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WRIGHT AIR DEVELOPMENT CENTER

ON

NOTCH SENSITIVITY OF HEAT-RESISTANT ALLOYS AT ELEVATED TEMPERATURES

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SUMMARY

Work previously reported for this investigation under Contract AF 18(600)-62 showed a qualitative relationship between notched-bar rupture properties and relaxation characteristics. The question remained unanswered as to how the data obtained might be affected by residual stresses developed during preparation of notches. Moreover, the relative dependence of notch behavior upon relaxation strength, ductility, or other material property was not established.

Tests are now in progress to study effects of notch preparation methods on notched-bar rupture data. No general conclusions have yet been drawn.

Smooth and notched bars from three different heats of Waspaloy are being tested at 1350°F in an attempt to learn whether variations in other properties can be related to reported differences in notch behavior between heats. Tests on smooth bars only show material from the one vacuum-melted heat being used to have higher creep strength and rupture strength than do the two heats melted in air.

A tentative quantitative analysis of notch behavior has been developed, based on initial stress distribution around a notch and on changes of this stress pattern during testing. Incomplete calculations indicate satisfactory agreement with observed behavior for conditions of notch strengthening examined to date.

INTRODUCTION

This report covers progress made during the third quarter of 1954 under Contract AF 18(600)-62 in a study of factors affecting notch sensitivity of heat-resistant alloys.

Past work has shown a qualitative relationship between notched-bar rupture properties and relaxation characteristics, but did not establish relative dependence of notch behavior upon relaxation strength, ductility, or other material properties. Further studies are to investigate the relative significance on notch behavior of variations in such properties. For this purpose two additional heats of Waspaloy have now been obtained. Notch strength at 1350°F of one is reputed to be greater than for the heat tested previously, while the notch strength of the other may be lower. Differences between heats in short-time tensile properties, creep-rupture properties, ductility in the rupture test, and relaxation rates are to be compared with corresponding differences in notch behavior. Specimens are to be compared both for conventional heat treatment and for this treatment without the 4-hour age at 1550°F. It has been reported that some heats of this alloy without the 1550°F aging treatment are notch weakened at test conditions where notch strengthening may be expected for conventional heat treatment.

One question left unanswered by past experiments was the magnitude of effects on notch rupture strength from residual stresses developed during machining of the notches. The present report gives results obtained to date in tests with bars after various notch-preparation methods. It also introduces a tentative method for quantitative analysis of notch behavior based on initial stress distribution and changes of stress pattern during testing of notched bars.

CURRENT STATUS OF THE INVESTIGATION

Study of Methods of Notch Preparation

It was reasoned that any residual stresses at the notch root would be largely eliminated for high stresses where extensive yielding occurs at the base of the notch during loading. Also for alloys with low relaxation strength stress concentrations should be reduced quickly and notch strength should not be much affected even though residual stresses augmented the initial level at the notch root. In accordance with these considerations, tests have been limited to a condition of high yield strength and low relaxation rate (Inconel X-550 at 1350°F). A single notch root radius (0.005 in.) and a single stress (40,000 psi) are being studied first. This condition appeared to give about the greatest degree of notch embrittlement in tests using ground notches.

Tests are still in progress at this first condition. A few experiments with other notch-root radii and/or other nominal stress levels will probably be necessary before definite conclusions may be drawn as to the magnitude of effects on test results caused by variations in notch preparation practices.

Comparison of Notch Behavior Differences and Smooth-Bar Property Differences for Different Heats of the Same Alloy

In order to reduce effects of prior history, material from three heats of Waspaloy under study were given the same hot rolling into bars of a common size from which specimen blanks are to be cut.

Limited creep-rupture tests have been completed for all three alloys with the two different heat treatments planned. Notched-bar rupture tests are being delayed until effects of various notch-preparation methods have been evaluated.

The only observation of note on tests so far is that the one vacuum-melted heat has higher creep strength and rupture strength than do the two heats melted in air.

Development of Correlations Between Notch Behavior and Smooth-Bar Properties

A tentative analysis explaining notched-bar behavior in terms of initial stress distribution and rate of change of this stress with time has been worked out in detail and is covered in a later section of this report. Incomplete calculations now in progress indicate that notch strengthening can be satisfactorily explained by this analysis for the conditions examined to date. Further calculations are to extend to cases of notch weakening.

