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NOTCH SENSITIVITY OF HIGH-TEMPERATURE ALLOYS

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## FOREWORD

This report was prepared by the University of Michigan under USAF Contract AF 33(616)-5775 which was in force from April 1958 through June 1959. Initiated under Project 7360, Task No. 73605, the research was administered under the direction of the Materials Laboratory, Directorate of Laboratories, Wright Air Development Center. Dr. A. J. Herzog acted as project engineer.

The main purpose of the report was to combine results obtained during the above contract with more-extensive findings of two previous contracts (AF 18(600)-62 and AF 33(616)-3380) to evaluate the current state of knowledge with respect to creep-rupture of notched specimens. Recent technical literature was heavily drawn upon in this evaluation.

## ABSTRACT

Critical examination of all available results on creep-rupture of notched specimens disclosed:(1) notch strengthening for all alloys under proper conditions, (2) maximum strengthening for intermediate notch acutities, (3) two general patterns of stress-rupture time data relative to the smooth-bar curve.

Analysis of observed behavior suggests notch strengthening requires stress redistribution by yielding and creep, and is associated with the multi-axial stress pattern produced by the notch.

Both maximum principal stress and the shear-stress invariant are hypothesized to influence rupture of notched specimens through their respective effects on crack initiation and propagation.

Research is proposed on crack development during creep-rupture in notched tension bars and in biaxial plate specimens, to verify the suggested explanation for notch behavior.

## PUBLICATION REVIEW

This report is published for the exchange and stimulation of ideas. Its publication does not necessarily imply approval by the Air Force of the findings or conclusions contained herein.

FOR THE COMMANDER:

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## NOTCH SENSITIVITY OF HIGH-TEMPERATURE ALLOYS\*

Research studies at the University of Michigan under WADC sponsorship have sought to determine basic reasons for the varied elevated-temperature stress-rupture behavior of notched specimens loaded by axial tension. Such specimens are characterized by multi-axial stresses, initially concentrated near the notch. Both the present analytical study and earlier experimental research were based on the concept that the rupture strength of a notched specimen is governed by whether or not the localized high stress can be sufficiently reduced by localized yielding at the notch upon load application and through subsequent creep-relaxation during the test before too large a portion of the total rupture life is expended.

Experimental background was obtained by determining rupture times for eight different alloys with a variety of notch geometries. Tests conducted with smooth (un-notched) specimens of these same alloys established pertinent short-time tension properties and creep-rupture properties at temperatures used for the notched-bar studies. Approximate correlation of these data was found possible on the assumption that rupture life under complex-stress creep depends on the effective stress of the shear-stress invariant theory. Publication by others of findings that rupture life was determined by the largest single principal stress raised questions as to the validity of this entire analysis to explain notch behavior.

A critical re-examination has now been made of the data on rupture behavior of notched specimens, including all available published work bearing on the problem. Explanations for the observed trends seem possible in terms of a dual theory of failure in which initiation of fracture rests chiefly with the largest principal stress, while propagation of fracture is controlled by creep rate which depends on the level of effective stress.

The present report outlines the general trends displayed by available data, develops an explanation of these trends and discusses further research to verify basic behavior patterns which became apparent during the study.

### Stress Distributions in Notched Specimens

All notched specimens tested at the University of Michigan were patterned after types for which the distribution of stresses in the elastic range is firmly established - namely, the cylindrical bar with

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a deep circumferential "V" notch and the flat strip with "V"-form edge notches. (See Ref. 1). When a small axial load is applied to such a specimen with a finite notch-root radius the stresses all remain elastic and are in direct proportion to the applied load. Due to the symmetry involved, one principal stress parallels the specimen axis and the other two principal-stress directions lie in the plane of the notch root. The peak local stress level occurs for the axial (longitudinal) component, just at the notch root; the axial stress declines steadily between the notch root and the specimen axis under elastic conditions. In round notched specimens the distribution of tangential (hoop) stress follows a pattern similar to that for the axial component except with lower values. At the free surface of the notch, the radial component for cylindrical bars or the transverse component for a flat notched strip can have no value. This radial or transverse stress component rises to a maximum some short distance beneath the notched surface and then declines toward the center line of the specimen. Typical flat specimens are sufficiently thin that any stress in the thickness direction may be neglected.

During loading to nominal stress levels that will cause rupture in a reasonable test period, localized yielding will occur near the notch root for most notch geometries employed for acceptance purposes or research investigations. Analysts disagree as to the precise form of the resulting stress pattern, but all seem in agreement that a small magnitude of short-time plastic strain permits substantial redistribution of stress concentrations and relief of elastic stress peaks. Qualitative findings of three recent studies (Refs. 2, 3, 4) of stresses in a cylindrical notched bar after loading into the plastic zone are compared in Figure 1. Though details differ, in each case the peak stress is an axial tension at a radius somewhat removed from the notch, while the maximum shear stress is highest at the notch root.

Sachs and Lubahn earlier (Ref.5) interpreted results of room-temperature tensile tests on steels to say that two percent of gross plastic strain in a specimen with a V notch having a root radius of 0.062 inch or sharper should reduce the stress concentration factor to near unity. This is a much higher order of overall strain than would be encountered in the usual notched-bar rupture test, yet some degree of stress concentration still was interpreted to exist; i. e., the axial component still exceeded the nominal stress over an outer portion of the notched cross section.

Before the bar underwent general yielding, extensive dispersion of the initial concentration of stress near the notch root was shown by Southwell and Allen (Ref. 6) to also occur in a long flat tension



bar notched symmetrically on opposite sides. In the plastic region formed across the minimum section of the bar, the longitudinal tensile stress was calculated to rise steadily with distance in from the root, while the lateral tension increased from zero at the notch to a maximum at the plastic boundary.

Although the foregoing paragraphs refer to experiments and analytical treatments for room-temperature conditions, no obvious restrictions prevent the results from being applied to elevated temperatures, provided the necessary elastic properties are determined at the temperature of interest and provided the rate of loading is sufficiently rapid to negate time-dependent effects.

### Creep-Relaxation During a Notched-Bar Rupture Test

Whether the stresses after loading are still all elastic or whether some plastic straining has taken place, redistribution of the existing stress pattern should occur by creep relaxation at elevated test temperature. The material in any small portion of the specimen should tend to creep at some specific rate determined only by the prior history of that bit of material and by the current stress level and orientation. Studies on a variety of alloys under different patterns of complex stress suggest use of either the shear-stress invariant or the maximum shear stress as a suitable measure of the stress combination determining rate of creep under combined stresses, with slight favor for the former. (See Refs. 7 through 19). These criteria were both originally proposed to correlate time-independent yielding of ductile metals. By the criterion of Tresca, only the largest and smallest principal stresses are involved, in the form of the maximum shearing stress,  $(S_1 - S_3)/2$ . All three principal stresses ( $S_1 > S_2 > S_3$ ) combine to give the equivalent stress  $\bar{S}$  of the shear-stress invariant theory:

$$2(\bar{S})^2 = (S_1 - S_2)^2 + (S_1 - S_3)^2 + (S_2 - S_3)^2.$$

This theory, generally attributed to von Mises (Ref. 20), and extended by Hencky (Refs. 21 and 22) and by Huber (Ref. 23), goes by many names including "distortion-energy" (Gestaltänderungsenergie), "octahedral shear stress", and "maximum shear-strain energy".

For the purposes of the present discussion, identification of the exact stress combination controlling creep is probably not necessary -- particularly since the two theories most in favor give quite similar answers. Immediately on loading each small element of the specimen

will have a unique creep rate corresponding to the proper effective stress acting on the element. The component of plastic creep strain in any given direction may result in elongation (creep) of the element, but could alternately replace part of the elastic strain associated with the existing stress, resulting in decline or "relaxation" of the stress level in the element. Because a small relative change in stress produces a large difference in creep rate, overall deformations are limited to the order of elastic strains in a notched specimen having high stress only at localized portions of the cross section. Should gross creep deformation develop over the entire cross section, the effective stress may be presumed to have become substantially uniform across the specimen.

At each distance from the axis in the slice of metal containing the plane of the notch, the alloy is thus subjected to a unique history of initial stress and strain and to subsequent continuous modification of the stress and strain patterns with time. Simultaneous microscopic and submicroscopic structural alterations are produced in the metal according to the heat-treatment conditions of the alloy and its metallurgical stability at the test temperature and under the variety of stress-strain histories imposed at different locations in the slice. General criteria for initiation and propagation of fracture during creep are still not well defined, but at some critical location rupture finally occurs causing renewed shifts in the stress pattern, build-up of the average stress level and eventual separation of the specimen into two pieces. This final separation of the notched bar is the event normally recorded in rupture testing.

#### Types of Notched-Bar Rupture Behavior

Rupture-test results for un-notched specimens are shown by a single straight line or a pair of intersecting straight lines on a graph of the logarithm of the stress versus the logarithm of the rupture time. For all reported studies where sufficient data have been obtained to establish a clear pattern for rupture tests with a given notch geometry, either of two general curve shapes relative to the corresponding smooth-bar data seem to have resulted:

(1.) One large segment of the reported data was characterized by roughly parallel stress-rupture time curves for smooth and notched specimens. (Case 1 of Figure 2).

(2.) For the remainder of the reported conditions the rupture curves for notched bars at the higher stresses fall off faster than for smooth bars, but at lower stresses (i. e., longer rupture times) the notched-bar curves change slope until they either parallel that for smooth bars

or become less steep than the smooth-bar curve. Case 2 of Figure 2 illustrates this behavior for notch weakening over only a limited range of nominal stress.

Available data from studies at the University and elsewhere on notched-bar rupture behavior have been assembled in Table 1 according to which of the above behavior patterns the data most closely followed. The bulk of these notched-bar rupture data relate to cylindrical specimens with a circumferential "V" notch, but limited tests are included for flat specimens with "V" notches at the edges. The flank angle of the notch was  $60^\circ$  in nearly all instances and the cross section at the notch 50% of the shank cross section. Different experimenters employed widely-different values of root radius at the base of the notch, with consequent variation in notch severity.

Perhaps the most-frequent characterization of notch severity has been the theoretical elastic stress concentration factor,  $K_t$ . This factor is the ratio of the axial stress component which would exist at the notch root under elastic loading to the nominal axial stress over the notched cross section for the same axial load. The value of  $K_t$  is customarily determined by mathematical analysis such as that of Neuber (Ref. 1). When the geometry of the part permits, the stress concentration factor can be evaluated by empirical tests on scale models made of plaster or similar material with elastic-brittle properties.

Notch configuration is also expressed in terms of the ratio ( $r/d$ ) of the notch root radius to the minor diameter at the base of the notch for a constant stated ratio of minor to major diameter. "Sharp" notches correspond to small values of  $r/d$  and this ratio becomes larger as the notch is made duller. In order to have a number which increases with the stress concentration, some authors prefer to define "notch sharpness" as the ratio  $d/2r$ , where  $d$  is again the minor diameter and  $r$  the radius of curvature of the notch root. (This latter ratio is the same as the triaxiality parameter ( $a/r$ ) used by Bridgman (Ref. 24) in his analysis of stress distribution at the neck of a tension specimen, in which  $a$  was the radius of the bar at the neck and  $r$  the radius of curvature of the necked surface).

The examples in Table 1 are divided about equally between the two general cases. For several of the materials the same alloy is found under both headings. More-complete coverage of possible test conditions might extend this finding of dual behavior to other members of the list, or might even show that the two apparent behaviors are really only different portions of a single more-general behavior pattern exhibited by all materials.

Although no differentiation was made in Table 1, alloys displaying curves of a given characteristic shape included widely-varying degrees of notch sensitivity as measured by comparative rupture times for notched and un-notched (smooth) specimens at like nominal stresses.

An alloy is commonly referred to as "notch ductile" or "notch strengthened" if the notched specimen exhibits longer life than the comparable smooth specimen at like temperature and nominal stress; i.e., at equal levels of stress as computed from the applied load divided by the minimum cross section at the start of the test. Should the rupture life of the notched specimen be less than that without a notch, the material is said to be "notch brittle" or "notch weakened".

