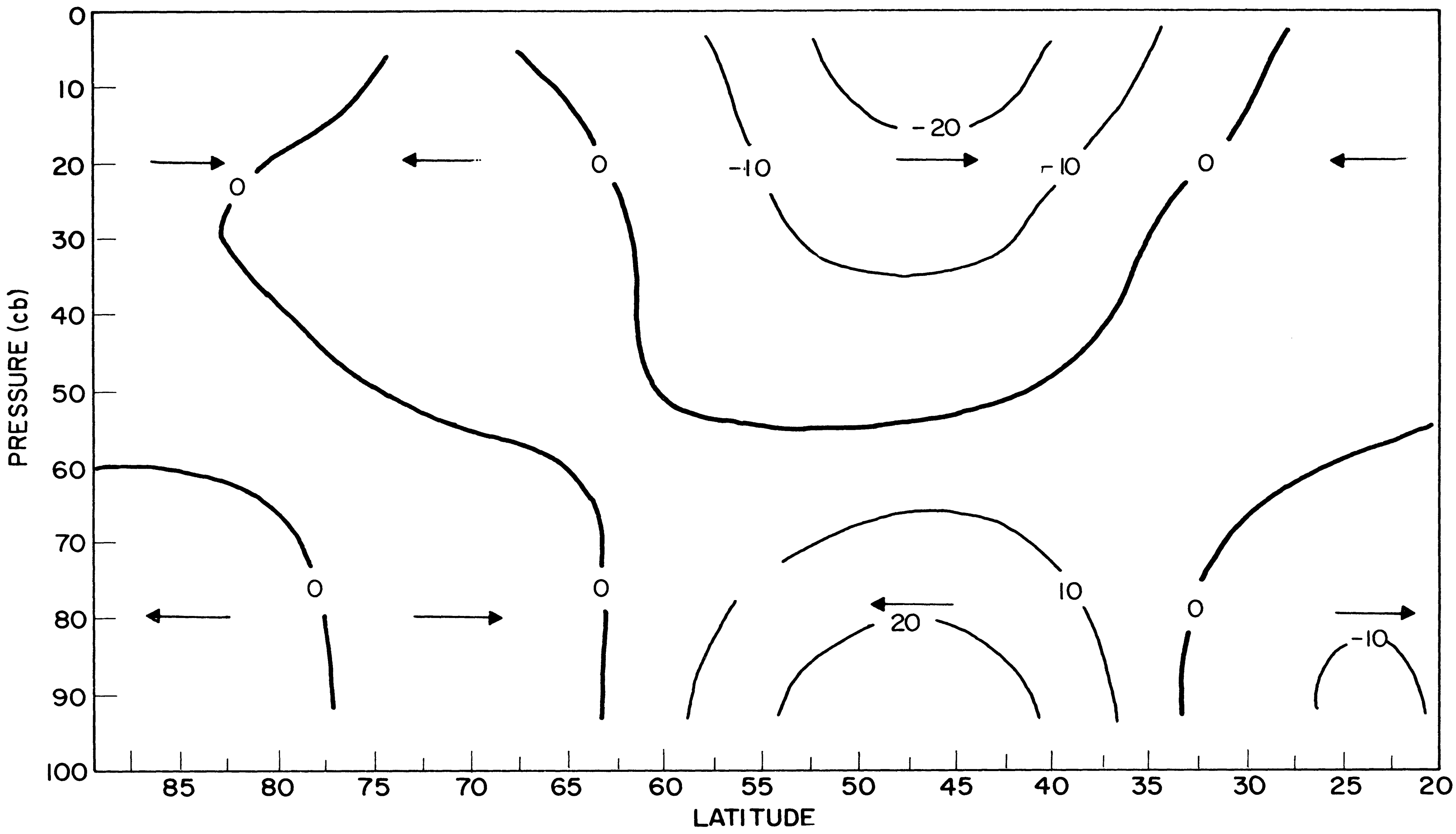

Figure 7. The zonally averaged meridional velocity, v_z, produced by $f((Tv)_z)$ for the month of January 1963. Arrangement and units as in figure 3.

lations in figures (2) and (3). The intensities of the circulations, however, differ as one would expect.

Figures (6) and (7) illustrate, respectively, the distributions of the vertical and meridional velocities due to $f((Tv)_z)$ for January 1963. Comparing figures (6) and (7) with the corresponding figures (2) and (3) we notice the similarity in circulation pattern except in a small polar region north of 82.5°N. The ascending motion in the polar region in figure (6) could be a spurious result. This may be attributed to the uncertainty in the data in that region, especially, in computing the eddy transfer of sensible heat.

Since $f((uv)_z)$ and $f((Tv)_z)$ force the circulation in a similar manner, it will be interesting to find their individual contribution to the total intensity of the circulation. Therefore we have computed the mass flux, $M^*(\varphi)$, across any latitude φ by a formula similar to equation (1.1), but here

$$M^*(\varphi) = \frac{2\pi a \cos\varphi}{g} \int_{p_o}^{p} v_z \, dp \qquad \ldots(3.1)$$

where p is the pressure at an altitude at which the mass circulation reverses its direction in the troposphere.

Figure (8) gives the mass circulation, as a function of latitude in units 10^6 tons sec^{-1}, forced by the two terms

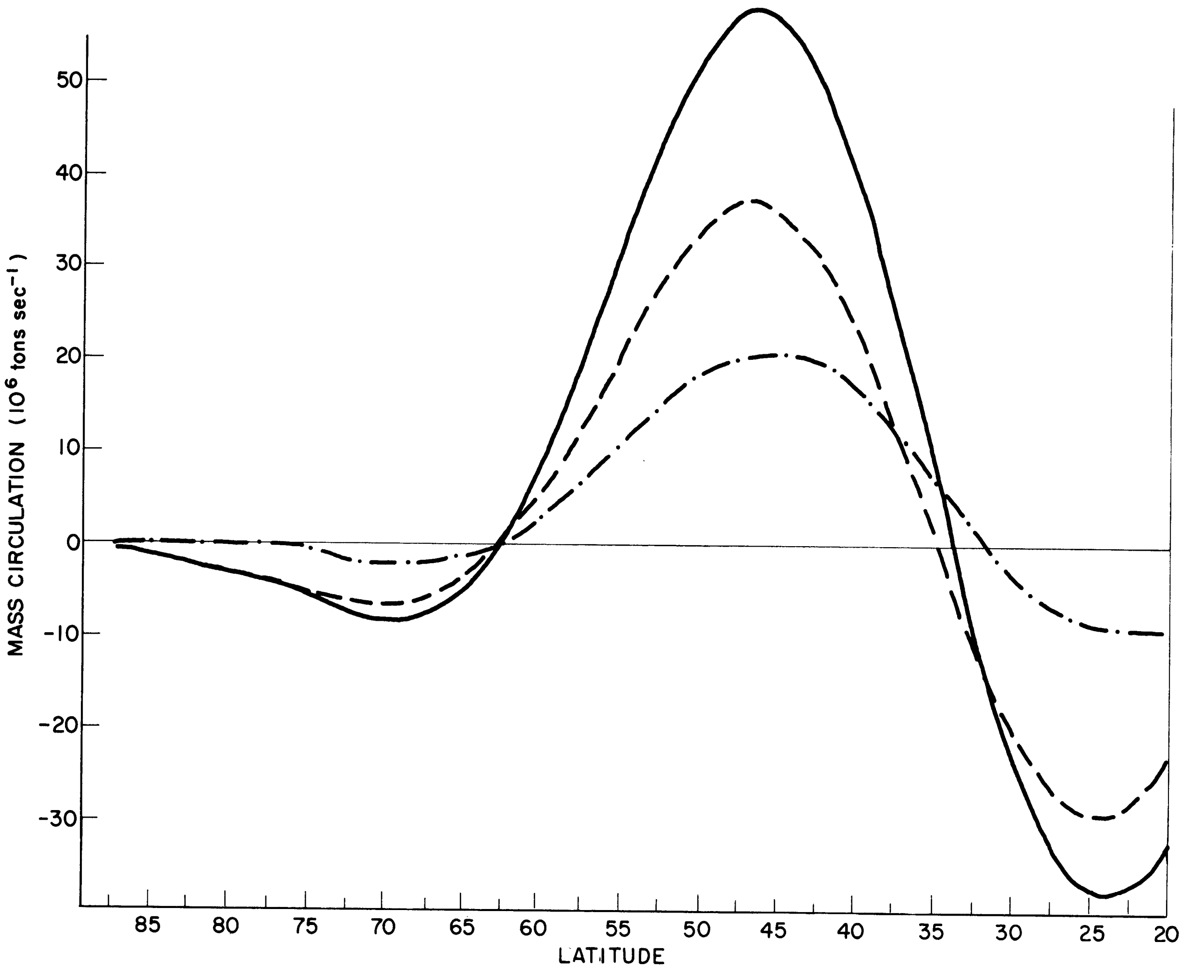

Figure 8. The mass circulation in the lower troposphere for the month of January 1963 as a function of latitude in the unit 10^6tons sec^{-1}. Solid curve: The mass circulation produced by both $f((uv)_z)$ and $f((Tv)_z)$. Dashed curve: The mass circulation produced by $f((uv)_z)$. Dash-dotted curve: The mass circulation produced by $f((Tv)_z)$.

separately and collectively. The positive mass circulation, in the middle latitudes, indicates the northward flow in the lower troposphere and the equal amount of southward flow in the upper troposphere, whereas the opposite is true for the negative circulation in the lower and higher latitudes. The dashed curve shows the mass circulation due to $f((uv)_z)$ while the dash-dotted curve gives that due to $f((Tv)_z)$. Their combined effect is shown by a continuous curve. The proportion of the mass circulation due to $f((uv)_z)$ or $f((Tv)_z)$ to the total is a function of latitude. But by and large $f((uv)_z)$ explains 2/3 of the total mass circulation while only 1/3 of it is explained by $f(Tv)_z)$.

3.3 MEAN MERIDIONAL CIRCULATIONS IN WAVE NUMBER REGIME

It was mentioned earlier that the data for computing $f((uv)_z)$ and $f((Tv)_z)$ were available as a function of wave number including wave number 1 to 15. It is however cumbersome to present the effect of all 15 waves. We have therefore grouped the 15 components in three groups. The first, consisting of wave numbers 1 to 4, represents the planetary or long waves. The next group, consisting of wave numbers 5 to 8, is usually called medium waves, while the final group, consisting of wave numbers 9 to 15, represents the short waves.

The effect of the long, medium and short waves on the vertical velocity, ω_z, is shown in figures (9), (11), (13), respectively. Similarly figures (10), (12), (14) give the distribution of the meridional velocity, v_z, for the corresponding wave groups. Comparison between the circulation pattern of different scales shows a striking similarity. Exceptions, however, exist in the polar region for small scales of motion. This, again, can be attributed to the uncertainty in data, especially in the polar region. The similar circulation pattern for different scales of motion is not surprising because eddy transports for both momentum and heat have similar distributions for all the three wave groups (Wiin-Nielsen, Brown and Drake (1964)).

The striking difference lies in the intensity of the circulation for different scales. In order to compare the contributions of different scales, we shall refer to figure (15). Here the dotted curve gives the distribution of the mass circulation in the lower troposphere as a function of latitude due to the long waves, dashed curve for the medium waves and the dash-dotted curve for the short waves. The total effect is shown by the continuous curve. Comparing the absolute values of the mass circulation over the entire region, 20°N to 87.5°N, it is found that the long waves account for 86% of the total

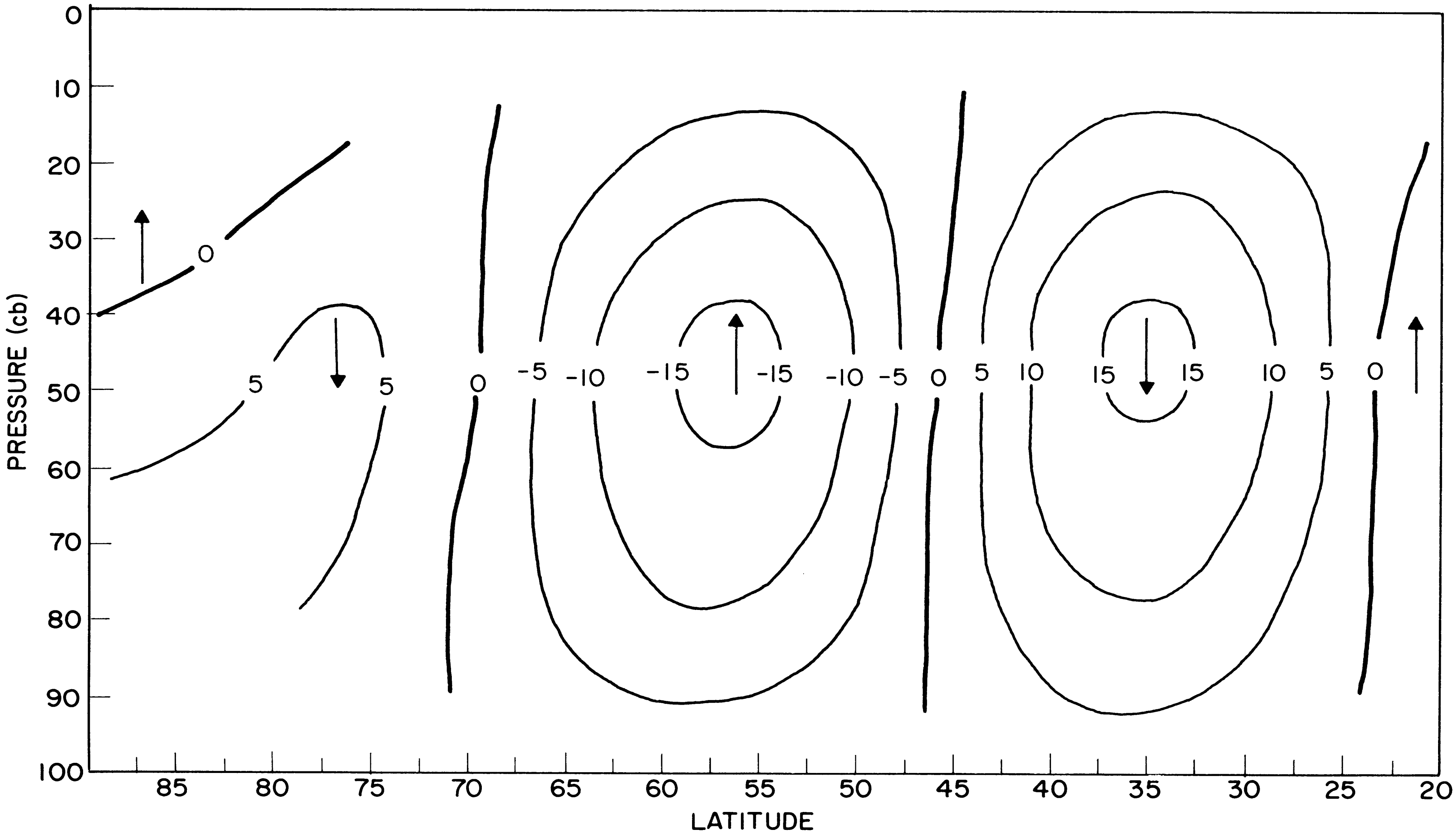

Figure 9. The zonally averaged vertical velocity, ω_z, produced by the planetary waves for the month of January 1963. Arrangement and units as in figure 2.

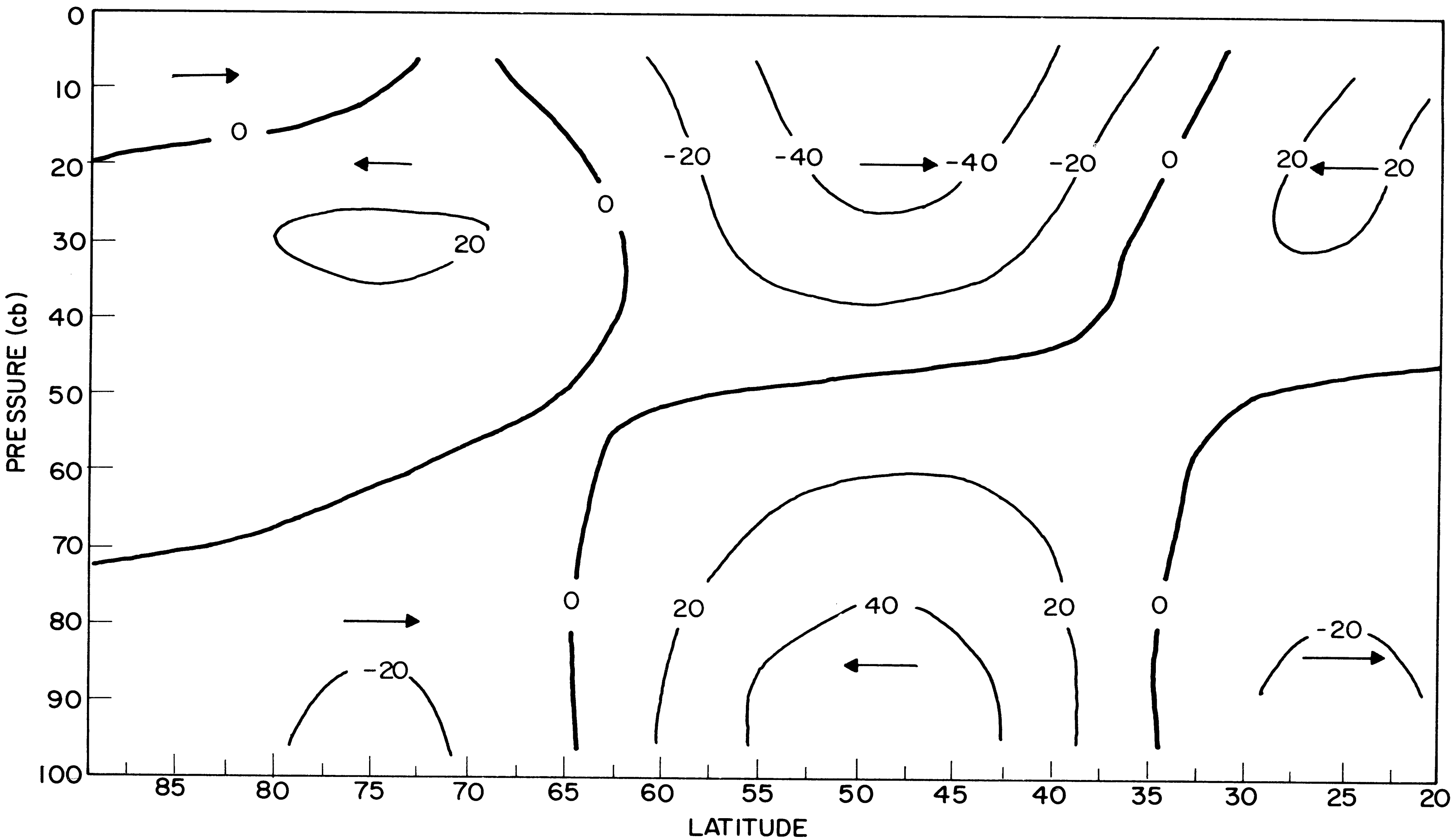

Figure 10. The zonally averaged meridional velocity, v_z, produced by the planetary waves for the month of January 1963. Arrangement and units as in figure 3.

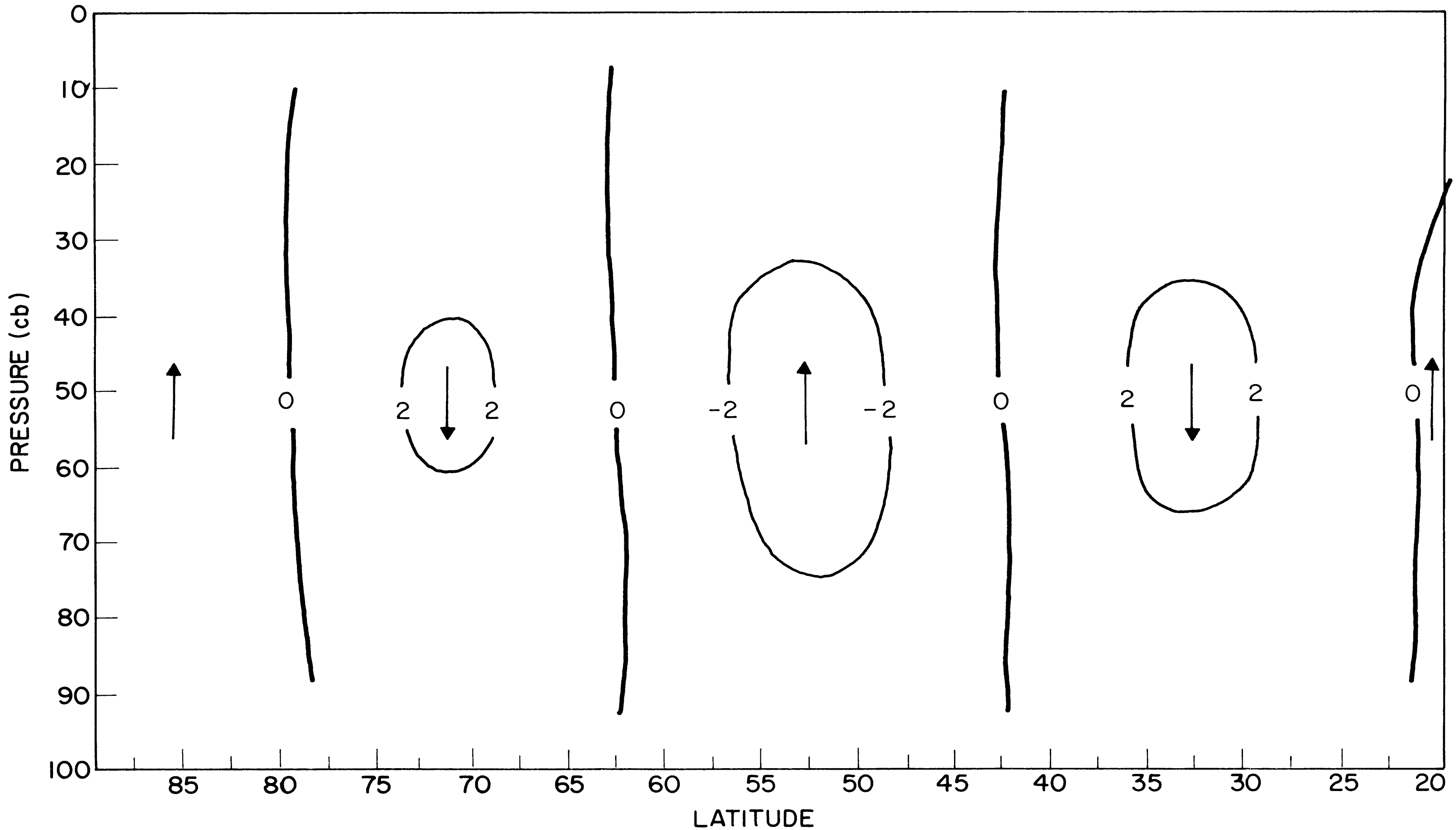

Figure 11. The zonally averaged vertical velocity, ω_z, produced by the medium waves for the month of January 1963. Arrangement and units as in figure 2.

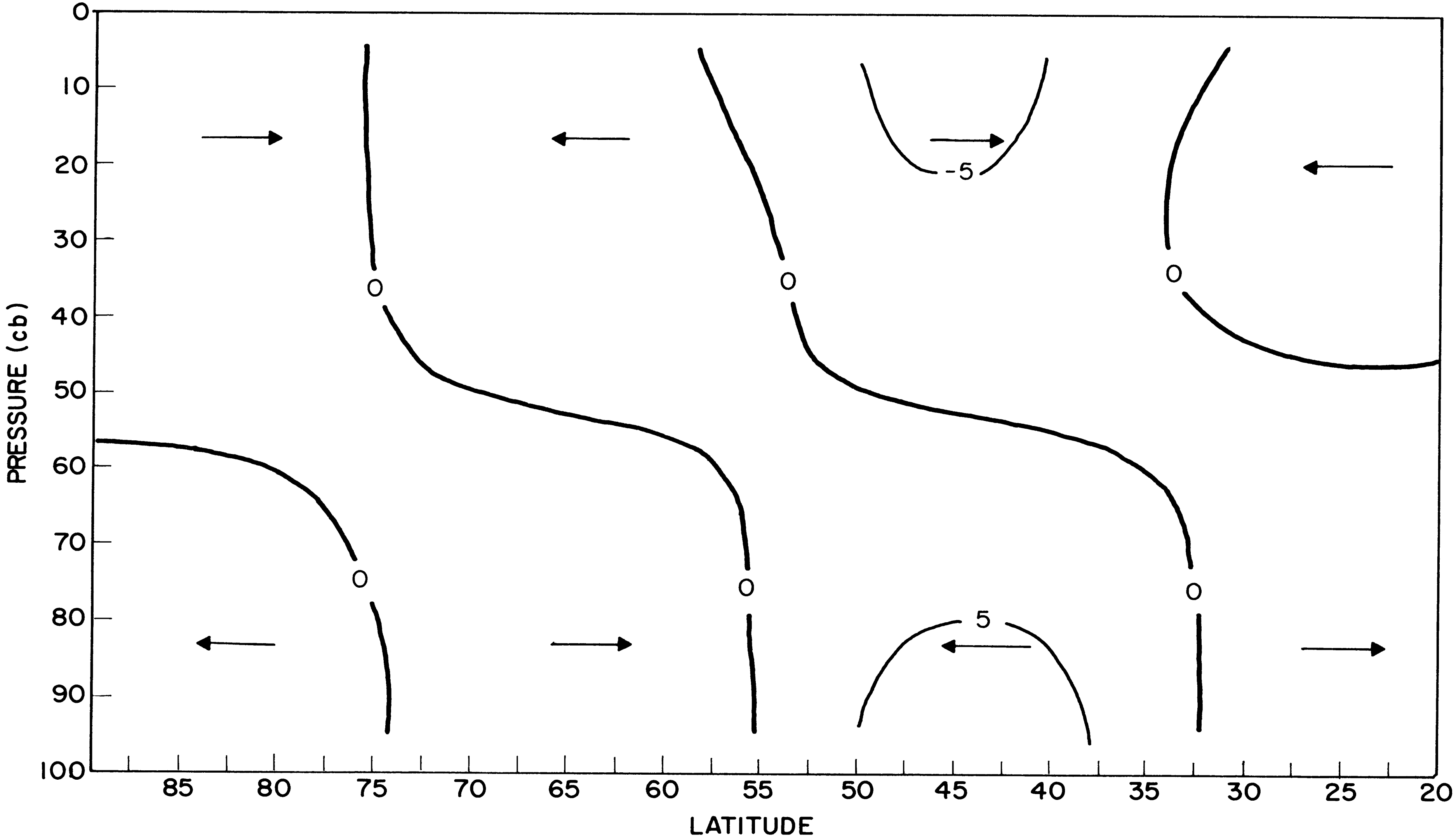

Figure 12. The zonally averaged meridional velocity, v_z, produced by the medium waves for the month of January 1963. Arrangement and units as in figure 3.

