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by

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FOREWORD

This report was prepared by the Radiation Laboratory of the Department of Electrical Engineering of The University of Michigan. The work was performed under Contract No. F 19628-67-C-0190, "Inverse Scattering Investigation" and covers the period 3 April - 3 July 1967. Dr. Vaughan H. Weston is the Principal Investigator and the contract is under the direction of Professor Ralph E. Hiatt, Head of the Radiation Laboratory. The contract is administered under the direction of the Electronic Systems Division, Air Force Systems Command, United States Air Force, Laurence G. Hanscom Field, Bedford, Massachusetts 01730, by Lt. H.R. Betz, ESSXS. This quarterly report was submitted by the authors on 18 July 1967.

This technical report has been reviewed and is approved.

Prior to release of this report to CFSTI (formerly OTS) it must be reviewed by the ESD Office of Public Information, ESTI, Laurence G. Hanscom Field, Bedford, Massachusetts 01730.

Approving Air Force Authority

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ABSTRACT

The problem in question consists of determining means of solving the inverse scattering problem where the transmitted field is given and the received fields are measured, and this data is used to discover the nature of the target.

The problem of what information can be determined about the body if the scattering matrix (phase and amplitude) is known only over an angular sector and measured in the far field, is studied further. Asymptotic analysis is used to show that in the high frequency case, portions of a piecewise smooth, convex surface can be found when knowledge of the bistatic scattered field is confined to a small cone.

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FURTHER COMMENTS ON THE FAR FIELD INFORMATION LIMITED TO A SOLID ANGLE

It was pointed out in the last quarterly that the near field could be expressed in terms of the far field quantity $\underline{E}_o(\theta, \phi)$ which is related to the scattered far field electric intensity

$$\underline{E} = \frac{e^{ikR}}{R} \underline{E}_o(\underline{\theta}, \phi) \ , \tag{1.1}$$

by the relation

$$\underline{E}(\underline{x}) = \frac{ik}{2\pi} \int_0^{2\pi} \int_0^{\theta_o - i\infty} e^{i\underline{k} \cdot \underline{x}} \underline{E}_o(\alpha, \beta) \sin\alpha \, d\alpha d\beta \ , \tag{1.2}$$

and a discussion was presented indicating the zone of space in which $\underline{E}(\underline{x})$ could be found when knowledge of $\underline{E}_o(\theta, \phi)$ is confined to a solid angle. In practice, the scattered far field (for a fixed transmitter position), will be measured at a set of N points $\underline{\theta}_n = (\theta_n, \phi_n)$ where n=1, 2 . . . N, located in the solid angle $0 \leqslant \phi \leqslant 2\pi$ and $0 \leqslant \theta \leqslant \theta_o$. With this in mind, there arises several considerations in connection with any computational procedure for determining a portion or portions of the surface of the body from the finite set of measurements, namely; the choice of near field representation, the location of the origin of the coordinate system, and the restrictions on the portions of the surface of the body which can be determined.

With regard to the choice of representation, there are three which are essentially equivalent. The first is the plane wave representation given above. If the number N is very large and the points are sufficiently dense, then numerical integration of

$$\underset{=}{\tilde{E}}(\underline{x}) = \frac{ik}{2\pi} \int_0^{\theta_o} \int_0^{2\pi} e^{i\underline{k}\cdot\underline{x}} \, \underline{E}_o(\alpha,\beta) \sin\alpha \, d\alpha \, d\beta \tag{1.3}$$

can be performed. This technique would have to be employed for high frequencies, in which case $\underline{E}_o(\alpha,\beta)$ would vary rapidly in the solid angle. Further details on this approach will be discussed at the end of this section.

For N finite (order of 20 or less) a polynomial fit may be made to $\underline{E}_o(\theta,\phi)$. One practical representation would involve the spherical harmonics

$$Y_{\substack{e\\o}mn}(\theta,\phi) = P_n^m(\cos\theta) \begin{matrix}\cos m\phi\\ \sin m\phi\end{matrix} , \tag{1.4}$$

in which case the far field quantity $\underline{E}_o(\theta,\phi)$ would be expressed in the following form

$$k\hat{\underline{\theta}} \cdot \underline{E}_o(\theta,\phi) = \sum_{\nu=1} \left[a_\nu S_\nu(\theta,\phi) + b_\nu T_\nu(\theta,\phi) \right] \tag{1.5}$$

$$k\hat{\underline{\phi}} \cdot \underline{E}_o(\theta,\phi) = \sum_{\nu=1} \left[-a_\nu T_\nu(\theta,\phi) + b_\nu S_\nu(\theta,\phi) \right] \tag{1.6}$$

where the summation over ν indicates summation over $n=1, \ldots \infty$, $m=0, 1, \ldots n$, and over both even and odd components. The functions S_ν and T_ν are given by the relations

$$S_{\substack{e\\o}mn} = \frac{1}{\sin\theta} \frac{\partial}{\partial\phi} Y_{\substack{e\\o}mn} = \mp \frac{mP_n^m(\cos\theta)}{\sin\theta} \begin{pmatrix}\cos m\phi\\ \sin m\phi\end{pmatrix} \tag{1.7}$$

$$T_{\substack{e\\o}mn} = \frac{\partial Y_{\substack{e\\o}mn}}{\partial \theta} = \frac{\partial P_n^m(\cos\theta)}{\partial \theta} \begin{pmatrix} \cos m\phi \\ \sin m\phi \end{pmatrix} \quad . \tag{1.8}$$

Thus given $\underline{E}_o(\theta, \phi)$ at the N points $\left\{\theta_n, \phi_n\right\}$, one can express $\underline{E}_o(\theta, \phi)$ in the form given be Eqs. (1.5) and (1.6) where the summation is from $\nu = 1$ to $\nu = N$. The unknown coefficients a_ν, b_ν ($\nu = 1, \ldots, N$) are found from the set of 2N linear algebraic equations. When the half-angle θ_o, of the measurement cone is less than $\pi/4$, it may be best in the practical treatment, to solve for the constants a_ν and b_ν indirectly, by using a modified set of functions $\tilde{S}_\nu$ and $\tilde{T}_\nu$ in place of S_ν and T_ν. The functions $\tilde{S}_\nu$ and $\tilde{T}_\nu$ which will be linear combinations of S_ν and T_ν are chosen so that they behave like θ^P as θ approaches zero. In this way the series