Experimental findings of researches cited in Table 1 amply demonstrate that such terms as the above cannot adequately characterize an alloy. Rather, the type and degree of response to a notch varies with test temperature and stress level, with the heat treatment employed and, particularly, with notch geometry.

#### Influence of Notch Acuity and Alloy Ductility

The most thorough study to date on effects of notch geometry (Ref. 26) presented results in terms of the parameter  $r/d$ . The first test series of that study employed a constant shank diameter ( $D=5/8$  inch) and minimum diameter ( $d = 15/32$  inch), but the notch root radius ( $r$ ) varied. For dull notches the rupture strength increased with notch severity (decreasing  $r/d$ ). With increasingly severe notches a maximum in rupture strength was found, followed by a decline in strength for the sharpest notches tested. Figure 3 (reproduced from Figure 9 of Reference 26) shows the influence of notch-root radius on the 1000-hour strength at 1200°F for Refractaloy 26 in three conditions of heat treatment (Alloys A, B, C) and for a 12 Cr, 3 W steel at 1000°F. A similar result would obtain for the 100-hour or other rupture strength since the curves for the several notch geometries were all roughly parallel to the smooth-bar curve.

The same pattern of rising and then falling strength with notch acuity is shown in Figure 4 for the 100-hour rupture strength of a variety of other alloys tested under WADC sponsorship. (See Refs. 31, 34, 35, 36). These results include findings for both round (cylindrical) and flat (strip) specimens and include several examples of "Case 2" behavior of Table 1. Rupture strengths are presented as a function of the theoretical stress concentration factor, with the smooth-bar data plotted at  $K_t = 1$ .

More such plots could be prepared from results available in Reference 28. Although the same general trends seem to be followed by all these materials, the relative degree of notch strengthening at the respective maxima differed greatly among the alloys. Moreover, the specific response to a given notch geometry differed among materials or for a given material under different test conditions. Both the triaxial and biaxial cases (i. e., both round and flat notched specimens) exhibited the trends noted.

The rupture tests whose behavior under variable notch acuity has been cited are believed to include all principal results available in the readily-accessible technical literature. Perhaps the most important feature of the assembled test results is the apparent universality of notch strengthening --- for every alloy at every test temperature, without exception, notch strengthening was obtained for certain of the test conditions when a wide range of notch acuities was evaluated.

Other features common to some portion of the data of Table 1 have been reported from time to time, but in most instances exceptions are in evidence or else alternate patterns of behavior seem to exist. The very necessity for listing two characteristic cases in Table 1 illustrates this point. A third possible type of notch-rupture curve at 1200°F (one straight-line segment parallel to the smooth-bar plot at high stresses, changing to a second straight line steeper than the lower-stress plot for un-notched specimens) was put forth for  $K_t = 4.1$  and 5.7 in A-286 with conventional heat treatment. (See Fig. 4 of Ref. 31, Part II). However, re-examination of the figure indicates that a more faithful representation of the data points is offered by a "Case 2" curve, as shown in Figure 5 of the present report.

Considerable evidence, albeit indirect, suggests that the occurrence of a minimum notch-strength ratio (notched-bar strength/smooth-bar strength) at intermediate rupture times reflects certain metallurgical changes and that comparative freedom from such changes results in "normal" (Case 1) behavior. W. F. Brown, Jr., and his associates, have reported on extensive tests with sharp notches ( $r < 0.001-0.002$  inch) for a number of precipitation-strengthened alloys, showing striking similarities between comparable curves of ductility and of notch-strength ratio versus rupture time. (See Refs. 37 and 43).

In view of the similarity to familiar curves of tensile elongation of an age-hardening aluminum alloy plotted as a function of aging temperature and time, the observed notch-rupture trends were ascribed to precipitation reactions which progress during the test, first increasing the strength (and lowering ductility) and later resulting in

softening from over-aging. These views were shared by Siegfried in his discussion of Reference 43, stating that notch sensitivity was found to be associated with precipitation reactions for a complex austenitic alloy. In this instance the rate of the precipitation reaction was found to depend on the prevailing stress level. No evidence of a similar precipitation process could be found with a 94% Sn - 6% Cd alloy giving a "Case 2" notch-rupture pattern for creep at room temperature.

One material studied extensively by Brown was the low-alloy steel 17-22 A(S). At high test stresses (i. e., at early rupture times) smooth specimens failed through the body of the grains after extensive elongation, and notch strengthening was found for test temperatures from 900° to 1200°F. For longer tests at the lowest temperature, where aging reactions are expected to be negligible, fractures in smooth specimens were transgranular and notch strengthening prevailed. Test temperatures of 1000°, 1100° and 1200°F were marked by intercrystalline cracking in smooth bars and by transition to notch weakening for tests lasting longer than about 200 hours, 20 hours and half an hour, respectively. It was stated that while these results may be coincidental, they do suggest that any precipitation reaction responsible for notch rupture sensitivity also contributes toward intercrystalline failure of the metal and is therefore present in the grain boundaries. For the sharp notches used by Brown the volume of metal at the base of the notch subjected to intense stress concentration was extremely small, so that the presence of intercrystalline cracks at the point where fracture initiated would be difficult to ascertain.

Be it merely an accompanying reflection of the existing metallurgical condition of the alloy or be it a basic contributor to notch sensitivity, low ductility has long been associated with notch weakening (Ref. 44), but recent workers in the field have repeatedly shown that no single value or range of rupture elongation in smooth bars can be used by itself to predict notch-rupture behavior. Among the extreme cases proving this statement are data of Brown et al. for Haynes 88 alloy at 1350°F and of Voorhees and Freeman for A-286 alloy at 1200°F. (Refs. 37 and 31, respectively). Brown found notch weakening with sharp notches at nominal stress levels for which the reduction of area in smooth bars was consistently above 25%, while for the A-286 with a 2200°F solution temperature uniform notch strengthening was found for notches with  $K_t$ 's up to 3.0, despite smooth-bar elongations of 1% or less for rupture times over 100 hours.

These examples are not to be interpreted as evidence against a positive relationship between an alloy's ductility and its response to the presence of a notch. Siegfried has suggested that notch sensitivity

can be related to smooth-bar ductility only in terms of the relative amounts of uniform ("proportional") elongation and of the necking elongation (corresponding to the reduction of area), by correlations similar to those of Kuntze for notched specimens in short-time tension tests (Ref. 45). But the work of Brown cited above demonstrated that sensitivity to sharp notches should be suspected whenever the smooth-bar ductility decreases rapidly with increasing times to rupture, and is often encountered when the smooth-bar ductility is less than 1 or 2 percent.

Examples in the literature of such apparent relations between ductility and notch-rupture behavior are many, particularly when sharp notches were employed; generally-poorer agreement has been found for dull notches. A Cr-Ni-Mo steel at 500°C showed transition to notch weakening while the smooth-bar reduction of area was still 10-15%, but after a drop from 60-70% reduction of area in short-time tension tests (Ref. 37). Ductility varied ten-fold in smooth-bar tests at 1350°F for Inconel X-550 alloy which displayed both minimum notch-strength ratio and minimum reduction of area at about 300 hours rupture time (Ref. 35). At 1100°F a sharp notch of  $K_t=8$  in Type 321 stainless steel displayed a change from notch strengthening to mild notch weakening and back to notch strengthening as the smooth-bar elongations changed only from 28% to 17-18% to 32% (Ref. 29). The same reference reported transition from notch strengthening to notch weakening in Type 304 stainless steel with  $K_t=3.7$  at a test temperature of 1100°F, corresponding to a drop in smooth-bar elongation from 26-42% to 11-13%.

An alloy's ductility and the notch geometry seem to be involved together in determining not only whether the notched-bar life will be less than or will exceed the smooth-bar performance, but also the shape of the notched-bar rupture curve. S-816 alloy and its Ta modification, with consistently-good smooth-bar ductility, evidenced "Class 1" behavior in Table 1 for temperatures ranging from 1200° to 1650°F. In contrast, three stainless steels (Types 304, 316, 321) with higher ductilities at 1500°F than at 1100°F showed "Class 1" behavior at the higher temperature but a minimum notch-strength ratio at the lower temperature for tests at fixed  $K_t$ . The influence of varying notch acuity at constant smooth-bar elongation is illustrated by Siegfried's tests at 1202° and 1346°F on one austenitic steel. At the lower temperature the two sharpest of three notches investigated had "Class 2" curves and the dullest notch gave results parallel to smooth specimens; at the higher test temperature only

the sharpest notch produced a "Class 2" pattern. It might also be recalled that Figure 5 of the present report shows a change from the one behavior to the other as notch acutities increase.

### Initiation and Propagation of Fracture in Notched Bars

Presumably ductility and notch geometry should also exert important effects on the initiation and propagation of fracture, supplementing the effects of time or stress level which seem to relate to the mode of fracture in smooth-bar rupture tests. For notched bars the short-time yield properties enter as well, since most reported tests have been at conditions leading to localized yielding at the notch root to give the modified patterns of initial stress mentioned earlier. Mirkin and Trunin (Ref. 4) made a careful study of notched rupture specimens, some run to fracture and others removed from testing before complete failure took place. Microhardness patterns and photomicrographs obtained for successive slices cut parallel to the specimen axis were interpreted to show that initial cracks or failure thresholds were of an intergranular nature and were located in the region of maximum longitudinal stress at the inner edge of the zone of plastic deformation formed near the notch upon load application. Intergranular cracks at a depth of 0.2-0.4 mm were noted for both short and long tests, in both a pearlitic steel and an austenitic alloy, and for several notch configurations in specimens with a minimum diameter of 7 mm. As the failure spread outward toward the surface at the notch root, fracture of the outer 0.05-0.1 mm proceeded through the body of the grains in all cases. Based on their observations that failures in the notch zone all began and extended initially along grain boundaries, as is characteristic for long-time failures of smooth specimens, Mirkin and Trunin strongly implied that notched-bar rupture curves should always parallel the second branch of the usual smooth-bar plot. But at least four sets of data referred to in Table 1 were noted to exhibit a corresponding definite change in slope of the rupture plots for both un-notched and notched specimens; additional examples in the tabulation surely permit as good representation by a pair of intersecting lines as by a single line.

In the main, initiation of notched-bar rupture as internal grain-boundary cracks has been confirmed by the other investigations reported to date. In Garofalo's work on stainless steels (Ref. 29) both notched and un-notched specimens failed predominantly by an intergranular mode, so that the mode of failure cannot necessarily be ascribed to the triaxial stressing caused by the notch. Photomicrographic studies of bars taken from tests interrupted at successively longer times clearly showed the start of fracture at a depth from the notch of about 2-4% of the minimum radius and intergranular propagation toward the surface and toward the axis. Similar findings were



reported for conditions of notch strengthening and of notch weakening. One excellent picture of a traverse of the rupture contour showed appreciable plastic deformation near the notch root and about midway between the notch and the specimen center, as attested by elongated grains sloping inward toward the axis. At the center and between the regions of disturbed material no such appreciable deformation was in evidence. Indications are that early intergranular cracks spread outward until the consequently-overloaded surface layers failed quickly in a ductile manner, but that the inward movement of the crack was arrested for some reason. Apparently the shift of load toward the core material resulted in new grain-boundary cracks in the region of high triaxiality around the axis. The metal with large deformation between the two areas of intergranular fracture was surmised to comprise the last portion of the specimen to pull apart after the entire load had shifted to this sole remaining part of the original cross section.

From his observations, Siegfried (Refs. 28, 46) concurs that cracks beginning in the interior of a notched specimen are intergranular in nature, but he also admits of cracks starting from the notch root which are in part intergranular and in part a mixture of intergranular and transgranular types.