The step-wise analysis developed is too detailed and lengthy to serve as a quality control method. An empirical expression relating notch strengthening or weakening to easily-obtained smooth-bar properties is being sought. As yet but limited effort has been placed on this subject and no general criteria of notch weakening or strengthening have been found.

EXPERIMENTAL RESULTS

New Rupture Data at 1500°F for S-816 with Special Heat Treatment

Previous reports indicate notch weakening under some conditions for S-816 with the following heat treatment: 2325°F, 1 hour, WQ + 13.5% Reduction at 1200°F + 1400°F, 12 hours, AC. To better establish smooth-bar rupture properties at 1500°F for comparison with notch characteristics at that temperature, the following additional data have now been obtained:

Stress Level (psi)	Rupture Life (hours)
65,000	0.2
50,000	2.5
25,000	1053.2

Notched-Bar Rupture Life at a Single Nominal Stress, but with Different Methods of Notch Preparation

Three techniques were employed to finish the notch root after prior turning to within 0.020-inch of the final root diameter:

(1.) Turning with a formed lathe tool.

(2.) Rough grinding to within about 0.005 inch, followed by a quick light finish grind to dimension, with flood cooling at all times.

(3.) Turning to within about 0.005 inch on diameter with a specially-formed lathe tool, then lapping with a wire 0.009-0.010 inch in diameter, using a fine lapping compound, as illustrated in Figure 1. The most highly stressed portion of the worked metal left at the notch root by the turning operation can be quickly and easily removed in a few minutes by this form-lapping technique.

Specimens tested had the notches prepared at any of three stages in the heat treatment:

(a) after conventional solution and aging treatments.

(b) after solution, but before aging steps.

(c) before either solution or aging.

(d) one pair of bars notched after a complete conventional heat treatment was placed in a 1600°F furnace 20 minutes and then air cooled prior to testing at 1350°F.

Test results are shown in Table 1.

TABLE I

NOTCHED-BAR RUPTURE TESTS FOR SPECIMENS WITH DIFFERENT METHODS OF NOTCH PREPARATION

(All specimens tested at 40,000 psi stress and 1350°F. Shank Diameter 0.600", Notch Diameter 0.424", Notch Root Radius 0.005", Notch Angle 60°)

Order and Method of Notch Preparation	Rupture Life, Hours
Conventional H. T. + Turned Notch	176.4
Conventional H. T. + Ground Notch	110.7
Conventional H. T. + Lapped Notch	74.5
Conventional H. T. + Turned Notch + 20 min. in 1600°F Furnace	168.6
Conventional H. T. + Ground Notch + 20 min. in 1600°F Furnace	87.5
[a](2150°F, 1 hr, AC + Turned Notch + Conv. Ages)	76.4
[a](2150°F, 1 hr, AC + Ground Notch + Conv. Ages)	30.7
[a](As Received + Turned Notch + Conv. H. T. in Atmosphere)	50.2
[a](As Received + Ground Notch + Conv. H. T. in Atmosphere)	65.1

a. Test to be repeated. Furnace coil burned after 3 (±0.5) hours of the 1600°F age. Cooled to room temperature and given usual aging at 1350°F before testing.

The data indicate a moderate increase in notch strength for turned notches over ground notches when specimens are machined after all heat treatments are completed. The lapped notch gave a somewhat shorter rupture life than even the ground notch.

A short "stress relief" at 1600°F after notching appeared to have little effect on relative strengths of turned and ground notches. Longer stress reliefs are not planned at this time in view of probable alteration of material properties before appreciable relaxation of residual stresses.

Comments on other results appear premature, but solution and aging after notch preparation seems to give lives of the same magnitude as for bars carefully machined after heat treatment.

It might be noted that the notch ground after conventional treatment ruptured at almost the identical time (109.4 hours) reported by Carlson, MacDonald, and Simmons (1) for the same stress and notch geometry. A number of the tests are being repeated where the aging time at 1600°F was uncertain due to furnace burnout.