$$k\underline{\hat{\theta}} \cdot \underline{E}_o = \sum_{\nu=1} \left[\tilde{a}_\nu \tilde{S}_\nu + \tilde{b}_\nu \tilde{T}_\nu \right]$$

$$k\underline{\hat{\phi}} \cdot \underline{E}_o = \sum_{\nu=1} \left[-\tilde{a}_\nu \tilde{T}_\nu + \tilde{b}_\nu \tilde{S}_\nu \right]$$

behaves locally as a Taylor series in the variable θ. The general expressions for $\tilde{S}_\nu$ and $\tilde{T}_\nu$ will not be given here, however their explicit expressions will be given for cases m=1 and n=1, 2, 3, along with the corresponding behavior for $\theta \longrightarrow 0$,

$$\tilde{S}_{\substack{e\\o}11} = S_{\substack{e\\o}11} \sim 0(1)$$

$$\tilde{S}_{\substack{e\,12\\o}} = S_{\substack{e\,12\\o}} - 3S_{\substack{e\,11\\o}} \sim 0(\theta^2)$$

$$\tilde{S}_{\substack{e\,13\\o}} = S_{\substack{e\,13\\o}} - 5S_{\substack{e\,12\\o}} - 9S_{\substack{e\,11\\o}} \sim 0(\theta^4) \quad .$$

The near field is given by the relation

$$\underline{E}(\underline{x}) = \sum_{\nu=1}^{N} \frac{i}{2\pi} \int_0^{2\pi} \int_0^{\frac{\pi}{2}-i\infty} e^{i\underline{k}\cdot\underline{x}} \left\{ \underline{\hat{\alpha}} \left[a_\nu S_\nu + b_\nu T_\nu \right] + \hat{\beta} \left[-a_\nu T_\nu + b_\nu S_\nu \right] \right\} \sin\alpha \, d\alpha \, d\beta \, .$$

Integrating by parts it can be placed in the following form

$$\underline{E}(\underline{x}) = \frac{-i}{2\pi} \sum_{\nu=1}^{N} \int_0^{2\pi} \int_0^{\frac{\pi}{2}-i\infty} a_\nu Y_\nu(\alpha,\beta) \left\{ \frac{\partial}{\partial\beta} \left[e^{i\underline{k}\cdot\underline{x}} \underline{\hat{\alpha}} \right] - \frac{\partial}{\partial\alpha} \left[e^{i\underline{k}\cdot\underline{x}} \sin\alpha \, \hat{\beta} \right] \right\} d\alpha \, d\beta$$

$$\frac{-i}{2\pi} \sum_{\nu=1}^{N} \int_0^{2\pi} \int_0^{\frac{\pi}{2}-i\infty} b_\nu Y_\nu(\alpha,\beta) \left\{ \frac{\partial}{\partial\beta} \left[e^{i\underline{k}\cdot\underline{x}} \underline{\hat{\beta}} \right] + \frac{\partial}{\partial\alpha} \left[e^{i\underline{k}\cdot\underline{x}} \sin\alpha \, \underline{\hat{\alpha}} \right] \right\} d\alpha \, d\beta \, .$$

The integral representation can be simplified on using the following results:

$$\frac{\partial}{\partial\beta} \left[e^{i\underline{k}\cdot\underline{x}} \underline{\hat{\alpha}} \right] - \frac{\partial}{\partial\alpha} \left[e^{i\underline{k}\cdot\underline{x}} \sin\alpha \, \underline{\hat{\beta}} \right] = ik\sin\alpha \, e^{i\underline{k}\cdot\underline{x}} \left[\underline{\hat{\alpha}}(\underline{x}\cdot\hat{\beta}) - \underline{\hat{\beta}}(\underline{x}\cdot\underline{\hat{\alpha}}) \right]$$

$$= -i \, \underline{k} \wedge \underline{x} \, e^{i\underline{k}\cdot\underline{x}}$$

$$= -\underline{\nabla} \wedge \left[e^{i\underline{k}\cdot\underline{x}} \, \underline{x} \right] \quad ,$$

$$\frac{\partial}{\partial\beta}\left[e^{i\underline{k}\cdot\underline{x}}\,\hat{\underline{\beta}}\right]+\frac{\partial}{\partial\alpha}\left[e^{i\underline{k}\cdot\underline{x}}\sin\alpha\,\hat{\underline{\alpha}}\right]=\sin\alpha\, e^{i\underline{k}\cdot\underline{x}}\left\{ik\left[\hat{\underline{\beta}}(\underline{x}\cdot\hat{\underline{\beta}})+\hat{\underline{\alpha}}(\underline{x}\cdot\hat{\underline{\alpha}})-2\hat{\underline{k}}\right.\right\}$$

$$=\frac{i}{k}\left[-\underline{k}\wedge\underline{k}\wedge\underline{x}+2ik\right]e^{i\underline{k}\cdot\underline{x}}$$

$$=\frac{i}{k}\,\underline{\nabla}\wedge\underline{\nabla}\wedge\left[e^{i\underline{k}\cdot\underline{x}}\,\underline{x}\right]\ .$$

From the following relationship

$$\psi_{\substack{e\\o}mn}(\underline{x})=h_n^{(1)}(kR)\,Y_{\substack{e\\o}mn}(\theta,\phi)$$

$$=\frac{(-i)^n}{2\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty}e^{i\underline{k}\cdot\underline{x}}\,Y_{\substack{e\\o}mn}(\alpha,\beta)\sin\alpha\, d\alpha\, d\beta\ ,$$

the near field can be expressed in terms of the spherical vector wave functions

$$\underline{m}_{\substack{e\\o}mn}=\underline{\nabla}\wedge\left[\psi_{\substack{e\\o}mn}\,\underline{x}\right]$$

$$\underline{n}_{\substack{e\\o}mn}=\frac{1}{k}\,\underline{\nabla}\wedge\underline{\nabla}\wedge\left[\psi_{\substack{e\\o}mn}\,\underline{x}\right]$$

as follows

$$\underline{E}(R,\theta,\phi)=\sum_{\nu=1}^{N}\left\{A_\nu\,\underline{m}_\nu+B_\nu\,\underline{n}_\nu\right\} \qquad (1.9)$$

where

$$\left.\begin{aligned} A_{\substack{e\\o}mn} &= i^{n+1}\, a_{\substack{e\\o}mn} \,, \\ \text{and} \qquad B_{\substack{e\\o}mn} &= i^{n}\, b_{\substack{e\\o}mn} \,. \end{aligned}\right\} \tag{1.10}$$