Two examples for conditions of notch strengthening examined by Davis and Manjoine showed irregular fractures with forking cracks running near a 45° angle from the axis (Ref. 26). However, for these samples, as well as for intergranular fractures normal to the specimen axis in notch-weakened bars, six locations chosen at random for photomicrographs all had the fracture surface directed into the body of the specimen, making an acute angle with respect to the specimen axis. Apparent absence of any matching obtuse angle was taken to imply that both forms of crack started below the surface and progressed outward. This probability was again supported by a noticeable degree of plastic deformation in material at the notch root, far greater than would be expected to result from the initial yielding when the load was applied.

Finally, the study of Carlson et al. (Ref. 34) revealed distinct intergranular cracks starting below the surface for Inconel X-550 alloy at 1500°F, while for Waspaloy at the same temperature notch-strengthened specimens showed intergranular fracture originating "at or near the surface" and a number of visible unconnected internal cracks. The same authors presented two photomicrographs for S-816 alloy tested at 1350°F. One picture of a notch from an interrupted test appears to show ductile failures originating from the notch surface. However, a possible explanation put forward for these cracks and for their failure to continue into the specimen was that the cracks were associated with a shallow layer of metal deformed during notch preparation and that

at the test conditions surface cracks would normally neither form nor extend in ductile material like that comprising the bulk of the notched cross section.

The question of whether notched-bar fractures in uniform material do or do not always start within the specimen below the notch surface appears not to be resolvable by the data available. An overwhelming majority of the results surveyed here support an affirmative answer. However, the reported findings possess one serious shortcoming --- the combination of nominal stress and notch acuity for all the specimens examined has always been such that plastic yielding has taken place when the load was added, so that the maximum principal stress has been shifted from the notch root to fibers within the body of the specimen. Cracks should then normally be expected to start at the notch root only if fracture is controlled by the shearing stress which still has its peak value at the notch after the plastic strain. Should the largest principal stress be the key factor in determining the location of initial fracture, failure would be expected to originate beneath the surface for all the reported cases in which examination for cracks was made.

The A-286 alloy tested at 1200°F by the present authors under a prior Air Force contract permits rupture times of the order of a thousand hours with notch geometries and load levels for which the proportional limit of the material is not exceeded at any point in the specimen. Careful examination, in the manner of Mirkin and Trunin, of microsections of specimens from interrupted tests run under these conditions should reveal how general are the past findings. Moreover, the results should be of considerable theoretical interest to metallurgists and engineers concerned with creep-rupture phenomena and their application.

## EXPLANATIONS FOR NOTCH STRENGTHENING AND THEIR SHORTCOMINGS

Nothing inherently new should be introduced into the properties of an alloy merely by introduction of a notch into the specimen. Any influence of the notch on rupture behavior logically must involve either (a) some effect of the resulting stress concentration, with the attendant stress gradients and the increase in local strains for any given axial load, or (b) an effect of the multiaxiality of the complex stress pattern produced when a notched bar is loaded.

Concentration of the acting stress near a notch seems well able to account for notch weakening, but the only apparent effect favoring an increase of strength from this factor would seem to be the local work hardening in the region of initial plastic loading strain. However, such work hardening fails on several counts as an adequate general explanation for notch strengthening:

(1) Under suitable nominal stress level and notch acuity, notch strengthening appears to be a universal characteristic for all types of alloys, whereas plastic working produces diverse changes in creep-rupture strengths, either favorable or adverse, depending upon the particular alloy and test conditions.

(2) Notch strengthening has been observed --- at least for A-286 alloy at 1200°F --- for conditions of presumably complete freedom from gross short-time plastic deformation. (Ref. 31, Part II).

(3) Un-notched specimens of Waspaloy at 1350°F were found to suffer a loss in rupture strength after momentary over-loading at temperature to produce plastic pre-strains of the order of 1%. At nominal stresses where plastic deformations of this magnitude would result near the root on loading, notched-bar rupture times fell far short of the smooth-bar results. Abrupt change to notch strengthening was noted when the nominal stress was lowered to the point where the initial plastic strains at the notch root fell below a few tenths of a percent (Ref. 31, Part I).

The present authors have approached notch strengthening with the belief that it could be explained by the beneficial effects to be expected from the multiaxial state of stresses around a notch. Secondary tensions acting perpendicular to the largest principal stress had long been known to raise the permissible level of this largest stress

before yielding or fracture occurred in ductile metals at room temperature. When such studies were extended to temperatures at which creep occurs, similar findings were reported (Refs. 7-15) for the stresses to cause yielding and short-time fracture at elevated temperatures or to produce a given secondary creep rate. The best correlation to this mass of data appeared to be by the shear-stress invariant theory (von Mises criterion).

With no contrary results then in evidence, this theory also seemed a logical guess as the stress combination to correlate rupture time under creep conditions. Such a premise was adopted by the present authors in their considerations of notch sensitivity of heat-resistant alloys (Ref. 35). In that three-part report, notched-bar rupture behavior was reasoned to be but the composite reflection of an initial stress concentration, the redistribution of that stress by any plastic deformation of loading or subsequent creep-relaxation during the test, and the cumulative "consumption" of available rupture life as creep progressed. Any influence from the notch should be associated with the variable stress-strain history it causes in the metal fibers of the specimen. If the point-to-point conditions in a notched bar could be followed throughout the test, the rupture life should be amenable to correlation in terms of normal smooth-bar properties.

Part 1 of reference 35 demonstrated qualitative agreement between notch strengthening and rates of stress leveling in relaxation tests on un-notched specimens. Rudiments of an engineering statement of creep-rupture behavior under variable stress were also established through a series of experiments in which a conventional creep specimen was run under one constant stress for a portion of the rupture life and then changed to other constant stress levels for additional portions of the test. Rupture in these tests occurred approximately when the sum of the fractions

$$\left( \frac{\text{Actual time at a given stress}}{\text{Rupture life for this stress}} \right) \text{ reached unity.}$$

Furthermore, the general shape of the creep curve during any portion of these multiple-stress tests seemed to depend only on the momentary stress and stage of the test, independent of the stress levels of prior portions of the test.

The second report in the series proposed a tentative stepwise mathematical procedure to analyze notched-bar rupture behavior in terms of experimental properties of smooth specimens. This procedure assumed quantitative addibility of rupture-time fractions and considered the effective stress of the shear-stress invariant theory to be a suitable measure of the stress controlling creep-rupture behavior under

multiaxial stresses. Hand calculations completed for one condition of notch strengthening and one of notch weakening indicated reasonable agreement with experimental findings. In these calculations, momentary creep properties were read off plots of stress versus creep rate, with the cumulative percent of expired life shown as a parameter.

The final report under reference 35 included extension of the investigation to flat specimens with notches at the edges. These specimens, with biaxial stressing compared with the triaxial stressing of round notched bars, consistently exhibited rupture times slightly below those for round specimens with the same theoretical stress concentration factors. However, no radical new behavior was found. The elevated-temperature rupture characteristics of all notched specimens under steady tensile load were concluded to depend on three major factors:

1. The distribution and level of the initial stress pattern, determined by the notch configuration and tensile characteristics of the alloy.
2. The rate at which the variable creep rates at different locations in the cross section could relax the peak stress originally concentrated near the notch. (Under multiaxial stressing, the effective stress can easily become less than the nominal value for alloys with low creep resistance).
3. Rupture characteristics of the material for the stress-strain-time histories experienced by different fibers in the notched bar. (If too large a portion of the total life were used up at the initial high stresses, the remaining service should be short even for a low final stress level.)

Calculated versus experimental notch-bar rupture times were compared for thirteen conditions chosen at random from among those tested in the program. Agreement was satisfactory for materials tested under conditions where they were metallurgically stable. In some cases where agreement was less favorable, observed deviations could be explained with the aid of fragmentary data on changes in creep-rupture behavior of smooth specimens subjected to a history of variable stress, designed to approximate conditions at different positions of a notched specimen. However, for none of the original alloys studied was sufficient stock available to permit complete evaluation of the material's smooth-bar behavior under all types of stress history to be expected in the notched rupture specimens studied.

Research under a second contract (Ref. 31) was designed primarily to overcome this deficiency by permitting extensive work on a single alloy. For this particular study, A-286 alloy was chosen in the

belief that both notch strengthening and notch weakening should be obtainable with the same lot of material by merely altering the solution temperature used in the heat treatment. Stock produced by the vacuum consumable electrode process was selected since it represented the most advanced form of the alloy available and should typify materials which would be produced for future applications.

Survey tests at 1200°F included specimens solution treated at temperatures covering the range from 1650° to 2300°F. For stresses of 60,000-70,000 psi, smooth and notched specimens alike exhibited a steady increase in rupture times for increasing solution temperatures up to about 2000 - 2200°F and then the strength fell for both types of specimens. Quite unexpected was the finding that for a moderate notch acuity ( $K_t=1.9$ ) the same order of notch strengthening was obtained for all solution temperatures, despite a drop in smooth-bar rupture ductility from 8-10% for the 1650°F treatment to 1-2% for solution at 2150-2200°F.

Major effort was directed toward more intensive investigation for solution temperatures of 1800° and 2200°F, which gave roughly comparable smooth-specimen rupture lives but widely-different rupture ductilities. The higher solution temperature resulted in a coarse grain size but was selected to study notch behavior for conditions where elongation at fracture would be 1% or less in long-time tests on un-notched specimens.

Rupture-test results for 1800°F solution temperature (plotted in Fig. 5 and listed in Table 2) show the varied notch-rupture characteristics obtained with the wide range of notch acuities investigated. The number of tests on un-notched specimens in this same condition was sufficient and the scatter of results small enough to establish a good set of experimental creep properties under both constant and variable stress. The normal creep and rupture characteristics could also be stated in mathematical forms which lend themselves to machine computation.

A trial program for the IBM 650 electronic computer was patterned to duplicate the hand calculation method of Reference 35. Internal checks were added to the program in attempts to fix the time intervals considered in the successive calculation cycles, so as to minimize over-corrections and consequent prediction of vacillating stresses in the notched bar.

The complexity of the equations for creep properties as a unique function of both the current stress level and the cumulative portion of rupture life "used up", together with need for repetitive comparisons

between stored values, made the calculations relatively slow for a machine with magnetic-drum storage. The half minute or so of running time for each calculation cycle limited the total number of steps which could be reasonably permitted for each set of input conditions. The time increments considered were therefore too large to approximate the stepless creep-relaxation process of an actual notched specimen, and errors of unknown magnitude were perforce introduced by the assumption employed that the properties of the metal at the start of each time interval held for that entire interval. Moreover, when time intervals were selected to give a reasonably-small number of computation cycles, the program used did not eliminate an occasional false indication of a negative effective stress in late stages of the calculations.

Of a total of fifteen sets of conditions for notch rupture tests fed into the computer, six reached an indication of a negative stress at a computed time far short of the experimental rupture life. The computer automatically stopped with an error indication when the logarithm of the stress was called for during determination of the normal rupture life at the current stress level. Eight of the remaining nine runs were discontinued after the computed residual levels of effective stress had ceased to make significant changes or when the computed test duration already far exceeded the actual life and the cumulative fraction of expired rupture life showed no probability of approaching 100% in an acceptable number of computer cycles. Table 3 lists the experimental rupture life, test stress and theoretical stress concentration factor for the fifteen conditions considered. Next is listed the cumulative percentage of rupture life calculated to have been expended in the metal immediately adjacent to the notch root during a total time equal to the actual rupture life. The final two columns present the computed time and cumulative percentage of life expired at the state of calculations when the computer run terminated.

Looking only at these last two columns, one set of calculations indicates rupture to be imminent (the calculated cumulative portion of expired rupture life has reached 99.9%) at a cumulative time which is some twenty times as great as the experimental rupture life. In eight other sets of calculations the cumulative portion of life computed to have been expended was still only 92.8-98.9% at cumulative times ranging from 1.6 to 66-fold greater than the actual rupture life. Furthermore, the calculations for most runs indicated a negligibly-slow approach to 100% of life expenditure at the time when the computer run was stopped.