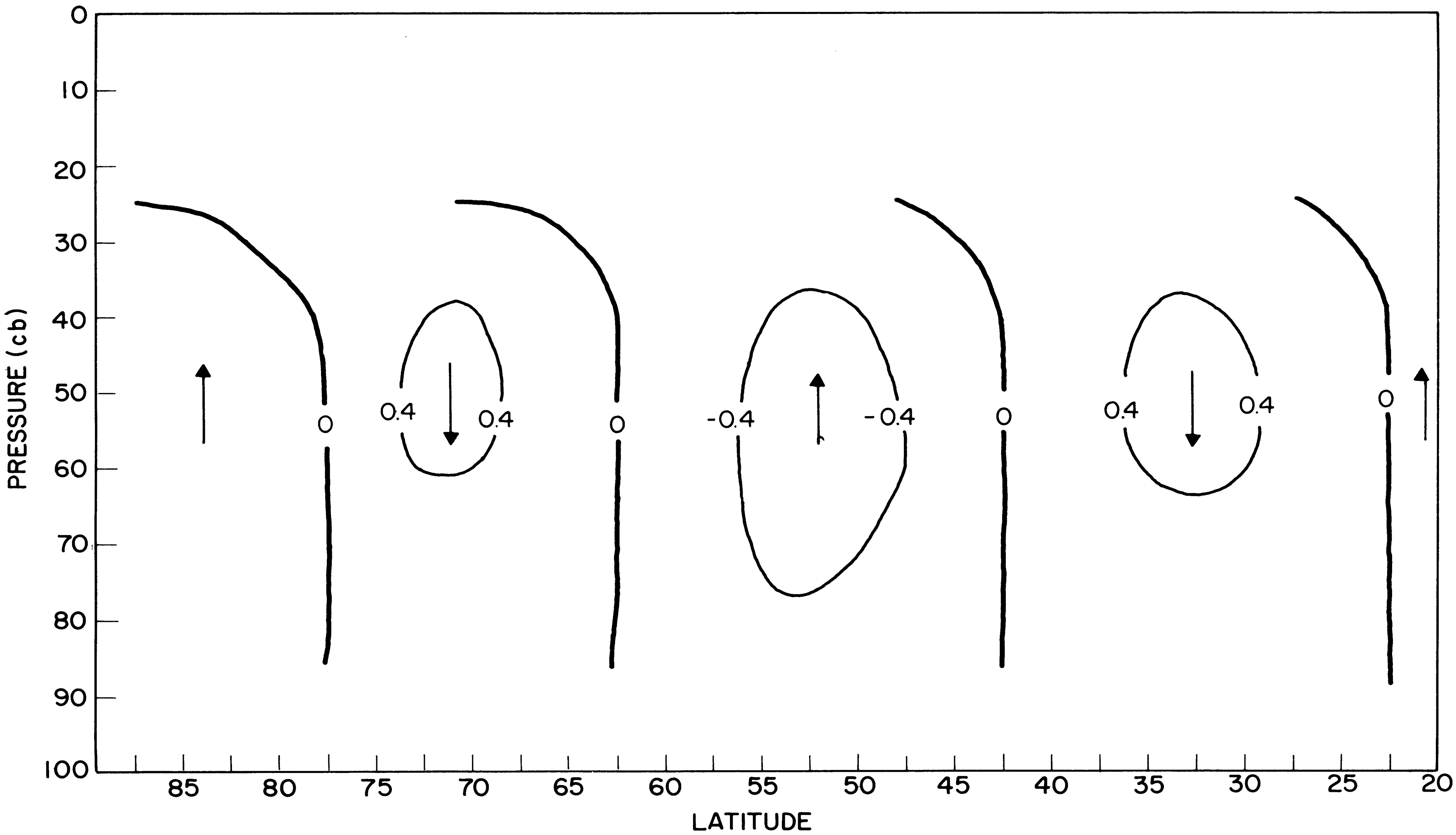

Figure 13. The zonally averaged vertical velocity, ω_z', produced by the short waves for the month of January 1963. Arrangement and units as in figure 2.

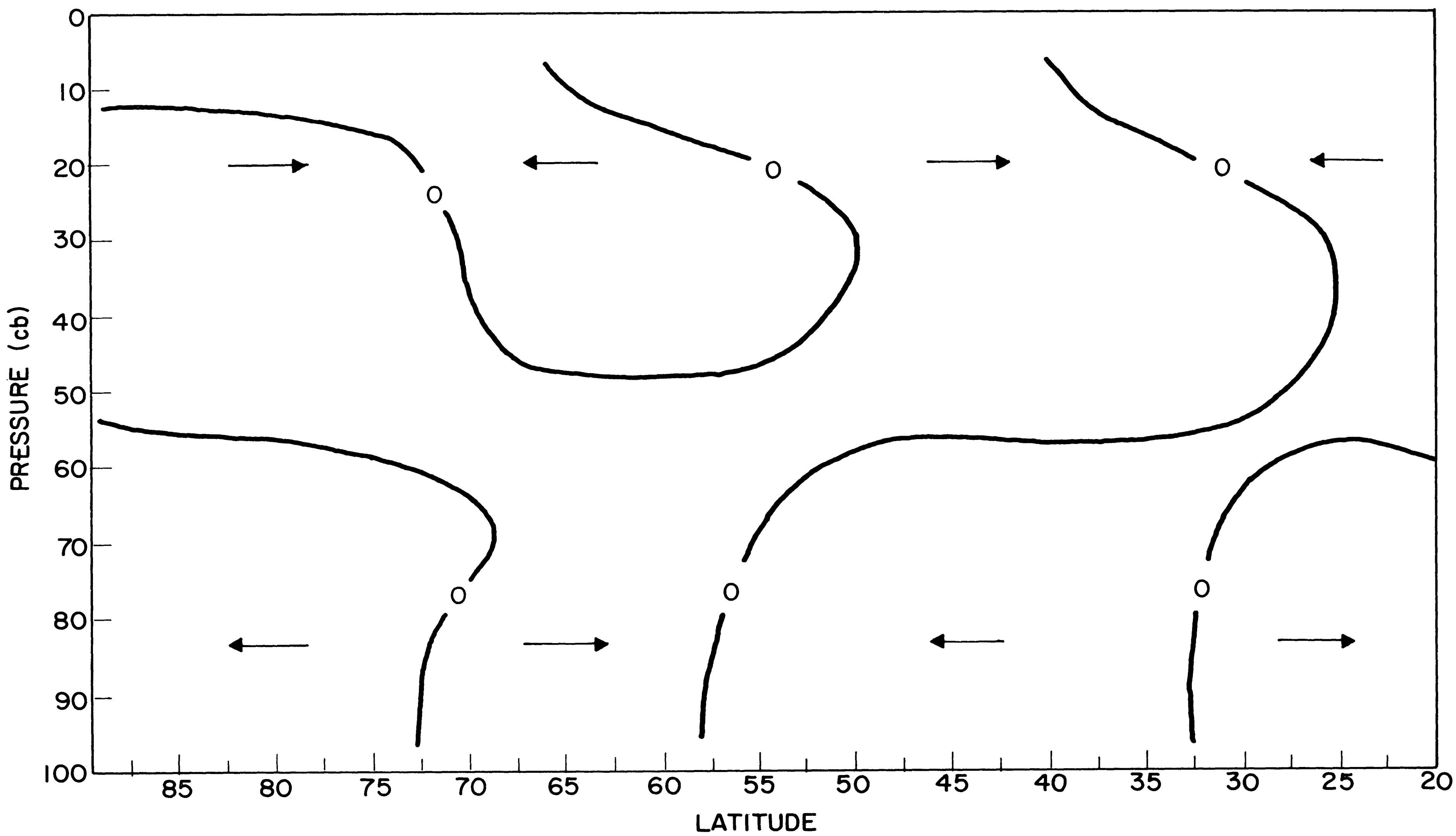

Figure 14. The zonally averaged meridional velocity, v_z', produced by the short waves for the month of January 1963. Arrangement and units as in figure 3.

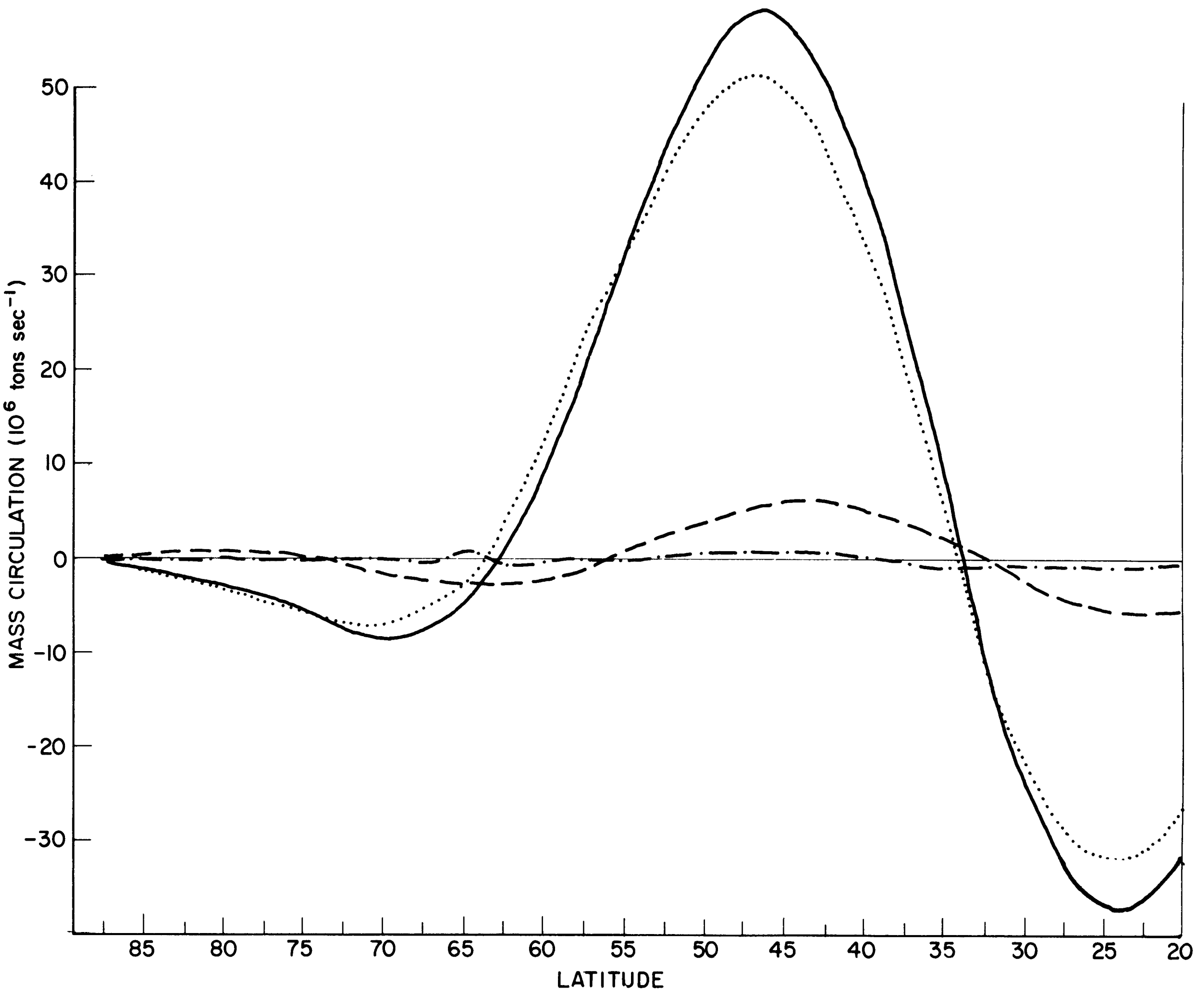

Figure 15. The mass circulation in the lower troposphere for the month of January 1963. Solid curve: The mass circulation produced by all scales of motion. Dotted curve: The mass circulation produced by the planetary waves. Dashed curve: The mass circulation produced by the medium waves. Dash-dotted curve: The mass circulation produced by the short waves. Arrangement and units as in figure 8.

mass circulation, while the medium waves explain 12% and the remaining 2% are due to the short waves.

3.4 SEASONAL VARIATIONS OF MEAN MERIDIONAL CIRCULATIONS

The investigation was carried out for four months, January, April, July and October of 1962, to represent the four seasons of the year. These computations were arranged in the same manner as for January 1963, to obtain the separate effects of $f((uv)_z)$ and $f((Tv)_z)$, and also to find meridional circulations in the wave number regime. It is unnecessary to present all these results, but certain interesting points may be summarized as follows:

(i) The conclusions drawn, in section 3.2, about the relative importance of $f((uv)_z)$ and $f((Tv)_z)$ for the month of January 1963, hold good irrespective of the season in the year 1962.

(ii) In January 1962 the ratio of the contribution by long, medium and short wave groups, to the total circulation was 0.55, 0.31 and 0.14, respectively, as compared to 0.86, 0.12, 0.02 in the month of January 1963. The extreme dominance of the very long waves was characteristic of January 1963 (Wiin-Nielsen, Brown and Drake (1964)).

(iii) For April, July and October 1962 the meridional circulations for different scales of motion had, by and large, the same proportion but different from that of January 1962 or 1963. Here the long waves explained only 32% of the total circulation, whereas the medium waves explained 57% and the remaining 11% are accounted for by the short waves.

The medium waves represent in general the baroclinically unstable waves in the atmosphere. It is satisfactorily established by empirical studies (Wiin-Nielsen, Brown and Drake (1963) and (1964)) that these scales dominate the eddy transfer processes in the spring, summer and fall seasons. The above results are thus in agreement with earlier studies.

For the discussion of the seasonal variation of the mean meridional circulation we shall consider the circulation forced by the total forcing function and summed over all scales of motion.

Figures (16), (18), (20) and (22) show the distribution of the vertical velocity, as a function of latitude and pressure, for the months of January, April, July and October 1962, respectively. Similarly the figures (17), (19), (21) and (23) illustrate the corresponding distribution of the meridional velocity for the respective months. Since the major features of the meridional circulations are the same for all the representative months of the seasons, we shall avoid the repetitious description of these figures, and discuss only certain points regarding the seasonal variation of the circulation.

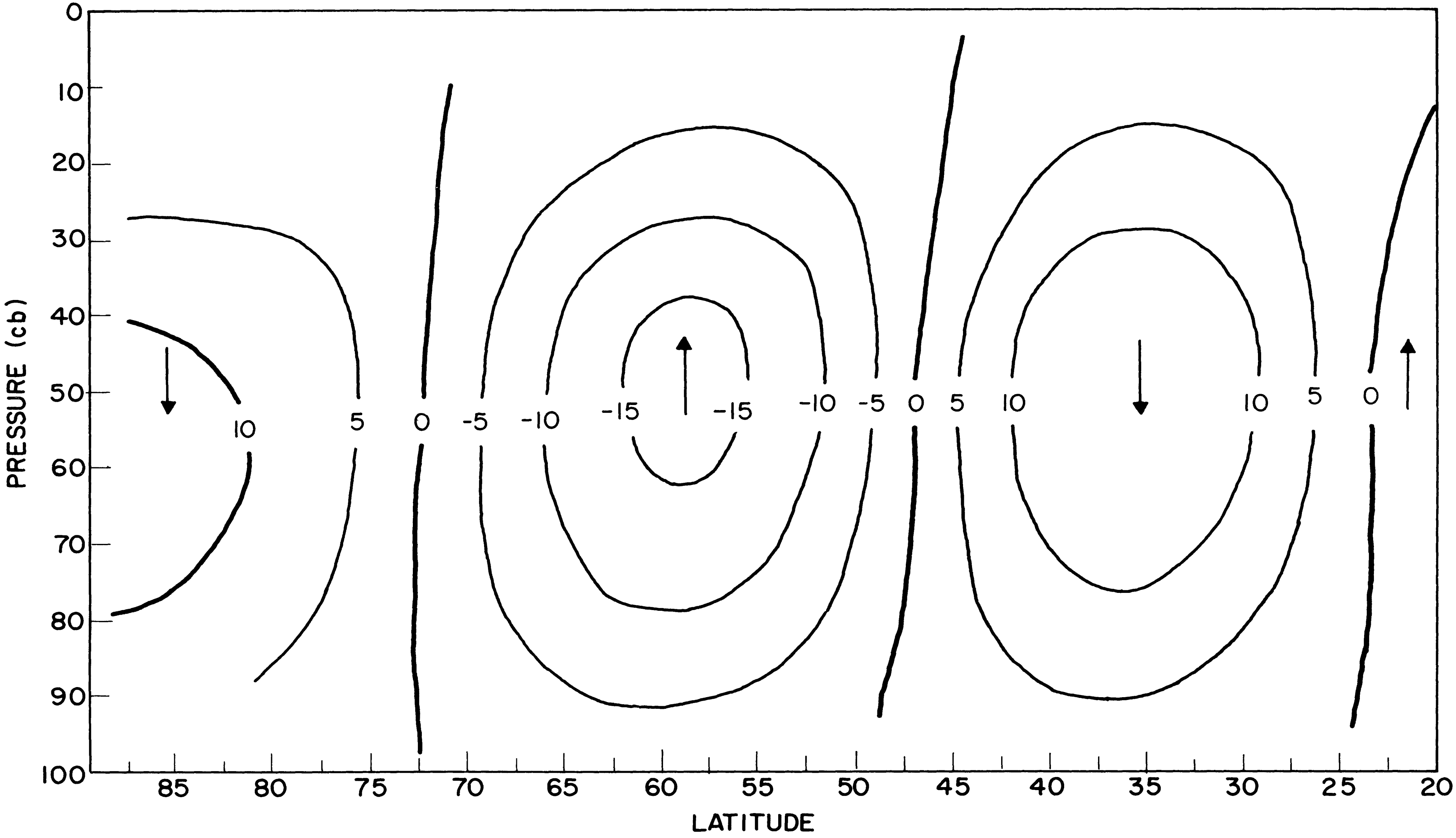

Figure 16. The zonally averaged vertical velocity, ω_z, for the month of January 1962. Arrangement and units as in figure 2.

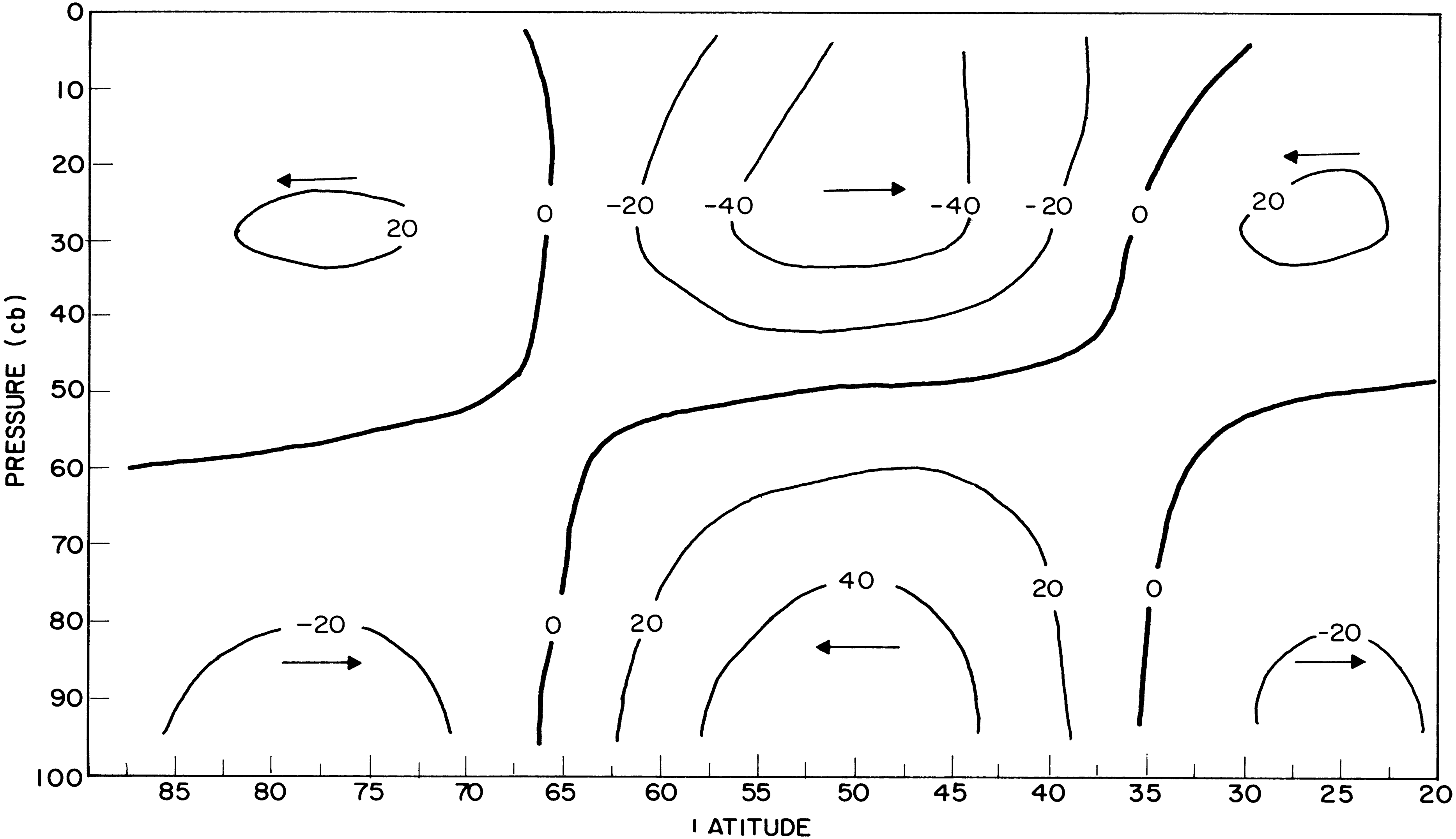

Figure 17. The zonally averaged meridional velocity, v_z, for the month of January 1962. Arrangement and units as in figure 3.

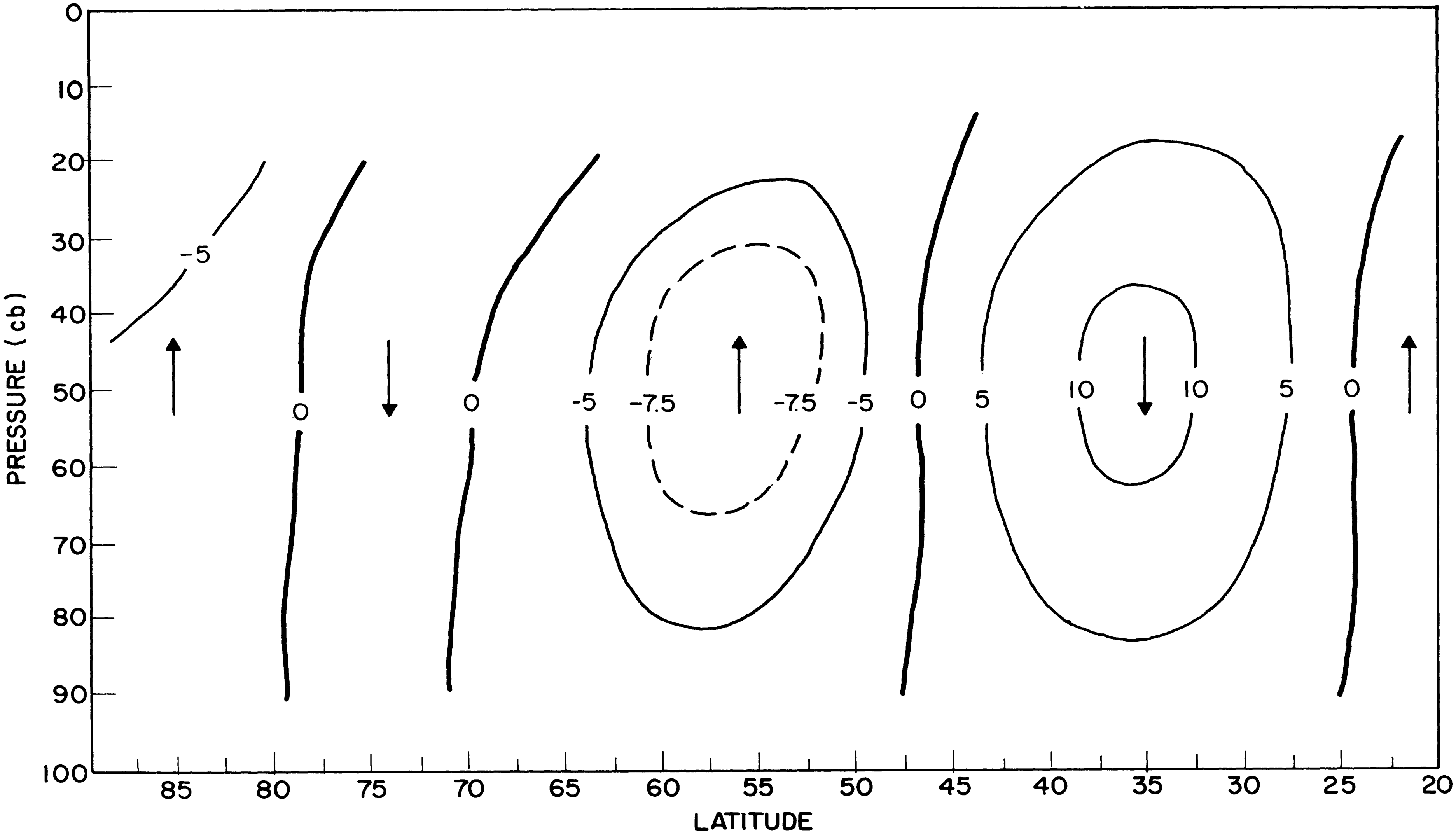

Figure 18. The zonally averaged vertical velocity, ω_z, for the month of April 1962. Arrangement and units as in figure 2.

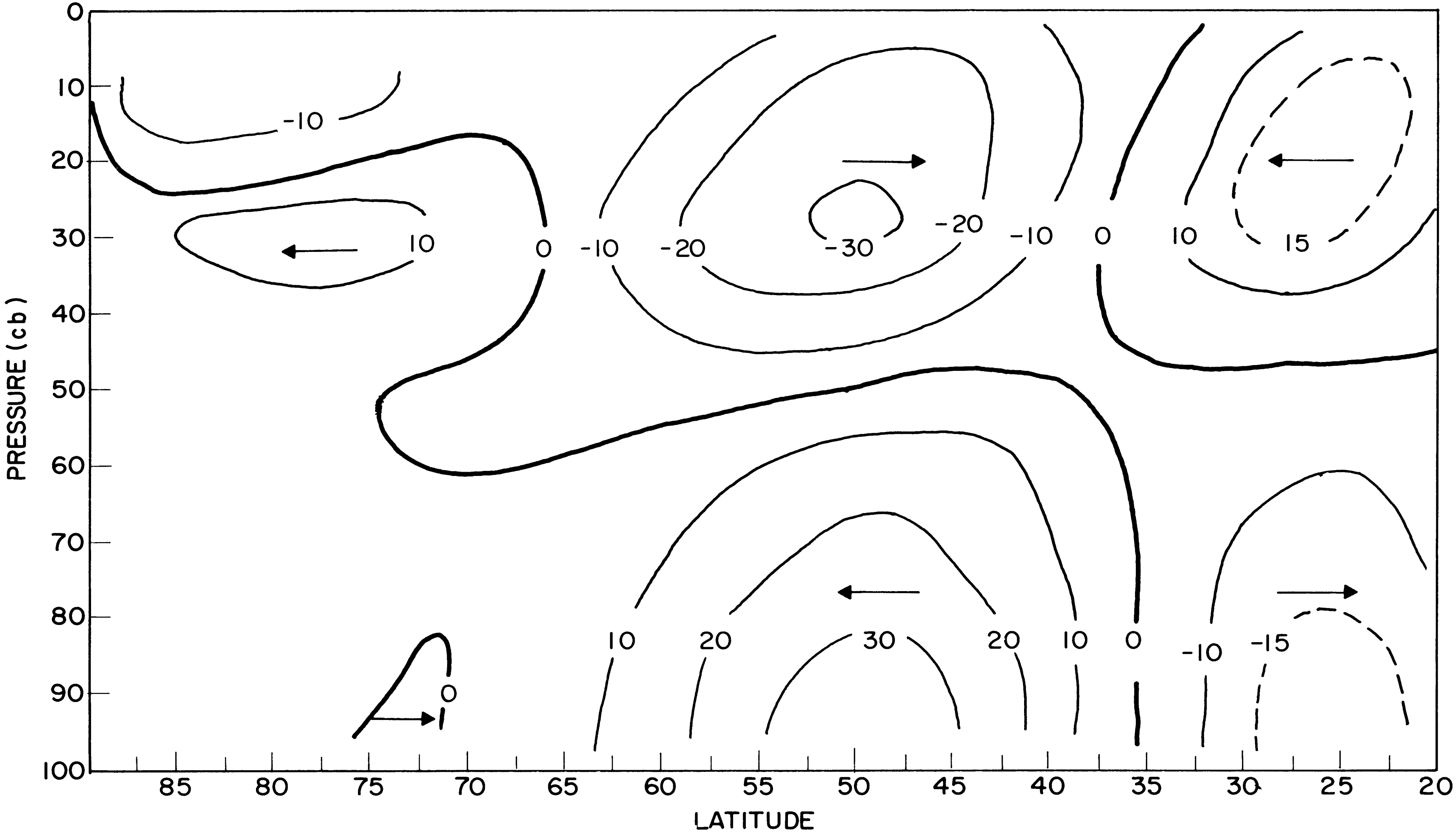

Figure 19. The zonally averaged meridional velocity, v_z, for the month of April 1962. Arrangement and units as in figure 3.