Alternatively, if some type of polynomial representation or Taylor series was derived for $\underline{E}_o(\theta, \phi)$ by curve fitting to the set of measured values $\underline{E}_o(\theta_n, \phi_n)$ n=1, 2 ... N, then the partial derivatives of $\underline{\hat{\theta}} \cdot \underline{E}_o$ and $\underline{\hat{\phi}} \cdot \underline{E}_o$ with respect to θ and ϕ could be calculated. To indicate how this could be used, take the near field point to be on the z axis. Expression (1.2) becomes

$$\underline{E}(0, 0, R) = \frac{ik}{2\pi} \int_0^{2\pi} \int_0^{\frac{\pi}{2} - i\infty} e^{ikR\cos\alpha}\, \underline{E}_o(\alpha, \beta) \sin\alpha \, d\alpha \, d\beta \tag{1.11}$$

after bending the contour from $\theta_o - i\infty$ to $\frac{\pi}{2} - i\infty$. Changing the variable of integration α to t by the substitution

$$\cos\alpha = 1 + it$$

the above expression becomes

$$\underline{E}(0, 0, R) = \frac{k}{2\pi} e^{ikR} \int_0^{2\pi} \int_0^{\infty} e^{-kRt}\, \underline{E}_o \, dt \, d\beta \tag{1.12}$$

which reduces to the form,

$$\frac{e^{ikR}}{R}\left\{\left[\sum_{n=0}^{N}\int_0^{2\pi}\frac{1}{(kR)^n}\left(\frac{\partial^n E_o}{\partial t^n}\right)_{\alpha=0} d\beta + \frac{1}{(kR)^{N+1}}\int_0^{\infty}\int_0^{2\pi} e^{-kRt}\,\frac{\partial^{n+1} E_o}{\partial t^{n+1}}\,dt d\beta\right]\right\} \tag{1.13}$$

upon integrating by parts. This has the form

$$\frac{e^{ikR}}{R}\sum_{n=0} R^{-n}\,\underline{E}_n(\theta,\phi)$$

which is equivalent to Wilcox's Expansion. This shows that the plane wave expansion is equivalent to Wilcox's expansion. When the number of data points N is large, and sufficiently dense, then numerical integration of quantity

$$\underline{\tilde{E}}(\underline{x}) = \frac{ik}{2\pi}\int_0^{\theta_o}\int_0^{2\pi} e^{i\underline{k}\cdot\underline{x}}\,\underline{E}_o(\alpha,\beta)\sin\alpha\, d\alpha\, d\beta \tag{1.14}$$

can be performed. This technique would be best employed in the high frequency region in which case $\underline{E}_o(\alpha,\beta)$ would vary rapidly in the solid angle. In using this technique there would be two sources of errors; (1) the neglect of the remaining integral taken over the range of α from θ_o to $\frac{\pi}{2} - i\infty$ which is required in the exact results, and (2) the interpolation and approximation to $\underline{E}_o(\alpha,\beta)$ in the cone $0 \leqslant \alpha \leqslant \theta_o$. The latter source of error depends upon the density of the number of measurements in the cone $0 \leqslant \theta \leqslant \theta_o$, and can be minimized by increasing the number of data points. The former source of error is more fundamental and will be examined in some detail here.

The example of high-frequency scattering by a sphere will be treated first, where the dominant contribution in a cone $0 \leqslant \theta \leqslant \theta_o$ in the back-scattered region, is the geometric optics contribution.

The incident field is taken in the form

$$\underline{E}^i = \hat{\underline{x}}\, e^{-ikz} \quad .$$

The geometric optics field in the far zone may be written

$$\underline{E}_o^s(\alpha,\beta) = -\frac{a}{2}\,\hat{\underline{e}}(\alpha,\beta)\, e^{-2ika\cos(\alpha/2)} \tag{1.15}$$

where $\hat{\underline{e}}$ is the unit vector

$$\hat{\underline{e}}(\alpha,\beta) = \hat{\underline{x}}(\cos\alpha\cos^2\beta + \sin^2\beta) - \hat{\underline{y}}\,(1-\cos\alpha)\sin\beta\cos\beta - \hat{\underline{z}}\sin\alpha\cos\beta\,. \tag{1.16}$$

It will be assumed that the far field is measured at a sufficiently dense number of points in the cone $0 \leqslant \theta \leqslant \theta_o$, such that a reasonable approximation to)1.15) can be obtained. Employing relation (1.14) the near field is given approximately by

$$\tilde{\underline{E}}^s(\underline{x}) = -\frac{ika}{4\pi}\int_0^{2\pi}\int_0^{\theta_o} e^{ikf(\alpha,\beta)}\,\hat{\underline{e}}(\alpha,\beta)\sin\alpha\, d\alpha\, d\beta \quad , \tag{1.17}$$

where

$$f(\alpha,\beta) = r\left[\sin\theta\sin\alpha\cos(\phi-\beta) + \cos\theta\cos\alpha\right] - 2a\cos\frac{\alpha}{2} \quad .$$

As $k \rightarrow \infty$, the dominant contribution to the integral arises from the vicinity of the stationary phase point ($\beta = \phi$, $\alpha = \alpha_o$) where α_o satisfies the equation

$$r\sin(\alpha_o - \theta) = a\sin\left(\frac{\alpha_o}{2}\right)$$

provided that $0 \leqslant \alpha_o \leqslant \theta_o$. By means of first order stationary phase evaluation we obtain immediately

Expression (1.21) thus can be reduced to the following form

$$\underline{E}^{s}(r,\theta,\phi) \sim -\frac{ka}{2}\sqrt{\frac{\sin\theta_o}{2\pi kr\sin\theta}}\; e^{ikg(\theta_o)} \left\{ e^{-i\frac{\pi}{4}}\,\underline{B}(\theta_o) - e^{i\frac{\pi}{4}}\,\underline{B}(-\theta_o) \right\} \tag{1.23}$$

where

$$\underline{B}(\theta_o) = \frac{e^{i\rho(\theta_o)}\left[\underline{A}_2(\theta_o) + \underline{A}_1(\theta_o) + \underline{A}_o(\theta_o)\right]}{kr\sin(\theta-\theta_o) + ka\sin\left(\frac{\theta_o}{2}\right)} \quad . \tag{1.24}$$

On the surface of the sphere $r=a$, outside the cones $0 \leqslant \theta \leqslant \frac{\theta_o}{2}$ and $\pi - \frac{\theta_o}{2} \leqslant \theta \leqslant \pi$, the dominant behavior of the amplitude is given by the factors

$$\frac{1}{\sqrt{ka}}\left[\sin(\theta-\theta_o) + \sin\left(\frac{\theta_o}{2}\right)\right]^{-1} ,$$

and

$$\frac{1}{\sqrt{ka}}\left[\sin(\theta+\theta_o \quad - \sin\left(\frac{\theta_o}{2}\right)\right]^{-1} .$$

It is seen that on this portion of the sphere expression (1.23) does not agree with (1.18) the amplitude of which is unity. In addition in the shadow region of the sphere in the cone $\pi - \theta_o/2 \leqslant \theta \leqslant \pi$, where the fields should vanish in the high frequency region, representation (1.23) cannot be used as there is a stationary phase point, in which case the asymptotic evaluation of (1.17) will give fields with amplitude the order of unity.