ANALYSIS OF CHANGES IN STRESS DISTRIBUTION FOR A NOTCHED BAR DURING LOADING AND DURING CREEP TO RUPTURE

This investigation has been based on the belief that variable response of materials to notches at elevated temperatures must be closely related to initial stress concentrations and their change with time, as controlled by creep and relaxation. At the outset it was reasoned that a notch introduces nothing inherently new into properties of an alloy, but only changes the stress-strain history of fibers in the notched bar. If one were to reproduce in a smooth bar the history experienced by a fiber of a notched bar, the life of each should be the same.

In the following development attention is focused on a thin circular section containing the plane of a circumferential notch. It is proposed to follow changes with time of the stress levels in representative fibers located in this disk at various radii from the axis of the notched bar.

Consideration is being limited to deep notches with a 60° included angle and with notch root radii between 0.005 and 0.100 inches. The diameter at the notch has been chosen to give a notch cross section half that of the shank. The widely adopted analysis of Neuber (See Ref. 2) for a deep hyperbolic notch should give a very close solution for stresses in the elastic region. When this analysis is applied to a notch with 0.005 inch radius of curvature at the root of a notch in a specimen with minimum diameter of 0.424 inches, the fiber nearest the notch root is found to have an axial stress slightly more than six times the nominal axial stress. For the same fiber the hoop stress (in the circumferential direction) is about twice the nominal axial stress.

General Behavior During Plastic Flow

One may inquire as to how high the tensile load may be raised before this most highly stresses fiber will yield plastically, and how high an initial localized stress concentration may be achieved in a bar with a deep sharp notch. The general plastic behavior of metals at room temperature is treated by Hill. (See Ref. 3). There is no reason immediately apparent why his mathematical development should not remain valid at elevated temperatures for changes occurring so rapidly that creep effects are relatively small, provided the necessary physical constants are evaluated at the temperature under consideration. The following observations are taken from Hill's treatment:

(1.) By experiment, the extent of yielding is but little affected by a moderate superimposed hydrostatic pressure. From this observation it may be reasoned that the component of plastic strain in a given direction depends not on the magnitude of total stress, but rather on the "deviator" stress in the given direction. This deviator stress (S'_{ij}) is defined as the component of total stress less the arithmetical average of the three principal stresses.

(2.) The start of yielding appears to depend only on differences between individual principal stresses ($S_1 > S_2 > S_3$). For ductile materials the best correlations to date seem to indicate the following combination of individual stresses as giving the proper measure of the effective stress $(\bar{S})$ at onset of yielding:

$$2\,(\bar{S})^2 = (S_1-S_2)^2 + (S_2-S_3)^2 + (S_3-S_1)^2 \,. \qquad (1)$$

In pure tension the value of $\bar{S}$ is simply the axial stress S_1.

A similar yield criterion in terms of strains can be written:

$$(9/2)(\bar{e})^2 = (e_1-e_2)^2 + (e_1-e_3)^2 + (e_2-e_3)^2 \,, \qquad (2)$$

where $\bar{e}$ is the axial strain for pure tension and e_1, e_2, e_3 are the three principal strains. (Some authors prefer to use the octahedral shear stress and strain, given respectively by $\frac{\bar{S}\sqrt{2}}{3}$ and $\bar{e}\sqrt{2}$

(3.) When an increment of plastic strain occurs, the directions of the principal components of this strain coincide with the axes of the <u>total</u> principal stresses existing at that moment, independent of the direction of the added increment of stress. Moreover, the magnitudes of plastic strain increments depend on the existing total stresses and <u>not</u> on the stress increment. As a consequence, the plastic-strain history must be followed in a step-wise manner. After each small plastic deformation the new stress pattern is determined. Then the strain pattern may be evaluated for the next stress addition.

The incremental plastic strain (de^p_{ij}) in any direction is related to the incremental effective strain $(\overline{de^p})$ by the following relationship developed on page 39 of reference 3:

$$de^p_{ij} = \overline{de}^p\,(3/2)(S'_{ij}/\bar{S}) \,. \qquad (3)$$

(4.) If an element "unloads", i.e., if the effective stress in a fiber decreases, all changes follow the laws of elasticity until such time as the effective stress is again raised to the value from which unloading began.