Viewed from these results alone, the calculation attempts would seem to have little expectation of success in permitting one to predict rupture times of notched bars from smooth-bar data. But the remaining computed results listed in Table 3 offer brighter hope. If one discounts the two runs for the sharpest notch ( $K_t=10$ ), with their uncertainty in initial stress levels, computed life expenditures of from 77.4 to 93.7% were obtained at a cumulative time equal to the experimental rupture life in seven runs; had the computed rate of rupture-life consumption been some 10-30% greater than that calculated by the procedure employed, correlation of actual and predicted results should have been satisfactory for these runs.

Mild stress gradients at the lower  $K_t$ 's increased the difficulties from over-corrections, so that of the seven runs at  $K_t=1.8$  or 1.27 only one reached a computed cumulative time in excess of the actual rupture life. In the six remaining abbreviated runs the portion of rupture life computed to have been expended near the notch by the time the computer halted ranged from 56.5 to 70.4% at cumulative times as short as 5-10% of the actual rupture time. Again, a moderate change in the computed rate of life expenditure should bring the calculated results into reasonable agreement with experiment.

So despite failure of the computer program used to predict the correct notched-bar rupture times from the data supplied, the results suggest that the general approach employed does in fact offer a step toward explaining notch behavior. The computed rapid expenditure of available rupture life at early times and the very slow expenditure of life computed for later portions of a test should be adequate to account for both notch weakening and notch strengthening according to the alloy's ability to relax a stress concentration before the bulk of the rupture life has been used up.

Part of the discrepancy between experiment and calculations may be assumed to lie with the input data for creep-rupture properties of the material, but the consistent trend toward too long a calculated life is more probably associated either with some faulty assumption in the analysis or with failure to adequately approach the stepless nature of the actual stress relaxation, both in time and in space.

For the calculations cited in Table 3 the specimen cross section at the notch was imagined to be divided into six concentric rings, each half the area of the adjacent one toward the axis. In all fifteen cases considered the largest expenditure of rupture life was indicated for the narrow ring next to the notch, even though the peak effective stress was often computed to have shifted inward toward the axis. Only by remote chance would the center of this ring always be the most severe location



from the standpoint of the rate at which rupture life was expended. If the ring were subdivided calculations for some sub-ring might predict failure at a time significantly earlier than that computed for the original scheme with only six rings. An increase in the number of rings considered to eight or nine would narrow the area included in the outermost ring by a factor of four or eight.

As for time increments, the limit of 50-100 placed on the number of cycles in the calculations performed meant that the available life "used up" in one step could be as much as several per cent. Under such conditions the assumption of uniform creep rate over the time interval could be considerably in error. This error is probably not so serious as it might first appear since too high a calculated creep-relaxation rate in one time interval results in too low a computed residual stress (and consequently too low a creep rate) for the following time interval. However, the shortest practicable time increments are to be desired. Change to a more advanced model of computer such as the IBM 704 would permit finer division of both the cross section and the time increments, and should still shorten the running time per set of input conditions.

#### Uncertainties in Factors Controlling Rupture Life.

Although the refinements mentioned are probably necessary before a rigorous evaluation can be made of the calculation procedure, they alone would not be expected to correct the apparent 10-30% error in the calculation of rupture-life expenditure. A more reasonable source of error of this magnitude rests with the assumption that the effective stress of the shear-stress invariant theory determines rupture time in creep under multiaxial stressing.

Definitive research is still needed on the factors controlling rupture during complex-stress creep. The first published study seems to be that of Johnson and Frost (Ref. 47) on rupture of a 0.5% Mo steel at 550°C and a commercially-pure copper at 250°C. One thin-wall tubular specimen of each material was subjected to pure axial tension and one to pure torsion. Three additional specimens of the steel and two of the copper were tested under combined tension-torsion loading. Of various stress combinations examined for their correlation against the logarithm of rupture time, only for maximum principal stress was a continuous relation found, and that relation was linear. Figure 6 shows reproductions of the reported plots for maximum principal stress and for the octahedral shear stress which is directly related to the effective stress of the shear-stress invariant theory. These results were unexpected and, as Johnson and Frost remarked, cut across all ideas previously expressed on the matter.

A later study on multiaxial-stress creep of Inconel at 1500°F was reported to support the maximum principal stress criterion for time-dependent fracture (Ref. 48). However, when these data for combined axial tension and internal pressure are plotted in the manner of Johnson and Frost, at least as good correlation is found by either the effective stress or maximum shear stress as by maximum principal stress. (See Figure 7). A still later paper by Davis (Ref. 49) concluded that rupture tests on tubular specimens of Type 316 stainless steel at 1200°F for pure internal pressure and for equal biaxial tensions correlate favorably with results of uniaxial tension tests if a comparison is made on the basis of the von Mises effective stress.

Some logical explanation must be assumed to exist for these diverse indications. In each instance the experiments appear to have been carefully planned and executed and presumably the same results would be found if the tests were duplicated elsewhere. In no two of these investigations was the same type of alloy used, but the criterion for creep-rupture is hoped to be independent of the particular material employed. The most evident remaining variable is specimen thickness.

The tubular specimens used by Johnson and Frost had a 0.020 or 0.030-inch wall, providing an average of six grains to the wall thickness. This number of grains was stated to have proved sufficient to provide a characteristic cross section in previous combined-stress work. Kennedy, Harms and Douglas increased their wall thickness to 0.060 inch, while the specimen of Davis was relatively thick (9/32 or 3/8 inch). Change from a very thin wall to a moderately-thin wall to a relatively-thick wall is thus seen to correspond to a change from fracture controlled by the largest principal stress, to control by either principal stress or effective stress, to control by effective stress alone.

One possible hypothesis based on these observations is that the maximum principal stress determines the location, orientation and time for the start of an initial crack, while the creep strain determined by the effective-stress level controls the rate of crack propagation and therefore the time of complete separation in a specimen of appreciable section size. Such a hypothesis is in agreement with studies cited earlier in which the first crack was always found to open up normal to the axis and in the region of highest principal stress. The high degree of triaxiality and consequent low tendency for creep in the region between the initial crack and the axis, together with the somewhat higher effective stress toward the notch root, could also account for the fact that the initial crack quickly reached the surface while its inward movement seemed to become arrested. Confirmation also follows from the observation of Davis that in his thick pressure tubes the destructive cracks started at the outer surface where the peak tensile stress was to be found rather than at the bore where higher effective stress and greater strain occurred.

Most of the notched-bar rupture behaviors discussed earlier seem capable of qualitative interpretation by the proposed hypothesis of dual influence from the maximum principal stress and from the effective stress.

With a dull notch, the peak axial stress component near the notch root is only moderately larger than the nominal stress. If the maximum principal stress governs crack initiation, cracks should not form much earlier than for a smooth specimen. The mild triaxiality present results in lower effective stress than in a smooth specimen, reducing the tendency for a crack to propagate. Rupture life would consequently be expected to exceed that for un-notched specimens.

For greater notch acuity, the initial level of the peak axial stress in the immediate area of the notch is higher, but over most of the cross section the three principal stresses at any point now approach one another in value. The small differences between principal stress components markedly lowers the effective stress in this region and should retard crack propagation to a far greater extent than for milder notches. Once the notch acuity has been increased to where substantial equality of the principal stresses at a given point is attained, any sharper notch can produce little further lowering of the effective stress for a given load, but the level of the maximum principal stress near the notch will continue to rise with  $K_t$ . From these considerations one would logically expect a peak notch-strength ratio at intermediate notch acuities if principal stress and effective stress are both involved in the rupture mechanism of notched specimens.

With sharper notches the higher peak axial stress should promote earlier crack formation, but in a middle range of notch acuities sufficient time still exists for creep relaxation to level the stress gradient in the region of initial crack propagation so as to bring about a lowering of the effective stress to less than the nominal value in this critical area. With an extremely sharp notch the first crack may be supposed to form during the time when rapid stress leveling is still in progress at the region of crack initiation. The creep causing this stress redistribution is thus occurring at a time and place favoring rapid propagation of any crack that forms.

Ability of an alloy to relax stress concentrations readily should prolong the time before the first crack opens by permitting relief of microscopic stress concentrations at triple points in the structure and should limit the rate of subsequent crack propagation by early reduction of effective stress levels to low values. Conversely, an alloy resistant to early relaxation should be prone to early crack development and extension before effective stresses in the critical region can drop below the nominal stress.

For the same history of maximum principal (tensile) stress, crack formation should occur at about the same time in notched bars of various shapes, but once a crack begins the effective stress for a flat (biaxial) notched specimen should be enough higher than for a round (triaxial) notched specimen to produce the lesser tendency for notch strengthening observed for flat specimens.

The hypothesis suggested above even seems capable of accounting for some previously unexplained results for notch rupture tests with step-up stressing, listed in Table 10 of Reference 35, Part 3. In these tests long-time exposure of a notched specimen to a low initial stress resulted in greatly extended rupture life at later high stress, compared with the notch strength under constant high stress throughout the test. In several instances notched specimens of Wasp-alloy at 1350°F became decidedly notch strengthened when subjected to step-up stress history, whereas with constant application of the final stress they would be severely notch weakened. The behavior now appears quite reasonable if the long time period permitted substantial stress redistribution but was not sufficiently prolonged to initiate cracks at the mild conditions prevailing under the low early stress.

It might be pointed out that no fixed limit to the attainable notch-strength ratio is apparent for the proposed explanation of notch strengthening, in contrast to the limit of 2.0 suggested by Sachs and Lubahn for the technical cohesive strength in short-time tests under uniform triaxiality from a very deep and sharp notch machined in a soft metal. (See Ref. 50). That such a limit does not apply to the notch-strength ratio under creep-rupture conditions would seem to be demonstrated by tests on Type 321 stainless steel at 1500°F, where notch-strength ratios of 2.3 or 2.4 were found for  $K_t=8$ . (Ref. 29).

## SUGGESTED FURTHER EXPERIMENTAL STUDIES

Critical examination of the extensive available data relating to notched-bar rupture behavior has revealed certain areas of important theoretical interest which still have not been covered experimentally. In addition, hypotheses developed to explain observed behaviors make broad implications for interpretation of creep-rupture under other patterns of complex stress.

A further program of basic research designed to re-examine aspects of crack initiation and propagation seems amply warranted by its probable contribution to our ability to interpret the large body of notched-bar rupture data being accumulated and to suggest new tests to provide the most useful information for application to advanced designs with their need for a minimum safety factor. In particular, proof of the validity or lack of validity should be established for the proposed hypothesis of dual control of notched-bar rupture life by the maximum principal stress and the effective stress of the shear-stress invariant theory.

### Interrupted Tests to Study Crack Initiation and Propagation.

Remarkable unanimity exists among investigators with respect to the location where cracks have been noted to start in notched-bar rupture tests and the subsequent progress of the crack. These reported studies covered a variety of alloys and test conditions but lacked generality since not a single example was included for which initial stresses all remained elastic, nor was the biaxial case of a notched strip treated. Perhaps the greatest shortcoming of investigations to date has been failure to relate crack incidence and growth quantitatively with existing stress patterns and histories at the zone of fracture.

As was noted earlier, in the absence of short-time plastic deformation at the notch during load application the highest principal stress for early times would always be the axial component at the notch root. This situation would exist for either round or flat notched specimens, but in the case of the round bar a tangential component exists at the notch while no secondary stress acts right at the notch root in a strip specimen. A smooth specimen in simple tension, too, has no secondary stress at the surface, but neither does it have such stresses at other locations in the cross section.

At least for A-286 alloy at 1200°F, rupture tests with mild notches could be run in which the largest principal stress at early times was at the surface. Partial confirmation of the conclusions of Johnson and Frost would result from a finding that the first cracks now formed at

the surface. By suitably adjusting the load to offset relaxation of the stress near the notch root, approximately the same maximum principal stress could be maintained for an extended period at the surface of both round and flat notched bars and of smooth specimens. If the principal stress determines the start of fracture the first cracks for all three of these specimen types should develop at roughly equal times. By testing a number of each type specimen and carefully examining sections from bars run to different times, one should be able to state whether the quite-different effective stress in these three specimen configurations appeared to have a direct influence on the rate of spread of cracks.