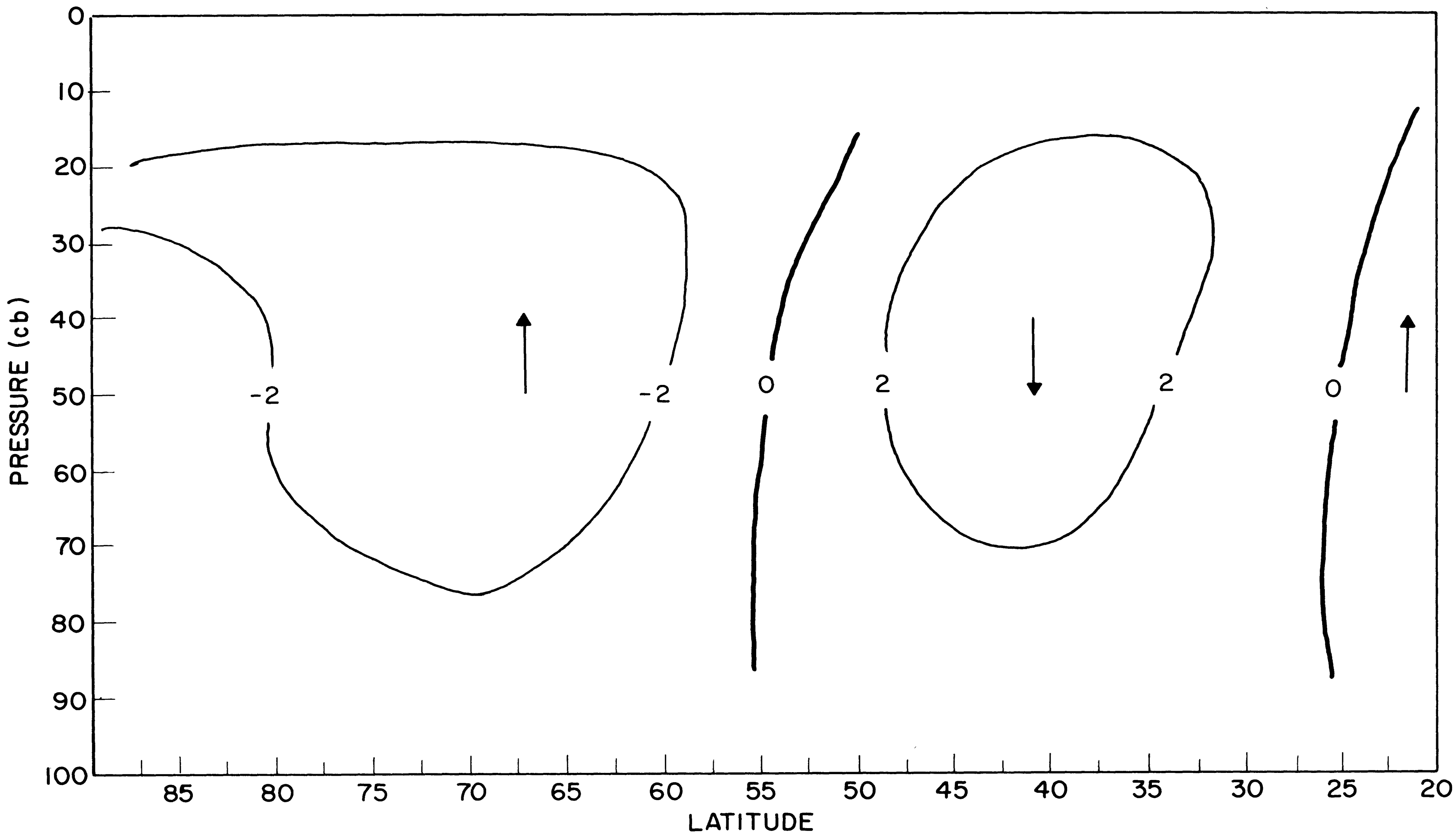

Figure 20. The zonally averaged vertical velocity, ω_z, for the month of July 1962. Arrangement and units as in figure 2.

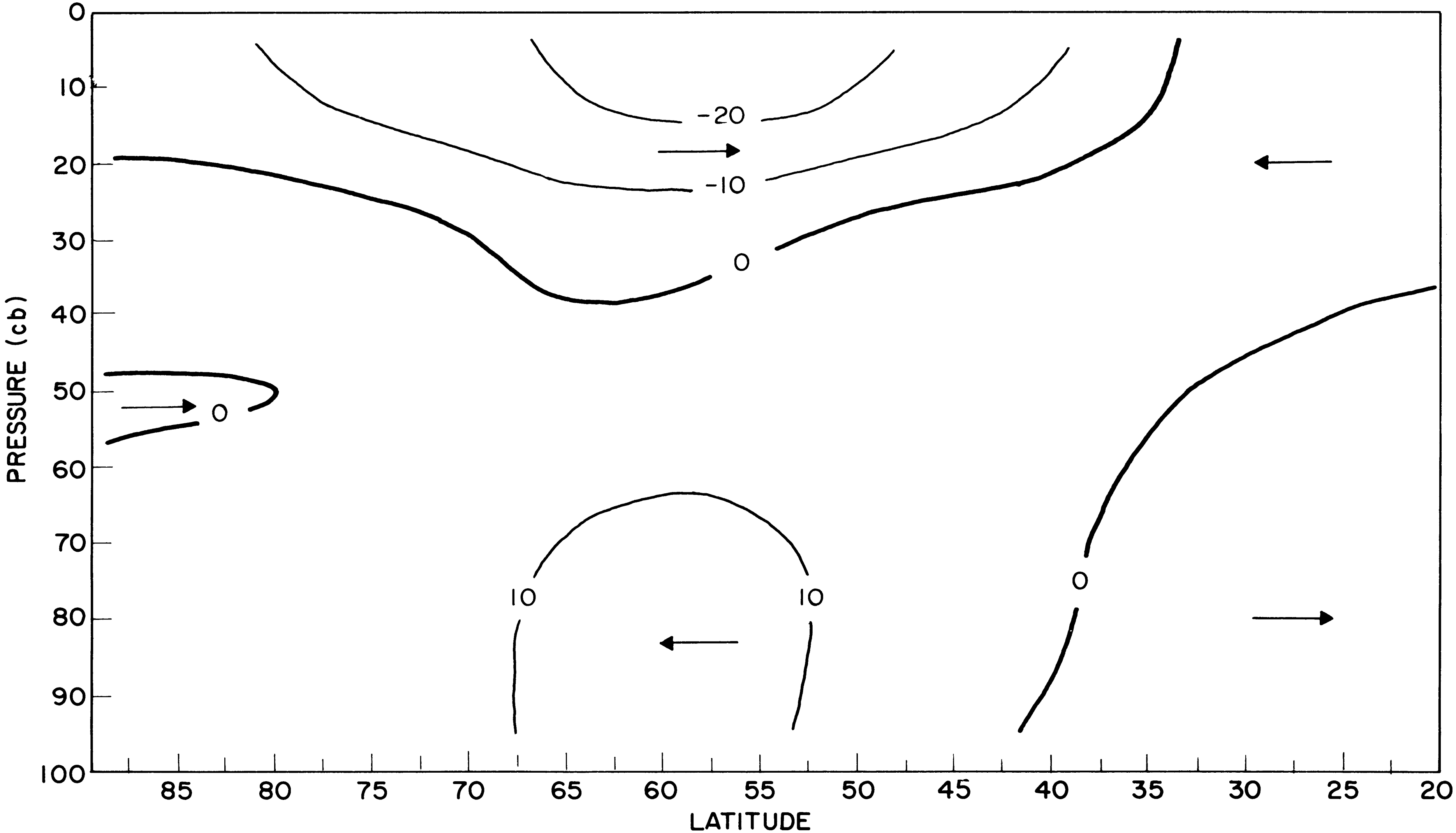

Figure 21. The zonally averaged meridional velocity, v_z, for the month of July 1962. Arrangement and units as in figure 3.

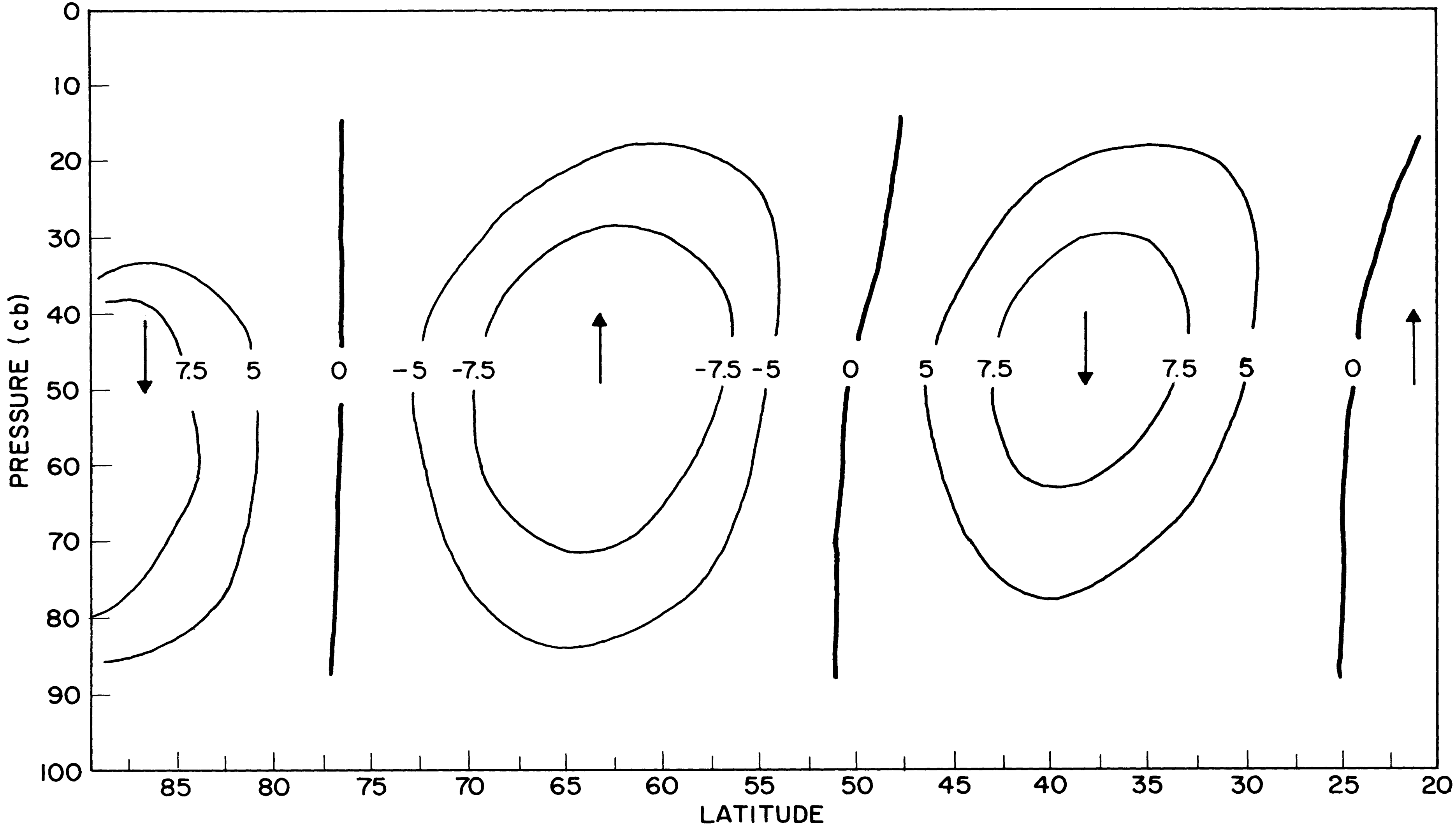

Figure 22. The zonally averaged vertical velocity, ω_z, for the month of October 1962. Arrangement and units as in figure 2.

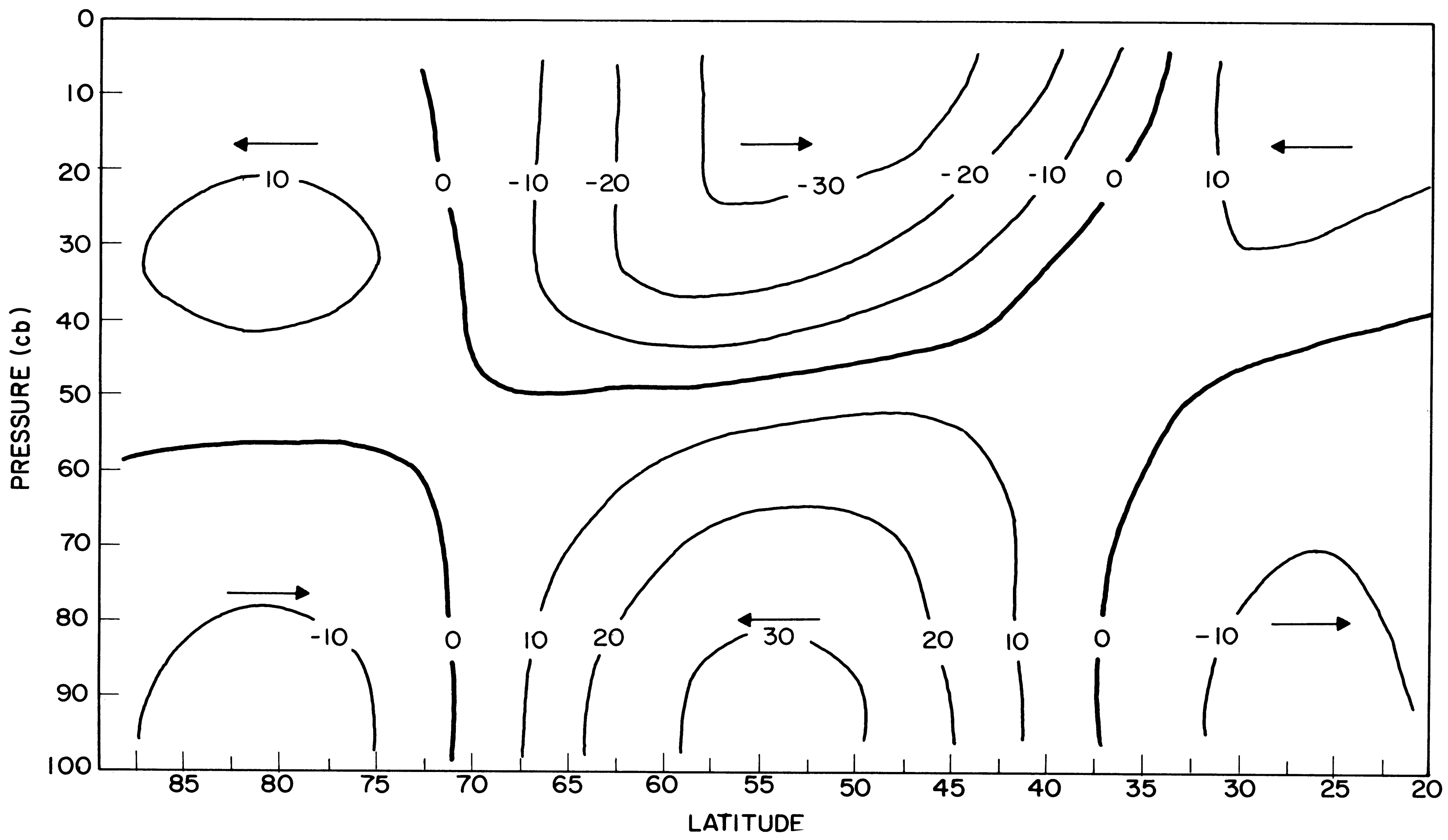

Figure 23. The zonally averaged meridional velocity, v_z, for the month of October 1962. Arrangement and units as in figure 3.

(i) The three-cell mean meridional circulations prevail in all the seasons, except possibly during summer.

(ii) There seems to be a trend to move these cells northward from winter to spring and to reach their extreme northern position in the summer, when the polar direct cell has almost disappeared. From summer to fall these cells move southwards regaining their extreme southern position in winter, completing an annual cycle of oscillation.

(iii) The intensity of the circulation gradually decreases from winter to the spring and attains its minimum in summer. The intensity starts to increase from summer to fall and reaches its maximum in winter. The mass circulations across the latitudes for January, April, July and October 1962 are shown in figure (24). Comparing the absolute values of the mass circulations, we find that the circulations in January are three times as intense as those in July. Further that the intensity of the circulations in April is almost the same as in October, but it is twice as much as that in July.

3.5 THE EFFECT OF THE LOWER BOUNDARY CONDITIONS ON MEAN MERIDIONAL CIRCULATIONS

So far we have discussed the results of mean meridional circulation calculations based on an adiabatic and frictionless model with the simplified boundary conditions, that the

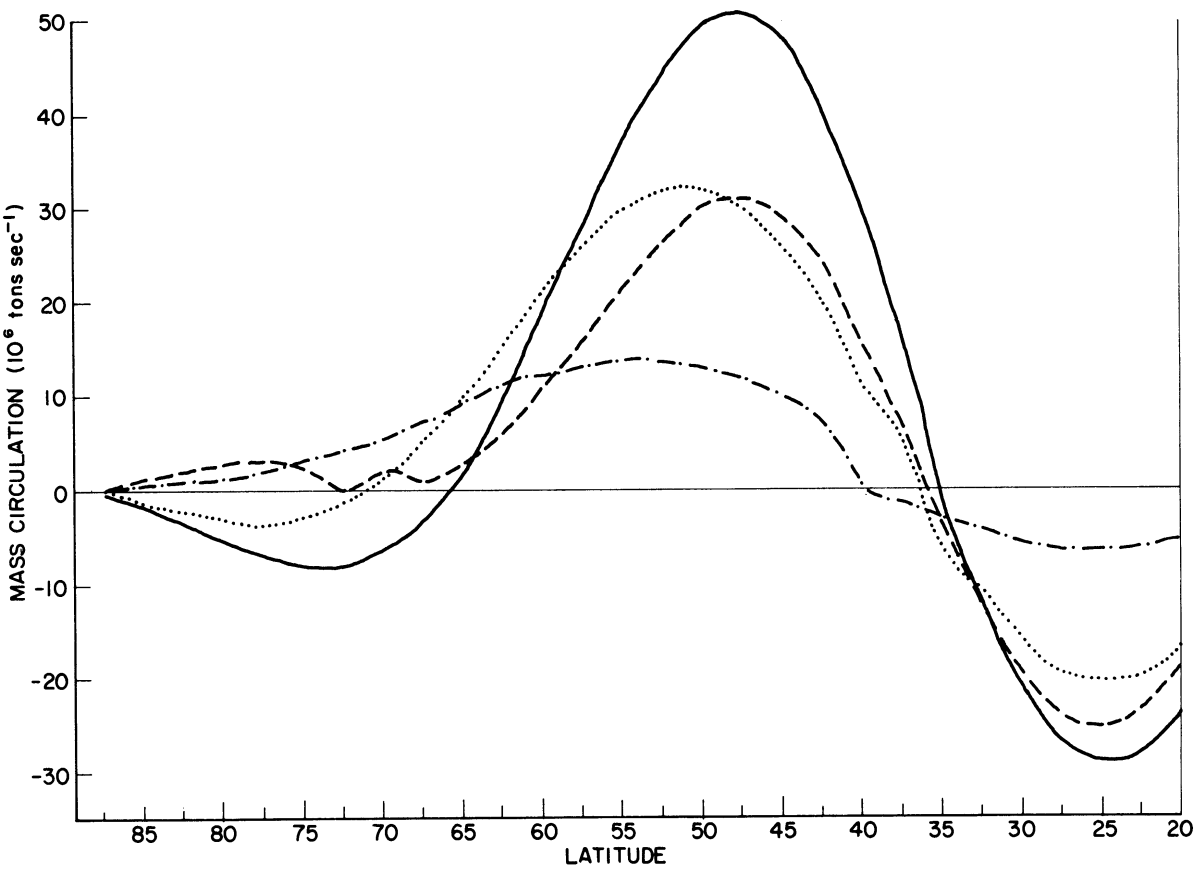

Figure 24. Solid curve: The mass circulation for the month of January 1962. Dashed curve: The mass circulation for the month of April 1962. Dash-dotted curve: The mass circulation for the month of July 1962. Dashed curve: The mass circulation for the month of October 1962. Arrangement and units as in figure 8.

vertical velocity, ω_z, is zero at the lower and at the upper boundary of the atmosphere. It is clear from section 2.1 that a realistic model should include the diabatic heating, the viscous forces and the effect of topography and friction at the lower boundary. In addition one should also have a greater vertical resolution in the data since the present method of solution uses finite differencing in the vertical. However, the earlier discussions (section 2.2) pointed out that it is not possible to incorporate the diabatic heating and viscous forces in a realistic manner, but the effects of topography and friction at the lower boundary can be computed from equation (2.13). Further we shall recall here that a greater vertical resolution in data was available in January 1964.

It will be of interest to compute the vertical velocity at the top of the frictional layer which is the combined effect of the topography and the surface friction, even if we are going to include only the effect of topography in the lower boundary condition.

The vertical velocity due to the topography, ω_{mz}, evaluated from equation (2.12), using the surface geostrophic winds for January 1964, is shown in figure (25a) as a function of latitude. The units are: 10^{-5} mb sec^{-1}, hence the negative values indicate

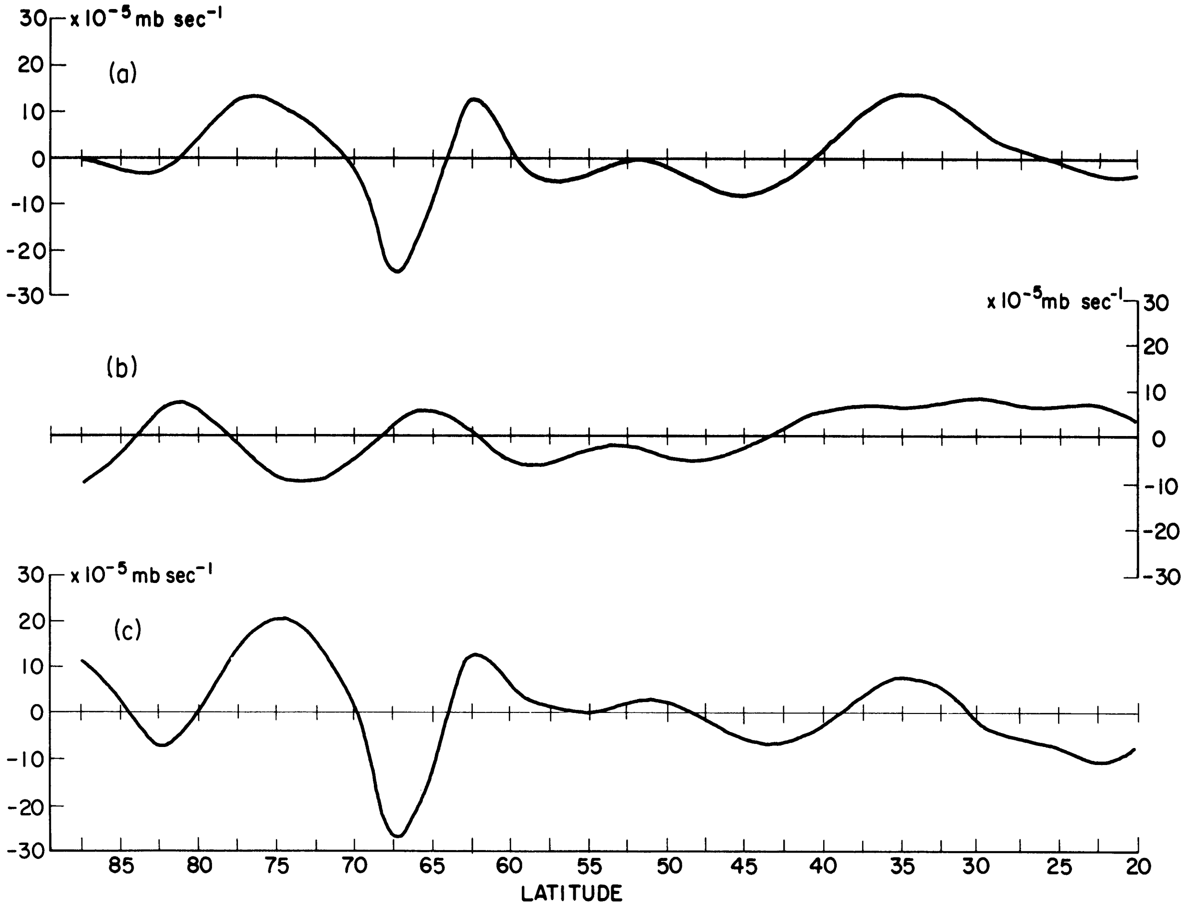

Figure 25. (a) The zonally averaged vertical velocity produced by the topography of the earth, (b) The zonally averaged vertical velocity produced by the skin friction, (c) The zonally averaged vertical velocity at the top of the frictional layer, as a function of latitude for the month of January 1964 in the unit 10^{-5}mb sec^{-1}.

a rising motion while the positive values indicate sinking motion. Such estimates, to the knowledge of the writer, have not been made in the past. A comparison is therefore not possible and hence a further discussion of these results is necessary.

Any physical description of these results requires the knowledge of the distribution of the meridional component and the topography over the region under consideration. It will therefore be interesting to consider the mean distribution of the meridional component which would force the air over an obstacle to produce a vertical motion. Tucker (1957) computed the time-mean distribution of the meridional component near the ground for the winter months, over the Northern Hemisphere. We shall now consider these results together with the distribution of the topography to obtain the qualitative distribution of the large scale vertical velocity. There is a net northerly flow in the latitude belt 20-25°N. Such a flow faces upslope mountains over the African continent and the Indian peninsula to produce an upward vertical motion, but there is a northerly flow down the slopes of the mountains over Burma to produce descending motion. Since the region of the ascending motion is much larger than the region of descending motion the zonal average will be an ascending motion. This agrees with our results in figure (25a). In the latitude belt 25-40°N there is a northerly flow down the slopes of Southern Rockies and a southerly flow down the slopes of Himalayas resulting in a net sinking motion. This explains the sinking motion in that latitude belt in figure (25a). In the latitude belt 40-60°N there is a southerly flow up the slopes of Northern Rockies and northerly flow up the slopes of Alps to produce an ascending motion, but there are regions in the northern Europe and Russia having a southerly flow down the slopes of mountains. The integrated effect around the latitude circle can still result as ascending motion. Such a feature is also seen in the figure. The results north of 60°N may be difficult to justify, especially their magnitudes, because the meridional component is very irregular in this region and hence the term $(p_o v_o)_z$ in equation (2.12) becomes difficult to evaluate. Further the finite differencing exaggerates the errors.