Stress Patterns During Loading of a Notched Cylindrical Bar

The above general behavior patterns apply to all classes of plastic flow. However, specific expressions for the distribution of stresses and strains have been published for only a limited number of examples. Lacking a rigorous analytical solution for notched tensile bars with axial symmetry, one must resort to approximate methods.

Fried and Sachs (Ref. 4) took hardness traverses on sections from large notched bars of carbon steel pulled to fracture at room temperature. Contour lines of constant hardness in the fractured bars were observed to correspond closely to the photo-elastic pattern showing lines of constant shear stress in flat notched specimens pulled under tension within the elastic range. From this coincidence it was concluded that "...in a notched body (under load) the lines of constant maximum shear stress are almost identical in the plastic and in the elastic state."

At conditions studied in the present program, changes in notch geometry during loading have been shown previously to be too small to affect elastic stress concentration factors significantly. Under these circumstances, the distribution of deviator stress components corresponding to the elastic state will be assumed to continue during the small plastic strains of the loading period.

When a notched specimen is stressed below the elastic limit for all fibers, nearly all the energy associated with straining is recoverable. During plastic deformations, energy is used in changing shape of the body, with unknown energy losses in the process. Neglecting any energy loss during plastic deformation, the attainable plastic strain in any fiber would appear to be that value at which the area under the actual stress-strain curve just equals the elastic strain energy which would have been expended had elastic conditions persisted throughout loading. This approximate method should satisfy the needs of the present analysis.

Application of Plasticity Correlations to Creep and Rupture

The usual methods of handling plastic flow at room temperature seem to be adequate to correlate creep rates under triaxial loading provided an experimental relation is available between stress and creep rate for the state of strain present. Johnson (Ref. 5) compared creep rates of magnesium at 20°C for flat plates pulled in two directions and for combined tension-torsion runs in thin tubes. The two systems were designed to give like values of effective stress, but with the principal stress components in the two cases differing by a constant value of hydrostatic stress. Creep rates were observed to be the same for these two quite different stress patterns. For tests on thin cylinders of four alloys at two or three temperatures each, a plot of octahedral stress versus octahedral strain gave common curves for pure shear, pure tension and variable ratios of shear and tension.

Similar good agreement between theory and experiment was found by Soderberg (Ref. 6) for thin steel cylinders under internal pressure when creep components were compared with corresponding deviator stresses.

The situation is less clear with respect to rupture under combined stresses at creep conditions. From short-time fracture tests on an aluminum alloy in combined tension and torsion, Johnson and Frost (Ref. 7) concluded: "...the criterion appears to be between the octahedral stress and the maximum shear stress, and is certainly not a direct function of the maximum principal stress." But incomplete tests on 0.5% Mo steel and on copper indicated the criterion of fracture might be the maximum principal (tensile) stress of the system imposed.

Proposed Step-wise Treatment of the Creep-Relaxation Process

When a notched bar is held under constant load at elevated temperature, complex changes in stress and strain throughout the bar may be expected, with gradual leveling of initial stress gradients. In the actual bar the stress levels will vary smoothly from point to point without discontinuities, but to facilitate calculations the cross section will be divided into a sufficient number of concentric rings such that conditions at the centroid of any given ring are quite representative of that entire ring. Further, the actual continuous change in stress pattern will be replaced by an equivalent series of time intervals over each of which the creep rate and stress in a given fiber may be considered nearly constant.

Immediately on loading, a fiber in the notched bar has a unique creep rate determined by the initial effective stress and effective strain. The corresponding creep rates in the three principal directions may then be calculated from a modification of Equation (3):

$$\dot{e}_i^p = \dot{\bar{e}}^p \, (3/2)(S_i' / \bar{S}) \quad . \qquad (4)$$

(The dot over a symbol represents "rate," a prime indicates a deviator component, and a bar over a symbol refers to effective stress or strain.)

The component of plastic strain in any direction may result in elongation (creep) of the body, but it could also replace initial elastic strain, with resultant drop in the stress level of the fiber (relaxation). How the total plastic deformation splits between creep and relaxation depends on the extent of stress gradients in the structure.

In a conventional tensile creep bar, where all fibers are subjected to the same stresses until necking occurs, the body can creep as a unit with no reaction of one fiber on another. Such is not the case, however, in a notched bar where the stresses vary continuously from one fiber to the next.