Similar studies might be appropriate for other types of tests mentioned below.

#### A Check Into the Apparent Role of Section Thickness

Section thickness appeared to be the only variable to explain the diverse conclusions as to criteria for rupture of cylindrical specimens under multiaxial loading. If the seeming lack of influence on rupture life from effective stress in the case of thinnest wall can be attributed to absence of any appreciable distance through which the crack can propagate under the action of creep, then the same situation should prevail for a very thin section in which multiaxial stresses are produced by a notch. A plate specimen with grooves across the face should behave similarly to other notched specimens (i. e., should be able to exhibit notch strengthening under suitable conditions) so long as the thickness at the base of the grooves is reasonably large. Deepening of the grooves to produce a very thin layer between the two roots should result only in notch weakening, no matter what the material's yield or creep characteristics may be, provided the interpretation placed on Johnson and Frost's data in an earlier section is correct.

An ideal material for survey tests of this type would be an alloy for which a wide range of notch geometries produced notch strengthening.

A more direct verification of the presumed effect of wall thickness in tests of thin cylinders might appear to be provided by further tests with specimens of that type covering a range of wall thicknesses. Unfortunately, such tests would entail serious experimental difficulties at creep conditions. Any significant increase in wall thickness for a small diameter specimen would cause a non-uniform stress pattern through the material, comparable to the stress gradient near a notch. If the diameter is increased in proportion to the wall thickness, troubles are met in obtaining uniform material properties throughout the large test piece and in maintaining the desired temperature over the entire

test section. A simpler procedure would involve the use of plate specimens which can be loaded independently in two directions.

#### Rupture of Plate Specimens with Variable Biaxial Loading.

It is desired to demonstrate whether or not the largest principal stress (or a related parameter) exerts a major influence on the start of cracking, while some combination of principal stresses governs the rate at which a crack spreads to eventual specimen fracture. For most direct application to the analysis of rupture in notched specimens, the test to be devised should permit independent variation of the stress level and the degree of triaxiality. The principal directions should preferably remain fixed and the stress components over the entire gauge section should be uniform and readily determined for any loading condition. Since observations of others may have been clouded by effects of absolute dimensions, any specimen chosen for study should permit a substantial thickness. Finally, provision for continuous measurement of creep strains in the principal directions would be very desirable. A biaxial-stress plate specimen with edge loading seems best able to meet these combined demands.

If a rectangular plate is loaded at the edge boundaries by tensile or compressive forces, the stress distribution should be purely biaxial and uniform at a sufficient distance from the points of load application. Furthermore, with concentric loading the principal-stress directions remain constant regardless of the ratios of the applied loads.

Inability to get uniform biaxial stress to extend completely across the gauge section has made the plate specimen of limited value to past experimenters. Much of this difficulty is hoped to be circumvented by going to a two-stage test, with provision for machining away troublesome portions of the specimens between stages. (Only the portion of the gauge section where fracture is finally to occur need have been subjected to a known history of uniform stressing).

In any specimen of plate form one of two undesirable conditions seems always to exist under biaxial loading: either (1) a portion of the gauge section is subjected to a highly-complex state of stress concentration at loading points or (2) a portion of the gauge section is at a free edge and therefore subjected to only a uniaxial state of stress. The stress across the gauge section due to any one applied load can be made to be completely uniform: the method of applying a second load, orthogonal to the first, produces the above undesirable conditions.

The specimen design illustrated in Figure 8 should permit substantially-uniform biaxial tension-tension in a middle portion of the gauge section away from the side boundaries. In this tentative design, a longitudinal load is applied through the two innermost pairs of 1/4-inch diameter holes at either end, while sideways load acts through one 1/4-inch and two 3/16-inch holes in line at each corner. Four 5/32-inch diameter holes are provided to permit attachment of extension rods for the creep measurements. Tests of several types, starting with this basic specimen design, are suggested to provide experimental insight into prime aspects of creep-rupture behavior:

1.) The proposition has been advanced that the time of initiation of cracking depends only on the level of the largest principal stress. Groups of tests are proposed in each of which the longitudinal stress would be constant and at the same level for three specimens. One of the three specimens would be subjected to longitudinal stress alone; the other two would have a transverse tension or a transverse compression, respectively, of magnitude equal to some common fraction of the longitudinal stress level. These three different stress patterns would be maintained for a length of time equal to that at which initial cracking was found in preliminary simple-tension tests, after which time the specimens would be cooled under load. All three specimens would then be removed from the test units and re-machined to a form such as that indicated by the dotted line on the drawing. The modified specimens would be re-loaded with the axial tension alone and brought back to the test temperature. No consistent difference in final rupture time should be found for the three types of initial loading if the maximum principal stress alone determines crack initiation. Prior experience for many materials in the form of conventional round specimens indicates that interruption of the test has little or no measurable effect on creep rupture so long as cooling and reheating are performed with the load in place.

Observations of final rupture time could be supplemented by measurement of creep strains in the two stress directions and by examination of extra biaxial-stress specimens to learn whether the time of first crack formation is indeed the same as for simple tension.

In the case of biaxial tests, lack of a secondary stress at the free edge will result in a different effective stress there compared with the effective stress in central portions. Just as for a notched specimen, this gradient in effective stress



can be expected to produce differences in creep rate across the gauge section and to promote redistribution of stress from the desired pattern of uniform longitudinal stress component. However, effects on the stress level of the portion of gauge section to be retained for the second stage of the test can be computed and appropriate corrections made in the loading force to maintain the average longitudinal stress at the proper value.

2.) The second postulate of the proposed dual failure theory states that progression of a crack, once it initiates, should depend on the effective stress of the shear-stress invariant theory or on some similar parameter reflecting the presence of secondary stress components. Tests which are the inverse of those just discussed should provide a useful check. After a common first stage in simple longitudinal tension to permit crack initiation under identical conditions, a secondary tension or compression stress would be added to certain specimens. Despite the lack of uniform biaxiality all the way across the gauge section, the differences in pattern from one type loading to another should result in discrete ranges of rupture times for groups of specimens in which the transverse force was tensile, zero and compressive, respectively.

Interruption of tests during the second stage to survey for crack progression across the specimen would be anticipated to show that for the same average effective stress the rate of crack propagation was the same. With the first crack able to start at any random location, a large number of such specimens would have to be examined under the imperfect conditions of the above tests to learn whether the most rapid spread of the crack was in the region of largest effective stress. One expedient to alleviate some of the need for numerous tests would be to introduce a shallow mild notch at each side edge of the gauge section to localize the crack initiation, while still having little influence on the stress pattern near the center of the specimen. Now the area and direction in which the crack spread is to be compared would be the same in all tests and the factors controlling crack propagation should be easier to demonstrate.

### Extensions to and Application of Suggested Experiments:

Successful completion of the experiments outlined should provide the answers originally sought on the influence of secondary stresses on rupture phenomena. Although the immediate need is not so apparent, one additional facet of the fundamentals of creep-rupture behavior appears to lend itself to study with the proposed plate specimen, namely, the effect of changing the direction of loading at constant magnitude of stress.

Exploratory tests to investigate addibility of rupture-life fractions when a simple tension acts in one direction for part of the test and in an orthogonal direction for the remainder should add to our insight into the rupture process. Combination tests in which the direction and magnitude of the loads are both changed with time are virtually unlimited in their possible variety. No method suggested to date seems able to produce a controlled uniform state of triaxial tension, but the proposed biaxial plate specimen should allow a close approximation to the stress-time history computed to exist at any location in a notched strip specimen, so that analytical and experimental times for rupture at that location can be compared.

Once the results from the suggested series of research are known, their introduction into a program for the IBM 704 computer is highly recommended, with initial emphasis on comparing predicted and experimental rupture behavior of notched strip specimens.

## SUMMARY AND CONCLUSIONS

Results from past studies on rupture behavior of notched specimens have been critically examined along with available published data to (1) determine general trends, (2) outline an explanation for observed behaviors and (3) ascertain what further research is needed to clarify remaining questions.

For all cases where data covered a wide range of nominal stress and notch acuity, notch strengthening appeared to be universal under at least some conditions of testing.

Individual sets of data followed either of two general patterns:

- (1.) the stress-rupture time curves for notched specimens were parallel to that of smooth specimens, or
- (2.) the curves of notch-strength ratio showed minimum values at intermediate rupture times.

For either pattern the notch-strength ratio increased with notch acuity to a maximum and then declined. Alloy ductility altered this behavior only in detail, with tendency toward notch weakening when relatively sharp notches combined with low ductility.

Critical analysis of these trends in terms of existing theory provides the rudiments of an explanation for the observed behavior:

- 1.) A notch results in concentration of both principal and effective stresses; by any prevailing theory of failure, notch weakening would always be anticipated in the absence of stress redistribution by initial short-time yielding or subsequent creep-relaxation.
- 2.) Its universal occurrence among all alloys studied indicates that notch strengthening results from conditions imposed in the specimen by the notch rather than from a particular property of the alloy. Of these conditions the triaxial nature of the stress pattern is the most logical factor to promote longer rupture life.
- 3.) The most serious uncertainty between theory and test arises from limited data on tubular specimens indicating that the maximum principal stress alone might determine rupture time. Notch strengthening is difficult to explain if this indication is general since the first crack in a notched specimen would normally be expected to form at a

shorter time than does rupture in a smooth specimen. However, further analysis of the existing data suggests the maximum principal stress controls rupture time only in very thin sections and that for heavier sections the stress controlling rupture is a close approximation to the effective stress which investigators agree determines creep rate under complex stresses.

4.) Assuming that the maximum principal stress plays a major role in the initiation of rupture, notch strengthening may be reasoned to arise as a result of a reduction in the rate of crack propagation under the existing complex stress pattern.

5.) Minimum notch-strength ratios at intermediate times appear to be associated with structural changes during testing, particularly damage due to plastic yielding during loading. Rupture curves which parallel the smooth-specimen curve appear to be for cases where such structural changes are not pronounced or where they affect both smooth and notched specimens in like degree.

6.) The increase in notch-strength ratio to a maximum and then a decline with increasing notch acuity is probably related to the crack propagation characteristics determined by differences in stress distribution for high versus low notch acuities. Such behavior could be anticipated where triaxiality and stress level have opposing effects on rupture life, since once the notch has been made sharp enough to produce substantial equality of the three principal stresses at points over most of the cross section, any sharper notch will simply raise the level of these stresses with little further approach toward uniform triaxiality.

7.) Ductility of smooth rupture specimens might also be related to notch behavior through its role as an imperfect indicator of crack propagation characteristics.

Attempts to calculate notch rupture life from smooth-specimen properties on the basis of effective stress invariably predicted longer than actual life, apparently because of inability to include the separate features of crack initiation and propagation.

A research program designed to check the crack propagation aspects of notch behavior seems warranted. This program should include:

(1) interrupted tests on smooth and notched specimens to permit examination to fix the time of first cracking, the location of this initial crack and the progress of cracks through the specimen,

- (2) tests to verify the hypothesis that as specimen thickness is increased the criterion of failure shifts from the maximum principal stress towards the effective stress, and
- (3) investigation of fracture processes under variable complex stress, using plate specimens loaded to give simple uniaxial tension and either tension-tension or tension-compression for different portions of the test.