The second term on the right hand side of equation (2.13) (without the sign) gives the effect of skin friction on the vertical motion, ω_{frz}. The computed results of this term are shown in figure (25b) as a function of latitude, in the units 10^{-5} mb sec^{-1}. We notice a sinking motion in the latitude belt 20-40°N, a region of strong frictional divergence, while an ascending motion is found in the latitude belt 50-60°N, a region of frictional convergence. Thus these results are in qualitative agreement with considerations based on conservation of mass. Again the results north of 60° latitude are affected by the ill defined distribution of zonal wind.

The vertical velocity, ω_{fz}, at the top of the frictional layer computed from equation (2.13), is shown in the figure (25c) as a function of latitude, in the units: 10^{-5} mb sec^{-1}. The distribution of ω_{fz} is very similar to that of ω_{mz}, indicating a dominating influence of the mountains at the top of the frictional layer. Except in the polar regions, ω_{fz} is, by and large, in phase with the meridional circulations computed from an adiabatic and frictionless model.

We shall now discuss the mean meridional circulation for January 1964 with the lower boundary condition $\omega_z = \omega_{mz}$ and upper boundary condition $\omega_z = 0$. It may be mentioned here that although in figure (25a) ω_{mz} is presented from 20°N to 87.5°N it was also computed over the region equator to 20°N from the extrapolated data for the meridional component. The value at the pole was set to zero. Hence it is possible to express ω_{mz} in a series of Legendre polynomials to determine $A_n(1)$. The forcing function $M(\mu, p_*)$ was represented by the first ten polynomials. It was therefore necessary to express ω_{mz} with the same number of polynomials. This helped to smooth out the errors in ω_{mz} due to the finite differencing.

The vertical velocity, ω_z, as a function of latitude and pressure for January 1964, in units 10^{-5} mb sec^{-1}, is shown in figure (26). The interesting feature of this figure is that the level of maximum vertical velocity is much closer to the observed level of non-divergence in the atmosphere (600 mb) as compared to that of January 1962 or 1963. Here the stratospheric circulations in the polar region are clearly seen.

The meridional velocity, v_z, as a function of latitude and pressure for January 1964 in the units cm sec^{-1} is shown in figure (27). The tropospheric circulations have a pattern similar to those found in January 1962 or 1963. In the polar stratosphere a complete circulation of an indirect cell is seen. There is also an indication of a stratospheric reverse circulation in the subtropical region.

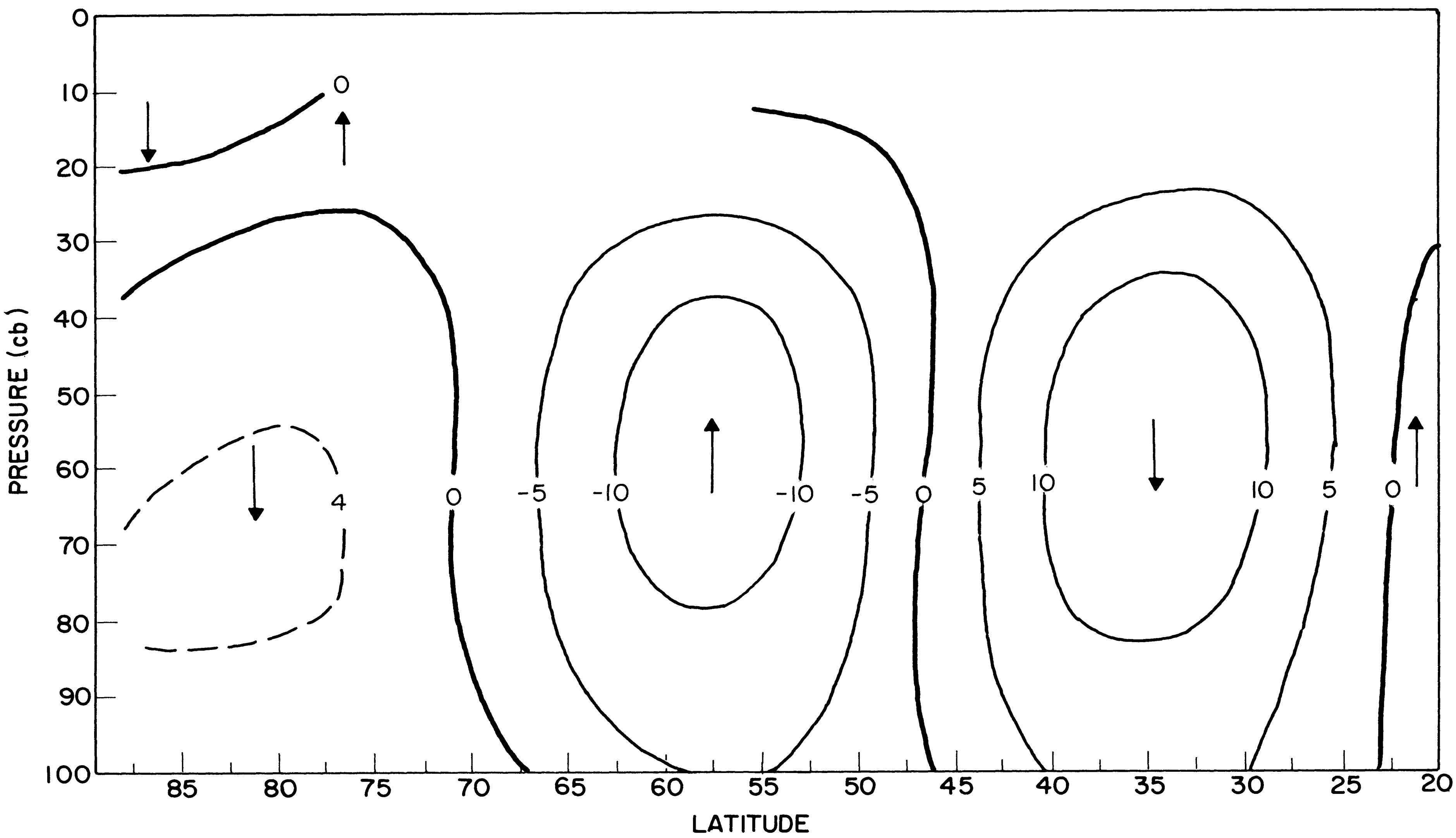

Figure 26. The zonally averaged vertical velocity, ω_z, for the month of January 1964. Arrangement and units as in figure 2.

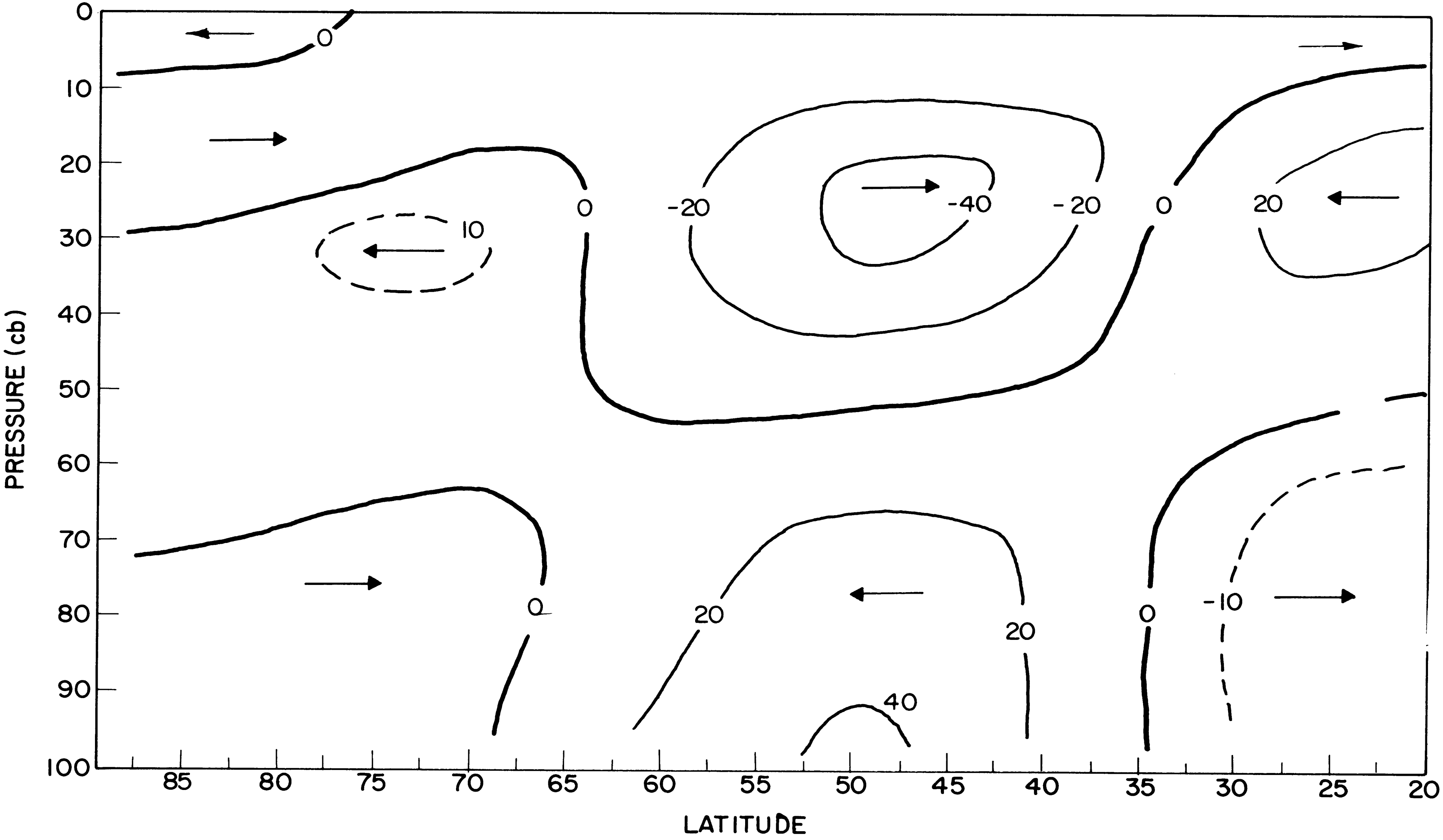

Figure 27. The zonally averaged meridional velocity, v_z, for the month of January 1964. Arrangement and units as in figure 3.

4. ENERGETICS OF THE ZONAL FLOW

4.1 PRELIMINARY REMARKS

Several investigations have been made in recent years of the energy cycle of the atmosphere. Lorenz (1955) introduced the subdivision of both available potential and kinetic energies into their zonal and eddy (deviation from zonal) contributions. Besides the energy levels of these quantities, the conversions between the zonal and eddy available potential energy, $C(A_z, A_e)$, and between the eddy and zonal kinetic energy, $C(K_e, K_z)$, can be computed from routine observed data. Saltzman and Fleisher (1960) computed $C(K_e, K_z)$ from height data for 500 mb. Wiin-Nielsen, Brown and Drake (1963, 1964) made an extensive study of both $C(A_z, A_e)$ and $C(K_e, K_z)$ for different scales of motion and described also their vertical distribution. Krueger, Winston and Haines (1965) made a study of $C(A_z, A_e)$ over a 5 year period. A comparison between these investigations shows good agreement on the magnitude and direction of these conversions.

The conversion between the zonal available potential energy and the zonal kinetic energy, $C(A_z, K_z)$, and the conversion between the eddy available potential energy and the eddy kinetic energy, $C(A_e, K_e)$ depend on the distribution of vertical motions in the atmosphere. Since the vertical velocities are impossible to observe for the large scale motion, it is necessary to compute them indirectly. The National Meteorological Center obtains the vertical velocities as a by-product of baroclinic numerical prediction model on the routine basis. Wiin-Nielsen (1959c), Saltzman and Fleisher (1961) and Krueger, Winston and Haines (1965) computed $C(A_z, K_z)$ and $C(A_e, K_e)$ for a layer 850-500 mb using these vertical velocities at 600 mb. These investigations on $C(A_z, K_z)$ and $C(A_e, K_e)$ agree with each other to some extent. Jensen (1961) computed the vertical velocities by a so-called adiabatic method to describe the vertical distribution of $C(A_e, K_e)$. Apparently these results are in error as pointed out by Wiin-Nielsen (1964). Since the results of these calculations depend entirely on the method of computing the vertical velocities, it is desirable to pursue further studies on the conversions. A discussion of the vertical variation of $C(A_z, K_z)$ is presented in section 4.4.

The generation of the zonal available potential energy, $G(A_z)$, and the eddy available potential energy, $G(A_e)$ depend on the distribution of diabatic heating in the atmosphere. Wiin-Nielsen and Brown (1960) were the first to obtain estimates of $G(A_z)$ and $G(A_e)$ computing them for a layer of 800-600 mb. Brown (1964) extended the study to four seasons of the year 1961 with an improvement in the lower boundary condition.

We have made an attempt here to compute a diabatic heating in the meridional plane and hence to obtain a vertical distribution of $G(A_z)$. The results are described in the following section.

4.2 DIABATIC HEATING

The dominant heating factors in the atmosphere are radiation, latent heat due to the phase change of water and the turbulent exchange of heat between the earth and the atmosphere. The routine observed data in the atmosphere do not permit a direct calculation of the diabatic heating by the above processes. But one should be able to estimate it indirectly from the physical laws governing the atmosphere. Wiin-Nielsen and Brown (1960) used a closed system of equations, the thermodynamic equation and the vorticity equation to solve for two unknowns: the vertical velocity and the diabatic heating, the other terms involved in the equations were computed from the observed data. Our attempt to compute the diabatic heating in the meridional plane is based on a similar procedure.

The thermodynamic equation in the mean state, in the same notation as before, is

$$\vec{v}\cdot\nabla\left(\frac{\partial\Psi}{\partial p}\right) + \frac{\bar{\sigma}\omega}{f_o} = -\frac{R}{C_p p f_o} H \qquad \ldots(4.1)$$

Taking the zonal average and rearranging the terms equation (4.1) becomes (see Appendix A)

$$H_z = \frac{gp \ln(\frac{p_2}{p_1})}{2\pi a^2 \cos\varphi\ (\Delta p)^2} \frac{\partial}{\partial\varphi}(TH_{\Delta p}(\varphi)) - \frac{C_p p\bar{\sigma}\omega_z}{R} \quad \ldots(4.2)$$

We have already discussed in section (2.2) that the vertical velocity, ω_z, obtained by solving equation (2.19) cannot be used to estimate the diabatic heating term from the steady state thermodynamic equation. It was therefore necessary to compute ω_z by some other indirect method. ω_z was computed by numerically integrating the steady state zonally averaged vorticity equation from the top (where $\omega_z = 0$) to the bottom of the atmosphere. These results show an unusually large ω_z at the lower levels due to the truncation errors involved in finite differencing. We have therefore used ω_z computed from equation (2.19) forced only by $f((uv)_z)$. It may be noted here that this method of computing ω_z is not the same as that used by Wiin-Nielsen and Brown (1960). The static stability $\bar{\sigma}$ was assumed, as before, to be a function of pressure alone. Knowing $TH_{\Delta p}(\varphi)$ and ω_z, H_z was computed for the layers: 850-700, 700-500, 500-300, 200-100 mb for January 1963.

Before going into a discussion of the final results, it may be worth while to compare the preliminary results for a layer with those obtained by Brown (1964). Brown computed the diabatic heating on a daily basis, to include the effect of standing and transient motion, and then averaged over the whole month for a layer 800-400 mb. Our results include only the effect of standing motion for the layer 700-500 mb. A comparison of $H_z(\varphi)$ for April, July, October 1962 and January 1963, in the units $10^{-2} kjt^{-1} sec^{-1}$, is shown in figure (28). The continuous curve shows our results while Brown's results are shown by a broken curve. Considering the difference in method of computations and the layer which they represent there is a striking similarity between the two sets of curves. This supports the validity of the results for diabatic heating in the meridional plane for January 1963 shown in figure (29). $H_z(\varphi, p)$ is given in the units $10^{-2} kjt^{-1} sec^{-1}$. The maximum heating occurs in the latitude belt 30-40°N in the lower troposphere where the effect of the turbulent transfer of heat from the earth's surface and the latent heat of phase change of water can be the dominant factors. A gradual decrease in the upper troposphere is due to the influence of radiational cooling. North of 40°N the net effect is a cooling. In the higher latitudes there are two distinct minima, in the lower troposphere. One around 80°N can be due to the ice cover over land and sea if the turbulent transfer of heat from the earth is the dominant factor. Another minimum around 60°N as compared to the belt near 70°N may be difficult to explain physically. Nevertheless,

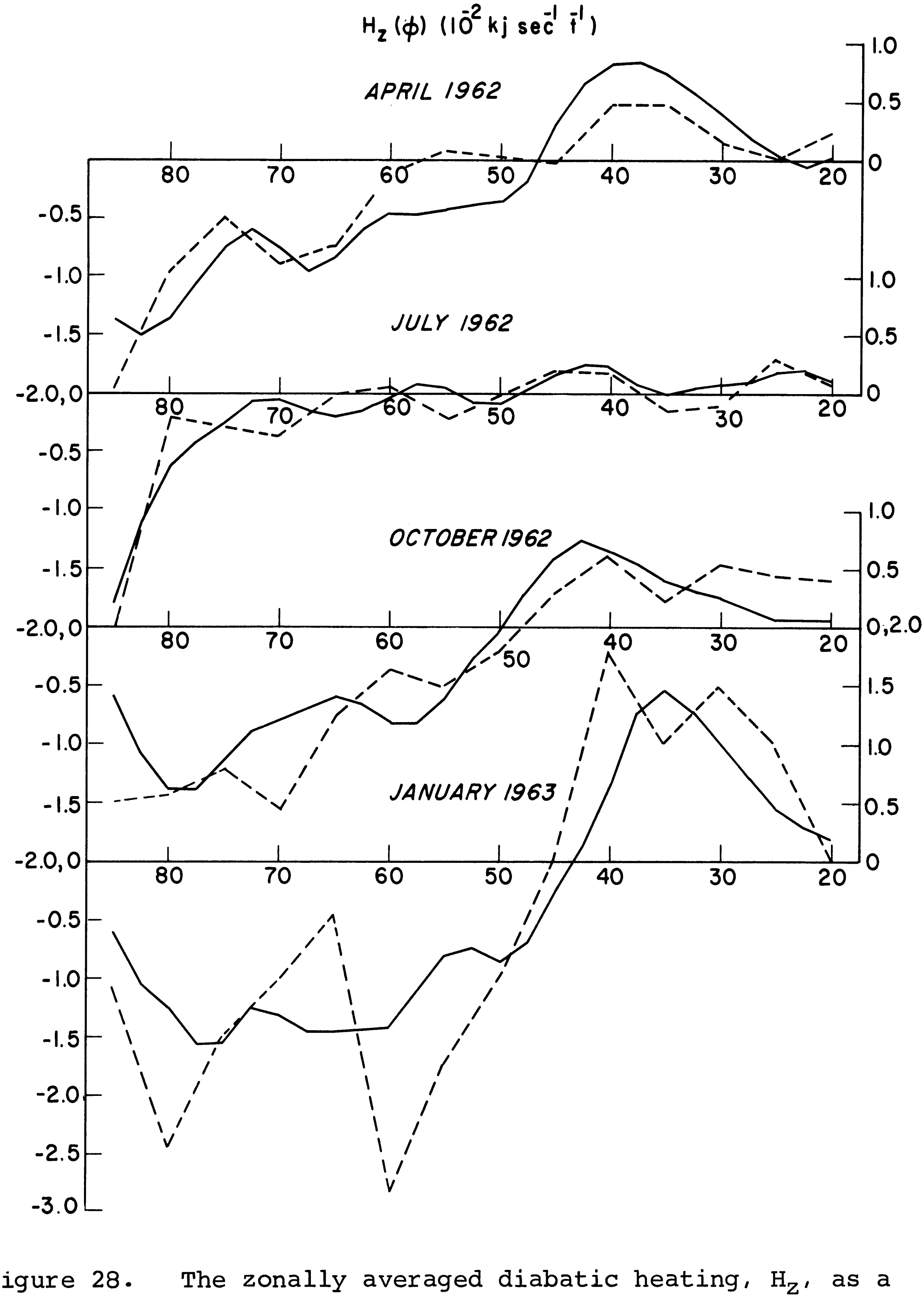

Figure 28. The zonally averaged diabatic heating, H_z, as a function of latitude for the months of April, July, October 1962 and January 1963 in the unit 10^{-2}kj sec^{-1}t^{-1}. Solid curves: H_z for the layer: 700-500 mb computed in the present study. Dashed curve: H_z for the layer: 800-400 mb according to Brown.

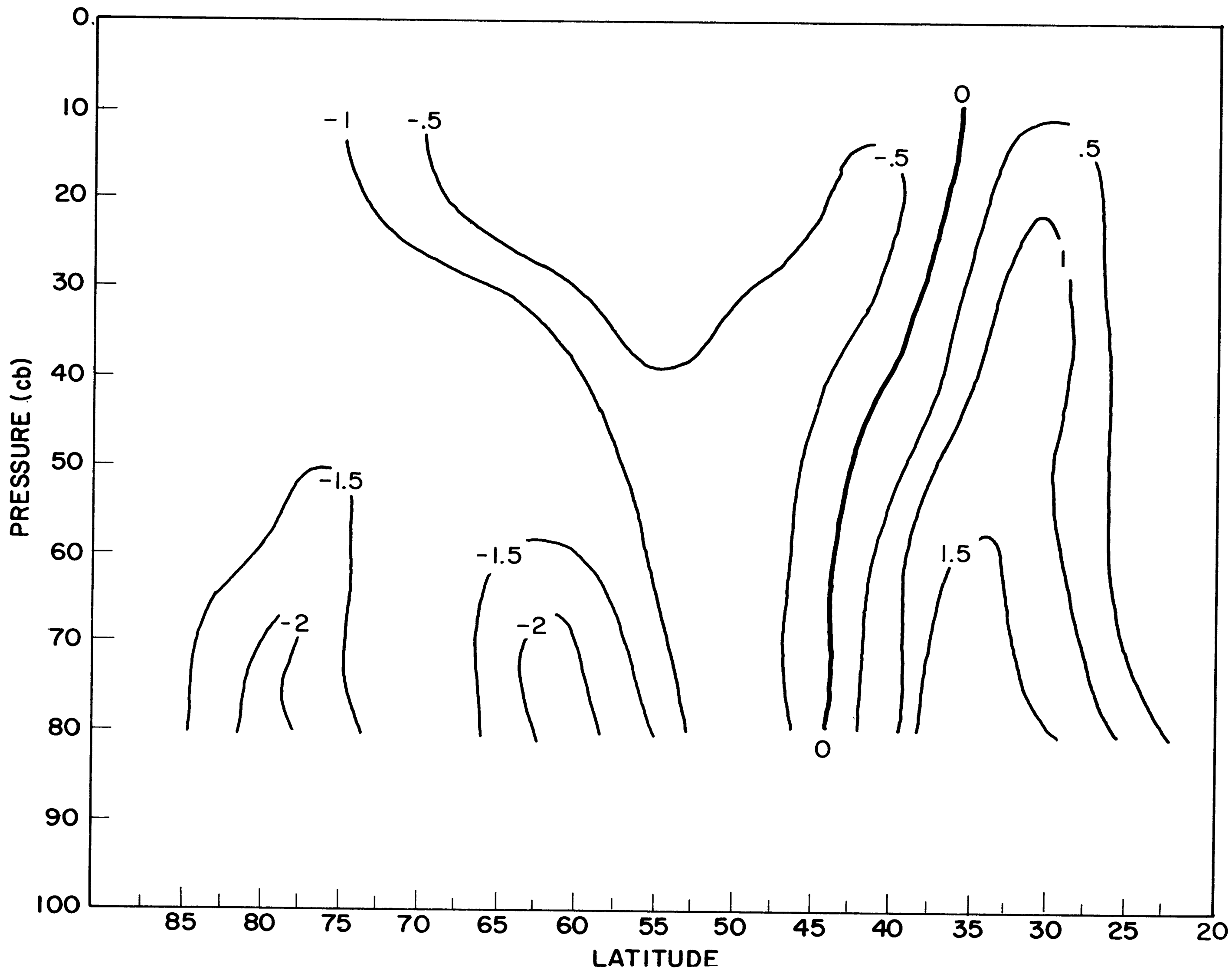

Figure 29. The zonally averaged diabatic heating as function of latitude and pressure for the month of January 1963 in the unit 10^{-2}kj $sec^{-1}t^{-1}$.

it should be mentioned that an Atlas of the heat balance of the earth by Budyko (1963) did show such a feature around 60°N and 70°N. In the higher troposphere the effect of the turbulent transfer of heat diminishes so that the cooling is only due to radiation.

4.3 THE GENERATION OF ZONAL AVAILABLE POTENTIAL ENERGY

The concept of available potential energy was originally introduced by Margules (1903) and its mathematical definition was given by Lorenz (1955). The available potential energy is a measure of the amount of energy available for conversion into kinetic energy; it can be defined as the excess of total potential energy (potential and internal) above the amount which would exist if the isentropic surfaces were horizontal. The generation of available potential energy depends upon the mass integral of the product of the deviations of diabatic heating and temperature from the area average of these quantities. This definition of the generation of available potential energy, G(A), can be expressed symbolically as:

$$G(A) = \frac{R}{gC_p} \int_0^{p_o} \int_S \frac{1}{\bar{\sigma}} \frac{1}{p} \alpha' H' dS \, dp \qquad \ldots(4.3)$$

Here the prime quantities are the deviations from their area average and S is the total area of the sphere. It is easily seen from equation (4.3) that the generation of available potential energy is the covariance between temperature and heating. Thus available potential energy is produced when warm air is heated and cold air is cooled.

Following Lorenz, G(A) can be considered as the sum of the zonal available potential energy, $G(A_z)$, and the eddy available potential energy, $G(A_e)$. We shall be dealing here only with the contribution from the former for which the expression can be written as:

$$G(A_z) = \frac{R}{gC_p} \int_0^{p_o} \int_S \frac{1}{\bar{\sigma}} \frac{1}{p} \alpha_z' H_z' \, dS \, dp \qquad \ldots(4.4)$$

Using hydrostatic equilibrium we can write

$$\alpha_z' = -\left(\frac{\partial \Phi}{\partial p}\right)_z' \qquad \ldots(4.5)$$

Substituting (4.5) in (4.4), the integral for $G(A_z)$, per unit area, over the latitude belt between 20°N (= φ_1) and 87.5°N (= φ_2), and for a layer of thickness Δp becomes

$$\frac{1}{S} G_{\Delta p}(A_z) = \frac{R}{C_p \bar{\sigma} p (\sin \varphi_2 - \sin \varphi_1)} \int_{\varphi_1}^{\varphi_2} h_z' H_z' \cos \varphi \, d\varphi \qquad \ldots(4.6)$$

where h_z' is the deviation from the area average of zonally averaged thickness (in meters) of the layer. $H_z(\varphi, p)$ was obtained for January 1964 as a function of latitude and pressure as described in the previous section and h_z' was computed from the mean observed data. Hence the final results of $G(A_z)$ include only the effect of time averaged motion. The integral in equation (4.6) was evaluated by Simpson's quadrature formula.

