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Technical Report

PRESSURE DISTRIBUTION ON AND FLOW OF SAND PAST A RIGID WHEEL

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INTRODUCTION

The analytical approach to the relationship between load, drag, and sinkage of a rigid wheel based on the soil relationships developed by M. G. Bekker are outlined in Refs. 1 and 2. The results obtained with these methods have considerable accuracy, particularly as regards sinkage in sand;[2] the predicted values, however, do appear to diverge both at high and low loads, and particularly so when plastic soils are employed. The accuracy obtained, however, is sufficient for most engineering evaluations at the present time since there still remain many other unknowns in the soil relationship.

This paper is an attempt to contribute to the understanding of the fundamental nature of the process and the relationship between the variables, together with the manner in which the soil moves during the passage of a rigid wheel. These comments pertain to the use of a fine dry sand as the soil.

The present theory of wheel action assumes that the soil is compacted vertically by the passage of the wheel and that the pressure against the wheel surface at any depth below the free soil surface is given by a relation of the form $P = \left(\frac{k_c}{b} + k_\phi\right) Z^n$, where k_c and k_ϕ are cohesive and frictional moduli, and b is the narrowest dimension of the bevameter. Such an approach leads to a pressure distribution of the form shown in Fig. 1, where A B C D represents the normal pressure p_n exerted on the wheel face to produce the vertical component whose value is given by

$$p_v = \left(\frac{k_c}{b} + k_\phi\right) Z^n$$

It is seen that this method of attack, generally attributed to Bernstein and Letoshnev,[3,4] results in a discontinuity at the point of ground contact D where, theoretically, the wheel leaves the surface. But surface E D is also a free surface, and an instantaneous change of stress, at the surface, of the type shown does not appear reasonable since sinkage relative to E D is zero at D. Although the soil in this region is considered to have been compacted by the passage of the load and thus will have different values of k_c and k_ϕ compared with the original soil, any such change in soil properties does not enter into the relationships developed. Secondly, when the loads are high, the sinkage is great, which means that soil compaction would be of considerable magnitude. However, a well-settled sand is not readily compressible, at least not to the degree required by the relationships given, and, observing a wheel in motion through a typical dry sand, it is quite apparent that effects other than compaction do exist. It is believed that some compaction and/or elasticity exist at light loads, and in such cases the analytical treatment should take this into account. Under such small sinkages there is no particular problem in transporting loads of such magnitude over soils; the troubles all begin when sinkage begins to assume considerable magnitude. The effects on the soil observed due to the increasing load shows a general heaving of the soil over a considerable area round the wheel contact. This of course means that the sinkage of the wheel does not produce a corresponding compaction of the soil. When motion occurs, this heaving is transmitted ahead of the wheel at all times, approximately along the lines of the Rankine theory, but with motion there is also considerable bulldozing occurring ahead of the wheel with the result that a "bow wave" is formed. This "bow wave" in the case of sand builds up to a definite form and the slopes of the surface exceed the angle of repose of the soil, with the result that sand on top of the wave moves both forward and laterally relative to the wheel so that the displaced sand is eventually disposed of by flowing past the sides of the wheel and falling into the rut behind the wheel made by the sinkage. There is thus a flow process in addition to the one of compaction.

Some of these facts are discussed in detail below, and the requirements for a theory are established which will indicate a method of approach with conditions closer to those actually existing than is now the case.

CYCLOIDAL ACTION

In a theoretical analysis it is possible to assume some idealized conditions. Let it be assumed that it is possible for a towed wheel loaded with a weight of W lb, sinking to a depth of Z in., to roll without slipping on the soil (experiment has indicated slippage occurs even at quite light loads).

Under such assumptions, it follows that a particle of soil located in the free surface of the sand, at point "a" of Fig. 2, will travel along the path a a' a", a portion of a cycloid. Thus the soil is compacted the vertical distance ab and bulldozed a horizontal distance ba".

The equation for the path aa" is given by

$$\begin{aligned} x &= D/2\ (\theta - \sin\theta) \\ y &= D/2\ (1 - \cos\theta) \end{aligned} \qquad (1)$$

Let the wheel be rotated one revolution from position A to B. Then the soil at a' will have work done on it in moving from a' to a"; a corresponding particle with the wheel at position B has already had work done on it from a_1 to a_1' or a total distance of aa"; consideration of all other particles will also show that the work done in one complete revolution of the wheel is that of overcoming the resistance along the complete path such as c c' c". It follows that the work done on the soil echg in one revolution is the same as that done on a rectangular block edfg. In effect the block edfg is moved forward the distance ba" and depressed through the vertical distance ab.

The above method of approach does not take into account any flow past the wheel, but does show how a "bow wave" can be formed despite the assumption of compaction only. This analysis perhaps represents the process for small sinkages with some degree of accuracy, though even the smallest sinkages tend to show some small degree of flow.

Consider the above approach in a little greater detail:

On the present general assumption, viz. that the sand ahead of the wheel is undisturbed as far as wheel action is concerned, the horizontal component of the cycloidal motion of the soil must all be accounted for by compaction. Unless compaction exists to the degree assumed, the horizontal displacement produced by the cycloidal motion must result in a heaving of the soil ahead of the wheel and the effective value of Z will change. If compaction were zero, a volume of sand equal to that given in Eq. (2) must be displaced ahead of the wheel.

$$\text{Horizontal displacement} = bZ\pi D \text{ cu in/rev} \tag{2}$$

With zero compaction, flow of the sand must occur past the wheel uniformly along the length of the path; it follows that the area of the mounds produced, above the original soil surface, at the center of the wheel must have a total cross-sectional area given by:

$$\begin{aligned}\text{Cross-sectional area} &= \frac{bZ\pi D}{\pi D}\\ &= bZ \text{ sq in.}\end{aligned} \tag{3}$$

This is illustrated in Fig. 3. For this to occur, the bulldozing effect must build up a bow wave of sufficient magnitude so that flow of the sand in the lateral direction of the wheel must occur at the same rate as it is displaced, and the bow wave ahead of the wheel must have inclined sides at an angle greater than ϕ (the angle of repose); then flow of the sand will occur both in the direction of motion and at right angles to it.

On the assumption that compaction does exist to some extent, but of insufficient magnitude to account for the total displacement by the wheel, we can write:

$$\text{Total displacement} = \text{Compaction + flow}$$

$$= bZ\pi D \text{ cu in./rev} \qquad (4)$$

To express the compaction factor in terms of W, bZ, D, etc., much more would have to be known regarding soils than is the case at the present time such as:

(1) Depth of soil affected by the load;
(2) Degree of compaction possible for the soil in question;
(3) The maximum load pressure applied to the soil by load W;
(4) Effects of load on surrounding soil as well as that immediately below tread.

Present evidence indicates that compaction is of rather small magnitude, e.g., a lightly loaded wheel resting on the surface of sand does sink a small amount, but at the same time the soil round the wheel also heaves to a small extent despite the low load.

Little is known about the maximum local soil stress, item (3), at the present time. Bernstein's relationships give a distribution of the type shown in Fig. 1, with a maximum equal to D.C. occurring under the centerline of the wheel; however, as already noted, such a distribution of stress does not seem to be a possible solution in actual practice. To approach this problem as a whole, it becomes necessary to obtain information regarding the relative effects of flow and compaction together with the stress pattern over the load-bearing surface of the wheel. Some preliminary test work was carried out at The University of Michigan under the auspices of the Office of Ordnance Research to provide some answers to this problem, with the following results.

FLOW OF SAND PAST A RIGID WHEEL

A number of tests were run at various loads with a given wheel towed at given speeds, and the shape of the mounds of sand left as a result of the motion were measured. The shapes measured ahead of and along the sides of the wheel were those left when the wheel had been brought to rest; thus they differ slightly from those existing when in actual motion.

The wheel employed in these tests was a model 12-1/2-in. diamter with a face of 3-1/2 in. or an aspect ratio of $\alpha = 0.28$. It was loaded with 25, 50, and 100 lb, respectively.

If the rut left behind the wheel during its normal motion is considered (in this case the results do represent the actual results produced by the wheel and its load when moving at velocity), traces such as those shown in Fig. 4 (a, b, and c for loads of 25, 50, and 100 lb, respectively) are obtained.

To estimate the approximate displacements of the sand, the shape of the mounds were considered to consist of flat surfaces extended so that the intersection of the sloping surfaces was at a point in place of the actual rounded condition of the mound itself. The error introduced by this approximation is considered to be small. By calculation it can be shown that:

Wheel load = 25 lb

Area of sand above original soil surface = 2.0 sq in.

Area of trough in sand below original soil surface = 2.35 sq in.

Compaction = 0.35 sq in.

Angle ϕ_1 = 32° approx

Angle ϕ_2 = 14° approx

Wheel load = 50 lb

Area of sand above original soil surface = 2.81 sq in.

Area of trough in sand below original soil surface = 3.02 sq in.

Compaction = 0.21 sq in.