Summing up, it is seen that the approximate representation (1.3) will give the correct fields in the high frequency case, for a portion of the illuminated face of the scatterer.

It is of interest to extend the analysis to scattering surfaces with edges. In this case the scatterer that will be taken will be the flat plate. For convenience the geometric optics result for the scattered field will be employed. For a plane wave of unit amplitude incident normally to a square flat plate, of length and width a, and situated on the z=0 plane of a Cartesian coordinate system, the physical optics scattered field is given by

$$\underline{E}_o(\alpha,\beta) = \frac{-i2}{\pi k u v} \sin\left(\frac{ka}{2} u\right) \sin\left(\frac{ka}{2} v\right) \left[\hat{\underline{\alpha}} \cos\alpha \cos\beta - \hat{\underline{\beta}} \sin\beta\right] \tag{1.25}$$

where $u = \sin\alpha \cos\beta$

$v = \sin\alpha \sin\beta$.

Expression (1.14) will be used for the near field calculation, with measurements of the scattered field again confined to the cone $0 \leqslant \theta \leqslant \theta_o$. The near field will be calculated on the x-axis in which case

$$\underline{E}(x,0,0) = \frac{1}{\pi 2} \int_0^{\theta_o} \int_0^{2\pi} \frac{e^{ik_x x}}{uv} \sin\left(\frac{kau}{2}\right) \sin\left(\frac{kav}{2}\right) \left[\hat{\underline{\alpha}} \cos\alpha \cos\beta - \hat{\beta} \sin\beta\right] \sin\alpha \, d\alpha \, d\beta$$

which reduces to

$$\underline{E} = \frac{1}{\pi 2} \hat{\underline{x}} \iint_A e^{ikxu} \sin\left(\frac{ka}{2} u\right) \sin\left(\frac{ka}{a} v\right) \frac{(1-u^2)\,du\,dv}{uv\sqrt{1-u^2-v^2}} \quad . \tag{1.27}$$

The integration in the u-v plane is over the area of the circle centered at the origin with radius $\sin\theta_o$. The above integral can be expressed in the form

$$\underline{E} = \hat{\underline{x}} \frac{1}{2\pi^2} \int_{-\sin\theta_o}^{\sin\theta_o} \left\{ \sin\left[u(kx + \frac{ka}{2})\right] + \sin\left[u(-kx + \frac{ka}{2})\right]\right\} \frac{g(u)}{u} \, du \qquad (1.28)$$

where

$$g(u) = (1-u^2) \int_{-v_o}^{v_o} \frac{\sin(\frac{ka}{2} v)}{v\sqrt{1-u^2-v^2}} \, dv$$

with $\quad v_o = \sqrt{\sin\theta_o^2 - u^2}$.

When $(kav_o)/2 >> 1$, the asymptotic behavior of $g(u)$ for large ka is given by

$$g(u) \sim (1-u^2) \left[\frac{\pi}{\sqrt{1-u^2}} - \frac{4\cos(\frac{ka}{2} v_o)}{ka \cos\theta_o v_o} \right] .$$

Since the following integral has the asymptotic behavior for $|\beta| \longrightarrow \infty$

$$\frac{1}{\pi} \int_{-\sin\theta_o}^{\sin\theta_o} \frac{\sin u\beta}{u} g(u) \, du \sim g(o) \operatorname{sgn}\beta ,$$

it follows that for $\quad |x| < \frac{ka}{2}$ (i.e. a point on the plate)

$$\underline{E} \sim \hat{\underline{x}} \left[1 - \frac{4\cos(\frac{ka}{2}\sin\theta_o)}{\pi \, ka \sin\theta_o \cos\theta_o} \right] ,$$

otherwise $\underline{E}$ will vanish for a point outside the plate.

Provided that

$$ka \sin\theta_o >> 1$$

which implies that the half-angle of the cone of observation must not be too small, the approximation expression for the near field given by Eq. (1.14) will give the correct expression for the scattered field points on the plate which are at least a wavelength away from the edges (indicate by the condition that $|\beta| = \left| x \pm \frac{ka}{2} \right| >> 1$).

From the above analysis, it is seen that the approximate expression (1.14) will give good results in the high frequency case, provided that the given data is sufficiently dense over the measurement cone $(0 \leqslant \theta \leqslant \theta_o)$. These remarks do not include, at present, surfaces which contain cavities or protuberances. A study will have to be made of these cases. From a numerical standpoint, expression (1.14) could be difficult to use in the high frequency case. Great care has to be taken in employing expression (1.14) because the integrand contains a rapidly varying exponential, which could lead to serious errors in the employment of any straightforward numerical procedure.

II

FORMAL PROCEDURE FOR DETERMINING POINTS ON SURFACE

Points which may lie on the portion of a perfectly conducting body can be determined from a knowledge of the field incident upon the body and the far field scattered by the body. Specifically, if a transmitter and associated receivers are located such that they subtend a solid angle Ω with respect to the scattering body then the points which can be determined will be included in the solid angle Ω. Since the criteria used for determining points on the surface is only a necessary condition, it is possible that the criteria may be satisfied by more than one point along a particular ray. However, the location of a point which lies on the surface is, of course, independent of frequency and thus the ambiguity can be removed by multifrequency measurements. Points which may determine other portions of the surface can be determined by additional transmitter siets and their associated receivers. Thus the data for the inverse scattering problem may consist of several multifrequency transmitter sites and their associated receivers. The form of the input data, the necessary coordinate rotation and the formal procedure for determining points which may lie on the surface are discussed in the paragraphs that follow.