$G(A_z)$ as a function of pressure, in the units $10^{-5}kjm^{-2}sec^{-1}cb^{-1}$ are shown in the figure (30) and the contribution to $G(A_z)$ from each layer, in the units $10^{-4}kjm^{-2}sec^{-1}$ is given in table I. The results show that the major portion of the production of the zonal available potential energy occurs in the lower troposphere. From the table it is clear that 4/5 of the total generation of zonal available potential energy takes place in the lower half of the troposphere. $G(A_z)$ decreases rapidly in the upper troposphere and finally becomes negative

Table I. The generation of zonal available potential energy for January 1964

Units: $10^{-4}kjm^{-2}sec^{-1}$

Layer	1000 -850	850 -700	700 -500	500 -300	300 -200	200 -150	150 -100	Total
$\frac{1}{S}G_{\Delta p}(A_z)$	5.65	6.46	10.28	3.32	0.62	0.06	-0.04	26.4

in the layer 150-100 mb. The maximum production of zonal available potential energy in the lower troposphere is due to the fact that relatively warm air in the low latitudes is heated

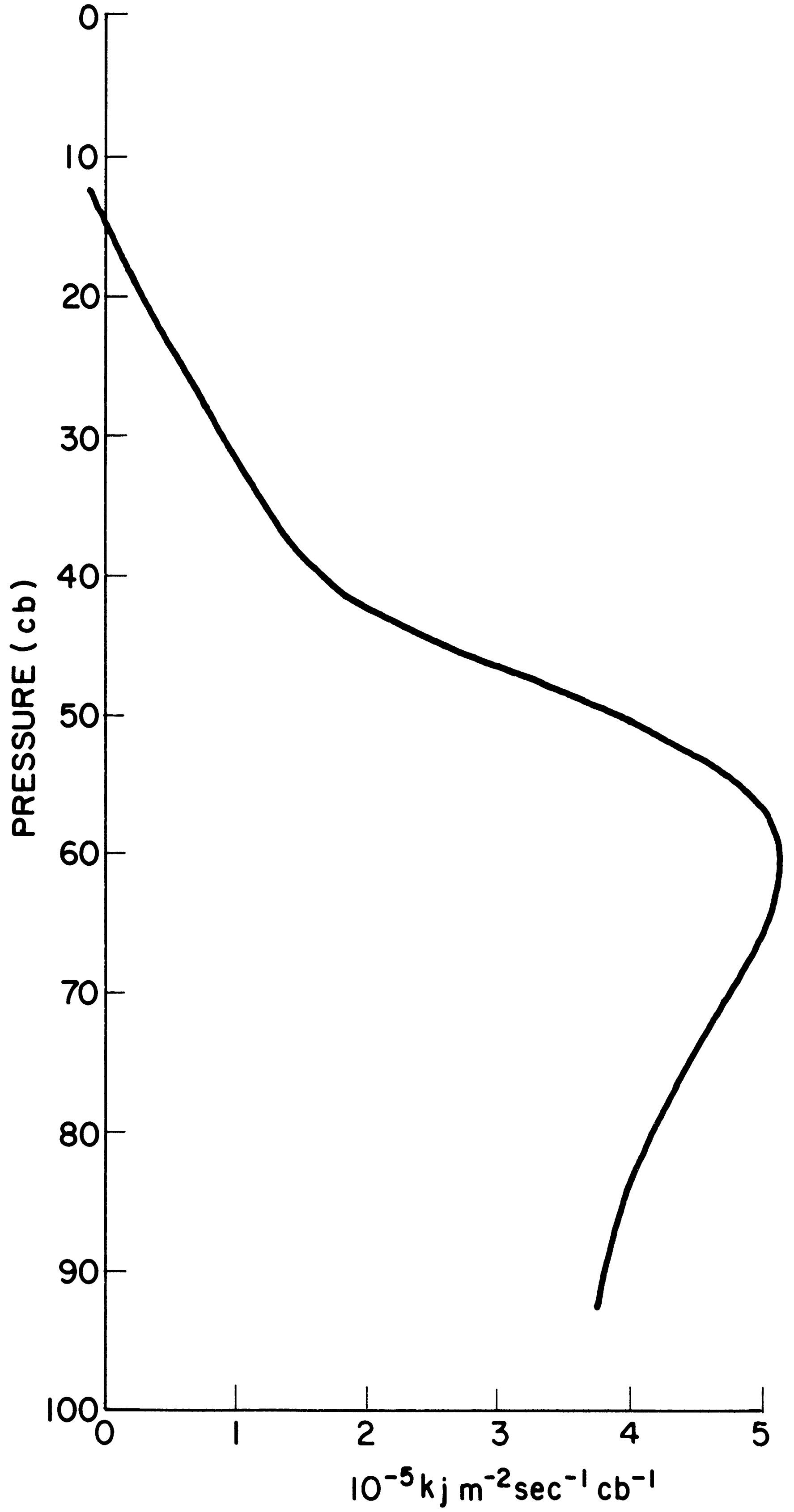

Figure 30. The generation of zonal available potential energy as a function of pressure for the month of January 1964 in the unit $10^{-5} kjm^{-2} sec^{-1} cb^{-1}$.

and that relatively cool air in the higher latitude is further cooled. In the higher troposphere the temperature gradient decreases. The tropopause level in the higher latitude is around 300 mb above which the temperature gradient reverses the sign. The heating has the same distribution as in the lower troposphere. As a result generation of zonal available potential energy decreases. The layer 150-100 mb lies in the stratosphere where the temperature and heating are out of phase by almost 180 degrees hence the covariance between them becomes negative. The negative value in this layer agrees with the study on the energetics of the lower stratosphere by Oort (1964).

$G(A_z)$ for the entire atmosphere over the latitude belt 20°N to 87.5°N was $26.4 \times 10^{-4} kjm^{-2} sec^{-1}$, for January 1964. Wiin-Nielsen and Brown (1960) estimated $G(A_z)$ to be $50.0 \times 10^{-4} kjm^{-2} sec^{-1}$ as the effect of standing as well as transient motion and $48.0 \times 10^{-4} kjm^{-2} sec^{-1}$ as the effect of standing motion, for January 1959. Comparing the latter estimate with our results it appears that our estimate differs approximately by a factor of two. Wiin-Nielsen and Brown computed $G(A_z)$ for the layer 800-400 mb where it has its maximum value according to our results and the estimate for the entire troposphere (1000-200 mb) was obtained assuming a constant distribution in the vertical. Hence their results are over-estimated. If we make the same assumption using the value of $G(A_z)$ for the layer 700-500 mb we obtain 51.4 x $10^{-4} kjm^{-2}sec^{-1}$, for the entire atmosphere (1000-0 mb) and 41.1 x $10^{-4} kjm^{-2}sec^{-1}$ for the troposphere (1000-200 mb). Thus these results are in very good agreement. Brown (1964) computed $G(A_z)$ for January 1962 and 1963. His results are 36.1 and 39.4 x $10^{-4} kjm^{-2}sec^{-1}$ for January 1962 and 1963 respectively. Hence these results are also in close agreement. Phillips (1956) obtained 21.3 x $10^{-4} kjm^{-2}sec^{-1}$ from his two level quasi-geostrophic model for general circulation, while Smagorinsky (1963) obtained 22.1 x $10^{-4} kjm^{-2}sec^{-1}$ from his primitive equation model for general circulation. These results are also in close agreement with those derived in this study.

4.4 CONVERSION OF THE ZONAL AVAILABLE POTENTIAL ENERGY TO THE ZONAL KINETIC ENERGY

It was first suggested by Margules (1903) that the transformation process which produces kinetic energy in the atmosphere is the simultaneous rising of warm air and sinking of cold air. This transformation process can be mathematically defined as a mass integral of the product of the deviations of the vertical motion and temperature from their area average. Symbolically we may write:

$$C(A,K) = -\frac{1}{g}\int_0^{p_o}\int_S \omega'\alpha' dS\ dp \qquad \ldots(4.7)$$

The contribution from the zonal flow to C(A,K) in hydrostatic equilibrium is

$$C(A_z,K_z) = \frac{1}{g}\int_0^{p_o}\int_S \omega_z'\left(\frac{\partial\Phi_z}{\partial p}\right)' dS\ dp \qquad \ldots(4.8)$$

or

$$= -\frac{1}{g}\int_0^{p_o}\int_S \Phi_z'\left(\frac{\partial\omega_z}{\partial p}\right)' dS\ dp \qquad \ldots(4.9)$$

In view of the equation of continuity, (4.9) becomes

$$C(A_z,K_z) = \frac{2\pi a}{g}\int_0^{p_o}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \Phi_z' \frac{\partial}{\partial\varphi}(v_z\cos\varphi)\, d\varphi\ dp \qquad \ldots(4.10)$$

or

$$= \frac{2\pi a^2}{g}\int_0^{p_o}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f u_{gz} v_z \cos\varphi\ d\varphi\ dp \qquad \ldots(4.11)$$

where u_{gz} is the zonal average of west-east component of the geostrophic wind. The conversion per unit area over the region 20°N and 87.5°N in a layer of thickness Δp can be written as

$$\frac{1}{S}C_{\Delta p}(A_z,K_z) = \frac{\Delta p}{g(\sin\varphi_2 - \sin\varphi_1)}\int_{\varphi_1}^{\varphi_2} f u_{gz} v_z \cos\varphi\ d\varphi \ \ldots(4.12)$$

From our results on mean meridional circulations it is possible to compute $C(A_z,K_z)$ from equation (4.8) as well as from equation (4.12). The latter equation was used since it permits a calculation for eight layers while the former only for seven layers. Computations were carried out on a daily basis to include the effect of transient and standing motion and also from mean monthly values to obtain the effect of standing motion. The results are illustrated in figure (31) as a function of pressure, in the units $10^{-6} kjm^{-2} sec^{-1} cb^{-1}$. The broken curve shows the effect of transient and standing motion while the continuous curve refers to the standing motion alone. These results show that the effect of transient eddies is small. The interesting feature of the figure is that the zonal available potential energy is transformed into zonal kinetic energy in the lower troposphere where the baroclinicity is more pronounced and available potential energy is maximum while the reverse is true in the upper troposphere and lower stratosphere where the kinetic energy is maximum. Evidence that zonal kinetic energy is converted to zonal available potential energy in the stratosphere was indicated by White and Nolan (1959), and Oort (1964).

$C(A_z, K_z)$ for the entire atmosphere over the latitude belt 20°N to 87.5°N was $-2.4 \times 10^{-4} kjm^{-2} sec^{-1}$ for January 1964. Wiin-Nielsen (1959c) obtained $1.03 \times 10^{-4} kjm^{-2} sec^{-1}$ for January 1959 over the same region. His computations were based on the vertical velocity at 600 mb and the thickness of the layer: 850-500 mb. He further assumed a parabolic distribution in the

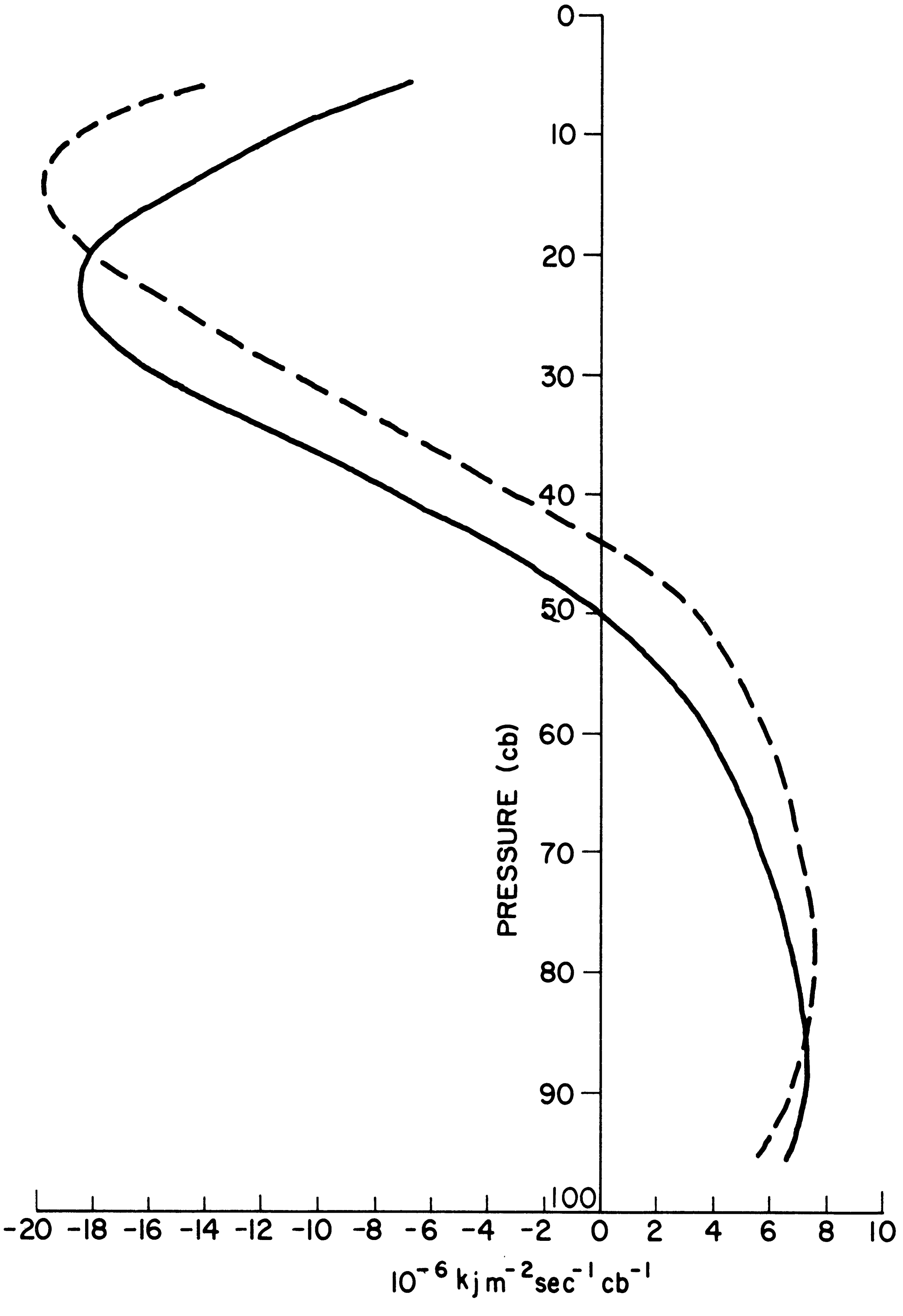

Figure 31. The conversion between zonal available potential energy and zonal kinetic energy, $C(A_Z,K_Z)$, as a function of pressure for the month of January 1964 in the unit 10^{-6}kjm^{-2}sec^{-1}cb^{-1}. Solid curve: $C(A_Z,K_Z)$ due to the standing motion. Dashed curve: $C(A_Z,K_Z)$ due to the standing and transient motion.

vertical for the vertical velocity having zero value at 1000 mb and 200 mb. $C(A_z, K_z)$ was thus estimated for the troposphere assuming a constant distribution in the vertical. If we make a similar assumption on the average value between 850-500 mb we find $2.1 \times 10^{-4} kjm^{-2} sec^{-1}$. This value agrees fairly well with Wiin-Nielsen's results considering the period for which they represent. Saltzman and Fleisher (1961) estimated $3.5 \times 10^{-4} kjm^{-2} sec^{-1}$ for the winter of 1959 using vertical velocities at 600 mb and thickness for the 850-500 mb layer over the Northern Hemisphere by extrapolating the data in the equatorial region. These results cannot be compared to our calculations because Palmen, Riehl and Vuorela (1958) showed that $C(A_z, K_z)$ is positive everywhere in the vertical over the tropical region. Jensen (1961) presented data for the vertical velocity, computed by the adiabatic method, and temperature for different layers for January 1958. Wiin-Nielsen (1964) used these data to evaluate the integral in equation (4.8) to show that these results should represent $C(A_z, K_z) - G(A_z)$ rather than $C(A_z, K_z)$. Since we have estimates of both $G(A_z)$ and $C(A_z, K_z)$ in the vertical such a comparison is shown in table II. The conversion computed from Jensen's data is indicated by an asterisk in the table. Considering the differences in method

Table II A comparison between $C^*(A_z, K_z)$ and $C(A_z, K_z) - G(A_z)$

Units: $10^{-4} kjm^{-2} sec^{-1}$

Layer	1000 -850	850 -700	700 -500	500 -300	300 -200	200 -100
$C^*(A_z, K_z)$	-10.53	-11.23	-8.04	-1.81	0.22	-0.23
$G(A_z, K_z) - G(A_z)$	-4.61	-5.34	-9.20	-3.92	-2.08	-2.01

of computation and the periods which they represent these results are not inconsistent with the conclusions drawn by Wiin-Nielsen.

5. DAILY VARIATIONS OF MERIDIONAL CIRCULATIONS AND ENERGY CONVERSION

5.1 PRELIMINARY REMARKS

The atmosphere is constantly in a state of complex motion, changing with time. The periodicities, such as the annual cycle of variations, are well hidden by the transient motion on a day-to-day basis. The periodicities like atmospheric tides and diurnal variation of temperature are hardly detectable in the large scale upper air flow. There is considerable doubt as to whether any detectable periodicities occur in the tropospheric weather data (Ward and Shapiro (1961)). However, there exists an oscillation of the general circulation, especially in the middle and the high latitudes, known as the index cycle (Namias and Clapp (1951)). The index cycle refers to the oscillation between strong and weak westerlies in the middle latitudes. High index represents a state of circulation with strong westerlies in the middle latitudes and relatively weak eddies, whereas a low index has relatively weak westerlies but well developed eddies (Smagorinsky (1963)). Therefore the eddy transport of momentum and heat must be relatively less intense during the high index period and relatively more intense during the low index period. The meridional circulations in our present model are forced by these eddy transport processes and hence one should expect similar changes in the intensity of the mean meridional circulations. Further the different stages in the index cycle take place gradually and hence one should expect the mean meridional circulation to be a smooth function of time.

To date, the study of variations in the meridional circulations on a daily basis has not been undertaken. This is so because the indirect method of computations is based on the steady state models. However, Defant and van de Boogaard (1963) and van de Boogaard (1964) computed meridional circulations by the direct method for one synoptic observation between equator and 40°N. Their results show good agreement with other investigators' (Palmen, Riehl and Vuorela (1958)) results based on seasonal averages. Therefore it seems that the meridional circulations computed on a daily basis may have circulation patterns consistent with the mean monthly circulations, discussed earlier.

5.2 DAILY VARIATIONS OF MERIDIONAL CIRCULATIONS

We shall recall that our method of computing the meridional circulation is based on a diagnostic equation. Hence it is possible to compute the meridional circulations whenever the forcing function is known.

The eddy transfers of zonal momentum and sensible heat were computed from the height data for two observations per day (00Z and 12Z) for January 1964. The meridional circulations were computed by the method described in section (2.3) with the boundary conditions that $\omega_z = 0$ at the top and bottom of the atmosphere.

To present these results as a function of time we have computed the mass circulation in the lower troposphere from equation (3.1). The mass circulations as a function of latitude and time in the units 10^6 tons sec^{-1} are illustrated in figure (32). The negative values indicate the equatorward mass flow while the positive values indicate the poleward flow. The curve, representing the zero mass circulation in the lower latitudes, shows the northern extent of the tropical direct cell. One might note here that if the tropical cell moves toward the north the intensity of the circulation in middle latitudes decreases. Such a behavior was also noticed in the seasonal variation of the meridional circulation (section 3.4). The curve for zero mass circulation in the higher latitudes shows the northern extent of the middle latitude reverse cell. This curve is very irregular and on certain days the reverse cell extends all the way up to the north pole. Considering the variation of the intensity of the circulation we notice that there are five distinct periods of maximum intensity.

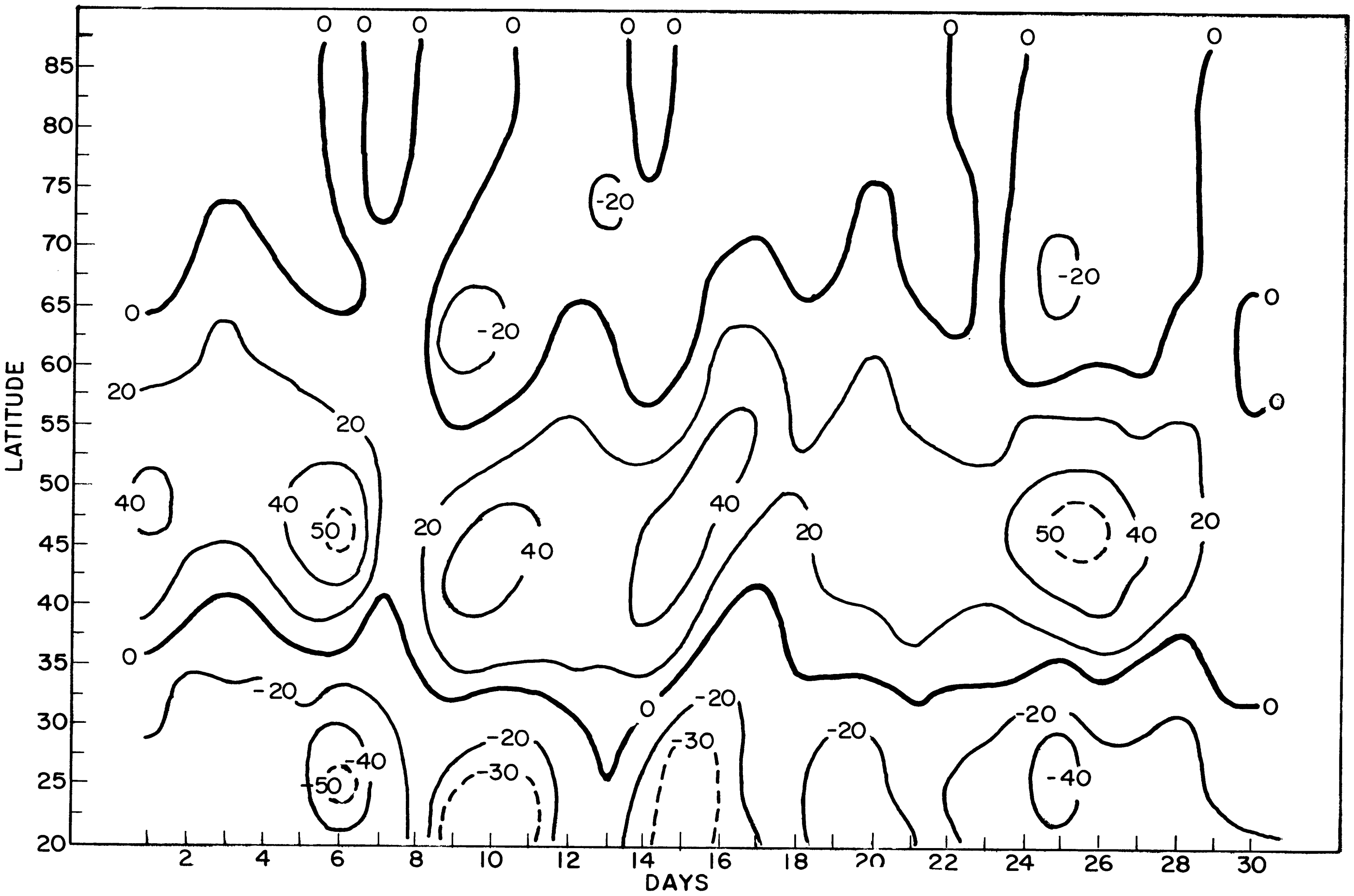

Figure 32. The daily variation of the mass circulation in the lower troposphere as a function latitude and time for the month of January 1964 in the unit 10^6tons sec^{-1}.

However there is no periodicity to their occurrence, but the time series analysis based on a long record of such data might show a substantial peak in the spectrum. The intensities of the eddy transports were checked from the computed data for the period on which the meridional circulation has relative maxima. It was found that there were indeed maximum values during these periods. An independent check can be made on the intensity of the meridional circulation considering the high and low index situations. The meridional circulation should be more intense during the low index situation than in the high index situation. One of the measures of the high and the low index is the magnitude of the maximum zonally averaged west-east component of wind, $u_z(max)$. Figure (33) shows $u_z(max)$ in m sec^{-1} as a function of time. Comparing the period of minimum strength of the zonal winds with figure (32) we notice that these periods agree with the periods of maximum intensity of the meridional circulation. Similarly, if we note the periods of maximum zonal winds we notice that these periods agree with the periods of minimum intensity of the circulations. These results obtained on a daily basis indicate that the present model is sufficiently accurate to give reasonable estimates of the meridional circulations which may be used for further studies of the energetics of the atmosphere.

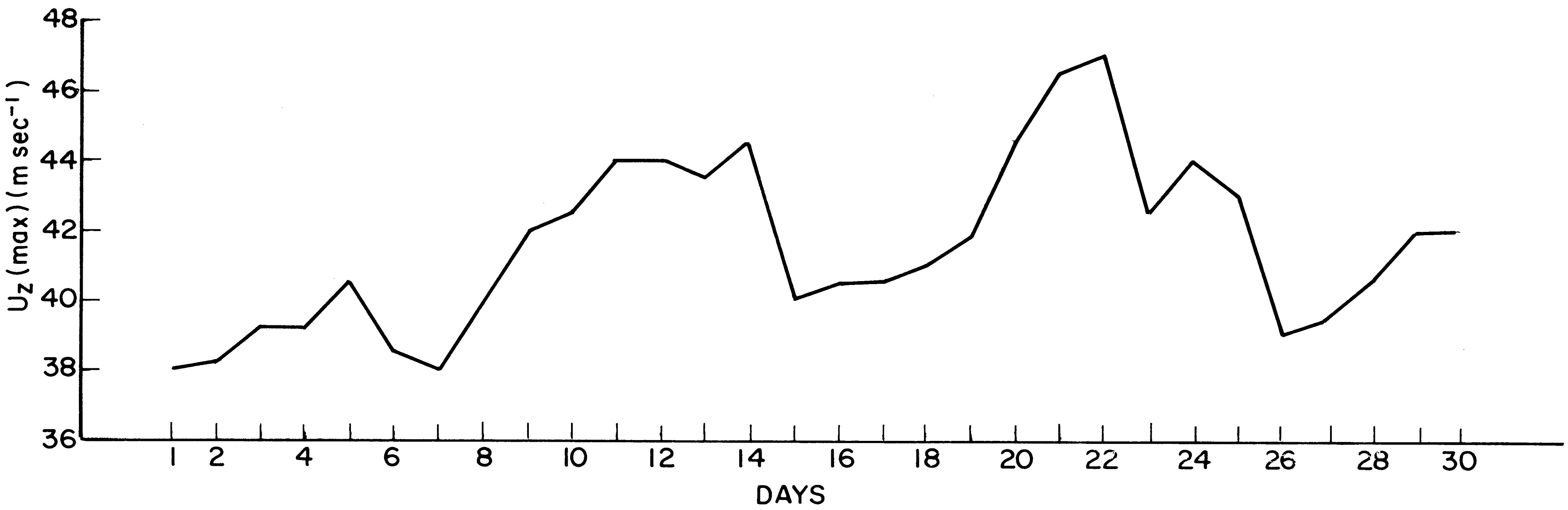

Figure 33. The variation of the daily values of the maximum zonal wind as a function of time for the month of January 1964 in the unit m sec^{-1}.