Angle ϕ_1 = 32° approx

Angle ϕ_2 = 24° approx

Wheel load = 100 lb

Area of sand above original soil surface = 2.76 sq in.

Area of trough in sand below original soil surface = 4.68 sq in.

Campaction = 1.92 sq in.

Angle ϕ_1 = 30° approx

Angle ϕ_2 = 33° approx

If the section of the mound at the vertical centerline of the wheel is considered, the soil surface appears as shown in Figs. 5a and b for the 50 and 100 lb, respectively. Calculating the area of the sand piled above the original surface of the soil in this case and comparing it with the area displaced by the wheel sinkage, the following is obtained:

Load, lb	Area of Mound, sq in.	Area of Displacement, sq in.	Difference, sq in.
50	5.0	5.7	0.7
100	9.6	10.7	0.9

Taking the dimensions of the bow wave shown in Figs. 6a and b for the two loads in question, we obtain:

Load, lb	Vol of Sand in Wave, cu in.	Displacement of Wheel per 1 in. Travel, in.
50	15.7	5.7
100	27.5	10.85

It is seen that the bow wave appears to be approximately three times the rate of displacement of the wheel per 1 in. of forward motion to provide sufficient slope to initiate the lateral motion necessary for flow round the wheel.

Comparing the hights of the mounds ahead and along the sides of the wheel, it is seen that the wave has heights of 2.3 in. and 3.4 in., respectively, while the sides are 1.6 in. and 2.0 in. while the mound behind the wheel amount to 0.85 and 0.9; there is a head to produce the necessary flow.

In addition, the flow past the wheel is seen to be roughly a constant amount less than the wheel displacement. This could be a measure of the compaction of the sand and indicates that compaction is only slightly affected by the change of maximum pressure to which the sand is subjected, from about 3 to 4-1/2 psi, in this case, a rather small change. The main increase in load-carrying capacity appears to come from the increasing area of contact of the wheel with the sand as a result of additional sinkage.

Looking at the pattern behind the wheel, the soil displaced above the original surface is in general somewhat smaller than the rut left by the wheel, again indicating that some compaction is occurring; at the high load, a considerable difference exists behind the wheel, despite reasonable agreement of the volumes when compared at the vertical center line of the wheel It should be emphasized again that accurate reading of the dimensions is difficult and the high-load results could be in error.

It follows that the theory assuming compaction of the soil alone under the action of the wheel does not represent the case with accuracy. Flow round the wheel in the case of a dry sand appears to be of greater importance. It is appreciated that vastly different conditions may exist in the case of a plastic soil. The effects of this change in nature of the process on the present system of calculations for load, drag, sinkage, etc., will be discussed later.

STRESS PATTERN ON WHEEL SURFACE

To measure the actual loading per unit area on the face of the wheel, a small pressure transducer was installed in the surface of the wheel, consisting of a plunger (1/4-in. diameter) attached to a differential transformer incorporated into an appropriate circuit whose output was led to a recorder so that a trace proportional to the variation of the load on the plunger could be obtained. A true point reading is not secured but at least the load recorded is fairly typical of the actual variations of the stress.

Figures 7, 8, and 9 show the results obtained, the first with the pressures plotted round the surface of the wheel showing the loaded area, the second with the same data plotted on an expanded angular base. The wheel employed in these tests was 12-1/2-in. diameter with a face width of 6-1/2 in. or an aspect ratio of $\alpha = 0.52$. Figure 7 shows the pressures with the pick-up at the center of the wheel face where the motion can be considered two-dimensional, while Fig. 9 shows the effects of placing it near the edge of the wheel where three-dimensional flow is undoubtedly occurring. Comparison of the diagram shows a definite change in the distribution of pressure for the two positions, but it does not appear to be of a serious nature for the locations recorded. In what follows, the central location only will be employed and the motion of the sand is considered to be of a two-dimensional nature.

One thing is immediately apparent from Fig. 7 that there is a definite change in the stress pattern between the light loads with small sinkage, and the heavier loads where sinkage and compaction are great.

Examine the stress diagram for the 10- and 20-lb loads. Stress begins at 13-1/2° before vertical and ends at 16-1/2° after for the 10-lb load, and at 20° and 15° for the 20-lb load. These values indicates a roughly elastic medium with little if any permanent compaction of the soil although a very shallow rut is left behind. At such light loads extremely small differences in soil level and small errors in the instrumentation could account for the differences in angular contact before and after the centerline. These loads are accompanied by a maximum soil stress of 1.22 and 1.6 psi, respectively.

The remaining diagrams, for the 40-, 60-, and 100-lb loads, show a stress pattern as follows.

Load	Angle at Which Stress Begins in Degrees Before B.D.C.	Angle Past B.D.C. at Which Stress Ends	Maximum Stress, psi
40	36	8	2.02
60	46	8	3.00
100	59.5	5	3.62

The soil no longer appears elastic to any great extent; in fact, the point of wheel contact ends, for all practical purposes, at one constant angle, 7°, if the stress curves are averaged out by using a straight line for the trailing side of the pressure curves, which Fig. 8 indicates as a possibility. Before any general conclusions are drawn, a more detailed investigation of the loadings is necessary.

Let us examine the loads applied to the wheel when in motion. These consist of the applied vertical load, W lb, plus the horizontal drag of the wheel, H lb, necessary to overcome the rolling resistance. It follows that there is a resultant force, R lb, acting at some angle θ° to the vertical as shown in Fig.

10. With respect to this resultant load, acting during motion of the wheel, the angular positions of beginning and end of the pressure diagrams become:

Load	Stress Begins Degrees Before Resultant R	Stress Ends Degrees After Resultant R
10	6.5	23.5
20	8.5	26.5
40	17.0	27.0
60	24.0	30.0
100	32.0	32.5

It will be observed that the majority of the contact area lies behind the direction of the resultant force when the loads are light, while at heavy loads the contact is roughly equally divided on either side of the resultant.

If the 10- and 20-lb loads are neglected, the remaining diagrams can be represented with considerable accuracy by a series of similar triangles, as shown superimposed in Fig. 8, with the maximum stress occurring at an average of 72% of the angular contact area measured from the leading edge of contact with the sand.

The similarity of the diagrams suggested some common constant relationship between depth of sinkage below the surface, the angle of contact, and the stress; various methods of calculation for such a relation were examined. Soil penetrations were carried out with the bevameter normal to the surface of the sand and inclined to it in the manner in which a wheel surface contacts the soil at high sinkage under a cycloidal motion. Typical results are shown in Fig. 11; it is observed that the slope of the log-log plots increases as the angle θ° to the soil surface reduces, and that all these lines meet in a common point, which appears to indicate that, given an infinitely large bed of homogeneous sand, the direction of application of the load does not matter after a considerable depth is reached. It would be assumed that past the common point of intersection one common line would represent all angles. The value of "n" varied from 1.0 to 1.3 for various sizes of the plate on the bevameter and the 90° position, to 2.4 for the 45° one. There is a problem as to where the penetration Z should be taken as zero when in an angular position—when the side of the foot first touches the soil or when the whole width of the plates contacts the soil. Plotting these data in the different ways, considerable differences in the value of n result. For example, if the 45° line is plotted on the basis of when the first contact is made with the sand, and also when corrected for the sinkage being considered zero when the centerline of the plate and bevameter is at the soil surface, then "n" changes from 2.4 to 1.74. When $\theta = 75°$, the change in value of "n" for similar plots is quite small, as would be expected. In any case, for all methods of plotting the value of "n" increases as the angle is reduced, compared with the normal 90° position.

Consideration of the conditions under which the soil is operating under a rolling wheel indicates that some variation in the value of the soil constants is to be expected before and after loading. As the wheel rolls forward, the uncompacted soil near the surface is loaded and compaction results, increasing presumably up to the point where the stress reaches a maximum. Further rotation of the wheel continues the displacement of the soil downward, if compaction is the only method in force, not with increasing stress as is generally assumed, but with a reducing one (which is not psssible according to the Bernstein theory). In a bevameter test the load p in $p = kZ^n$ continues to increase at all times as Z increases; thus there is a distinct difference in the action of the soil in the two cases. This suggested the following solution.

A static wheel subjected to a load W sinks some distance Z in the soil as shown in Fig. 12a. It is reasonable to suspect that the point value of the pressure against the wheel surface varies from zero at the surface to some maximum directly under the load, i.e., a symmetrical stress-strain curve, would exist and the value of "n" in the pressure-sinkage relationship for each side of the diagram would be identical, i.e., the soil is the same on each side of the center. In the case of a towed wheel, the conditions are such that from the original surface of the sand to the point of peak load the soil is being compacted, and a certain value of "n" exists for this type of action on the soil. Beyond the peak pressure, where maximum compaction has occurred, it is conceivable that new values of k and "n," say k_1 and "n_1," exist due to the compaction.