Consider three parallel bands with axial stresses $S_1 > S_2 > S_3$ at their respective centroids. Corresponding axial creep rates, if each band were separate from its neighbors, would be $C_1 > C_2 > C_3$. For continuity to be maintained between filaments, the same total deformation must exist on the two sides of the common interface. This does not say that the deformations at the two edges of a particular band will be the same. The creep rate at different points across any such band in a notched bar will deviate slightly from the rate at its centroid, but this latter value should be quite representative if the band chosen is not too wide.

When band (1) has a total creep in excess of band (2), the difference in plastic strain must be made up by elastic strains in the two bands so long as the fibers of (2) do not become stressed above their yield point. This elastic interaction gives a stress reduction (or relaxation) in band (1) and a stress rise in (2). The absolute values of these two elastic stress changes will be distributed inversely as the areas of the two bands concerned.

Sinultaneously, the elastic stress in band (2) is relaxing and that in (3) increasing due to a similar interaction at the 2-3 interface. The net stress change for band (2) can be taken as the difference between the gain from (1) and the loss to (3).

Using this analysis, the physical requirement of constant total axial load in a notched-bar test is automatically met since each time one band drops a certain portion of the axial load by relaxation, a neighbor picks up exactly the same amount of load. The procedure can be applied in turn to strain rates and stresses in all three principal directions.

On occasion the lower-stressed of two adjacent fibers will be above the elastic limit. In this event the stress interaction between the two bands will be reduced considerably below that for the pure elastic case. The load gain of one band will still be set equal to the load loss of the other, but the stress-strain relationships for the ring with rising stress level must be determined from the short-time tensile curve at the existing stress level.

Applying this approach to concentric rings in the plane of a circumferential notch, analysis will start with the two outermost rings and proceed inward toward the axis. The drop in load for each by relaxation and the gain in load by shift from its neighbor is found for each ring for the same short time interval. When all changes are known, the new stress levels in each ring are calculated and the process repeated.

During the first time interval considered, each ring has been subjected to some average stress level for the given length of time, so that a fraction of the total rupture life of each ring has been used up. Initial correlation attempts will assume that the portion of rupture life consumed during any interval equals the fraction:

$$\frac{\text{actual time at a given effective stress}}{\text{rupture time at this effective stress}}.$$

When the entire life has expired for any one fiber, failure of the notch bar will be considered to have initiated and rupture should be imminent.

Part of the data of Johnson and Frost referred to above indicates that rupture life depends on the largest single stress; i. e., on the axial component in the present case. If the analysis indicated here is correct, the axial component cannot fall below the nominal stress. This, plus the fact that other data refute the role of the largest principal stress as controlling rupture, was the reason for using the effective stress in the above fraction.

SAMPLE CALCULATIONS ILLUSTRATING PROPOSED CORRELATION METHOD

For the notches tested in this program the region of stress concentration occupies approximately the outer one tenth of the notched cross section. The notched section was divided into a central core covering one half of the total area, surrounded by four rings each with one tenth of the area, plus four outer rings each with one fortieth of the total area. These rings are designated consecutively by numbers from one to nine.

When a Waspaloy specimen with 0.040-inch root radius was loaded to a nominal stress of 35,000 psi at 1500°F, the stress distribution at the centroids of the several rings was estimated to be as follows:

Ring No.	Axial Stress, S_a (psi)	Tangential Stress, S_t (psi)	Radial Stress, S_r (psi)
9 (outermost)	73,300	22,900	2,180
8	69,400	23,150	5,760
7	65,900	23,400	9,100
6	61,200	22,700	10,640
5	51,150	20,900	13,330
4	40,800	18,400	14,900
3	35,200	17,400	15,160
2	31,000	16,200	14,950
1	23,650	13,870	13,650

These values were converted to deviator components and then combined to give effective stress levels using relationships previously cited:

$$S'_a = S_a - (1/3)(S_a+S_t+S_r) \quad , \text{ etc.}$$

$$\overline{S}^2 = (1/2)\{(S'_a-S'_t)^2+(S'_a-S'_r)^2+(S'_t-S'_r)^2\}$$

Results of the above calculations are included in Table II, below.