5.3 DAILY VARIATIONS OF CONVERSION FROM ZONAL AVAILABLE POTENTIAL ENERGY TO ZONAL KINETIC ENERGY

We have already discussed the vertical variation of $C(A_z, K_z)$ computed on a daily basis and then averaged over the whole month (January 1964). We shall consider here the daily variations of $C(A_z, K_z)$ as a function of pressure. Figure (34) shows $C(A_z, K_z)$ as a function of pressure and time in the units $10^{-6} kjm^{-2} sec^{-1} cb^{-1}$. The positive values indicate the conversion of zonal available potential energy to zonal kinetic energy while the negative values indicate the reverse process. It is interesting to note that during the whole month the zonal kinetic energy was produced in the lower troposphere while the zonal available potential energy was produced in the upper troposphere. This is due to the dominating influence of the indirect meridional cell in the middle latitude over the region 20°N to 87.5°N. There exists an oscillation of $C(A_z, K_z)$ in both the lower and upper troposphere. If we consider the absolute values, we note that there is a time lag of almost a day between the maximum amplitude of these oscillations in the lower and upper troposphere. Any relation between these oscillations to those of the zonal kinetic energy and zonal available potential energy can be obtained with a long record of such data. Such an attempt was not made in the present study.

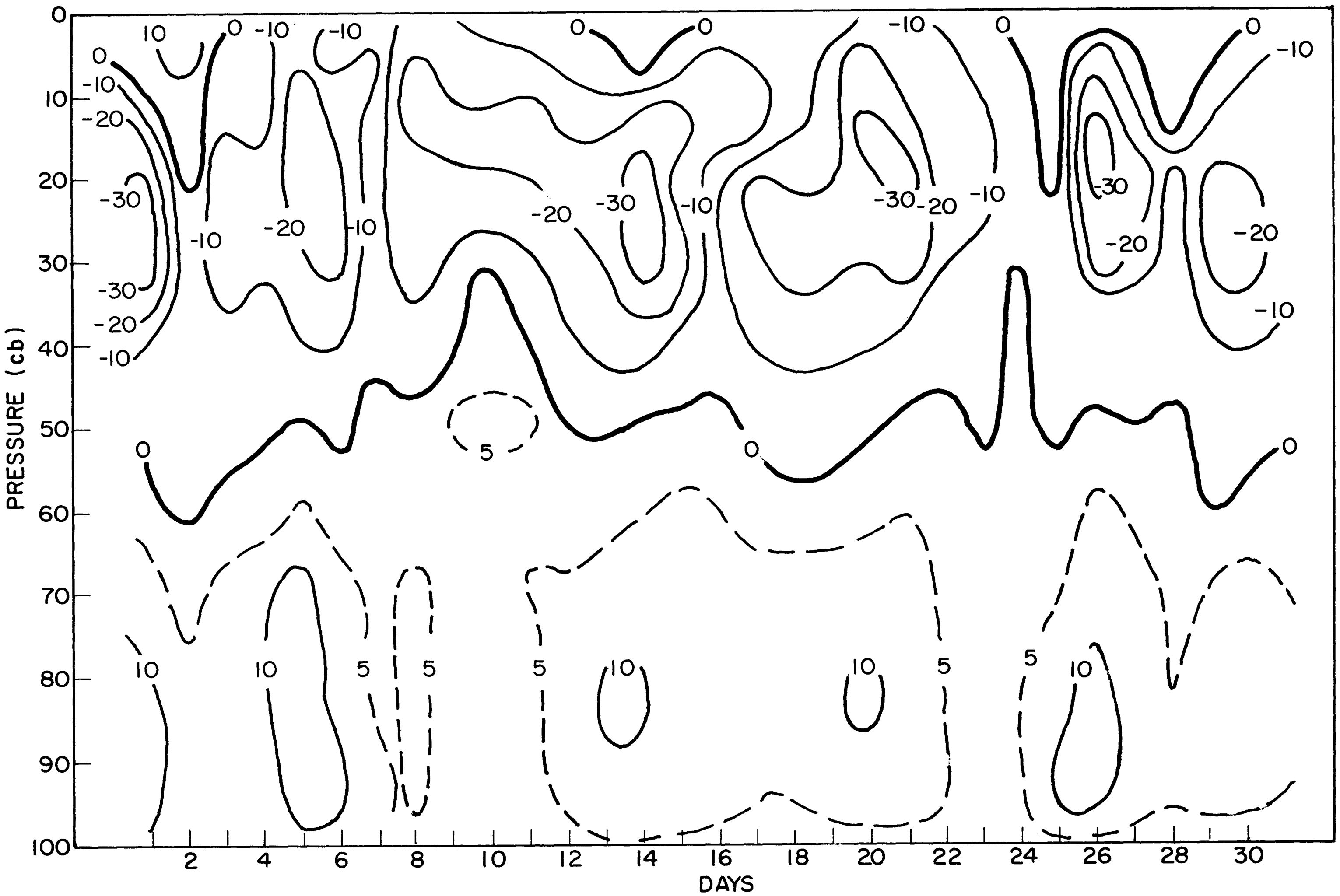

Figure 34. The daily variation of the energy exchanges $C(A_Z, K_Z)$ as a function of pressure and time for the month of January 1964 in the unit $10^{-6} kjm^{-2} sec^{-1} cb^{-1}$.

6. CONCLUSIONS

6.1 SUMMARY

We have shown that the mean meridional circulations produced by eddy transports of heat and momentum in a quasi-geostrophic, adiabatic and frictionless model are in agreement with those obtained by considering the balance of angular momentum in the atmosphere (Mintz and Lang (1955) and Holopainen (1966a)).

The magnitude of the mean meridional wind component, v_z, computed in the present study is less than 1 m sec^{-1}. The zonal average of the absolute value of the meridional wind component, $|v|_z$, is about 10 m sec^{-1}. On the average, therefore, v_z is at least one order of magnitude less than $|v|_z$. This justifies the assumption in the quasi-geostrophic theory that the mean meridional circulations are secondary processes.

Numerical prediction models based on the quasi-geostrophic theory have a tendency to shift the jet stream toward the north (Thompson (1961)). This is because such models neglect the mean meridional circulations so that the time rate of change of zonal wind is due mainly to the horizontal convergence of momentum. The effect of the mean meridional circulation on the time rate change of zonal wind (i.e. the term fv_z in the zonally averaged first equation of motion) is of the same order of magnitude as that of the convergence of zonal momentum and counteracts it in the upper troposphere (see figure 14, Wiin-Nielsen, Brown and Drake (1963)). For example the value of fv_z is -5×10^{-5}m sec^{-2}, at 45°N near 200 mb for January 1962 and the value for the horizonal convergence of momentum is 9×10^{-5}m sec^{-2} at the same latitude and pressure (see Wiin-Nielsen, Brown and Drake (1963)). The numerical prediction models based on the primitive equations include the mean meridional circulations and, therefore, should predict the jet position more accurately.

Wiin-Nielsen, Brown and Drake (1963) showed that the horizontal convergence of eddy transport of sensible heat decreases the zonal mean temperature south of 50°N and increases it north of that latitude. It follows from the computations of zonally averaged vertical velocity that the vertical flux of heat associated with the mean meridional circulation decreases the zonal mean temperature in the tropics, increases it in the latitude belt 25-45°N, decreases it in the latitude belt 45°-70°N and finally increases it in the polar region.

Considering the separate effects of momentum and heat transports on the mean meridional circulations, it is found that the former is twice as effective as the latter, irrespective of the season. It may be concluded from this that the mean meridional circulations over the region north of 20°N play a more important role in the angular momentum balance than in the heat balance.

The influence of the planetary scale motion on the mean meridional circulations during the winter months is much larger than the influence of the other scales of motion whereas the baroclinically unstable waves dominate the forcing mechanism during other seasons.

The seasonal variation of meridional circulations shows that the circulation cells move toward the pole from winter to summer with a decrease in their intensity. A comparison between the intensities of the circulation for different seasons shows that the intensity in the summer is 1/3 of that in the winter. During the spring and fall the intensity of the circulation is almost the same and has a value which is about 2/3 that of the winter season. This oscillation of the meridional cells is closely related to the annual oscillation of the general circulation.

The vertical velocity due to the skin friction and topography at the top of the frictional layer is, by and large, in phase with the vertical velocity computed from an adiabatic and frictionless model.

Net diabatic heating in the meridional plane is positive south of 40°N and negative north of that latitude during winter months. In the upper troposphere heating decreases gradually with height in the region of net heating, whereas the cooling decreases sharply with height in the region of net cooling.

The generation of zonal available potential energy is maximum in the lower troposphere, where the baroclinicity is large, and decreases sharply in the upper troposphere and finally becomes negative in the lower stratosphere.

The conversion from zonal available potential energy to zonal kinetic energy occurs in the lower troposphere where the zonal available potential energy is maximum while the reverse process takes place in the upper troposphere where kinetic energy is maximum.

The present method of computing meridional circulations is sufficiently sensitive to give reasonable estimates on a daily basis.

6.2 CERTAIN CRITICAL REMARKS

Even if the results of this study agree with those of other investigators, we are not in a position to say that these are the true values of the mean meridional circulations for the real atmosphere. The effects of the diabatic heating and friction, on the mean meridional circulations, are by no means negligible as compared to the effects of eddy processes. This was revealed from a pilot study made for a single layer. The diabatic heating alone produced a single cell "Hadley type" circulation while friction has an effect similar to the eddy processes and produced a three cell circulation.

It was mentioned earlier that the eddy transports of heat and momentum were computed from the objective height analysis obtained from the National Meteorological Center. Recently Holopainen (1966b) made a detailed study showing that the momentum transports computed from the objective height analysis are over-estimated as compared to those computed from the subjective analysis or from wind statistics. Such an over-estimation of the momentum transports must have affected our results to some extent.

In our formulation of the problem we have assumed that $f = f_o$ to be consistent with the quasi-geostrophic theory and to satisfy certain integral constraints (Wiin-Nielsen (1959a)). Such an assumption over-estimates the meridional circulations in the polar region and under-estimates them in the subtropical region.

7. SUGGESTIONS FOR FUTURE WORK

The role of meridional circulations in the general circulation is more important in the tropical region than in the middle of high latitudes. Because of the lack of data it was not possible for us to obtain reasonable estimates of the mean meridional circulations in the tropical region. It will be worth while to pursue the study with the present method of computation over the entire Northern Hemisphere, or even more so over the entire globe, when such data are available.

More realistic estimates of meridional circulations can be made by incorporating the effects of diabatic heating and friction in the atmosphere.

Equation (2.3) may be solved by expressing the vertical velocity, ω, as a series of associated Legendre polynomials to obtain a three-dimensional distribution of vertical velocity. Here also one will face the problem of data for the forcing functions in the tropics, which may be solved by using an internally consistent extrapolation procedure of the type done in the present study.

The meridional circulations are computed on a daily basis in the present study as a preliminary test of the accuracy of the model. Such computation for a much longer period of time should be made for further energetics studies or a time series analysis.

The time series analysis of the energy parameters can be made for different layers in the atmosphere. Attempts at such studies have been made. Wiin-Nielsen (1966) made a harmonic analysis of a time series of energy parameters based on mean monthly values over a year. His results show that there exists a time lag between the phase angles of the maximum generation of available potential energy and the maximum kinetic energy. Similarly there exists a time lag between the phase angles of the maximum kinetic energy and the maximum dissipation of kinetic energy. Carasso, Horn and Johnson (1965) made a cross-spectrum analysis between different energy parameters for the layer: 850-500 mb. Their results indicate a systematic transformation of available potential energy to kinetic energy at periods less than 10 days. The results of the latter study would have been entirely different if they had made a study for a different layer in the atmosphere. This is evident from the daily variation of $C(A_z,K_z)$ as a function of pressure (see figure (34)). Such a large variation in the vertical of any other energy parameter is also seen from the previous studies (Wiin-Nielsen, Brown and Drake (1963) and (1964)). It will be, therefore, worth while pursuing cross-spectrum analysis for different layers in the atmosphere.

APPENDIX A

DERIVATION OF EQUATION (2.5)

We shall rewrite equation (2.3) in the form

$$\bar{\sigma}\nabla_s^2\omega + f_o^2 \frac{\partial^2\omega}{\partial p^2} = f_o \frac{\partial}{\partial p}\{\frac{u}{a\cos\varphi}\frac{\partial}{\partial\lambda}(\zeta+f) + \frac{v}{a}\frac{\partial}{\partial\varphi}(\zeta+f)\}$$

$$- \frac{f_o}{a\cos\varphi}\frac{\partial}{\partial p}(\frac{\partial F_\varphi}{\partial\lambda} - \frac{\partial F_\lambda \cos\varphi}{\partial\varphi})$$

$$- f_o\nabla_s^2\{\frac{u}{a\cos\varphi}\frac{\partial}{\partial\lambda}(\frac{\partial\Psi}{\partial p}) + \frac{v}{a}\frac{\partial}{\partial\varphi}(\frac{\partial\Psi}{\partial p})\}$$

$$- \frac{R}{C_p}\frac{1}{p}\nabla_s^2 H \qquad \text{...(A.1)}$$

where $\nabla_s^2 \equiv \frac{1}{a^2\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2} + \frac{1}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}(\cos\varphi\frac{\partial}{\partial\varphi})$

Now, taking the zonal average as defined by equation (2.4) we obtain

$$\frac{\bar{\sigma}}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}(\cos\varphi\frac{\partial\omega_z}{\partial\varphi}) + f_o^2\frac{\partial^2\omega_z}{\partial p^2} =$$

$$f_o\frac{\partial}{\partial p}\{\frac{u}{a\cos\varphi}\frac{\partial}{\partial\lambda}(\zeta+f) + \frac{v}{a}\frac{\partial}{\partial\varphi}(\zeta+f)\}_z$$

$$+ \frac{f_o}{a\cos\varphi}\frac{\partial}{\partial p}(\frac{\partial F_{\lambda,z}\cos\varphi}{\partial\varphi})$$

$$- \frac{f_o}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}[\cos\varphi\frac{\partial}{\partial\varphi}\{\frac{u}{a\cos\varphi}\frac{\partial}{\partial\lambda}(\frac{\partial\Psi}{\partial p}) + \frac{v}{a}\frac{\partial}{\partial p}(\frac{\partial\Psi}{\partial p})\}_z]$$

$$- \frac{R}{C_p}\frac{1}{p}\frac{1}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}(\cos\varphi\frac{\partial H_z}{\partial\varphi}) \qquad \text{...(A.2)}$$

Let us consider the first term on the right hand side of equation (A.2). Since the advection is done by the non-divergent winds, we can write

$$\{\frac{u}{a\cos\varphi}\frac{\partial}{\partial\lambda}(\zeta+f) + \frac{v}{a}\frac{\partial}{\partial\varphi}(\zeta+f)\}_z =$$

$$\frac{1}{a\cos\varphi}\{\frac{\partial}{\partial\lambda}(u(\zeta+f)) + \frac{\partial}{\partial\varphi}(v(\zeta+f)\cos\varphi)\}_z$$

$$= \frac{1}{2\pi}\frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}(\cos\varphi\int_0^{2\pi} v(\zeta+f)\,d\lambda)$$

$$= \frac{1}{2\pi}\frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}(\cos\varphi\int_0^{2\pi} v\zeta\,d\lambda)$$

$$= \frac{1}{2\pi}\frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}\{\cos\varphi\int_0^{2\pi} v[\frac{1}{a\cos\varphi}(\frac{\partial v}{\partial\lambda} - \frac{\partial u\cos\varphi}{\partial\varphi})]d\lambda\}$$

$$= -\frac{1}{2\pi}\frac{1}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}(\int_0^{2\pi} v\frac{\partial u\cos\varphi}{\partial\varphi}\,d\lambda)$$

$$= -\frac{1}{2\pi}\frac{1}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}(\int_0^{2\pi}\frac{1}{\cos\varphi} v\cos\varphi\frac{\partial u\cos\varphi}{\partial\varphi}\,d\lambda)$$

$$= -\frac{1}{2\pi}\frac{1}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}(\int_0^{2\pi}\frac{1}{\cos\varphi}[\frac{\partial}{\partial\varphi}(uv\cos^2\varphi) - u\cos\varphi\frac{\partial v\cos\varphi}{\partial\varphi}]d\lambda)$$

$$= -\frac{1}{2\pi}\frac{1}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}(\int_0^{2\pi}\frac{1}{\cos\varphi}[\frac{\partial}{\partial\varphi}(uv\cos^2\varphi + u\cos\varphi\frac{\partial u}{\partial\lambda}]d\lambda)$$

$$= -\frac{1}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}(\frac{1}{\cos\varphi}\frac{\partial}{\partial\varphi}(uv)_z\cos^2\varphi) \qquad \ldots(A.3)$$

Similarly, we can write the third term on the right hand side of equation (A.2) as

$$\{\frac{u}{a\cos\varphi}\frac{\partial}{\partial\lambda}(\frac{\partial\Psi}{\partial p}) + \frac{v}{a}\frac{\partial}{\partial\varphi}(\frac{\partial\Psi}{\partial p})\}_z =$$

$$\frac{1}{a\cos\varphi}\{\frac{\partial}{\partial\lambda}(u\frac{\partial\Psi}{\partial p}) + \frac{\partial}{\partial\varphi}(v\frac{\partial\Psi}{\partial p}\cos\varphi)\}_z$$

$$= \frac{1}{2\pi}\frac{1}{a\cos\varphi}(\int_0^{2\pi}\frac{\partial}{\partial\varphi}(v\frac{\partial\Psi}{\partial p}\cos\varphi)d\lambda)$$

$$= \frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}\{(\frac{\partial\Psi}{\partial p}v)_z\cos\varphi\}$$

$$= \frac{1}{f_o\, a\cos\varphi}\frac{\partial}{\partial\varphi}\{(\frac{\partial\Phi}{\partial p}v)_z\cos\varphi\} \qquad \ldots(A.4)$$

Further we shall show that $(\frac{\partial\Phi}{\partial p}v)_z$ is related to the eddy transfer of sensible heat in the atmosphere.

The eddy transfer of sensible heat across a latitude wall is defined as:

$$TH(\varphi) = \frac{1}{g}\int_0^{p_o}\int_0^{2\pi} C_p Tv\, a\cos\varphi\, d\lambda\, dp$$

$$= \frac{2\pi C_p\, a\cos\varphi}{g}\int_0^{p_o}(Tv)_z\, dp$$

where p_o = 100 cb is taken as the surface pressure.

The eddy transfer of sensible heat across a latitude circle for a layer of thickness Δp is

$$TH_{\Delta p}(\varphi) = \frac{2\pi C_p \; a \cos\varphi}{g} \Delta p (\tilde{T}\tilde{v})_z \qquad \ldots (A.5)$$

where $\tilde{T}$ and $\tilde{v}$ are the average values of T and v for the layer Δp.

In hydrostatic equilibrium we have

$$h = -\frac{R}{g}\tilde{T}\ln(p_2/p_1), \qquad \ldots (A.6)$$

where p_1 is the pressure at the lower boundary and p_2 is the pressure at the upper boundary of the layer and h is the thickness of the layer (in meters).

Writing $\frac{\partial\Phi}{\partial p}$ in the finite difference form, h can also be expressed as:

$$h = -\frac{\Delta p}{g}\left(\frac{\partial\Phi}{\partial p}\right) \qquad \ldots (A.7)$$

Hence from equations (A.6) and (A.7), we have

$$\tilde{T} = \frac{\Delta p}{R\ln\left(\frac{p_2}{p_1}\right)}\left(\frac{\partial\Phi}{\partial p}\right) \qquad \ldots (A.8)$$

In view of (A.8), equation (A.5) can be written as:

$$TH_{\Delta p}(\varphi) = \frac{2\pi a \cos\varphi \; C_p (\Delta p)^2}{gR \ln(\frac{p_2}{p_1})} (\frac{\partial\Phi}{\partial p} v)_z \qquad ...(A.9).$$

Therefore equation (A.4) becomes

$$\{\frac{u}{a \cos\varphi} \frac{\partial}{\partial\lambda}(\frac{\partial\Psi}{\partial p}) + \frac{v}{a} \frac{\partial}{\partial\varphi}(\frac{\partial\Psi}{\partial p})\}_z$$

$$\frac{gR \ln(\frac{p_2}{p_1})}{2\pi a^2 \cos\varphi \; f_o C_p (\Delta p)^2} \frac{\partial}{\partial\varphi}(TH_{\Delta p}(\varphi)) \qquad ...(A.10)$$

Finally, substituting (A.4) and (A.10) in equation (A.2) we have

$$\frac{1}{\cos\varphi}\frac{\partial}{\partial\varphi}(\cos\varphi \frac{\partial\omega_z}{\partial\varphi}) + \frac{f_o^2 a^2}{\bar\sigma}\frac{\partial^2\omega_z}{\partial p^2} =$$

$$- \frac{f_o}{\bar\sigma}\frac{\partial}{\partial p}\{\frac{1}{\cos\varphi}\frac{\partial}{\partial\varphi}[\frac{1}{\cos\varphi}\frac{\partial(uv)_z\cos^2\varphi}{\partial\varphi}]\}$$

$$+ \frac{f_o a}{\bar\sigma \cos\varphi}\frac{\partial}{\partial p}(\frac{\partial F_{\lambda,z}\cos\varphi}{\partial\varphi})$$

$$- \frac{gR \ln(\frac{p_2}{p_1})}{(\Delta p)^2 2\pi a^2 \cos\varphi \; C_p\bar\sigma}\frac{\partial}{\partial\varphi}\{\cos\varphi \frac{\partial}{\partial\varphi}(\frac{1}{\cos\varphi}\frac{\partial}{\partial\varphi} TH_{\Delta p}(\varphi))\}$$

$$- \frac{R}{C_p p\bar\sigma}\frac{1}{\cos\varphi}\frac{\partial}{\partial\varphi}(\cos\varphi \frac{\partial H_z}{\partial\varphi}) \qquad ...(A.11)$$

APPENDIX B

DERIVATION OF EQUATIONS TO COMPUTE EDDY TRANSFERS OF ZONAL MOMENTUM AND SENSIBLE HEAT AS A FUNCTION OF WAVE NUMBER

The basic data, of routine objective height analysis, on an octagonal grid, were available from the National Meteorological Center. These data were interpolated to obtain the values of the height field at latitude-longitude grid points.

The linear balance equation,

$$\nabla^2\Psi = \nabla\cdot(\frac{1}{f}\,\nabla\Phi) \qquad \ldots(B.1)$$

introduced by Shuman (1957b), was used to solve the stream function Ψ. The stream function was then expanded in Fourier series as:

$$\Psi = \sum_{n=0}^{15} (a_n(\varphi)\ \cos n\lambda + b_n(\varphi)\ \sin n\lambda) \qquad \ldots(B.2)$$

The eddy transfer of zonal momentum can be expressed as a function of wave numbers as:

$$\{(uv)_z\}_n = \frac{n}{2a^2\cos\varphi}\{a_n(\varphi)\ \frac{\partial b_n(\varphi)}{\partial\varphi} - b_n(\varphi)\ \frac{\partial a_n(\varphi)}{\partial\varphi}\} \qquad \ldots(B.3)$$

The eddy transfer of sensible heat was computed from the thickness field and the meridional wind for the layer.

Let the thickness field be expressed in terms of Fourier series as:

$$h = \sum_{n=0}^{15} (A_n(\varphi)\ \cos n\lambda + B_n(\varphi)\ \sin n\lambda) \qquad \ldots(B.4)$$

The mean value of the meridional component of the wind for the layer, $\tilde{v}$, is related to the mean stream function for the layer $\tilde{\Psi}$ as:

$$\tilde{v} = \frac{1}{a \cos \varphi} \frac{\partial \tilde{\Psi}}{\partial \lambda} \qquad \ldots(B.5).$$

Thus, the eddy transfer of sensible heat can be expressed as a function of the wave numbers by the formula:

$$\{TH_{\Delta p}(\varphi)\}_n = \frac{\pi \Delta p C_p n}{R \ln(\frac{p_1}{p_2})} \{A_n(\varphi)\tilde{b}_n(\varphi) - B_n(\varphi)\tilde{a}_n(\varphi)\} \qquad \ldots(B.6).$$

APPENDIX C

EXTRAPOLATION PROCEDURE

The eddy transfers of zonal momentum and sensible heat were extrapolated, from 20°N to the equator, by an even polynomial, $F(\varphi)$, of degree four. Such a polynomial is of the form:

$$F(\varphi) = a_o + a_1\varphi^2 + a_2\varphi^4 \qquad \text{...(C.1)}$$

If we assume that exchange of momentum and sensible heat is zero at the equator, (C.1) becomes

$$F(\varphi) = a_1\varphi^2 + a_2\varphi^4 \qquad \text{...(C.2)}$$

The eddy fluxes should be continuous and smooth at φ_1 (20°N). Therefore,

$$\varphi_1^2 a_1 + \varphi_1^4 a_2 = F(\varphi_1) \qquad \text{...(C.3)}$$

$$2\varphi_1 a_1 + 4\varphi_1^3 a_2 = \left(\frac{dF(\varphi)}{d\varphi}\right)_{\varphi=\varphi_1} = F'(\varphi_1) \qquad \text{...(C.4)}$$

or

$$a_1 = \frac{4F(\varphi_1) - \varphi_1 F'(\varphi_1)}{2\varphi_1^2} \qquad \text{...(C.5)}$$

$$a_2 = \frac{\varphi_1 F'(\varphi_1) - 2F(\varphi_1)}{2\varphi_1^4} \qquad \text{...(C.6)}$$

Substituting for a_1 and a_2 in equation (C.2) we have

$$F(\varphi) = \frac{4F(\varphi_1) - \varphi_1 F'(\varphi_1)}{2}\left(\frac{\varphi}{\varphi_1}\right)^2 + \frac{\varphi_1 F'(\varphi_1) - 2F(\varphi_1)}{2}\left(\frac{\varphi}{\varphi_1}\right)^4 \qquad \text{...(C.7)}$$

For computational purposes equation (C.7) can be written in a finite difference form as follows:

Let the derivative $F'(\varphi_1)$ be approximated by a forward differencing formula:

$$F'(\varphi_1) = \frac{F(\varphi_2) - F(\varphi_1)}{\Delta\varphi}$$

where $\Delta\varphi = 2.5$ degrees.

Further let $\varphi_1 = K\Delta\varphi$, $\varphi_2 = (K+1)\Delta\varphi$ and $\varphi = k\Delta\varphi$ so equation (C.7) can be written as a function of k

$$F(k) = \frac{(K+4)F(k) - KF(K+1)}{2}\left(\frac{k}{K}\right)^2 + \frac{KF(K+1) - (K+2)F(K)}{2}\left(\frac{k}{K}\right)^4 \qquad \text{...(C.8)}$$

It may be noted that in our case K = 8, and k = 0, 1, ..., 7.