Working on the assumption that the sand, up to the point of maximum pressure, has one set of soil properties, and a different set of values after the peak is reached (that is, the soil compaction has reached its limits for the load in question at the peak stress and despite further sinkage, relative to the original soil surface, there is a reduction of stress), calculations were made for the values of k and n for the curves of pressure before and after the peak pressure with the results shown in the table below and in Fig. 13:

Load	k	k_1	n	n_1
10	7.8	6.0	0.85	1.62
20	6.1	7.3	0.91	1.7
40	3.5	13.0	1.27	1.5
60	2.4	16.0	1.5	1.6
100	1.6	13.0	1.6	1.6

The method of plotting was that the point of contact of the original soil surface with the wheel at A of Fig. 12b formed the reference for the values of p and Z from which k and n were determined while the point C of the compacted surface CD was employed for a sinkage reference for k_1 and n_1, Z being measured along the line of OG normal to AE and CF, the zero for Z being E in the uncompacted case and F for the compacted portion of the curve. In effect, the soil contact with the wheel was that provided by a static wheel loaded with the resultant force R along OG but with two different soil properties, one on each side of the line of application of the load.

The log-log plots of the 40-, 60-, and 100-lb loadings could be averaged into the relationship given by $p_1 = 13.0\ Z_1^{1.5}$ with reasonable accuracy but in the case of the k and n values only "n" could be averaged with a moderate degree of accuracy at the value of n = 1.42; the k's appeared to have a definite variable value with the load. The 10- and 20-lb loads could be represented by the relationships:

$$p = 7.0\ Z^{0.9}$$

$$p_1 = 7.0\ Z_1^{1.65}$$

within a reasonable error.

The original uncompacted soil tested by the bevameter gave an average of $p = 3.9\ Z^{1.1}$. Thus the figures in the above table confirm some changes in the values of k and n as stress is applied. In fact, the log-log plots under the various loads did indicate a gradual departure from the straight-line relationship as the stress approached the maximum value, indicating a continued increase in "n" with the stress.

To establish and check this idea, a bevameter test was carried out on the original soil and again after the passage of a loaded wheel, the values of k and n obtained are shown below.

	k	n
Original soil	3.0	1.0
Compacted soil		
60-lb load	3.4	1.0
100-lb load	4.3	1.0

SOIL REACTION

The application of an external load and drag to a wheel result in a resultant load of R lb at some angle θ° to the vertical and must of course induce an equivalent reaction in the soil, as represented by the load diagrams of Figs. 7, 8, and 9.

To check how closely this measured reaction agreed with the load applied, the average load per square inch of contact area and its point of application was obtained from three of the diagrams with the following results.

Vertical Load	Resultant R	θ°	Radial Component of Soil Reaction lb	θ°
100	112.7	27.5	83	27.3
60	64.9	22.2	53.2	17.0
40	42.3	18.8	34.8	13.0

In view of all the various factors involved, the agreement between the applied load and the measured reaction can be considered reasonable.

EFFECT OF FRICTION ON SOIL REACTION

In the actual case with motion, there is an additional force involved: friction which results from the slippage occurring between the wheel and sand. This force is of course normal to the wheel surface at all points and would result in some modification of the supporting forces and the resultant point of application.

Tests were run to determine the magnitude of the tangential force necessary to cause slip of a rigid wheel under various loads.[3,4] These tests indicate that an average value for the coefficient of friction to be expected is of the order of 0.48; however, the relationship $F = \mu W$ does not represent the results too well, particularly at high loads where the sinkage is large. However, using this relationship for simplicity, when friction is added to the radial pressures the combined resultant soil forces become, for the 100-lb load:

Resultant Load	Soil Reaction, lb
112.7	99.2

Much more would have to be known regarding the variation of the frictional forces with pressure before an accurate estimate of the resultant reaction of the soil and its direction of application on the wheel surface is known.

DISCUSSION

The experimental work reported here establishes that the passage of a wheel over a dry sand does not result in compaction of the soil alone; there is considerable flow around the wheel. The theories in general use at present assume the former only, and thus the question arises how accurately the relationships developed by the use of compaction alone agree with the actual case.

The results reported here indicate a change in the effective values of k and n with load from a value of about 8.0 and 0.85 for the 10-lb load, to 1.6 and 1.6, respectively, at the higher loads.

The result is that the calculated theoretical sinkage at low loads will be higher, and at the high loads lower, than the values determined by the present theory with n = constant = 1.0 to 1.3. Comparison of these results with Ref. 2 indicates that the corrections are in the right direction, with the result that experiment and theory will agree to a higher degree of accuracy than is now the case, at least for sand.

One other aspect of this change from compaction to flow with compaction are the effects that will be introduced into determining the drag forces, etc. The drag at present is obtained theoretically by equating the work done on the wheel by the towing device, to the work of soil compression. In actual practice, soil flow (in the case of sand) appears to be the major factor, in place of compaction. With flow it has been shown that the pressure against the wheel surface is still of the form $P = kZ^n$ as with the compaction theory; thus, using values of "k" and "n" which do represent the actual conditions on the wheel surface, the forces to move the wheel must be the same, and the work done is unchanged, whether compaction or flow results. The correct solution to the analytical problem appears to depend more on the values of "k" and "n" existing in practice than on the exact nature of the process itself. The variation of the parameters from the Bekker soil-value system is sufficiently limited to permit the use of suitable correction factors, but more must be learned about a wider range of soils and wheels to obtain such factors. In fact, it is suggested that the values of k and n depend on the state of stress of the soil, as well as on its general physical properties. Tests for soils properties under a variable applied stress are planned, with the applied stress being of a flexible nature so that similar heaving of the soil, etc., can occur as with present test procedures.

The variable value of "n" with load does not simplify the problem. Actually, it is rather the opposite: it indicates soil properties that are a function of the applied stress, and the complications in handling the analytical portion of the problem will be multiplied. Perhaps this is not too important in these days of computers, but it would be a problem for calculations by any other means.

Many more tests will be necessary before the changes indicated here can be accepted for general application, tests not only of the soil itself but also of the loading created by the wheel and their effects on soil properties.

It may be assumed that soil properties will have to be secured under various stress levels as indicated above to proceed with any accurate theory or calculations; however, it has already been shown that the soil properties as measured by the Bekker system do give results within the required limits for many general engineering problems.

CONCLUSIONS

As a result of this work, it can be concluded that, for a towed wheel in a dry sand:

(1) Compaction effects are small.

(2) Flow of sand occurs as a result of bulldozing forming a "bow wave."

(3) The normal pressure of the sand against the surface of a rigid wheel is of the form $p = kZ^n$.

(4) The pressure of the sand on the surface of a rigid wheel can best be represented by two sets of soil values, one during the compaction phase, and the other as the stress is relieved.

(5) The assumption of either compaction or flow has little effect upon the theoretical relationships for sinkage, drag, etc., provided the correct average values for the soil constants are employed.

(6) No assumptions are made in the bevameter test concerning compaction or flow; thus the use of the soil constant for either process appears legitimate.

(7) Acknowledging flow as the main process for the conditions considered offers a more realistic approach to the problem without throwing out all existing analyses.

ACKNOWLEDGMENETS

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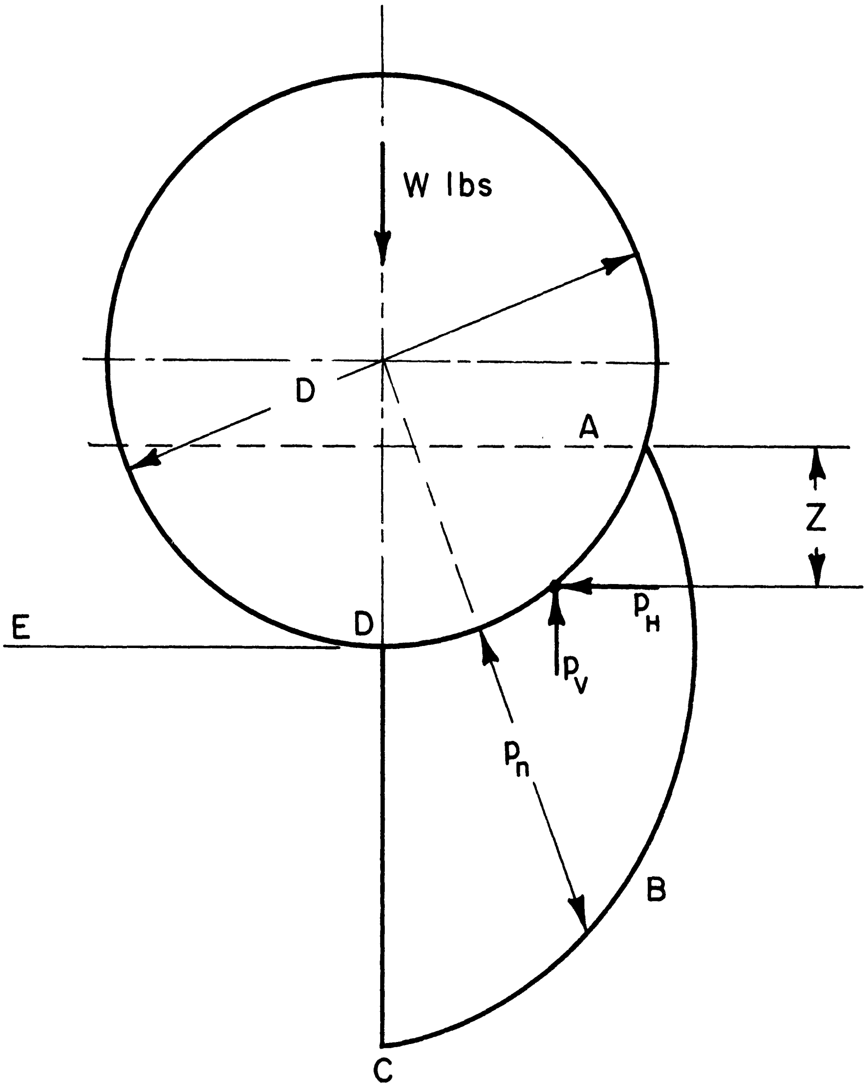


Fig. 1. Pressure against wheel face by Bernstein's equation.