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TABLE 1 - LISTING OF ALLOYS AND TEST CONDITIONS FOR WHICH REPORTED NOTCHED-BAR RUPTURE DATA FOLLOWED EITHER OF TWO CHARACTERISTIC BEHAVIOR PATTERNS RELATIVE TO RUPTURE CURVES FOR SMOOTH SPECIMENS

Alloy Designation	Test Temperature		Literature References	Notes	Alloy Designation	Test Temperature		Literature References	Notes
	(°F)	(°F)				(°F)	(°F)		
<u>Case 1: Notched Bar Curve Parallel to Smooth-Bar Curve</u>									
Russian Steel E. I. 10 (Pearlitic)	1022		4	(a)	SAE4340 Steel	900, 1100	37	(b)	
AMS 5616	600, 700		25	(b)	Ni-Ci-Mo Steel	932	27	(b, d)	
17-22A(S)	600, 700, 800		25	(b)	0, 13C, 0, 7Cr, 1, 6Ni, 0, 8Mo Steel	932	39	-	
12 Cr, 3W Steel	1000		26	(a)	AMS 5616	1100	37	(b)	
20 Cr, 1Mo, 0, 1C Steel	935		27	(b)	17-22A(S)	1100	35	(f, g)	
Four Austenitic Steels	1112		28	-	17-22A(S)	900, 1000, 1100, 1200	37, 40	(b)	
13 Cr-13 Ni-10 Co-3 Cb-Ta-2.5 W -2 Mo-0.4 C 1202;1346			28	(c)	17-22A(V);B50R310	1000	33	(d)	
Russian Steel E. I. 257 (Austenitic)	1076		4	(a)	Types 304, 316, 321 Stainless Steel	1100	29	-	
Types 304, 316, 321 Stainless Steel	1500		29	(e)	17-4PH	600, 700	25, 41	(b)	
19-9DL	1100		30	-	16-25-6	1100	33	(d)	
17-7PH(TH-1050)	900		31	(c)	16-25-6	1200	36	(a)	
16-25-6	1200		32	(d, e)	A-286	1200	42	(b)	
A-286	1200		31	(a, d, e)	13Cr-13Ni-10Co-3Cb+Ta-2, 5W - 2Mo-0, 4 C	1202, 1346	28	(b)	
K-42-B	1100		33	(d)	Waspaloy	1350	35	(b, d)	
Refractaloy 26	1200		26	(d)	Inconel X; Nimonic 80A	1350	38	-	
S-816; S-816+Ta	1200, 1350, 1500, 1600, 1650; 32, 34, 35, 36; (a, d, f)		32, 34, 35, 36; (a, d, f)	(a, d)	Inconel X	1350, 1425	37	(b)	
Waspaloy	1200, 1350, 1500		34, 35	(a, d)	Haynes 88	1350	37	(b)	
M-252	1350, 1500		32	-	Inconel X-550	1350, 1500, 1600	34, 36	-	
Inconel X	1500		37, 38	(b)	Inconel X-550	1350	35	(f)	
2024-T4 Aluminum Alloy	400		35	(a, e, f)	94 Sn, 6 Cd	Room	28	(a, d)	

Case 2: Relative Notched-Bar Strength Lowest at Intermediate Stresses

Notes:

- a). Data cover a range of notch acuities.
- b). For Sharp Notches.
- c). For Dull Notches.
- d). Data were included for two or more heat treatments or lots of alloy.
- e). A corresponding definite change in slope of the stress-rupture time plots was demonstrated by data for both smooth and notched specimens.
- f). Results were reported for both round and flat specimens.
- g). The most pronounced trend was provided by flat bars with a relatively-sharp Notch ( $K_t = 3.3$ ).

TABLE 2 - RESULTS AT 1200°F FOR A-286 WITH 1800°F SOLUTION TEMPERATURE  
 (Heat Treatment: 1 hr Solution, Oil Quench + Age at 1325°F, 16 hr, Air Cool)

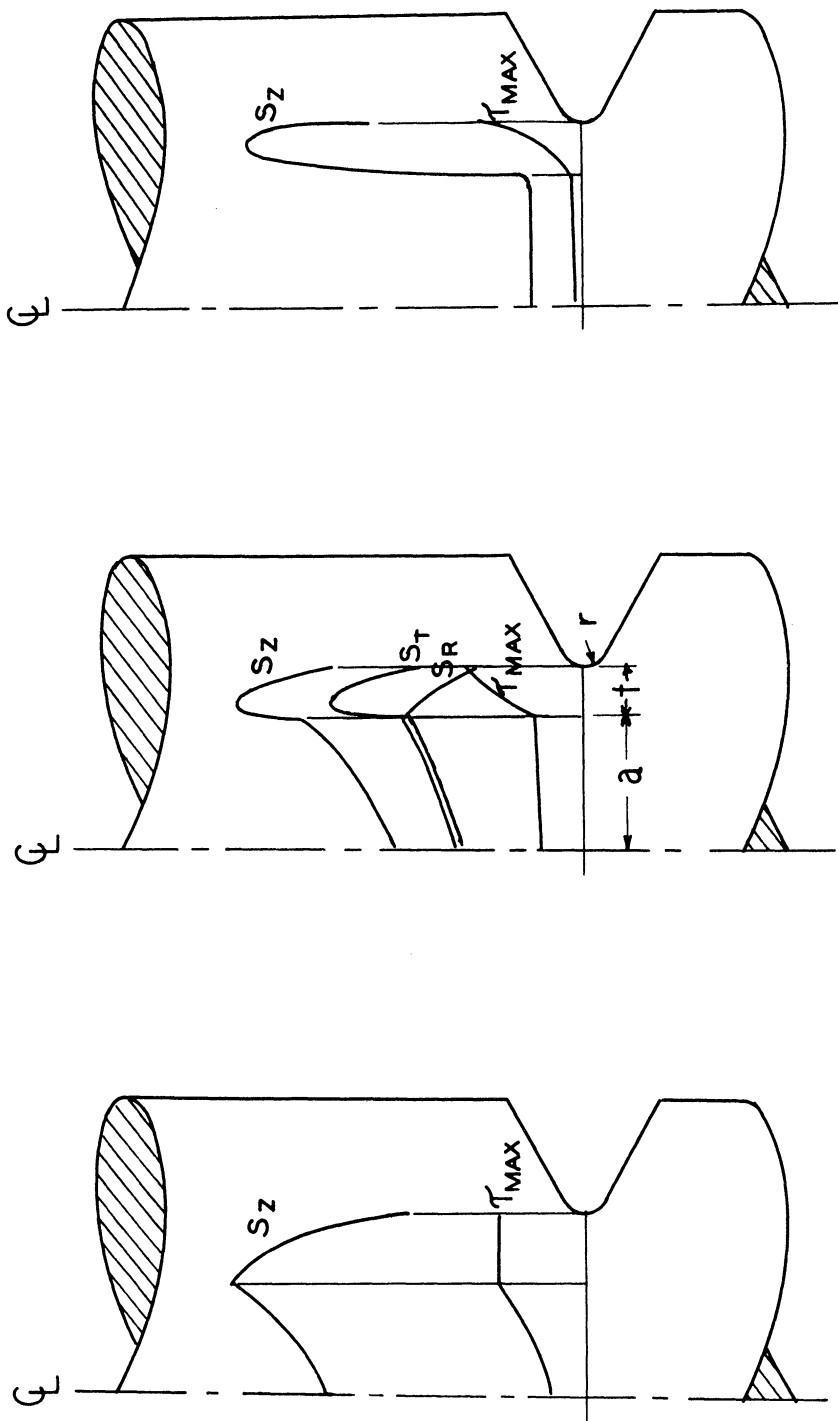
SMOOTH BARS				*NOTCHED BARS			
Stress (psi)	Rupture Life(hr)	Elongation at Rupture (%)	Reduction of Area(%)	Stress (psi)	Rupture Life(hr)	Stress (psi)	Rupture Life(hr)
100,000	0.26	17.	18.			$K_t = 3.0$	
90,000	1.45	8.	9.	70,000	97.2	70,000	167.5
80,000	4.25	8.	8.5	70,000	22.6	45,000	908.1
70,000	14.9	9.5	13.5	65,000	241.6	$K_t = 4.1$	
70,000	20.3	6.5	8.5	60,000	319.3	80,000	20.9
65,000	62.4	5.5	10.5	50,000	840.0	70,000	112.8
60,000	99.1	7.	8.5	47,500	1043.6	65,000	168.1
60,000	79.9	5.	10.			60,000	227.6
50,000	384.6	6.	8.	$K_t = 1.54$		50,000	363.0
45,000	615.5	4.	8.	70,000	215.3	45,000	490.4
45,000	771.3	3.5	4.5	65,000	292.3	40,000	508.8
40,000	1435.1	5.5	5.5	60,000	481.3	35,000	1651.5
				$K_t = 1.8$		$K_t = 8.7-10$	
				80,000	109.5	70,000	6.5
				70,000	239.6	65,000	6.65
				65,000	366.2	60,000	11.1
				60,000	543.4	40,000	451.9
				55,000	727.4		
				$K_t = 2.39$			
				62,000	426.0		

\* Nominal Notch Geometries (Inches)

Theoretical stress concentration factor,  $K_t$  : 1.27 1.54 1.82 2.39 3.0 4.1 5.7 8.7-10  
 Diameter of shank, D: 0.462 0.462 0.600 0.500 0.460 0.500 0.600 0.600  
 Minimum diameter, at base of notch, d: 0.326 0.327 0.424 0.350 0.325 0.350 0.424 0.424  
 Notch root radius, r: 0.237 0.108 0.081 0.032 0.017 0.009 0.005 0.0015-0.002

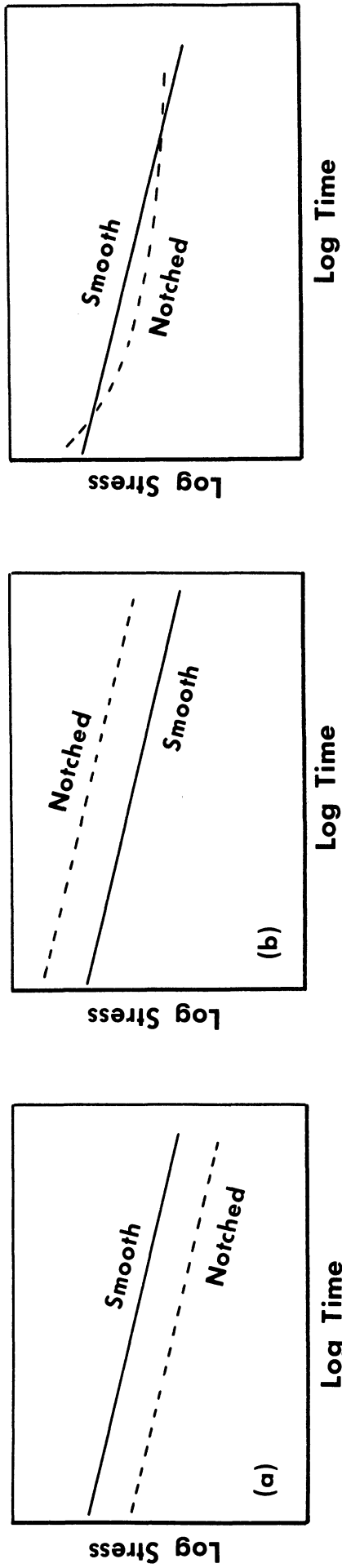
TABLE 3 - ACTUAL LIFE COMPARED WITH SELECTED VALUES FROM COMPUTATIONS  
FOR RUPTURE LIFE OF NOTCHED SPECIMENS OF A-286 AT 1200°F

K <sub>t</sub>	Stress, psi	Experimental Rupture Life, hrs.	Computed % of Life		Computed Conditions When	
			Expended Near Notch in Cumulative Time Equal To Experimental Life	Expended Near Notch in Cumulative Time Equal To Experimental Life	Calculations Were Terminated	Rupture Life Expended Near Notch, %
10	65,000	8	47.2	31,000	97.6	
10	50,000	130	65.6	9,775	92.8	
4.1	80,000	21	77.4	780	98.9	
4.1	60,000	225	88.2	8,025	98.0	
4.1	50,000	380	86.1	4,580	96.8	
4.1	35,000	1650	91.7	16,500	94.4	
3.0	70,000	170	86.9	3,400	99.9	
3.0	50,000	600	93.7	7,525	98.4	
1.8	80,000	110	88.9	142	95.5	
1.8	60,000	600	-	189	56.5	
1.27	50,000	1700	-	87.5	66.0	
1.27	70,000	50	-	20.6	58.4	
1.27	60,000	600	-	55.3	58.6	
1.27	50,000	850	-	218	70.4	
1.27	35,000	4000	-	1133	61.2	



a. After Uzhik, Ref. 2.      b. After Likhachev, Ref. 3,      c. After Mirkin and Trunin, Ref. 4.  
for 0.95 of Tensile Load

Fig. 1 - Qualitative Pattern of Stresses in a Cylindrical Notched Specimen after Loading into the Plastic Zone.