APPENDIX D

TABULATION OF RESULTS

TABLE D-1

VERTICAL VELOCITY, ω_z, FOR JANUARY 1963

Units: 10^{-5}mb sec^{-1}

Latitude	Pressure (cb)			
	77.5	60.0	40.0	25.0
87.5	2.9	1.1	- 2.8	- 5.4
85.0	3.5	2.4	- 1.0	- 4.0
82.5	4.3	4.1	1.5	- 2.0
80.0	5.0	5.7	4.1	0.2
77.5	5.3	6.7	6.0	2.0
75.0	4.8	6.6	6.7	3.0
72.5	3.3	5.1	5.8	2.9
70.0	1.0	2.2	3.2	1.7
67.5	- 1.9	- 1.9	- 0.8	- 0.6
65.0	- 5.2	- 6.5	- 5.8	- 3.6
62.5	- 8.2	-11.1	-10.8	- 6.7
60.0	-10.6	-14.8	-15.0	- 9.5
57.5	-11.9	-17.0	-17.7	-11.3
55.0	-11.8	-17.1	-18.2	-11.8
52.5	-10.2	-15.0	-16.3	-10.8
50.0	- 7.2	-10.9	-12.2	- 8.3
47.5	- 3.3	- 5.3	- 6.3	- 4.6
45.0	1.1	1.0	0.5	- 0.2
42.5	5.3	7.2	7.2	4.2
40.0	8.8	12.4	12.9	8.1
37.5	11.0	15.8	16.7	10.8
35.0	11.8	17.1	18.3	12.2
32.5	11.0	16.1	17.4	11.9
30.0	8.9	13.2	14.4	10.3
27.5	5.8	8.8	9.8	7.5
25.0	2.4	3.9	4.5	4.2
22.5	- 0.8	- 0.9	- 0.7	0.9
20.0	- 3.4	- 4.7	- 4.9	- 1.9

TABLE D-2

MERIDIONAL VELOCITY, v_z, FOR JANUARY 1963

Units: cm sec^{-1}

Latitude	Pressure (cb)				
	88.75	68.75	50.00	32.50	12.50
87.5	- 3.5	2.8	5.4	4.8	- 6.0
85.0	- 6.1	3.2	7.4	7.9	- 7.4
82.5	- 9.5	2.6	8.5	11.7	- 7.1
80.0	-13.3	0.8	8.7	16.0	- 5.2
77.5	-17.2	- 1.7	8.0	20.3	- 2.0
75.0	-20.3	- 4.4	6.6	24.0	1.7
72.5	-21.6	- 6.6	4.7	26.0	4.7
70.0	-20.3	- 7.7	2.7	25.7	6.0
67.5	-15.7	- 7.0	1.0	22.6	4.7
65.0	- 7.9	- 4.1	- 0.2	16.4	0.3
62.5	3.0	0.8	- 0.6	7.4	- 7.3
60.0	15.9	7.4	- 0.2	- 3.4	-17.3
57.5	29.5	14.9	0.8	-14.9	-28.7
55.0	42.2	22.4	2.3	-25.8	-40.0
52.5	52.3	28.8	4.0	-34.5	-49.7
50.0	58.4	33.2	5.5	-39.9	-56.3
47.5	59.6	34.8	6.6	-41.1	-58.7
45.0	55.6	33.4	7.1	-38.0	-56.3
42.5	46.7	29.0	6.9	-30.9	-49.3
40.0	34.1	22.2	5.9	-20.8	-38.5
37.5	19.3	13.8	4.4	- 9.1	-25.1
35.0	4.1	5.0	2.6	2.6	-10.8
32.5	- 9.6	- 3.3	0.7	12.7	2.7
30.0	-20.3	-10.0	- 1.0	19.9	14.1
27.5	-27.0	-14.6	- 2.3	23.6	22.1
25.0	-29.4	-16.6	- 3.0	23.5	26.4
22.5	-27.8	-16.2	- 3.2	20.1	26.9
20.0	-23.1	-13.8	- 3.0	14.3	24.3

TABLE D-3

VERTICAL VELOCITY, ω_z, FORCED BY $f((uv)_z)$ FOR JANUARY 1963

Units: 10^{-5}mb sec^{-1}

Latitude	Pressure (cb)			
	77.5	60.0	40.0	25.0
87.5	5.2	3.9	- 0.8	- 3.6
85.0	5.1	4.3	0.4	- 2.7
82.5	5.0	4.8	1.9	- 1.3
80.0	4.7	5.2	3.5	0.1
77.5	4.1	5.2	4.6	1.3
75.0	3.1	4.4	4.9	2.0
72.5	1.6	3.0	4.1	1.9
70.0	- 0.1	0.8	2.1	1.1
67.5	- 2.0	- 1.9	- 0.7	- 0.4
65.0	- 3.9	- 4.9	- 4.1	- 2.4
62.5	- 5.5	- 7.5	- 7.5	- 4.5
60.0	- 6.7	- 9.6	-10.2	- 6.3
57.5	- 7.1	-10.6	-11.9	- 7.4
55.0	- 6.7	-10.4	-12.1	- 7.6
52.5	- 5.5	- 8.8	-10.6	- 6.8
50.0	- 3.6	- 6.1	- 7.6	- 5.0
47.5	- 1.3	- 2.5	- 3.5	- 2.4
45.0	1.2	1.4	1.2	0.6
42.5	3.5	5.1	5.7	3.6
40.0	5.3	8.1	9.5	6.1
37.5	6.5	10.0	11.9	7.8
35.0	6.7	10.6	12.7	8.5
32.5	6.2	9.8	11.8	8.1
30.0	4.9	7.9	9.5	6.7
27.5	3.2	5.1	6.1	4.6
25.0	1.2	2.1	2.3	2.2
22.5	- 0.6	- 0.9	- 1.3	- 0.2
20.0	- 2.0	- 3.2	- 4.2	- 2.2

TABLE D-4

MERIDIONAL VELOCITY, v_z, FORCED BY $f((uv)_z)$ FOR JANUARY 1963

Units: cm sec^{-1}

Latitude	Pressure (cb)				
	88.75	68.75	50.00	32.50	12.50
87.5	- 6.4	2.1	6.4	5.3	- 4.0
85.0	- 9.5	2.4	8.7	8.3	- 5.0
82.5	-12.6	1.9	9.9	11.5	- 4.8
80.0	-15.3	0.6	9.9	14.9	- 3.5
77.5	-17.3	- 1.2	8.7	18.0	- 1.3
75.0	-18.2	- 3.2	6.7	20.4	1.1
72.5	-17.7	- 4.9	4.2	21.5	3.1
70.0	-15.5	- 5.7	1.8	20.8	4.0
67.5	-11.3	- 5.2	- 0.1	18.1	3.1
65.0	- 5.4	- 3.3	- 1.1	13.3	0.1
62.5	1.9	0.2	- 1.2	6.6	- 4.9
60.0	10.0	4.9	- 0.2	- 1.2	-11.5
57.5	18.0	10.1	1.6	- 9.4	-19.0
55.0	25.1	15.4	3.9	-17.1	-26.2
52.5	30.4	19.8	6.1	-23.1	-32.3
50.0	33.2	22.7	7.9	-26.8	-36.1
47.5	33.2	23.6	8.9	-27.6	-36.9
45.0	30.2	22.3	8.8	-25.4	-34.6
42.5	24.6	18.9	7.6	-20.5	-29.2
40.0	17.1	13.8	5.5	-13.5	-21.3
37.5	8.5	7.6	2.7	- 5.5	-11.8
35.0	- 0.1	1.1	- 0.3	2.5	- 2.0
32.5	- 7.7	- 4.8	- 3.0	9.3	7.1
30.0	-13.5	- 9.4	- 5.2	14.2	14.4
27.5	-17.1	-12.3	- 6.4	16.6	19.2
25.0	-18.3	-13.4	- 6.5	16.4	21.2
22.5	-17.2	-12.7	- 5.8	14.0	20.6
20.0	-14.4	-10.5	- 4.3	10.0	17.8

TABLE D-5

VERTICAL VELOCITY, ω_z, FORCED BY $f((Tv)_z)$ FOR JANUARY 1963

Units: 10^{-5}mb sec^{-1}

Latitude	Pressure (cb)			
	77.5	60.0	40.0	25.0
87.5	- 2.3	- 2.8	- 2.0	- 1.7
85.0	- 1.6	- 1.9	- 1.4	- 1.3
82.5	- 0.7	- 0.8	- 0.4	- 0.6
80.0	0.3	0.5	0.6	0.1
77.5	1.2	1.6	1.4	0.6
75.0	1.7	2.2	1.9	1.0
72.5	1.7	2.2	1.8	1.0
70.0	1.1	1.4	1.1	0.6
67.5	0.1	0.1	- 0.1	- 0.2
65.0	- 1.2	- 1.7	- 1.7	- 1.2
62.5	- 2.7	- 3.6	- 3.3	- 2.2
60.0	- 4.0	- 5.2	- 4.8	- 3.2
57.5	- 4.8	- 6.4	- 5.8	- 3.9
55.0	- 5.1	- 6.7	- 6.1	- 4.2
52.5	- 4.7	- 6.2	- 5.7	- 4.0
50.0	- 3.6	- 4.8	- 4.6	- 3.3
47.5	- 2.0	- 2.8	- 2.8	- 2.2
45.0	- 0.1	- 0.3	- 0.7	- 0.9
42.5	1.8	2.1	1.5	0.6
40.0	3.4	4.3	3.4	1.9
37.5	4.6	5.8	4.8	3.0
35.0	5.0	6.5	5.6	3.6
32.5	4.8	6.3	5.6	3.8
30.0	4.0	5.3	4.9	3.6
27.5	2.7	3.7	3.7	2.9
25.0	1.2	1.8	2.2	2.1
22.5	- 0.3	0.0	0.6	1.1
20.0	- 1.4	- 1.5	- 0.7	0.2

TABLE D-6

MERIDIONAL VELOCITY, v_z, FORCED BY $f((Tv)_z)$ FOR JANUARY 1963

Units: cm sec^{-1}

Latitude	Pressure (cb)				
	88.75	68.75	50.00	32.50	12.50
87.5	2.8	0.8	- 1.0	- 0.5	- 1.9
85.0	3.4	0.9	- 1.3	- 0.4	- 2.4
82.5	3.1	0.7	- 1.4	0.2	- 2.3
80.0	1.9	0.2	- 1.1	1.1	- 1.7
77.5	0.1	- 0.4	- 0.7	2.3	- 0.6
75.0	- 2.0	- 1.2	- 0.1	3.6	0.6
72.5	- 3.8	- 1.7	0.4	4.6	1.6
70.0	- 4.8	- 2.0	0.9	4.9	2.0
67.5	- 4.4	- 1.7	1.1	4.5	1.6
65.0	- 2.5	- 0.9	1.0	3.1	0.2
62.5	1.1	0.6	0.6	0.8	- 2.3
60.0	5.9	2.5	- 0.1	- 2.2	- 5.7
57.5	11.5	4.7	- 0.8	- 5.5	- 9.7
55.0	17.1	7.0	- 1.6	- 8.7	-13.8
52.5	21.9	9.0	- 2.2	-11.4	-17.5
50.0	25.2	10.5	- 2.4	-13.1	-20.2
47.5	26.4	11.2	- 2.3	-13.5	-21.7
45.0	25.4	11.1	- 1.7	-12.6	-21.7
42.5	22.1	10.2	- 0.7	-10.4	-20.2
40.0	17.0	8.5	0.5	- 7.3	-17.3
37.5	10.8	6.3	1.7	- 3.6	-13.3
35.0	4.2	3.8	2.9	0.1	- 8.9
32.5	- 1.9	1.4	3.8	3.4	- 4.4
30.0	- 6.7	- 0.6	4.2	5.7	- 0.3
27.5	- 9.9	- 2.2	4.1	7.0	3.0
25.0	-11.2	- 3.2	3.5	7.1	5.2
22.5	-10.6	- 3.5	2.5	6.1	6.3
20.0	- 8.7	- 3.3	1.3	4.3	6.5

TABLE D-7

VERTICAL VELOCITY, ω_z, DUE TO LONG WAVES FOR JANUARY 1963

Units: 10^{-5}mb sec^{-1}

Latitude	Pressure (cb)			
	77.5	60.0	40.0	25.0
87.5	5.8	4.9	0.5	- 4.0
85.0	5.9	5.4	1.6	- 3.0
82.5	5.8	6.0	3.0	- 1.4
80.0	5.5	6.3	4.4	0.2
77.5	4.8	6.0	5.2	1.4
75.0	3.5	4.9	5.0	2.1
72.5	1.6	2.8	3.6	1.9
70.0	- 0.8	- 0.2	1.0	0.7
67.5	- 3.4	- 3.9	- 2.7	- 1.3
65.0	- 6.1	- 7.7	- 6.8	- 3.8
62.5	- 8.3	-11.2	-10.8	- 6.4
60.0	- 9.9	-13.8	-13.9	- 8.6
57.5	-10.4	-14.9	-15.6	- 9.9
55.0	- 9.7	-14.3	-15.5	-10.2
52.5	- 7.9	-11.9	-13.4	- 9.1
50.0	- 5.1	- 8.1	- 9.5	- 6.8
47.5	- 1.6	- 3.1	- 4.3	- 3.6
45.0	2.0	2.3	1.5	0.2
42.5	5.4	7.3	7.1	3.9
40.0	8.1	11.4	11.7	7.2
37.5	9.6	13.9	14.7	9.5
35.0	9.9	14.6	15.8	10.5
32.5	9.0	13.4	14.8	10.2
30.0	7.1	10.7	12.0	8.7
27.5	4.4	6.9	8.0	6.3
25.0	1.6	2.7	3.4	3.5
22.5	- 1.1	- 1.3	- 0.9	0.6
20.0	- 3.2	- 4.4	- 4.4	- 1.8

TABLE D-8

MERIDIONAL VELOCITY, v_z, DUE TO LONG WAVES FOR JANUARY 1963

Units: cm sec^{-1}

Latitude	Pressure (cb)				
	88.75	68.75	50.00	32.50	12.50
87.5	- 7.1	1.4	6.2	8.4	- 4.5
85.0	-10.8	1.4	8.4	12.6	- 5.5
82.5	-14.4	0.6	9.7	16.7	- 5.3
80.0	-17.7	- 0.8	10.0	20.5	- 3.8
77.5	-20.1	- 2.6	9.2	23.4	- 1.5
75.0	-21.1	- 4.4	7.5	25.0	1.1
72.5	-20.2	- 5.7	5.4	24.7	3.0
70.0	-16.8	- 5.9	3.1	22.2	3.4
67.5	-10.8	- 4.5	1.0	17.3	1.6
65.0	- 2.2	- 1.4	- 0.4	10.1	- 2.7
62.5	8.3	3.3	- 1.0	1.1	- 9.6
60.0	19.8	9.2	- 0.8	- 8.8	-18.4
57.5	31.3	15.7	0.3	-18.7	-28.2
55.0	41.3	22.0	1.9	-27.4	-37.7
52.5	48.7	27.2	3.9	-33.8	-45.6
50.0	52.4	30.4	5.7	-37.0	-50.8
47.5	51.9	31.3	7.1	-36.6	-52.3
45.0	47.0	29.5	7.9	-32.6	-49.8
42.5	38.4	25.3	7.9	-25.4	-43.4
40.0	27.0	19.0	7.2	-16.1	-33.7
37.5	14.2	11.6	5.7	- 5.7	-22.1
35.0	1.5	3.8	3.9	4.3	- 9.7
32.5	- 9.7	- 3.3	1.9	12.7	1.9
30.0	-18.2	- 9.0	0.1	18.5	11.5
27.5	-23.3	-12.7	- 1.5	21.2	18.3
25.0	-24.7	-14.2	- 2.5	20.7	21.7
22.5	-22.9	-13.7	- 2.9	17.5	22.0
20.0	-18.6	-11.5	- 2.9	12.3	19.6

TABLE D-9

VERTICAL VELOCITY, ω_z, DUE TO MEDIUM WAVES FOR JANUARY 1963

Units: 10^{-5} mb sec^{-1}

Latitude	Pressure (cb)			
	77.5	60.0	40.0	25.0
87.5	- 2.4	- 2.9	- 2.6	- 1.3
85.0	- 1.9	- 2.3	- 2.0	- 1.0
82.5	- 1.2	- 1.4	- 1.1	- 0.6
80.0	- 0.4	- 0.3	- 0.2	- 0.1
77.5	0.4	0.7	0.8	0.4
75.0	1.1	1.5	1.5	0.8
72.5	1.5	2.0	1.9	1.0
70.0	1.5	2.0	1.9	1.0
67.5	1.3	1.7	1.5	0.7
65.0	0.8	1.0	0.8	0.3
62.5	0.1	0.1	- 0.1	- 0.2
60.0	- 0.6	- 0.9	- 1.0	- 0.7
57.5	- 1.3	- 1.7	- 1.8	- 1.1
55.0	- 1.7	- 2.3	- 2.3	- 1.4
52.5	- 1.9	- 2.6	- 2.4	- 1.5
50.0	- 1.8	- 2.4	- 2.2	- 1.3
47.5	- 1.4	- 1.8	- 1.7	- 1.0
45.0	- 0.8	- 1.0	- 0.8	- 0.5
42.5	- 0.1	- 0.1	0.1	0.0
40.0	0.6	0.8	1.0	0.6
37.5	1.1	1.6	1.7	1.1
35.0	1.5	2.1	2.1	1.4
32.5	1.6	2.2	2.2	1.5
30.0	1.5	2.1	2.0	1.4
27.5	1.2	1.6	1.6	1.2
25.0	0.7	1.0	0.9	0.9
22.5	0.3	0.4	0.3	0.5
20.0	- 0.2	- 0.2	- 0.3	0.1

TABLE D-10

MERIDIONAL VELOCITY, v_z, DUE TO MEDIUM WAVES FOR JANUARY 1963

Units: cm sec^{-1}

Latitude	Pressure (cb)				
	88.75	68.75	50.00	32.50	12.50
87.5	2.9	0.9	- 0.5	- 2.2	- 1.5
85.0	3.8	1.1	- 0.7	- 2.9	- 1.9
82.5	4.0	1.1	- 0.9	- 2.9	- 1.9
80.0	3.4	0.8	- 0.9	- 2.4	- 1.5
77.5	2.2	0.2	- 0.8	- 1.3	- 0.8
75.0	0.5	- 0.5	- 0.6	0.2	0.2
72.5	- 1.4	- 1.2	- 0.3	1.8	1.3
70.0	- 3.1	- 1.9	- 0.1	3.3	2.2
67.5	- 4.4	- 2.3	0.2	4.4	2.8
65.0	- 4.9	- 2.4	0.5	4.8	2.9
62.5	- 4.7	- 2.2	0.6	4.6	2.5
60.0	- 3.5	- 1.6	0.7	3.7	1.5
57.5	- 1.8	- 0.7	0.7	2.3	0.2
55.0	0.5	0.4	0.6	0.5	- 1.4
52.5	2.8	1.4	0.4	- 1.3	- 3.0
50.0	4.8	2.3	0.1	- 2.8	- 4.3
47.5	6.3	2.9	- 0.2	- 3.9	- 5.2
45.0	7.0	3.2	- 0.4	- 4.3	- 5.6
42.5	6.8	3.1	- 0.7	- 4.0	- 5.3
40.0	5.8	2.6	- 0.9	- 3.2	- 4.5
37.5	4.2	1.8	- 1.0	- 1.9	- 3.1
35.0	2.2	0.9	- 1.0	- 0.4	- 1.5
32.5	0.2	- 0.1	- 1.0	0.9	0.2
30.0	- 1.7	- 1.0	- 0.9	2.0	1.8
27.5	- 3.1	- 1.7	- 0.8	2.6	3.1
25.0	- 4.0	- 2.1	- 0.7	2.7	4.0
22.5	- 4.2	- 2.2	- 0.5	2.3	4.4
20.0	- 3.9	- 2.1	- 0.4	1.5	4.4

TABLE D-11

VERTICAL VELOCITY, ω_z, DUE TO SHORT WAVES FOR JANUARY 1963

Units: 10^{-5}mb sec^{-1}

Latitude	Pressure (cb)			
	77.5	60.0	40.0	25.0
87.5	- 0.5	- 0.9	- 0.7	0.0
85.0	- 0.4	- 0.7	- 0.6	0.0
82.5	- 0.3	- 0.5	- 0.4	0.1
80.0	- 0.1	- 0.3	- 0.2	0.1
77.5	0.0	- 0.0	0.1	0.1
75.0	0.2	0.2	0.2	0.1
72.5	0.3	0.4	0.4	0.1
70.0	0.3	0.4	0.4	0.0
67.5	0.2	0.3	0.3	0.0
65.0	0.1	0.2	0.2	- 0.1
62.5	0.0	0.0	0.0	- 0.2
60.0	- 0.2	- 0.2	- 0.2	- 0.3
57.5	- 0.3	- 0.3	- 0.3	- 0.3
55.0	- 0.4	- 0.5	- 0.4	- 0.3
52.5	- 0.4	- 0.5	- 0.5	- 0.2
50.0	- 0.3	- 0.5	- 0.4	- 0.1
47.5	- 0.2	- 0.4	- 0.3	0.0
45.0	- 0.1	- 0.2	- 0.2	0.1
42.5	0.0	0.0	0.0	0.2
40.0	0.2	0.2	0.2	0.3
37.5	0.3	0.3	0.3	0.3
35.0	0.3	0.4	0.4	0.3
32.5	0.3	0.4	0.4	0.2
30.0	0.3	0.4	0.3	0.1
27.5	0.2	0.3	0.2	0.0
25.0	0.1	0.2	0.1	- 0.1
22.5	0.0	0.0	- 0.1	- 0.2
20.0	- 0.1	- 0.1	- 0.2	- 0.2

TABLE D-12

MERIDIONAL VELOCITY, v_z, DUE TO SHORT WAVES FOR JANUARY 1963

Units: cm sec^{-1}

Latitude	Pressure (cb)				
	88.75	68.75	50.00	32.50	12.50
87.5	0.7	0.6	- 0.2	- 1.4	0.0
85.0	0.9	0.7	- 0.3	- 1.9	0.0
82.5	1.0	0.8	- 0.4	- 2.1	0.1
80.0	0.9	0.8	- 0.4	- 2.1	0.2
77.5	0.7	0.7	- 0.4	- 1.8	0.3
75.0	0.3	0.5	- 0.4	- 1.2	0.3
72.5	0.0	0.3	- 0.3	- 0.5	0.4
70.0	- 0.3	0.1	- 0.3	0.2	0.4
67.5	- 0.6	- 0.1	- 0.2	0.9	0.3
65.0	- 0.7	- 0.3	- 0.2	1.4	0.1
62.5	- 0.6	- 0.3	- 0.2	1.7	- 0.1
60.0	- 0.4	- 0.3	- 0.2	1.7	- 0.4
57.5	0.0	- 0.2	- 0.2	1.5	- 0.7
55.0	0.4	0.0	- 0.2	1.1	- 0.9
52.5	0.9	0.2	- 0.3	0.6	- 1.1
50.0	1.3	0.4	- 0.3	0.0	- 1.2
47.5	1.5	0.6	- 0.3	- 0.6	- 1.1
45.0	1.6	0.7	- 0.4	- 1.1	- 0.9
42.5	1.5	0.7	- 0.4	- 1.4	- 0.6
40.0	1.2	0.6	- 0.4	- 1.5	- 0.3
37.5	0.8	0.5	- 0.3	- 1.5	0.1
35.0	0.4	0.3	- 0.3	- 1.3	0.4
32.5	0.0	0.1	- 0.2	- 0.9	0.7
30.0	- 0.4	- 0.1	- 0.1	- 0.6	0.8
27.5	- 0.6	- 0.2	0.0	- 0.2	0.8
25.0	- 0.8	- 0.3	0.1	0.1	0.7
22.5	- 0.8	- 0.3	0.2	0.4	0.5
20.0	0.6	0.2	0.2	0.5	0.2

TABLE D-13

VERTICAL VELOCITY, ω_z, FOR JANUARY 1962

Units: 10^{-5}mb sec^{-1}

Latitude	Pressure (cb)			
	77.5	60.0	40.0	25.0
87.5	10.4	12.7	9.9	4.1
85.0	9.8	12.1	9.6	4.2
82.5	8.6	10.9	9.1	4.1
80.0	7.0	9.2	8.0	3.9
77.5	5.0	6.7	6.4	3.3
75.0	2.5	3.6	3.9	2.2
72.5	- 0.3	0.0	0.8	0.7
70.0	- 3.2	- 4.0	- 2.9	- 1.2
67.5	- 6.0	- 7.9	- 6.8	- 3.3
65.0	- 8.3	-11.4	-10.5	- 5.5
62.5	-10.0	-14.0	-13.5	- 7.3
60.0	-10.7	-15.4	-15.3	- 8.6
57.5	-10.4	-15.2	-15.6	- 9.1
55.0	- 9.0	-13.5	-14.3	- 8.6
52.5	- 6.6	-10.2	-11.3	- 7.1
50.0	- 3.5	- 5.8	- 6.9	- 4.9
47.5	0.0	- 0.7	- 1.8	- 2.0
45.0	3.4	4.4	3.5	1.0
42.5	6.3	8.9	8.4	3.9
40.0	8.5	12.2	12.1	6.3
37.5	9.6	14.1	14.3	7.8
35.0	9.5	14.2	14.7	8.4
32.5	8.4	12.6	13.3	8.1
30.0	6.4	9.8	10.5	6.8
27.5	3.9	6.1	6.8	4.9
25.0	1.2	2.1	2.7	2.7
22.5	- 1.2	- 1.5	- 1.1	0.6
20.0	- 3.0	- 4.3	- 4.1	- 1.2

TABLE D-14

MERIDIONAL VELOCITY, v_z, FOR JANUARY 1962

Units: cm sec^{-1}

Latitude	Pressure (cb)				
	88.75	68.75	50.00	32.50	12.50
87.5	-12.9	- 3.6	3.9	10.6	4.6
85.0	-18.5	- 5.5	5.4	15.4	6.9
82.5	-23.0	- 7.3	6.1	19.5	9.2
80.0	-26.0	- 8.8	6.1	22.3	11.3
77.5	-27.0	- 9.9	5.4	23.6	12.7
75.0	-25.7	-10.1	4.2	22.9	13.1
72.5	-21.7	- 9.2	2.5	19.8	12.1
70.0	-15.1	- 6.9	0.7	14.2	9.3
67.5	- 6.1	- 3.1	- 0.8	6.3	4.6
65.0	4.7	2.1	- 2.0	- 3.7	- 2.0
62.5	16.7	8.3	- 2.6	-14.8	- 9.9
60.0	28.7	15.1	- 2.5	-26.1	-18.7
57.5	39.5	21.8	- 1.8	-36.5	-27.5
55.0	48.1	27.5	- 0.6	-44.7	-35.3
52.5	53.5	31.7	0.9	-49.8	-41.2
50.0	55.0	33.6	2.5	-51.1	-44.4
47.5	52.4	33.1	3.9	-48.2	-44.5
45.0	45.9	30.0	4.9	-41.4	-41.4
42.5	36.2	24.8	5.4	-31.4	-35.3
40.0	24.3	17.9	5.4	-19.5	-27.0
37.5	11.6	10.2	4.9	- 6.9	-17.4
35.0	- 0.5	2.5	4.0	4.8	- 7.4
32.5	-10.9	- 4.2	2.9	14.5	1.7
30.0	-18.5	- 9.5	1.8	21.0	9.2
27.5	-22.9	-12.7	0.8	23.9	14.5
25.0	-23.9	-13.8	- 0.1	23.3	17.2
22.5	-22.0	-13.0	- 0.6	19.6	17.6
20.0	-17.9	-10.7	- 0.9	13.9	16.0

TABLE D-15

VERTICAL VELOCITY, ω_z, FOR APRIL 1962

Units: 10^{-5} mb sec^{-1}

Latitude	Pressure (cb)			
	77.5	60.0	40.0	25.0
87.5	- 2.4	- 4.7	- 5.7	- 9.5
85.0	- 1.8	- 3.7	- 4.5	- 8.1
82.5	- 1.0	- 2.2	- 2.7	- 6.0
80.0	- 0.1	- 0.7	- 0.8	- 3.5
77.5	0.5	0.6	0.8	- 1.2
75.0	0.8	1.2	1.7	0.7
72.5	0.5	1.0	1.8	1.8
70.0	- 0.3	0.0	0.8	2.0
67.5	- 1.5	- 1.7	- 1.0	1.4
65.0	- 2.9	- 3.9	- 3.3	0.0
62.5	- 4.3	- 6.0	- 5.8	- 1.7
60.0	- 5.4	- 7.8	- 7.8	- 3.5
57.5	- 5.9	- 8.7	- 9.0	- 4.8
55.0	- 5.7	- 8.5	- 9.0	- 5.5
52.5	- 4.7	- 7.1	- 7.7	- 5.2
50.0	- 2.9	- 4.6	- 5.1	- 4.1
47.5	- 0.8	- 1.3	- 1.7	- 2.2
45.0	1.6	2.3	2.1	0.1
42.5	3.9	5.7	5.8	2.5
40.0	5.6	8.4	8.7	4.7
37.5	6.6	10.0	10.5	6.2
35.0	6.8	10.3	10.9	6.8
32.5	6.0	9.2	9.9	6.5
30.0	4.6	7.0	7.6	5.4
27.5	2.7	4.1	4.5	3.7
25.0	0.6	0.9	1.1	1.6
22.5	- 1.3	- 2.0	- 2.0	- 0.4
20.0	- 2.8	- 4.2	- 4.4	- 2.0