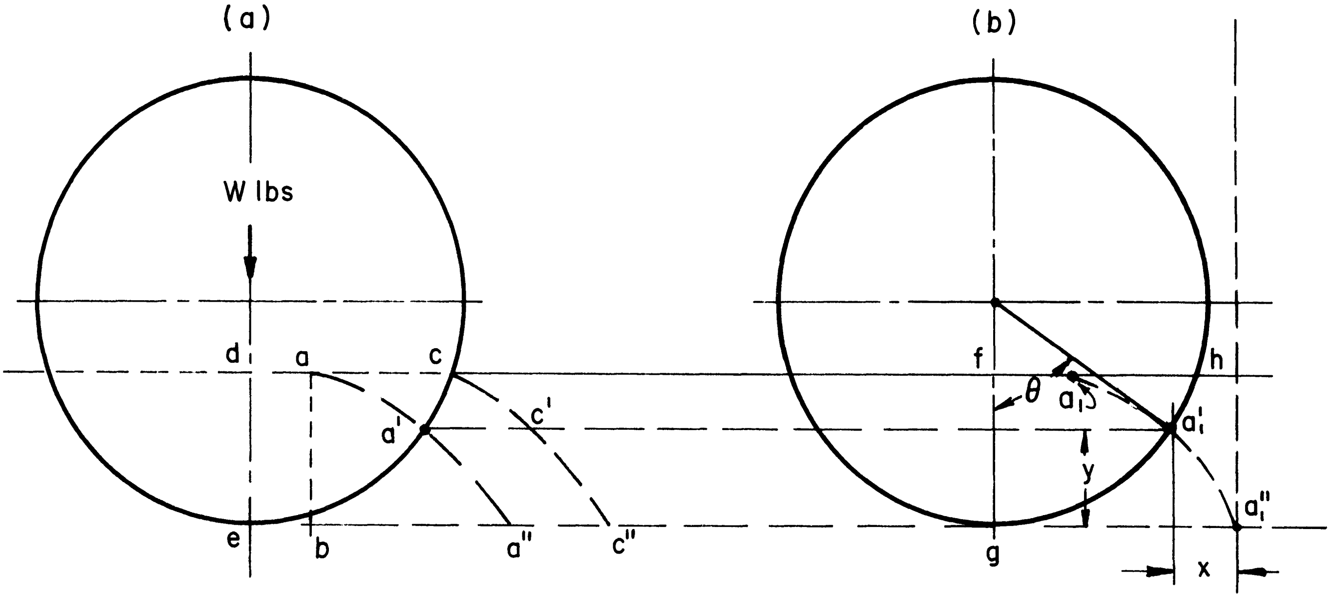


Fig. 2. Cycloidal motion.