2.1 Input data for the Inverse Scattering Problem.

The input data for the data for the Inverse Scattering problem must be given in terms of a coordinate system whose origin is in the interior of the scattering body as shown in Fig. 2-1. The orientation of the axes is arbitrary.

The following data is necessary at each frequency and for each transmitter:

1. There are m transmitters located at the points $T_i(R_{ti}, \theta_{ti}, \phi_{ti})$ for $i=1,2---M$

   where R_{ti} is the distance in meters from the origin in the transmitter location.

   θ_{ti} and ϕ_{ti} are the angular displacements measured in degrees as defined by Fig. 2-1.

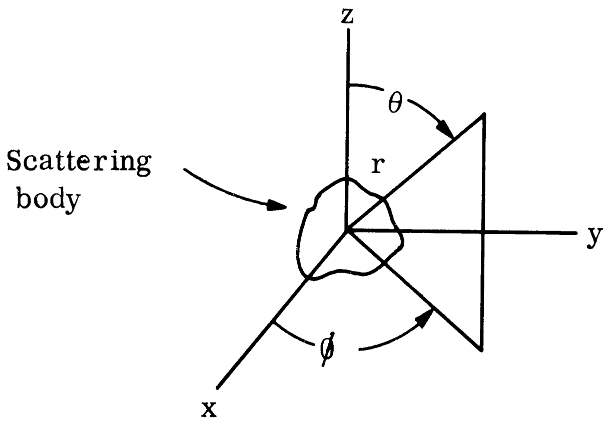

FIG. 2-1

2. The field at the origin due to each transmitter is $\underline{E}_{ti} = E_{ti} e^{i\alpha_{ti}} \hat{e}_i$

where

E_{ti} is the amplitude of the transmitted field at the origin measured in volts per meter.

α_{ti} is the phase of the transmitted field measured in radians at the origin of the coordinate system.

$\hat{e}_i = e_{i1} \hat{\theta}_{ti} + e_{i2} \hat{\phi}_{ti}$ is a unit vector specifying the polarization of the transmitted signal.

3. Associated with the i^{th} transmitter there are N receivers located at the points $P_{ij}(R_{ij}, \theta_{ij}, \phi_{ij})$

where

R_{ij} is the distance from the origin to the j^{th} receiver in meters.

θ_{ij} and ϕ_{ij} are the angular displacements of the j^{th} receiver location in degrees.

4. The scattered field at j^{th} receiver due to the i^{th} transmitter is

$$\underline{E}_{ij}^{s} = E_{\theta ij} e^{i\alpha_{\theta ij}} \hat{\theta}_{ij} + E_{\phi ij} e^{i\alpha_{\phi ij}} \hat{\phi}_{ij}$$

where

$E_{\theta ij}$ and $E_{\phi ij}$ are the amplitude, in volts per meter, of the θ and ϕ components of the scattered field respectively.

$\alpha_{\theta ij}$ and $\alpha_{\phi ij}$ are the phase, in radians, of the scattered field respectively.

2.2 Coordinate Transformation for Inverse Scattering.

2.2.1 Coordination Rotation.

The data for the inverse scattering problem will be given as specified in Section 2.1. In order that the data may be used in the inverse scattering problem a rotation of the coordinate system is necessary for each transmitter location such that in the new coordinates, denoted by primes, the transmitter lies on the positive z' axis and the positive x' axis is parallel to and in the direction of the polarization vector.

For the i^{th} transmitter the position vector to the transmitter and the polarization vector are

$$\underline{T}_i = T_i \hat{r}_{ti}$$

$$\hat{e}_i = e_{i1} \hat{\theta}_{ti} + e_{i2} \hat{\phi}_{ti} \tag{2.1}$$

where the subscript ti denotes the i^{th} transmitter. For convenience let

$$e_{i1} = \cos \gamma_i$$

$$e_{i2} = \sin \gamma_i \tag{2.2}$$

Using (2.1) and (2.2) the unit vectors $\hat{x}_i'$, $\hat{y}_i'$ and $\hat{z}_i'$ are

$$\hat{x}_i' = \cos\gamma_i\,\hat{\theta}_{ti} + \sin\gamma_i\,\hat{\phi}_{ti}$$
$$\hat{y}_i' = \hat{z}_i' \times \hat{x}_i' = -\sin\gamma_i\,\hat{\theta}_{ti} + \cos\gamma_i\,\hat{\phi}_{ti} \qquad (2.3)$$
$$\hat{z}_i' = \hat{r}_{ti}$$

In any coordinate system the unit vector $\hat{r}$, $\hat{\theta}$, $\hat{\phi}$ at a point are related to the unit vector $\hat{x}$, $\hat{y}$, $\hat{z}$ by

$$\hat{x} = \sin\theta\cos\phi\,\hat{r} + \cos\theta\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi}$$
$$\hat{y} = \sin\theta\sin\phi\,\hat{r} + \cos\theta\sin\phi\,\hat{\theta} + \cos\phi\,\hat{\phi} \qquad (2.4)$$
$$\hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$$

and

$$\hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$$
$$\hat{\theta} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z} \qquad (2.5)$$
$$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}\,.$$

The unit vector to the j^{th} receiver is from (2.5)

$$\hat{r}_{ij} = \sin\theta_{ij}\cos\phi_{ij}\,\hat{x}_i + \sin\theta_{ij}\sin\phi_{ij}\,\hat{y}_i + \cos\theta_{ij}\,\hat{z}_i$$
$$= \sin\theta_{ij}'\cos\phi_{ij}'\,\hat{x}_i' + \sin\theta_{ij}'\sin\phi_{ij}'\,\hat{y}_i' + \cos\theta_{ij}'\,\hat{z}_i' \qquad (2.6)$$