TABLE D-16

MERIDIONAL VELOCITY, v_z, FOR APRIL 1962

Units: cm sec^{-1}

Latitude	Pressure (cb)				
	88.75	68.75	50.00	32.50	12.50
87.5	2.9	3.7	1.5	7.0	-10.6
85.0	3.7	4.8	1.9	10.2	-14.3
82.5	3.7	5.2	2.0	12.8	-16.2
80.0	2.9	4.8	1.7	14.7	-16.1
77.5	1.8	3.7	1.0	15.4	-14.3
75.0	0.5	2.4	0.1	14.8	-11.1
72.5	- 0.1	1.2	- 1.0	12.7	- 7.6
70.0	0.2	0.6	- 2.0	8.9	- 4.4
67.5	2.0	1.0	- 2.8	3.6	- 2.4
65.0	5.4	2.5	- 3.3	- 3.0	- 2.2
62.5	10.3	5.1	- 3.4	-10.2	- 3.9
60.0	16.1	8.6	- 3.2	-17.5	- 7.5
57.5	22.3	12.5	- 2.6	-24.0	-12.3
55.0	27.8	16.2	- 1.7	-29.0	-17.6
52.5	32.0	19.2	- 0.9	-31.8	-22.4
50.0	33.9	20.8	- 0.1	-32.0	-25.8
47.5	33.2	20.7	0.5	-29.4	-27.1
45.0	29.7	18.7	0.8	-24.4	-25.8
42.5	23.7	15.0	0.6	-17.3	-21.9
40.0	15.9	10.0	0.2	- 9.1	-15.9
37.5	7.1	4.3	- 0.5	- 0.7	- 8.5
35.0	- 1.4	- 1.5	- 1.4	6.9	- 0.7
32.5	- 8.9	- 6.4	- 2.3	12.8	6.6
30.0	-14.3	-10.1	- 3.0	16.5	12.5
27.5	-17.2	-12.2	- 3.5	17.6	16.3
25.0	-17.6	-12.4	- 3.7	16.3	17.7
22.5	-15.6	-11.1	- 3.6	12.9	16.9
20.0	-11.9	- 8.6	- 3.3	8.3	14.4

TABLE D-17

VERTICAL VELOCITY, ω_z, FOR JULY 1962

Units: 10^{-5}mb sec^{-1}

Latitude	Pressure (cb)			
	77.5	60.0	40.0	25.0
87.5	- 0.3	- 0.9	- 0.6	- 2.9
85.0	- 0.5	- 1.1	- 1.0	- 2.9
82.5	- 0.8	- 1.5	- 1.5	- 2.9
80.0	- 1.1	- 2.0	- 2.2	- 3.0
77.5	- 1.4	- 2.5	- 2.9	- 3.1
75.0	- 1.7	- 2.9	- 3.5	- 3.2
72.5	- 1.9	- 3.2	- 4.0	- 3.2
70.0	- 2.0	- 3.3	- 4.2	- 3.2
67.5	- 1.9	- 3.2	- 4.2	- 3.2
65.0	- 1.7	- 2.9	- 3.9	- 3.1
62.5	- 1.4	- 2.3	- 3.3	- 2.8
60.0	- 0.9	- 1.6	- 2.5	- 2.4
57.5	- 0.4	- 0.8	- 1.5	- 1.8
55.0	0.1	- 0.0	- 0.4	- 1.1
52.5	0.6	0.8	0.6	- 0.3
50.0	1.0	1.5	1.6	0.6
47.5	1.3	2.0	2.4	1.4
45.0	1.5	2.4	2.9	2.0
42.5	1.5	2.5	3.2	2.6
40.0	1.5	2.4	3.2	2.9
37.5	1.3	2.2	2.9	2.9
35.0	1.0	1.8	2.5	2.7
32.5	0.8	1.4	1.9	2.3
30.0	0.4	0.9	1.3	1.8
27.5	0.2	0.4	0.6	1.1
25.0	- 0.1	- 0.1	0.1	0.5
22.5	- 0.3	- 0.4	- 0.4	- 0.1
20.0	- 0.4	- 0.6	- 0.7	- 0.5

TABLE D-18

MERIDIONAL VELOCITY, v_z, FOR JULY 1962

Units: cm sec^{-1}

Latitude	Pressure (cb)				
	88.75	68.75	50.00	32.50	12.50
87.5	0.4	0.9	- 0.4	4.2	- 3.2
85.0	0.9	1.4	- 0.5	5.7	- 4.8
82.5	1.5	2.1	- 0.4	6.5	- 6.5
80.0	2.5	3.0	- 0.0	6.4	- 8.2
77.5	3.8	4.1	0.5	5.5	-10.0
75.0	5.3	5.3	1.3	3.9	-11.9
72.5	7.0	6.6	2.2	2.0	-13.8
70.0	8.6	7.9	3.2	- 0.1	-15.8
67.5	10.1	9.0	4.3	- 2.0	-17.7
65.0	11.3	10.0	5.4	- 3.3	-19.4
62.5	12.0	10.7	6.3	- 4.1	-20.8
60.0	12.3	11.0	6.9	- 4.0	-21.9
57.5	11.9	10.9	7.4	- 3.2	-22.3
55.0	11.1	10.3	7.5	- 1.8	-22.1
52.5	9.7	9.4	7.3	- 0.0	-21.1
50.0	8.0	8.1	6.7	1.9	-19.4
47.5	6.0	6.6	5.9	3.7	-17.0
45.0	3.9	4.9	4.9	5.1	-13.9
42.5	1.8	3.2	3.8	6.0	-10.5
40.0	- 0.1	1.5	2.6	6.3	- 6.9
37.5	- 1.7	0.0	1.5	6.1	- 3.4
35.0	- 2.9	- 1.2	0.6	5.5	- 0.2
32.5	- 3.8	- 2.1	- 0.2	4.5	2.4
30.0	- 4.2	- 2.7	- 0.8	3.5	4.3
27.5	- 4.3	- 3.0	- 1.2	2.4	5.5
25.0	- 4.1	- 3.0	- 1.3	1.6	5.9
22.5	- 3.7	- 2.8	- 1.3	1.0	5.7
20.0	- 3.1	- 2.4	- 1.2	0.6	5.0

TABLE D-19

VERTICAL VELOCITY, ω_z, FOR OCTOBER 1962

Units: 10^{-5} mb sec^{-1}

Latitude	Pressure (cb)			
	77.5	60.0	40.0	25.0
87.5	7.5	9.9	8.2	2.9
85.0	6.6	8.6	7.2	2.6
82.5	5.0	6.6	5.6	2.1
80.0	3.1	4.0	3.5	1.4
77.5	0.9	1.1	1.0	0.4
75.0	- 1.3	- 1.9	- 1.6	- 0.7
72.5	- 3.3	- 4.6	- 4.2	- 2.1
70.0	- 4.9	- 7.0	- 6.5	- 3.5
67.5	- 6.0	- 8.7	- 8.4	- 4.8
65.0	- 6.5	- 9.5	- 9.6	- 5.8
62.5	- 6.4	- 9.5	-10.0	- 6.5
60.0	- 5.7	- 8.6	- 9.6	- 6.7
57.5	- 4.4	- 7.0	- 8.2	- 6.3
55.0	- 2.8	- 4.7	- 6.1	- 5.2
52.5	- 1.0	- 2.1	- 3.4	- 3.6
50.0	0.8	0.7	- 0.5	- 1.6
47.5	2.5	3.3	2.5	0.6
45.0	3.8	5.4	5.2	2.7
42.5	4.6	6.9	7.3	4.6
40.0	5.0	7.7	8.6	6.0
37.5	4.9	7.7	9.0	6.8
35.0	4.3	7.0	8.5	6.8
32.5	3.4	5.7	7.3	6.2
30.0	2.3	4.0	5.4	5.0
27.5	1.1	2.1	3.3	3.5
25.0	0.0	0.3	1.1	1.8
22.5	- 0.9	- 1.2	- 0.9	0.2
20.0	- 1.5	- 2.3	- 2.4	- 1.1

TABLE D-20

MERIDIONAL VELOCITY, v_z, FOR OCTOBER 1962

Units: cm sec^{-1}

Latitude	Pressure (cb)				
	88.75	68.75	50.00	32.50	12.50
87.5	- 9.3	- 3.8	2.5	9.7	3.3
85.0	-12.8	- 5.2	3.3	13.2	4.6
82.5	-14.7	- 6.1	3.7	15.2	5.4
80.0	-14.8	- 6.1	3.6	15.2	5.6
77.5	-13.0	- 5.3	3.0	13.3	5.0
75.0	- 9.2	- 3.6	2.2	9.5	3.4
72.5	- 3.9	- 1.0	1.3	4.2	0.6
70.0	2.7	2.4	0.5	- 2.0	- 3.3
67.5	9.9	6.3	0.1	- 8.6	- 8.2
65.0	17.0	10.5	0.2	-14.8	-14.0
62.5	23.5	14.6	0.9	-20.0	-20.0
60.0	28.7	18.2	2.1	-23.8	-25.9
57.5	32.2	21.0	3.6	-25.8	-31.1
55.0	33.6	22.7	5.3	-25.8	-34.9
52.5	32.9	23.1	7.0	-24.0	-36.9
50.0	30.2	22.0	8.2	-20.6	-36.8
47.5	25.7	19.7	8.8	-16.0	-34.4
45.0	19.9	16.2	8.7	-10.7	-29.8
42.5	13.4	11.8	7.9	- 5.3	-23.4
40.0	6.7	7.0	6.4	- 0.3	-15.9
37.5	0.5	2.3	4.3	3.9	- 7.8
35.0	- 4.9	- 2.1	2.0	7.0	0.0
32.5	- 8.9	- 5.7	- 0.3	8.8	6.9
30.0	-11.5	- 8.2	- 2.3	9.3	12.3
27.5	-12.6	- 9.5	- 3.9	8.7	15.9
25.0	-12.3	- 9.8	- 4.9	7.1	17.6
22.5	-11.0	- 9.0	- 5.3	5.0	17.5
20.0	- 9.0	- 7.6	- 5.1	2.7	15.9

TABLE D-21

VERTICAL VELOCITY, ω_z, FOR JANUARY 1964

Unit: 10^{-5}mb sec^{-1}

Latitude	Pressure (cb)							
	100.0	92.5	77.5	60.0	40.0	25.0	17.5	12.5
87.5	0.1	2.3	4.3	3.7	0.4	- 1.0	0.6	1.6
85.0	0.6	2.6	4.5	4.0	0.9	- 0.8	0.4	1.3
82.5	1.4	3.0	4.7	4.4	1.6	- 0.5	0.1	0.7
80.0	2.2	3.3	4.6	4.6	2.2	- 0.2	- 0.2	0.1
77.5	2.8	3.3	4.1	4.2	2.4	0.0	- 0.6	- 0.6
75.0	3.0	2.9	3.0	3.2	2.0	- 0.1	- 1.0	- 1.2
72.5	2.6	1.9	1.3	1.4	0.9	- 0.4	- 1.4	- 1.7
70.0	1.7	0.4	- 0.9	- 1.2	- 0.9	- 1.0	- 1.7	- 1.9
67.5	0.3	- 1.4	- 3.4	- 4.2	- 3.2	- 1.9	- 1.9	- 1.9
65.0	- 1.4	- 3.3	- 6.1	- 7.3	- 5.7	- 2.8	- 2.0	- 1.7
62.5	- 3.1	- 5.1	- 8.3	-10.1	- 8.1	- 3.6	- 2.0	- 1.4
60.0	- 4.5	- 6.5	- 9.8	-12.0	-10.0	- 4.3	- 1.8	- 0.9
57.5	- 5.3	- 7.1	-10.4	-12.7	-10.7	- 4.6	- 1.6	- 0.4
55.0	- 5.4	- 6.8	- 9.6	-12.0	-10.3	- 4.4	- 1.2	0.1
52.5	- 4.7	- 5.6	- 7.7	- 9.7	- 8.6	- 3.8	- 0.8	0.5
50.0	- 3.3	- 3.6	- 4.8	- 6.2	- 5.8	- 2.7	- 0.3	0.8
47.5	- 1.2	- 1.0	- 1.1	- 1.8	- 2.2	- 1.2	0.2	0.9
45.0	1.1	1.9	2.8	3.0	0.7	0.5	0.6	0.9
42.5	3.4	4.6	6.4	7.4	5.6	2.2	1.0	0.8
40.0	5.3	6.8	9.3	11.0	8.8	3.6	1.4	0.6
37.5	6.6	8.1	11.1	13.3	11.0	4.8	1.7	0.5
35.0	7.1	8.6	11.6	14.0	11.9	5.4	1.8	0.3
32.5	6.7	8.0	10.8	13.1	11.5	5.5	1.9	0.2
30.0	5.5	6.5	8.8	10.8	9.8	5.1	1.8	0.2
27.5	3.7	4.5	6.0	7.5	7.3	4.2	1.7	0.2
25.0	1.7	2.1	2.9	3.7	4.2	3.1	1.4	0.3
22.5	- 0.3	- 0.2	- 0.2	0.1	1.2	1.8	1.1	0.3
20.0	- 2.0	- 2.1	- 2.7	- 3.0	- 1.5	0.6	0.8	0.3

TABLE D-22

MERIDIONAL VELOCITY, v_z, FOR JANUARY 1964

Unit: cm sec^{-1}

Latitude	Pressure (cb)							
	96.25	85.00	68.75	50.00	32.50	21.25	15.00	6.25
87.5	- 8.3	- 3.8	1.0	4.6	2.6	- 6.0	- 5.9	3.6
85.0	-11.6	- 5.6	1.0	6.5	4.3	- 7.7	- 8.0	4.4
82.5	-14.0	- 6.7	0.7	7.9	6.4	- 7.7	- 9.0	4.2
80.0	-15.1	- 7.9	- 0.1	8.6	8.5	- 6.3	- 9.0	2.6
77.5	-14.6	- 8.4	- 1.1	8.7	10.5	- 3.6	- 8.1	- 0.1
75.0	-12.3	- 8.0	- 1.8	8.2	12.0	- 3.8	- 6.4	- 3.4
72.5	- 8.8	- 6.5	- 1.7	6.7	11.7	2.4	- 5.1	- 7.8
70.0	- 3.3	- 4.3	- 2.3	5.2	10.2	4.3	- 3.6	-11.5
67.5	3.3	- 0.4	- 0.8	3.2	6.6	4.1	- 3.1	-14.6
65.0	10.7	4.7	1.5	1.0	0.7	1.2	- 4.2	-16.8
62.5	18.3	10.5	5.0	- 1.0	- 6.8	- 4.1	- 6.5	-17.6
60.0	25.6	16.8	9.4	- 2.6	-15.2	-11.5	-10.0	-16.9
57.5	31.8	23.0	14.0	- 3.8	-23.9	-20.3	-14.5	-15.0
55.0	36.2	28.4	18.3	- 4.5	-31.7	-29.4	-19.6	-12.4
52.5	38.6	32.2	21.8	- 4.0	-37.6	-37.4	-24.4	- 9.2
50.0	41.0	34.1	24.2	- 3.4	-40.1	-42.7	-27.8	- 5.6
47.5	36.0	34.1	24.0	- 2.7	-40.3	-45.7	-30.8	- 3.5
45.0	31.3	30.7	22.4	- 1.2	-36.4	-44.4	-31.2	- 1.6
42.5	24.8	25.7	19.0	0.5	-29.5	-39.4	-29.5	- 0.7
40.0	17.1	17.4	14.0	2.0	-20.4	-37.1	-25.7	- 0.8
37.5	9.1	9.5	8.3	3.4	- 9.8	-20.3	-20.0	- 1.5
35.0	1.3	1.5	2.3	4.2	0.6	- 8.5	-13.1	- 2.7
32.5	- 5.5	- 5.6	- 3.3	4.5	9.8	3.3	- 5.5	- 4.0
30.0	-10.7	-11.2	- 7.9	4.3	16.8	13.6	2.2	- 4.9
27.5	-14.2	-14.9	-11.1	3.3	21.0	21.2	9.1	- 5.4
25.0	-15.7	-16.5	-12.6	2.1	22.2	26.8	14.9	- 5.1
22.5	-15.5	-17.2	-12.6	0.6	20.7	28.9	19.1	- 4.2
20.0	-14.1	-14.4	-11.4	- 1.0	17.0	28.1	21.7	- 2.9

TABLE D-23

DIABATIC HEATING, H_z, FOR JANUARY 1963

Unit: 10^{-2}kj $sec^{-1}t^{-1}$

Latitude	Pressure (cb)			
	77.5	60.0	40.0	25.0
87.5	- 0.5	- 0.2	- 0.7	- 0.3
85.0	- 1.3	- 0.6	- 0.8	- 0.3
82.5	- 1.9	- 1.0	- 0.3	0.0
80.0	- 2.0	- 1.2	- 0.5	- 0.2
77.5	- 1.9	- 1.5	- 1.1	- 0.7
75.0	- 1.6	- 1.5	- 1.2	- 1.1
72.5	- 1.3	- 1.3	- 1.2	- 0.9
70.0	- 1.2	- 1.3	- 1.4	- 0.5
67.5	- 1.4	- 1.4	- 1.3	- 0.4
65.0	- 1.6	- 1.4	- 1.1	- 0.5
62.5	- 2.0	- 1.4	- 1.0	- 0.5
60.0	- 2.2	- 1.4	- 0.8	- 0.4
57.5	- 1.9	- 1.1	- 0.4	- 0.3
55.0	- 1.4	- 0.8	0.0	- 0.1
52.5	- 0.9	- 0.8	- 0.1	0.0
50.0	- 0.7	- 0.9	- 0.6	- 0.1
47.5	- 0.5	- 0.7	- 0.8	- 0.2
45.0	- 0.3	- 0.3	- 0.7	- 0.4
42.5	0.2	0.1	- 0.6	- 0.6
40.0	0.9	0.7	- 0.2	- 0.6
37.5	1.5	1.2	0.3	- 0.3
35.0	1.7	1.5	0.9	0.3
32.5	1.6	1.3	1.3	0.8
30.0	1.4	1.0	1.2	0.9
27.5	1.1	0.7	0.7	0.6
25.0	0.7	0.5	0.3	0.3
22.5	0.4	0.3	0.1	0.1
20.0	0.2	0.2	0.2	0.1

TABLE D-24

MASS CIRCULATION AS A FUNCTION OF TIME FOR JANUARY 1964

Unit: 10^6 tons sec^{-1}

Lat. \ Time (day)	1	2	3	4	5	6	7	8	9	10
87.5	- 1	- 1	- 1	- 1	- 1	1	- 1	0	2	1
85.0	- 3	- 2	- 2	- 2	- 2	4	- 3	1	5	1
82.5	- 5	- 4	- 4	- 3	- 4	6	- 5	1	8	2
80.0	- 7	- 6	- 5	- 5	- 6	8	- 6	4	10	2
77.5	- 9	- 8	- 7	- 6	- 8	8	- 7	5	11	1
75.0	-11	- 9	- 8	- 7	- 9	8	- 8	6	9	- 4
72.5	-11	- 8	10	- 7	-10	2	- 8	8	3	- 8
70.0	-10	- 7	12	1	-10	2	8	10	0	-14
67.5	- 7	1	15	3	- 3	-11	10	12	- 3	-19
65.0	- 3	6	19	10	9	-12	12	13	-21	-23
62.5	6	12	22	15	10	10	13	15	-24	-23
60.0	14	18	26	20	14	7	15	15	-24	-19
57.5	23	24	28	26	23	11	17	15	- 7	-14
55.0	30	30	29	30	31	22	19	15	- 5	11
52.5	37	33	29	33	38	33	20	15	11	17
50.0	41	34	27	33	42	43	19	14	19	30
47.5	42	31	24	30	43	50	18	14	30	40
45.0	39	25	18	25	40	51	7	13	38	46
42.5	34	9	2	16	33	45	6	12	42	47
40.0	26	5	- 6	6	23	34	-11	7	40	42
37.5	17	-10	- 9	- 7	11	18	- 7	5	34	33
35.0	-10	-18	-16	-17	- 6	- 8	-11	1	24	20
32.5	-14	-27	-21	-24	-16	-25	-17	- 2	3	- 8
30.0	-18	-34	-24	-29	-25	-40	-22	- 9	- 7	-14
27.5	-24	-37	-26	-30	-31	-49	-26	-11	-19	-25
25.0	-28	-36	-24	-27	-32	-52	-27	-14	-29	-33
22.5	-28	-32	-21	-22	-29	-47	-26	-17	-35	-37
20.0	-26	-26	-16	-14	-23	-37	-24	-19	-39	-37

TABLE D-24 (continued)

MASS CIRCULATION AS A FUNCTION OF TIME FOR JANUARY 1964

Unit: 10^6 tons sec^{-1}

Time (day) / Lat.	11	12	13	14	15	16	17	18	19	20
87.5	- 1	- 2	- 2	1	- 1	- 1	- 2	- 2	0	0
85.0	- 1	- 4	- 6	4	- 2	- 4	- 6	- 4	0	- 1
82.5	- 2	- 8	-10	6	- 3	- 7	-11	- 7	- 1	- 2
80.0	- 2	-11	-14	7	- 4	- 9	-15	-10	- 1	- 2
77.5	- 2	-13	-18	1	- 5	-10	-17	-13	- 2	- 3
75.0	- 4	-14	-20	0	- 5	-10	-16	-13	- 2	3
72.5	- 6	-14	-20	- 8	- 6	- 7	-12	-13	- 2	4
70.0	- 9	-12	- 5	-12	- 8	- 2	7	-11	- 1	7
67.5	-12	- 9	-14	-16	-10	6	8	- 7	0	10
65.0	-13	1	- 9	-18	-10	15	17	4	4	13
62.5	-11	10	4	-19	- 9	24	27	8	5	18
60.0	- 9	14	6	-15	7	32	35	12	10	23
57.5	11	19	11	- 9	9	39	39	16	16	27
55.0	18	23	16	6	18	43	40	19	22	31
52.5	27	27	20	15	29	44	37	20	28	34
50.0	35	28	23	28	38	42	32	20	32	34
47.5	40	29	24	38	43	36	25	18	34	33
45.0	41	28	25	46	45	28	18	16	34	31
42.5	39	27	24	48	42	19	0	12	31	26
40.0	32	25	23	45	34	1	- 7	8	24	20
37.5	22	23	22	39	25	- 8	- 6	5	15	12
35.0	10	21	20	29	12	-15	- 7	4	4	4
32.5	- 6	2	18	18	-12	-22	- 8	- 7	-11	-14
30.0	-17	-16	16	- 1	-22	-26	- 9	- 8	-18	-19
27.5	-26	-16	3	-17	-31	-28	-11	- 7	-23	-23
25.0	-32	-17	1	-22	-36	-28	-13	- 9	-26	-25
22.5	-33	-16	- 7	-27	-36	-25	-16	-11	-26	-26
20.0	-32	-17	-10	-28	-33	-22	-20	-16	-25	-26

TABLE D-24 (continued)

MASS CIRCULATION AS A FUNCTION OF TIME FOR JANUARY 1964

Unit: 10^6 tons sec^{-1}

Lat. \ Time (day)	21	22	23	24	25	26	27	28	29	30
87.5	0	0	1	0	- 1	- 1	0	0	0	1
85.0	0	0	3	0	- 2	- 2	0	- 1	1	4
82.5	- 2	1	5	- 3	- 3	- 4	0	- 2	1	7
80.0	- 3	1	7	- 4	- 6	- 6	0	- 3	2	9
77.5	- 3	- 3	8	- 6	- 9	- 9	- 4	- 4	7	11
75.0	- 4	- 4	8	- 8	-13	-12	- 6	- 5	8	10
72.5	- 3	- 5	7	-10	-18	-15	- 8	- 6	10	9
70.0	- 3	- 5	6	-11	-22	-17	-10	- 7	11	6
67.5	- 4	- 6	5	-11	-23	-17	-12	- 8	12	2
65.0	2	- 4	6	-10	-22	-15	-11	2	14	- 6
62.5	8	- 2	8	- 8	-17	-10	- 9	10	14	- 7
60.0	11	5	9	- 9	-10	3	- 2	12	15	- 7
57.5	16	10	11	15	9	13	7	16	16	- 3
55.0	20	16	15	23	21	25	16	21	16	1
52.5	24	21	19	31	34	36	25	25	17	7
50.0	28	26	24	38	46	46	34	28	17	13
47.5	31	29	26	42	53	52	39	28	17	19
45.0	31	30	27	42	54	53	41	27	17	23
42.5	30	27	24	37	48	50	39	22	16	25
40.0	27	23	19	28	36	41	32	15	15	25
37.5	21	16	12	17	20	29	22	- 3	14	21
35.0	14	7	6	10	- 4	15	- 3	-11	1	15
32.5	1	- 5	- 9	-16	-17	- 7	- 9	-19	0	8
30.0	- 5	-10	-16	-24	-31	-16	-16	-27	-13	- 7
27.5	- 9	-16	-23	-31	-41	-25	-25	-31	-14	-12
25.0	-15	-20	-27	-34	-44	-30	-29	-33	-16	-15
22.5	-19	-20	-28	-33	-41	-31	-29	-30	-20	-19
20.0	-21	-19	-27	-29	-34	-29	-26	-27	-24	-22

TABLE D-25

$C(A_z, K_z)$ AS A FUNCTION OF PRESSURE AND TIME FOR JANUARY 1964

Unit: 10^{-6} kjm^{-2}sec^{-1}cb^{-1}

Days	Pressure (cb)							
	96.25	85.0	70.0	50.0	30.0	20.0	15.0	6.25
1	13	19	7	- 2	-30	-38	-26	12
2	8	7	3	- 5	- 5	1	5	15
3	8	9	6	- 3	-14	-14	-10	- 1
4	8	9	6	- 1	-10	-11	- 8	- 7
5	10	12	11	- 1	-18	-24	-24	-27
6	10	9	8	- 2	-22	-20	-11	- 8
7	4	5	4	1	- 2	- 5	- 7	-11
8	6	7	7	3	-12	-19	-20	-21
9	3	3	4	5	- 5	-19	-21	-16
10	3	3	2	5	2	-15	-22	-13
11	3	4	3	6	- 2	-19	-21	-19
12	5	6	5	0	-19	-25	-16	- 6
13	8	10	7	- 1	-16	-23	-16	- 8
14	9	10	8	1	-32	-41	-27	3
15	7	9	9	2	-25	-14	-21	- 4
16	7	8	7	2	- 6	- 8	-13	-13
17	5	8	6	- 1	-17	-24	-14	- 6
18	5	6	6	- 4	-21	-15	- 5	- 3
19	7	9	8	- 3	-20	-18	-12	-11
20	8	10	8	- 1	-18	-25	-33	-24
21	6	7	7	2	-24	-33	-27	-14
22	5	5	4	3	- 8	-16	-18	-14
23	3	4	3	- 1	- 8	-12	-11	- 2
24	3	5	3	1	9	- 5	- 5	- 1
25	0	8	4	- 1	- 2	6	5	9
26	10	11	9	3	-23	-37	-33	-15
27	8	8	7	0	-19	-25	-16	- 2
28	5	5	3	1	- 4	- 2	2	3
29	6	6	4	- 5	-23	-23	-10	13
30	6	7	6	- 4	-21	-20	-15	- 3

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