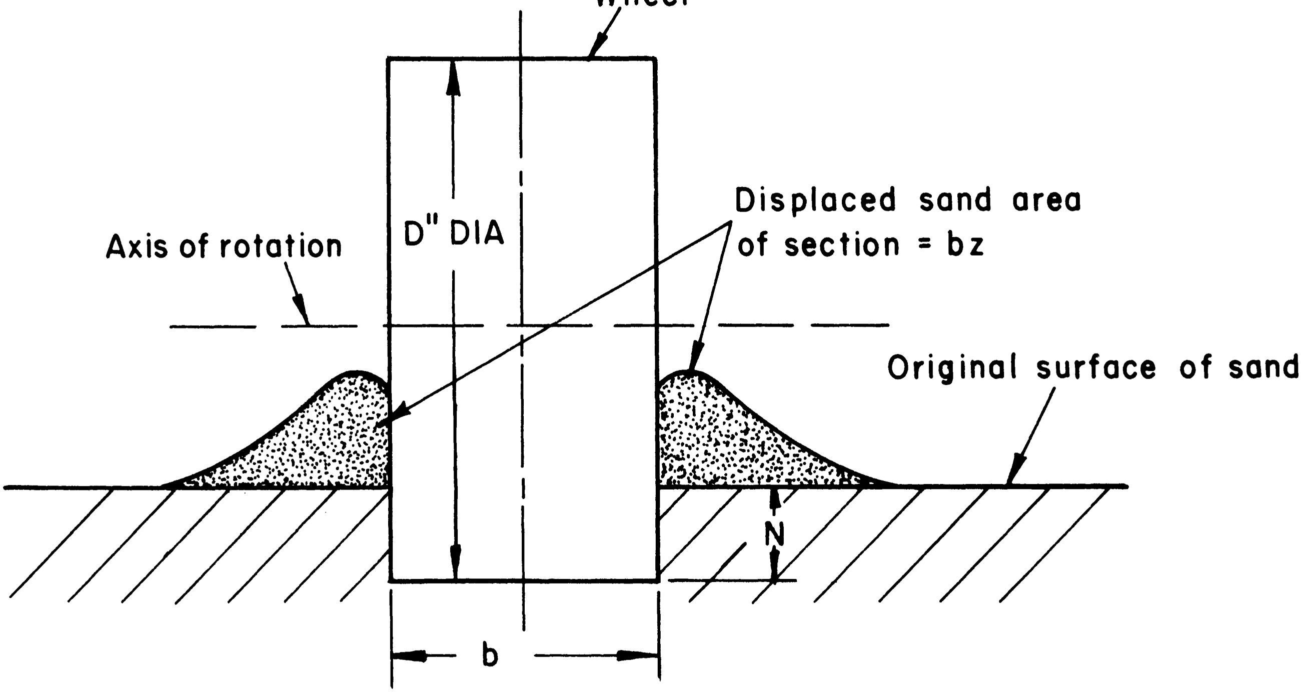


Fig. 3. Sinkage accompanied by displacement

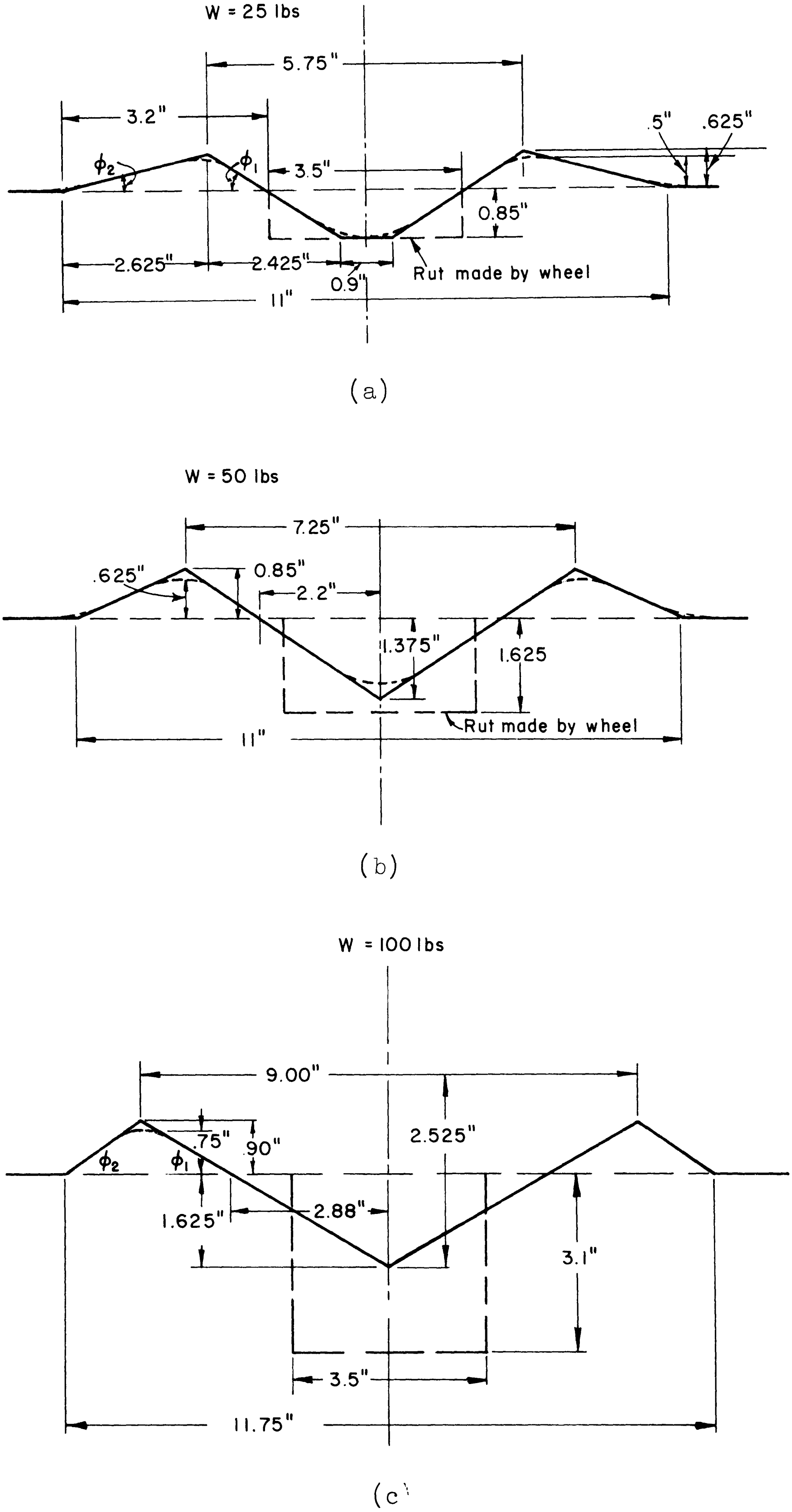


Fig. 4. Typical ruts left by wheel at various loads.

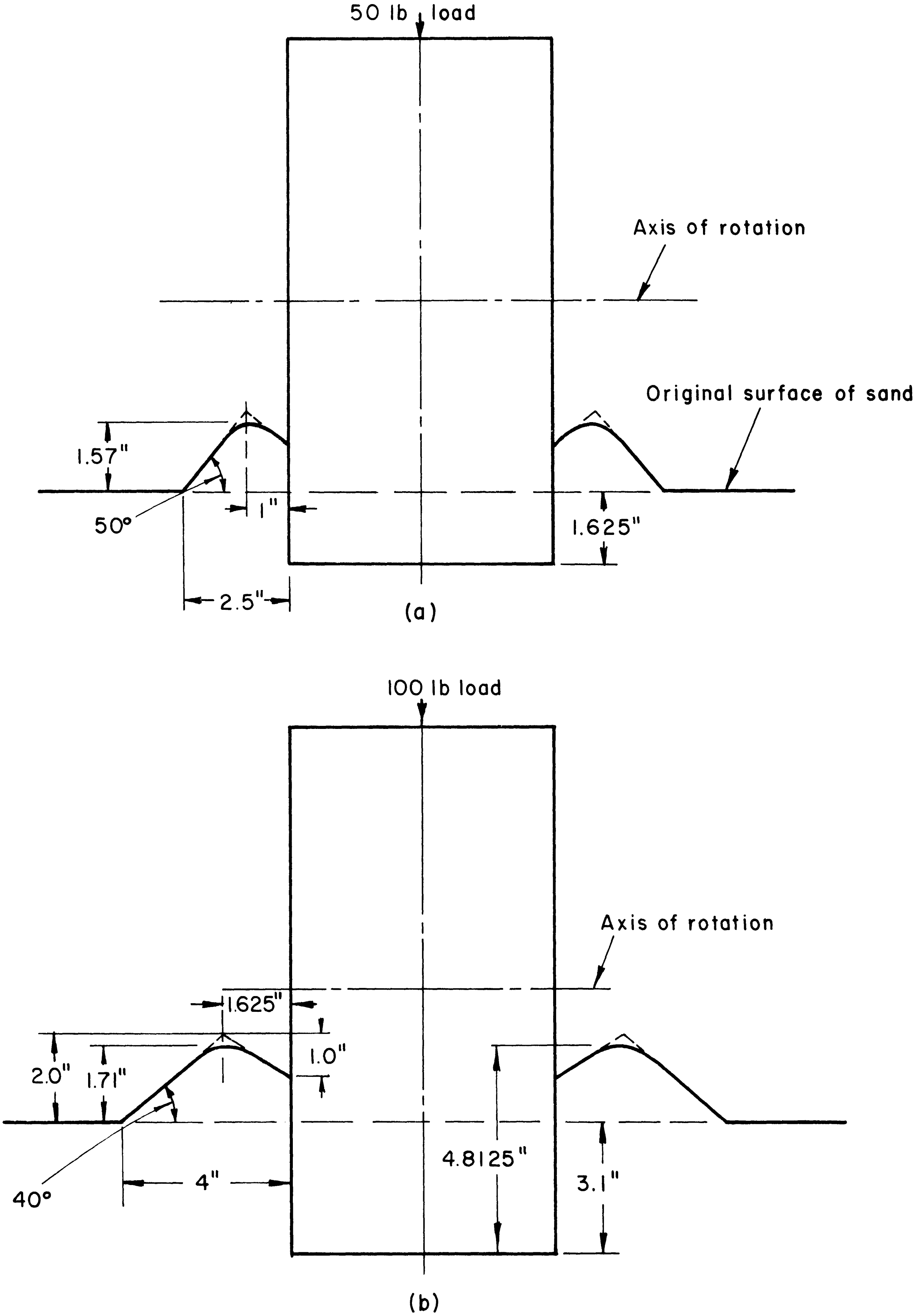


Fig. 5. Typical side displacements.

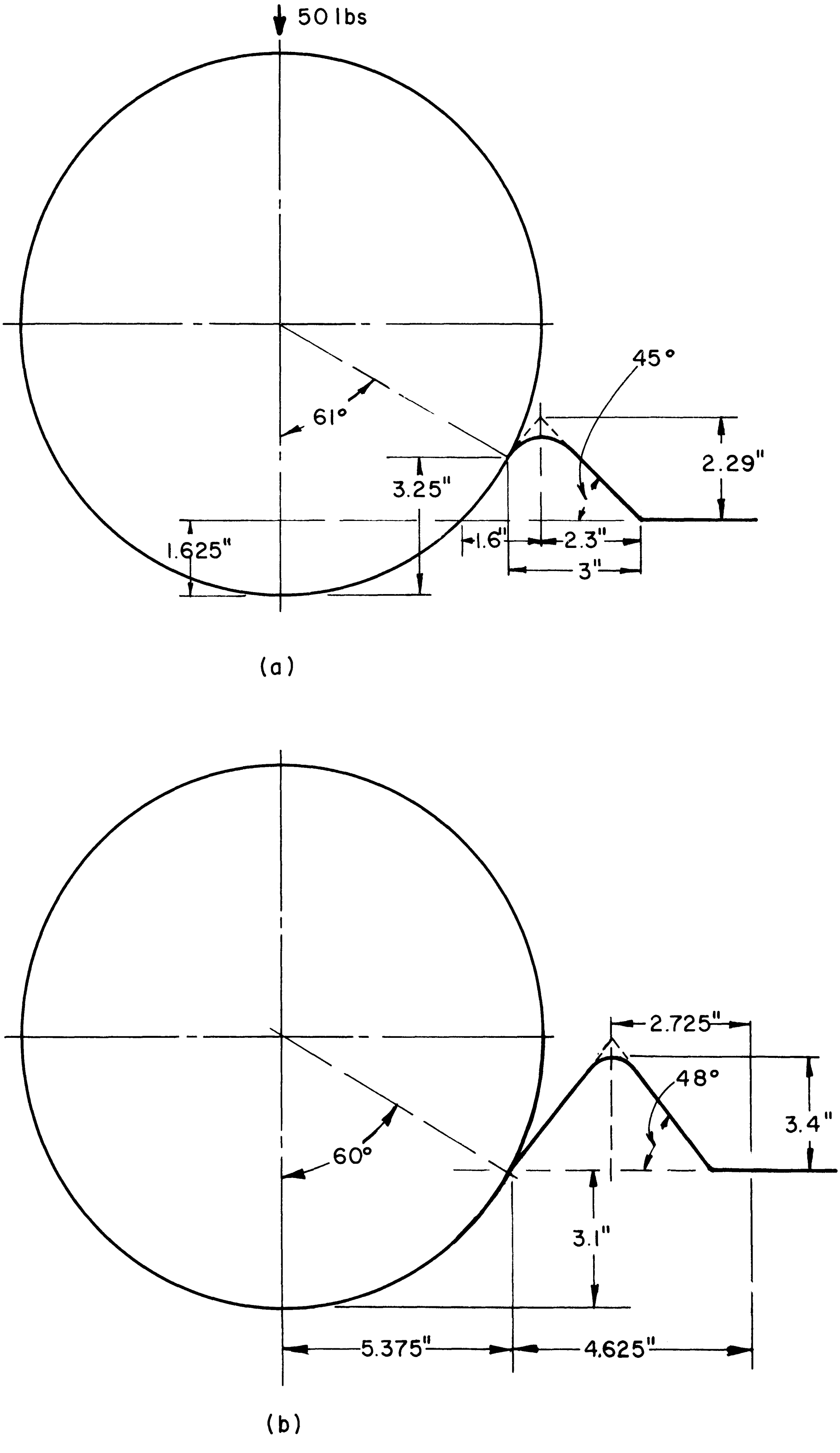


Fig. 6. Typical "bow waves."