TABLE II

INITIAL CONDITIONS AT THE CENTROIDS OF THE SEVERAL RINGS CONSIDERED IN THE ANALYSIS OF A NOTCHED BAR OF WASPALOY LOADED TO 35,000 psi AT 1500°F. (NOTCH ROOT RADIUS 0.040 IN.)

Ring No.	Stress Components (psi) S'_a	$-S'_t$	$-S'_r$	$\bar{S}$	Creep Rate (in/in/hr) $\dot{\bar{e}}^p$	Slope of Tensile Curve H' (psi/in/in.)
9	40,510	9,890	30,610	63,500	0.072	--
8	36,630	9,620	27,010	57,000	0.029	13.6×10^6
7	33,100	9,400	23,700	51,100	0.012	17.6×10^6
6	27,690	10,810	22,870	45,650	0.0054	21×10^6
5	22,690	7,560	15,130	34,600	0.00097	21×10^6
4	16,100	6,300	9,800	24,300	0.00019	21×10^6
3	12,630	5,170	7,410	19,000	0.000 074	21×10^6
2	10,280	4,520	5,770	15,100	0.000 034	21×10^6
1	6,590	3,190	3,410	9,900	0.000 009 7	21×10^6

During the experimental program on this alloy complete smooth-bar creep curves were obtained for a range of stress levels. These were cross plotted to show stress versus creep rate for different percentages of total life expired. Thus, one curve gave initial creep rates; a second showed minimum creep rates (approximately 5 to 15% of total life expired.) Other curves showed creep rates for times equal to 0.4, 0.6, and 0.8 of the rupture lives at the several creep stresses employed.

From such plots the effective creep rate $\dot{\bar{e}}^p$ can be found corresponding to the effective stress $\bar{S}$ for each ring. The initial creep rate for each ring has been added to Table II. Values from that tabulation will now be used to illustrate the steps in a typical calculation cycle.

Step 1.

For any given ring the deviator components of creep are distributed in proportion to the deviator stresses:

$$\dot{e}'_i = (3/2)(\dot{\bar{e}}^p)(S'_i / \bar{S}).$$

Thus for Ring 9:

$$\dot{e}'_a = (3/2)(0.072 \text{ in/in/hr})(40{,}510/63{,}500) = 0.0689 \text{ in/in/hr.}$$

$$\dot{e}'_t = (3/2)(0.072)(-9{,}890/63{,}500) = -0.0168$$

and for Ring 8:

$$\dot{e}'_a = (3/2)(0.029)(36{,}630/57{,}000) = 0.0279$$

$$\dot{e}'_t = (3/2)(0.029)(-9{,}620/57{,}000) = -0.00734$$

Step 2.

Perform subtractions for each component and each pair of rings:

Ring No.	$\dot{e}'_a$	$-\dot{e}'_t$
9	0.0689	0.01690
8	0.0279	0.00724
9-8	0.0410	0.00946

Step 3.

The differential creep-rate components may be converted to stress changes for a time interval chosen small enough that the assumption of constant conditions over the interval is not too far in error:

Stress component change (psi) = (Component creep rate difference) (in/in/hr)

(Time interval) (hr) (An area-modulus factor) (psi/in/in)

The generalized relationship between components of stress and strain in the elastic region may be expressed:

$S_{ij} = \lambda\, e_{ii}\, \delta_{ij} + 2G\, e_{ij}$, where λ is a proportionality constant and G the shear modulus.

By the assumption of plastic incompressibility inherent to the criterion of yielding already adopted in this analysis, e_{ii} can be set equal to zero. It is further noted that the plastic strains in relaxation are equal to but opposite in direction to the elastic strains replaced. Therefore, changes of principal stresses and principal strains for relaxation are related by $\Delta S_i = -2G\Delta e_i$.

Since differences in deviator components are the same as differences in total components, this same expression relates deviator changes. The factor 2G is related to the elastic modulus (E) and Poisson's ratio (n) by:

$$2G = \left(\frac{E}{1+n}\right).$$

Available data indicate that n has a value of about 0.32 for the temperatures and alloys of interest to this program.