From Eqs. (2.3), (2.4) and (2.5) the following relations between the unit vectors can be obtained

$$\hat{x}_i \cdot \hat{x}_i' = \cos\gamma_i\cos\theta_{ti}\cos\phi_{ti} - \sin\gamma_i\sin\phi_{ti}$$
$$\hat{y}_i \cdot \hat{x}_i' = \cos\gamma_i\cos\theta_{ti}\sin\phi_{ti} + \sin\gamma_i\cos\phi_{ti} \qquad (2.7)$$
$$\hat{z}_i \cdot \hat{x}_i' = -\cos\gamma_i\sin\theta_{ti}$$

$$\begin{aligned}
\hat{x}_i \cdot \hat{y}_i' &= -\sin\gamma_i \cos\theta_{ti} \cos\phi_{ti} - \cos\gamma_i \sin\phi_{ti} \\
\hat{y}_i \cdot \hat{y}_i' &= -\sin\gamma_i \cos\theta_{ti} \sin\phi_{ti} + \cos\gamma_i \cos\phi_{ti} \\
\hat{z}_i \cdot \hat{y}_i' &= \sin\gamma_i \sin\theta_{ti}
\end{aligned} \tag{2.8}$$

$$\begin{aligned}
\hat{x}_i \cdot \hat{z}_i' &= \sin\theta_{ti} \cos\phi_{ti} \\
\hat{y}_i \cdot \hat{z}_i' &= \sin\theta_{ti} \sin\phi_{ti} \\
\hat{z}_i \cdot \hat{z}_i' &= \cos\theta_{ti} \quad .
\end{aligned} \tag{2.9}$$

From Eq. (2.6) the angle θ'_{ij} can be expressed in terms θ_{ij} and ϕ_{ij} as

$$\hat{z}_i' \cdot \hat{r}_{ij} = \cos\theta'_{ij} = \sin\theta_{ij} \cos\phi_{ij}\, \hat{z}_i' \cdot \hat{x}_i + \sin\theta_{ij} \sin\phi_{ij}\, \hat{z}_i' \cdot \hat{y}_i + \cos\theta_{ij}\, \hat{z}_i' \cdot \hat{z}_i$$

$$0 \leqslant \theta'_{ij} < \pi \, . \tag{2.10}$$

Using the result in Eqs. (2.7), (2.8) and (2.9) in (2.10) we obtain

$$\cos\theta'_{ij} = \sin\theta_{ij} \sin\theta_{ti} \cos(\phi_{ij} - \phi_{ti}) + \cos\theta_{ij} \cos\theta_{ti} \quad . \tag{2.11}$$

The angle ϕ'_{ij} can be obtained from

$$\begin{aligned}
\hat{x}' \cdot \hat{r}_i = \sin\theta'_{ij} \cos\phi'_{ij} &= \sin\theta_{ij} \cos\phi_{ij}\, \hat{x}' \cdot \hat{x} + \\
&+ \sin\theta_{ij} \sin\phi_{ij}\, \hat{x}' \cdot \hat{y} + \cos\theta_i\, \hat{x}' \cdot \hat{z}
\end{aligned} \tag{2.12}$$

$$\hat{y}' \cdot \hat{r}_i = \sin\theta'_{ij} \sin\phi'_{ij} = \sin\theta_{ij} \cos\phi_{ij}\, \hat{y}' \cdot \hat{x} +$$

$$+ \sin\theta_{ij} \cos\phi_{ij}\, \hat{y}' \cdot \hat{y} + \cos\theta_{ij}\, \hat{y}' \cdot \hat{z} \ . \tag{2.13}$$

Using the result of (2.7), (2.8) and (2.9) in (2.12) and (2.13) we obtain

$$\begin{aligned} \sin\theta'_{ij} \cos\phi'_{ij} = {} & \sin\theta_{ij} \cos\gamma_i \cos\theta_{ti} \cos(\phi_{ij} - \phi_{ti}) \\ & + \sin\theta_{ij} \sin\gamma_i \sin(\phi_{ij} - \phi_{ij}) \\ & - \cos\theta_{ij} \cos\gamma_i \sin\theta_{ti} \end{aligned} \tag{2.14}$$

and

$$\begin{aligned} \sin\theta'_{ij} \sin\phi'_{ij} = {} & -\sin\theta_{ij} \sin\gamma_i \cos\theta_{ti} \cos(\phi_{ij} - \phi_{ti}) \\ & + \sin\theta_{ij} \cos\gamma_i \sin(\phi_{ij} - \phi_{ti}) \\ & + \cos\theta_{ij} \sin\gamma_i \sin\theta_{ti} \ . \end{aligned} \tag{2.15}$$

Using the value of θ'_{ij} determined by Eq. (2.11) Eqs. (2.14) and (2.15) determine ϕ'_{ij} in the range $0 \leqslant \phi'_{ij} < 2\pi$.

In a similar manner the inverse transformation can be obtained as

$$\cos\theta_{ij} = -\sin\theta'_{ij} \sin\theta_{ti} \cos(\phi'_{ij} + \gamma_i) + \cos\theta'_{ij} \cos\theta_{ti} \quad 0 \leqslant \theta_{ij} < \pi \tag{2.16}$$

and

$$\sin\theta_{ij}\cos\phi_{ij} = \sin\theta'_{ij}\cos\theta_{ti}\cos\phi_{ti}\cos(\phi'_{ij}+\phi_{ti})$$
$$-\sin\theta'_{ij}\sin\phi_{ti}\sin(\gamma_i+\phi'_{ij})$$
$$+\cos\theta'_{ij}\sin\theta_{ti}\cos\phi_{ti} \tag{2.17}$$

$$\sin\theta_{ij}\sin\phi_{ij} = \sin\theta'_{ij}\cos\theta_{ti}\sin\phi_{ti}\cos(\phi'_{ij}+\gamma_i)$$
$$+\sin\theta'_{ij}\cos\phi_{ti}\sin(\gamma_i+\phi'_{ij})$$
$$+\cos\theta'_{ij}\sin\theta_{ti}\sin\phi_{ti}\,. \tag{2.18}$$

Equations (2.17) and (2.18) uniquely determine ϕ_{ij} in the range $0 \leqslant \phi_{ij} < 2\pi$.

The components of the scattered field at the j^{th} receiver due to the i^{th} transmitter in the directions of the unit vectors $\hat{\theta}'_{ij}$ and $\hat{\phi}'_{ij}$ must be determined. It is convenient to remove a factor $(e^{ikR_{ij}})/R_{ij}$ before developing these expressions. The expression for the scattered field at the j^{th} receiver due to the i^{th} transmitter at a particular frequency is

$$\underline{E}^s_{ij} = E_{\theta_{ij}}\, e^{i\alpha_{\theta_{ij}}}\,\hat{\theta}_{ij} + E_{\phi_{ij}}\, e^{i\alpha_{\phi_{ij}}}\,\hat{\phi}_{ij}\,. \tag{2.19}$$

The scattered field can be written as

$$\underline{E}^s_{ij} = \frac{e^{ikR_{ij}}}{kR_{ij}} \underline{\mathcal{E}}_{ij} = \frac{e^{ikR_{ij}}}{kR_{ij}} \left[\mathcal{E}_{\theta_{ij}} \hat{\theta}_{ij} + \mathcal{E}_{\phi_{ij}} \hat{\phi}_{ij} \right] . \tag{2.20}$$