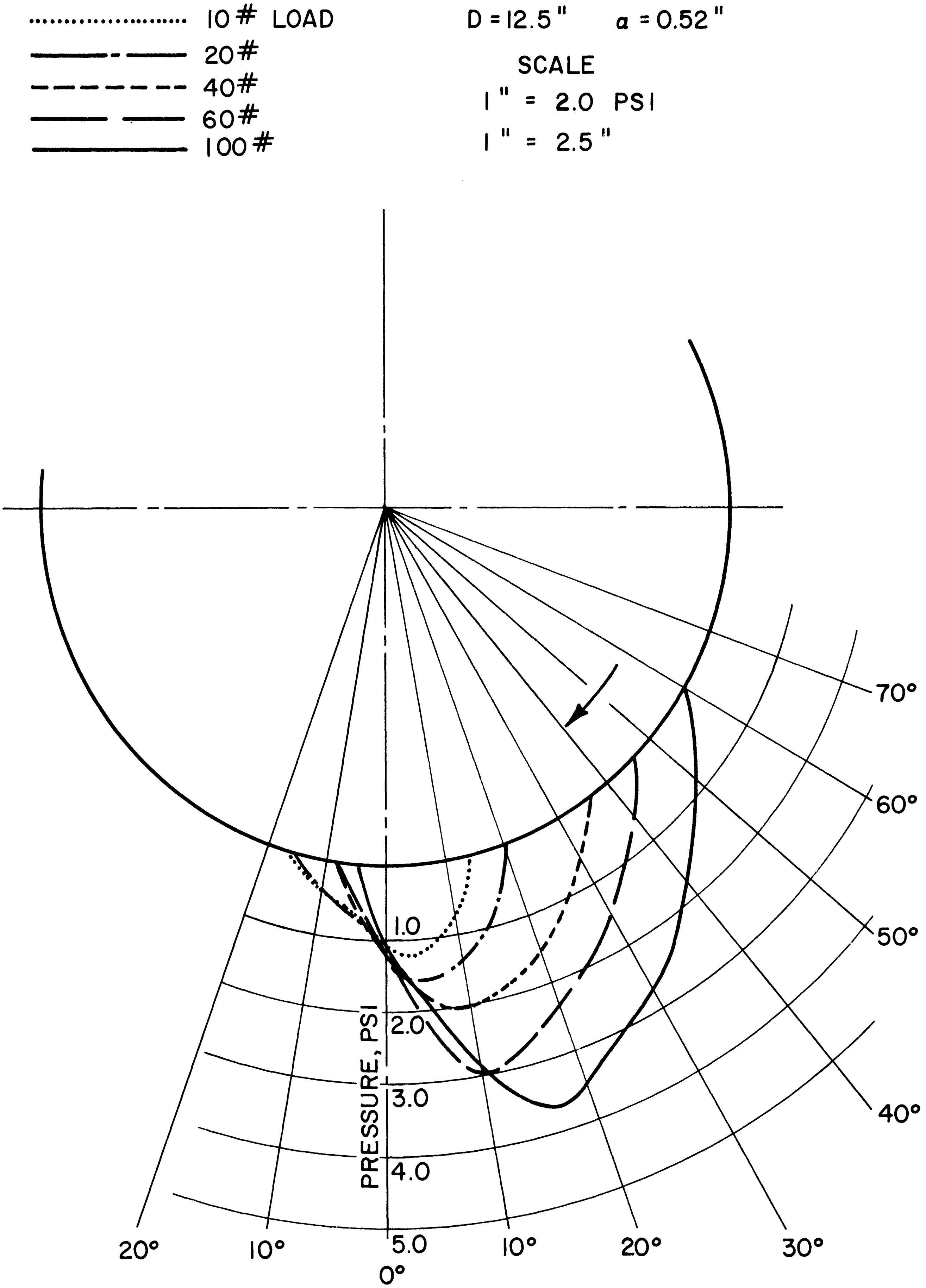


Fig. 7. Variation of loaded area and pressure under center of wheel.

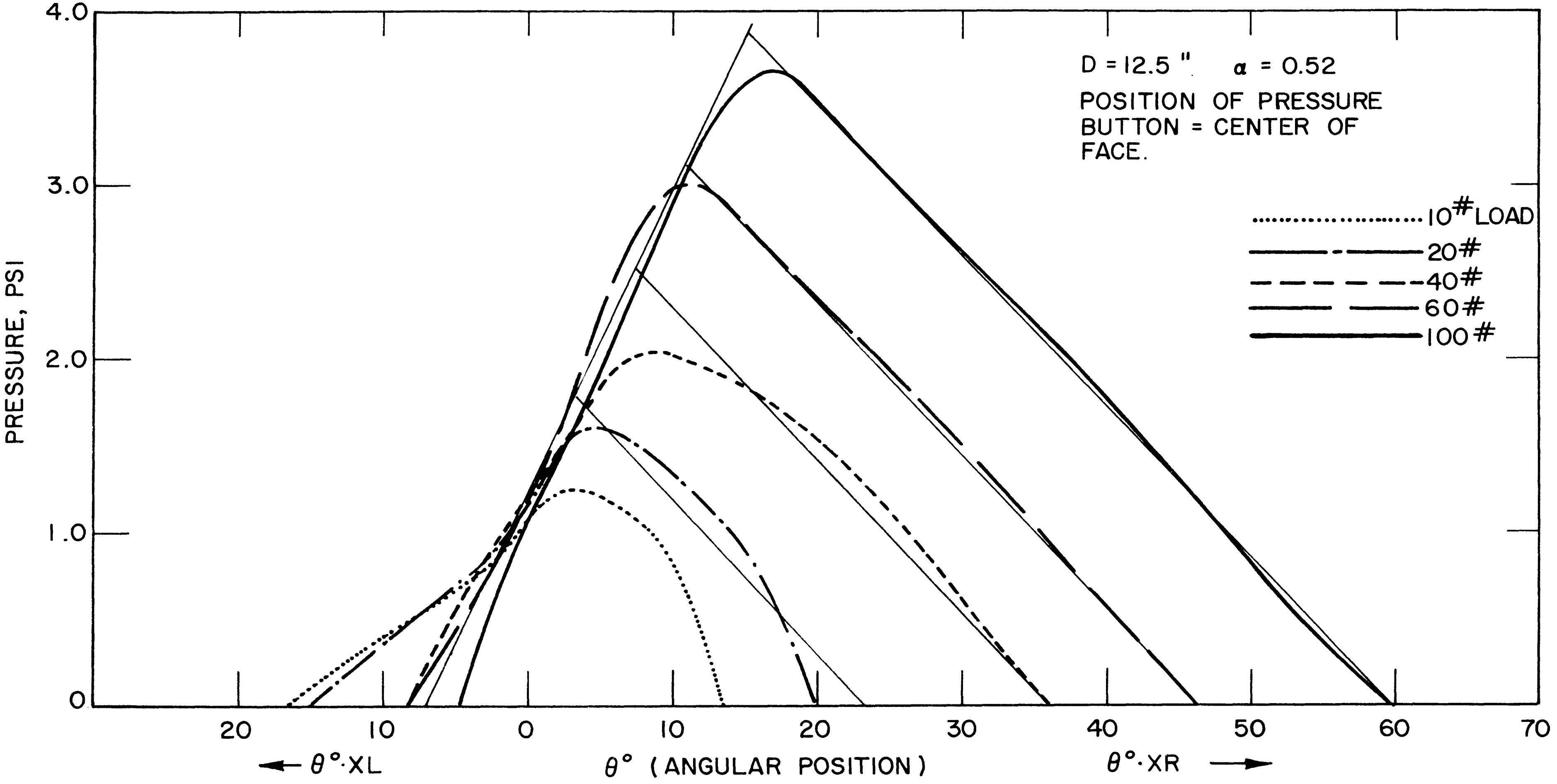


Fig. 8. Variation of angle of contact and pressure on wheel with load.