When the stress gain of the lower-stressed fiber puts it into the yield range, the resulting deformation for a given stress change is larger than for the elastic case by the ratio E/H' , where H' is the slope of the stress-strain curve at the stress concerned. In the following the symbol M will be defined for both the elastic and plastic cases as

$$M = H'/(1+n), \text{ with } H'=E \text{ for elastic stresses.}$$

Distribution of stress changes in adjacent fibers with different creep rates may be seen from consideration of Figure 2, which shows bands (1) and (2) with respective cross sections A_1 and A_2. In a given time interval, creep would change the lengths from ab to ac_1 and ac_2 if the bands could creep independently. The two rings will now be strained amounts δ_1 and δ_2 in opposite directions until the creep difference Δc is "made up"

$$\delta_1 - \delta_2 = \Delta c \qquad (I)$$

But the load picked up by (2) must equal that dropped by (1) to preserve boundary conditions.

$$\delta_1 A_1 M_1 + \delta_2 A_2 M_2 = 0$$

$$\delta_1 = -\delta_2 A_2 M_2 \,/\, A_1 M_1 \qquad \text{(II)}$$

Substituting (II) into (I):

$$-\delta_2 (1 + A_2 M_2 / A_1 M_1) = \Delta c$$

$$-\delta_2 = \Delta c \left(\frac{A_1 M_1}{A_1 M_1 + A_2 M_2} \right)$$

$$\delta_1 = \Delta c \left(\frac{A_2 M_2}{A_1 M_1 + A_2 M_2} \right)$$

The stress changes corresponding to these strains are:

$$\Delta S_1 = \delta_1 M_1 = \left(\frac{A_2 M_1 M_2}{A_1 M_1 + A_2 M_2} \right)(\Delta c)$$

$$\Delta S_2 = \delta_2 M_2 = -\left(\frac{A_1 M_1 M_2}{A_1 M_1 + A_2 M_2} \right)(\Delta c)$$

The area-modulus factors for rings (1) and (2) are thus seen to be, respectively:

$$+\frac{A_2 M_1 M_2}{A_1 M_1 + A_2 M_2} \quad \text{and} \quad -\frac{A_1 M_1 M_2}{A_1 M_1 + A_2 M_2}$$

For elastic strain distribution between rings of equal area, the area-modulus factor is simply $M/2 = G$.

At 1500°F Waspaloy has an elastic modulus of 21×10^6, while $H' = 13.6 \times 10^6$ at $\bar{S} = 57{,}000$ psi. Therefore,

$$M_9 = \frac{21 \times 10^6}{1.32} = 15.9 \times 10^6$$

$$M_8 = \frac{13.6 \times 10^6}{1.32} = 10.3 \times 10^6 \ .$$

For a time interval of 0.002 hour, the axial strain change in ring 9 is found to be:

$$(0.041 \text{ in/in/hr})(0.002 \text{ hr})\left(\frac{(-15.9)(10.3 \times 10^6)}{15.9 + 10.3}\right) \text{ psi/in/in} = -510 \text{ psi.}$$

The axial stress in Ring 8 is raised by 510 psi from the same 9-8 interaction while the 8-7 interaction would lower the axial stress in Ring 8 by 240 psi, determined by a calculation similar to the one shown. The net change in Ring 8 is thus +510 -240 = +270 psi during the first 0.002 hr.

Step 4.

Combine the new values of the deviator stress for each ring to get the new effective stress.

Step 5.

The percent of life expended during the above time interval is found for each ring.

(a) At the initial effective stress of 63,500 psi the rupture life of Ring 9 would be 1.3 hr.

(b) At the new effective stress (62,800 psi) after the first time interval, the rupture life is 1.5 hr.

(c) $\frac{0.002 \text{ hr}}{(1.3 + 1.5)/2}$ 100% = 0.14% of life expended in Ring 9 during the first 0.002 hr of the notched-bar test.

Step 6.

Knowing the new effective stresses and the cumulative expenditure of life to date, the new effective creep rate for each ring may be read from the plot of stress versus creep rate and the process repeated for a second interval.