Case 1 - Notched-Bar Curve Parallel to Smooth-Bar Curve.

Case 2 - Notch-Strength Ratio a Minimum at Intermediate Times.

**Fig. 2 - Characteristic Patterns of Notched-Bar Rupture Curves Relative to Smooth-Bar Data.**



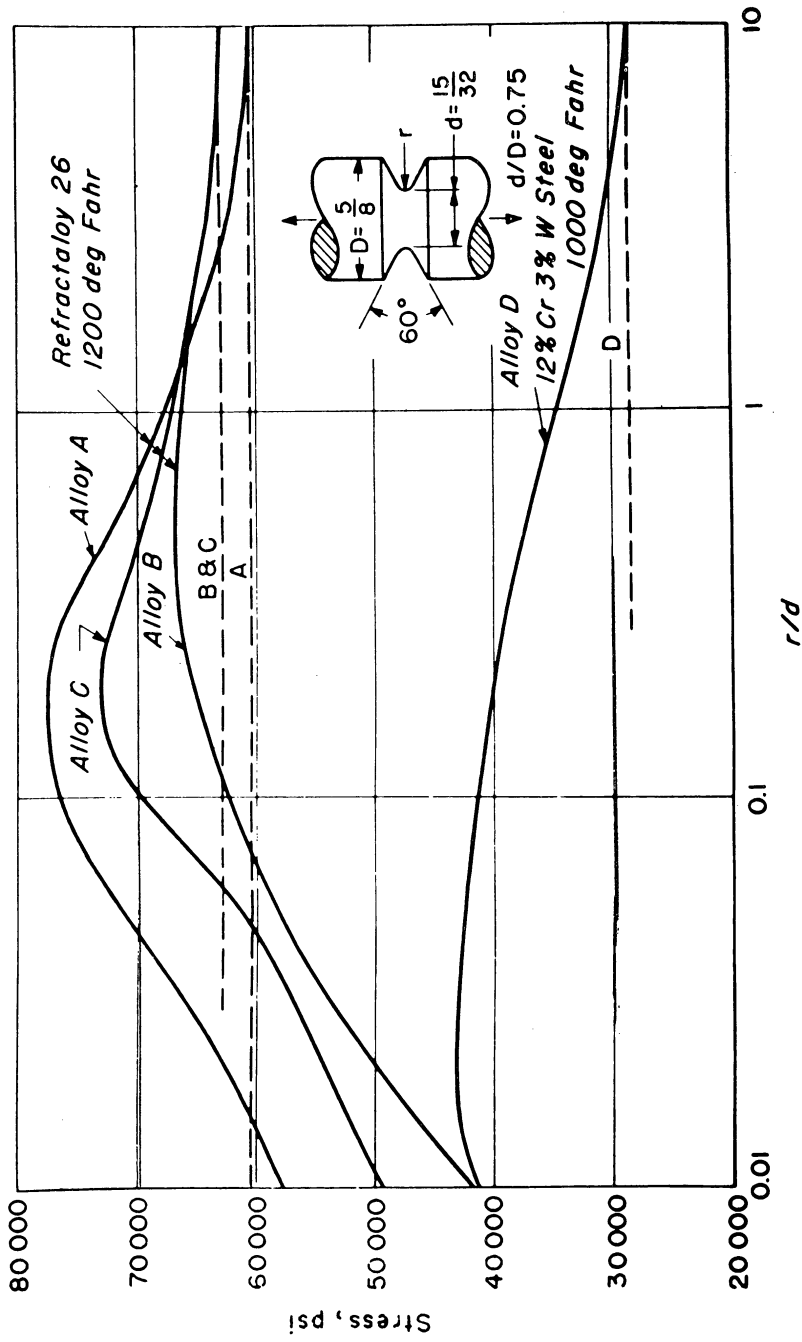


FIG. 3—Influence of Radius of Curvature on 1000-hr Rupture Strength of Notched Bars  
(After Davis and Manjoine, Ref. 26)

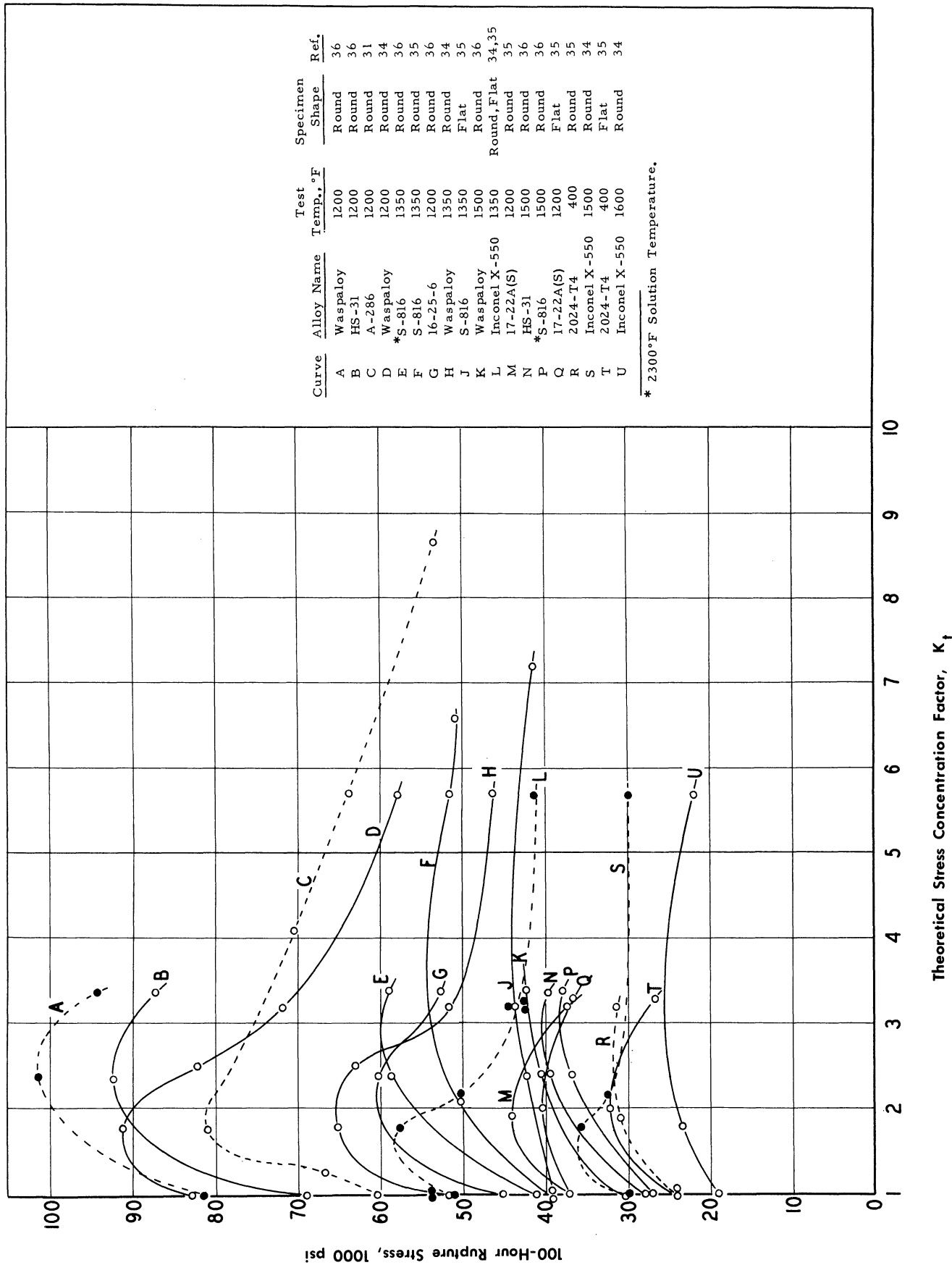


Fig. 4 - Effect of Notch Acuity on 100-Hour Rupture Strength of Round and Flat Bars of Several Alloys.