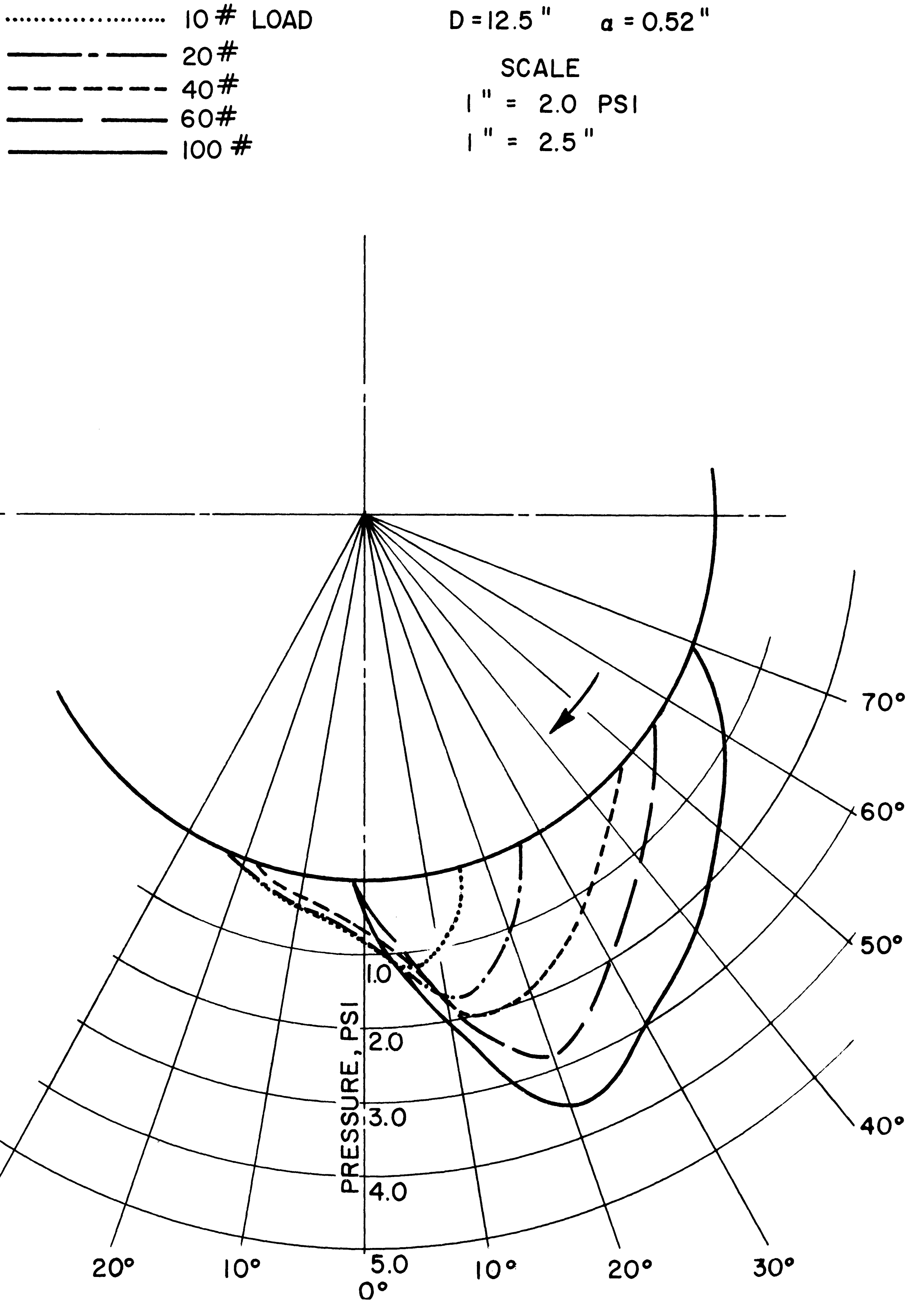
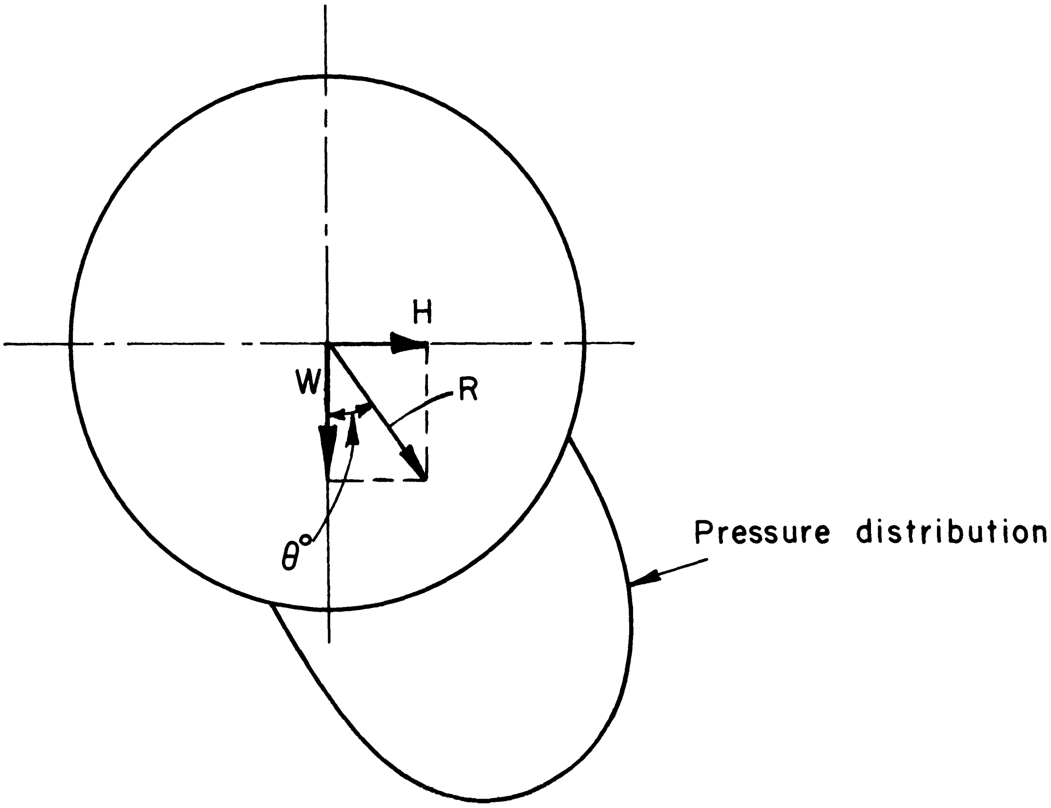


Fig. 9. Variation of loaded area and pressure near edge of wheel.

Dia = 12.5 Width = 6.5 Aspect Ratio = 0.52



Load (lb)	Drag (lb)	Resultant (lb)	θ°	Sinkage (in.)
10	1.25	10.07	7°	0.17
20	4.10	20.4	11.5°	0.25
40	13.65	42.2	19°	0.57
60	24.46	64.8	22°	0.97
100	52.00	112.6	27.5°	2.31

Fig. 10. Resultant forces on rigid wheel.