The quantities $\mathcal{E}_{\theta_{ij}}$ and $\mathcal{E}_{\phi_{ij}}$ can be determined from (2.19) and (2.20) and are

$$\mathcal{E}_{\theta_{ij}} = kR_{ij} E_{\theta_{ij}} e^{i(\alpha_{\phi_{ij}} - kR_{ij})} \tag{2.21}$$

$$\mathcal{E}_{\phi_{ij}} = kR_{ij} E_{\phi_{ij}} e^{i(\alpha_{\phi_{ij}} - kR_{ij})} . \tag{2.22}$$

The desired components of the scattered field are

$$\mathcal{E}_{\theta'_{ij}} = \hat{\theta}'_{ij} \cdot \underline{\mathcal{E}}_{ij} = \mathcal{E}_{\theta_{ij}} \hat{\theta}'_{ij} \cdot \hat{\theta}_{ij} - \mathcal{E}_{\phi_{ij}} \hat{\theta}'_{ij} \cdot \hat{\phi}_{ij} \tag{2.23}$$

and

$$\mathcal{E}_{\phi'_{ij}} = \hat{\phi}'_{ij} \cdot \underline{\mathcal{E}}_{ij} = \mathcal{E}_{\theta_{ij}} \hat{\phi}'_{ij} \cdot \hat{\theta}_{ij} + \mathcal{E}_{\phi_{ij}} \hat{\phi}'_{ij} \cdot \hat{\phi}_{ij} . \tag{2.24}$$

The products of the unit vector can be obtained from Eqs. (2.3), (2.4) and (2.5). The results are

$$\mathcal{E}_{\theta'_{ij}} = \mathcal{E}_{\theta_{ij}} \left\{ \cos\theta'_{ij} \cos(\gamma_i + \phi'_{ij}) \left[\cos\theta_{ti} \cos\theta_{ij} \cos\phi_{ij} \cos(\phi_{ti} - \phi_{ij}) \right.\right.$$

$$\left. + \sin\theta_{ti} \sin\theta_{ij} \right] + \cos\theta'_{ij} \sin(\gamma_i + \phi'_{ij}) \sin(\phi_{ij} - \phi_{ti}) \cos\theta_{ij}$$

$$\left. - \sin\theta'_{ij} \left[\sin\theta_{ti} \cos\theta_{ij} \cos(\phi_{ti} - \phi_{ij}) - \cos\theta_{ti} \sin\theta_{ij} \right] \right\}$$

$$+ \mathcal{E}_{\phi_{ij}} \left\{ \cos\theta'_{ij} \left[\cos(\gamma_i + \phi'_{ij}) \cos\theta_{ti} \sin(\phi_{ti} - \phi_{ij}) \right.\right.$$

$$\left.\left. + \sin(\gamma_i + \phi'_{ij}) \cos(\phi_{ti} - \phi_{ij}) \right] - \sin\theta'_{ij} \sin\theta_{ti} \sin(\phi_{ti} - \phi_{ij}) \right\} \tag{2.25}$$

$$\mathcal{E}_{\phi'_{ij}} = \mathcal{E}_{\theta_{ij}} \left\{ -\sin(\phi'_{ij} + \gamma_i) \left[\cos\theta_{ti} \cos\theta_{ij} \cos(\phi_{ti} - \phi_{ij}) \right.\right.$$

$$\left.\left. + \sin\theta_{ti} \sin\theta_{ij} \right] + \cos(\gamma_i + \phi'_{ij}) \cos\theta_{ij} \sin(\phi_{ij} - \phi_{ti}) \right\}$$

$$+ \mathcal{E}_{\phi_{ij}} \left\{ -\sin(\phi'_{ij} + \gamma_i) \cos\theta_{ti} \sin(\phi_{ti} - \phi_{ij}) \right.$$

$$\left. + \cos(\gamma_i + \phi'_{ij}) \cos(\phi_{ti} - \phi_{ij}) \right\} \quad . \tag{2.26}$$

2.3 Points on the Surface.

The scattered field due to the i^{th} transmitter along a ray from the origin to the j^{th} receiver at a particular frequency can be written as

$$\underline{E}^s = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \Big[(-i)^{n+1} a_{emn} \underline{m}_{emn} + (-i)^{n+1} a_{omn} \underline{m}_{omn}$$

$$+ (-i)^n b_{emn} \underline{n}_{emn} + (-i)^n b_{omn} \underline{n}_{omn} \Big] \tag{2.27}$$

where

$$\underline{m}_{\substack{e\\o}mn} = \mp \frac{m}{\sin\theta'_{ij}} h_n^{(1)}(kr'_{ij}) P_n^m(\cos\theta'_{ij}) {}^{\sin}_{\cos} m\phi'_{ij} \hat{\theta}'_{ij}$$

$$- h_n^{(1)}(kr'_{ij}) \frac{\partial}{\partial\theta'_{ij}} P_n^m(\cos\theta'_{ij}) {}^{\cos}_{\sin} m\phi'_{ij} \hat{\phi}'_{ij} \tag{2.28}$$

and

$$\underline{n}_{\substack{e\\o}mn} = \frac{n(n+1)}{kr'_{ij}} h_n^{(1)}(kr'_{ij}) P_n^m(\cos\theta'_{ij}) {}^{\cos}_{\sin} m\phi'_{ij} \hat{r}'_{ij}$$

$$+ \frac{1}{kr'_{ij}} \frac{\partial}{\partial r'_{ij}} \left[r'_{ij} h_n^{(1)}(kr'_{ij}) \right] \frac{\partial}{\partial\theta'_{ij}} P_n^m(\cos\theta'_{ij}) \hat{\theta}'_{ij}$$

$$\pm \frac{m}{kr'_{ij} \sin\theta'_{ij}} \frac{\partial}{\partial r'_{ij}} \left[r'_{ij} h_n^{(1)}(kr'_{ij}) \right] P_n^m(\cos\theta'_{ij}) \hat{\phi}'_{ij} \quad . \tag{2.29}$$

The coefficients a_{emn}, a_{omn}, b_{emn} and b_{omn} are unknown and to be determined.