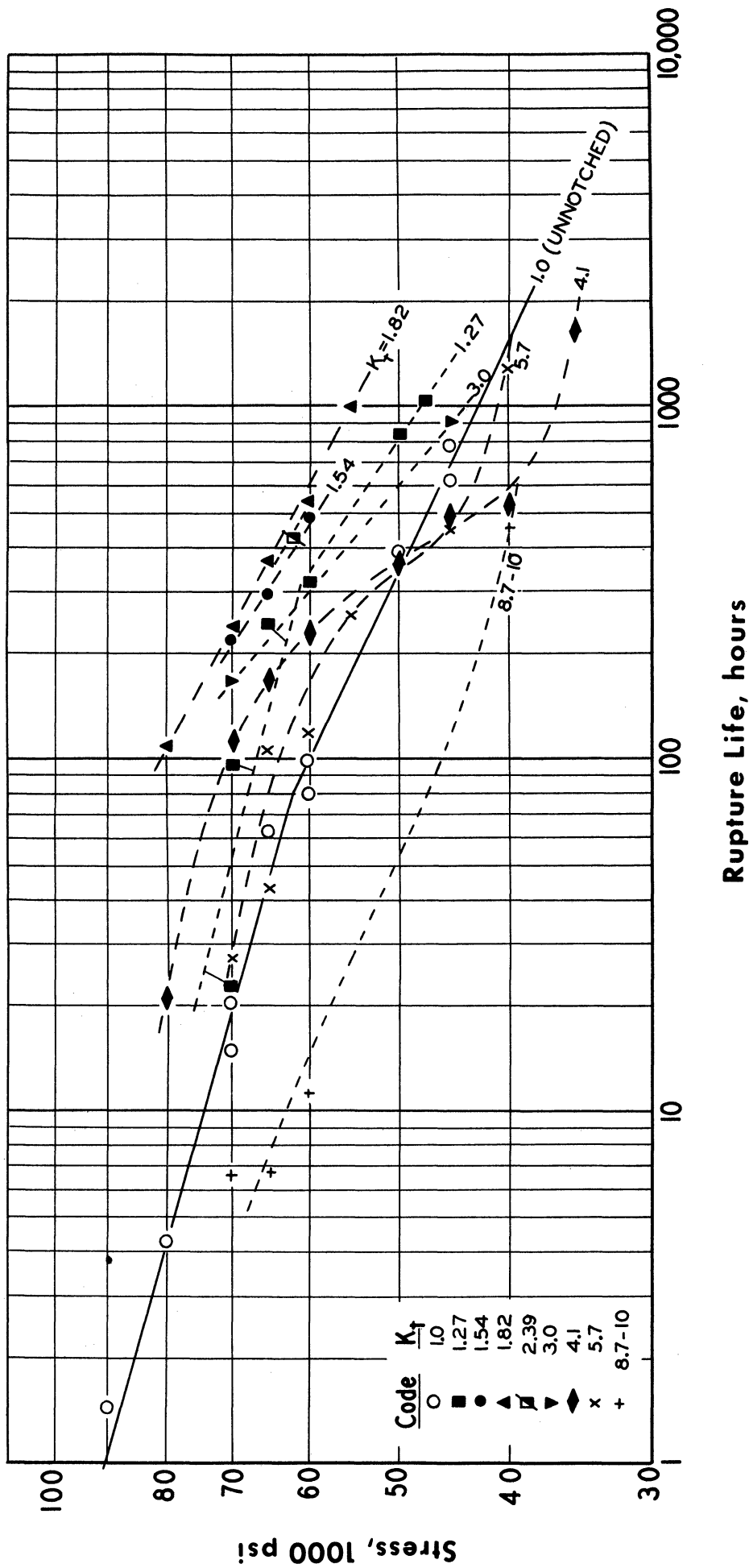
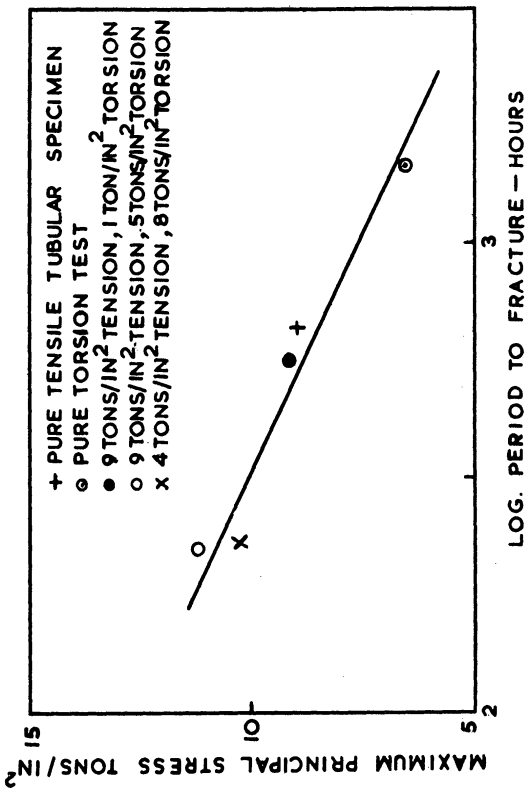
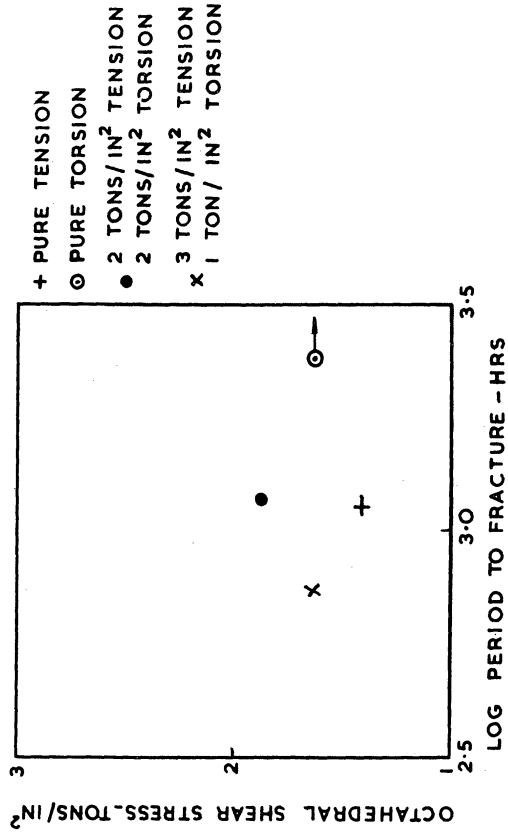
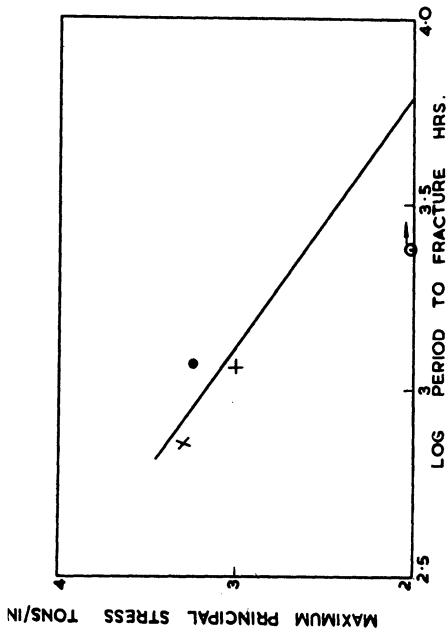
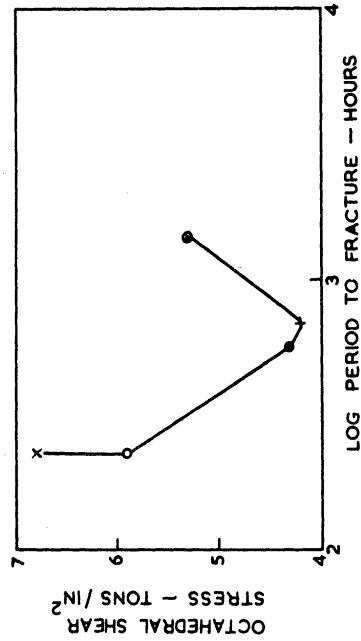


Fig. 5 - Rupture Properties at 1200°F for Smooth and Notched Specimens of A-286 Alloy.



0.5% Molybdenum Steel at 550°C



Commercially-Pure Copper at 250°C

Fig. 6 - Results of Johnson and Frost for Rupture of Thin Tubes under Combined Tension and Torsion.

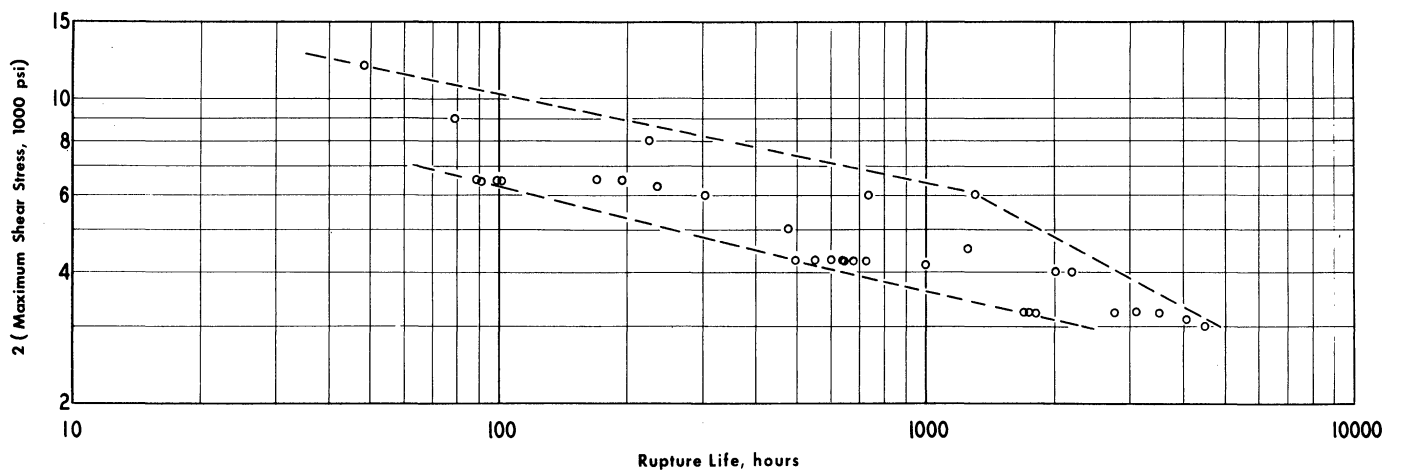
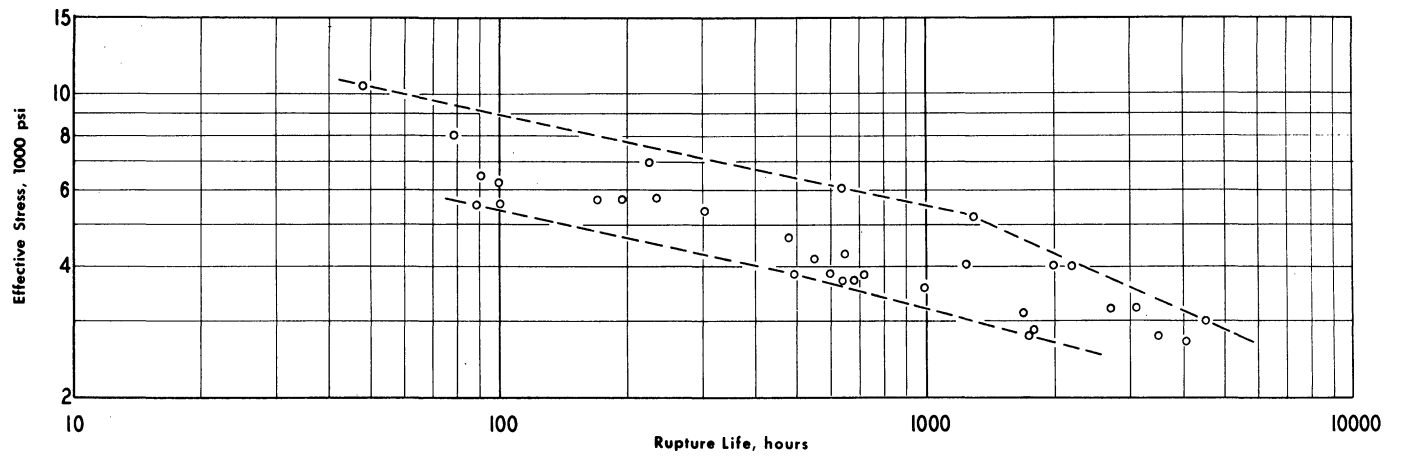
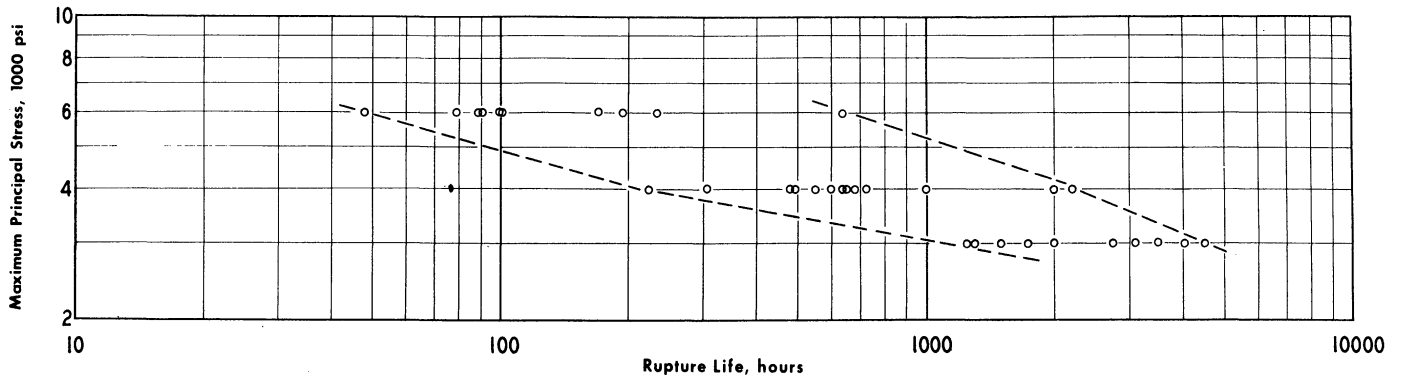


Fig. 7 - Rupture Data of Ref. 48 Plotted in the Manner of Johnson and Frost.



<p style="text-align: center;">UNCLASSIFIED</p> <p>The University of Michigan Research Institute, Ann Arbor, Mich. NOTCH SENSITIVITY OF HIGH-TEMPERATURE ALLOYS, by H. R. Voorhees and J. W. Freeman. July 1959. 48p. incl. illustrations. tables. [Proj. 8 - (8-7360); Task 73605] (WADC TR 59-470) [Contract AF 33(616)-5775]</p> <p style="text-align: center;">Unclassified report</p> <p>All available results on creep-rupture of notched specimens are (over)</p>	<p style="text-align: center;">UNCLASSIFIED</p> <p>1. Alloys, High temperature 2. Metals-Creep 3. Metals-Fracture 4. Metals-Notch sensitivity</p> <p>I. Voorhees, H. R. II. Freeman, J. W. III. Wright Air Development Center</p> <p style="text-align: center;">Air Research and Development</p> <p style="text-align: center;">UNCLASSIFIED</p>
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