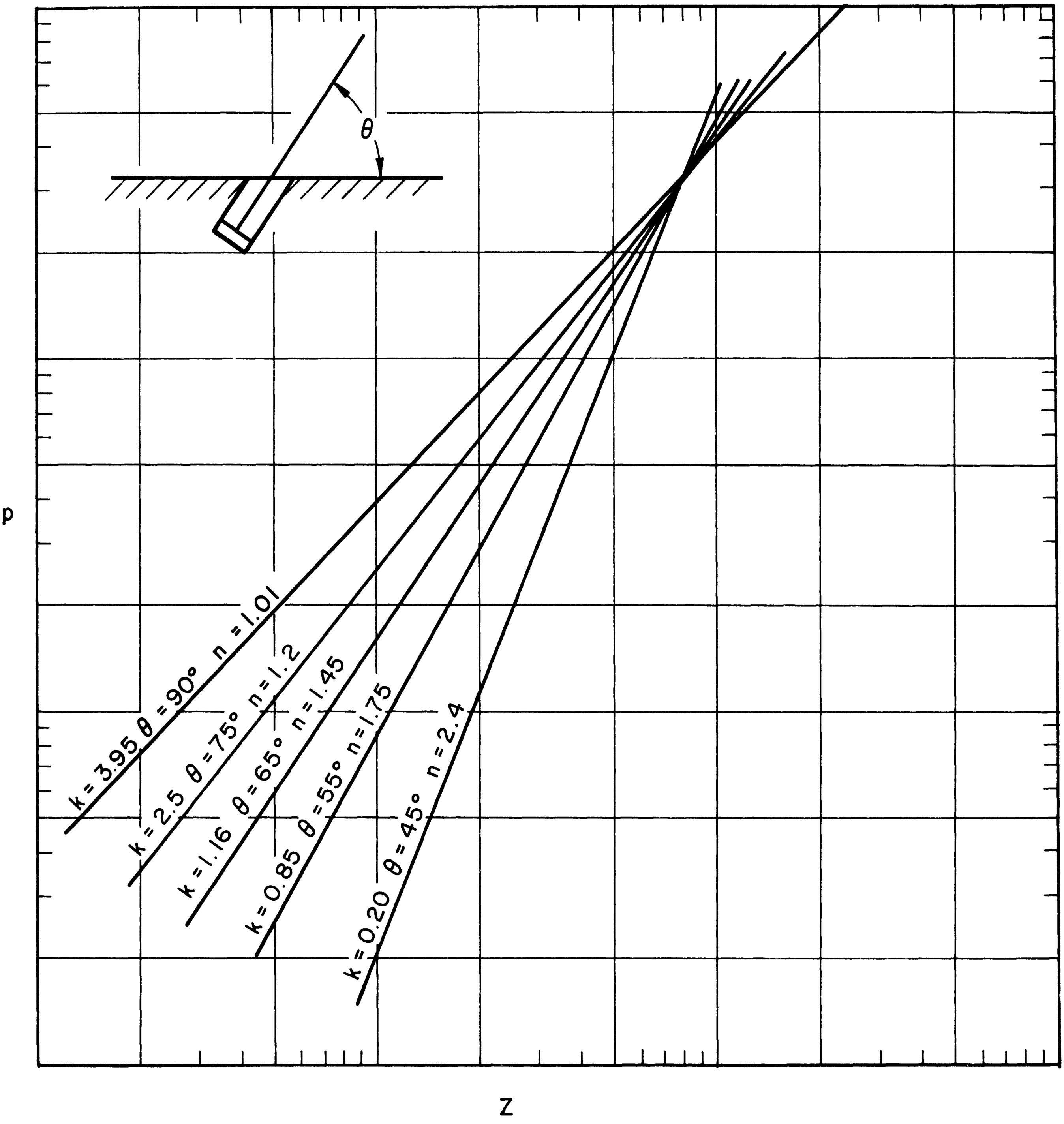
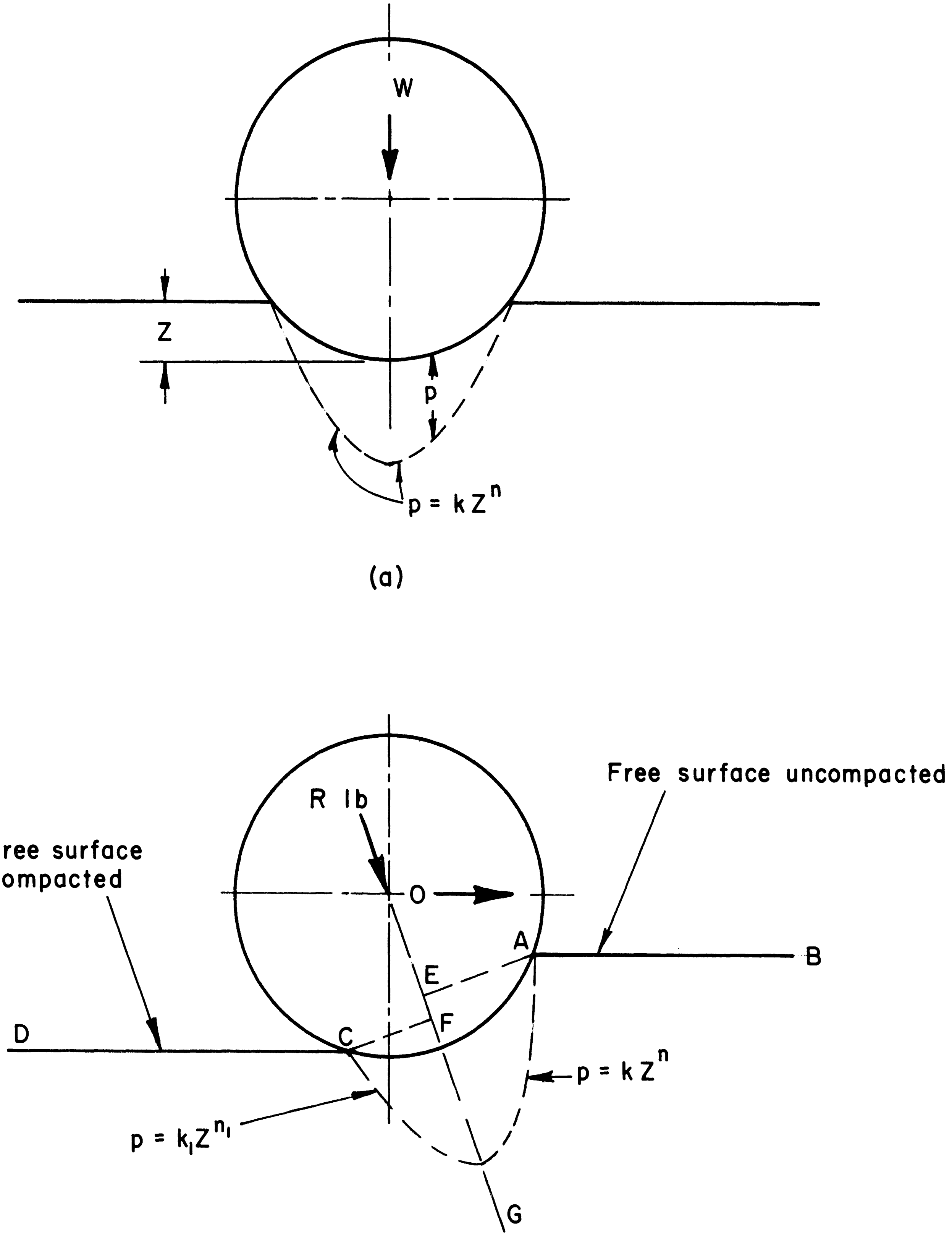


Fig. 11. Stress-sinkage relations for various angular penetrations.



(a)

(b)

Fig. 12. (a) Wheel loaded statically; (b) Load on wheel in motion.

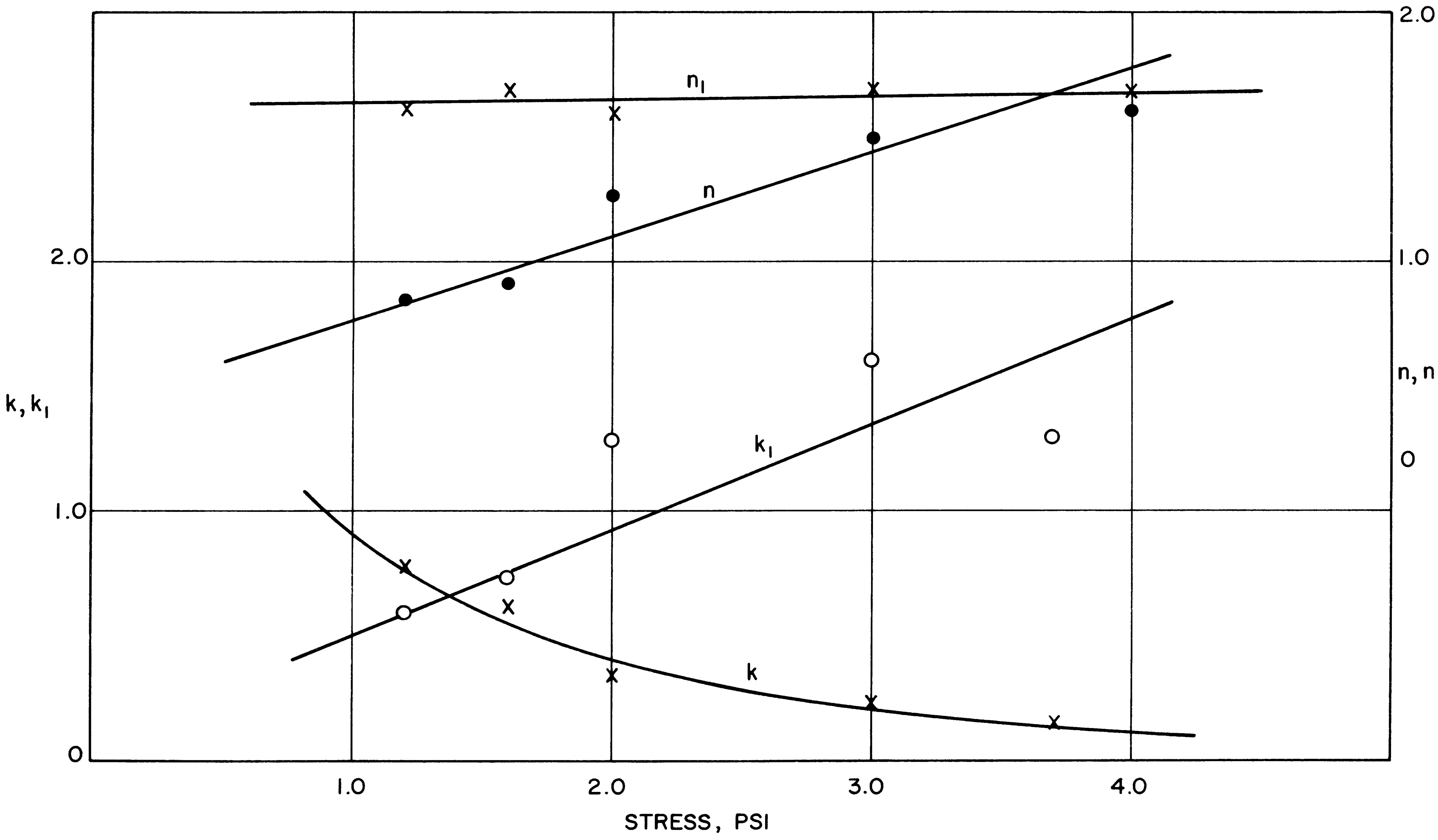


Fig. 13. Variation of soil constants with stress.