Assuming the receiver is in the far field of the scattering body and introducing the following notation

$$S_{\substack{e\\o}mn} = \mp \frac{m}{\sin\theta'_{ij}} P_n^m(\cos\theta'_{ij}) \begin{matrix}\sin\\ \cos\end{matrix}\, m\,\phi'_{ij} \tag{2.30}$$

and

$$T_{\substack{e\\o}mn} = \frac{\partial}{\partial\theta'_{ij}} P_n^m(\cos\theta'_{ij}) \begin{matrix}\cos\\ \sin\end{matrix}\, m\,\phi'_{ij} \tag{2.31}$$

the expression for the scattered field can be written approximately as

$$\underline{E}^s = \frac{e^{ikr'_{ij}}}{kr'_{ij}} \sum_{n=1}^{\infty} \sum_{m=0}^{n} \Big\{ \Big[a_{emn} S_{emn} + a_{omn} S_{omn} + b_{emn} T_{emn} + b_{omn} T_{omn} \Big] \hat{\theta}'_{ij} + \Big[-a_{emn} T_{emn} - a_{omn} T_{emn} + b_{emn} S_{emn} + b_{omn} S_{omn} \Big] \hat{\phi}'_{ij} \Big\} \tag{2.32}$$

From (2.25), (2.26) and (2.32) we obtain

$$\mathcal{E}_{\theta'_{ij}} = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(a_{emn} S_{emn} + a_{omn} S_{omn} + b_{emn} T_{emn} + b_{omn} S_{omn} \right) \tag{2.33}$$

$$\mathcal{E}_{\phi'_{ij}} = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(-a_{emn} T_{emn} - a_{omn} T_{emn} + b_{emn} S_{emn} + b_{omn} S_{omn} \right) . \tag{2.34}$$

It is convenient to write Eqs. (2.33) and (2.34) as a single sum as follows:

$$\mathcal{E}_{\theta'_{ij}} = \sum_{\nu=1}^{\infty} (a_\nu S_\nu + b_\nu T_\nu) \tag{2.35}$$

$$\mathcal{E}_{\phi'_{ij}} = \sum_{\nu=1}^{\infty} (-a_\nu T_\nu + b_\nu S_\nu) \tag{2.36}$$

where

$a_1 = a_{e10}$

$a_2 = a_{o10}$

$a_3 = a_{e11}$ etc.

In general, the subscripts e or o, and the values of m and n can be determined for a given ν as follows:

1. n is determined by $(n-1)(n+2) \leqslant \nu \leqslant n(n+3)$
2. If $\nu-(n-1)(n+2)$ is even use o, odd use e
3. m is given by the integer or next highest integer to $\frac{\nu-(n-1)(n+2)}{2} -1$.

Assuming that the series in (2.35) and (2.36) can be approximated by a finite number of terms a system of 2N linear equations (where N is the number of receivers) can be formed to determine the a_ν and b_ν. There will be N equations of the form

$$\mathcal{E}_{\theta'_{ij}} = \sum_{\nu=1}^{N} (a_\nu S_\nu + b_\nu T_\nu) , \tag{2.37}$$

one for each value of the index j.

Similarly, there will be N equations of the form

$$\mathcal{E}_{\phi'_{ij}} = \sum_{\nu=1}^{N} \left(-a_\nu T_\nu + b_\nu S_\nu \right) . \tag{2.38}$$

The system of linear Eqs. (2.37) and (2.38) can be solved by standard methods and results used in Eq. (2.27). The scattered field is approximately

$$\underline{E}^s = \sum_{n=1}^{N(\nu)} \sum_{m=0}^{n} \Big[(-i)^{n+1} a_{emn} \underline{m}_{emn} + (-i)^{n+1} a_{omn} \underline{m}_{omn} + (-i)^n b_{emn} \underline{n}_{emn} + b_{omn} \underline{n}_{omn} \Big] . \tag{2.39}$$

From Section 2.1 the incident field along a ray to the jth receiver is

$$\underline{E}_{ti} = E_{ti}\, e^{i\alpha_{ti}}\, e^{-ikr'_{ij}} \left(\sin\theta'_{ij} \cos\phi'_{ij}\, r'_{ij} + \cos\theta'_{ij} \cos\phi'_{ij}\, \hat{\theta}'_{ij} - \sin\phi'_{ij}\, \hat{\phi}'_{ij} \right) . \tag{2.40}$$

From Eqs. (2.39) and (2.40) the total field is

$$\underline{E}_T = \underline{E}_{ti} + \underline{E}^s \quad . \tag{2.41}$$

The criteria for determining points which may lie on the surface of the scattering body is

$$\underline{E}_T \times \underline{E}_T^* = 0 . \tag{2.42}$$

In an actual calculation Eq. (2.42) may not be satisfied exactly and practically it may be more useful to look for minimum values of the function

$$\left| \underline{E}_T \times \underline{E}_T^* \right| .$$

III

PRESENT AND FUTURE PLANS

At present, analysis is being carried out to investigate the amount of data that is necessary in order to determine the shape and size of a dielectric body. A review of the techniques developed by Gelfand and Levitan and the technique developed by Kay for scalar scattering has been carried out. This review will be presented in detail in the next quarterly. The possibility of generalizing the results to the three dimensional vector case will be investigated.

Practical techniques for determining the surface of a perfectly conducting body are being carried out on a specific example. Theoretical condition $\underline{E} \times \underline{E}^{*} = 0$, based upon knowledge of the total field $\underline{E}$ in the vicinity of the scattering surface, was investigated numerically, and the results indicate that this condition may be very difficult to employ in any numerical scheme where only approximate values of the total field are known. To improve its use, a proper normalization factor, yet to be found, may have to be employed. However, alternative conditions can be used, such as in the high frequency case, where the approximate condition $|\underline{H}^{s}|^{2} = |\underline{H}^{i}|^{2}$ may be employed to find the illuminated portion of the body. This later condition appears to be much more practical in any numerical approach.

Further asymptotic analysis will be carried out with the approximate near field expression Eq. (1.14) to investigate what portions, if any, of concave surfaces can be determined when knowledge of the far field is confined to a cone $0 \leqslant \theta \leqslant \theta_{o}$.

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13. ABSTRACT

The problem in question consists of determining means of solving the inverse scattering problem where the transmitted field is given and the received fields are measured, and this data is used to discover the nature of the target.

The problem of what information can be determined about the body if the scattering matrix (phase and amplitude) is known only over an angular sector and measured in the far field, is studied further. Asymptotic analysis is used to show that in the high frequency case, portions of a piecewise smooth, convex surface can be found when knowledge of the bistatic scattered field is confined to a small cone.

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