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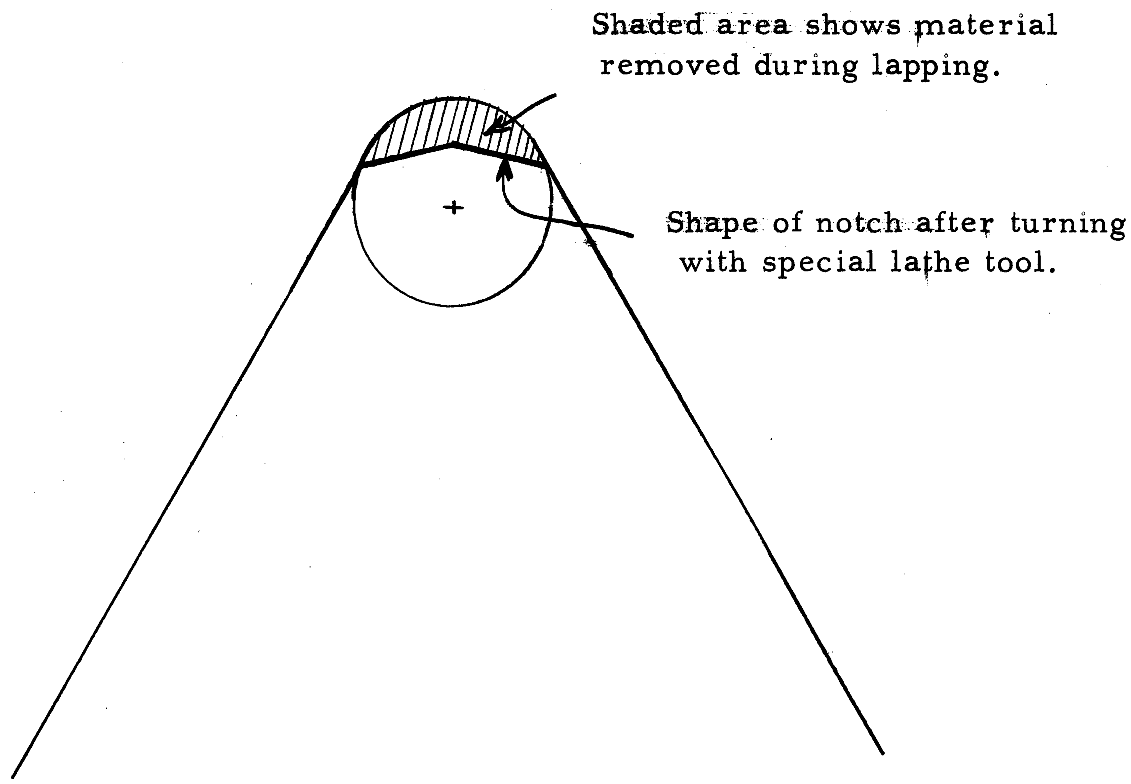

Figure 1. Sketch Illustrating Notch Preparation by Form Lapping.

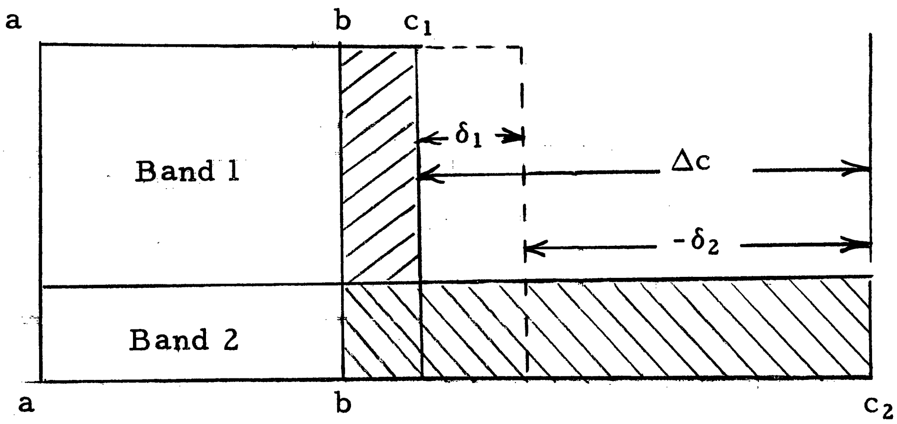

Figure 2. Schematic Representation of Stress-Strain Interactions at an Interface between Two Bands with Different Creep Rates.